
Some Issues in Quarkonium Production

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- Brief Review of NRQCD Factorization
- Status of a Proof of NRQCD Factorization
- Why is large p_T important?
- Why isn't the CSM a viable description of production?
- What are LP and NLP factorization (fragmentation)?
- What happens in NRQCD at LO in α_s ?
- What happens in NRQCD at NLO in α_s ?
- Why are there $3\frac{1}{2}$ sets of NLO predictions?
- What happens to J/ψ polarization in NLO?
- What do we expect beyond NLO in α_s ?
- How do we go beyond NLO in α_s ?
- Conclusions and Outlook

Brief Review of NRQCD Factorization

- **NRQCD Factorization Conjecture** (GTB, Braaten, Lepage (1995)):

The inclusive cross section for producing a quarkonium at large momentum transfer (p_T) can be written as

$$\sigma(H) = \sum_n F_n(\Lambda) \langle 0 | \mathcal{O}_n^H(\Lambda) | 0 \rangle.$$

- The $F_n(\Lambda)$ are the “short-distance” coefficients (SDCs).
 - The SDCs are essentially the partonic cross sections to make a $Q\bar{Q}$ pair convolved with the parton distributions.
- The $\langle 0 | \mathcal{O}_n^H(\Lambda) | 0 \rangle$ are the NRQCD long-distance matrix elements (LDMEs).
 - The LDMEs are the probability for a $Q\bar{Q}$ pair to evolve into a heavy quarkonium.

- The SDCs depend on the production process.
They can be calculated in QCD perturbation theory.
- The LDMEs are nonperturbative, but they are conjectured to be universal (process independent).
- The LDMEs have a known scaling with the heavy-quark velocity v .
 - $v^2 \approx 0.23$ for the J/ψ . $v^2 \approx 0.1$ for the $\Upsilon(1S)$.
 - The sum in the factorization formula is a “ v expansion.”
- In phenomenology, the v expansion in the factorization formula is truncated at a particular order in v .
- A key feature of NRQCD factorization: Quarkonium production can occur through color-octet, as well as color-singlet, $Q\bar{Q}$ states.
 - The color-singlet production LDMEs are simply related to color-singlet decay LDMEs.
 - The color-octet LDMEs must be determined from fits to measured production cross sections.
- If we drop all of the color-octet contributions and retain only the leading color-singlet contribution, then we have the color-singlet model (CSM).

- The current phenomenology of production of S -wave quarkonia (J/ψ , $\psi(2S)$, and $\Upsilon(nS)$) makes use of LDMEs through relative order v^4 :

$$\begin{aligned}\langle \mathcal{O}^H(^3S_1^{[1]}) \rangle & (O(v^0)), \\ \langle \mathcal{O}^H(^1S_0^{[8]}) \rangle & (O(v^3)), \\ \langle \mathcal{O}^H(^3S_1^{[8]}) \rangle & (O(v^4)), \\ \langle \mathcal{O}^H(^3P_J^{[8]}) \rangle & (O(v^4)).\end{aligned}$$

- Calculations show that the $^3S_1^{[1]}$ contributions are negligible.
- The $\langle \mathcal{O}^H(^3P_J^{[8]}) \rangle$ ($J = 0, 1, 2$) are related by the heavy-quark spin symmetry.
- Three color-octet LDMEs need to be determined phenomenologically for each state.

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- The current phenomenology of production of P -wave quarkonia (χ_{cJ}) makes use of LDMEs through relative order v^4 :

$$\begin{aligned}\langle \mathcal{O}^H(^3P_J^{[1]}) \rangle & (O(v^4)), \\ \langle \mathcal{O}^H(^3S_1^{[8]}) \rangle & (O(v^4)).\end{aligned}$$

- The $\langle \mathcal{O}^H(^3P_J^{[1]}) \rangle$ ($J = 0, 1, 2$) are related by the heavy-quark spin symmetry. They can be determined from potential models.
- Only one LDME ($\langle \mathcal{O}^H(^3S_1^{[8]}) \rangle$) has to be determined from phenomenology.

Status of a Proof of NRQCD Factorization

- Nayak, Qiu, Sterman (2005, 2006): Factorization holds through NNLO, up to corrections of relative order m_Q^2/p_T^2 .
- It is not known if this result generalizes to higher orders in α_s .
- An all-orders proof is essential because soft gluons can violate factorization, and the α_s that is associated with soft gluons is not small.
- In the absence of further theoretical progress, we must rely on experiment to prove or to disprove NRQCD factorization.

Why is large p_T important?

- Existing proofs of factorization for light hadrons all require p_T significantly greater than the hadron masses.
 - “Higher-twist” power corrections of order m_H^2/p_T^2 get out of control when $p_T \sim m_H$.
 - There are known violations of factorization at order m_H^4/p_T^4 .
- Phenomenologically, Drell-Yan factorization doesn't work until $p_T \geq 3m_H$.
- Suggests that we should require $p_T \geq 3m_{\text{quarkonium}}$.

Why isn't the CSM a viable description of production?

- The CSM is theoretically inconsistent.
 - Uncanceled infrared divergences at leading order in v for production of P -wave states and at higher orders in v for other states.
- The CSM predictions in NLO fall well below the observed cross sections.

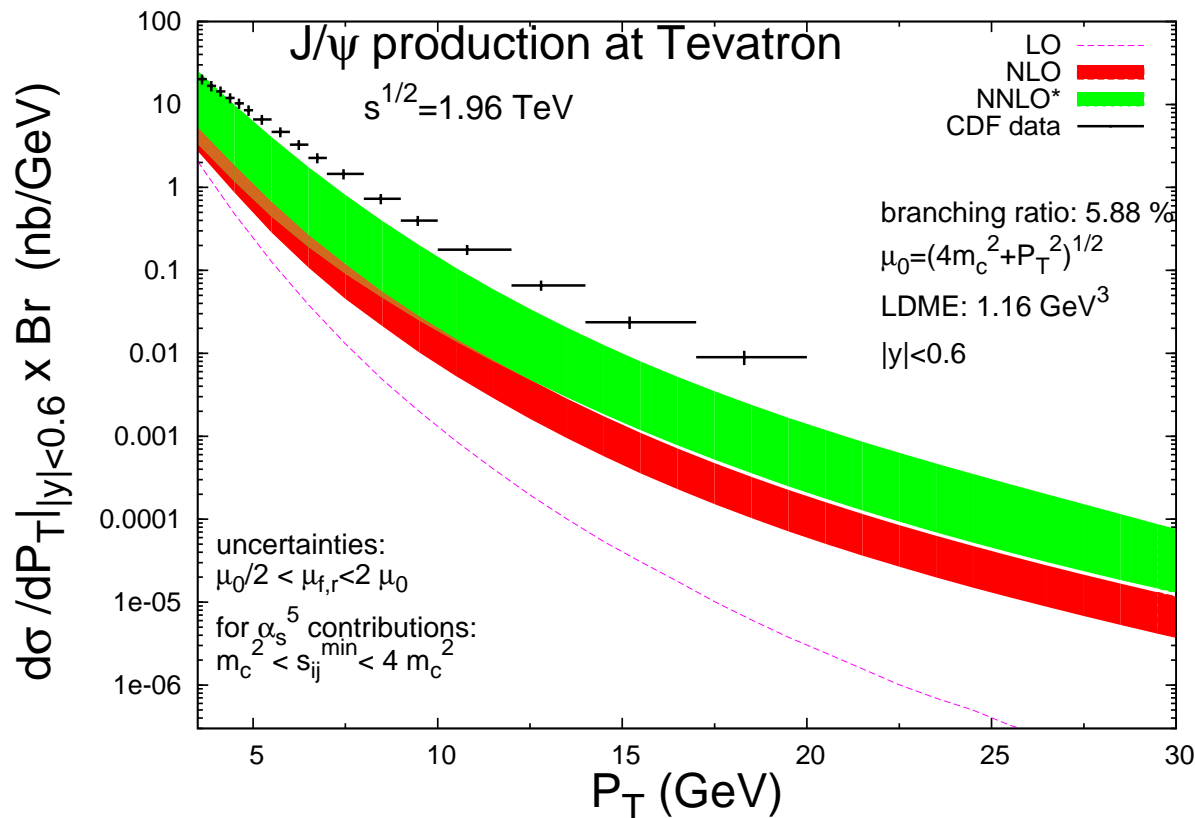


Figure courtesy of Pierre Artoisenet.

- The NNLO* calculation is an estimate based on real-emission contributions only.
- When the virtual contributions are added, the true NNLO contribution will likely be smaller.

- The CSM predictions in NLO do not describe the polarization data.

□ / • CDF data: Run I / II Helicity frame

..... CS, LO

— CS, NLO

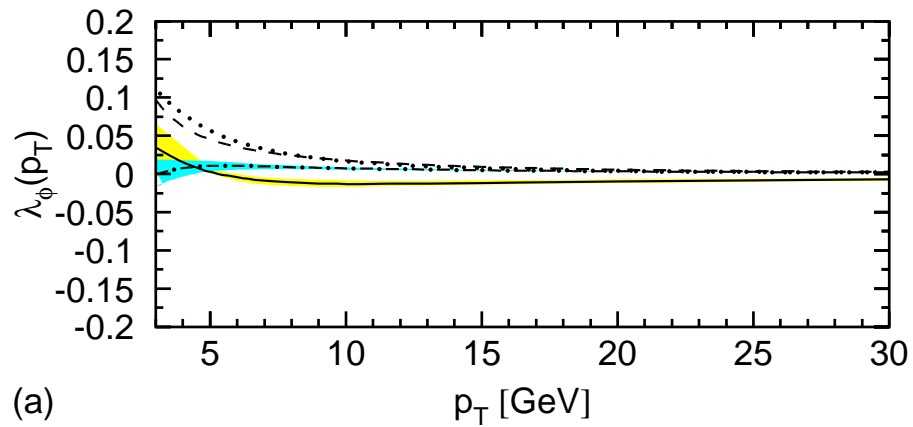
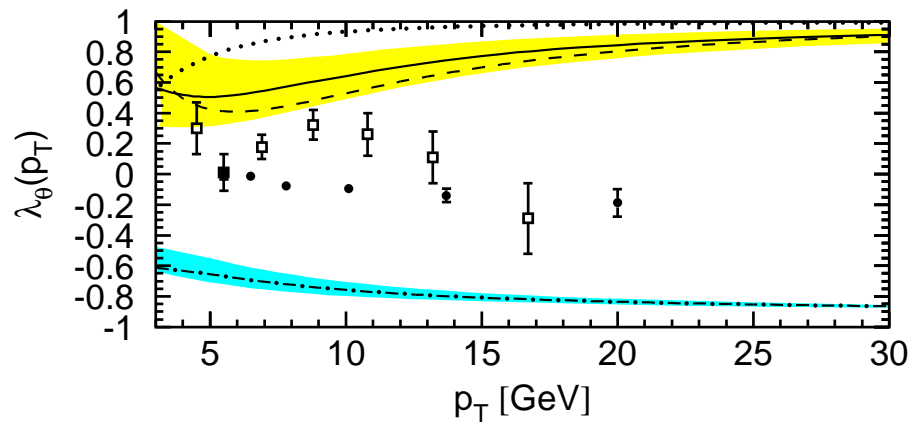
- - - CS+CO, LO

— CS+CO, NLO

$|y| < 0.6$

$\sqrt{s} = 1.96 \text{ TeV}$

$p\bar{p} \rightarrow J/\psi + X$



(a)

What are LP and NLP factorization (fragmentation)?

- **Leading power (LP) fragmentation:**

(Collins, Soper (1982))

$$d\sigma/dp_T^2 \sim 1/p_T^4$$

Convolution of $\text{parton}_1 + \text{parton}_2 \rightarrow \text{parton}_3 + X$ **scattering with**
 $\text{parton}_3 \rightarrow \text{quarkonium}$ **fragmentation functions.**

- **Next-to-leading power (NLP) fragmentation:**

(Kang, Qiu, and Sterman (2011); Fleming, Leibovich, Mehen, Rothstein (2012))

$$d\sigma/dp_T^2 \sim m_Q^2/p_T^6$$

Convolution of $\text{parton}_1 + \text{parton}_2 \rightarrow Q\bar{Q} + X$ **scattering with**
 $Q\bar{Q} \rightarrow \text{quarkonium}$ **fragmentation functions.**

- **Believed to hold to all orders in perturbation theory up to corrections of order**
 m_Q^4/p_T^8 .

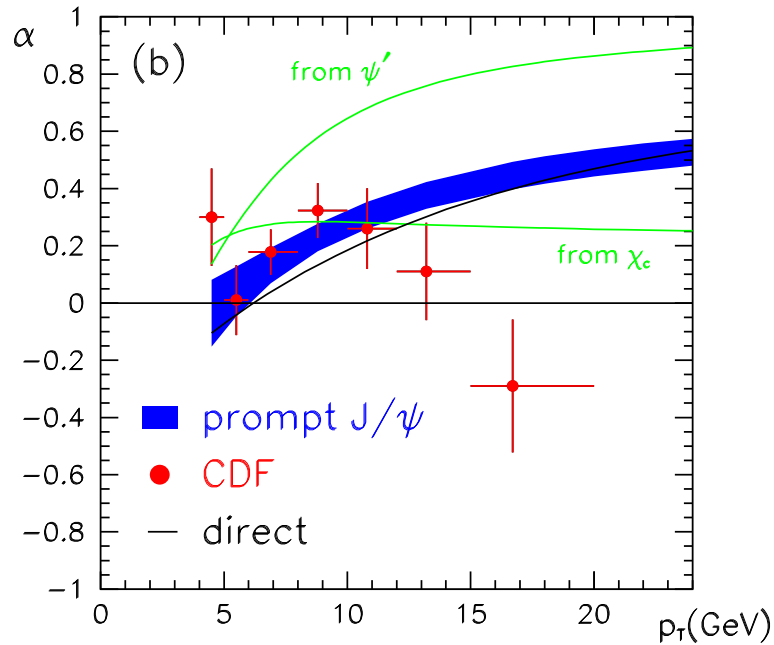
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- Not very predictive by itself because the nonperturbative fragmentation functions are unknown.
 - If NRQCD factorization holds, then the fragmentation functions can be written as a sum of NRQCD LDMEs times perturbatively calculable short-distance coefficients.
 - Then, the fragmentation approach provides powerful a way to identify and compute the contributions (LP and NLP) that are most important at high p_T .
 - Much simpler than a full perturbative calculation at any given order in α_s .
 - Also provides a framework in which to resum logs of p_T^2/m_Q^2 .

What happens in NRQCD at LO in α_s ?

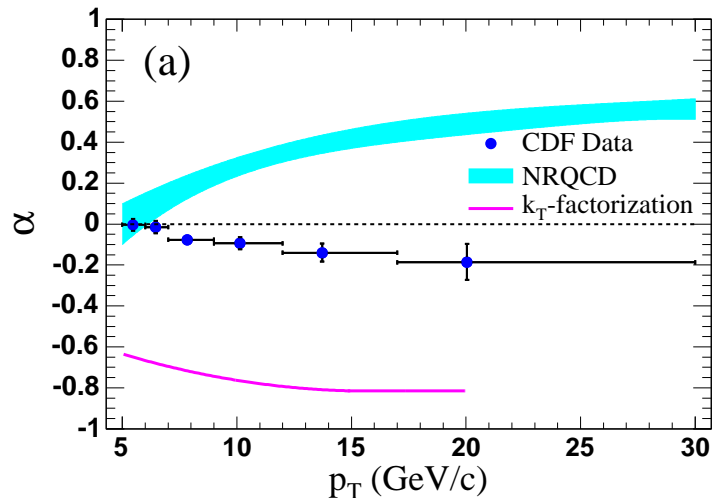
- Only the 3S_1 color-octet contribution has LP behavior in LO.
 - Dominates at large p_T .
- The 3S_1 color-octet contribution is transversely polarized.
- The prediction of large transverse polarization at large p_T is not borne out by the data.

J/ψ Polarization in LO

CDF Run I:



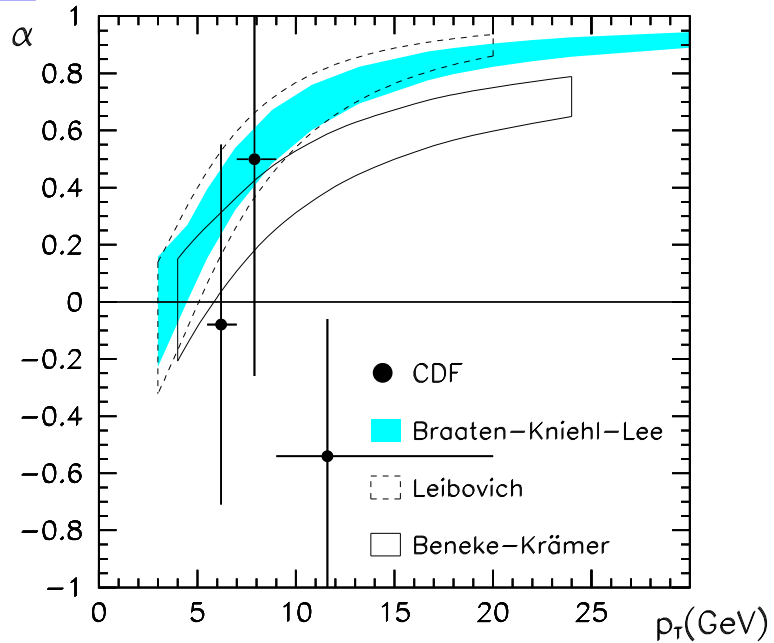
CDF Run II:



- LO NRQCD prediction shown (Braaten, Kniehl, Lee (1999)).
- Run I results are marginally compatible with the LO NRQCD prediction.
- The Run II results are incompatible with the LO NRQCD prediction.
- The Run I and Run II results are inconsistent with each other.

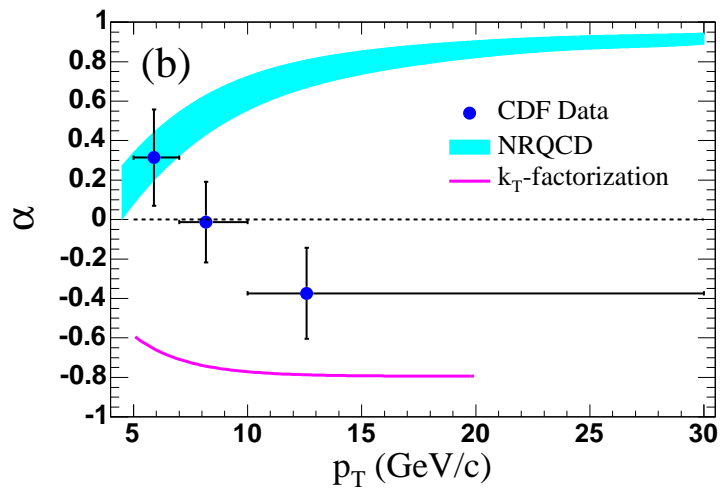
$\psi(2S)$ Polarization in LO

Run: I



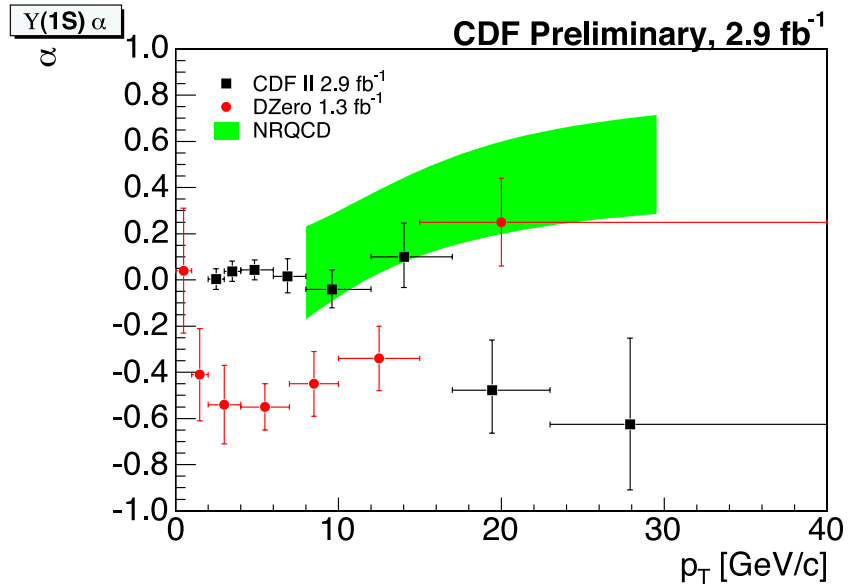
- The Run II data are incompatible with the LO NRQCD prediction.

Run: II

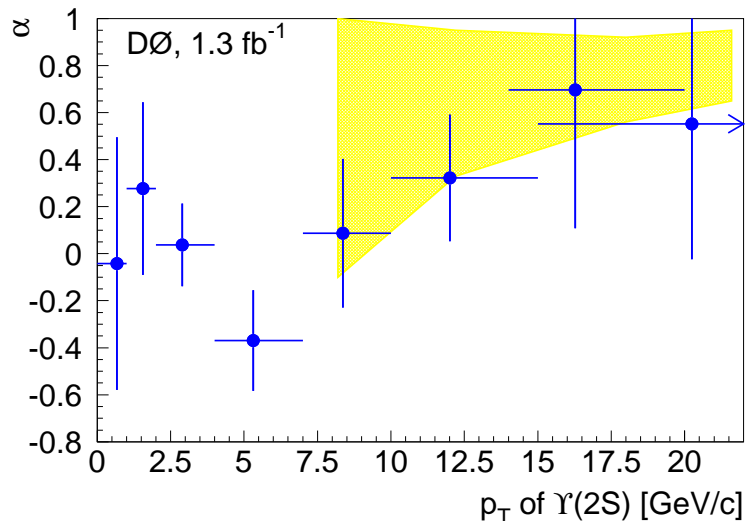


Υ Polarization in LO

$\Upsilon(1S)$ Polarization:

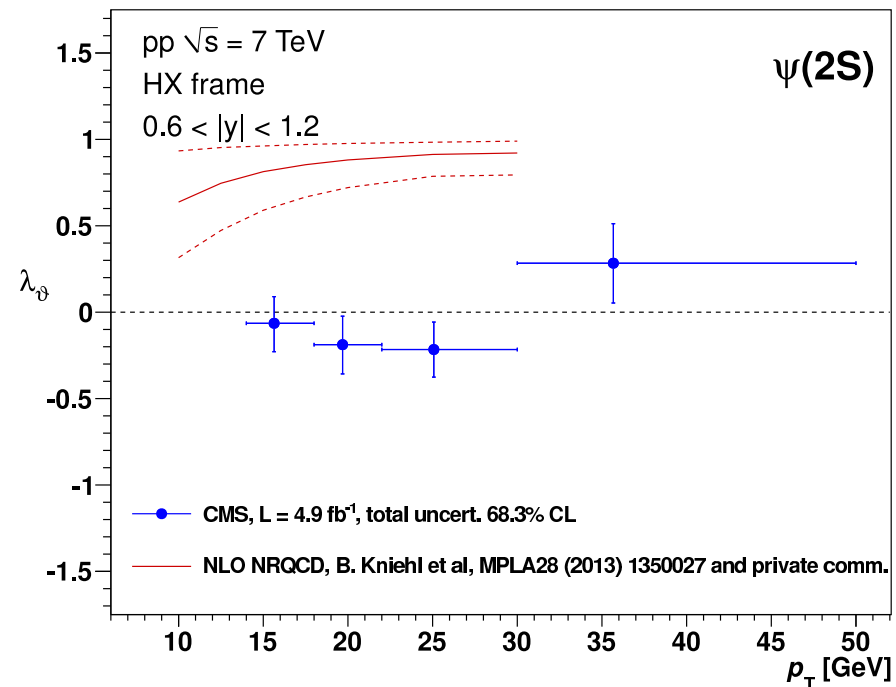
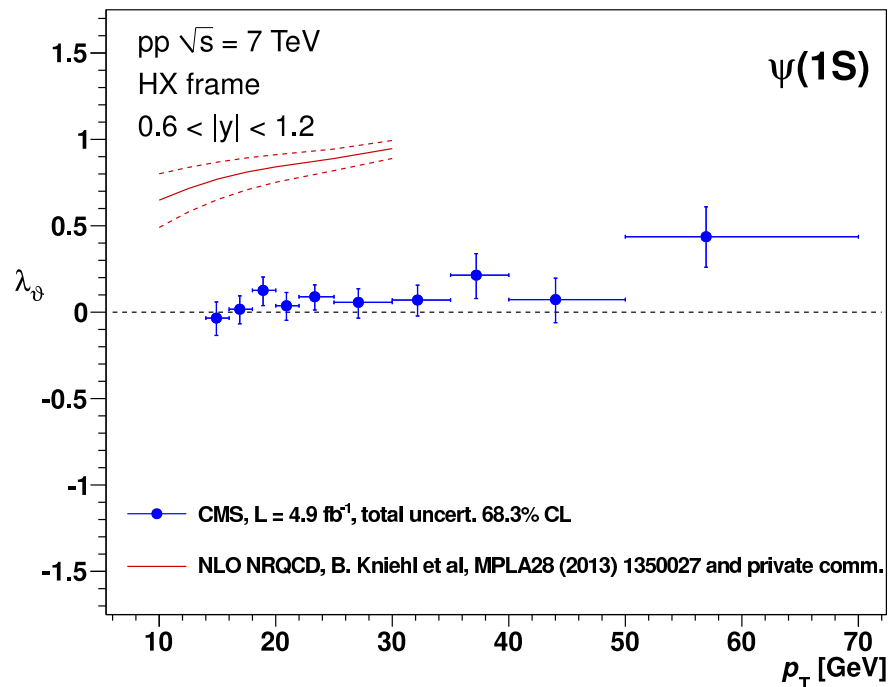


$\Upsilon(2S)$ Polarization:



- In the $\Upsilon(1S)$ case, the D0 results (red) are incompatible with the CDF results (black).
- Both the CDF and D0 results are incompatible with the LO NRQCD prediction (green) (Braaten and Lee (2000)), but in different regions of p_T .
- In the $\Upsilon(2S)$ case, the theoretical and experimental error bars are too large to make a stringent test.

- It was thought that these discrepancies between theory and experiment might not be definitive because
 - there are inconsistencies in the experimental data,
 - p_T might not be high enough for factorization to work.
- These ideas were blown out of the water by the CMS polarization measurement.



What happens in NRQCD at NLO in α_s ?

- There is a large k factor ~ -10 in the 3P_J color-octet channel.
- On the other hand, NLO corrections to the 3S_1 and 1S_0 color-octet channels are small.

Explanation

- Enhancements at high p_T from LP behavior can overcome a power of α_s .
- The color-octet 3P_J channel receives a large correction in NLO because it first shows LP behavior in NLO (gluon fragmentation).
- The color-octet 3S_1 channel receives a small correction in NLO because it already has LP behavior in LO (gluon fragmentation).
- The color-octet 1S_0 channel first shows LP behavior in NLO (gluon fragmentation).
But the NLO correction is numerically small because the gluon-fragmentation contribution happens to be small.
- In NLO, the 3P_J and 3S_1 color-octet channels can cancel.
 1S_0 dominance would lead to a prediction of small polarization.

Why are there $3\frac{1}{2}$ sets of NLO predictions?

- Three groups have carried out complete NLO calculations.
 - PKU group (Kuang-Ta Chao's group): Ma, Wang, Chao, Shao, Wang, Zhang
 - Hamburg group (Bernd Kniehl's group): Butenschön, Kniehl
 - IHEP group (Jianxiong Wang's group): Gong, Wan, Wang, Zhang
- All three groups agree on the SDCs for hadroproduction.
- However, they extract very different NRQCD LDMEs and make different predictions because of different assumptions about the data used in the fits.
- The PKU group (2010) fits the CDF J/ψ data for $p_T > 7$ GeV. They were able to determine only 2 linear combinations of LDMEs unambiguously:

$$M_{0,r_0} = \langle O^\psi(^1S_0^{[8]}) \rangle + (r_0/m_c^2) \langle O^\psi(^3P_0^{[8]}) \rangle = (7.4 \pm 1.9) \times 10^{-2} \text{ GeV}^3,$$

$$M_{1,r_1} = \langle O^\psi(^3S_1^{[8]}) \rangle + (r_1/m_c^2) \langle O^\psi(^3P_0^{[8]}) \rangle = (0.05 \pm 0.02) \times 10^{-2} \text{ GeV}^3.$$

$$r_0 = 3.9 \text{ and } r_1 = -0.56.$$

- The Hamburg group (2011) determined all 3 color-octet LDMEs by making a **global fit** to data with $p_T > 3$ GeV from the Tevatron, LHC, RHIC, HERA, LEP II, KEKB.

- They made use of their computations of NLO corrections to $p\bar{p}$, pp , ep , $\gamma\gamma$, and e^+e^- production.
- Their LDMEs are very different from those of the PKU group:

$$M_{0,r_0} = (2.17 \pm 0.56) \times 10^{-2} \text{ GeV}^3,$$

$$M_{1,r_1} = (0.62 \pm 0.08) \times 10^{-2} \text{ GeV}^3.$$

- The IHEP group (2012) fit the CDF J/ψ , $\psi(2S)$, and χ_{cJ} data for $p_T > 7$ GeV.

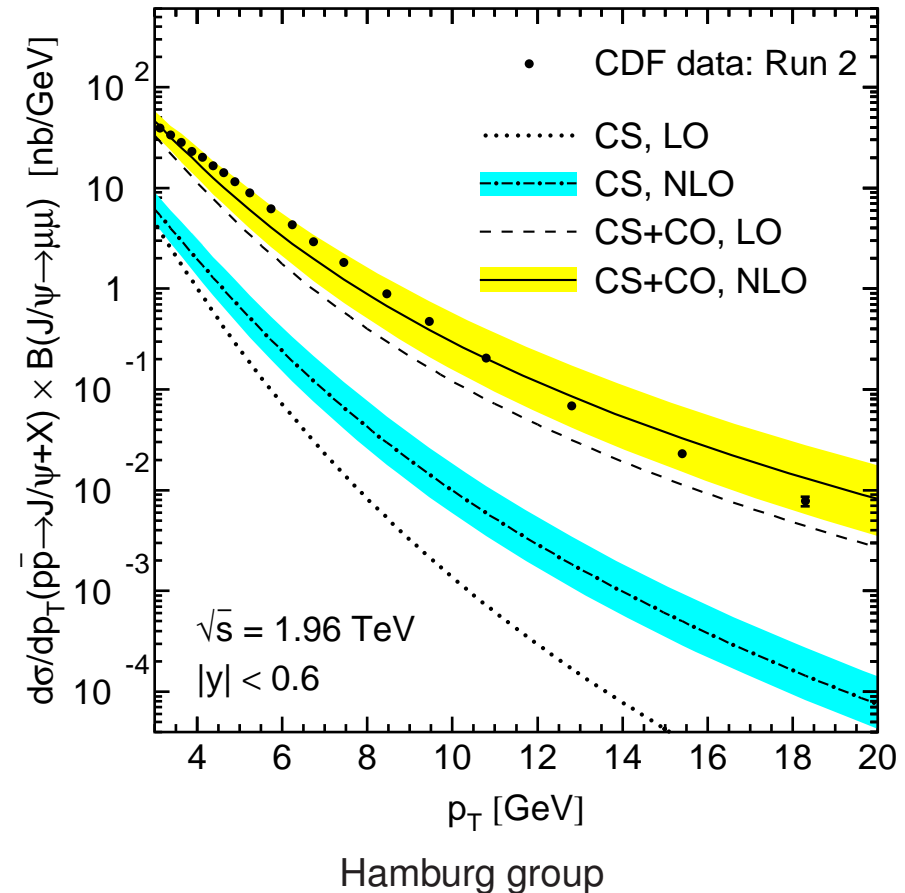
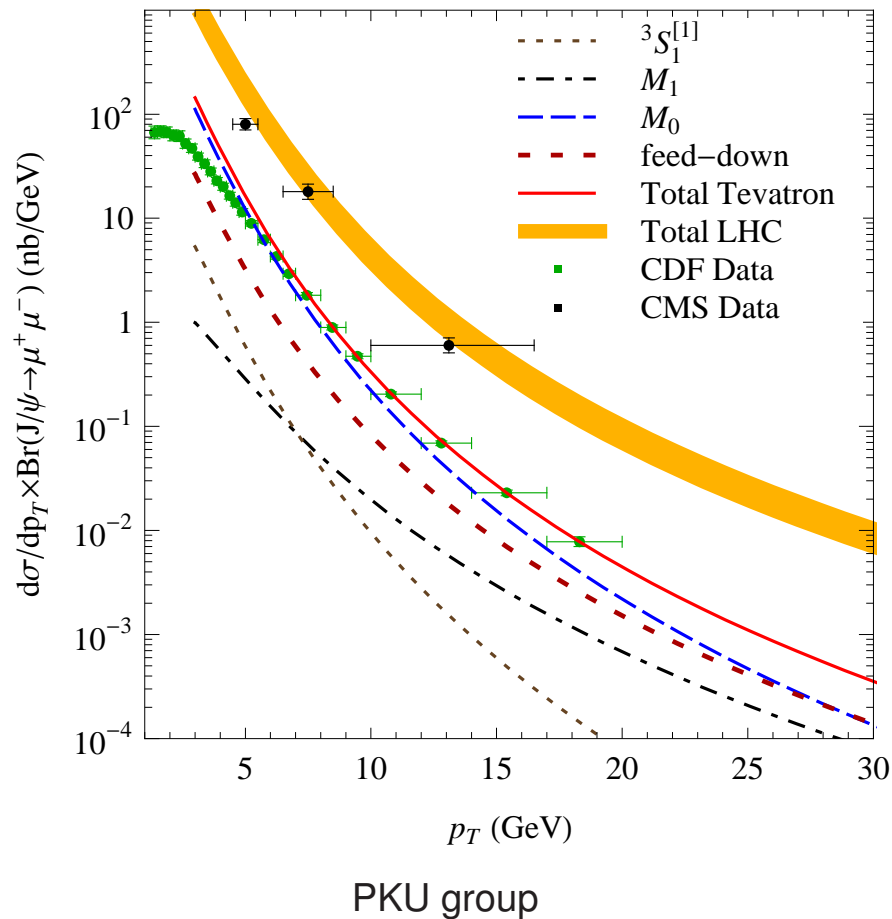
- They computed feeddown from $\psi(2S)$ and χ_{cJ} states and included it in their fit.
- They were able to determine all 3 color-octet LDMEs.
- They obtained a quality of fit and a result for the LDME linear combinations that is similar to that of the PKU group:

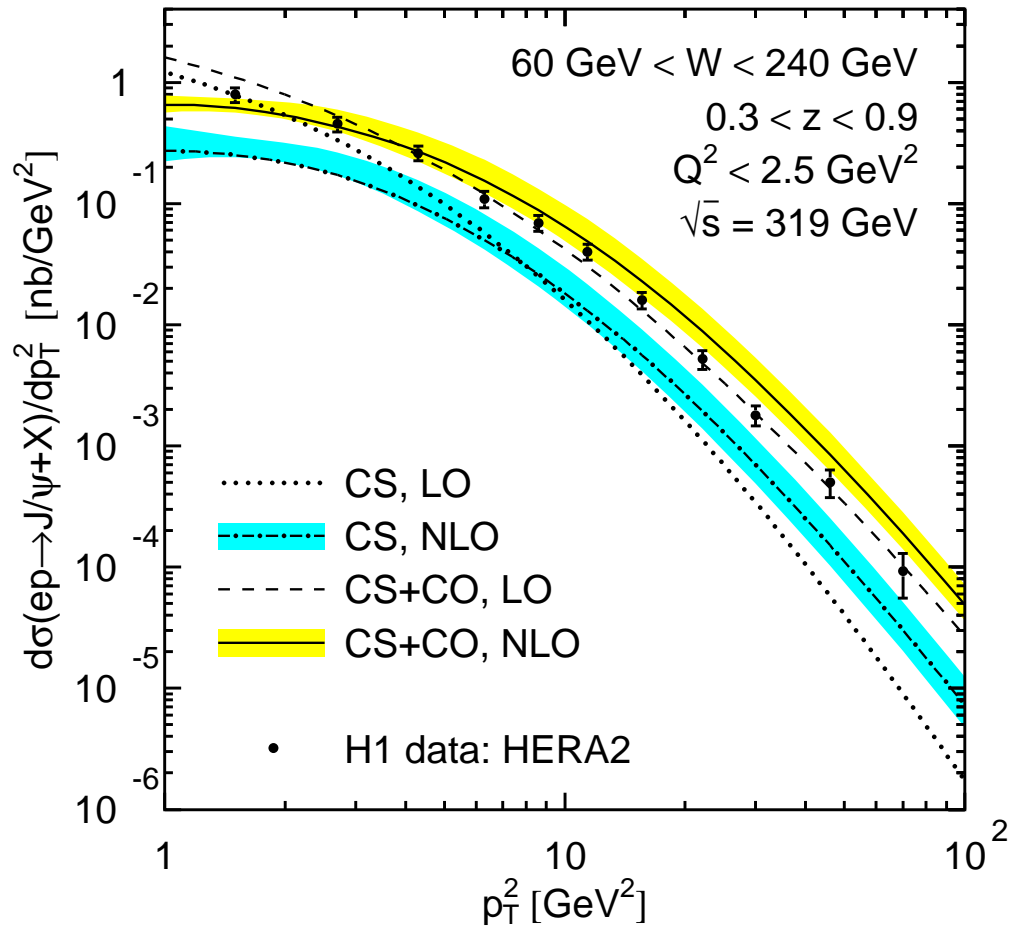
$$M_{0,r_0} = (6.00 \pm 0.98) \times 10^{-2} \text{ GeV}^3,$$

$$M_{1,r_1} = (0.07 \pm 0.02) \times 10^{-2} \text{ GeV}^3.$$

-
- Most of the difference between the Hamburg-group fit and the others comes from the use of HERA (H1 (2002, 2005)) data.
 - Most of the HERA data lie at $p_T \lesssim 3$ GeV.
 - Does NRQCD factorization hold at such low values of p_T ?

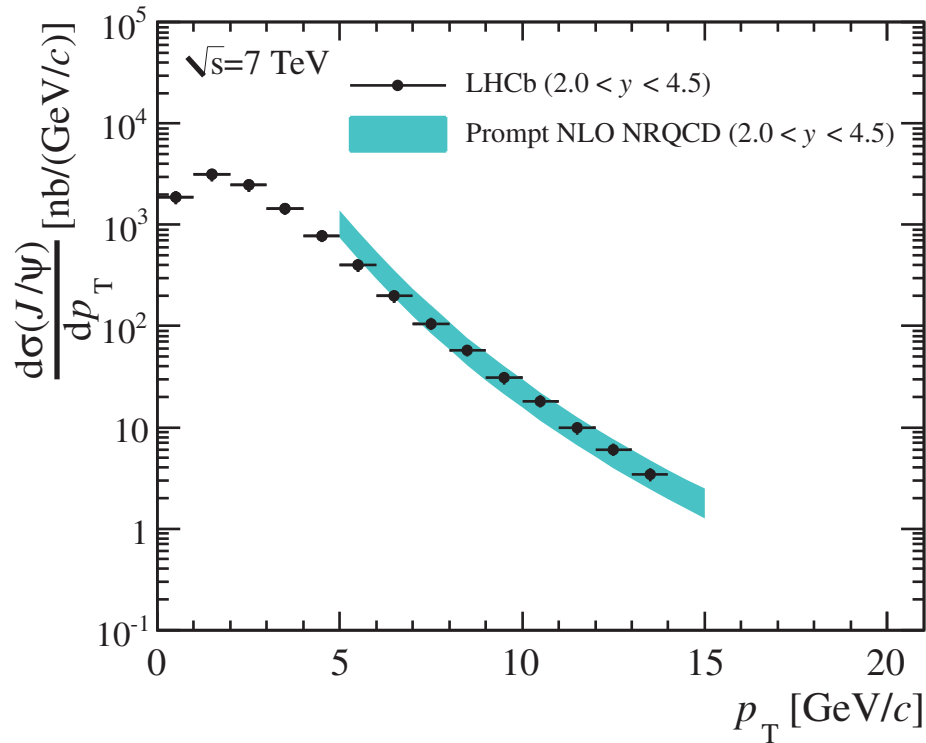
- Although the Hamburg-group fits agree with the data, within uncertainties, there are tensions in the shapes.
- The shape of the PKU-group fit agrees with the CDF data better than the shape of the Hamburg-group (2011) global fit.



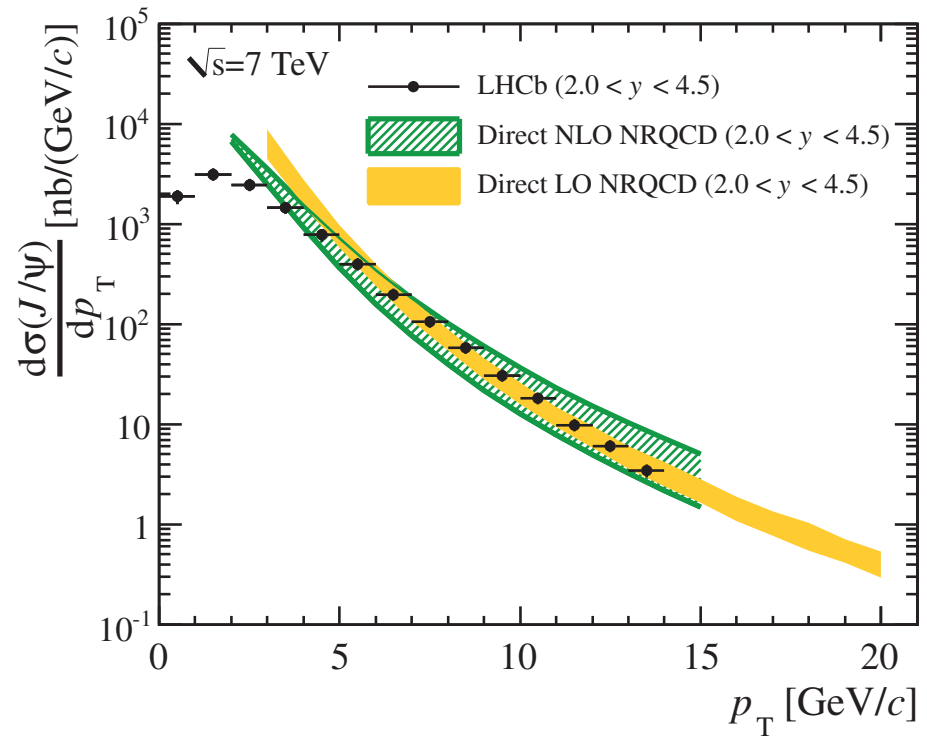


There is a slight discrepancy in shape between the Hamburg-group (2011) global fit and the H1 data.

- There is also a slight discrepancy in shape between the LHCb data and the Hamburg-group (2010) NLO fit to the Tevatron and HERA data.

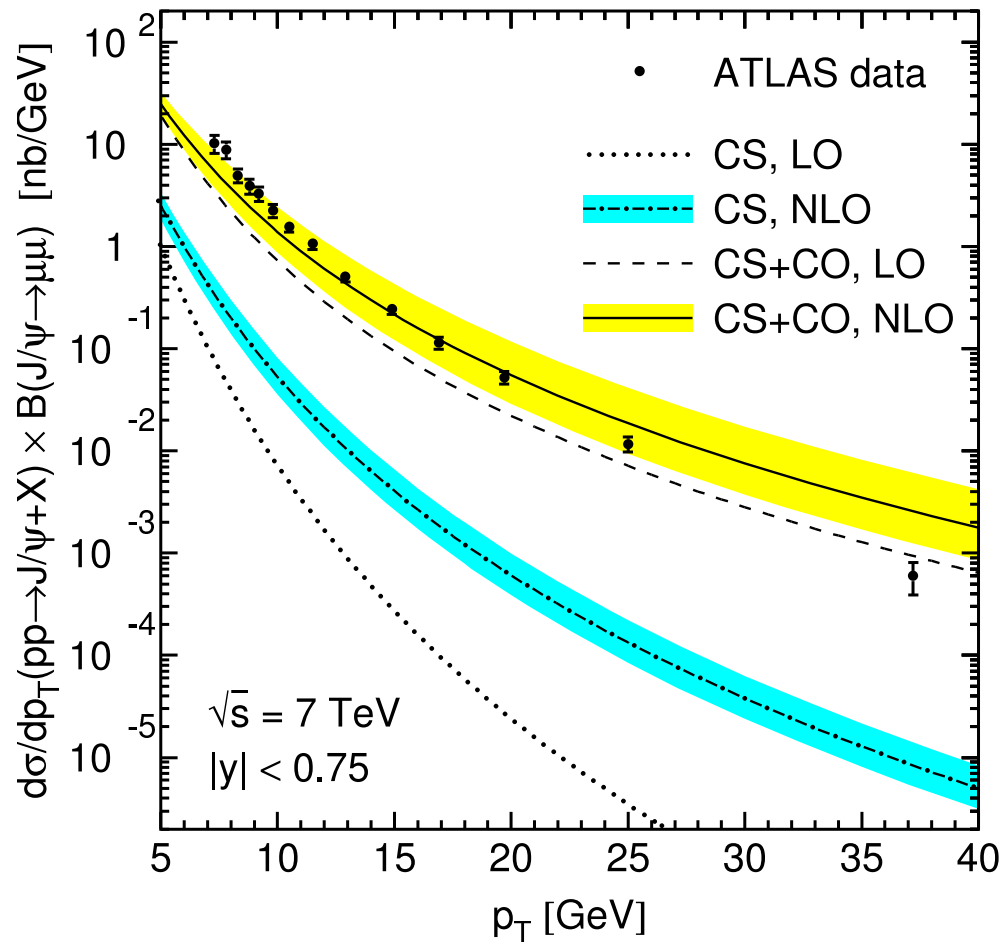


PKU group



Hamburg group

- The shape discrepancy between the Hamburg-group prediction and the data becomes more apparent at high p_T .



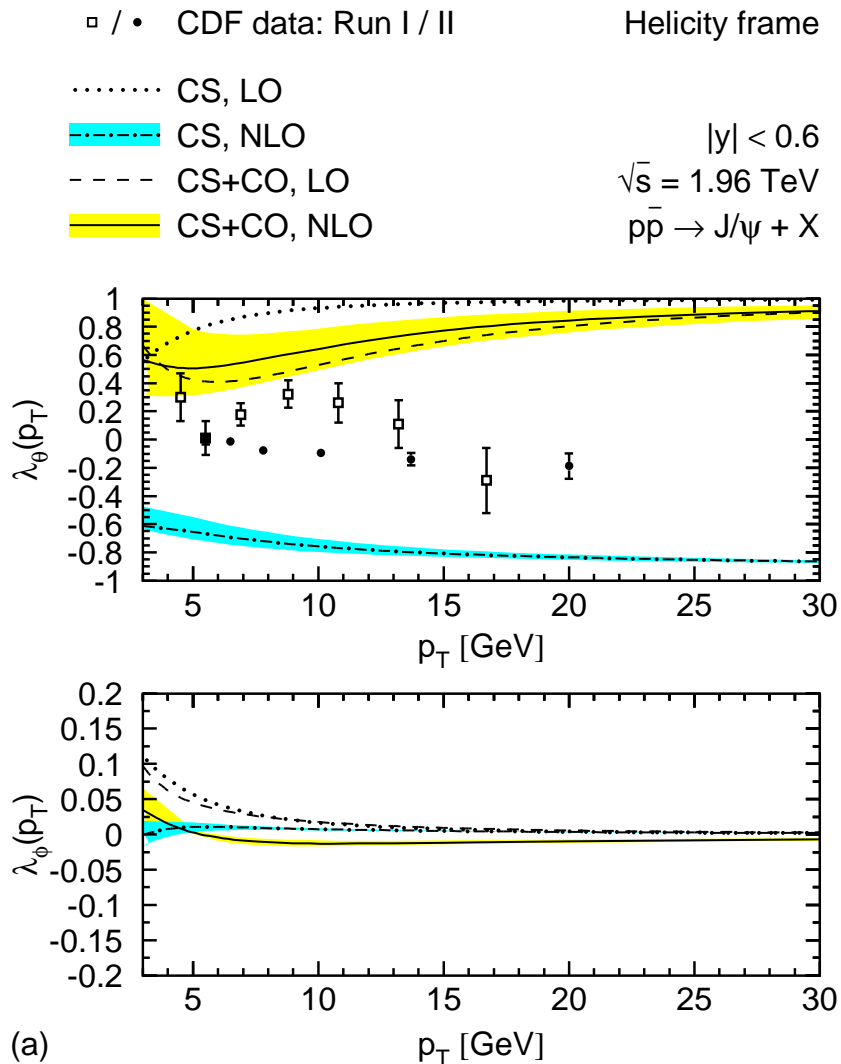
- ATLAS (2011) data.
- Not included in the Hamburg-group global fit.

- All of this suggests that NRQCD factorization may not work until p_T is much greater than the quarkonium mass.
- See Faccioli, Knünz, Lourenço, Seixas, Wöhri (2014) and Carlos Lourenço's talk for further details.

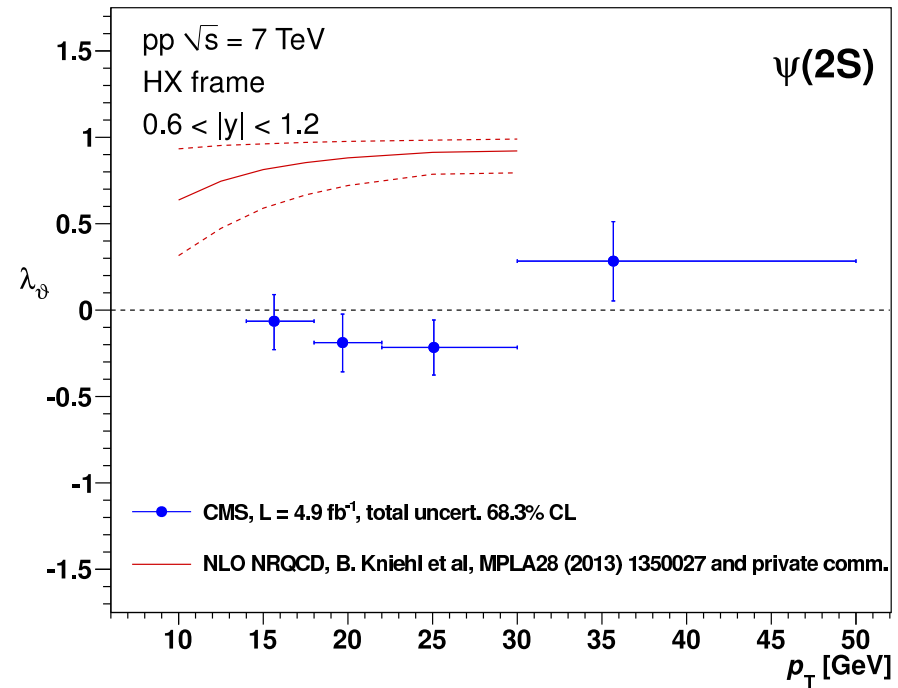
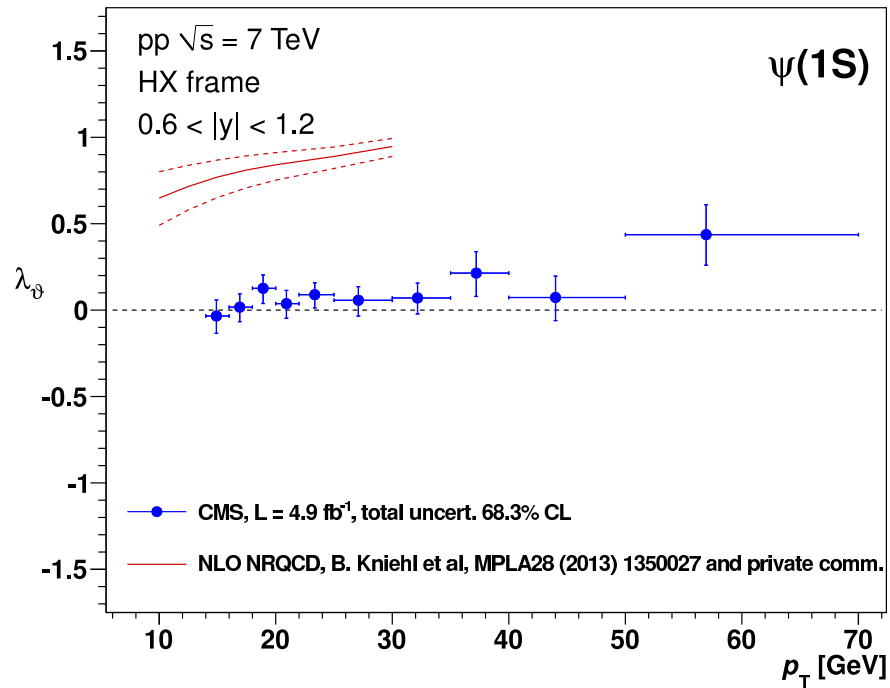
What happens to J/ψ polarization in NLO?

- In NLO, large corrections to the 3P_J color-octet channel at high p_T make it competitive with the 3S_1 color-octet channel.
- At high p_T , the 3P_J color-octet channel is mostly transversely polarized.
- In NLO, the transverse polarization from the 3P_J color-octet channel could cancel the transverse polarization from the 3S_1 color-octet channel.

Hamburg group (2011): NLO Prediction for J/ψ Polarization



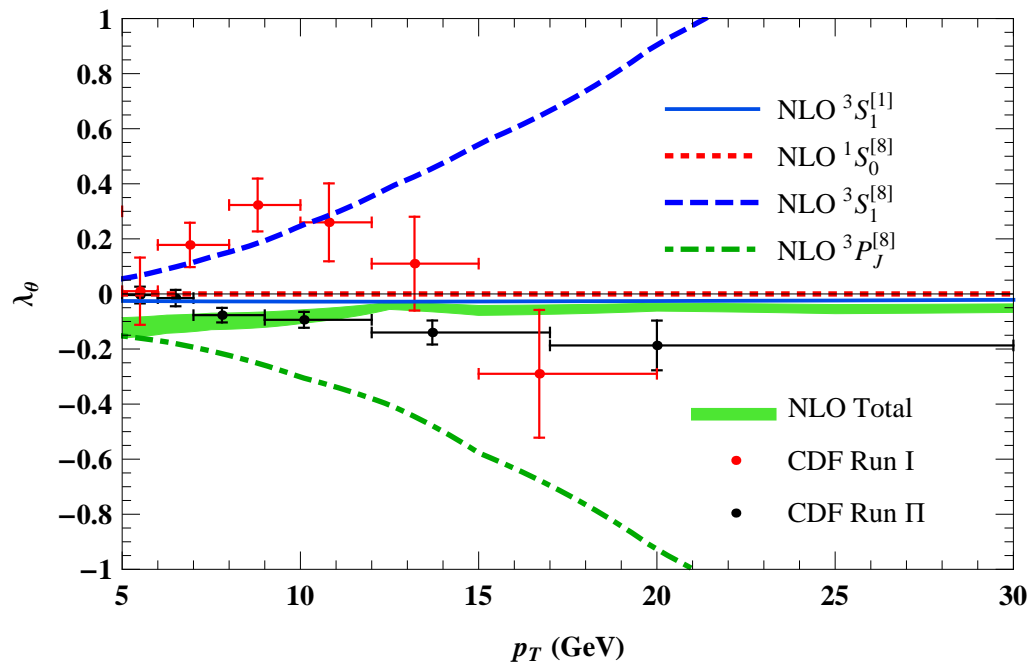
- Uses the Hamburg-group (2011) global fit of the J/ψ production cross sections.
- The contributions of the 3P_J and 3S_1 color-octet channels add to produce substantial polarization at high p_T .
- The prediction is in disagreement with the CDF data.



- The prediction based on the Hamburg-group (2011) global fit is also in disagreement with the CMS data.

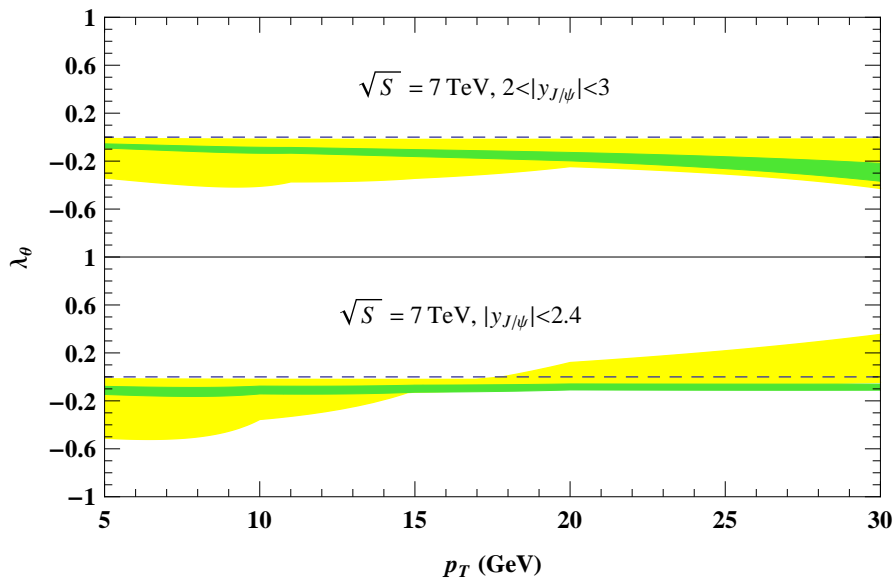
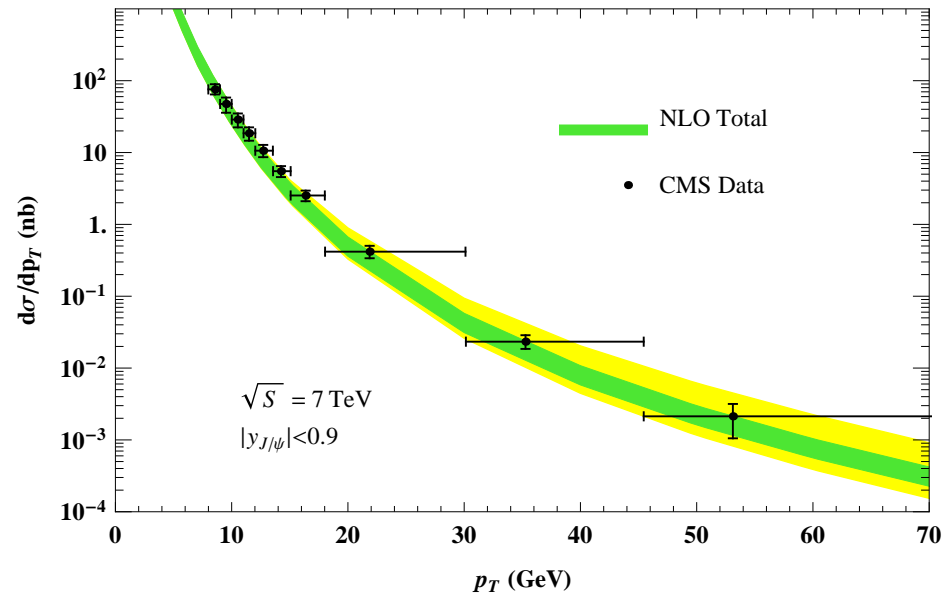
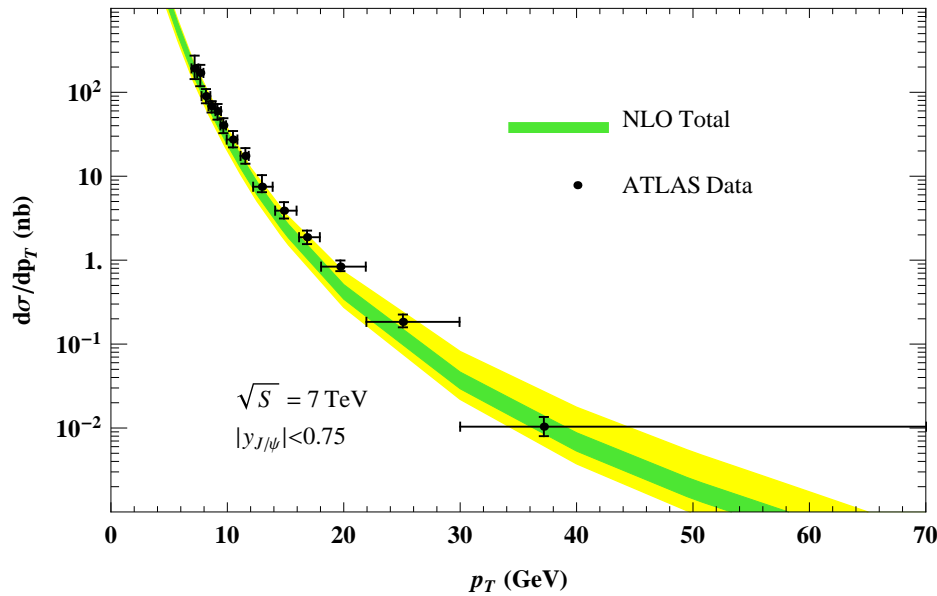
PKU group (2012): Fit to J/ψ Polarization in NLO

- This is the $\frac{1}{2}$ in the $3\frac{1}{2}$ sets of NLO predictions.
- Two LDME combinations are insufficient to predict the polarization.
- Fix all three LDMEs by including the CDF Run II J/ψ polarization measurement in the fit, as well as the CDF Run II measurements of $d\sigma/dp_T$.



- In this fit (solid green), the color-octet 3S_1 (dashed blue) and color-octet 3P_J (dashed green) contributions to the transverse polarization largely cancel.
- See Hee Sok Chung's talk and Carlos Lourenço's talk for more discussion of this cancellation.

- The PKU-group (2012) LDMEs still give reasonable predictions for the LHC p_T spectra.



- The PKU-group (2012) LDMEs predict a slightly longitudinal polarization at the LHC.

- However, the PKU-group LDMEs seem to be incompatible with the HERA data, even at $p_T = 7$ GeV.

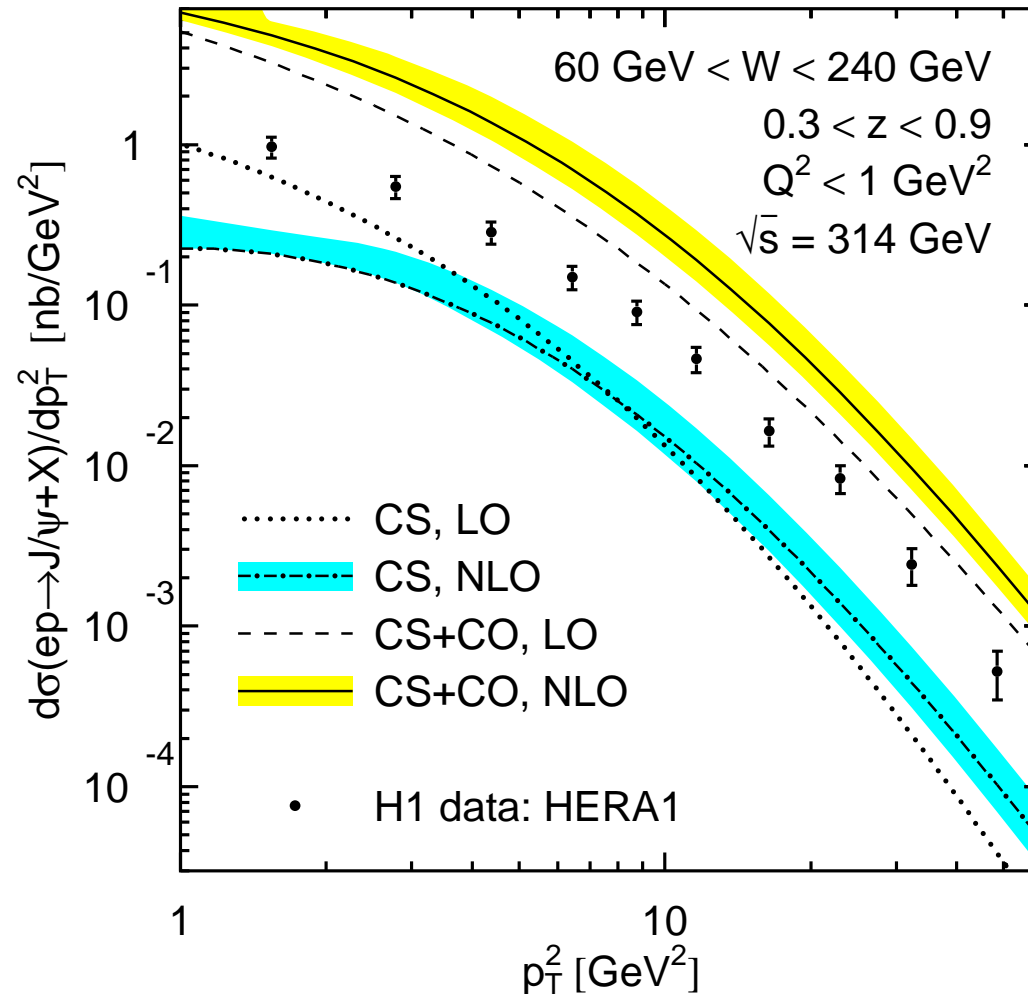


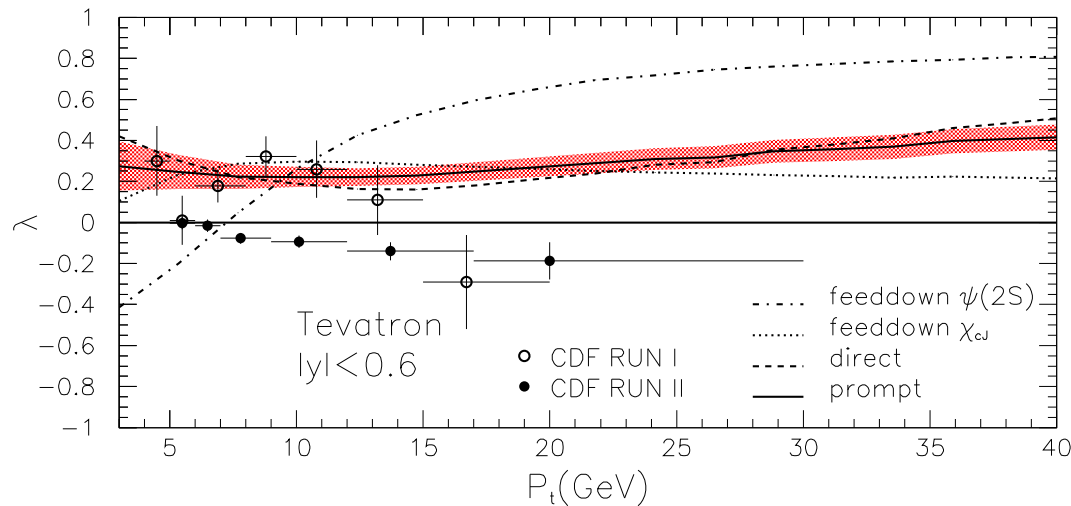
Figure courtesy of Mathias Butenschön.

- Is higher p_T needed in order to suppress non-factorizing contributions?

IHEP group (2013): NLO Prediction for J/ψ polarization

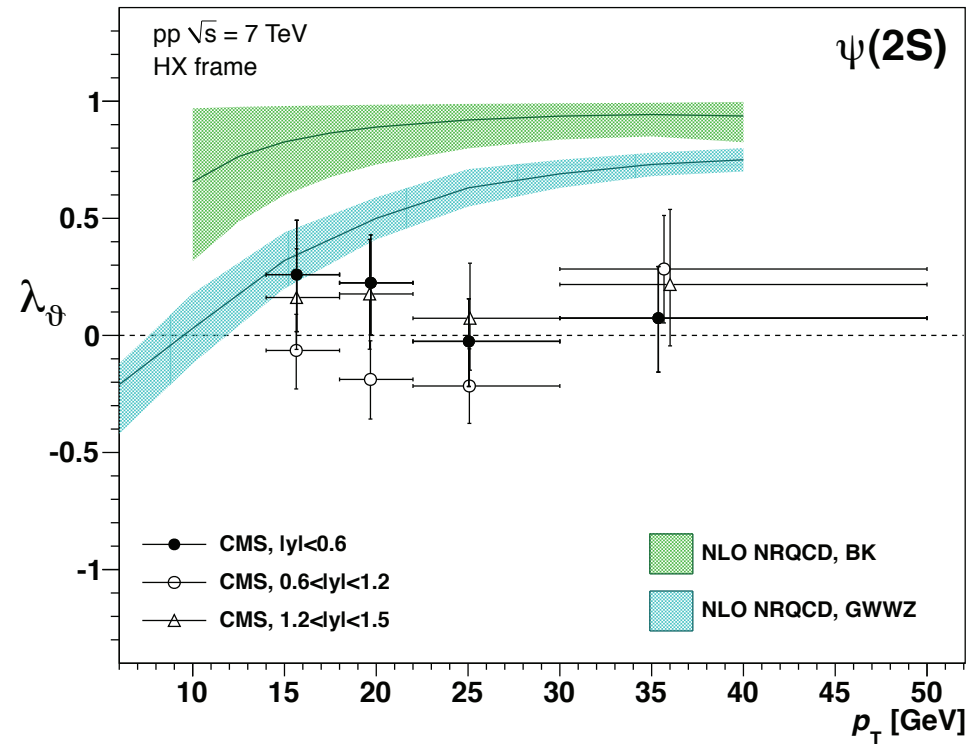
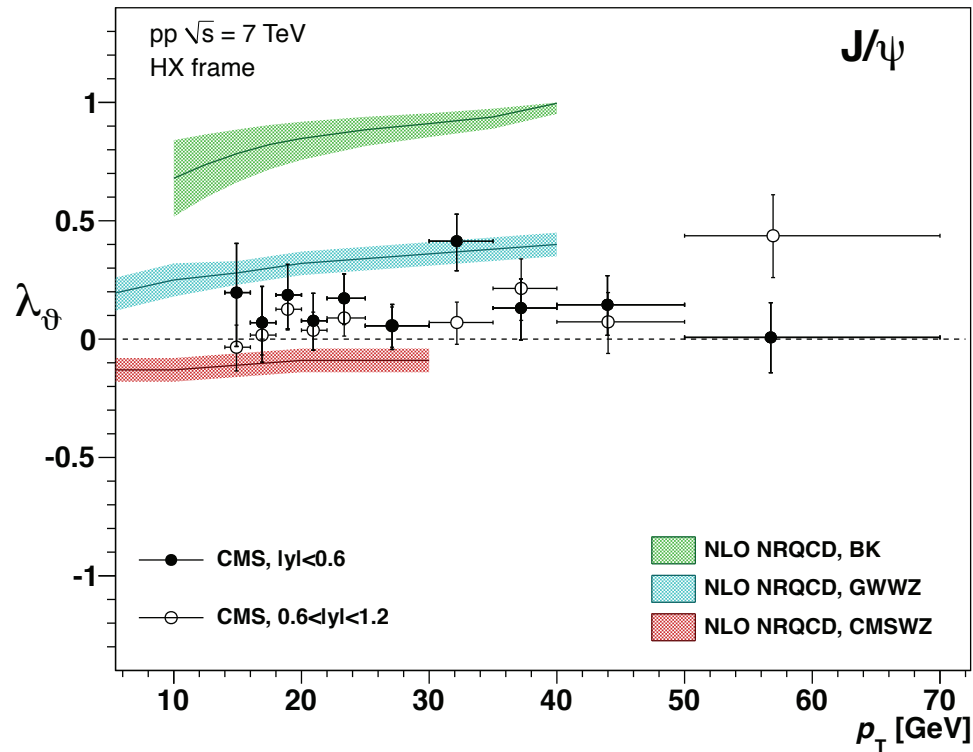
- Makes use of the LDMEs from a fit to the CDF (2005, 2009) and LHCb (2011, 2012) J/ψ , χ_{cJ} , and $\psi(2S)$ production cross sections.
- Effects of feeddown from χ_{cJ} and $\psi(2S)$ states calculated and included in fits and polarization predictions.

J/ψ Polarization at CDF:



- The prediction for CDF shows less transverse polarization than that of the Hamburg group.
- Agrees with the CDF Run I data, but not with the CDF Run II data.

J/ψ Polarization at CMS:



- The IHEP-group prediction (blue band) shows less transverse polarization than the Hamburg-group prediction (green band).
- Still in disagreement with the CMS data.

Conclusion about NLO Polarization

- NLO calculations either fail to make a polarization prediction (PKU) or make predictions that disagree with the data (Hamburg, IHEP).

What do we expect beyond NLO in α_s ?

- Since all three color-octet channels have LP behavior at NLO, we expect further corrections to have “normal” k factors: factor of two or less.
- Exception: The LO and NLO corrections in the 3P_J channel have opposite signs and cancel completely at $p_T \approx 7$ GeV.
 - NNLO corrections could be very important in this channel.

How do we go beyond NLO in α_s ?

- Full NNLO calculations are probably not feasible at present.
- Use LP and NLP factorization to simplify the calculation.
- LP and NLP factorization are only valid for p_T significantly larger than the quarkonium mass.
- Preliminary indications are that this approach, at last, leads to correct polarization predictions.

(GTB, Chung, Kim, Lee (2014))

See Hee Sok Chung's talk for details.

Conclusions and Outlook

- Theoretical ideas on production have undergone a major transformation in recent years.
- Much of this transformation has been driven by high-quality collider measurements.
 - The CSM does not agree with cross sections or polarization data.
 - NRQCD factorization at LO does not agree with the polarization data.
 - NRQCD factorization at NLO either does not agree with the polarization data or does not make a prediction.
- Interactions between theory and experiment have been essential.
 - There were surprises in the theory that likely would not have been discovered yet without experiment to keep theory honest.
 - Theory has motivated new measurements.

What do we need from theory?

- Calculations at NLO for more processes
 - double charmonium production
 - associated charmonium production (see Rong Li's talk)
- Ideas for additional experimental tests of theory
- Use of LP and NLP factorization to go to higher orders in α_s
(See the talks of Hong Zhang, Pierre Artoisenet, and Hee Sok Chung.)
- Extension of the theory to lower values of p_T
Resummation of logs of m_Q^2/p_T^2 might be useful.
- A proof or disproof of NRQCD factorization

What do we need from experiment?

- **Measurements at the highest accessible values of p_T in order to minimize effects of non-factorizing contributions**
It would be very useful for LHCb to make measurements at higher values of p_T .
- **Measurements at low p_T**
 - Will stimulate theoretical work to extend NRQCD factorization to low p_T .
 - Provide tests of theoretical frameworks that are orthogonal to collinear factorization.
(See Raju Venugopalan's talk.)
- **Measurements of direct-production cross sections and polarizations in order to avoid confusion from feeddown effects**
- **Measurements of all three polarization parameters in at least one polarization frame**
- **Measurements of χ_{cJ} cross sections and polarizations**
Particularly interesting because only one LDME must be determined from phenomenology.

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- **Measurements of $\Upsilon(nS)$ cross sections and polarizations**
Test NRQCD in a new regime in which v^2 is much smaller than for charmonium systems.
 - The v expansion should work much better for $\Upsilon(nS)$ than for J/ψ .
 - Different v^2 may mean different relative sizes of LDMEs.
 - $m_b \gg m_c$: Non-factorizing contributions may be suppressed only at higher values of p_T .
 - **Measurements of additional production processes**
 - jet substructure (see Tom Mehen's talk)
 - double-charmonium production
 - $J/\psi + Z$, $J/\psi + W^\pm$ (see James Catmore's talk)
 - $J/\psi + \text{jet}$

Outlook

- After more than 40 years of theoretical and experimental effort, we may, at last, be on the verge of understanding quarkonium production.
(See the talks of Hee Sok Chung and Carlos Lourenço.)
- The continuing interaction between theory and experiment will be crucial to the ultimate success of this effort.

Backup Slides

Why isn't the CEM viable as a description of production?

- The Color Evaporation Model (CEM) says that rate to produce a quarkonium is proportional to the rate to produce a $Q\bar{Q}$ pair, regardless of the quantum numbers of the $Q\bar{Q}$ pair or the quarkonium.
 - Not plausible in quantum field theory: Different $Q\bar{Q}$ states will have different overlaps with a given quarkonium state.
- The CEM requires an *ad hoc* modification, k_T smearing, in order to describe the data reasonably well.
- Nevertheless, because of its simplicity, the CEM is a useful way to describe production when a fundamental theory is not necessary, e.g. in studies of production in media.

Why isn't the k_T -Factorization Approach getting more attention?

- The k_T -Factorization Approach could, in principle, yield valid results. But...
- it relies on k_T -dependent parton distributions, which are poorly determined;
- calculations are usually carried out within the CSM;
- calculations are usually carried out only in LO.

$$\underline{e^+e^- \rightarrow J/\psi + X(\text{non-}c\bar{c})}$$

- Belle (2009):

$$\sigma(e^+e^- \rightarrow J/\psi + X(\text{non-}c\bar{c})) = 0.43 \pm 0.09 \pm 0.09 \text{ pb.}$$

- NLO calculation (Zhang, Ma, Wang, Chao (2009), Butenschön and Kniehl (2011)):

$$\sigma(e^+e^- \rightarrow J/\psi + X(\text{non-}c\bar{c})) = 0.99_{-0.17}^{+0.35} \text{ pb} \quad (\mu = \sqrt{s}/2).$$

- NRQCD LDMEs from the Butenschön-Kniehl (2011) global fit.
- Includes feeddown estimate of 0.29 pb from Zhang, Ma, Wang, Chao (2009).
- The comparison with the Belle data favors the Butenschön-Kniehl value of M_{0,r_0} .

Comments

- The most recent Belle (2009) measurements give

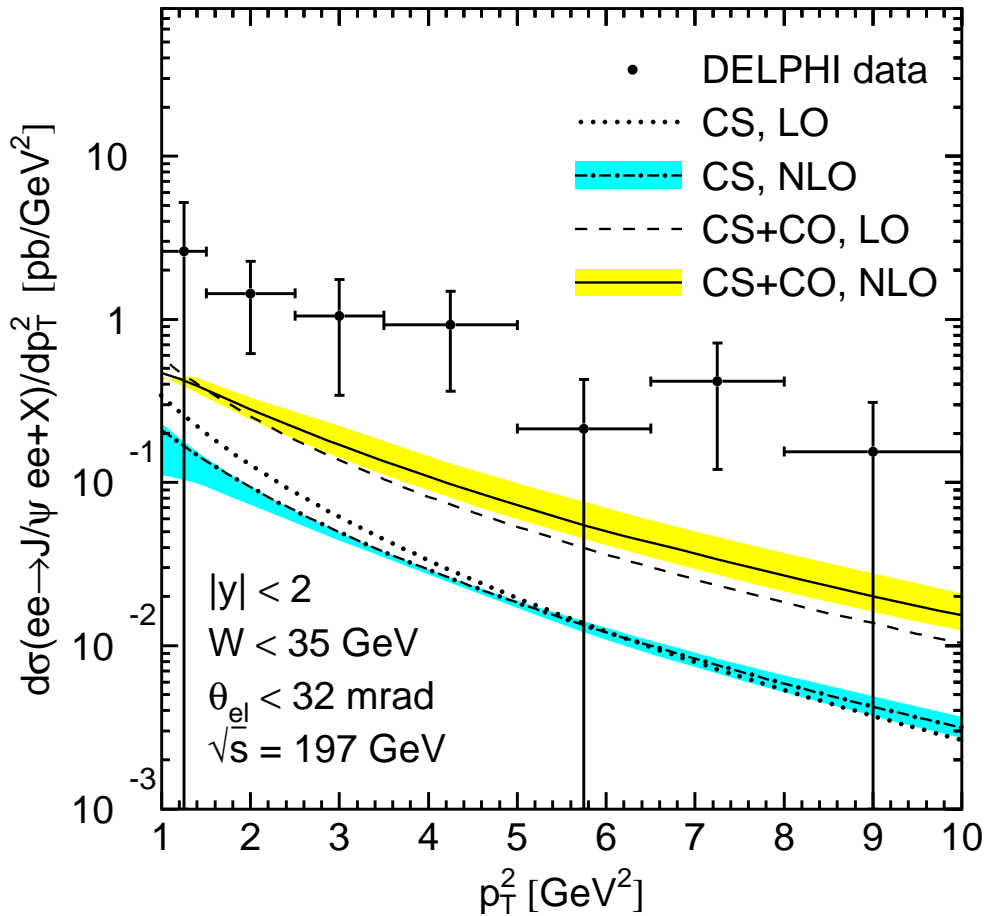
$$\begin{aligned} \sigma(e^+e^- \rightarrow J/\psi + X) &= \sigma(e^+e^- \rightarrow J/\psi + c\bar{c} + X) + \sigma(e^+e^- \rightarrow J/\psi + X(\text{non-}c\bar{c})) \\ &= 1.17 \pm 0.12_{-0.12}^{+0.13} \text{ pb.} \end{aligned}$$

-
- However, BaBar (2001) obtained

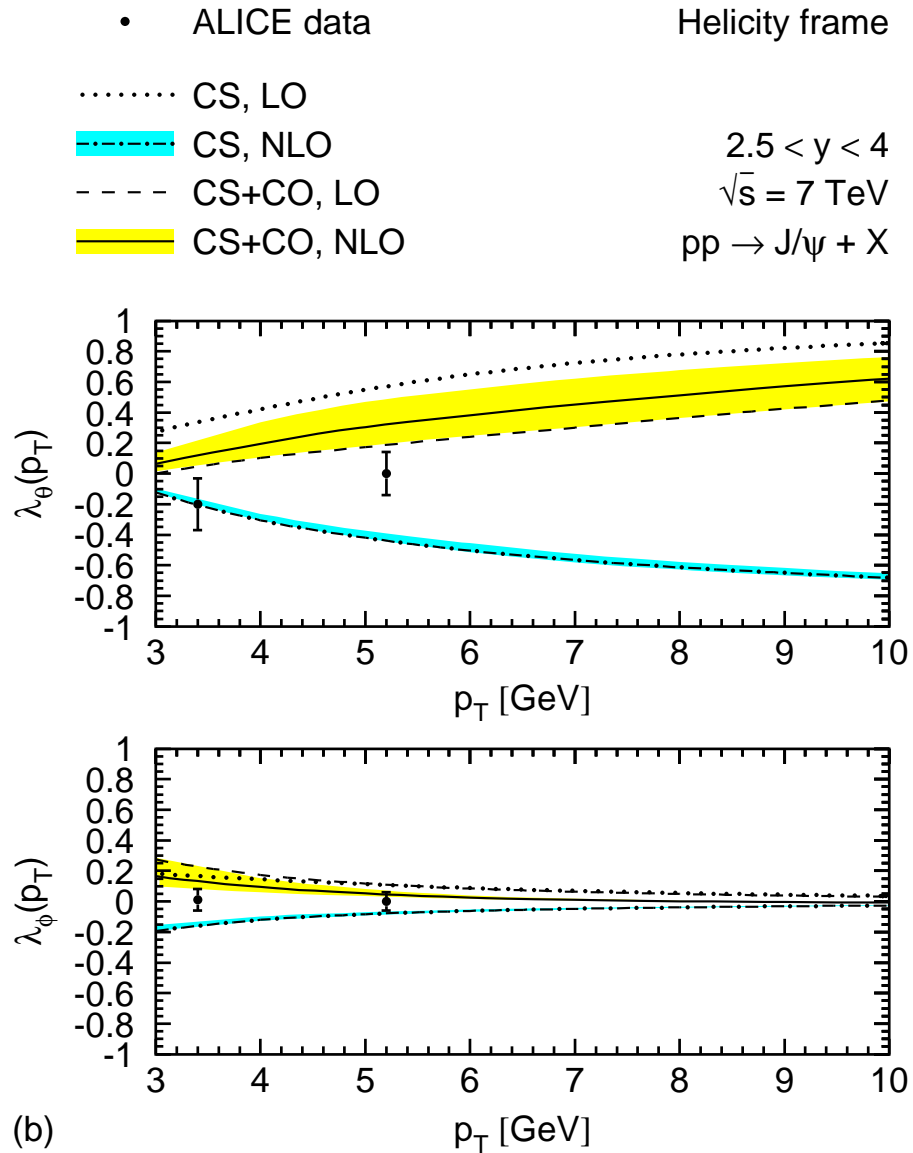
$$\sigma(e^+e^- \rightarrow J/\psi + X) = 2.52 \pm 0.21 \pm 0.21 \text{ pb.}$$

- Most of the data are at $p_T \lesssim 3 \text{ GeV}$. Does factorization hold at such small values of p_T ?

J/ψ Production in $\gamma\gamma$ Scattering at LEP II



- The DELPHI (2003) data are slightly incompatible with the prediction of the Butenschön and Kniehl (2011) global fit.
- The error bars are large, especially at high p_T .
- Factorization may not hold at low values of p_T .

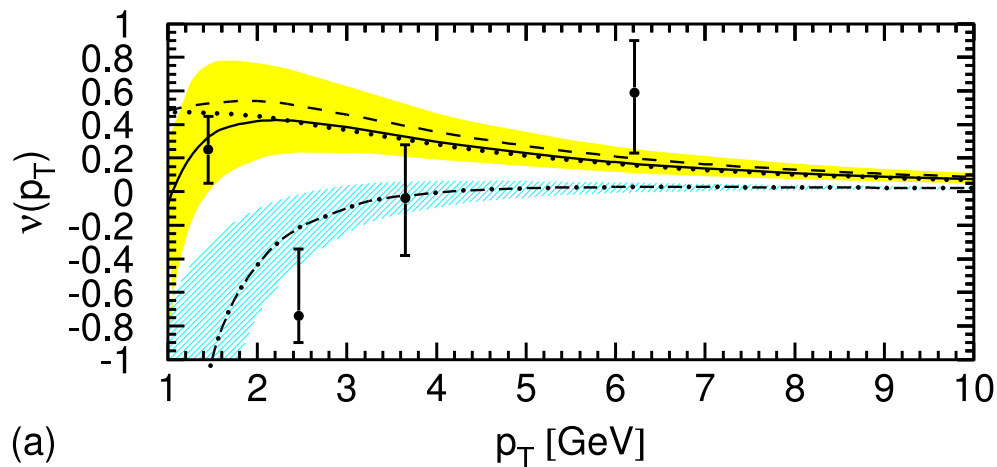
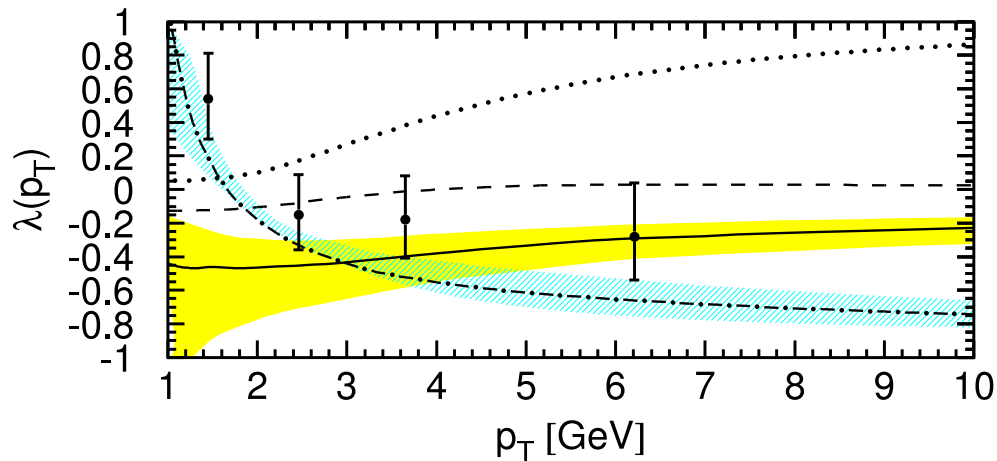


- The prediction from the Butenschön and Kniehl (2011) global fit is in agreement with the ALICE (2012) data.
- But the theory is for direct production, while the ALICE data includes production in B -meson decays and feed-down from χ_{cJ} states and the $\psi(2S)$.

- The Butenschön and Kniehl (2011) global fit can also be used to predict the polarization in inelastic J/ψ photoproduction at HERA.

- H1 data Helicity frame

- CS, LO
- -.-.- CS, NLO $60 \text{ GeV} < W < 240 \text{ GeV}$
- - - - CS+CO, LO $0.3 < z < 0.9$
- ——— CS+CO, NLO $Q^2 < 2.5 \text{ GeV}^2$



(a)

- The data are roughly compatible with the theory at large p_T , but the error bars are large.

• H1 data Collins-Soper frame

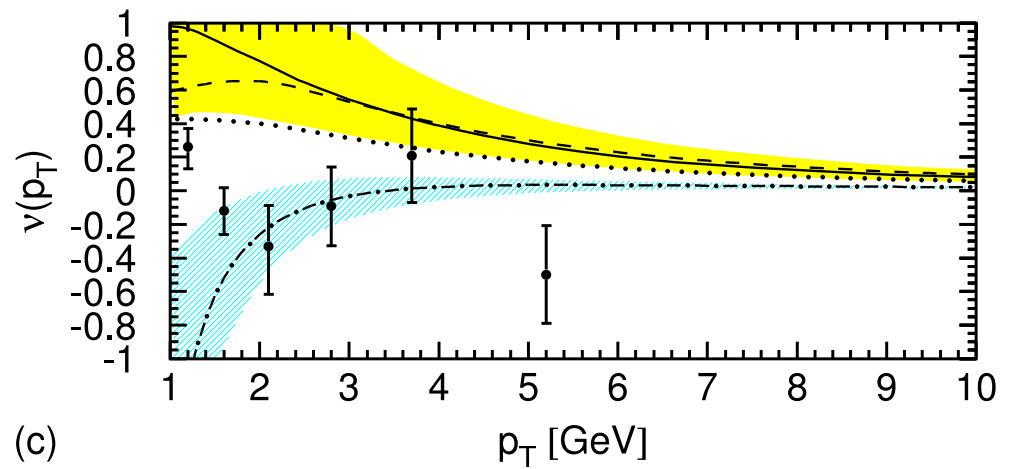
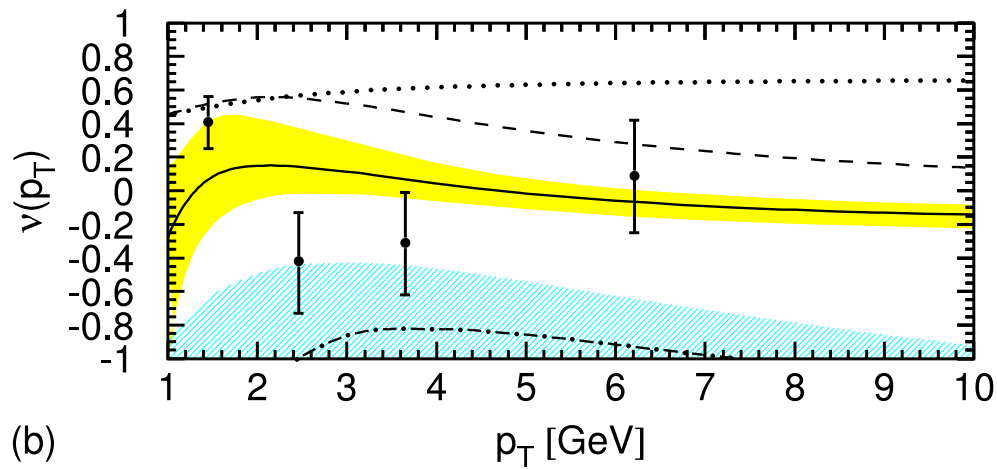
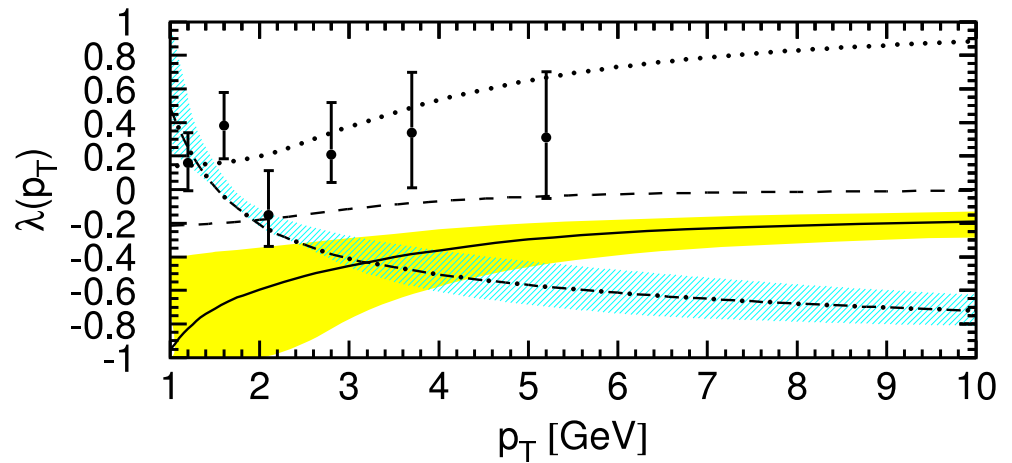
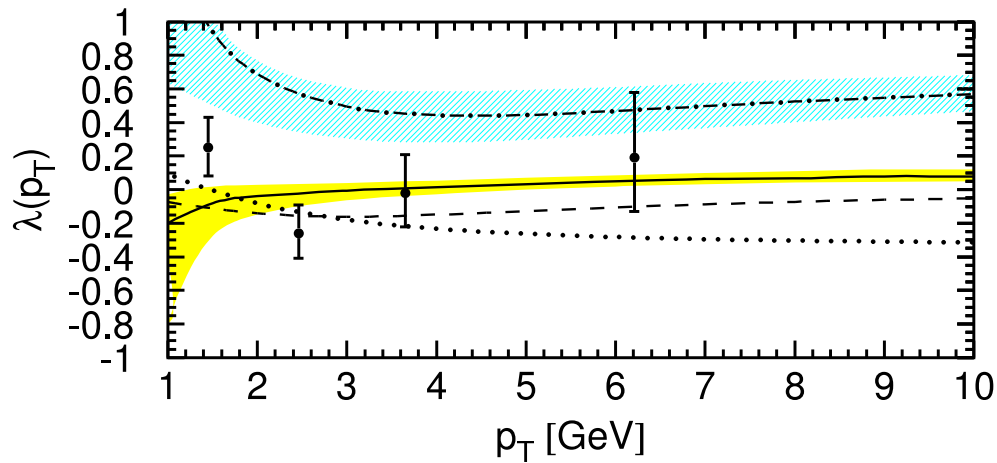
• ZEUS data (till z=1) Target frame

..... CS, LO
 -.-.- CS+CO, LO
 CS, NLO
 CS+CO, NLO

$60 \text{ GeV} < W < 240 \text{ GeV}$
 $0.3 < z < 0.9$
 $Q^2 < 2.5 \text{ GeV}^2$

..... CS, LO
 -.-.- CS+CO, LO
 CS, NLO
 CS+CO, NLO

$50 \text{ GeV} < W < 180 \text{ GeV}$
 $0.4 < z < 0.95$
 $Q^2 < 1 \text{ GeV}^2$



(b)

(c)