Top theory
What role for precision in top-quark physics?

Sven-Olaf Moch

Universität Hamburg & DESY, Zeuthen

Large Hadron Collider Physics Conference, New York, June 06, 2014
Plan

Theory status
- Top-quark pair-production
- Single top-quark production
- Electroweak effects
- Asymmetries

Big question
- Top-quark mass
Why precision?

**Long history of indirect searches for new physics**

- In absence of direct evidence for new physics look for deviations in precision measurements
- Constraints for parameter space through precision tests
  - e.g., LEP electroweak precision data predicts low Higgs mass $M_H$

$$\begin{align*}
\Delta \chi^2 &= \Delta \alpha_{\text{had}}^{(5)} \\
0.02750 \pm 0.00033 \\
0.02749 \pm 0.00010 \\
\text{incl. low } Q^2 \text{ data}
\end{align*}$$

Limit $M_H = 152$ GeV

[Graph showing precision limits with LEP and LHC excluded regions.]

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After the Higgs boson discovery

Electroweak precision fits

- Extension to MSSM [Heinemeyer, Hollik, Stöckinger, Weiglein, Zeune ’12]
- Relations for radiative corrections known through two loops
  - need for precise input parameters: $M_W$ and top-quark mass $m_t$
Electroweak vacuum stability

- Phase diagram if Standard Model extrapolated up to Planck scale $M_P$
  - relation between Higgs mass $m_H$ and top quark mass $m_t$
  - condition of absolute electroweak vacuum stability dependent on $m_t$
- Precision on $m_t$ decisive for “fate of universe”

Bezrukov, Kalmykov, Kniehl, Shaposhnikov ‘12;
Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice et al. ‘12;
Alekhin, Djouadi, S.M. ‘12; Masina ‘12;
Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia ‘13;
[many people]
Top-quark pair-production

Rates and shapes

• Requirements for precision of theory predictions
  • driven by accuracy of experimental data
  • QCD corrections at least to NLO in QCD
• NNLO QCD corrections mandatory if data accuracy of $O(10\%)$ or better
  • inclusive cross section for top-quark pairs at LHC8
  • expected for differential distributions in run II
• Electroweak corrections become important as well, since order $\alpha_s^2 \sim \alpha_{EW}$
Total cross section

Exact result at NNLO in QCD

Czakon, Fiedler, Mitov ’13

- NNLO perturbative corrections (e.g. at LHC8)
  - $K$-factor (NLO $\rightarrow$ NNLO) of $\mathcal{O}(10\%)$; scale stability of $\mathcal{O}(\pm 5\%)$
- Beyond NNLO
  - theory improvements with soft gluon resummation [many people]
  - $K$-factor (NNLO $\rightarrow$ resummed) small; scale stability further improved
Top-quark mass from total cross section

• Intrinsic limitation of sensitivity in total cross section

\[ \left| \frac{\Delta \sigma_{t\bar{t}}}{\sigma_{t\bar{t}}} \right| \simeq 5 \times \left| \frac{\Delta m_t}{m_t} \right| \]

• QCD factorization for cross section

\[ \sigma_{pp \to X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij \to X}(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2) \]

• joint dependence on non-perturbative parameters: parton distribution functions \( f_i \), strong coupling \( \alpha_s \), masses \( m_X \)

Correlations are essential

• Cross section at LHC has correlation of \( m_t, \alpha_S(M_Z) \) and gluon PDF

\[ \sigma_{t\bar{t}} \sim \alpha_s^2 m_t^2 g(x) \otimes g(x) \]

• effective parton \( \langle x \rangle \sim 2m_t/\sqrt{s} \sim 2.5 \ldots 5 \cdot 10^{-2} \)

• fit with fixed values of \( m_t \) and \( \alpha_S(M_Z) \) carries significant bias
  Czakon, Mangano, Mitov, Rojo ‘13

• fit with PDF re-weighting and fixed values of \( m_t \) insufficient
  Beneke, Falgari, Klein, Piclum, Schwinn, Ubiali, Yan ‘12
Going differential

Shapes

• NNLO QCD corrections mandatory if data accuracy of $O(10\%)$ or better
• Case for:
  • inclusive cross section for top-quark pairs at LHC8
  • expected for differential distributions in run II

Strategy

• In absence of complete NNLO QCD predictions can resort to approximations
• Exploit Sudakov resummation of soft gluon emission
  • physical cross sections convolution with parton luminosity dominated by threshold logarithms
• Approximate NNLO results in (semi-inclusive) kinematics
  Kidonakis, Sterman ‘96; . . . ; Ahrens, Ferroglia, Neubert, Pecjak, Yang ‘11; Kidonakis ‘12; Ferroglia, Pecjak, Yang ‘13
• Upshot: single variable differential distributions in $p_T^t$, $y^t$, $m_{t\bar{t}}$ available
Going differential

Shapes

- NNLO QCD corrections mandatory if data accuracy of $\mathcal{O}(10\%)$ or better
- Case for:
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- Upshot: single variable differential distributions in $p_T^t$, $y^t$, $m_{t\bar{t}}$ available
Distributions at (approximately) NNLO

- Approximate NNLO results show trend in right direction for comparison with data
- Great opportunity for high statistics measurements in run II
  - fast and reliable numerical codes are essential (interface to fastNLO or Applgrid)

Guzzi, Lipka, S.M. ‘14

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Tools 4 you: available codes

Total cross section

• NNLO QCD corrections and soft gluon resummation
  • Hathor Aliev, Lacker, Langenfeld, Moch, Uwer, Wiedermann [arXiv:1007.1327]
  • Top++ Czakon, Mitov [arXiv:1112.5675]
  • TOPIXS Beneke, Falgari, Klein, Piclum, Schwinn, Ubiali, Yan [arXiv:1206.2454]

Differential distributions

• NLO QCD corrections (in association with jets, leptons, . . .)
• Fixed order with on-shell top-quarks or decays
  • MCFM Campbell, Ellis [arXiv:1204.1513]
• Matched to parton shower
  • MC@NLO Frixione, Webber [hep-ph/0305252]
  • POWHEG-BOX Alioli, Nason, Oleari, Re [arXiv:1002.2581]
    with contributions of [many people]
  • aMC@NLO Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Shao, Stelzer, Torrielli, Zaro [arXiv:1405.0301]
**Electroweak corrections**

- **Electroweak corrections (ratio of $\sigma_{\text{EW}}/\sigma_{\text{LO}}$)**
  - Beenakker, Denner, Hollik, Mertig, Sack, Wackeroth ‘94; Bernreuther, Fücker ‘05;
  - Kühn, Scharf, Uwer ‘06

![Graph](image)

- **Left:** $\sigma_{\text{EW}}/\sigma_{\text{LO}}$ as function of total cms energy (effect depends on Higgs mass and Higgs-Yukawa coupling)

- **Right:** $\sigma_{\text{EW}}/\sigma_{\text{LO}}$ as function of top-quark mass: $\mathcal{O}(2\%)$ with respect to $\sigma_{\text{LO}}$ negative contribution to total cross section

- **New:** NLO EW corrections now included in update of Hathor (v2.1)
  - Kühn, Scharf, Uwer ‘13

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Top theory – p.12
Single top-quark production

- Study of charged-current weak interaction of top quark
- $s$-channel production

- $t$-channel production
  - $bg$-channel at NLO enhanced by gluon luminosity

- $Wt$-production
  - contributes at LHC (small at Tevatron)
Theory status

- QCD corrections known
  - complete NLO  
    Harris, Laenen, Phaf, Sullivan, Weinzierl ‘02; Sullivan ‘04; 
    Campbell, Frederix, Maltoni, Tramontano ‘09; [+ many people]
  - implementations merged with parton shower
    - in MC@NLO  
      Frixione, Laenen, Motylinski, Webber, White ‘09
    - in POWHEG  
      Aioli, Nason, Oleari, Re ‘09
- approximate NNLO corrections  
  Kidonakis ‘11, ‘13

Treatment of heavy quarks

- Scheme with $n_l = 4$ light flavors + heavy quark of mass $m_b$ at low scales
  - no mass singularities for $m_b, m_t \gg \Lambda_{QCD}$, no (evolving) PDFs
- Scheme with $n_l = 5$ light flavors
  - $b$ PDF for $Q \gg m_b$ generated perturbatively
Single top-quark $t$-channel production

- **New:** Approximate NNLO QCD corrections in Hathor v2.0
  Kant, Kind, Kintscher, Lohse, Martini, Mölbitz, Rieck, Uwe [to appear]
QCD corrections at NNLO

- **New:** computation of NNLO QCD corrections by Brucherseifer, Caola, Melnikov ‘14
  - fully differential, with cuts on $p_T$
- QCD corrections treated in structure function approach
  - non-factorizable contributions neglected
    (neglected diagrams $O(1/N_c^2)$ suppressed)
- QCD corrections to $t$-channel single top quark production at LHC8

<table>
<thead>
<tr>
<th>$p_\perp$</th>
<th>$\sigma_{\text{LO}}, \text{pb}$</th>
<th>$\sigma_{\text{NLO}}, \text{pb}$</th>
<th>$\delta_{\text{NLO}}$</th>
<th>$\sigma_{\text{NNLO}}, \text{pb}$</th>
<th>$\delta_{\text{NNLO}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 GeV</td>
<td>$53.8^{+3.0}_{-4.3}$</td>
<td>$55.1^{+1.6}_{-0.9}$</td>
<td>+2.4%</td>
<td>$54.2^{+0.5}_{-0.2}$</td>
<td>−1.6%</td>
</tr>
<tr>
<td>20 GeV</td>
<td>$46.6^{+2.5}_{-3.7}$</td>
<td>$48.9^{+1.2}_{-0.5}$</td>
<td>+4.9%</td>
<td>$48.3^{+0.3}_{-0.02}$</td>
<td>−1.2%</td>
</tr>
<tr>
<td>40 GeV</td>
<td>$33.4^{+1.7}_{-2.5}$</td>
<td>$36.5^{+0.6}_{-0.03}$</td>
<td>+9.3%</td>
<td>$36.5^{+0.1}_{+0.1}$</td>
<td>−0.1%</td>
</tr>
<tr>
<td>60 GeV</td>
<td>$22.0^{+1.0}_{-1.5}$</td>
<td>$25.0^{+0.2}_{+0.3}$</td>
<td>+13.6%</td>
<td>$25.4^{+0.1}_{+0.2}$</td>
<td>+1.6%</td>
</tr>
</tbody>
</table>
QCD corrections at NNLO

- New: computation of NNLO QCD corrections Brucherseifer, Caola, Melnikov ‘14
  - fully differential, with cuts on $p_T$
- QCD corrections treated in structure function approach
  - non-factorizable contributions neglected (neglected diagrams $O(1/N_c^2)$ suppressed)

\[
\begin{array}{cccc}
q & \rightarrow & q' \\
\quad & b & \quad & t \\
a) \\
q & \rightarrow & q' \\
\quad & b & \quad & t \\
b) \\
q & \rightarrow & q' \\
\quad & b & \quad & t \\
c) \\
q & \rightarrow & q' \\
\quad & b & \quad & t \\
d) \\
\end{array}
\]

- QCD corrections to $t$-channel single anti-top quark production at LHC8

<table>
<thead>
<tr>
<th>$p_\perp$ (GeV)</th>
<th>$\sigma_{\text{LO}}, \text{pb}$</th>
<th>$\sigma_{\text{NLO}}, \text{pb}$</th>
<th>$\delta_{\text{NLO}}$</th>
<th>$\sigma_{\text{NNLO}}, \text{pb}$</th>
<th>$\delta_{\text{NNLO}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$29.1^{+1.7}_{-2.4}$</td>
<td>$30.1^{+0.9}_{-0.5}$</td>
<td>$+3.4%$</td>
<td>$29.7^{+0.3}_{-0.1}$</td>
<td>$-1.3%$</td>
</tr>
<tr>
<td>20</td>
<td>$24.8^{+1.4}_{-2.0}$</td>
<td>$26.3^{+0.7}_{-0.3}$</td>
<td>$+6.0%$</td>
<td>$26.2^{+0.01}_{-0.1}$</td>
<td>$-0.4%$</td>
</tr>
<tr>
<td>40</td>
<td>$17.1^{+0.9}_{-1.3}$</td>
<td>$19.1^{+0.3}_{+0.1}$</td>
<td>$+11.7%$</td>
<td>$19.3^{+0.2}_{+0.1}$</td>
<td>$+1.0%$</td>
</tr>
<tr>
<td>60</td>
<td>$10.8^{+0.5}_{-0.7}$</td>
<td>$12.7^{+0.03}_{+0.2}$</td>
<td>$+17.6%$</td>
<td>$12.9^{+0.2}_{+0.2}$</td>
<td>$+1.6%$</td>
</tr>
</tbody>
</table>
Charge asymmetry

- Top-quark charge asymmetry arises from interference between real emission and virtual contributions
  - interference term does not contribute to total cross section (Furry’s theorem)
- Asymmetric contribution for un-integrated $t\bar{t}$ phase space
  - virtual corrections
  - real corrections
- Asymmetry is genuine quantum effect
  - theory predictions effectively leading order only
  - interpretation requires good understanding of the quantum level
Forward-backward asymmetry at Tevatron

• Top-quark forward-backward asymmetry $A_{FB}$ definitions at Tevatron with
  $\Delta y_{t\bar{t}} = y_t - y_{\bar{t}}$

$$A_{FB}^{t\bar{t}} = \frac{N(\Delta y_{t\bar{t}} > 0) - N(\Delta y_{t\bar{t}} > 0)}{N(\Delta y_{t\bar{t}} > 0) + N(\Delta y_{t\bar{t}} > 0)}$$

- Consistent picture from theory predictions include QCD+EW corrections
  Kühn, Rodrigo ‘11; Hollik, Pagani ‘11 or soft gluon resummation
  Almeida, Sterman, Vogelsang ‘08; Ahrens, Ferroglia, Neubert, Pecjak, Yang ‘11

- Two-loop QCD corrections needed for assessment of theory uncertainty
Charge asymmetry at LHC

• Top-quark charge asymmetry $A_C$ at LHC with $\Delta|y| = |y_t| - |y_\bar{t}|$

$$A_C = \frac{N(\Delta|y| > 0) - N(\Delta|y| > 0)}{N(\Delta|y| > 0) + N(\Delta|y| > 0)}$$

• no forward backward charge asymmetry, because initial state at LHC is $P$ symmetric

ATLAS+CMS Preliminary $\sqrt{s} = 7$ TeV
TOPLHCWG March 2014

<table>
<thead>
<tr>
<th></th>
<th>(stat)</th>
<th>(syst)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CMS [PLB 717 (2012) 129]</td>
<td>0.004 ± 0.010 ± 0.011</td>
<td></td>
</tr>
<tr>
<td>ATLAS [JHEP 1402 (2014) 107]</td>
<td>0.006 ± 0.010 ± 0.005</td>
<td></td>
</tr>
<tr>
<td>ATLAS+CMS</td>
<td>0.005 ± 0.007 ± 0.006</td>
<td></td>
</tr>
<tr>
<td>Theory (NLO+EW) [JHEP 1201 (2012) 063]</td>
<td>0.0115 ± 0.0006</td>
<td></td>
</tr>
</tbody>
</table>

• Consistency with theory predictions, but again two-loop QCD corrections needed
Top quark mass

What is the value of the top quark mass? 

\[ m_t = ? \]
Quark masses in Standard Model

- Higgs boson gives mass to matter fields via Higgs-Yukawa coupling
  - large top quark mass $m_t$

QCD

- Classical part of QCD Lagrangian
  $$L = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_b + \sum_{\text{flavors}} \bar{q}_i (i\gamma\partial - m_q)_{ij} q_j$$
  - field strength tensor $F^a_{\mu\nu}$ and matter fields $q_i, \bar{q}_j$
  - covariant derivative $D_{\mu,ij} = \partial_{\mu} \delta_{ij} + ig_s (t_a)_{ij} A^a_{\mu}$

- Formal parameters of the theory (no observables)
  - strong coupling $\alpha_s = g_s^2 / (4\pi)$
  - quark masses $m_q$

Challenge

- Suitable observables for measurements of $\alpha_s, m_q, \ldots$
  - comparison of theory predictions and experimental data
Quark mass renormalization

Pole mass

- Based on (unphysical) concept of top-quark being a free parton

\[ \phi - m_q - \Sigma(p, m_q) \bigg|_{p^2 = m_q^2} \]

- heavy-quark self-energy \( \Sigma(p, m_q) \) receives contributions from regions of all loop momenta – also from momenta of \( \mathcal{O}(\Lambda_{QCD}) \)

- Ambiguity \( \Delta m_q \) in definition of pole mass up to corrections \( \mathcal{O}(\Lambda_{QCD}) \)

Bigi, Shifman, Uraltsev, Vainshtein ’94; Beneke, Braun ’94; Smith, Willenbrock ’97

- lattice QCD bound: \( \Delta m_q \geq 0.7 \cdot \Lambda_{QCD} \simeq 200 \text{ MeV} \) Bauer, Bali, Pineda ’11

Short distance mass

- Short distance masses (\( \overline{MS} \), 1S Hoang, Teubner ’99, PS Beneke ’98, …) probe at scale of hard interaction: \( m_{\text{pole}} = m_{\text{short distance}} + \delta m \)

- \( \overline{MS} \) mass definition \( m(\mu_R) \) realizes running mass (scale dependence)

- Conversion between \( m_{\text{pole}} \) and \( \overline{MS} \) mass \( m(\mu_R) \) in perturbation theory

Gray, Broadhurst, Gräfe, Schilcher ’90; Chetyrkin, Steinhauser ‘99; Melnikov, v. Ritbergen ‘99
Top quark mass

What is the value of the top quark mass ?

\[ m_t = ? \]
### Some Answers

Some answers are extracted from the page, including details on top quark properties and their measurements from various experiments. The page contains a table and a graph illustrating the top quark mass measurements from different experiments:

- **Tevatron+LHC** $m_{\text{top}}$ combination - March 2014, $L_{\text{int}} = 3.5 \text{ fb}^{-1} - 8.7 \text{ fb}^{-1}$

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$m_{\text{top}}$ (GeV) ± Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDF RunII, l+jets</td>
<td>172.85 ± 1.12 (0.52 ± 0.49)</td>
</tr>
<tr>
<td>CDF RunII, di-lepton</td>
<td>170.28 ± 3.69 (1.95 ± 3.13)</td>
</tr>
<tr>
<td>CDF RunII, all jets</td>
<td>172.47 ± 2.01 (1.43 ± 0.95)</td>
</tr>
<tr>
<td>CDF RunII, $E_{T}^{\text{miss}}$+jets</td>
<td>173.93 ± 1.85 (1.26 ± 1.05)</td>
</tr>
<tr>
<td>D0 RunII, l+jets</td>
<td>174.94 ± 1.50 (0.83 ± 0.47)</td>
</tr>
<tr>
<td>D0 RunII, di-lepton</td>
<td>174.00 ± 2.79 (2.36 ± 0.55)</td>
</tr>
<tr>
<td>ATLAS 2011, l+jets</td>
<td>172.31 ± 1.55 (0.23 ± 0.72)</td>
</tr>
<tr>
<td>ATLAS 2011, di-lepton</td>
<td>173.09 ± 1.63 (0.64 ± 1.50)</td>
</tr>
<tr>
<td>CMS 2011, l+jets</td>
<td>173.49 ± 1.06 (0.27 ± 0.33)</td>
</tr>
<tr>
<td>CMS 2011, di-lepton</td>
<td>172.50 ± 1.52 (0.43 ± 1.46)</td>
</tr>
<tr>
<td>CMS 2011, all jets</td>
<td>173.49 ± 1.41 (0.69 ± 1.23)</td>
</tr>
<tr>
<td>World comb. 2014</td>
<td>173.34 ± 0.76 (0.27 ± 0.24)</td>
</tr>
</tbody>
</table>

The graph shows the m$_{\text{top}}$ distribution with confidence intervals and statistical uncertainties. The top quark properties in ATLAS are discussed by F. Derue.

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**Top theory – p.24**
World combination

Experiment: ATLAS, CDF, CMS & D0 coll. 1403.4427

\[ m_t = 173.34 \pm 0.76 \text{ GeV} \]
World combination

**Experiment:** ATLAS, CDF, CMS & D0 coll. 1403.4427

\[ m_t = 173.34 \pm 0.76 \text{ GeV} \]

In all measurements considered in the present combination, the analyses are calibrated to the Monte Carlo (MC) top-quark mass definition.
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**Theory:**
That is, we can state as the final result for the likely relation between the top quark mass measured using a given Monte Carlo event generator ("MC") and the pole mass as

\[ m_{\text{pole}} = m_{\text{MC}} + Q_0 \left[ \alpha_s(Q_0) c_1 + \ldots \right] \]

where \( Q_0 \sim 1 \text{ GeV} \) and \( c_1 \) is unknown, but presumed to be of order 1 and, according to the argument above, presumed to be positive.

A. Buckley et al. arXiv:1101.2599
**Monte Carlo mass**

- Intuition tells, that Monte Carlo mass is pole mass due to kinematics\[ E_q^2 - p^2 = m_q^2 \]
- Quantum mechanics considers particles in potential
  - heavy quarks interact with potential due to gluon field
- String hadronization model based on potential energy of heavy-quark pair (colored sources)
**Static energy of heavy-quark**

- Consider static energy $E_{\text{stat}}$ of heavy-quark pair
  - well-defined problem in perturbation theory at large scales $R$
  - use lattice at small scales
- QCD corrections to static energy receives from
  - heavy-quark self-energy $\Sigma(p, m_q)$
  - heavy-quark potential $V(R)$
- Renormalon ambiguities in QCD corrections can cancel between $\Sigma(p, m_q)$ and $V(R)$
  
  $E_{\text{stat}} = 2m_q + 2\Sigma(m_q, m_q) + V(R)$
  
  $= 2m_{\text{pole}} + V(R)$
  
  $= 2m^{\text{MSR}}(R) + \left(2\Sigma^{\text{fin}}(R, R) + V(R)\right)$

- Extraction of a short-distance mass $m^{\text{MSR}}(R)$ from the quarkonium spectrum is free of an ambiguity of order $\mathcal{O}(\Lambda_{QCD})$

Hoang, Smith, Stelzer, Willenbrock ‘98
Conversion of Monte Carlo mass to pole mass

- Running of \( m^{\text{MSR}}(R) \) mass
  Hoang, Stewart '08

**Strategy**

- Identify Monte Carlo mass with short distance mass at low scale \( \mathcal{O}(1) \) GeV
  \( m^{\text{MC}} \rightarrow m^{\text{MSR}}(R) \) with \( R \simeq 1 \ldots 9 \) GeV

- Choice 1: run \( m^{\text{MSR}}(R) \) from low scale to \( R = m_t \): \( m^{\text{MSR}}(R) \rightarrow m(m) \) and convert from \( m(m) \) to pole mass

<table>
<thead>
<tr>
<th>( m^{\text{MSR}}(1) )</th>
<th>( m^{\text{MSR}}(3) )</th>
<th>( m^{\text{MSR}}(9) )</th>
<th>( \overline{m}(\overline{m}) )</th>
<th>( m_{1p}^{\text{pl}} )</th>
<th>( m_{2p}^{\text{pl}} )</th>
<th>( m_{3p}^{\text{pl}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>173.72</td>
<td>173.40</td>
<td>172.78</td>
<td>163.76</td>
<td>171.33</td>
<td>172.95</td>
<td>173.45</td>
</tr>
</tbody>
</table>

- Choice 2: convert from \( m^{\text{MSR}}(R) \) at low scale directly to pole mass

<table>
<thead>
<tr>
<th>( m^{\text{MSR}}(1) )</th>
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<td>173.72</td>
<td>173.87</td>
<td>173.98</td>
</tr>
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</table>
Conversion of Monte Carlo mass to pole mass

Summary

\[ m_{\text{pole}} = 173.34 \pm 0.76 \text{ GeV (exp)} + \Delta m(\text{th}) \]

with

\[ \Delta m(\text{th}) = \pm 0.7 \text{ GeV (} m^{\text{MC}} \rightarrow m^{\text{MSR}}(3\text{GeV}) \text{)} + 0.5 \text{ GeV (} m(m) \rightarrow m_{\text{pole}} \text{)} \]
Alternative mass measurements

- Rates and shapes of distributions offer possibility for top mass determination with well-defined renormalization scheme
- Requirements:
  - theory predictions at least to NLO in QCD
  - sufficient sensitivity to $m_t$ (kinematics)
- Observables (examples):
  - inclusive cross section
  - shapes of distributions; e.g., $m_{lb}$ distribution or jet rates
Total cross section with running mass

Comparison pole mass vs. $\overline{\text{MS}}$ mass

Dowling, S.M. ‘13

- NNLO cross section with running mass significantly improved
  - good apparent convergence of perturbative expansion
  - small theoretical uncertainty form scale variation
Top cross section data in ABM12 fit

- Fit with correlations
  - $g(x)$ and $\alpha_s(M_Z)$ already well constrained by global fit (no changes)
  - for fit with $\chi^2/NDP = 5/5$ obtain value of $m_t(m_t) = 162.3 \pm 2.3$ GeV (equivalent to pole mass $m_t = 171.2 \pm 2.4$ GeV) Alekhin, Blümlein, S.M. ‘13
  - $\chi^2$-profile steeper for pole mass (bigger impact of top-quark data and greater sensitivity to theoretical uncertainty at NNLO)
Differential cross sections

NLO in QCD

Running mass for differential distributions show same features, e.g. $p_T$-distribution \[ \text{Dowling, S.M. ‘13} \]
Top-quark pairs with one jet

- Mass measurement from shape of distribution for invariant mass of $t\bar{t} + 1\text{jet}$ system

  - variable $\rho_s = \frac{2 \cdot m_0}{\sqrt{s_{t\bar{t}+1\text{jet}}}}$ with fixed scale $m_0 = 170 \text{ GeV}$

- Normalized-differential $t\bar{t} + \text{jet}$ cross section

  $$R(m_t, \rho_s) = \frac{1}{\sigma_{t\bar{t}+1\text{jet}}} \frac{d\sigma_{t\bar{t}+1\text{jet}}}{d\rho_s}(m_t, \rho_s)$$

- significant mass dependence for $0.4 \leq \rho_s \leq 0.5$ and $0.7 \leq \rho_s$
Mass sensitivity

• Differential cross section $\mathcal{R}(m_t, \rho_s)$
  • good pertubative stability, small theory uncertainties, small dependence on experimental uncertainties, . . .

• Sensitivity to top-quark mass very good
  $$\left| \frac{\Delta \mathcal{R}}{\mathcal{R}} \right| \simeq (m_t S) \times \left| \frac{\Delta m_t}{m_t} \right|$$

• increased sensitivity for system $t\bar{t} + \text{jet}$ compared to $t\bar{t}$
Summary

Theory predictions for rates and shapes

- Precision predictions of inclusive and differential observables for LHC measurements
- QCD corrections at NNLO + electroweak corrections at NLO
- Quality of perturbative expansion depends on scheme for top-quark mass
- Short-distance masses $\overline{MS} m(m)$ or $m_{MSR}(R)$ show better convergence and scale stability

Top-quark mass

- Top quark mass is parameter of Standard Model Lagrangian
- Measurements of $m_t$ require careful definition of observable
- Correlations in data analysis are important, e.g. with $\alpha_s$ and PDFs
- Relation of Monte Carlo mass $m^{MC}$ to pole mass with additional theory uncertainty $\Delta m_t$

Future tasks

- Joint effort theory and experiment