Optics Design Techniques for the suppression of emittance growth induced by CSR

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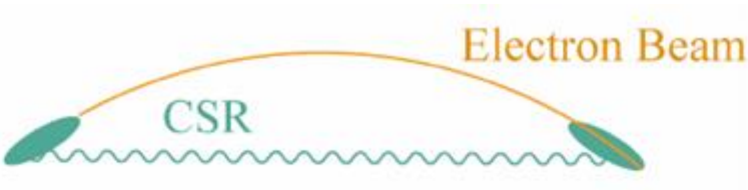
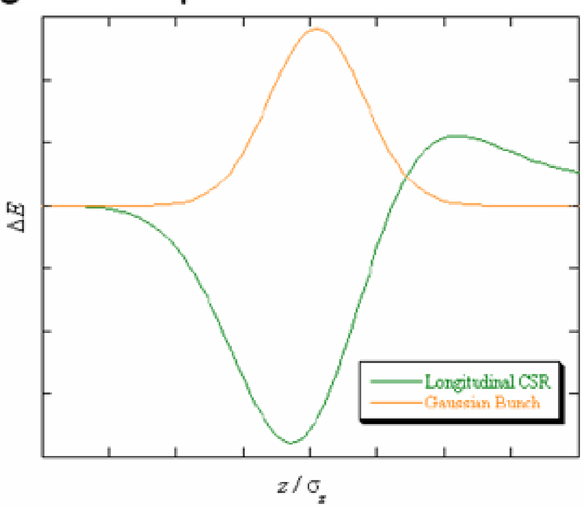
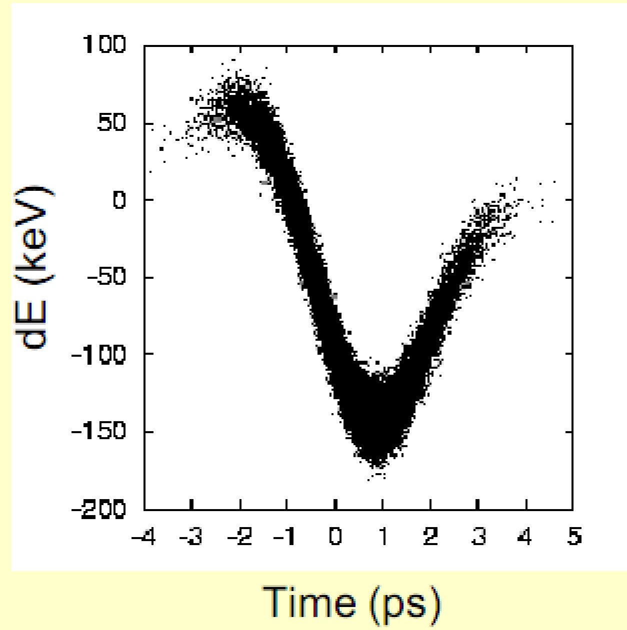
Mechanism of emittance growth due to CSR effect



(1) CSR emission from the bunch tail catches up with the bunch head

(2) Energy change depending on the longitudinal position





energy change
in a bending path

CSR study paper selection

- **Theoretical studies**
  - Y. Derbenev, et al., TESLA-FEL 95-05, 1995
  - B. Carlsten and T. Raubenheimer, Phys. Rev. E 51, 1453 (1995)
  - Y. Derbenev and V. Shiltsev, SLAC-PUB-7181, 1996
  - E. Saldin, E. Schneidmiller, and M .Yukov, Nucl. Instrum. Methods Phys. Res., Sec A 398, 373 (1997)
- **Experimental support**
  - M. Dohlus, and T. Limberg, Nucl. Instrum. Methods Phys. Res., Sec A 393, 494 (1997)
  - H. Braun, F. Chautard, R. Corsini, T. Raubenheimer, and P. Tenenbaum, Phys. Rev. Lett. 84, 658 (2000)
  - S. Mitri, et al., Phys. Rev. ST Accel. Beams 15, 020701 (2012)
- **CSR cancellation studies**
  - D. Douglas, JLAB-TN-98-012, 1998
  - R. Hajima, Nucl. Instrum. Methods Phys. Res., Sec A 528 (2004) 335-339
  - S. Mitri, m. Cornacchia, and S. Spampinati, Phys. Rev. Lett. 014801 (2013)

Linear Approximation of CSR effect

$$\Delta E_{rms} \cong 0.22 \frac{eQL_b}{4\pi\varepsilon_0 \rho^{2/3} \sigma_z^{4/3}}$$

Bending path

Bunch length

Bending radius

If we assume that:

1. The bunch length **σ_z** doesn't change a lot along the transport line
2. The Transient CSR effect if not large (**uniform bending field**)
3. Bending angles of the dipoles are not very large, **< 10°**

The CSR induced energy spread growth can be **linearized**

If with const. ρ

$$\Delta E(csr) / E_0 \cong k\rho^{1/3}\theta \quad or \quad \Delta E(csr) E_0 \cong \kappa\theta$$

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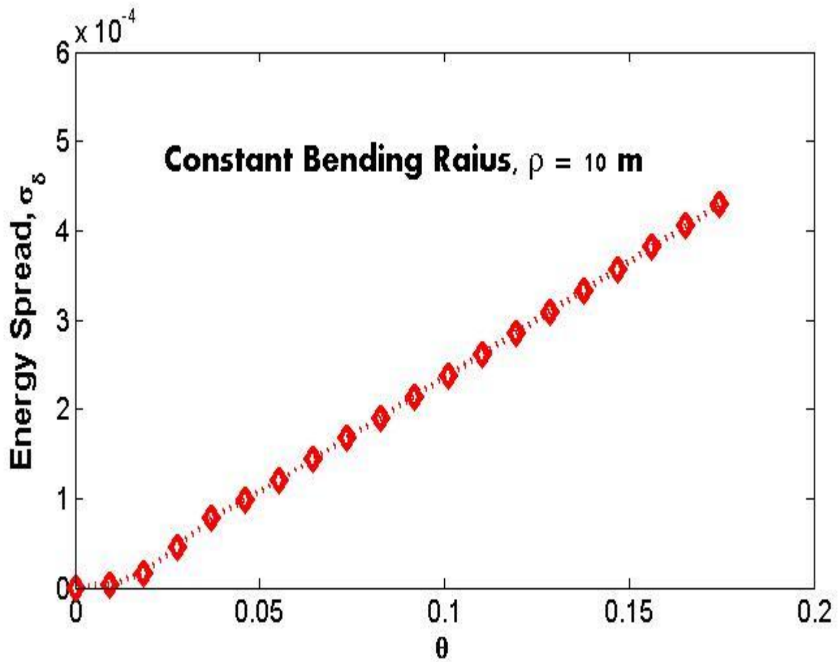
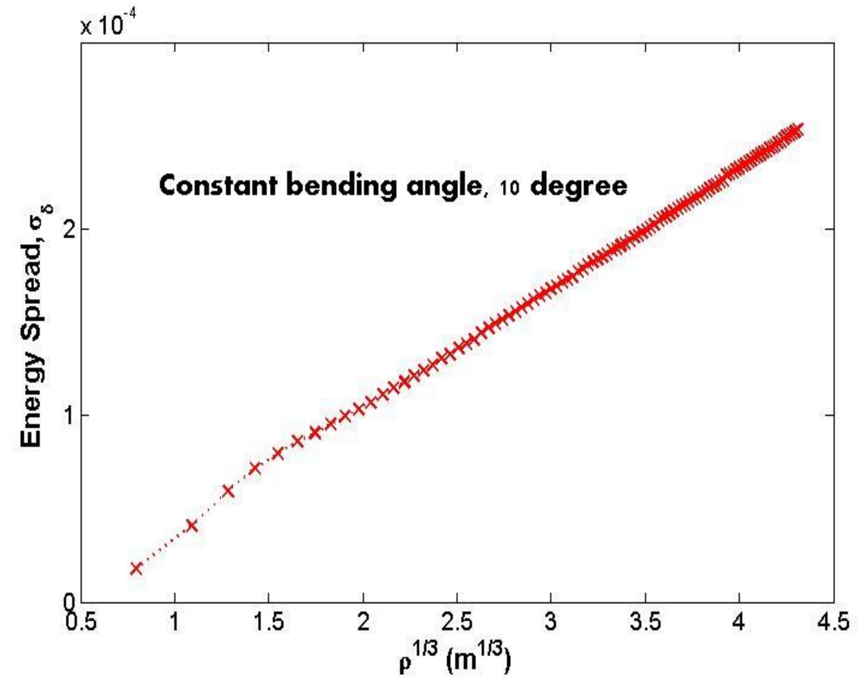
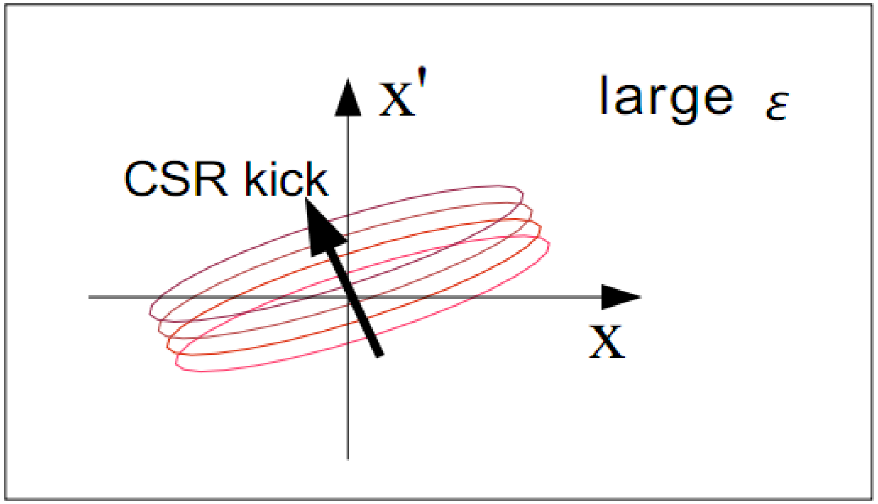




In the cases 1.5° < θ < 10°, 4 m < ρ < 80 m, this approximation applies well.

Envelope matching method

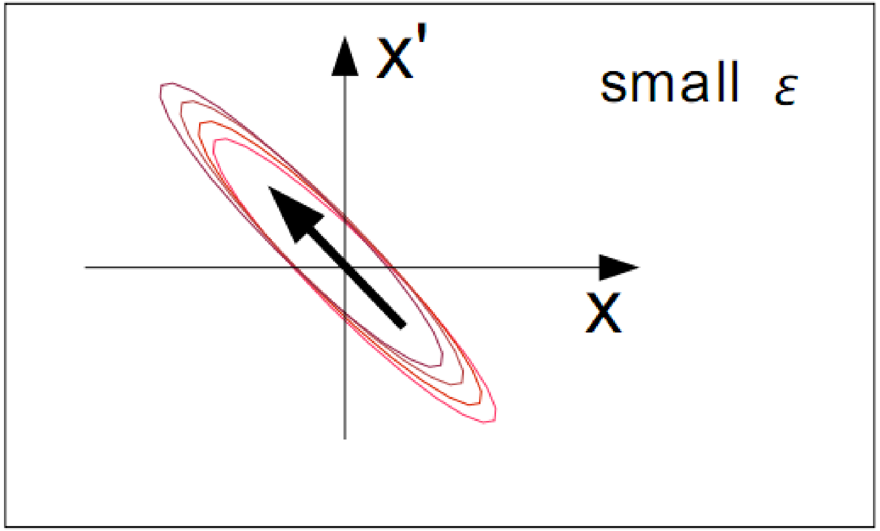
The concept of envelope matching is introduced by Dr. Hajima. In this method, the emittance growth in a achromatic cell can be considered as a stretch of the transverse phase space.



coincidence between the CSR kick and the phase ellipse orientation.

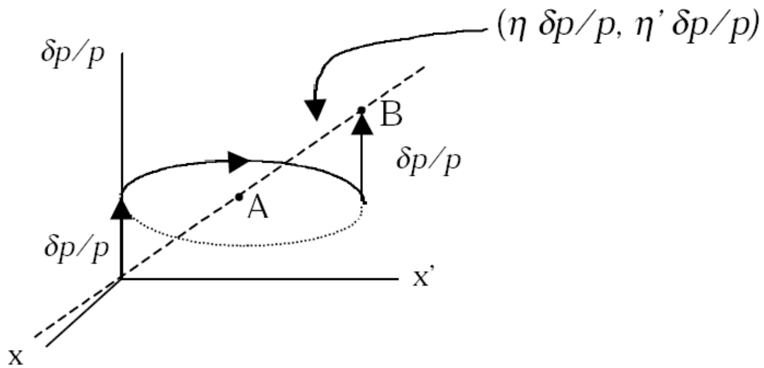
↓

minimum emittance growth



R. Hajima, JJAP 42, L974 (2003).

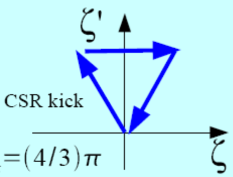
Cell-to-cell phase matching method



Periodic transport, with half integer phase advance between two identical periods. Electrons experience the same CSR kicks at two periods. With –I transportation, the CSR kicks are cancelled.

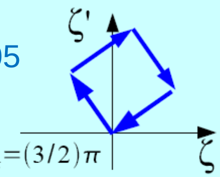
D. Douglas, JLAB-TN-98-012, 1998

Or cancellation after a series of periodic cells



$\Delta\psi_x = (4/3)\pi$

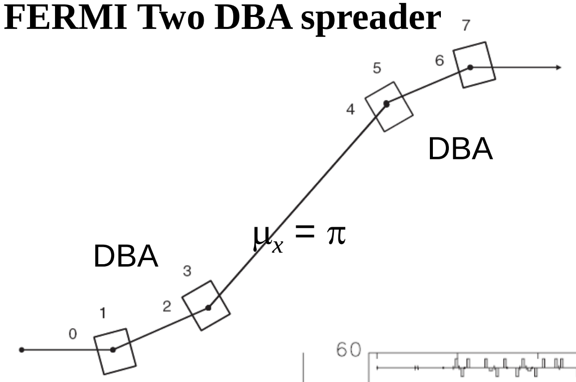
R. Hajima, 2005

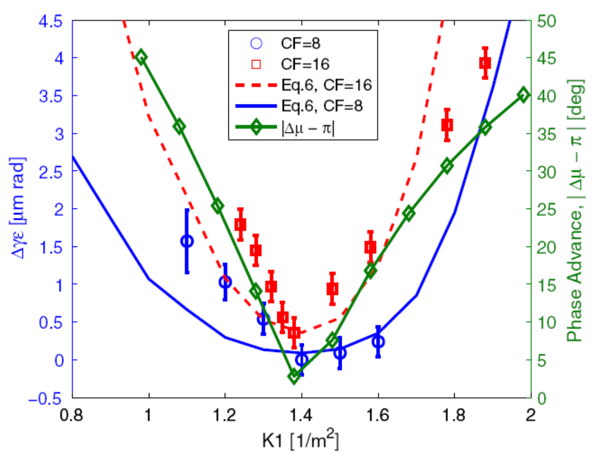


$\Delta\psi_x = (3/2)\pi$

The CSR kick is canceled, if $\Delta\psi_x(1) + \Delta\psi_x(2) + \ldots = 2N\pi$

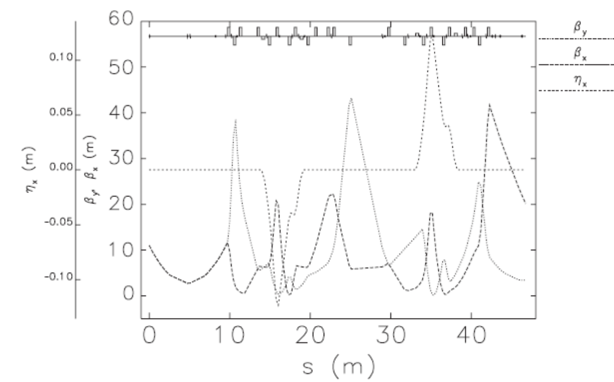
FERMI Two DBA spreader





When the phase advance between two DBAs is π, the CSR induced emittance growth can be minimized.

S. Mitri, et al., PRL, 014801, 2013



Envelope Matching method	Phase Matching method
1. Strong constraints on the Twiss parameters 2. Its effects relies on the shape of the phase ellipse	1. It requires identical cells, and special layout 2. Accurate phase advance between cells

The kind of optics design method we need

1. The CSR induced emittance growth can be suppressed within a single achromatic module

2. Constraints on the magnets rather than on the twiss parameters

3. Easy to design and use

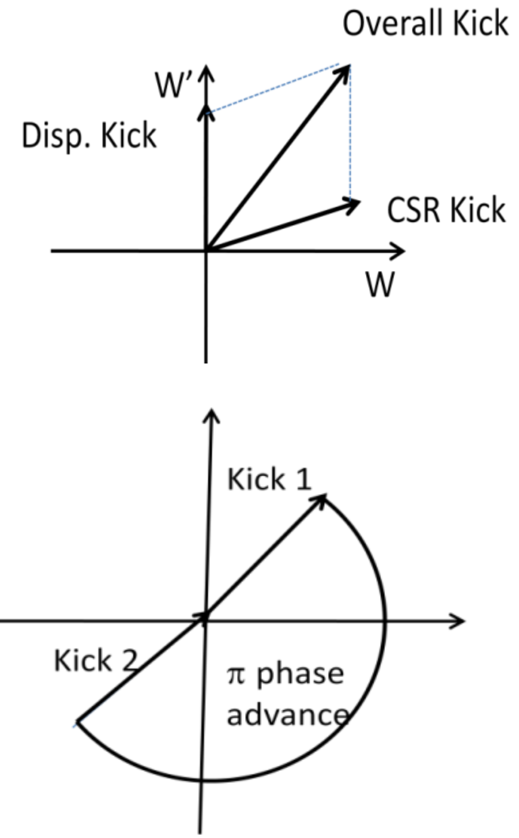
Methods to treat the linearized CSR

Matrix analysis

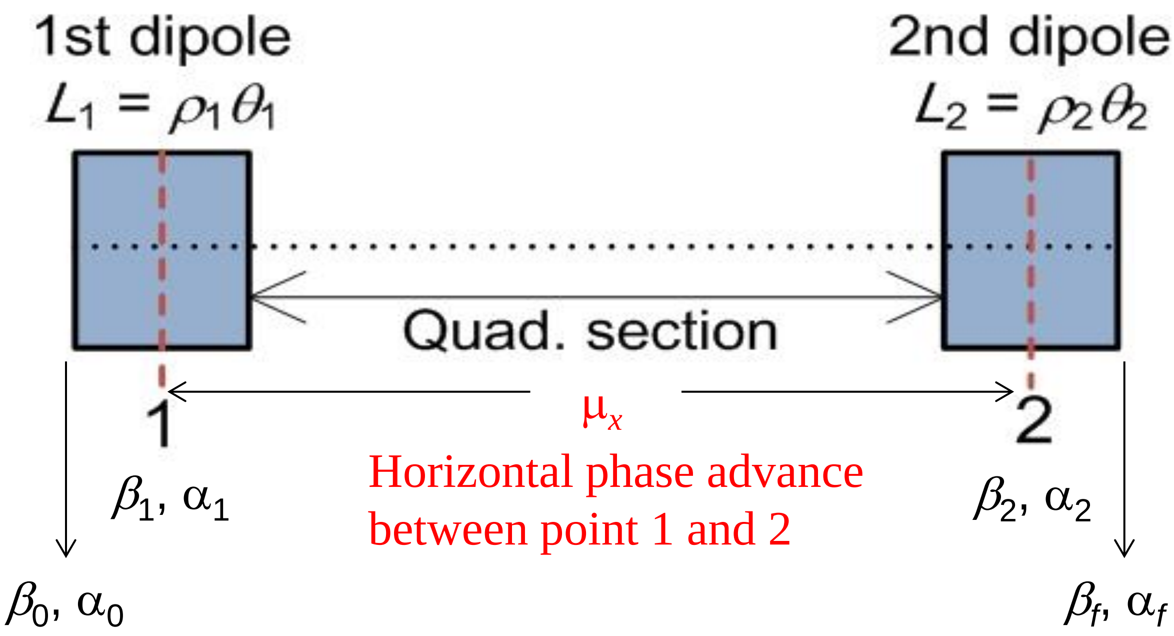
$$x'' = -\frac{x}{\rho^2} + \frac{1}{\rho}\left(\delta_0 + \underbrace{\delta_{CSR} + \kappa\left[s - s_0\right]}_{\text{for CSR}}\right)$$

$$R_{bend} = \begin{pmatrix} \cos\theta & \rho\sin\theta & \rho(1-\cos\theta) & \rho(1-\cos\theta) & \rho^2(\theta-\sin\theta) \\ -\rho^{-1}\sin\theta & \cos\theta & \sin\theta & \sin\theta & \rho(1-\cos\theta) \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & \rho\theta \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Single kick discription



Two-dipole achromat



Through straightforward derivation, one can find that the **Achromatic Condition** is

$\mu_x = \pi$ (modulus 2π) if the dipoles deflect beam in the **same** direction,
$\mu_x = 0$ (modulus 2π) if the dipoles deflect beam in the **opposite** direction.

CSR-cancellation condition in linear regime

$$r_{15} = [\frac{C_2(2S_1 - C_1\theta_1)}{c} + S_2^2\theta_1 r + (\theta_2 - \sin\theta_2) r^{4/3}]\rho_1^{4/3} + \frac{cS_2(\alpha_1 - \alpha_2)(2S_1 - C_1\theta_1)}{\beta_1}\rho_1^{7/3} r,$$

$$r_{25} = [S_2(2r^{1/3}S_2 + C_2\theta_1) - \frac{S_1(2S_1 - C_1\theta_1)}{r}]\rho_1^{1/3} + \frac{cC_2(\alpha_1 - \alpha_2)(2S_1 - C_1\theta_1)}{\beta_1}\rho_1^{4/3},$$

Solutions of $r_{15} = r_{25} = 0$

$$r^* \equiv (\frac{\rho_2}{\rho_1})^* = sign(\frac{\theta_2}{\theta_1})(\frac{S_1(2S_1 - C_1\theta_1)}{S_2(2S_2 - C_2\theta_2)})^{3/4} \cong (\frac{\theta_1}{\theta_2})^3 \cong \frac{1}{c^3}, \quad or \quad L_1\theta_1^2 \cong L_2\theta_2^2,$$

A

$$(\frac{\alpha_1 - \alpha_2}{\beta_1})^* = \frac{\theta_1 + r^{*1/3}\theta_2}{\rho_1(-2 + \theta_1 C_1 / S_1)} \cong -\frac{12}{\rho_1\theta_1} = -\frac{12}{L_1}.$$

B

The matrix between the centers of the dipoles

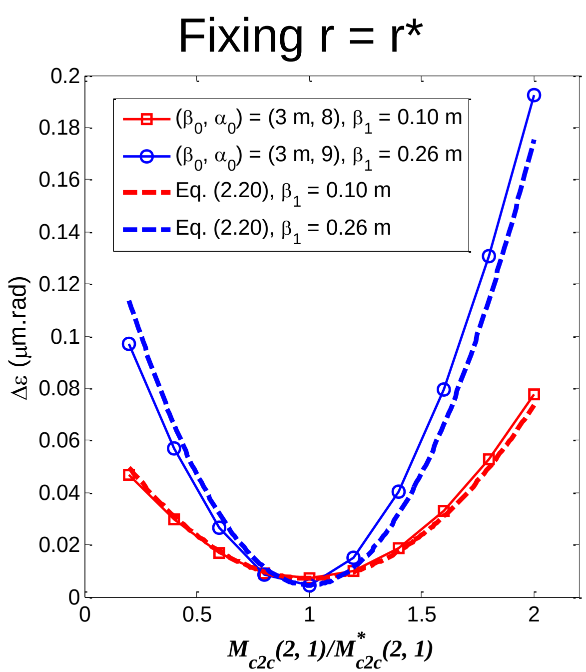
$$M_{c2c}^* \cong \begin{pmatrix} -\frac{1}{c} & 0 & \rho_2(1-C_2) + \frac{\rho_1}{c}(1-C_1) \\ \frac{12}{L_1}c & -c & -\frac{12\rho_1 c(1-C_1)}{L_1} \\ 0 & 0 & 1 \end{pmatrix}$$

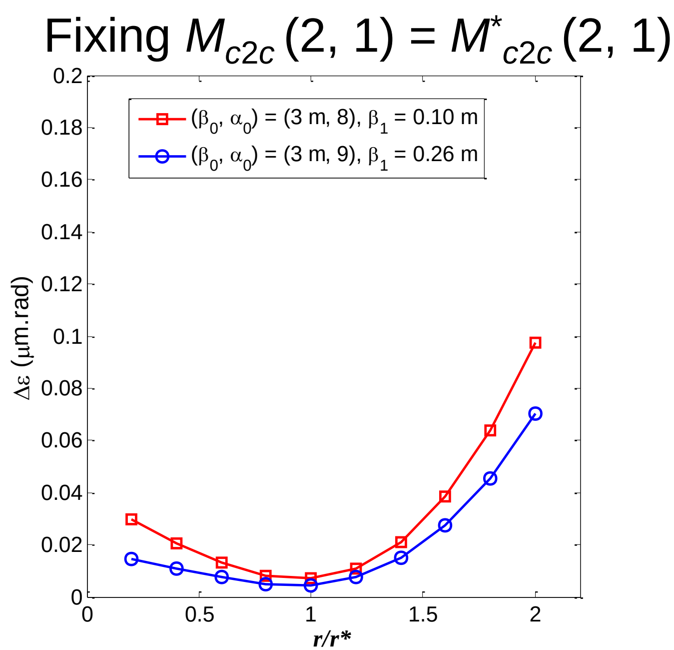
$c \equiv S_2/S_1$, $S_1 = \sin(\theta_1/2)$, $S_2 = \sin(\theta_2/2)$, $C_1 = \cos(\theta_1/2)$, $C_2 = \cos(\theta_2/2)$

One example

- θ_1 = 6 degrees, θ_2 = 4 degrees, and ρ_1 = 8 m

➡ $c \cong 2/3$, $r^* \cong 27/8$, $\rho_2^* \cong 27$ m, and $M^*_{c2c}(2, 1) \cong 30/\pi$





If only condition A ($r = r^*$, $L_1\theta_1^2 = L_2\theta_2^2$) is fulfilled (dashed lines in left plot)

$$\Delta\varepsilon|_{r=r^*} = k_{rms}^2 S_1^2(\theta_1 + r^{*1/3}\theta_2)^2 \rho_1^{2/3}\beta_1[1-(\frac{\alpha_1-\alpha_2}{\beta_1})/(\frac{\alpha_1-\alpha_2}{\beta_1})^*]^2.$$

Two identical bending magnet cases

$$r^* \equiv (\frac{\rho_2}{\rho_1})^* = sign(\frac{\theta_2}{\theta_1})(\frac{S_1(2S_1 - C_1\theta_1)}{S_2(2S_2 - C_2\theta_2)})^{3/4} \cong (\frac{\theta_1}{\theta_2})^3 \cong \frac{1}{c^3}, \quad or \quad L_1\theta_1^2 \cong L_2\theta_2^2,$$

A

$$(\frac{\alpha_1 - \alpha_2}{\beta_1})^* = \frac{\theta_1 + r^{*1/3}\theta_2}{\rho_1(-2 + \theta_1 C_1 / S_1)} \cong -\frac{12}{\rho_1\theta_1} = -\frac{12}{L_1}.$$

B

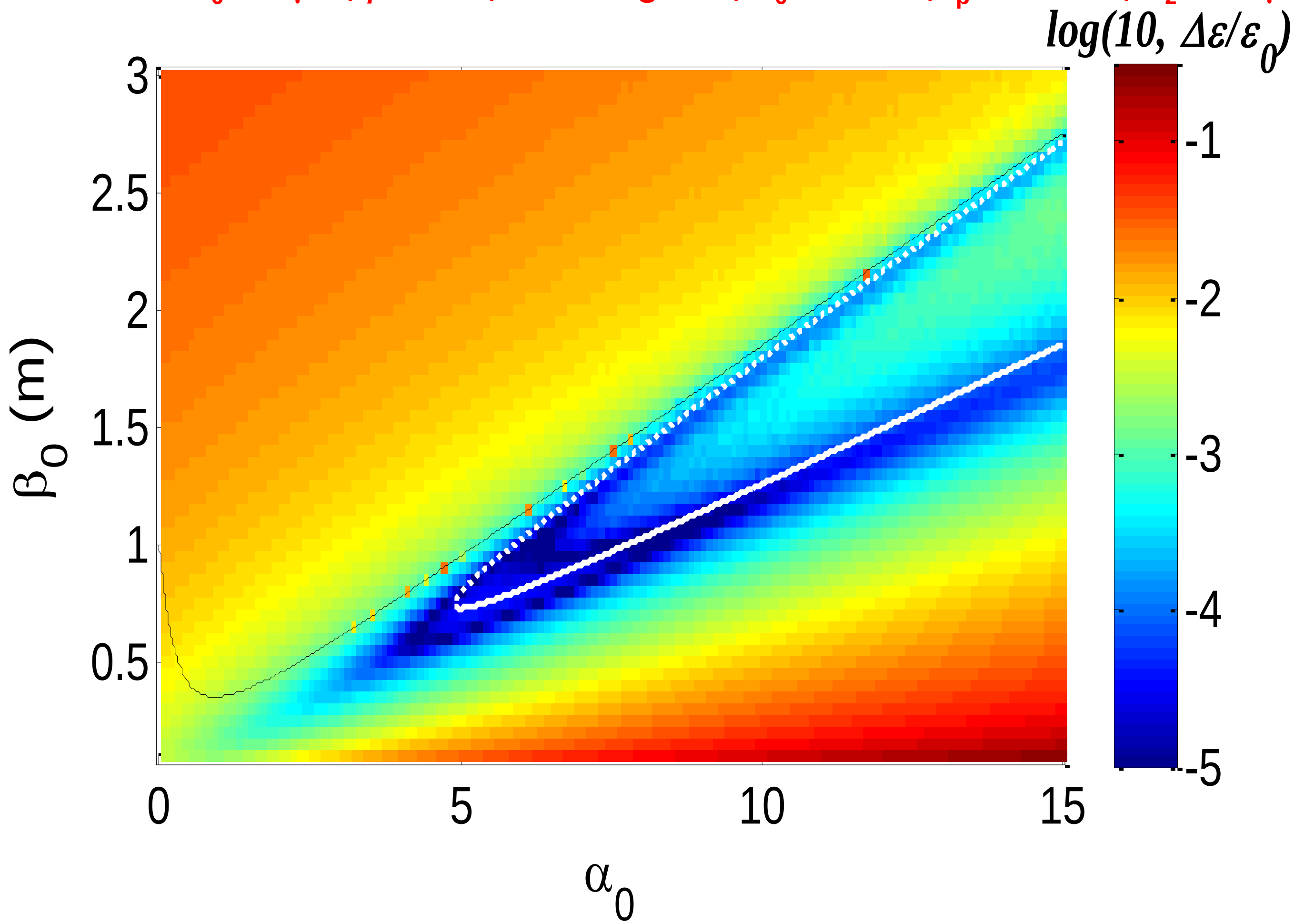
If the Twiss parameters are also symmetric, we can get:

$$\beta_1 = -1/6\alpha_1 L$$

CSR cancellation vs. Envelop matching

- **The proposed conditions have higher efficiency in CSR-suppression than envelope matching method (black line).**

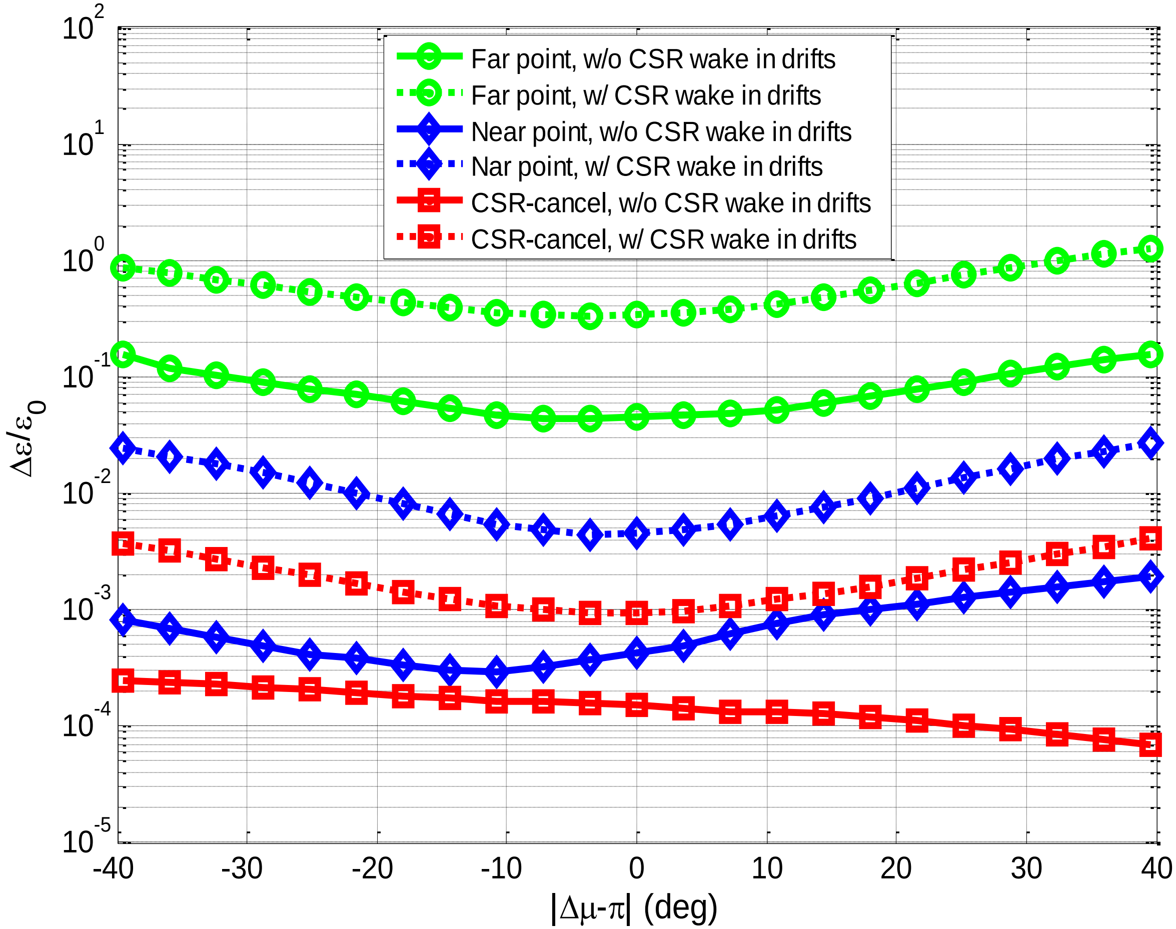
Initial emittance ε_0 = 2 μm, ρ = 6 m, θ = 3 degrees, E_0 = 1 GeV, I_p = 5000 A, σ_z = 30 μm, σ_δ = 0.05%.

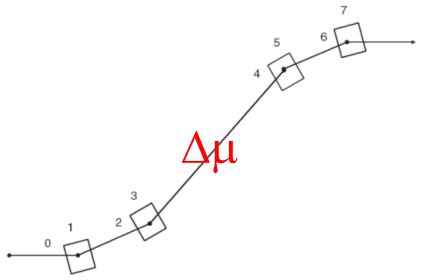


CSR cancellation vs. Phase matching

- **The proposed conditions have higher efficiency in CSR-suppression than envelope matching method (green lines).**

Two identical DBAs, $\Delta\mu$ is the phase advance between two DBAs.





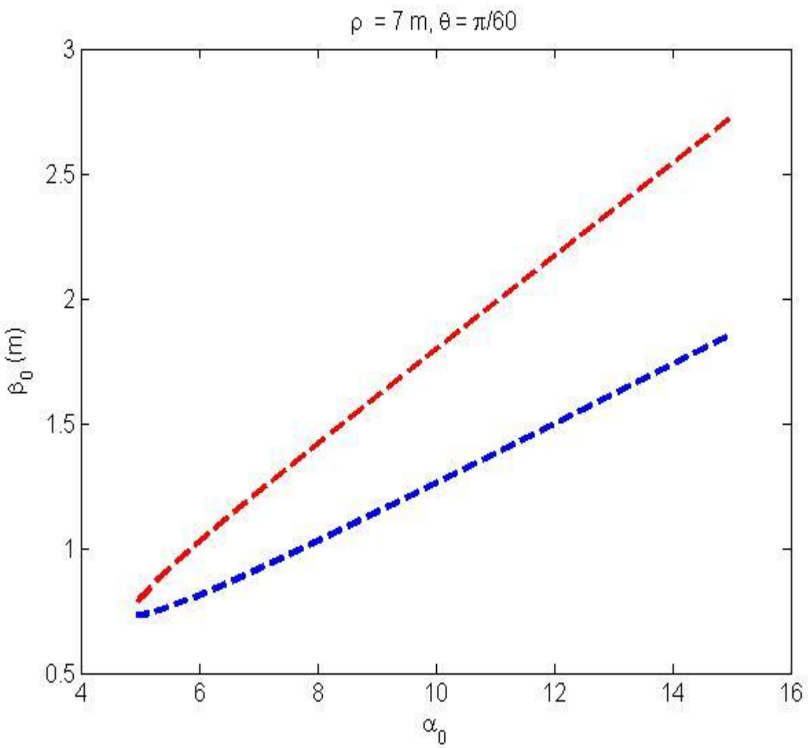
Green lines: (β_0, α_0) far away from the $\beta_{0,1}$ curve

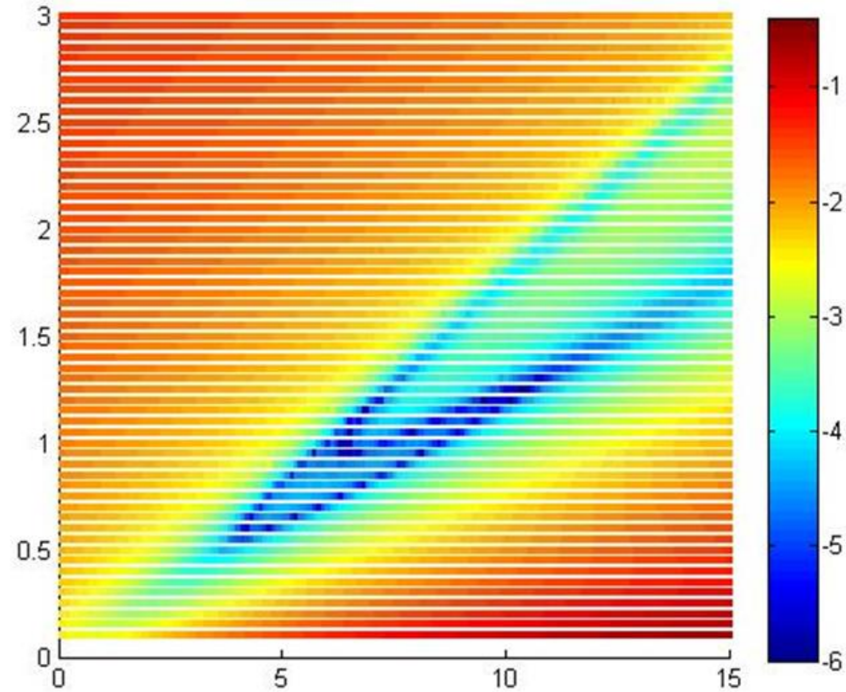
Blue lines: (β_0, α_0) near the $\beta_{0,1}$ curve

Red lines: (β_0, α_0) on the $\beta_{0,1}$ curve

Theoretical and Numerical simulation results

Initial emittance: 2 μm.rad, bunch length 30 mm, peak current 5000 A





The CSR cancellation constraints on the Twiss parameters at the entrance of the DBA cell.

Thanks for your attention!