Heavy Ion Theory

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\[ R_{Au} \]

\[ \text{Splat} \]

\[ \frac{R_{Au}}{\gamma} \]
Goal: Extract the medium properties of QGP from this!

Looks like a lot of particles $\sim 15000$, but it a lot smaller than $6 \times 10^{23}$!
1. The “critical” energy density and temperature are

\[ e_c \simeq 1 \text{ GeV/fm}^3 \quad T_c \simeq 160 \text{ MeV} \]

2. What are its transport properties? Shear viscosity?

Need reach an energy density of \( e_c \) over a \underline{Large} volume for \underline{Long} enough.
The most important data – collective correlations

Flow $\equiv$ correlations between the particles over the full phase-space that doesn’t decrease with increasing multiplicity
Flow harmonics and fluctuations:

Quantify the init. e-dense w. eccentricity:

$$\epsilon_2 = -\frac{\langle r^2 \cos(2(\phi_s - \Phi_2)) \rangle}{\langle r^2 \rangle}$$

and orientation angle $\Phi_2$:

$$\Phi_2 = \text{orientation angle of ellipse}$$

Expand the observed momentum spectra in a fourier series:

$$\frac{dN}{d\phi_p} = N (1 + 2v_2 \cos(2(\phi_p - \Psi_2)))$$
Higher harmonics in the initial and final state:

Quantify the initial e-dense with triangularity:

\[ \epsilon_3 = -\frac{\langle r^3 \cos(3(\phi_s - \Phi_3)) \rangle}{\langle r^3 \rangle} \]

and orientation angle \( \Phi_3 \):

\[ \Phi_3 = \text{orientation angle of triangle} \]

Expand the observed momentum spectra in a Fourier series:

\[ \frac{dN}{d\phi_p} = N (1 + 2v_2 \cos(2(\phi_p - \Psi_2)) + 2v_3 \cos(3(\phi_p - \Psi_3)) + \ldots) \]
Data on flow coefficients: (measure $\sqrt{\langle v_n^2 \rangle}$)

$$\frac{1}{p_T} \frac{dN}{dp_T dφ} = \frac{1}{p_T} \frac{dN}{dp_T} (1 + 2v_2(p_T) \cos(2φ) + ....)$$

Flow is large $X:Y \sim 2.0 : 1$
Probability distributions of the harmonics (ATLAS):

Probability distribution is somewhat non-gaussian towards peripheral collisions

$$\langle v_3^4 \rangle - 2 \langle v_3^2 \rangle^2 \neq 0$$
Strong correlations between $v_2$ and $v_3$ and $v_5$ and their angles.
More Correlations

Very rich pattern of correlations amounts the observed flow harmonics
Data recapitulation: Collective flow everywhere!

Flow \equiv an all-particle correlation over a large phase space.

1. Flow harmonics \( v_1, v_2, v_3, v_4, v_5 \ldots \) are large.

2. Fluctuations in the flow are large and (somewhat) non-gaussian.

3. There are strong dynamical correlations between the harmonics.

A hydrodynamic theory of QGP needs to reproduce these trends!
Hydrodynamics:

- For hydrodynamics need:
  \[
  \frac{\ell_{\text{mfp}}}{R_{\text{Au}}} \ll 1 \quad \text{and} \quad \frac{\ell_{\text{mfp}}}{c\tau} \ll 1
  \]

- How to define \( \ell_{\text{mfp}} \)?
  \[
  \ell_{\text{mfp}} \sim \frac{\eta}{e + p} \quad \text{and} \quad e + p = sT
  \]

Condition:

\[
\frac{\eta}{s} \times \frac{1}{\tau T} \ll 1
\]

Medium Property \quad \text{Experimental Property} \sim 1/2

To have a strong hydrodynamic response need the shear viscosity to entropy ratio small:

\[
\eta/s < \frac{1}{2}
\]
What does $\eta/s < \frac{1}{2}$ mean theoretically?

- **Perturbation theory:**
  (Baym and Pethick, Arnold, Moore, Yaffe)
  - Kinetic theory of quarks and gluons + soft gauge fields + collinear emission

\[ \frac{\eta}{s} \simeq 0.3 \left( \frac{0.5}{\alpha_s} \right)^2 \hbar \]

- **$\mathcal{N} = 4$ Super Yang Mills at strong coupling**
  (Kovtun, Son, Starinets, Policastro)
  - No quasi-particles.

\[ \frac{\eta}{s} = \frac{\hbar}{4\pi} = 0.08 \hbar \quad \Longrightarrow \quad \text{A useful limit} \]

The plasma can't be very weakly coupled to support hydrodynamics in heavy ion collisions!

Measuring the shear viscosity will determine what QGP is like.
Viscous Hydrodynamic Equations

\[ T^{\mu\nu} = e u^\mu u^\nu + p g^{\mu\nu} + \pi^{\mu\nu} \]

- **Stress tensor**
- **Ideal fluid**
- **Viscous corrections**

- **First order in gradients Navier stokes theory**

\[ \pi = \pi^{\mu\nu}_{(1)} \equiv -\eta \left( \nabla^\mu u^\nu + \nabla^\nu u^\mu - \frac{2}{3} \Delta^\mu \nabla \cdot u \right) \]

- **Second order**

\[ \pi^{\mu\nu} = \pi^{\mu\nu}_{(1)} + O(\epsilon^2) \]

- Second derivatives

- **For example:**

\[ \pi^{\mu\nu} = \pi^{\mu\nu}_{(1)} - \tau_\pi D_t \pi^{(\mu\nu)} + \text{Other 2nd derivs} \]
Qualitative effect of viscosity on the fluctuations

Initial

Final Ideal

Final Visc.

Viscosity damps fluctuations!
Basic linear response theory for $v_2$ and $v_3$

In a linear response approximation the $v_2$ comes from geometric correl in the initial state:

$$v_2 e^{i2\Psi_2} = k_2 \epsilon_2 e^{i2\Phi_2}$$

Linear response to deformation

So the rms $v_2$ is a combination of the linear response coefficients and geometry

$$\sqrt{\langle v_2^2 \rangle} = v_2 \{2\} = k_2 \times \sqrt{\langle \epsilon_2^2 \rangle}$$

Response coefficients $k_2, k_3$ depend on $\eta/s$, harmonic order, system size in specific ways
Hydrodynamic Simulations of $v_n$ with $\eta/s = 0.2$

(Gale, Jeon, Schenke, Tribedy, Venugopalan)

viscous corrections and system size

momentum dependence of corrections

Hydro fits the $v_n$ data due to the unique-characteristics of viscous hydro
Hydrodynamic simulations of $v_n$ fluctuations

(Gale, Jeon, Schenke, Tribedy, Venugopalan)

- Hydro reproduces the fluctuations in the $v_n$

$$\eta/s \simeq 0.2$$

- Non-gaussianity because the $\epsilon_n$ is bounded:

$$\epsilon_n < 1$$

(Ollitrault, Yan)

The eccentricity fluctuates in a universal (non)-gaussian way.

Can use to find $\langle \epsilon_n \rangle$ in a mod. independent way!
In a linear response approximation the $v_5$ comes from geometric correl in the initial state:

$$v_5 e^{i5\Psi_5} = k e_5 e^{i5\Phi_5}$$

Linear response to deformation
The non-linear response provides additional mode mixing correlating 2-3-5

\[ v_5 e^{i5\Psi_5} = k_{\text{lin}} \epsilon_5 e^{i5\Phi_5} + k_{\text{non-lin}} \epsilon_2 \epsilon_3 e^{i2\Phi_2 + 3\Phi_3} \]

Linear response to 5-deformation
Non-linear response due to 2-3 mixing
Origin of the event-plane correlations:

\[ \langle \cos(2\Psi_2 + 3\Psi_3 - 5\Psi_5) \rangle \]

The relative amplitudes of the linear and non-linear response is determined by hydrodynamics with the right \( \eta/s \)!
Hydrodynamic Summary

- The hydrodynamic expansions is converging wonderfully.
- A rich pattern to viscous corrections. Many more measurements.
  - Flow Fluctuations
  - Non-linear response
  - Event classes
  - Longitudinal structure
- We need to extract $\eta/s$ with a global analysis and a careful systematic-error budget

Overall consistency makes the bounds $\frac{1}{4\pi} < \frac{\eta}{s} < \frac{4}{4\pi}$ quite believable
Thermalization of QGP at weak coupling
Gluons at small $x$ are over-occupied – can be treated as a classical field.

1. After the collisions the classical gluon field evolves

2. Classical evolution is unstable (the UV catastrophe):
   - There is a turbulent self-similar cascade of non-abelian fields to the UV

3. After the field strength dilutes, we use the QCD-Boltzmann equation to evolve:
   
   A first principles approach from ultra-high energy wave-functions to equilibration!
- Describe the non-expanding case. (But the expanding case is done!)
- Initially modes are highly occupied to up to the saturation momentum.

\[ f_g \equiv \langle a_p^\dagger a_p \rangle = \frac{1}{\alpha} \theta(Q_s - |p|) \]

- Evolve on a big periodic lattice (256 × 256 × 4096)
Find a fixed point of the classical evolution:

\[ f(t, p) = (Q_s t)^\alpha f_S((Q_s t)^\beta) \]

UV cascade w. universal exponents given by \( \alpha = -4/7 \) and \( \beta = -1/7 \) from kinetics
Then use the QCD-Boltzmann Equation

- Kinetics ≡ scattering + screening + collinear emission

\[ \lambda = 4\pi \alpha_s N_c \]

\[ \text{Equilibrium Value} \]

\[ \text{Mean gluon momentum relative to equilb. value} \]

\[ \text{Rescaled time: } \lambda^2 Tt \left(1+C_2 \log \lambda^{-1}\right) \]
Thermalization:

- Find that the overpopulated gluon fields (in box) thermalize on a time scale:
  \[ \lambda = 4\pi \alpha_s N_c \]

  \[ \tau_{eq} = \frac{72.0}{1 + 0.12 \log(\lambda^{-1})} \frac{1}{\lambda^2 T} \]

  - precisely defined
  - fit to numerics

- Expanding case more complex but same ingredients:
  1. Universal classical cascade to the UV characterized by exponents
  2. Boltzmann equation with collinear emission follows the classical evolution

- Best estimate:
  \[ \tau_{eq} \approx 0.5 \text{ fm} \left( \frac{2.0 \text{ GeV}}{Q_s} \right) \left( \frac{0.3}{\alpha_s} \right)^{13/5} \]
Summary – two aspects of QGP and its equilibrium description:

1. Rich flow phenomenology together with a convergent hydrodynamic description.

2. Beautiful weak coupling picture for the thermalization process
   - Pressing need to find signatures which test this thermalization picture
   - It is hard because equilibrium forgets the past!

Thank you!