η_c, J/ψ, h_c DECAY CONSTANTS FROM LATTICE AND QCD SUM RULES

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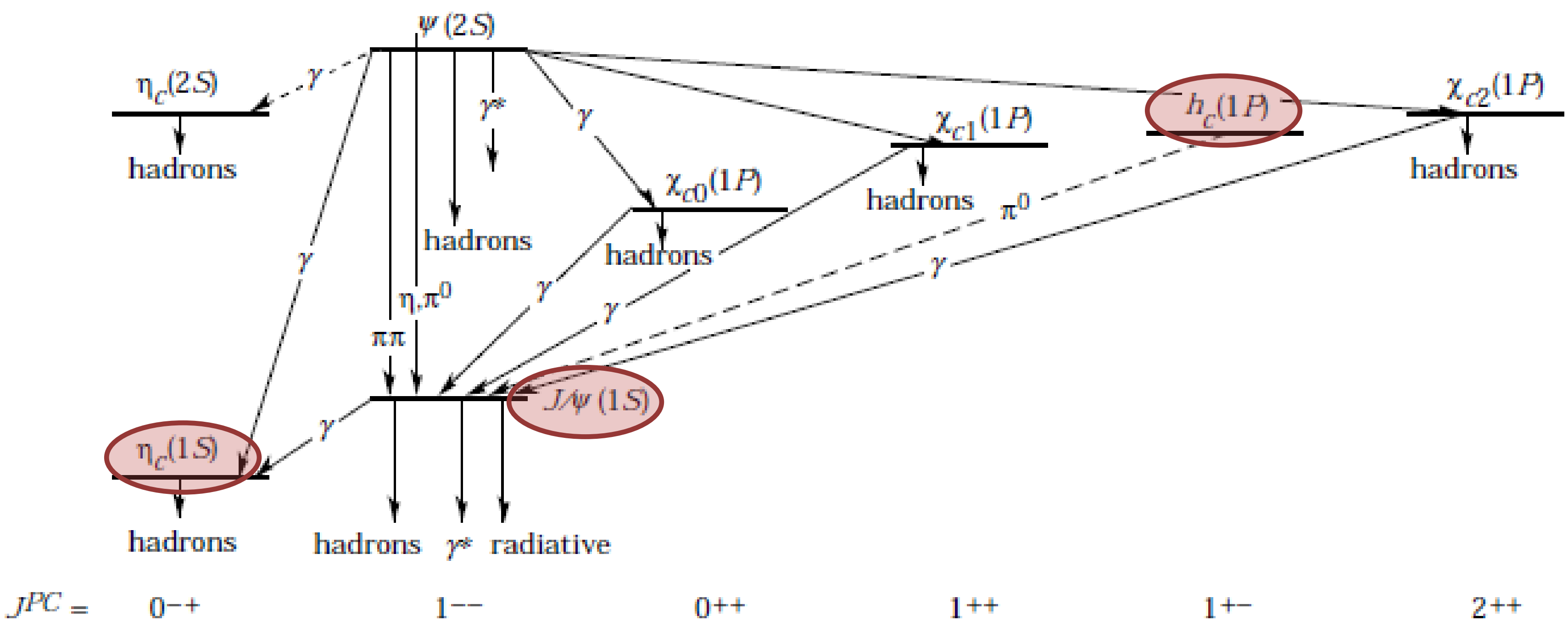
NUMEROUS NEW CHARMONIA STATES OBSERVED IN THE LAST 10 YEARS

State	m (MeV)	Γ (MeV)	J^{PC}	Process (mode)	Experiment (#σ)	Year	Status
$h_c(1P)$	3525.41 ± 0.16	<1	1^{+-}	$\psi(2S) \to \pi^0 (\gamma\eta_c(1S))$	CLEO [8–10] (13.2)	2004	OK
				$\psi(2S) \to \pi^0 (\gamma ...)$	CLEO [8–10] (10), BES [11] (19)		
				$p\bar{p} \to (\gamma\eta_c) \to (\gamma\gamma\gamma)$	E835 [12] (3.1)		
				$\psi(2S) \to \pi^0 (...)$	BESIII [11] (9.5)		
$\eta_c(2S)$	3638.9 ± 1.3	10 ± 4	0^{-+}	$B \to K (K_S^0 K^- \pi^+)$	Belle [13,14] (6.0)	2002	OK
				$e^+e^- \to e^+e^- (K_S^0 K^- \pi^+)$	BABAR [15,16] (7.8),		
					CLEO [17] (6.5), Belle [18] (6)		
				$e^+e^- \to J/\psi (...)$	BABAR [19] (np), Belle [20] (8.1)		
$\chi_{c2}(2P)$	3927.2 ± 2.6	24 ± 6	2^{++}	$e^+e^- \to e^+e^- (D\bar{D})$	Belle [21] (5.3), BABAR [22,23] (5.8)	2005	OK
$X(3872)$	3871.68 ± 0.17	< 1.2	$1^{++}/2^{-+}$	$B \to K (\pi^+\pi^- J/\psi)$	Belle [36,37] (12.8), BABAR [38] (8.6)	2003	OK
				$p\bar{p} \to (\pi^+\pi^- J/\psi) + ...$	CDF [39–41] (np), D0 [42] (5.2)		
				$B \to K (\omega J/\psi)$	Belle [43] (4.3), BABAR [23] (4.0)		
				$B \to K (D^{*0}\overline{D}^0)$	Belle [44,45] (6.4), BABAR [46] (4.9)		
				$B \to K (\gamma J/\psi)$	Belle [47] (4.0), BABAR [48,49] (3.6)		
				$B \to K (\gamma\psi(2S))$	BABAR [49] (3.5), Belle [47] (0.4)		
				$pp \to (\pi^+\pi^- J/\psi) + ...$	LHCb [50] (np)		
$X(3915)$	3917.4 ± 2.7	28^{+10}_{-9}	$0/2^{?+}$	$B \to K (\omega J/\psi)$	Belle [51] (8.1), BABAR [52] (19)	2004	OK
				$e^+e^- \to e^+e^- (\omega J/\psi)$	Belle [53] (7.7), BABAR [23] (np)		
$X(3940)$	3942^{+9}_{-8}	37^{+27}_{-17}	$?^{?+}$	$e^+e^- \to J/\psi (D\overline{D}^*)$	Belle [54] (6.0)	2007	NC!
				$e^+e^- \to J/\psi (...)$	Belle [20] (5.0)		
$G(3900)$	3943 ± 21	52 ± 11	1^{--}	$e^+e^- \to \gamma (D\overline{D})$	BABAR [55] (np), Belle [56] (np)	2007	OK
$Y(4008)$	4008^{+121}_{-49}	226 ± 97	1^{--}	$e^+e^- \to \gamma(\pi^+\pi^- J/\psi)$	Belle [57] (7.4)	2007	NC!
$Z_1(4050)^+$	4051^{+24}_{-43}	82^{+51}_{-55}	?	$B \to K (\pi^+\chi_{c1}(1P))$	Belle [58] (5.0), BABAR [59] (1.1)	2008	NC!
$Y(4140)$	4143.4 ± 3.0	15^{+11}_{-7}	$?^{?+}$	$B \to K (\phi J/\psi)$	CDF [60,61] (5.0)	2009	NC!
$X(4160)$	4156^{+29}_{-25}	139^{+113}_{-65}	$?^{?+}$	$e^+e^- \to J/\psi (D\overline{D}^*)$	Belle [54] (5.5)	2007	NC!
$Z_2(4250)^+$	4248^{+185}_{-45}	177^{+321}_{-72}	?	$B \to K (\pi^+\chi_{c1}(1P))$	Belle [58] (5.0), BABAR [59] (2.0)	2008	NC!
$Y(4260)$	4263^{+8}_{-9}	95 ± 14	1^{--}	$e^+e^- \to \gamma (\pi^+\pi^- J/\psi)$	BABAR [62,63] (8.0)	2005	OK

THE CHARMONIUM SYSTEM - states below $D_{(s)}D_{(s)}$ thresholds:



CHARMONIA & CHARMONIA DECAY CONSTANTS

- for understanding the features of quark confinement
- testing the validity of various quark models
- describing weak processes involving charm states

η_c, J/ψ, h_c → $f_{\eta c}$, $f_{J/\psi}$, f_{hc}

DECAY CONSTANTS ARE DEFINED AS :

$$\langle 0|\bar{c}(0)\gamma_\mu\gamma_5 c(0)|\eta_c(p)\rangle = -i f_{\eta_c} p_\mu \ ,$$ P –pseudoscalar current

$$\langle 0|\bar{c}(0)\gamma_\mu c(0)|J/\psi(p,\lambda)\rangle = f_{J/\psi} m_{J/\psi} e^\lambda_\mu \ ,$$ V- vector current

$$\langle 0|\bar{c}(0)\sigma_{\mu\nu} c(0)|J/\psi(p,\lambda)\rangle = i f^T_{J/\psi}(\mu)(e^\lambda_\mu p_\nu - e^\lambda_\nu p_\mu) \ ,$$

$$\langle 0|\bar{c}(0)\sigma_{\mu\nu} c(0)|h_c(p,\lambda)\rangle = i f_{h_c}(\mu)\varepsilon_{\mu\nu\alpha\beta} e^\alpha_\lambda p^\beta \ ,$$

T – tensor current

Only J/ ψ can be directly measured:

$$\Gamma(J/\psi \to e^+e^-) = \frac{4\pi\alpha_{\rm em}}{3m_{J/\psi}}\frac{4}{9} f^2_{J/\psi}$$

Other decay constants have to be extracted from or once known are used in
- various charmonia radiative decays
- two-body nonleptonic B- & D-decays to charmonia

will come to that later….

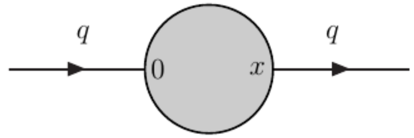
CALCULATION OF DECAY CONSTANTS :

nonperturbative objects → TWO NONPERTURBATIVE METHODS:

- **QCD SUM RULES**
- **LATTICE QCD**

we are going to compare the results obtained by these two methods

QCD SUM RULES



$$\Pi_{\mu\nu}(q) = i\int dx\; e^{iqx}\langle 0|\mathcal{T}\left[V_\mu^\dagger(x)V_\nu(0)\right]|0\rangle = \left(q_\mu q_\nu - g_{\mu\nu}q^2\right)\Pi_V(q^2) \qquad V_\mu = \bar{c}\gamma_\mu c$$

$$\Pi_P(q^2) = i\int dx\; e^{iqx}\langle 0|\mathcal{T}\left[P^\dagger(x)P(0)\right]|0\rangle\,, \quad P = 2m_c\; i\bar{c}\gamma_5 c$$

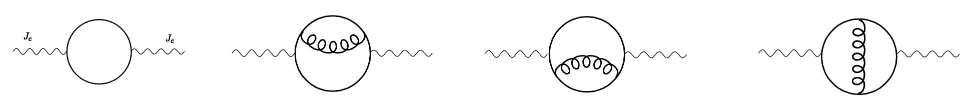
$$\Pi_{\mu\nu\rho\sigma}(q) = i\int dx\; e^{iqx}\langle 0|\mathcal{T}\left[T_{\mu\nu}^\dagger(x)T_{\rho\sigma}(0)\right] = P^-_{\mu\nu\rho\sigma}\underbrace{\Pi_-(q^2)}_{J/\psi(1^{--})} + P^+_{\mu\nu\rho\sigma}\underbrace{\Pi_+(q^2)}_{h_c(1^{+-})}$$

→ NEW CALC. & RESULTS

"OPE" or "theoretical" side:

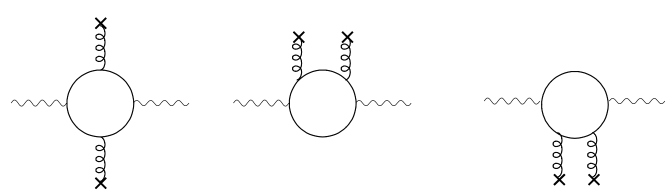
$$\Pi_i(q^2) = \frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}\Pi_i(s)}{s-q^2}ds \equiv \int_0^\infty \frac{\rho_i(s)}{s-q^2}ds$$

- $\rho_i^{\text{pert}}(s) = \rho_i^{(0)}(s) + \frac{\alpha_s}{\pi}\rho_i^{(1)}(s)$



- $\Pi_i^{\text{non-pert}}(q^2) = C_i^{\text{G}}(Q^2)\langle\frac{\alpha_s}{\pi}G^2\rangle\Big|_{Q^2=-q^2}$ $\qquad \langle\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^{\mu\nu\; a}\rangle \equiv \langle\frac{\alpha_s}{\pi}G^2\rangle$

$$C_i^{\text{G}}(Q^2) \propto 1/Q^{2n_i}$$



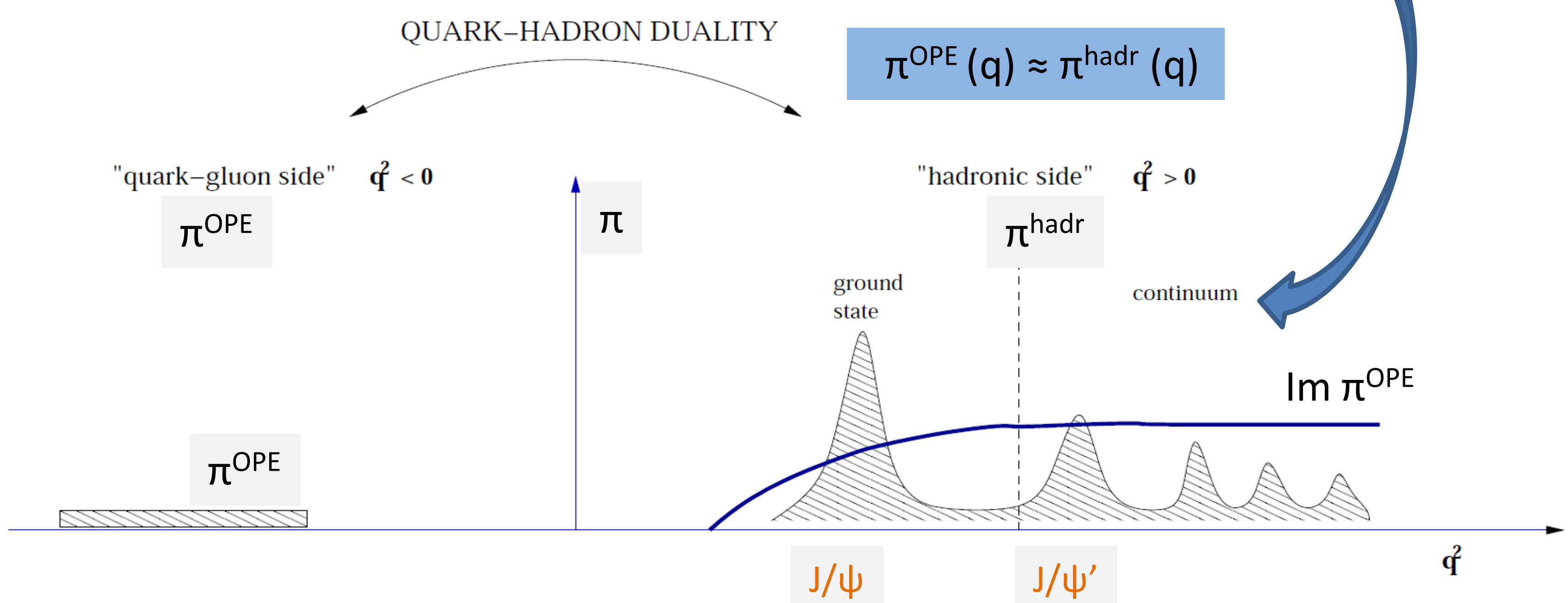
“hadronic” or “phenomenological” side:

$$\Pi_i(q^2) = \frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}\Pi_i(s)}{s-q^2}ds \equiv \int_0^\infty \frac{\rho_i(s)}{s-q^2}ds$$

$$\rho(s) = \sum_{H_q} |\langle 0|j(0)|H_q\rangle|^2\,\delta(s-E_H^2)$$

$$\rho(s) \;=\; |\langle 0|j(0)|H_0\rangle|^2\;\delta(s-M_H^2) + \sum_{H'} |\langle 0|j(0)|H'\rangle|^2\,\delta(s-E_{H'}^2)$$

ρ^{pert}



MOMENT SUM RULES:

$$\mathcal{M}_n(Q_0^2) = \frac{1}{n!}\left(\frac{d}{dq^2}\right)^n \Pi_i(q^2)\Big|_{q^2=-Q_0^2}$$

$$Q_0^2 = 4m_c^2\xi$$

$$\mathcal{M}_n^{\text{theo. } i}(\xi) = \mathcal{M}_n^{\text{pert.}}(\xi) + \mathcal{M}_n^{\text{non-pert.}}(\xi)$$
$$= \frac{1}{(4m_c^2)^n}\int_0^1 \frac{2v(1-v^2)^{n-1}\rho_i(v)}{[1+\xi(1-v^2)]^{n+1}}\,dv + \frac{1}{n!}\left(-\frac{d}{dQ^2}\right)^n C_i^{\text{G}}(Q^2)\,\langle\frac{\alpha_s}{\pi}G^2\rangle\Big|_{Q^2=Q_0^2=4m_c^2\xi}$$

$$\mathcal{M}_n^{\text{phen. } i}(Q_0^2) = \sum_{k=0}^{\infty}\frac{|\langle 0|J^i(0)|H_k\rangle|^2}{(m_{H_k}^2+Q_0^2)^{n+1}}$$

$$\mathcal{M}_n^{\text{phen. } V}(Q_0^2) = \frac{f_{J/\psi}^2}{\left(m_{J/\psi}^2+Q_0^2\right)^{n+1}} + \int_{s_0^\psi}^{\infty}\frac{\rho_V^{\text{pert.}}(s)ds}{(s+Q_0^2)^{n+1}}$$

$$\mathcal{M}_n^{\text{phen. } P}(Q_0^2) = \frac{\left(f_{\eta_c}m_{\eta_c}^2\right)^2}{(m_{\eta_c}^2+Q_0^2)^{n+1}} + 4m_c^2\int_{s_0^{\eta_c}}^{\infty}\frac{\rho_P^{\text{pert.}}(s)ds}{(s+Q_0^2)^{n+1}}$$

$$\mathcal{M}_n^{\text{phen. } +}(Q_0^2) = \frac{f_{h_c}^2}{(m_{h_c}^2+Q_0^2)^{n+1}} + \int_{s_0^{h_c}}^{\infty}\frac{\rho_+^{\text{pert.}}(s)ds}{(s+Q_0^2)^{n+1}}$$

$$\mathcal{M}_n^{\text{phen. } -}(Q_0^2) = \frac{[f_{J/\psi}^T(\mu)]^2}{\left(m_{J/\psi}^2+Q_0^2\right)^{n+1}} + \int_{s_0^{\psi T}}^{\infty}\frac{\rho_-^{\text{pert.}}(s)ds}{(s+Q_0^2)^{n+1}}$$

“ONE RESONANCE + CONTINUUM” RULE

$$s_0^\psi \in [3.3^2, 3.65^2]\ \text{GeV}^2$$

$$s_0^{\eta_c} \in [3.1^2, 3.5^2]\ \text{GeV}^2$$

$$s_0^{h_c} \in [3.6^2, 4.0^2]\ \text{GeV}^2$$

FINAL QCD SR EXPRESSIONS

$$\widetilde{\mathcal{M}}^i_n(\xi, s_0) = \frac{1}{(4m_c^2)^n} \int_0^{v[s_0^i]} \frac{2v(1-v^2)^{n-1}\rho_i^{\text{pert.}}(v)}{[1+\xi(1-v^2)]^{n+1}}\, dv + \frac{1}{n!}\left(-\frac{d}{dQ^2}\right)^n C_i^{\mathrm{G}}(Q^2)\, \langle \frac{\alpha_s}{\pi} G^2 \rangle \Big|_{Q^2 = 4m_c^2\xi}$$

$$v[s_0] = \sqrt{1 - 4m_c^2/s_0}$$

CHARMONIA MASSES and DECAY CONSTANTS:

$$m^2_{J/\psi} = -4m_c^2\xi + \frac{\widetilde{\mathcal{M}}^V_n(\xi, s_0^\psi)}{\widetilde{\mathcal{M}}^V_{n+1}(\xi, s_0^\psi)}, \qquad f_{J/\psi} = \left(m^2_{J/\psi} + 4m_c^2\xi\right)^{\frac{n+1}{2}} \left[\widetilde{\mathcal{M}}^V_n(\xi, s_0^\psi)\right]^{1/2}$$

$$m^2_{\eta_c} = -4m_c^2\xi + \frac{\widetilde{\mathcal{M}}^P_n(\xi, s_0^{\eta_c})}{\widetilde{\mathcal{M}}^P_{n+1}(\xi, s_0^{\eta_c})}, \qquad f_{\eta_c} = \left(m^2_{\eta_c} + 4m_c^2\xi\right)^{\frac{n+1}{2}} \left[\widetilde{\mathcal{M}}^P_n(\xi, s_0^{\eta_c})\right]^{1/2} \frac{2m_c}{m^2_{\eta_c}}$$

$$m^2_{h_c} = -4m_c^2\xi + \frac{\widetilde{\mathcal{M}}^+_n(\xi, s_0^{h_c})}{\widetilde{\mathcal{M}}^+_{n+1}(\xi, s_0^{h_c})}, \qquad f_{h_c}(\mu_0) = \left(m^2_{h_c} + 4m_c^2\xi\right)^{\frac{n+1}{2}} \left[\widetilde{\mathcal{M}}^+_n(\xi, s_0^{h_c})\right]^{1/2} \Big|_{\mu_0 = m_c\sqrt{1+4\xi}}$$

$$m^2_{J/\psi} = -4m_c^2\xi + \frac{\widetilde{\mathcal{M}}^-_n(\xi, s_0^\psi)}{\widetilde{\mathcal{M}}^-_{n+1}(\xi, s_0^\psi)}, \qquad f^T_{J/\psi}(\mu_0) = \left(m^2_{J/\psi} + 4m_c^2\xi\right)^{\frac{n+1}{2}} \left[\widetilde{\mathcal{M}}^-_n(\xi, s_0^\psi)\right]^{1/2}$$

$$\mu^2 = m_c^2 + Q_0^2 \qquad Q_0^2 = 4m_c^2\xi$$

PARAMETERS $m_c^{MS}(m_c)$ =1.275(15) GeV, $<\alpha_s/\pi\ G^2>$ =0.009(7) GeV^4 , s_0^i , ξ

CONDITIONS

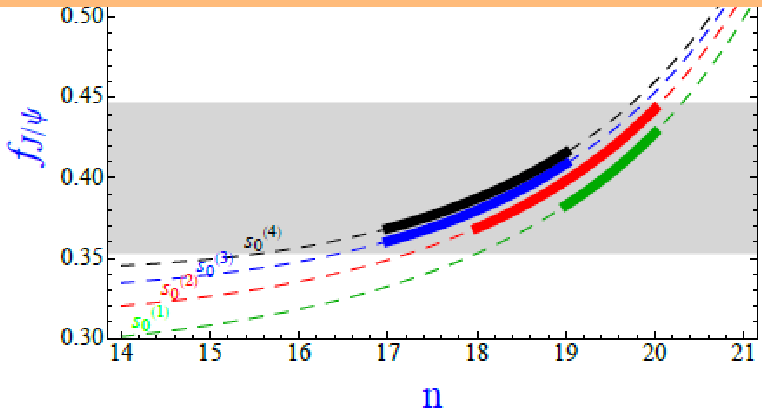
✓ m_{calc} (η_c, J/ψ) = m_{ex} (η_c, J/ψ) up to < 1 % / m_{calc} (h_c) = m_{ex} (h_c) up to < 5 %

✓ $\rho^{pert}(NLO)/\ \rho^{pert}(LO) < 30$ %

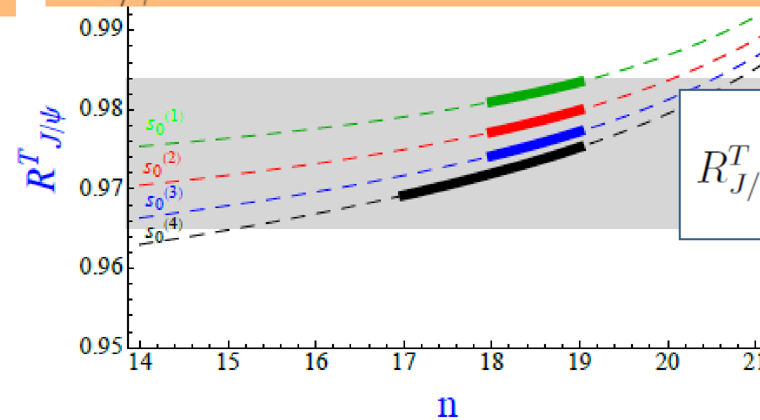
✓ $\rho^{non\text{-}pert}/\ \rho^{pert} < 50$ %

→ STABILITY WINDOW

$$f_{J/\psi} = (335 \div 447)\ \text{MeV} = (401 \pm 46)\ \text{MeV}$$

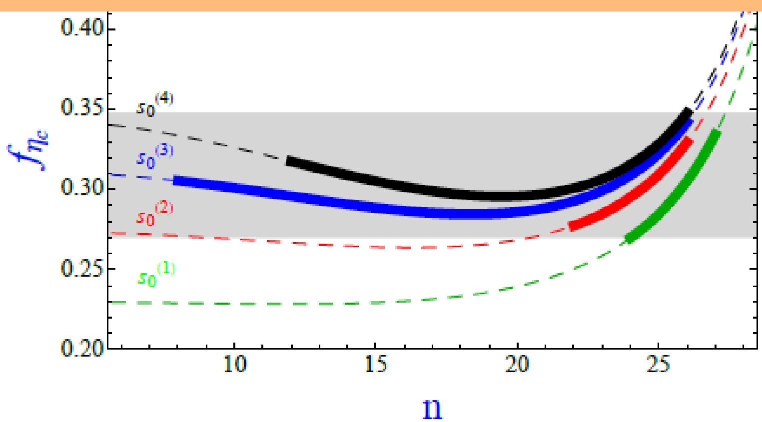


$$R^T_{J/\psi} = (0.965 \div 0.984) = 0.975 \pm 0.010$$

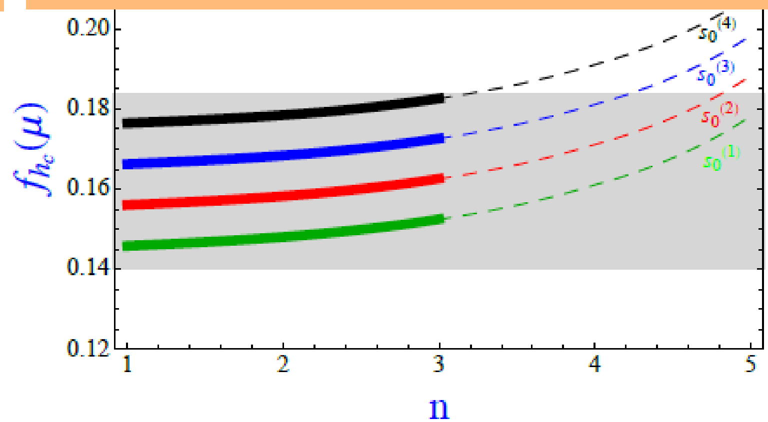


$$R^T_{J/\psi} = \frac{f^T_{J/\psi}(\mu)}{f_{J/\psi}}$$

$$f_{\eta_c} = (270 \div 348)\ \text{MeV} = (309 \pm 39)\ \text{MeV}$$



$$f_{h_c}(2\ \text{GeV}) = (140 \div 184)\ \text{MeV} = (162 \pm 22)\ \text{MeV}$$



LATTICE QCD

- unquenched ($N_f = 2$) dynamical light quark simulations
- use the gauge field configurations generated by ETMC coll. at four spacing points with the maximally twisted mass term:

$$S = a^4 \sum_x \bar{\psi}(x) \left\{ \frac{1}{2} \sum_\mu \gamma_\mu \left(\nabla_\mu + \nabla^*_\mu \right) - i\gamma_5 \tau^3 r \left[m_{\rm cr} - \frac{a}{2} \sum_\mu \nabla^*_\mu \nabla_\mu \right] + \mu_c \right\} \psi(x)$$

$$\psi(x) = [c(x) \;\; c'(x)]^T$$

$$J^{PC} = 0^{-+} \qquad P = 2\mu_c \, \bar{c}\gamma_5 c' \,,$$

$$J^{PC} = 1^{--} \qquad V_i = Z_A \, \bar{c}\gamma_i c' \quad \text{or} \quad T_{0i} = Z_T(\mu) \, \bar{c}\sigma_{0i} c' \,,$$

$$J^{PC} = 1^{+-} \qquad T_{ij} = Z_T(\mu) \, \bar{c}\sigma_{ij} c' \qquad i, j \in (1,2,3) \,,$$

DECAY CONSTANTS

$$\langle 0 | P | \eta_c(\vec{0}) \rangle = f_{\eta_c} m^2_{\eta_c} \,,$$

$$\langle 0 | V_i | J/\psi(\vec{0}, \lambda) \rangle = f_{J/\psi} m_{J/\psi} e^\lambda_i \,,$$

$$\langle 0 | T_{0i}(\mu) | J/\psi(\vec{0}, \lambda) \rangle = -i f^T_{J/\psi}(\mu) m_{J/\psi} e^\lambda_i \,,$$

$$\langle 0 | T_{ij}(\mu) | h_c(\vec{0}, \lambda) \rangle = -i f_{h_c}(\mu) m_{h_c} \varepsilon_{ijk} e^\lambda_k \,.$$

CORRELATION FUNCTIONS – for calculation of masses and decay constants:

$$\langle\sum_{\vec{x}} P(\vec{x};t)P^\dagger(0;0)\rangle = -4\mu_c^2 \sum_{\vec{x}} \langle \mathrm{Tr}\left[S_c(\vec{0},0;\vec{x},t)\gamma_5 S'_c(\vec{x},t;\vec{0},0)\gamma_5\right]\rangle$$

$$\xrightarrow{t \gg 0} \frac{\cosh[m_{\eta_c}(T/2-t)]}{m_{\eta_c}} \left|\langle 0|P(0)|\eta_c(\vec{0})\rangle\right|^2 e^{-m_{\eta_c}T/2},$$

$$\langle\sum_{\vec{x}} V_i(\vec{x};t)V_i^\dagger(0;0)\rangle = -Z_A^2 \sum_{\vec{x}} \langle \mathrm{Tr}\left[S_c(\vec{0},0;\vec{x},t)\gamma_i S'_c(\vec{x},t;\vec{0},0)\gamma_i\right]\rangle$$

$$\xrightarrow{t \gg 0} \frac{\cosh[m_{J/\psi}(T/2-t)]}{m_{J/\psi}} \left|\langle 0|V_i(0)|J/\psi(\vec{0},\lambda)\rangle\right|^2 e^{-m_{J/\psi}T/2}$$

$$\langle\sum_{\vec{x}} T_{0i}(\vec{x};t)T_{0i}^\dagger(0;0)\rangle = -Z_T^2 \sum_{\vec{x}} \langle \mathrm{Tr}\left[S_c(\vec{0},0;\vec{x},t)\sigma_{0i} S'_c(\vec{x},t;\vec{0},0)\sigma_{0i}\right]\rangle$$

$$\xrightarrow{t \gg 0} \frac{\cosh[m_{J/\psi}(T/2-t)]}{m_{J/\psi}} \left|\langle 0|T_{0i}(0)|J/\psi(\vec{0},\lambda)\rangle\right|^2 e^{-m_{J/\psi}T/2}$$

$$\langle\sum_{\vec{x}} T_{ij}(\vec{x};t)T_{ij}^\dagger(0;0)\rangle = -Z_T^2 \sum_{\vec{x}} \langle \mathrm{Tr}\left[S_c(\vec{0},0;\vec{x},t)\sigma_{ij} S'_c(\vec{x},t;\vec{0},0)\sigma_{ij}\right]\rangle$$

$$\xrightarrow{t \gg 0} \frac{\cosh[m_{h_c}(T/2-t)]}{m_{h_c}} \left|\langle 0|T_{ij}(0)|h_c(\vec{0},\lambda)\rangle\right|^2 e^{-m_{h_c}T/2}$$

Hadron masses $\Longrightarrow$ **$a*m_H$** ($H = \eta_c$, J/ψ, h_c) - from a constant of the plateau of m_H^{eff} :

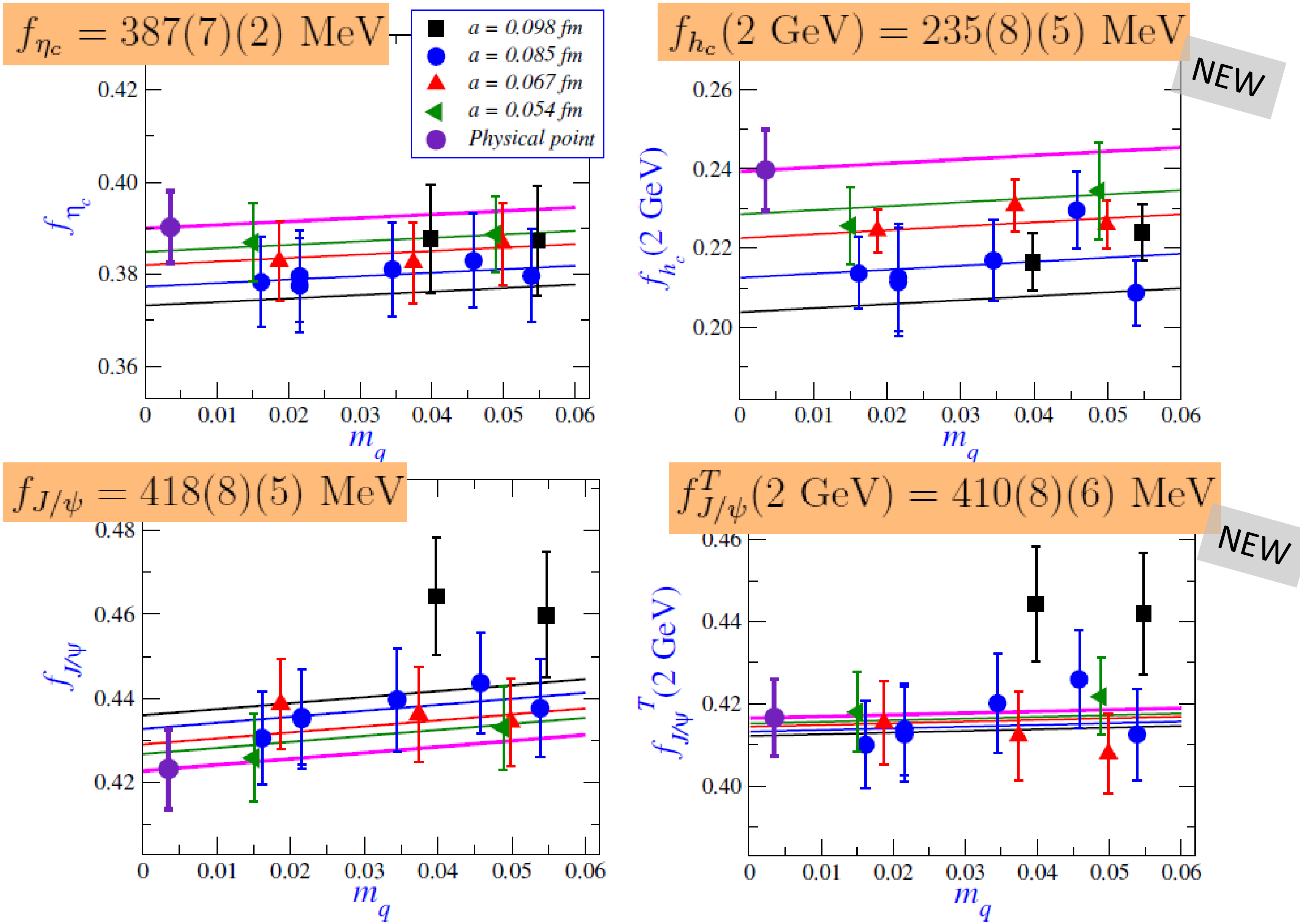
$$\frac{\cosh\left[m_H^{\text{eff}}(t)\left(\frac{T}{2}-t\right)\right]}{\cosh\left[m_H^{\text{eff}}(t)\left(\frac{T}{2}-t-1\right)\right]}=\frac{C_J(t)}{C_J(t+1)}$$

Decay constants - extrapolation of decay constants obtained at four lattice spacing to the continuum limit:

$$f_H = f_H^{\text{cont.}}\left[1+b_H m_q + c_H\frac{a^2}{(0.086\ \text{fm})^2}\right]$$

-dependence on the sea quark masses
& on the lattice spacings

m_q & a-dependence of results:



CONCLUSIONS

- we have calculated four decay constants of charmonium states $f_{\eta c}$, $f_{J/\psi}$, $f_{J/\psi}^{T}$, f_{hc}

QCD SR

– one resonance + continuum

$m_c = 1.275(15)$ GeV ,

$\langle \alpha_s/\pi\, GG \rangle = 0.009(7)\ \mathrm{GeV}^4$

Lattice QCD

– unquenched ($N_f = 2$) simulations using twisted mass QCD (generated by ETMC coll.) at four spacing points

QCD SR		Lattice QCD
$f_{J/\psi} = (401 \pm 46)$ MeV	**GOOD AGREEMENT**	$f_{J/\psi} = (418 \pm 9)$ MeV
$f_{J/\psi}^{T}$ (2 GeV) = (391 ± 45) MeV		$f_{J/\psi}^{T}$ (2 GeV)= (410 ± 10) MeV
$R_{J/\psi}^{T}$ (2 GeV) = 0.975 ± 0.010		$R_{J/\psi}^{T}$ (2 GeV) = 0.981 ± 0.008
$f_{\eta c} = (309 \pm 39)$ MeV	**NOT SO GOOD AGREEMENT**	$f_{\eta c} = (389 \pm 7)$ MeV
$f_{hc} = (162 \pm 22)$ MeV		$f_{hc} = (235 \pm 9)$ MeV

NOT SO GOOD AGREEMENT

- $\langle \alpha_s/\pi\, GG \rangle$ fixed in the vector channel - problem?
- more hadronic resonances needed ?

PHENOMENOLOGICAL IMPLICATIONS

$\eta_c \to \gamma\gamma^{(*)}$

$$\Gamma(\eta_c \to \gamma\gamma) = \frac{4\pi\alpha_{\rm em}^2}{81} m_{\eta_c}^3 |F_{\gamma\eta_c}(0)|^2$$

➢ Babar coll. : **$F_{\gamma\eta_c}(Q^2)$** from $d\sigma(e^+ e^- \to e^+ e^- \eta_c)/dQ^2$ which is driven by $\gamma\gamma^* \to \eta_c$

in the range $Q^2 = (0,50)$ GeV^2 - data are **very well described by a single pole**:

$m_{pole} \approx m_{\eta_c} = 2.9(1)(1)$ GeV

- both photons on-shell $\eta_c \to \gamma\gamma$: $F_{\gamma\eta_c}(0)$ has a pole structure:

$$\Gamma(\eta_c \to \gamma\gamma) = \frac{4\pi\alpha_{\rm em}^2}{81} m_{\eta_c}^3 \left(\frac{f_{\eta_c}}{m_{\eta_c}^2(1+\delta)}\right)^2$$

Experimentally: $\Gamma^{\rm exp}(\eta_c \to \gamma\gamma) = 5.0(4)$ keV

Using our f_{η_c}(lattice) = 0.387(8) → δ = 0.15 (5) GeV^2

Factorization approximation: $\delta = 0$ → the result larger than exp. value:

$$\Gamma^{\rm fact.}(\eta_c \to \gamma\gamma) = (6.64 \pm 0.27)\ {\rm keV}$$

DIFFERENT MODELS FOR $F_{\gamma\,\eta_c}(0)$ vs $\delta = 0.15\,(5)\ \mathrm{GeV}^2$

- **perturbative QCD**

$$F_{\eta_c\gamma}(0) \simeq \frac{4 f_{\eta_c}}{m^2_{\eta_c} + 2\langle \mathbf{k}^2_\perp \rangle}$$

(Feldmann, Kroll, 9709203)

$\delta = 0.15\,(5)\ \mathrm{GeV}^2$ - too large value to be interpreted as $\sqrt{\langle \mathbf{k}^2_\perp \rangle} = 0.81(14)\ \mathrm{GeV}$

(mean transverse momentum of c-quark)

- **heavy quark symmetry**

$$2\langle \mathbf{k}^2_\perp \rangle \to b_{\eta_c} m_{\eta_c} \qquad b_{\eta_c} = 2m_c - m_{\eta_c}$$

(Lansberg,Pham, 0603113)

$\delta = 0.15\,(5)\ \mathrm{GeV}^2$ - too large value to be interpreted as $b_{\eta_c} = \delta m_{\eta_c} = 0.46(16)\ \mathrm{GeV}$

- **the nearest vector meson dominance (VDM)**

from lattice: $V^{J/\psi \to \eta_c}(0) = 1.92(3)(2)$

(Becirevic, Sanfilippo, 1206.1445)

$$F^{\mathrm{VMD}}_{\eta_c\gamma}(0) = 2\frac{f_{J/\psi}}{m_{J/\psi}}\,\frac{2V^{J/\psi\to\eta_c}(0)}{m_{J/\psi} + m_{\eta_c}}$$

with our f_{η_c} **(lattice) = 0.387(8)**

$$\Gamma(\eta_c \to \gamma\gamma) = 6.0(4)\ \mathrm{keV}$$

again too large when compared to the experimental value

B → K nonleptonic decays

- class II decays in the generalized factorization approximation:

$$\Gamma(B \to J/\psi K) = \frac{G_F^2 |V_{cb} V_{cs}^*|^2}{32\pi m_B^3} \lambda^{3/2}(m_B^2, m_{J/\psi}^2, m_K^2)\ a_2^2\ f_{J/\psi}^2 \boxed{\left[f_+^{B\to K}(m_{J/\psi}^2)\right]^2},$$

$$\Gamma(B \to \eta_c K) = \frac{G_F^2 |V_{cb} V_{cs}^*|^2}{32\pi m_B^3} (m_B^2 - m_K^2)^2 \lambda^{1/2}(m_B^2, m_{\eta_c}^2, m_K^2)\ a_2^2\ f_{\eta_c}^2 \boxed{\left[f_0^{B\to K}(m_{\eta_c}^2)\right]^2}$$

$$\frac{B(B \to \eta_c K)}{B(B \to J/\psi K)} \sim \boxed{\left(\frac{f_{\eta_c}}{f_{J/\psi}}\right)^2} \boxed{\left(\frac{f_0^{B\to K}(m_{\eta_c}^2)}{f_+^{B\to K}(m_{J/\psi}^2)}\right)^2}$$

our result: $f_{\eta_c}/f_{J/\psi} = 0.926(6)$

$$\boxed{\frac{f_+^{B\to K}(m_{J/\psi}^2)}{f_0^{B\to K}(m_{\eta_c}^2)} = 1.53(10)\big|_{B^\pm-\text{mode}},\ 1.56(13)\big|_{B^0-\text{mode}}}$$

- ✓ in agreement with LCSR calculation (Duplancic,Melic, 0805.4170)
- ✓ in agreement with quentched lattice QCD (Becirevic et al, 1205.5811)
- ✓ differs from the uquenched lattice study with NR QCD for heavy quarks = 1.37(2) (Bouchard et al, 1306.0434)

CONCLUSION - PHENOMENOLOGICAL RESULTS

- **$\eta_c \to \gamma\gamma$**
  using our $f_{\eta c}$ experimental result is not reproduced – nonfactorizable effects?

- **$B \to K$ nonleptonic decays**

- using our $f_{\eta c}/ f_{J/\psi}$ QCD SR results, LCSR results and old lattice QCD results
  – experimental $f_+^{B\to K}(m^2_{J/\psi})/ f_0^{B\to K}(m^2_{\eta c})$ is well reproduced
- just a limit for $BR(B^+ \to h_c K^+) < 3.8 \times 10^{-6}$ - it would be interesting to measure this BR

MORE RESEARCH IS NEEDED:

- **η_c decay constant** in QCD SR – at NLO, with the inclusion of higher hadronic resonances (Becirevic, Melic, in progress)
- **$\eta_c \to \gamma\gamma$** in QCD SR at NLO
- **$J/\psi \to \eta_c\gamma$, $h_c \to \eta_c\gamma$** in lattice QCD (Becirevic et al, in progress) and QCD SR at NLO