LHeC Workshop 2014

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QCD at N3LO in Hadronic Collisions

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Motivation

Measuring fundamental parameters and finding deviations from the Standard Model will require extremely high precision from both experiment and theory.

At the LHC the need for precision has driven us to compute NLO QCD corrections to most and even NNLO QCD corrections to some processes.

The LHC Higgs cross section and coupling measurements are now exceeding the precision of the NNLO uncertainties. Mostly because the Higgs cross section has a particularly bad perturbative convergence!

To improve on the theoretical uncertainties the N3LO correction will be necessary.

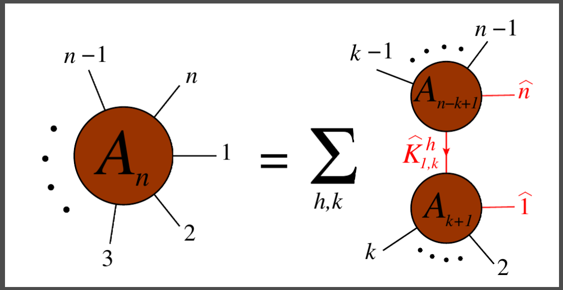
Brief Overview

- Status of higher order QCD in hadronic collisions
  - LO & NLO
  - NNLO
- Towards LHC Higgs production at N3LO
  - Challenges
  - Weaponry: Reverse Unitarity, IBPs, Diff. Eqts, Asympt. Exp
  - Current Status
- Conclusions

Perturbative QCD at LO and NLO

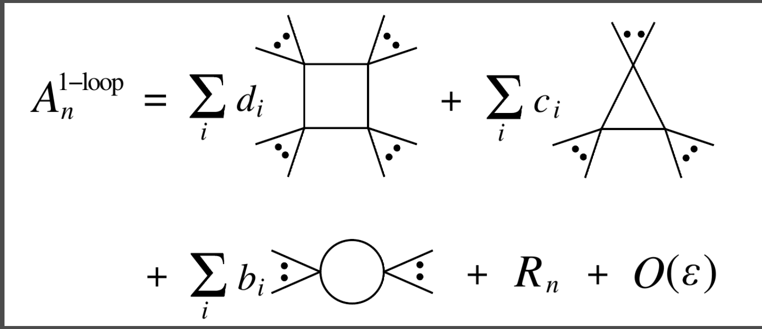
LO:

- BG and BCFW Recursion,
  Analytical formulas from N=4 SYM
- Current frontier ~10 jets at LHC



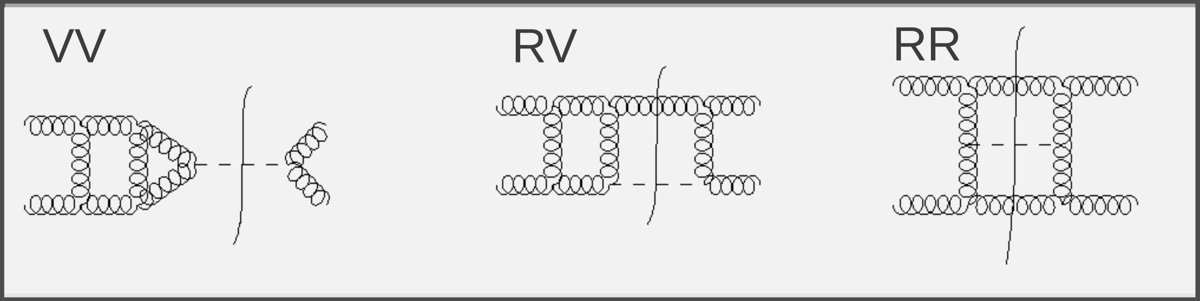
NLO:

- 1-loop amplitudes can be extracted from trees via Unitarity/OPP
- Subtraction of IR divergences of the real emission fully automated FKS/Dipoles
- Current frontier ~ 5 jets at LHC



Perturbative QCD at NNLO

Is still a **big** challenge!



VV:
Still don't know all master integrals for 2 → 2!
Problems related to number of internal and external mass scales and non-planarity.

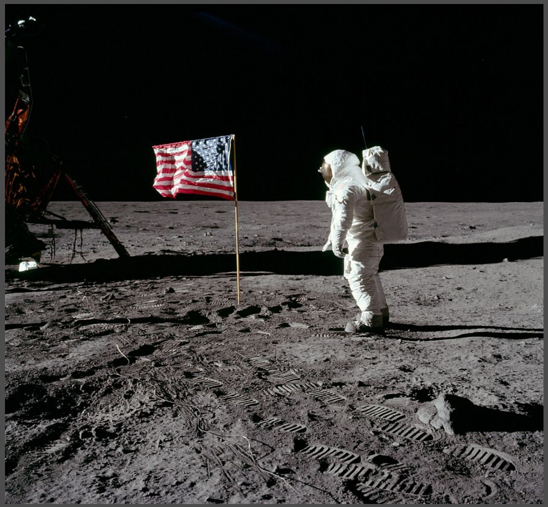
RV:
One loop amplitudes are known. One loop counter terms are also known.

Numerical stability is still an issue!

RR:
Trees known
Infra-red divergences become very complicated.
Subtraction has been Achieved for 2 → 1&2

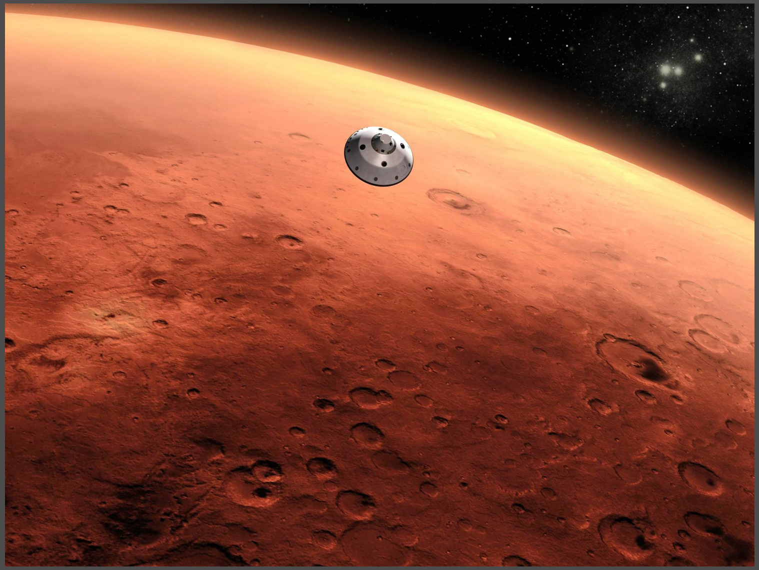
Still no truly satisfactory subtraction scheme!

Calculations which have been done:
H,W, Z, WH, ttbar (inclusive only), diphoton, gg->gg, gg->Hg



We just about reached NNLO.

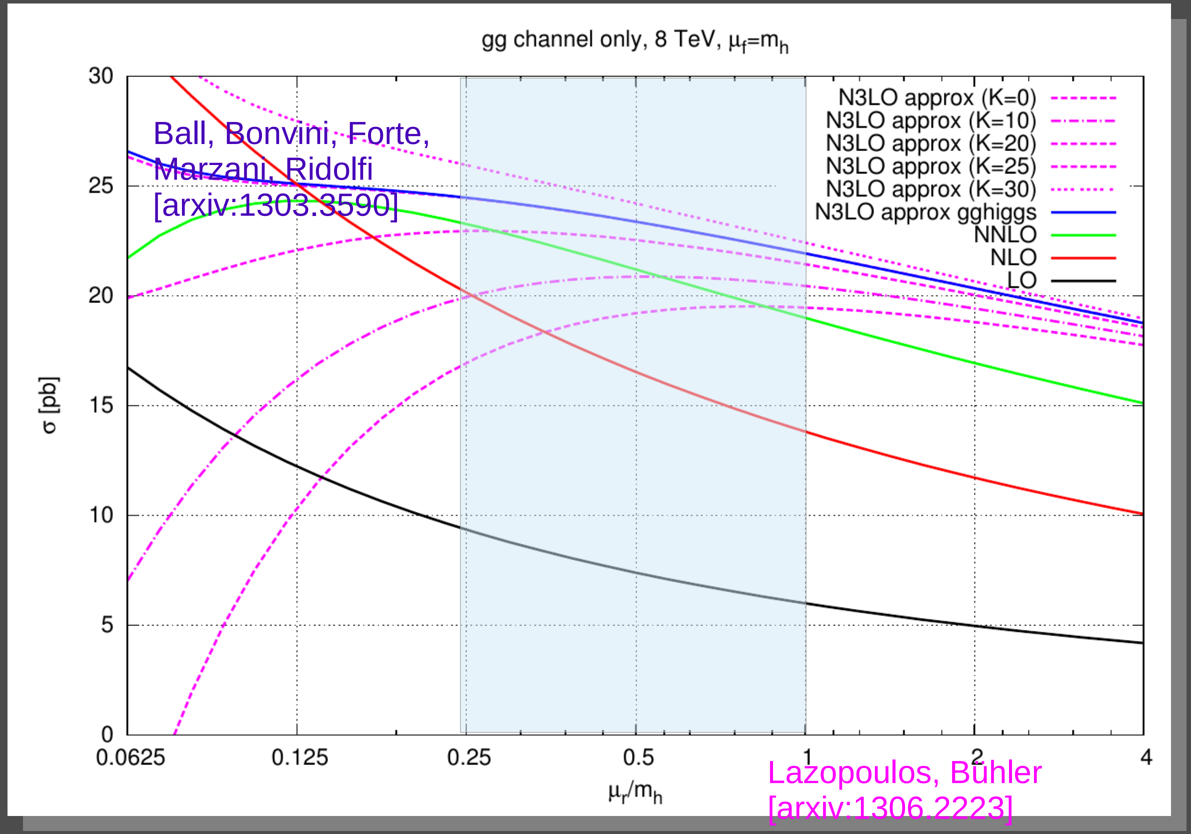
Are we sure we are ready for N3LO?



What is the Scale uncertainty of the LHC Higgs Cross section?

$$\sigma^{\rm N3LO}_{PP\to H+X} = \alpha_s(\mu_R)^5 \left[K + f(\sigma^{\rm lower\ orders}_{PP\to H+X}, \log\mu_R) \right]$$

N3LO approx
Soft logs +
BFKL



Lazopoulos, Bühler
[arxiv:1306.2223]

The ultimate precision at N3LO for LHC Higgs production

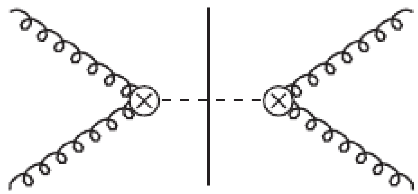
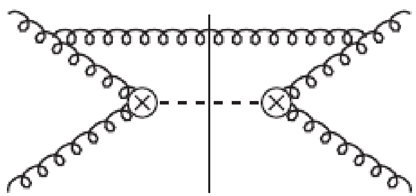
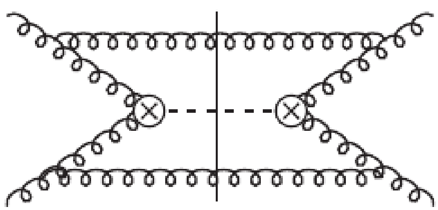
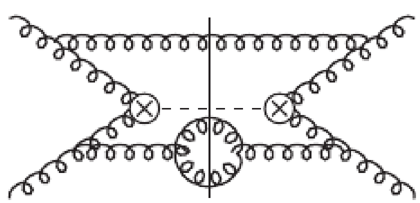
Order	Cross section [pb]	$\sigma/\sigma_{\mathrm{NNLO}}$	$\sigma/\sigma_{\mathrm{LO}}$
LO	$10.31\,^{+26.9\%}_{-16.6\%}$	0.51	1.00
NLO	$17.41\,^{+20.8\%}_{-12.7\%}$	0.86	1.69
NNLO	$20.27\,^{+8.3\%}_{-7.1\%}$	1.00	1.97
$\mathrm{N^3LO}$ (K=0)	$18.53\,^{+1.2\%}_{-7.9\%}$	0.91	1.80
$\mathrm{N^3LO}$ (K=5)	$19.23\,^{+0.3\%}_{-5.1\%}$	0.95	1.87
$\mathrm{N^3LO}$ (K=10)	$19.92\,^{+0.0\%}_{-2.6\%}$	0.98	1.93
$\mathrm{N^3LO}$ (K=15)	$20.62\,^{+0.4\%}_{-2.2\%}$	1.02	2.00
$\mathrm{N^3LO}$ (K=20)	$21.31\,^{+2.0\%}_{-3.1\%}$	1.05	2.07
$\mathrm{N^3LO}$ (K=30)	$22.70\,^{+6.0\%}_{-4.9\%}$	1.12	2.20
$\mathrm{N^3LO}$ (K=40)	$24.09\,^{+9.6\%}_{-6.5\%}$	1.19	2.34

Challenges at N3LO

- No calculation has been done before at this order for a hadron collider
- Quantity and complexity of integrals challenges our current methodology for higher order calculations.
- Problem of infra-red divergences even more pronounced! Most singular limits unknown! IR poles up to $\frac{1}{\epsilon^6}$.
- For LHC Higgs production an analytical evaluation of the inclusive cross section should be feasible, since partonic cross section depends only on a single parameter

$$z = \frac{m_H^2}{\hat{s}}$$

Growth of Complexity of real emissions

LO		1 diagram	1 integral
NLO		10 diagrams	1 integral
NNLO		381 diagrams	18 integrals
N3LO		26565 diagrams	~200 integrals

[Graphics by Claude Duhr]

Tools for N3LO: Reverse Unitarity and differential equations

- Use reverse unitarity to derive Integration by parts integrals and set of Master Integrals:

$$\delta_+(q^2) \to \left(\frac{1}{q^2}\right)_c \equiv \frac{1}{2\pi i}\,\mathrm{Disc}\,\frac{1}{q^2} = \frac{1}{2\pi i}\left(\frac{1}{q^2+i0} - \frac{1}{q^2-i0}\right)$$

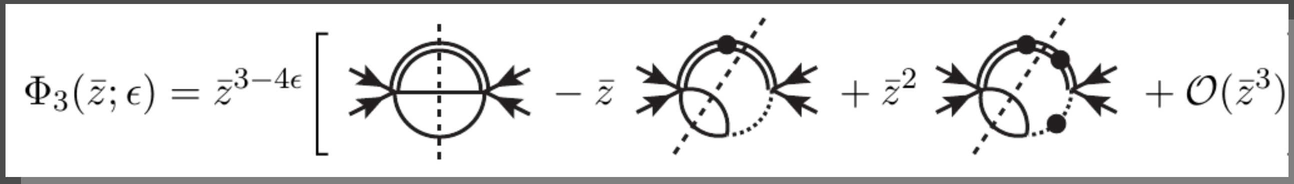
- Establish a system of differential equations:

$$\frac{\partial}{\partial z}\mathbf{F}_i(z,\epsilon) = \sum_j c_{ij}(z,\epsilon)\,\mathbf{F}_j(z,\epsilon)$$

- Solve boundary condition from the soft limit $z \mapsto 1$

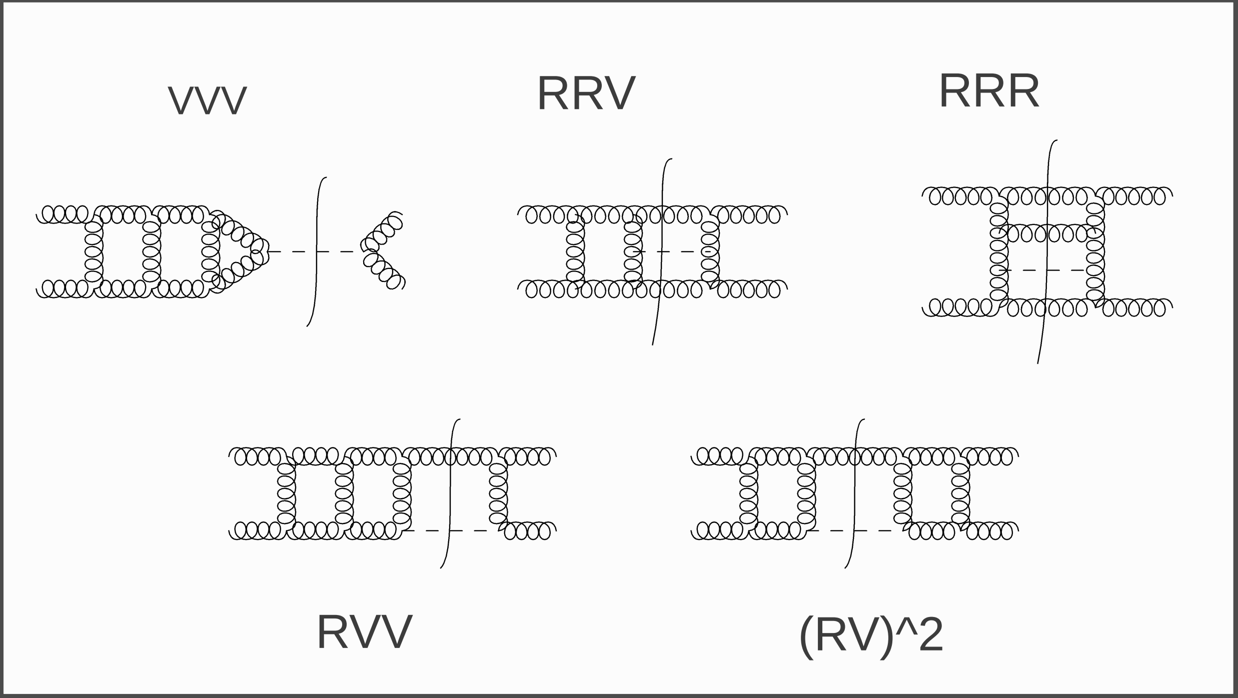
Tools for N3LO: Soft expansion

- The soft limits of the master integrals can be found by using an asymptotic expansion.
- They are related, via a much smaller set of master integrals, which can be found via IBPS.
- It turns out that these soft masters can also be used to compute higher order coefficients in the soft expansion:



- This expansion is fast converging and therefore yields a very powerful alternative method to the differential equations.

Status of N3LO



VVV:

- Known [Baikov, Chetyrkin, Smirnov, Smirnov, Steinhauser; Gehrmann, Glover, Huber, Ikizlerli, Studerus]

RVV: in progress

- 2-loop amplitude known up to O(ε) [...]
- One loop soft current known [Duhr, Gehrmann; Li, Zhu]
- Soft limit known [in progress]

(RV)^2:

- Known [Anastasiou, Duhr, Dulat, FH, Mistlberger; Kilgore]

RRV: in progress

- Soft limit almost finished

RRR:

- Know first two terms in soft expansion [Anasatsaiou, Duhr, Dulat, Mistlberger]

Collinear/UV counterterms:

- known [Pak, Rogal, Steinhauser; Anastasiou, Buehler, Duhr, FH; Höschele, Hoff, Pak, Steinhauser, Ueda; Buehler, Lazopoulos]

+ UV and collinear counter terms

→ We are now very close to assembling the soft approximation of the N3LO cross-section

Conclusions and Outlook

- Summarised the current status of higher order QCD computations, and reported on recent progress towards N3LO.
- While LO and NLO computations are now highly automated (limited only by current computer power) NNLO still is a big challenge.
- For inclusive Higgs cross section the soft limit at N3LO is being assembled as we speak. Higher coefficients in the soft expansion are in close reach.
- For the full cross section we will still need a few years.
- The techniques which we are now developing for the LHC Higgs production should also be useful for other processes at the LHC and possible future Colliders.
- For sure the Drell Yann process contains the same set of Master integrals.
- Also inclusive N3LO QCD corrections to DIS should be feasible with the same methods.