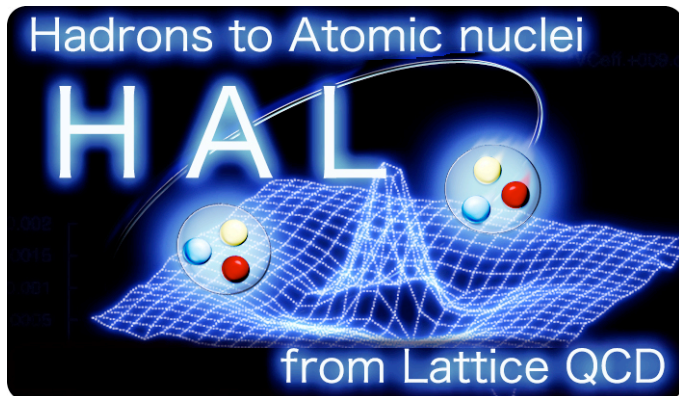


Charmed Tetra-quark states in Lattice QCD – Interactions from HAL QCD method –

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for HAL QCD Collaboration



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F. Etminan (Univ. of Birjand)

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N. Ishii, K. Murano (RCNP)

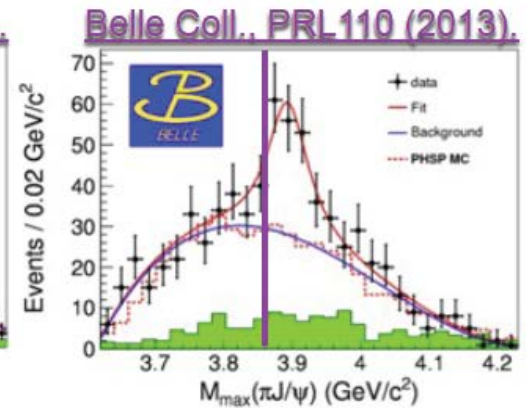
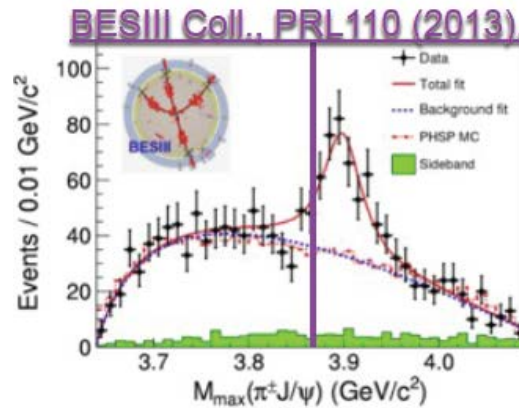
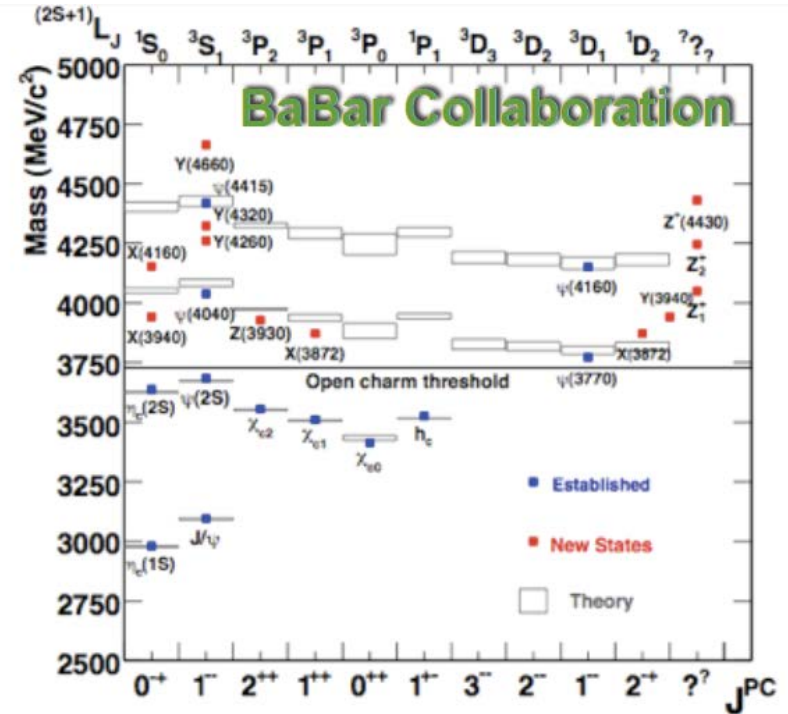
T. Miyamoto, H. Nemura, K. Sasaki, M. Yamada (Univ. of Tsukuba)

New hadronic DoF in charmed system ?

- Discoveries of XYZ mesons
 - What is the relevant effective DoF ?
 - Hadronic molecules ?
 - Diquarks ?
 - Gluonic excitations ?
 - (or threshold effects ?)
 - What kind of physics is hidden behind ?
 - The role of heavy quark sym ?
- Lattice QCD study of exotic states

[Our approach]

- Determine interactions and extract the physics
- exotic states



[Zc\(3900\)](#)

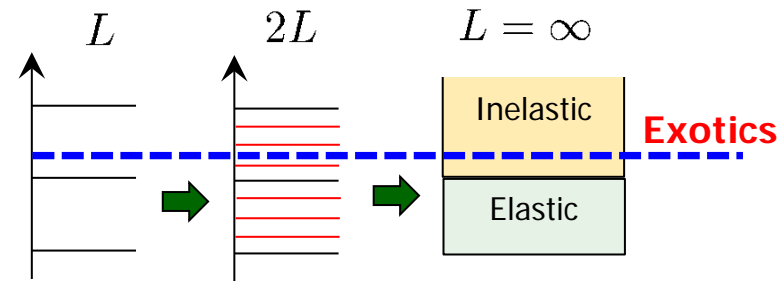
Interactions on the Lattice

- **Traditional approach: Luscher's method**

- Phase shift & B.E. from temporal correlation in finite V
- Spectrum of all states (incl. “continuum” states) have to be identified
- Coupled channel: challenging

Talks by Prelovsek, Liu, etc.

M.Luscher,
CMP104(1986)177
CMP105(1986)153
NPB354(1991)531



- **HAL QCD method**

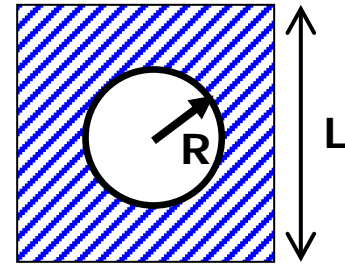
- Potential from spacial (& temporal) correlation
- Phase shift & B.E by solving Schrodinger eq in infinite V
- All states (incl. “continuum” states) are automatically classified w.r.t. asymptotic states and their interactions
- Coupled channel: straightforward

HAL QCD method

- Potential is constructed so as to reproduce the scattering phase shifts (or, S-matrix)
- Nambu-Bethe-Salpeter (NBS) wave function

$$\psi(\vec{r}) = \langle 0 | H_1(\vec{x} + \vec{r}) H_2(\vec{x}) | H_1 H_2, W_k \rangle_{\text{in}}$$

$$\psi_l(r) \simeq e^{i\delta(k)} \frac{\sin(kr - l\pi/2 + \delta(k))}{kr} \quad (r > R)$$

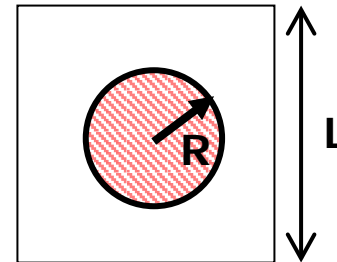


M.Luscher (1991), C.-J.Lin et al. (2001), CP-PACS Coll. (2005), Ishizuka (2009)

- Define (energy-independent) potential from wave func.

$$(\nabla^2 + k^2)\psi(r) = m \int dr' U(\mathbf{r}, \mathbf{r}') \psi(r'), \quad r < R$$

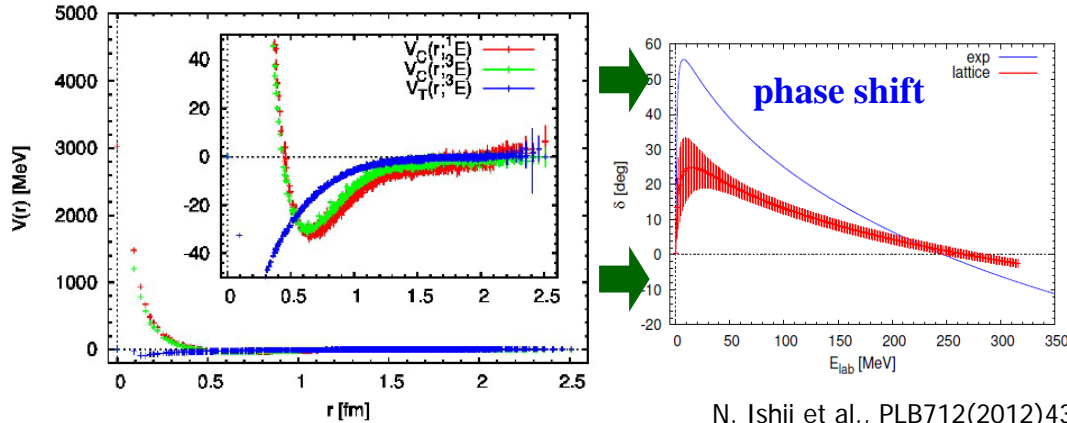
- **U** is not observable but faithful to phase shift



- All information of elastic scatt. states encoded in **U**
- Extension to coupled channel system straightforward

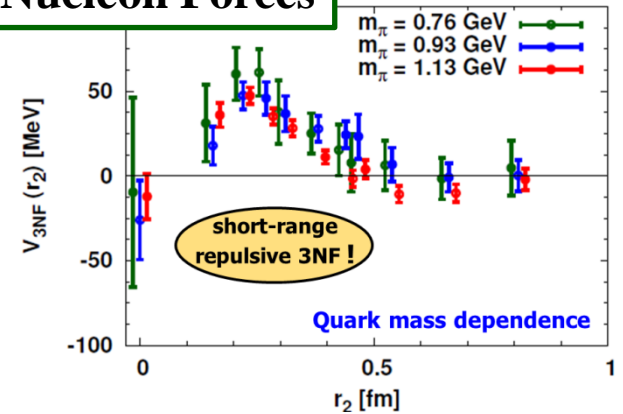
Successful applications to various interactions

NN forces (central & tensor)



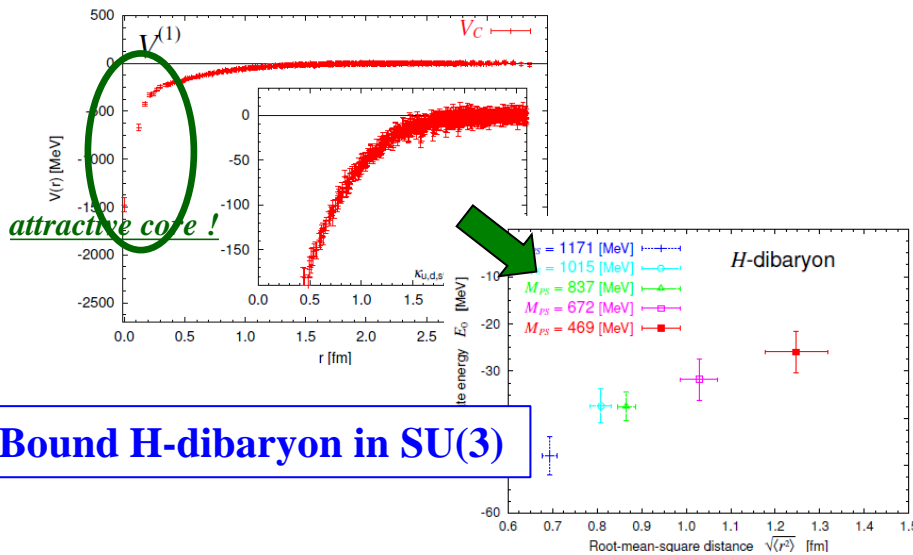
N. Ishii et al., PLB712(2012)437

Three-Nucleon Forces



T.D. et al., PTP127(2012)723

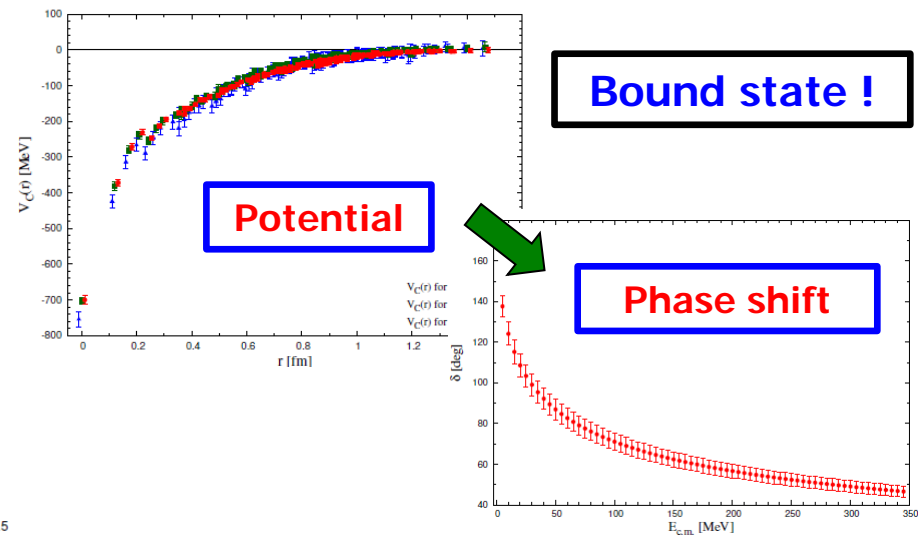
H-dibaryon



Bound H-dibaryon in SU(3)

T. Inoue et al., PRL106(2011)162002

N Ω forces



F. Etminan et al., NPA928(2014)89

Search of charmed exotics on Lattice

- $T_{cc} (ccu^{\text{bar}}d^{\text{bar}})$, $T_{cs}(csu^{\text{bar}}d^{\text{bar}})$
 - Manifestly exotic
 - Could be good probe of (ud)-diquark DoF (spin frozen by heavy quark sym)

- Z_c states
 - Many experiments
 - Exotic w/ OZI rule

Color-magnetic int. in diquark

$$\langle v_{ij} \rangle = -\langle (\vec{\lambda}(i) \cdot \vec{\lambda}(j)) (\vec{\sigma}(i) \cdot \vec{\sigma}(j)) \rangle$$

$\langle v_{ij} \rangle$	C=1	C=8	C=3 ^{bar}	C=6
S=0	-16	2	-8	4
S=1	16/3	-2/3	8/3	-4/3

→ Good (I=0) / bad (I=1) diquark

Lattice QCD setup

- Nf=2+1 dynamical clover fermion + RG improved gauge action
 - Relativistic Heavy Quark (RHQ) action for charm
 - $a^{-1}=2.176\text{GeV}$, $a=0.0907\text{fm}$
 - $32^3 \times 64$ lattice, **L=2.9fm**
 - ~400configs
 - (disconnected diagram neglected)

S. Aoki et al. (PACS-CS Coll.)
PRD79(2009)034503

Y. Namekawa et al.,
PRD84(2011)074505

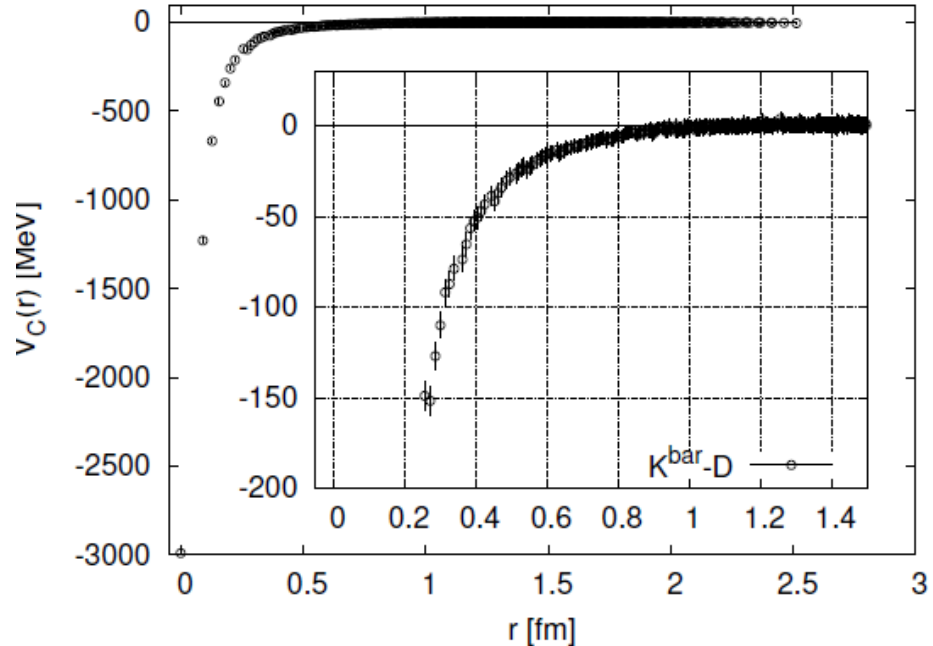
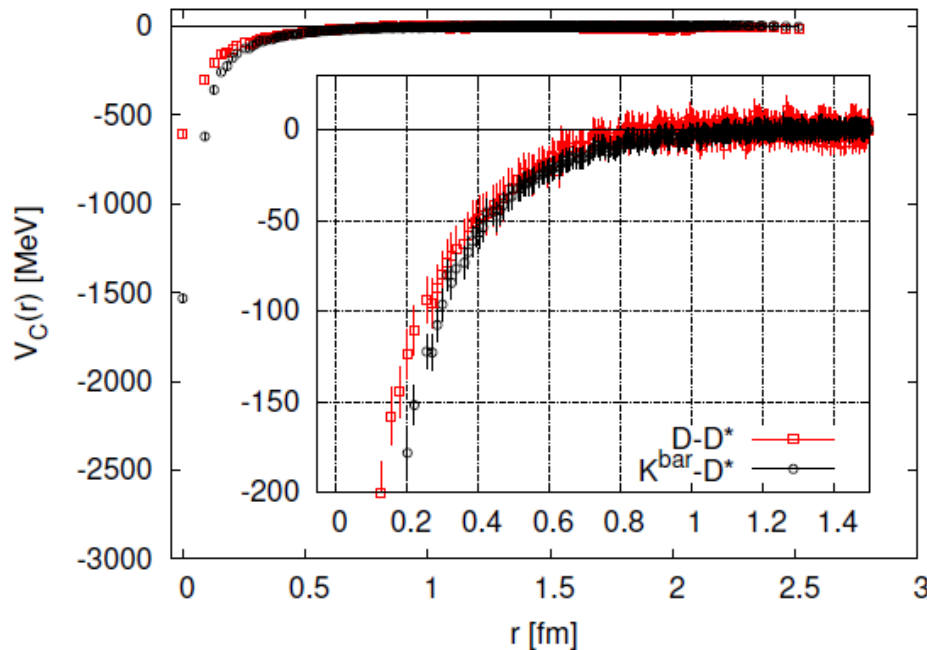
Masses:	[on the lattice]	[PDG]
• $m(\pi)$	= 0.41, 0.57, 0.70 GeV	0.14
• $m(K)$	= 0.64, 0.71, 0.79 GeV	0.50
• $m(D)$	= 1.90, 1.95, 2.00 GeV	1.87
• $M(D^*)$	= 2.05, 2.10, 2.16 GeV	2.01
• $M(\eta_c)$	= 2.99, 3.01, 3.02 GeV	2.98
• $M(J/\psi)$	= 3.10, 3.12, 3.02 GeV	3.10

T_{cc}, T_{cs} states w/ "good" diquark

$$T_{cc} (J^P=1^+, I=0) \Leftrightarrow D-D^*$$

$$T_{cs} (J^P=1^+, I=0) \Leftrightarrow K^{\text{bar}}-D^*$$

$$T_{cs} (J^P=0^+, I=0) \Leftrightarrow K^{\text{bar}}-D$$



$$m_\pi=410\text{MeV}$$

Attraction !

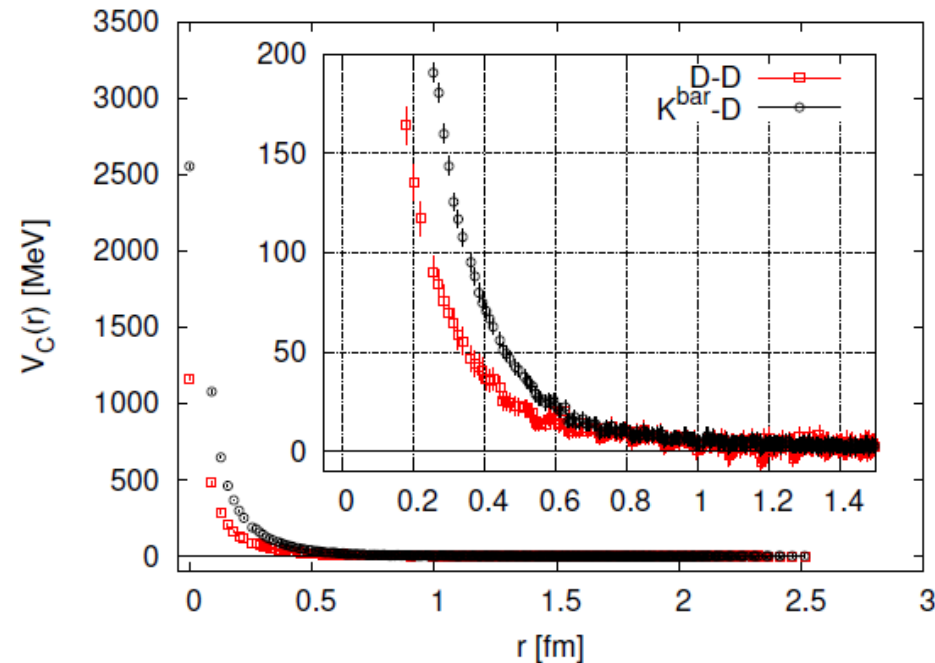
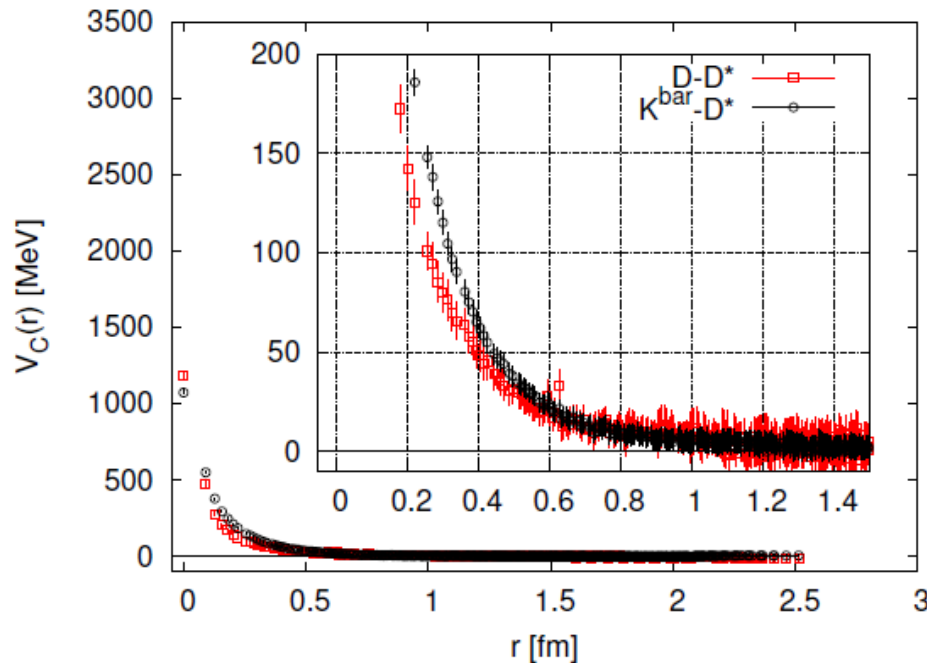
Tcc, Tcs states w/ "bad" diquark

$$T_{cc} (J^P=1^+, I=1) \Leftrightarrow D-D^*$$

$$T_{cc} (J^P=0^+, I=1) \Leftrightarrow D-D$$

$$T_{cs} (J^P=1^+, I=1) \Leftrightarrow K^{\text{bar}}-D^*$$

$$T_{cs} (J^P=0^+, I=1) \Leftrightarrow K^{\text{bar}}-D$$



$$m_\pi=410\text{MeV}$$

Repulsion !

Tcc, Tcs states w/ **good** diquark

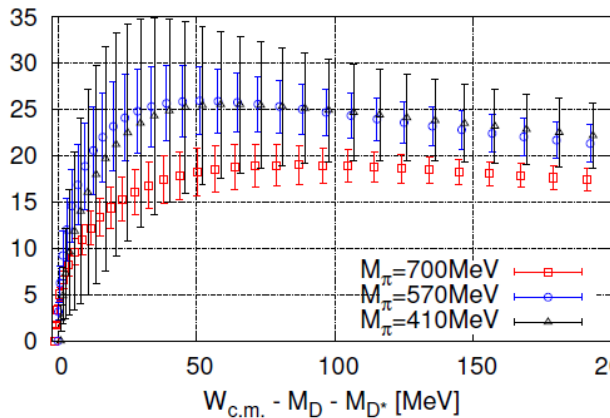
Phase Shift

Tcc ($J^P=1^+, I=0$)
 $\Leftrightarrow D-D^*$

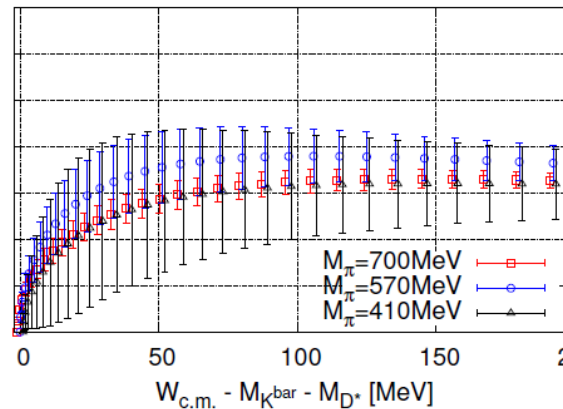
Tcs ($J^P=1^+, I=0$)
 $\Leftrightarrow K^{\text{bar}}-D^*$

Tcs ($J^P=0^+, I=0$)
 $\Leftrightarrow K^{\text{bar}}-D$

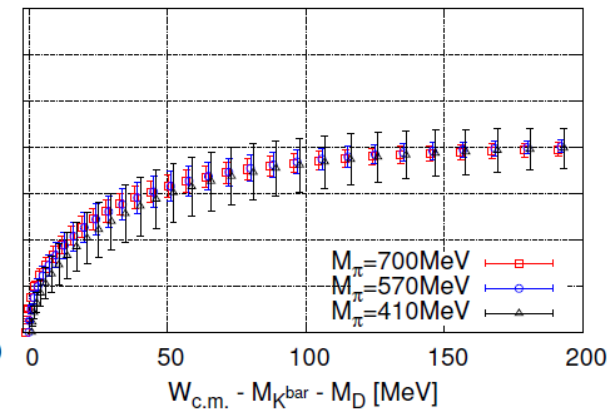
(c) D-D* phase shift



(b) K^{bar}-D* phase shift



(a) K^{bar}-D phase shift



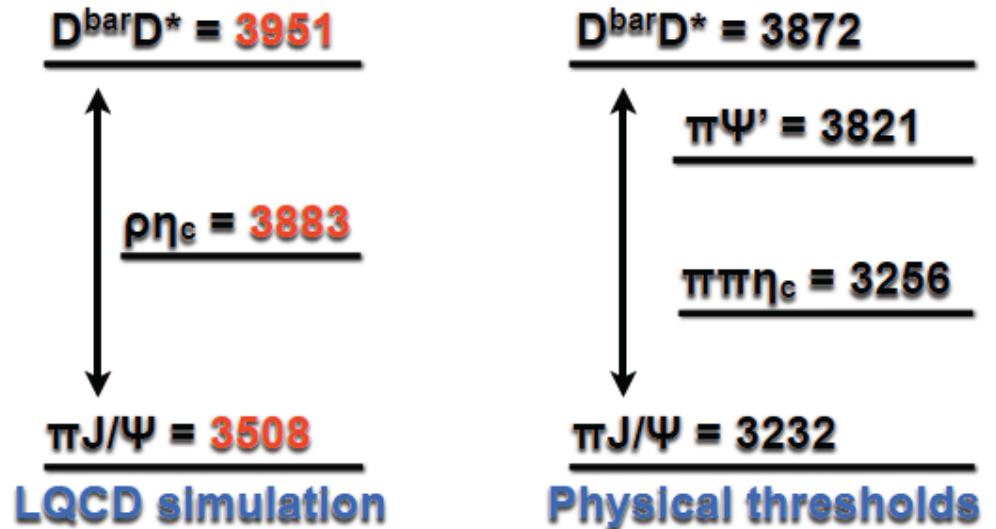
- Strong Attraction, but no bound state for $m_\pi > 400\text{MeV}$
- More attractive for lighter quark mass
- T_{bb} has a bound state if we combine m_b & potential in Tcc

Zc(3900) search on the Lattice

Can "isolated" $D^{\text{bar}} D^*$ molecular state exist in QCD ?

Or, does dynamical coupling play an important role ?

◆ Thresholds in $I^G J^P = 1^+ 1^+$ channel



$m_\pi = 410 \text{ MeV}$

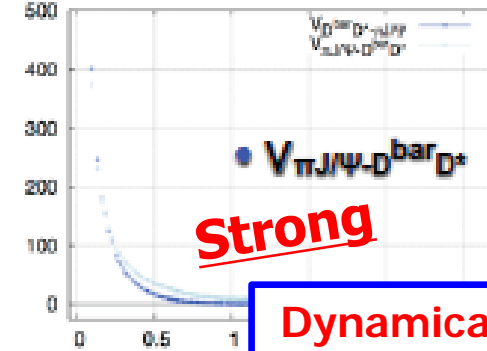
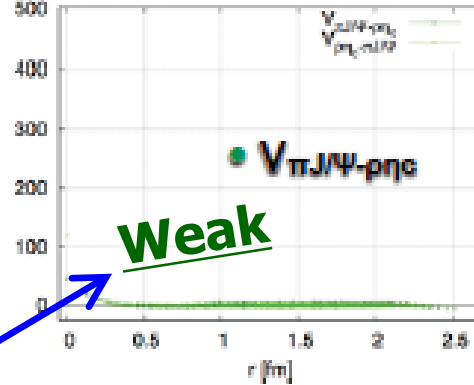
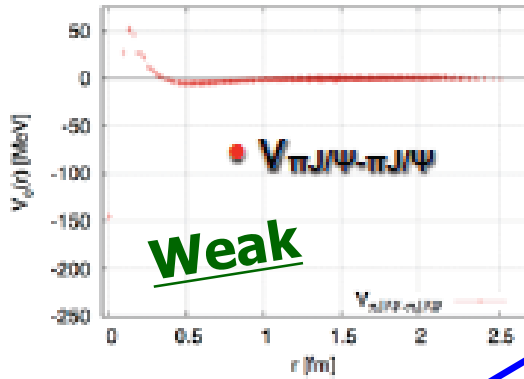
- $M_{\pi \psi'} > M_{D^{\text{bar}} D^*}$ due to heavy pion mass
- $\rho \rightarrow \pi \pi$ decay not allowed w/ $L=3\text{fm}$ lattice

3x3 coupled channel potential

$\pi J/\psi$

$\rho\eta_c$

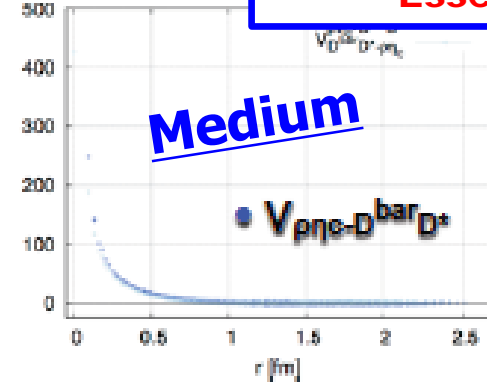
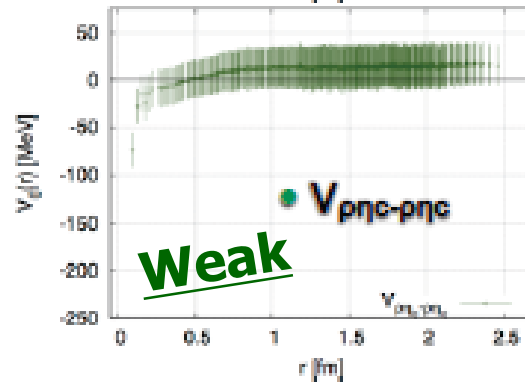
$D^{\text{bar}}D^*$



$\pi J/\psi$

Dynamical Coupling Essential

Heavy Quark Sym



$\rho\eta_c$

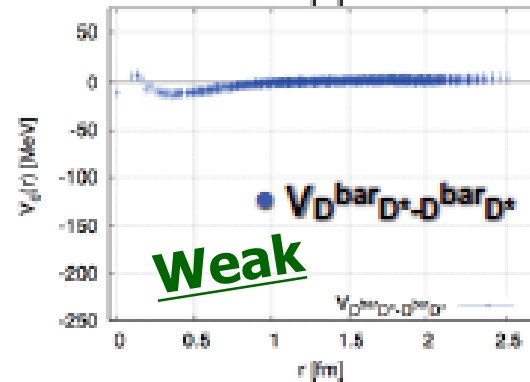
Weak diagonal potentials

(Small $V_{\pi J/\psi - \pi J/\psi}$, $V_{\rho\eta_c - \rho\eta_c}$ & $V_{D^{\text{bar}}D^* - D^{\text{bar}}D^*}$)

Strong off-diagonal $D^{\text{bar}}D^*$ potentials

$\Rightarrow V_{D^{\text{bar}}D^* - \pi J/\psi} > V_{D^{\text{bar}}D^* - \rho\eta_c} \gg V_{\pi J/\psi - \rho\eta_c}$

✓ strong charm quark exchange interactions

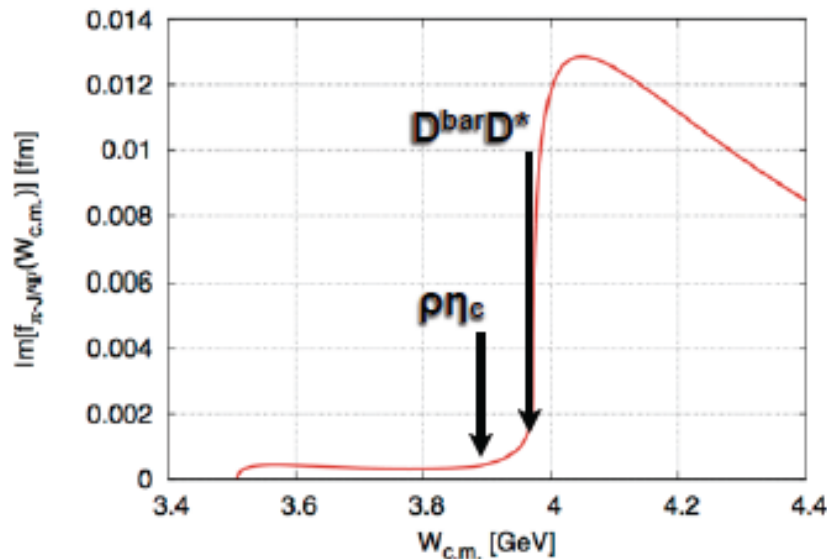


$D^{\text{bar}}D^*$

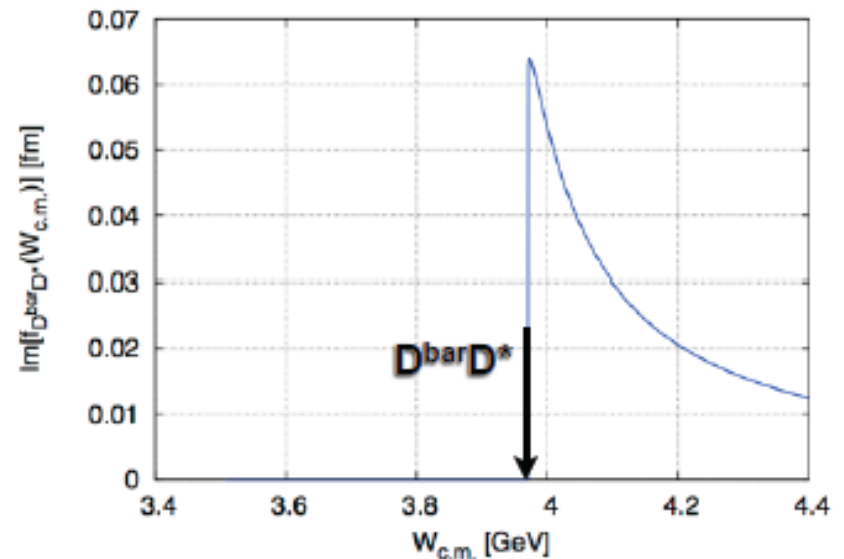
2-body invariant mass

- **2-body invariant mass** : $\rho_i(W_{c.m.}) \equiv |S_{ii}(W_{c.m.}) - 1|^2/k_i \propto \text{Im}f_{ii}(W_{c.m.})$
($i = \pi J/\Psi, \bar{D}D^*$)

- **$\pi J/\Psi$ invariant mass ($m_\pi=410\text{MeV}$)**



- **$\bar{D}D^*$ invariant mass ($m_\pi=410\text{MeV}$)**

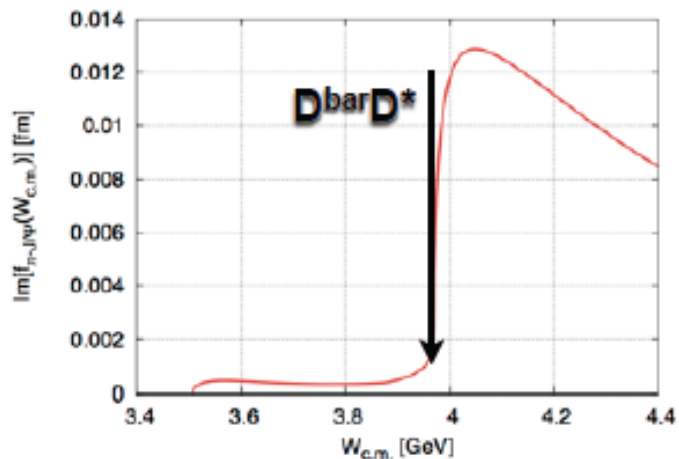


✓ **enhancement near $\bar{D}D^*$ threshold due to large $\pi J/\Psi$ - $\bar{D}D^*$ coupling**

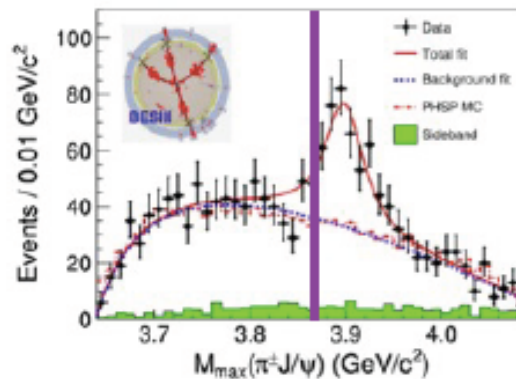
- **broad tail in $\pi J/\Psi$ invariant mass**
- **sharp enhancement in $\bar{D}D^*$ invariant mass**

LQCD results & EXP. results

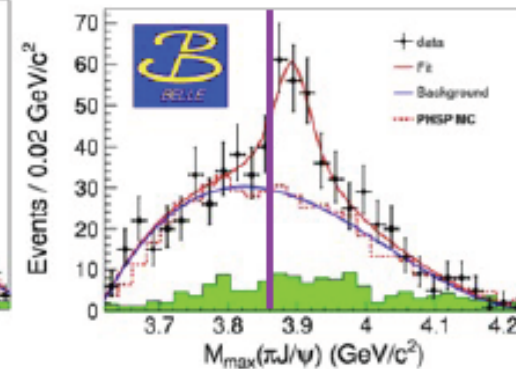
• $\pi J/\psi$ invariant mass ($m_\pi=410\text{MeV}$)



• $e^+e^- \rightarrow \pi(\pi J/\psi)$ @ 4.26 GeV

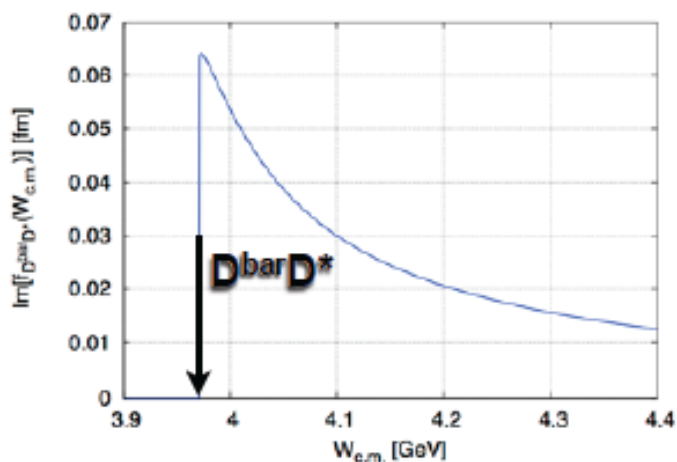


BESIII Coll., PRL 110 (2013).

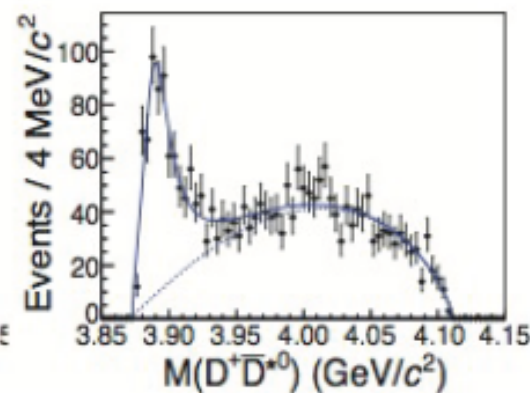
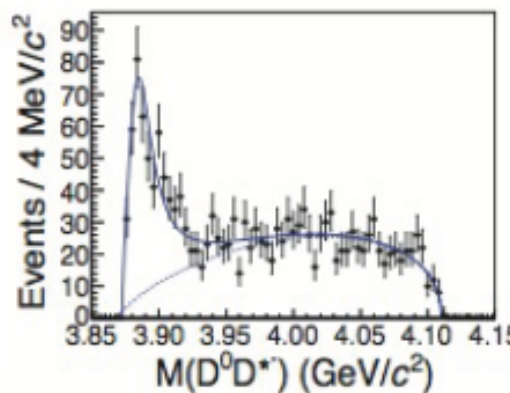


Belle Coll., PRL 110 (2013).

• $D\bar{D}^*$ invariant mass ($m_\pi=410\text{MeV}$)



• $e^+e^- \rightarrow \pi^{+/-} (D\bar{D}^*)_{+/-}$



BESIII Coll., PRL 112 (2014).

Y. Ikeda et al. (HAL QCD Coll.)

c.f. S. Prelovsek et al., arXiv:1405.7623

No Zc found < 4.2 GeV in 13 lowest states on lattice

Summary

- Charmed tetraquark states from HAL QCD method
 - Coupled channel potential is a powerful tool to probe (exotic) states in the continuum
- Tcc-tetraquark
 - Tcc ($J^P=1^+, I=0$) strong attraction; qualitatively consistent w/ diquark-picture
 - Tbb ($J^P=1^+, I=0$) : good candidate for a bound state
- Zc(3900)
 - Strong coupling exists between $D^{\text{bar}} D^*$ and $\pi J/\psi$, while heavy quark sym suppresses $\rho \eta_c - \pi J/\psi$
 - Possibility of complex pole ? Search in progress
- Outlook
 - Challenge toward Physical point

