Heavy Hybrids in pNRQCD

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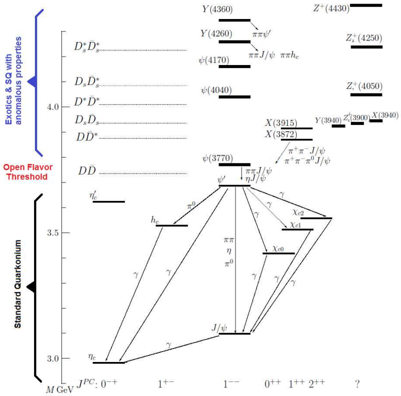
- In the last decade many new unexpected states have been found close or above threshold.



Voloshin

2008

- The states that do not fit Quarkonium potential models are called Exotics and labeled Xs, Ys and Zs.
- This states are candidates for **non traditional hadronic states**, including four constituent quark or an excited gluon constituent.
- There is an **ongoing experimental effort** to study normal and Exotic quarkonium: new states, production mechanisms, decays and transitions, precision and high statistics data.
- BaBar, Belle2, BESIII, LHCb and Panda (under construction).

Quarkonium Hybrids

What are quarkonium Hybrids?

- A quarkonium hybrid consists of Q, $\bar{Q}$ in a color octet configuration and a gluonic excitation g.

Born-Oppenheimer Hybrids:
The heavy quarks are nearly static, and the gluons adapt nearly instantaneously.

Born-Oppenheimer approximation Heavy Hybrids

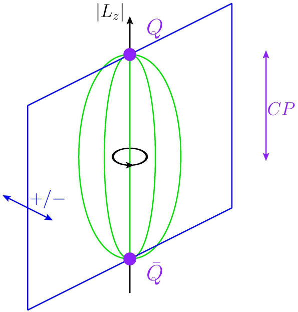
- The gluonic static energies can be defined in NRQCD and computed on the lattice or, in the short range, using pNRQCD.
- The hybrid state energy levels are obtained solving the Schrödinger equation with $H_{kin} + E_g$.
- H_{kin} acts on the gluon wave functions. Additional approximations are needed, because the gluonic wave functions are not available.
- The mixing terms have to be taken into account because the static energies are degenerate at short distances.

Pioneered by Juge, Kuti, Morningstar 1999

Symmetries of the static system

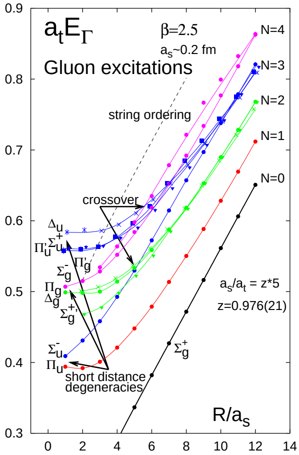
Static states classified by symmetry group $D_{\infty\,h}$
Representations labeled Λ_η^σ

- Λ rotational quantum number $|\hat{\mathbf{n}} \cdot \mathbf{K}| = 0, 1, 2 \ldots$ corresponds to $\Lambda = \Sigma, \Pi, \Delta \ldots$
- η eigenvalue of CP: $g \mathrel{\hat{=}} +1$ (gerade), $u \mathrel{\hat{=}} -1$ (ungerade)
- σ eigenvalue of reflections
- σ label only displayed on Σ states (others are degenerate)



- The static energies correspond to the irreducible representations of $D_{\infty\,h}$.
- In general it can be more than one state for each irreducible representations of $D_{\infty\,h}$, usually denoted by primes, e.g. Π_u, Π_u', Π_u''...

Lattice data on hybrid static energies



Juge, Kuti, Morningstar 2002

- Σ_g^+ is the ground state potential that generates the standard quarkonium states.
- The rest of the static energies correspond to excited gluonic states that generate hybrids.
- The two lowest hybrid static energies are Π_u and Σ_u^-, they are nearly degenerate at short distances.
- The static energies have been computed in quenched lattice QCD, the most recent data by Juge, Kuti, Morningstar, 2002 and Bali and Pineda 2003.
- Quenched and unquenched calculations for Σ_g^+ and Π_u were compared in Bali et al 2000 and good agreement was found below string breaking distance.

Radial wave functions and and Schrödinger equation

The **hybrid masses** are obtained by solving the Schrödinger equation with $H_{kin} + E_{n\Lambda_\eta^\sigma}$

- Let us define Hybrid state as:

$$|N; j, m, s, l; n, \eta; \epsilon\rangle = \sum_{\Lambda} \int dr \, |j, m, s, l; n, \Lambda, \eta; \epsilon\rangle \, \frac{\psi_N^\Lambda(r)}{r} \, .$$

- $H_{kin} = -\frac{\partial_r^2}{2\mu} + \frac{L_{Q\overline{Q}}^2}{2\mu r^2}$ acts in principle on **both** the gluonic and heavy quark parts.
- The radial derivative acting on the gluonic state can be ignored.

Angular derivative

In spherical coordinates $\mathbf{K} = K_\theta \hat{\boldsymbol{\theta}} + K_\phi \hat{\boldsymbol{\phi}} + K_n \hat{\mathbf{n}}$, and the rising and lowering operators $K_\pm = K_\theta \pm i K_\phi$.

$$\begin{aligned} \mathbf{L}_{Q\overline{Q}}^2 &= (\mathbf{L} - \mathbf{K})^2 = \mathbf{L}^2 - 2\mathbf{L}\cdot\mathbf{K} + \mathbf{K}^2 \\ &= \mathbf{L}^2 - 2K_n^2 + \mathbf{K}^2 - L_- K_+ - L_+ K_- \, , \end{aligned}$$

- Acting on our J^{PC} eigenstates the first two terms give $l(l+1)$ and $-2\Lambda^2$.
- $\mathbf{K}^2 - L_- K_+ - L_+ K_-$ is not directly determined since we do not have the gluonic wave functions.
- $L_- K_+ + L_+ K_-$ mixes different channels.

To determine the last two terms we use:

1. $L^2_{Q\overline{Q}}/2\mu r^2$ is most important in the short range.
2. The static symmetry group $D_{\infty h}$ is extended to $O(3) \times C$ in the limit $r \to 0$.

$\Rightarrow$ approximate $\mathbf{K}^2 - L_- K_+ - L_+ K_-$ for its short range behavior.

$\mathbf{K}^2$

- $\langle \mathbf{K}^2 \rangle = k(k+1)$ with k from the K^{PC} multiplet corresponding to $n\Lambda^\sigma_\eta$ in the $r \to 0$ limit.
- For the three the lowest static energies, Σ^+_g, Π_u and Σ^-_u, pNRQCD tells us these are 0^{++}, 1^{+-} and 1^{+-} respectively.

$L_- K_+ + L_+ K_-$

- If we ignore the mixing terms, for $\Lambda > 0$, there are two degenerate states with opposite parity. If we include them the degeneracy is lifted and we obtain an effect called Λ-doubling.
- The mixing involves gluonic static energies with λ and $\lambda \pm 1$.
- We will only consider mixing between static energies that are nearly degenerate.
- We have considered the mixing through **coupled Schrödinger equations**.

Gluonic static energies in pNRQCD

EFT of QCD for Quarkonium

- Quarkonium systems are non–relativistic bound states.
- **Multiscale system**: $m \gg \mathbf{p}_Q \gg E_b$, and Λ_{QCD}. m is the heavy–quark mass.
- We can exploit the **scale hierarchies** by building an **Effective Field Theory** (EFT).

pNRQCD for Hybrid static energies

- The short range $\mathbf{p}_Q \sim 1/r \gg \Lambda_{QCD}$ behavior of the static energies can be studied in weakly-coupled pNRQCD.
- In this region pNRQCD is obtained integrating out $\mathbf{p}_Q$ (perturbative) and Λ_{QCD} nonperturbative.

- The Gluelumps are the adjoint sources in the presence of a gluonic field

$$H\left(\mathbf{R},\mathbf{r},t\right)=H^{a}\left(\mathbf{R},t\right)\mathcal{O}^{a\dagger}\left(\mathbf{R},\mathbf{r},t\right)\ ,$$

Gluonic excitation operators up to dim 3

Λ_η^σ	K^{PC}	H^a
Σ_u^-	1^{+-}	$\mathbf{r}\cdot\mathbf{B}\,,\mathbf{r}\cdot(\mathbf{D}\times\mathbf{E})$
Π_u	1^{+-}	$\mathbf{r}\times\mathbf{B}\,,\mathbf{r}\times(\mathbf{D}\times\mathbf{E})$
$\Sigma_g^{+\prime}$	1^{--}	$\mathbf{r}\cdot\mathbf{E}\,,\mathbf{r}\cdot(\mathbf{D}\times\mathbf{B})$
Π_g	1^{--}	$\mathbf{r}\times\mathbf{E}\,,\mathbf{r}\times(\mathbf{D}\times\mathbf{B})$
Σ_g^-	2^{--}	$(\mathbf{r}\cdot\mathbf{D})(\mathbf{r}\cdot\mathbf{B})$
Π_g'	2^{--}	$\mathbf{r}\times((\mathbf{r}\cdot\mathbf{D})\mathbf{B}+\mathbf{D}(\mathbf{r}\cdot\mathbf{B}))$
Δ_g	2^{--}	$(\mathbf{r}\times\mathbf{D})^i(\mathbf{r}\times\mathbf{B})^j+(\mathbf{r}\times\mathbf{D})^j(\mathbf{r}\times\mathbf{B})^i$
Σ_u^+	2^{+-}	$(\mathbf{r}\cdot\mathbf{D})(\mathbf{r}\cdot\mathbf{E})$
Π_u'	2^{+-}	$\mathbf{r}\times((\mathbf{r}\cdot\mathbf{D})\mathbf{E}+\mathbf{D}(\mathbf{r}\cdot\mathbf{E}))$
Δ_u	2^{+-}	$(\mathbf{r}\times\mathbf{D})^i(\mathbf{r}\times\mathbf{E})^j+(\mathbf{r}\times\mathbf{D})^j(\mathbf{r}\times\mathbf{E})^i$

- We can see that in the short distance limit the $\Pi_u-\Sigma_u^-$, $\Pi_g-\Sigma_g^{+\prime}$, $\Delta_g-\Sigma_g^- -\Pi_g'$ and $\Delta_u-\Pi_u'-\Sigma_u^+$ multiplets must be degenerate.

Hybrid Static energies

- The hybrid static energy spectrum reads

$$E_H = 2m + V_H \,,$$

with

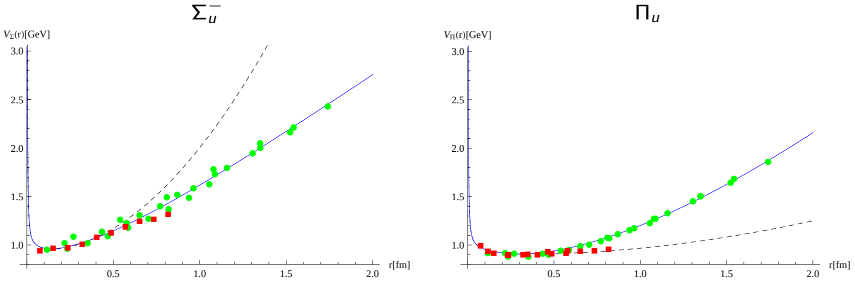
$$V_H = \lim_{T\to\infty} \frac{i}{T} \log \left\langle H^a(T/2)\mathcal{O}^a(T/2)H^b(-T/2)\mathcal{O}^b(-T/2)\right\rangle \,.$$

- Up to next–to–leading order in the **multipole expansion**.

$$V_H = V_o + \Lambda_H + b_H r^2 \,,$$

- $V_o(r)$ is the octet potential, which can be computed in perturbation theory.
  - We work in the **Renormalon Subtracted scheme** which improves the convergence of the octet potential.
- Λ_H corresponds to the **gluelump mass**.
  - It is a non-perturbative quantity, that has been determined in the lattice. $\Lambda^{RS}_{1+-} = 0.87(15)$ GeV Bali, Pineda 2004
  - It is the same for operators corresponding to different projections of the same gluonic operators.
- b_H is a non-perturbative quantity.
  - We are going to fix it through a fit to the static energies lattice data.
  - Breaks the degeneracy of the potentials.

Static Hybrid potentials



Lattice data:Bali, Pineda 2004; Juge, Kuti, Morningstar 2003, dashed line $V^{(0.5)}$, solid line $V^{(0.25)}$

$V^{(0.5)}$

Lattice data fitted for the $r = 0 - 0.5$ fm range, $b_\Sigma^{(0.5)} = 1.112\,\mathrm{GeV/fm^2}$, $b_\Pi^{(0.5)} = 0.110\,\mathrm{GeV/fm^2}$.

$$c_{\mathrm{BP}} = 0.105\,\mathrm{GeV}, \quad c_{\mathrm{KJM}} = -0.471\,\mathrm{GeV}\,,$$

$V^{(0.25)}$

- $r \leq 0.25$ fm: pNRQCD potential, $b_\Sigma^{(0.25)} = 1.246\,\mathrm{GeV/fm^2}$, $b_\Pi^{(0.25)} = 0.000\,\mathrm{GeV/fm^2}$.
- $r > 0.25$ fm: phenomenological potential, $\mathcal{V}'(r) = \frac{a_1}{r} + \sqrt{a_2 r^2 + a_3} + a_4$.

Hybrid state masses from $V^{(0.5)}$

Solving the coupled Schrödinger equations we obtain

GeV	$c\bar{c}$				$b\bar{c}$				$b\bar{b}$			
	m_H	$\langle 1/r \rangle$	E_{kin}	P_Π	m_H	$\langle 1/r \rangle$	E_{kin}	P_Π	m_H	$\langle 1/r \rangle$	E_{kin}	P_Π
H_1	4.05	0.29	0.11	0.94	7.40	0.31	0.08	0.94	10.73	0.36	0.06	0.95
H_1'	4.23	0.27	0.20	0.91	7.54	0.30	0.16	0.91	10.83	0.36	0.11	0.92
H_2	4.09	0.21	0.13	1.00	7.43	0.23	0.10	1.00	10.75	0.27	0.07	1.00
H_2'	4.30	0.19	0.24	1.00	7.60	0.21	0.19	1.00	10.87	0.25	0.13	1.00
H_3	4.69	0.37	0.42	0.00	7.92	0.42	0.34	0.00	11.09	0.50	0.23	0.00
H_4	4.17	0.19	0.17	0.97	7.49	0.25	0.14	0.97	10.79	0.29	0.09	0.98
H_5	4.20	0.17	0.18	1.00	7.51	0.19	0.15	1.00	10.80	0.22	0.10	1.00

Consistency test:

1. The potentials describe the lattice data well up to $r \lesssim 0.55 - 0.65$ fm which corresponds $\langle 1/r \rangle \gtrsim 0.36 - 0.30$ GeV.
2. The multipole expansion requires $\langle 1/r \rangle > E_{kin}$.

- Spin symmetry multiplets

H_1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

Hybrid state masses from $V^{(0.25)}$

Solving the coupled Schrödinger equations we obtain

GeV	$c\bar{c}$				$b\bar{c}$				$b\bar{b}$			
	m_H	$\langle 1/r \rangle$	E_{kin}	P_Π	m_H	$\langle 1/r \rangle$	E_{kin}	P_Π	m_H	$\langle 1/r \rangle$	E_{kin}	P_Π
H_1	4.15	0.42	0.16	0.82	7.48	0.46	0.13	0.83	10.79	0.53	0.09	0.86
H_1'	4.51	0.34	0.34	0.87	7.76	0.38	0.27	0.87	10.98	0.47	0.19	0.87
H_2	4.28	0.28	0.24	1.00	7.58	0.31	0.19	1.00	10.84	0.37	0.13	1.00
H_2'	4.67	0.25	0.42	1.00	7.89	0.28	0.34	1.00	11.06	0.34	0.23	1.00
H_3	4.59	0.32	0.32	0.00	7.85	0.37	0.27	0.00	11.06	0.46	0.19	0.00
H_4	4.37	0.28	0.27	0.83	7.65	0.31	0.22	0.84	10.90	0.37	0.15	0.87
H_5	4.48	0.23	0.33	1.00	7.73	0.25	0.27	1.00	10.95	0.30	0.18	1.00
H_6	4.57	0.22	0.37	0.85	7.82	0.25	0.30	0.87	11.01	0.30	0.20	0.89
H_7	4.67	0.19	0.43	1.00	7.89	0.22	0.35	1.00	11.05	0.26	0.24	1.00

Consistency test:

1. The multipole expansion requires $\langle 1/r \rangle > E_{kin}$.

Conclusion:

- $V^{(0.25)}$ yields more consistent results.
- As expected the Born–Oppenheimer program works better in bottomonium than charmonium

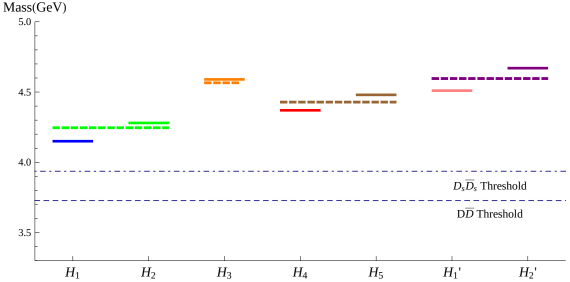
- Spin symmetry multiplets

H_1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u
H_6	$\{3^{--}, (2,3,4)^{-+}\}$	Σ_u^-, Π_u
H_7	$\{3^{++}, (2,3,4)^{+-}\}$	Π_u

Λ–doubling effect

- In Braaten *et al* 2014 a similar procedure was followed to obtain the hybrid masses.
- No Λ–doubling effect mixing terms where included, and phenomenological potentials fitting the lattice data.
- We can compare the results to estimate the size of the Λ–doubling effect.

Charmonium sector



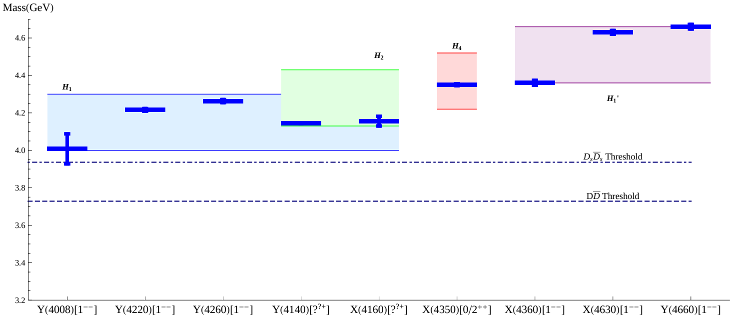
Braaten *et al* 2014 results plotted in dashed lines.

- The mixing lower the mass o f the $H_1(H_4)$ multiplet with respect to $H_2(H_4)$.

Identification with experimental states

Most of the candidates have 1^{--} or $0^{++}/2^{++}$ since the main observation channels are production by e^+e^- or $\gamma\gamma$ annihilation respectively.

- Charmonium states (Belle, CDF, BESIII, Babar):



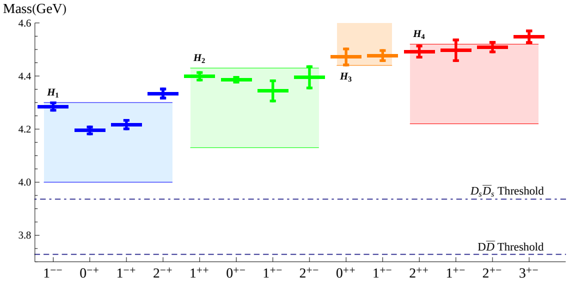
- Bottomonium states: $Y_b(10890)[1^{--}]$, $m = 10.8884 \pm 3.0$ (Belle). Possible H_1 candidate, $m_{H_1} = 10.79 \pm 0.15$.

However, except for $Y(4220)$, all other candidates observed decay modes violate Heavy Quark Spin Symmetry.

Comparison with direct lattice computations

Charmonium sector

- Calculations done by the Hadron Spectrum Collaboration using unquenched lattice QCD with a pion mass of 400 MeV. Liu *et all* 2012
- They worked in the constituent gluon picture, which consider the multiplets H_2, H_3, H_4 as part of the same multiplet.
- Their results are given with the η_c mass subtracted.



Error bands take into account the uncertainty on the gluelump mass ± 0.15 GeV

Split (GeV)	Liu	$V^{(0.25)}$
$\delta m_{H_2-H_1}$	0.10	0.13
$\delta m_{H_4-H_1}$	0.24	0.22
$\delta m_{H_4-H_2}$	0.13	0.09
$\delta m_{H_3-H_1}$	0.20	0.44
$\delta m_{H_3-H_2}$	0.09	0.31

- Our masses are $0.1 - 0.14$ GeV lower except the for the H_3 multiplet, which is the only one dominated by Σ_u^-.
- Good agreement with the mass gaps between multiplets, in particular the Λ-doubling effect ($\delta m_{H_2-H_1}$).

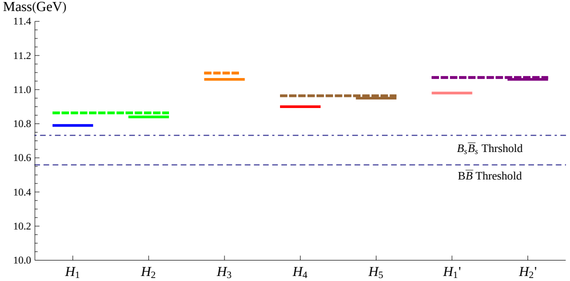
Conclusions

- We have computed the heavy hybrid masses using a QCD analog of the Born-Oppenheimer approximation including the Λ–doubling terms by using coupled Schröringer equations.
- The static energies have been obtained combining pNRQCD for short distances and lattice data for long distances.
- A large set of masses for spin symmetry multiplets for $c\bar{c}$, $b\bar{c}$ and $b\bar{b}$ has been obtained.
- Λ–doubling effect lowers the mass of the multiplets generated by a mix of static energies, the same pattern is observed in direct lattice calculations and QCD sum rules.
- Mass gaps between multiplets in good agreement with direct lattice computations, but the absolute values are shifted.
- Several experimental candidates for Charmonium hybrids belonging to the H_1, H_2, H_4 and H_1' multiplets.
- One experimental candidate to the bottomonium H_1 multiplet.

Thank you for your attention

Λ–doubling effect

Bottomonium sector



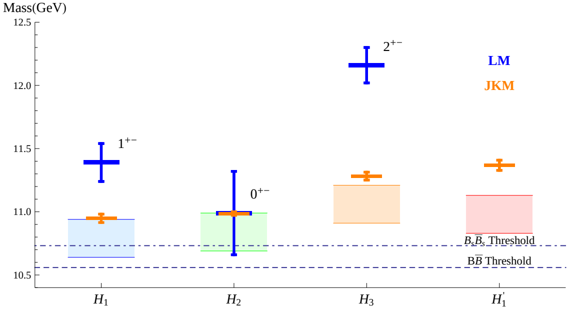
Braaten *et al* 2014 results plotted in dashed lines.

- The mixing lowers the mass of the $H_1(H_4)$ multiplet with respect to $H_2(H_4)$.

Comparison with direct lattice computations

Bottomonium sector

- Calculations done by Juge, Kuti, Morningstar 1999 and Liao, Manke 2002 using quenched lattice QCD.
- Juge, Kuti, Morningstar 1999 included no spin or relativistic effects.
- Liao, Manke 2002 calculations are fully relativistic.



Error bands take into account the uncertainty on the gluelump mass ± 0.15 GeV

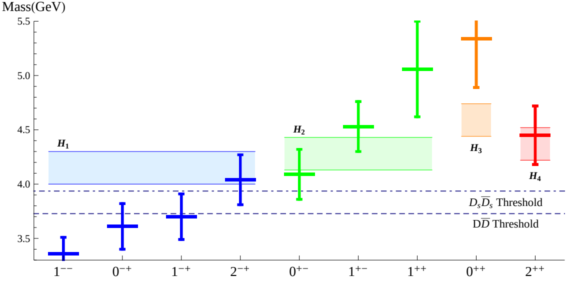
Split (GeV)	JKM	$V^{(0.25)}$
$\delta m_{H_2-H_1}$	0.04	0.05
$\delta m_{H_3-H_1}$	0.33	0.27
$\delta m_{H_3-H_2}$	0.30	0.22
$\delta m_{H_1'-H_1}$	0.42	0.19

- Our masses are $0.15 - 0.25$ GeV lower except the for the H_1' multiplet, which is larger by 0.36 GeV.
- Good agreement with the mass gaps between multiplets, in particular the Λ-doubling effect ($\delta m_{H_2-H_1}$).

Comparison with QCD sum rules

- A recent analysis of QCD sum rules for hybrid operators has been performed by Chen *et al* 2013, 2014 for $b\bar{b}$ and $c\bar{c}$ hybrids, and $b\bar{c}$ hybrids respectively.
- Correlation functions and spectral functions were computed up to dimension six condensates which stabilized the mass predictions compared to previous calculations which ony included up to dimension 4 condensates.

Charmonium sector Chen *et al* 2013

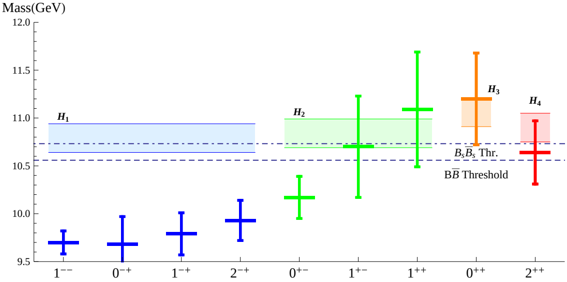


Error bands take into account the uncertainty on the gluelump mass ± 0.15 GeV

- The spin average of the H_1 multiplet is 0.4 GeV lower than our mass.
- H_2, H_3 and H_4 multiplets are incomplete.
- Large uncertainties compared to direct lattice calculations.

Comparison with QCD sum rules

Bottomonium sector Chen *et al* 2013



Error bands take into account the uncertainty on the gluelump mass ± 0.15 GeV

- The spin average of the H_1 multiplet is 0.98 GeV lower than our mass.
- H_2, H_3 and H_4 multiplets are incomplete.
- Large uncertainties compared to direct lattice calculations.