

Light Quark Mass Dependence of the $\mathbf{X(3872)}$ in an Effective Field Theory

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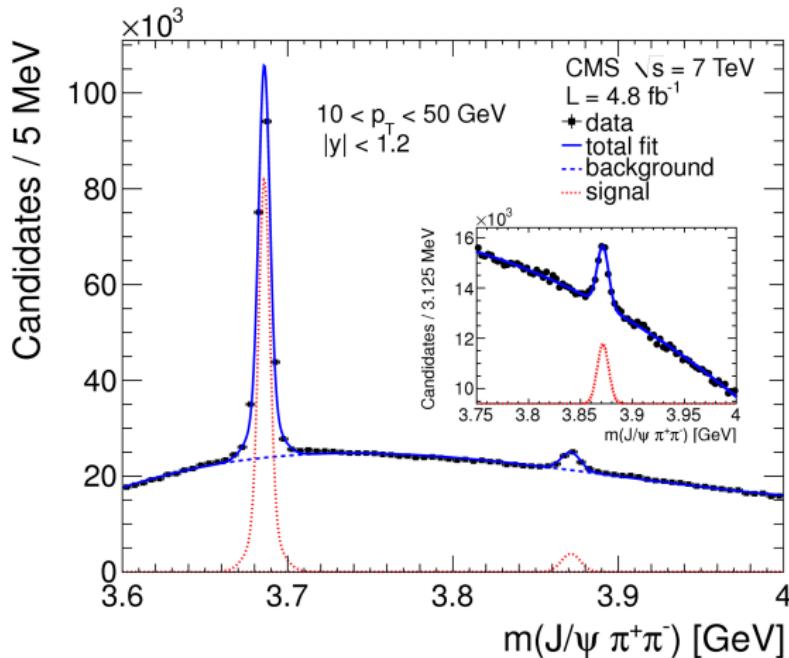
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- 1 Introduction and Motivation
- 2 XEFT and the $\bar{D}^0 D^{*0}$ Scattering Amplitude
- 3 Binding Energy and Scattering Length
- 4 Conclusion and Outlook

Introduction and Motivation

- First observation by the Belle Collaboration [Choi et al., 2003]
- Determination $J^{PC} = 1^{++}$ by LHCb [Aaij et al., 2013]



[Chatrchyan et al., 2013]

- Interpretations: tetraquark, charmonium, hadronic molecule
- Mass of the $X(3872)$ close to $D^0\bar{D}^{*0}$ threshold

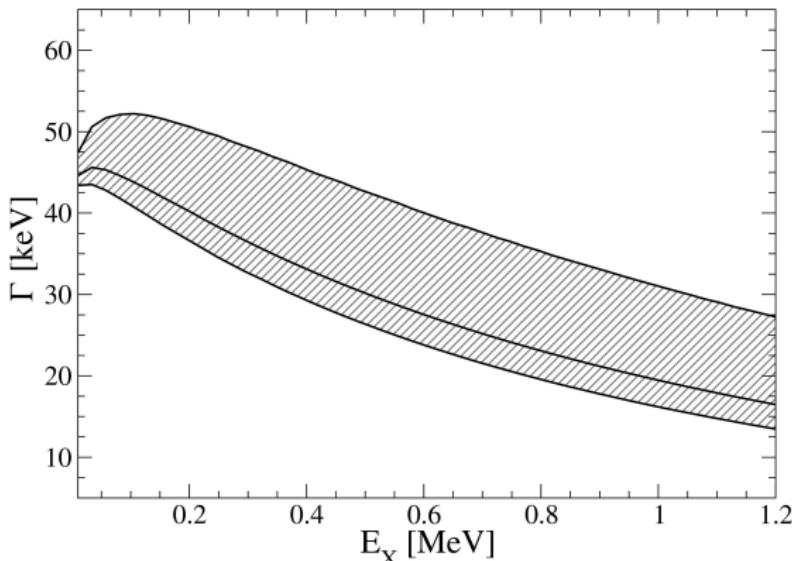
Particle Content of the $X(3872)$

$$X = \frac{1}{\sqrt{2}} (\bar{D}^0 D^{*0} + D^0 \bar{D}^{*0})$$

- Recent observation of a candidate for the X on the lattice [Prelovsek and Leskovec, 2013]
- Performed on rather small lattices for large quark masses
- Previous work:
 - Unitarized heavy meson ChPT: no sensitivity to contact interactions [Wang and Wang, 2013]
 - Non-relativistic Faddeev-type three-body equations: contact interactions essential [Baru et al., 2013]

Basics of XEFT

- Universal properties due to small binding energy
 $E_X = m_{D^*} + m_D - M_X = (0.17 \pm 0.26) \text{ MeV}$
- Corrections calculable in XEFT [Fleming et al., 2007]



Decay rate for $X \rightarrow D^0 \bar{D}^0 \pi^0$ as a function of E_X

Basics of XEFT

- Similar to KSW theory for NN scattering [Kaplan et al., 1998]
 - Includes pions perturbatively
 - Unnaturally large NNLO coefficients [Fleming et al., 2000]
- Nearness of $D^0 D^{*0}$ hyperfine splitting and pion mass induces small mass scale $\mu^2 = \Delta^2 - m_\pi^2$
- Mass scale μ , $D^{(*)0}$ and pion momenta and binding momentum of same order $Q \ll m_\pi, m_D, m_{D^*}$
- Pions and $D^{(*)0}$ mesons treated non-relativistically
- Integrated out charged $D^{(*)\pm}$ mesons
 - Effective field theory: $1/a$ suppression [Braaten and Kusunoki, 2004]
 - Charmonium- hadronic molecule hybrid: charged states small contribution [Takizawa and Takeuchi, 2013]
- Takes finite width of the D^{*0} into account

XEFT Lagrangian

$$\begin{aligned}\mathcal{L} = & \mathbf{D}^\dagger \left(i\partial_0 + \frac{\vec{\nabla}^2}{2m_{D^*}} \right) \mathbf{D} + D^\dagger \left(i\partial_0 + \frac{\vec{\nabla}^2}{2m_D} \right) D \\ & + \bar{\mathbf{D}}^\dagger \left(i\partial_0 + \frac{\vec{\nabla}^2}{2m_{D^*}} \right) \bar{\mathbf{D}} + \bar{D}^\dagger \left(i\partial_0 + \frac{\vec{\nabla}^2}{2m_D} \right) \bar{D} + \pi^\dagger \left(i\partial_0 + \frac{\vec{\nabla}^2}{2m_\pi} + \delta \right) \pi \\ & + \frac{g}{\sqrt{2}f} \frac{1}{\sqrt{2m_\pi}} \left(D\mathbf{D}^\dagger \cdot \vec{\nabla} \pi + \bar{D}^\dagger \bar{\mathbf{D}} \cdot \vec{\nabla} \pi^\dagger \right) + \text{h.c.} \\ & - \frac{C_0}{2} \left(\bar{\mathbf{D}}D + \mathbf{D}\bar{D} \right)^\dagger \cdot \left(\bar{\mathbf{D}}D + \mathbf{D}\bar{D} \right) \\ & + \frac{C_2}{16} \left(\bar{\mathbf{D}}D + \mathbf{D}\bar{D} \right)^\dagger \cdot \left(\bar{\mathbf{D}}\vec{\nabla}^2 D + \mathbf{D}\vec{\nabla}^2 \bar{D} \right) + \text{h.c.} \\ & - \frac{D_2\mu^2}{2} \left(\bar{\mathbf{D}}D + \mathbf{D}\bar{D} \right)^\dagger \cdot \left(\bar{\mathbf{D}}D + \mathbf{D}\bar{D} \right) + \dots,\end{aligned}$$

Power Counting in XEFT



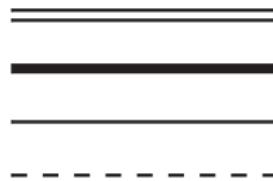
$$\sim Q^{-1}$$



$$\sim Q^0$$



$$\sim Q^0$$



$$\sim Q^{-2}$$



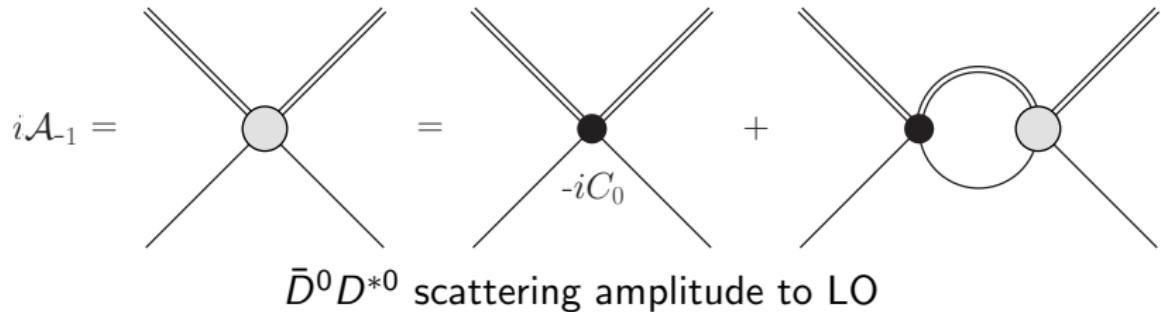
$$\sim Q^5$$



$$\sim Q^1$$

$$-i \frac{g}{\sqrt{2}f} \frac{1}{\sqrt{2}m_\pi} (\boldsymbol{\varepsilon} \cdot \mathbf{p}_\pi)$$

LO Scattering Amplitude

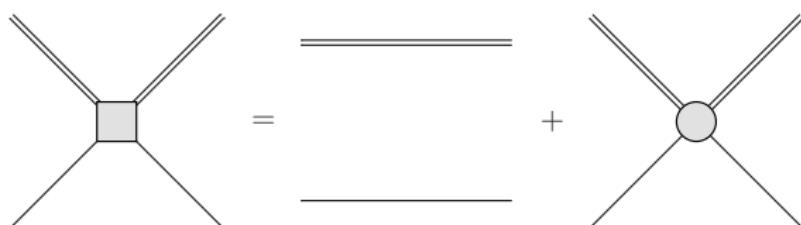
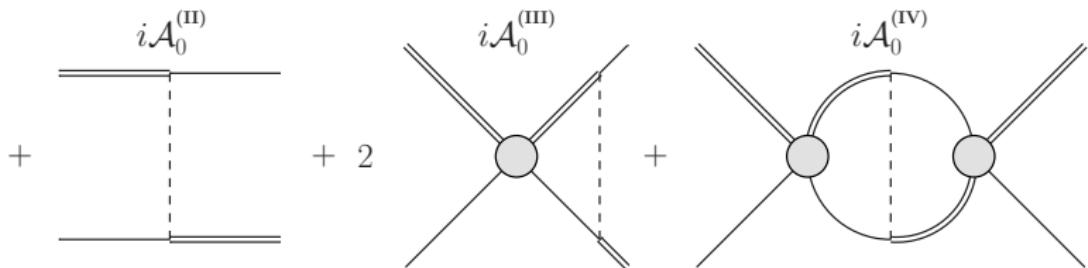
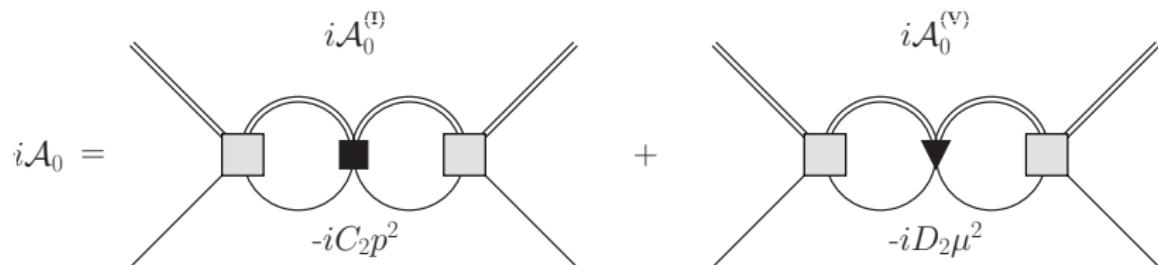


$$i\mathcal{A}_{-1} = \frac{2\pi i}{M_{DD^*}} \frac{1}{-\gamma + \sqrt{-2M_{DD^*}E - i\epsilon}}$$

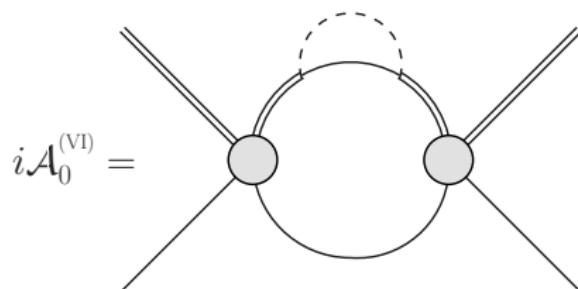
$$\gamma \equiv \frac{2\pi}{M_{DD^*} C_0(\Lambda)} + \Lambda$$

$$\text{Pole at } -E = \frac{\gamma^2}{2M_{DD^*}}$$

NLO Contributions to the Scattering Amplitude



Infrared Divergences in XEFT



$$i\mathcal{A}_0^{(\text{VI})} =$$

$$i\mathcal{A}_0^{(\text{VI})} = \frac{ig^2}{6f^2} \frac{1}{p} (i\Lambda - \mu) \frac{\mu^2}{2} \left(\frac{M_{DD^*}}{2\pi} \right)^2 \mathcal{A}_{-1}^2$$

- Infrared divergent
- Renormalization scale dependent
- Pion bubbles give contribution to the D^* self energy

Resummation for the D^{*0} Propagator

$$iG = \text{---} = \text{---} + \text{---} \Sigma^{\text{OS}} \text{---}$$

Full D^{*0} propagator

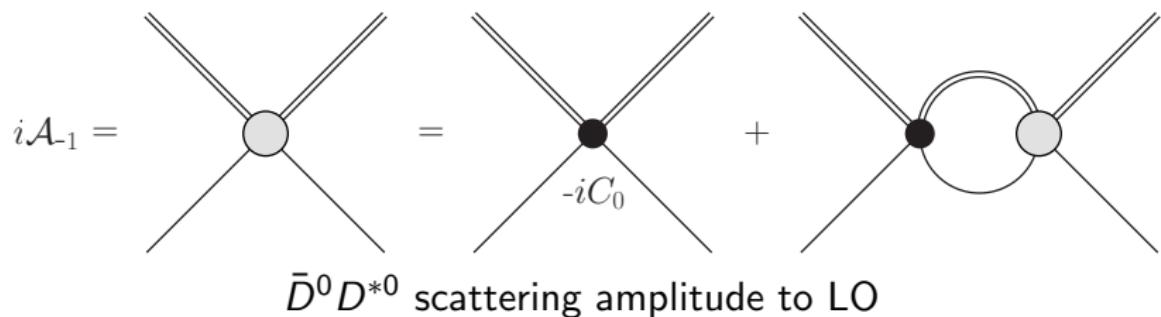
$$iG = \frac{i}{p_0 - p^2/2m_{D^*} + \Sigma^{\text{OS}} + i\epsilon}$$

$$i\Sigma^{\text{OS}} = \text{---} \Sigma^{\text{OS}} \text{---} = \text{---} \text{---} + \text{---} \times \text{---}$$

$$\Sigma^{\text{OS}} = \frac{g^2}{24\pi f^2} i\mu^3$$

- Purely imaginary for $m_\pi < \Delta$, induces decay width for D^{*0}
- Real valued for $m_\pi \geq \Delta$, induces mass shift for D^{*0}

LO Scattering Amplitude

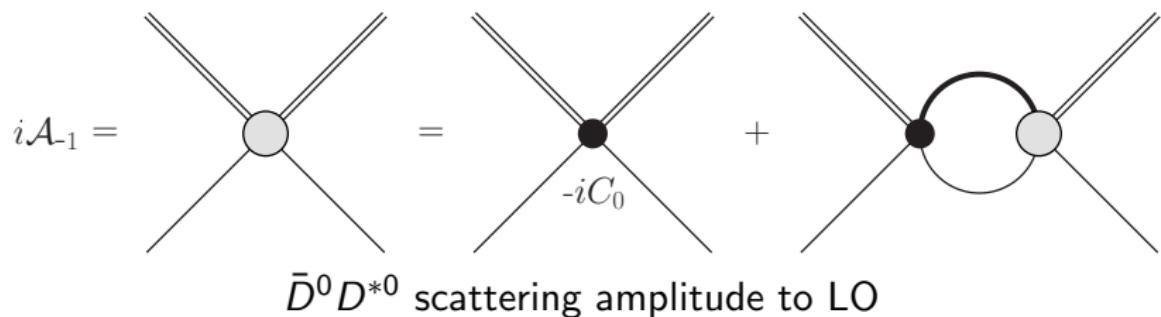


$$i\mathcal{A}_{-1} = \frac{2\pi i}{M_{DD^*}} \frac{1}{-\gamma + \sqrt{-2M_{DD^*}E - i\epsilon}}$$

$$\gamma \equiv \frac{2\pi}{M_{DD^*} C_0(\Lambda)} + \Lambda$$

$$\text{Pole at } -E = \frac{\gamma^2}{2M_{DD^*}}$$

LO Scattering Amplitude

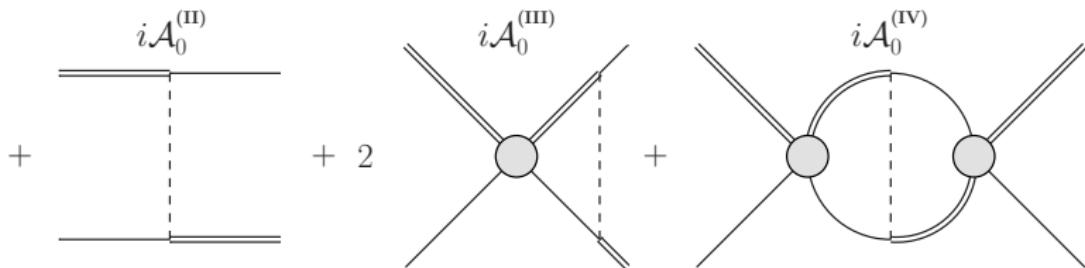
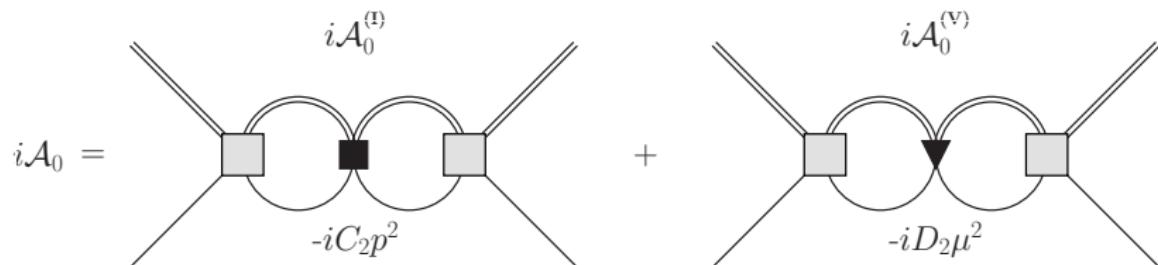


$$i\mathcal{A}_{-1} = \frac{2\pi i}{M_{DD^*}} \frac{1}{-\gamma + \sqrt{-2M_{DD^*}E - 2M_{DD^*}\Sigma^{\text{os}}} - i\epsilon}$$

$$\gamma \equiv \frac{2\pi}{M_{DD^*} C_0(\Lambda)} + \Lambda$$

$$\text{Pole at } -E = \frac{\gamma^2}{2M_{DD^*}} + \Sigma^{\text{os}}$$

NLO Contributions to the Scattering Amplitude



NLO Contributions to the Scattering Amplitude

$$i\mathcal{A}_0 = i\mathcal{A}_0^{(I)} + i\mathcal{A}_0^{(V)}$$

$i\mathcal{A}_0^{(I)}$

$-iC_2 p^2$

$i\mathcal{A}_0^{(V)}$

$-iD_2 \mu^2$

$$+ i\mathcal{A}_0^{(II)} + 2 i\mathcal{A}_0^{(III)} + i\mathcal{A}_0^{(IV)}$$

$i\mathcal{A}_0^{(II)}$

$+ 2$

$i\mathcal{A}_0^{(III)}$

$i\mathcal{A}_0^{(IV)}$

$$= \text{---} +$$

=

NLO Scattering Amplitudes

$$i\mathcal{A}_{-1} = \frac{2\pi i}{M_{DD^*}} \frac{1}{-\gamma + \eta}$$

$$i\mathcal{A}_0^{(I)} = \frac{-iC_2}{C_0^2} \left(p^2 + 2M_{DD^*} \Sigma^{\text{os}} \frac{-\eta + \Lambda}{-\gamma + \Lambda} \right) \mathcal{A}_{-1}^2$$

$$i\mathcal{A}_0^{(II)} = \frac{ig^2}{6f^2} \left(1 + \frac{\mu^2}{4p^2} \log \left(1 - \frac{4p^2}{\mu^2} \right) \right)$$

$$i\mathcal{A}_0^{(III)} = \frac{ig^2}{3f^2} \left((-\eta + \Lambda) + \frac{i\mu^2}{2p} \log \left(1 + \frac{2p}{i\eta + \mu - p} \right) \right) \frac{M_{DD^*}}{2\pi} \mathcal{A}_{-1}$$

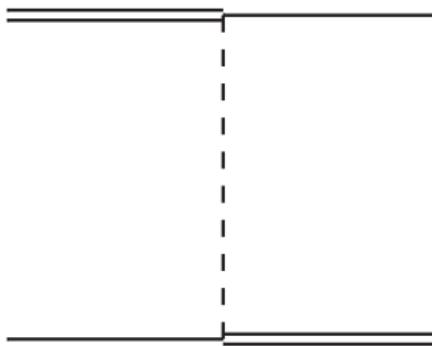
$$i\mathcal{A}_0^{(IV)} = \frac{ig^2}{6f^2} \left((-\eta + \Lambda)^2 + \mu^2 \left(\text{log} \left(\frac{\Lambda}{2\eta - i\mu} \right) + 1 + R \right) \right) \left(\frac{M_{DD^*}}{2\pi} \right)^2 \mathcal{A}_{-1}^2$$

$$i\mathcal{A}_0^{(V)} = \frac{-iD_2\mu^2}{C_0^2} \mathcal{A}_{-1}^2$$

$$\eta \equiv \sqrt{-p^2 - 2M_{DD^*} \Sigma^{\text{os}} - i\epsilon}$$

$$R \equiv \frac{1}{2} \left(-\gamma_E + \log \left(\frac{\pi}{4} \right) + \frac{2}{3} \right)$$

One-Pion Exchange



$$i\hat{\mathcal{A}}_0^{(II)}{}_{ij} = \frac{ig^2}{2f^2} \frac{(\boldsymbol{\varepsilon}_i \cdot \mathbf{p}_\pi)(\boldsymbol{\varepsilon}_j \cdot \mathbf{p}_\pi)}{\mathbf{p}_\pi^2 - \mu^2}$$
$$\xrightarrow{\text{S-wave}} \delta_{ij} \cdot \frac{ig^2}{6f^2} \left(1 + \frac{\mu^2}{4p_\pi^2} \log \left(1 - \frac{4p_\pi^2}{\mu^2} \right) \right) \equiv \delta_{ij} \cdot i\mathcal{A}_0^{(II)}$$

- Separate amplitudes $\hat{\mathcal{A}}_{ij} = \delta_{ij} \cdot \mathcal{A}$

Effective Range Expansion

- Relation between scattering amplitude and S-matrix

$$S - 1 = e^{2i\delta_s} - 1 = i \frac{p M_{DD^*}}{\pi} \mathcal{A}$$

- Apply effective range expansion

$$p \cot \delta_s = ip + \frac{2\pi}{M_{DD^*} \mathcal{A}} = -\frac{1}{a_s} + \frac{1}{2} r_s p^2 + \dots$$

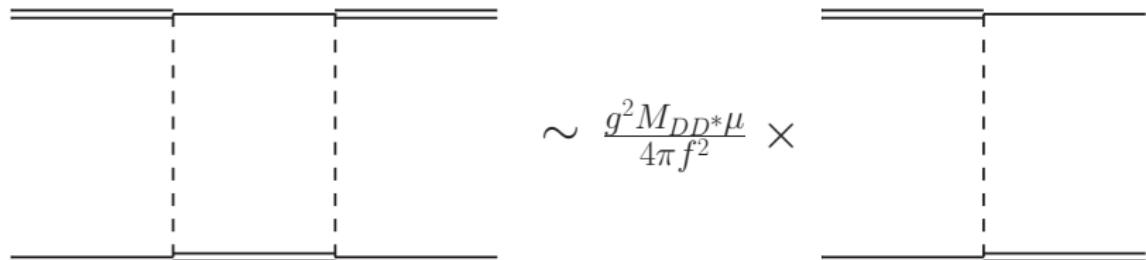
- OPE in coordinate space oscillatory [Suzuki, 2005]

$$\frac{ig^2}{2f^2} \frac{(\boldsymbol{\varepsilon}_i \cdot \mathbf{p}_\pi)(\boldsymbol{\varepsilon}_j \cdot \mathbf{p}_\pi)}{\mathbf{p}_\pi^2 - \mu^2}$$

$$\xrightarrow{\text{F.T.}} \frac{ig^2}{8\pi f^2} (\boldsymbol{\varepsilon}_i \cdot \boldsymbol{\varepsilon}_j - 3(\boldsymbol{\varepsilon}_i \cdot \hat{\mathbf{r}})(\boldsymbol{\varepsilon}_j \cdot \hat{\mathbf{r}})) \frac{\cos(\mu r) + \mu r \sin(\mu r)}{r^3} + \dots$$

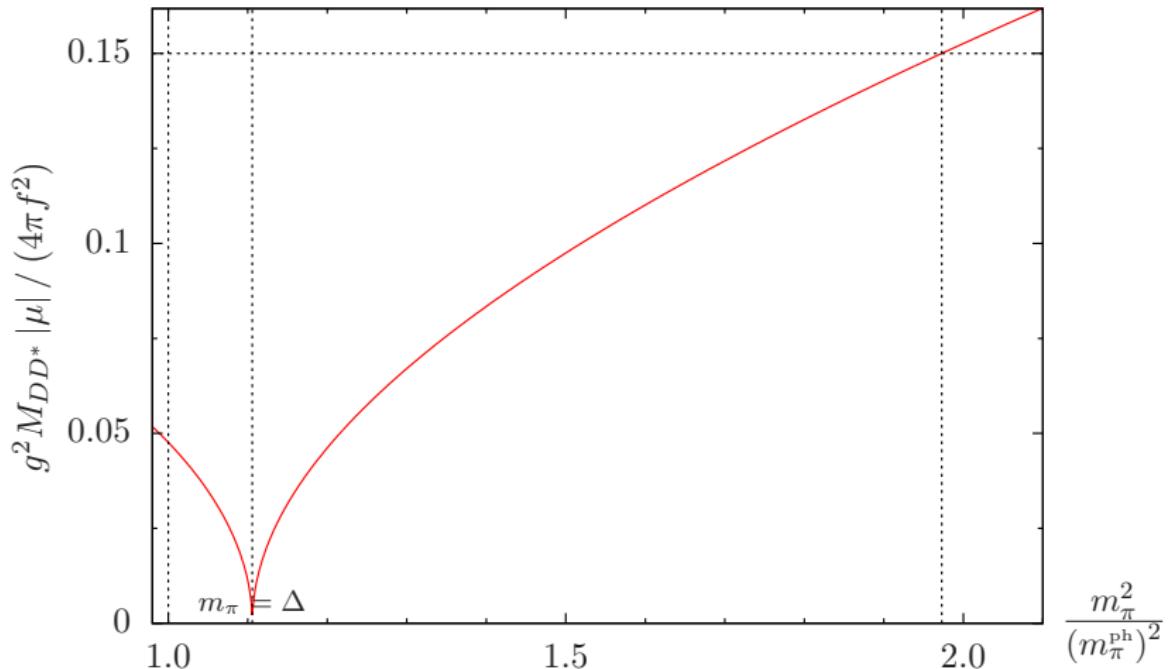
- Effective range expansion only valid up to order p^0

Suppression of the Two-Pion Exchange

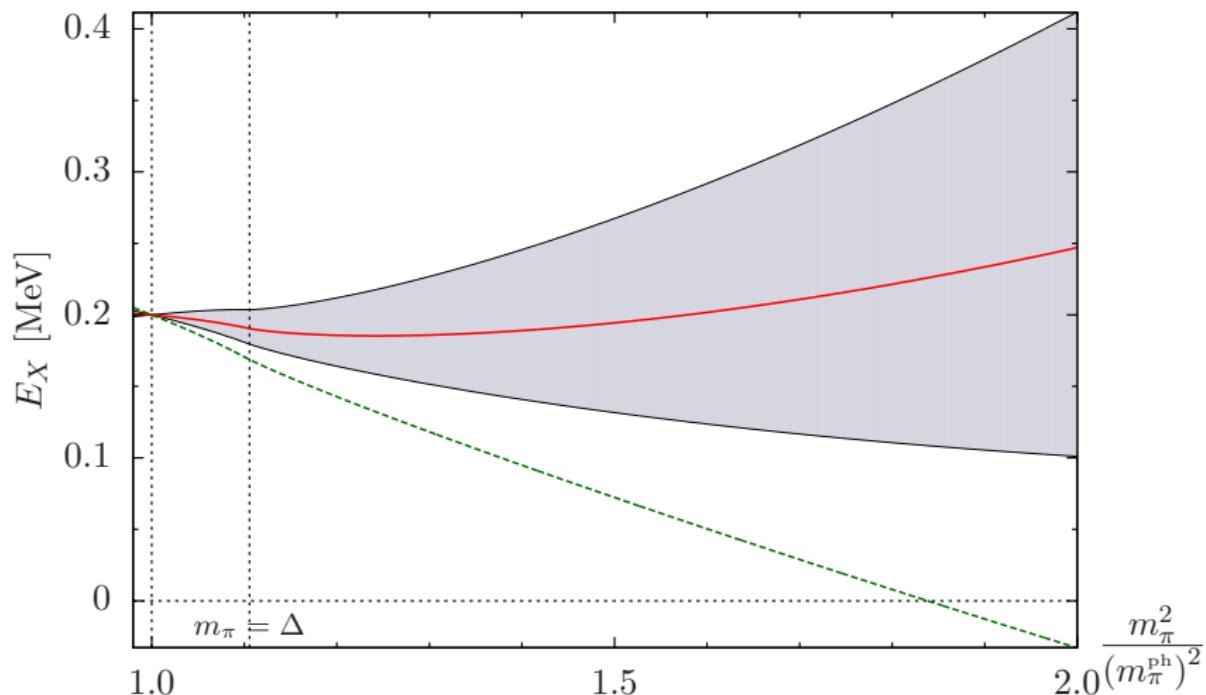


- Expansion factor in KSW for NN scattering
$$\left(\frac{g_A^2 M_N m_\pi}{8\pi f^2}\right)^{\text{ph}} \sim 0.5$$
 [Kaplan et al., 1998]
- Expansion factor in XEFT for DD^* scattering
$$\left(\frac{g^2 M_{DD^*} \mu}{4\pi f^2}\right)^{\text{ph}} \sim 0.05$$
 [Fleming et al., 2007]
- Quark mass dependent → estimate range of validity

Expansion factor



Results for the Binding Energy

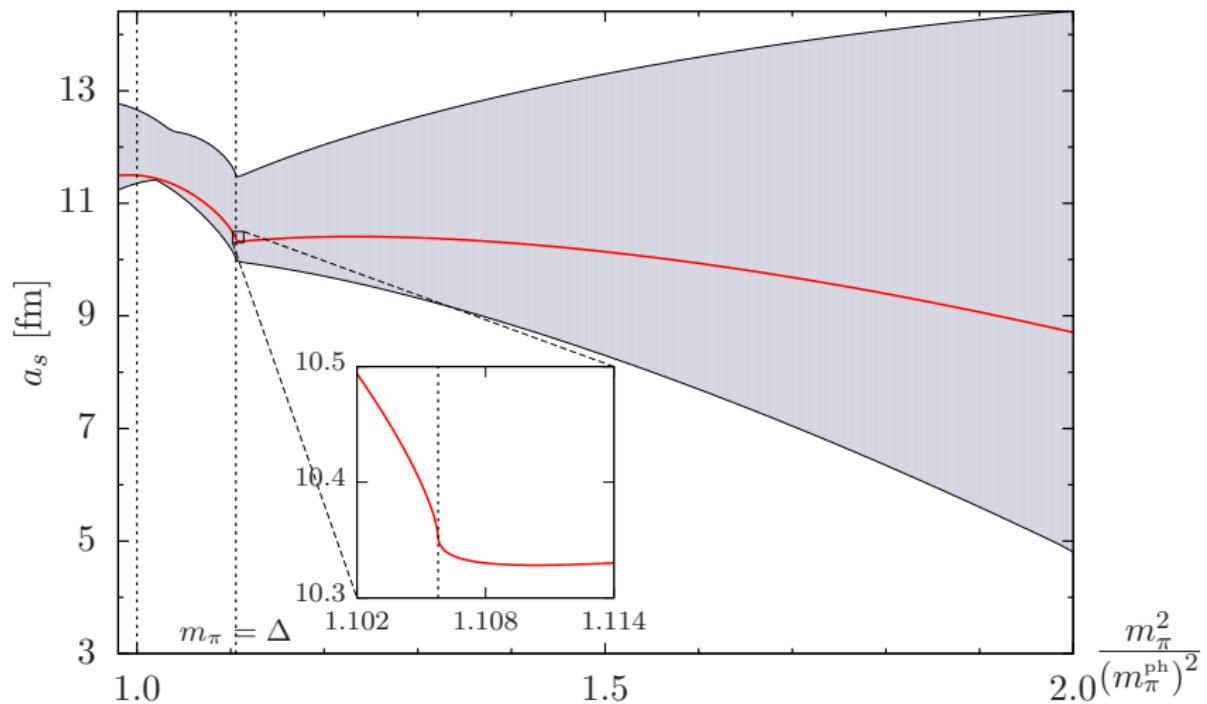


Red: LO contact interaction and OPE only

Bounds: Natural ranges for NLO coefficients

Green: Unnaturally large NLO coefficient

Results for the Scattering Length



Red: LO contact interaction and OPE only
Bounds: Natural ranges for NLO coefficients

Conclusion and Outlook

Conclusion

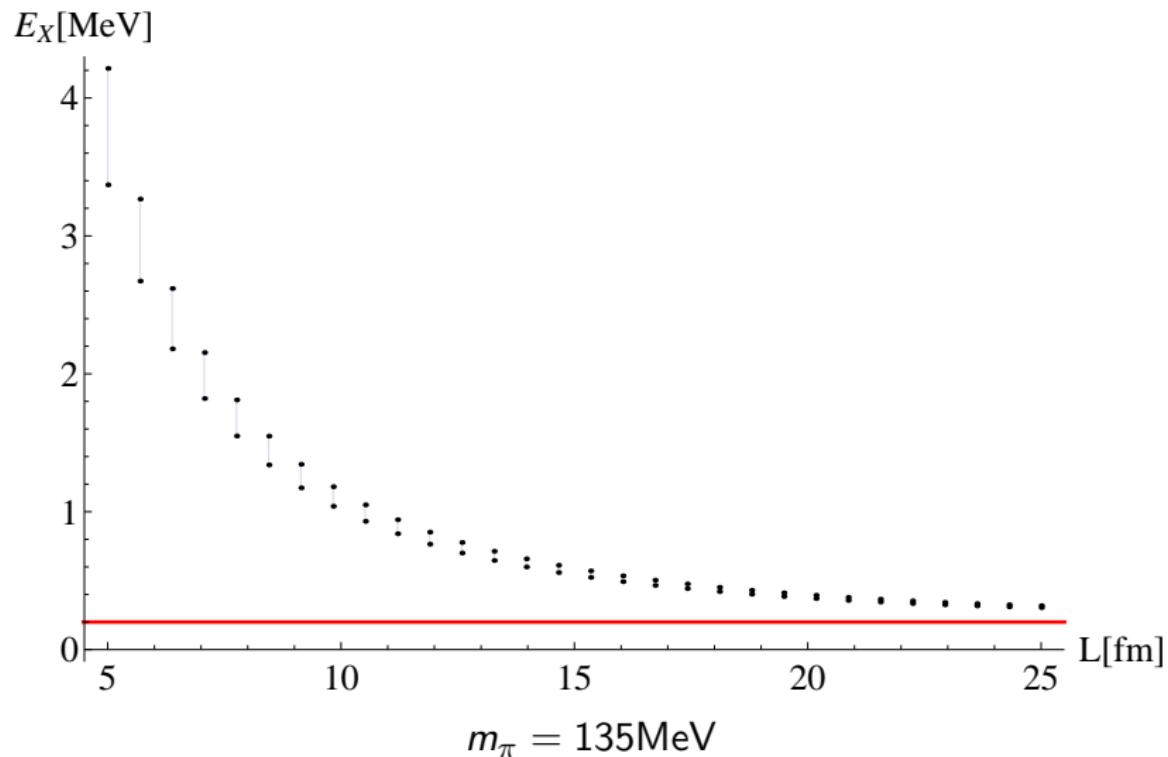
- XEFT applicable to calculate chiral extrapolations analytically
- Quark mass dependent contact interaction essential for renormalization
- $X(3872)$ should be observable on the lattice
- High sensitivity of scattering length (cusp effect)
- Qualitative agreement with results from non-relativistic Faddeev-type three-body equations [Baru et al., 2013]
- Discrepancy with results from unitarized heavy meson ChPT [Wang and Wang, 2013]

Outlook

- Extension to NNLO; Inclusion of charged D -mesons
- Relativistic pion fields for extrapolation to chiral limit
- Calculation of finite volume effects

Outlook: finite volume effects

- Binding energy of the X , $E_X \lesssim 0.5$ MeV
 $\Rightarrow S$ -wave Scattering length $a_s \gtrsim 5$ fm
- Recent simulation on lattice with a spatial size $L \approx 2$ fm
[Prelovsek and Leskovec, 2013]
- Finite volume corrections essential
- Periodic boundary conditions
 \Rightarrow allowed loop momenta $\mathbf{q} = \frac{2\pi\mathbf{n}}{L}$, $\mathbf{n} \in \mathbb{Z}^3$
- Replace integrals by sums $\int \frac{d^3 q}{(2\pi)^3} \rightarrow \frac{1}{L^3} \sum_{\mathbf{q}=\frac{2\pi\mathbf{n}}{L}}$
- For $m_\pi \gg \Delta$ use pw expansion to include effects of pions
 \Rightarrow analogous procedure as in pionless EFT [Beane et al., 2004]
- Close to and below threshold evaluate diagrams with pions explicitly



Dots: Binding energy in the finite volume

Lines: Binding energy in the infinite volume

Renormalization of C_2 and D_2

$$C_2 = \frac{M_{DD^*}}{2\pi} \frac{r_0}{2} (C_0)^2 \equiv c_2 (C_0)^2$$

$$D_2 = \frac{6f^2}{g^2} \left(\frac{2\pi}{M_{DD^*}} \right)^2 \left(d_2 + \log \left(\frac{\Lambda}{\mu^{\text{ph}}} \right) - R \right) (C_0)^2$$