# Longitudinal Dynamics
# CAS 2014 - Prague

```
(%i2)  E0 : 0.93826E9 $
       c  : 2.9979e8 $
       rho_PS :  70 $
       R_PS   : 100 $
       R_PSB  :  25 $
       h_PSB  :   1 $
       h_PS   :   8 $
       a_PS   : 0.027  $
       P_ext  : 26e9  $
```

1) What are the relativistic beta and gamma for the protons at injection into the PS?

```
(%i11) E_kin_inj : 1.4E9$
```

```
(%i12) E_tot_inj : E_kin_inj + E0 $
       float(%) *1e-9*'GeV;
```

(%o13) 2.338 $GeV$

```
(%i14) y_inj : E_tot_inj / E0 ;
```

(%o14) 2.492

```
(%i15) b_inj : sqrt(1-1/ y_inj ^2);
```

(%o15) 0.916

2) Calculate the magnetic field in the PS at injection and ejection.

```
(%i16) P_inj : b_inj * E_tot_inj $
       float(%) *1e-9*'GeV;
```

(%o17) 2.142 $GeV$

Field in Tesla

```
(%i18) B_inj : P_inj *1e-9 / (0.3 * rho_PS);
```

(%o18) 0.102

```
(%i19) B_ext : P_ext *1e-9 / (0.3 * rho_PS);
```

(%o19) 1.238

3) At PS injection, calculate and compare the RF frequency in the PS and in the PSB.

```
(%i20) v_RF_PS_inj : b_inj * c * h_PS / (2 * %pi * R_PS) $
       float(% *1e-6*'MHz);
```

(%o21) 3.496 $MHz$

```
(%i22) v_RF_PSB : b_inj * c * h_PSB / (2 * %pi * R_PSB) $
       float(% *1e-6*'MHz);
```

(%o23) 1.748 $MHz$

So the frequency in the PS is twice higher.

4) What is the synchronous phase in the PS at injection?

No acceleration, before transition -> phi_s = 0;

5) Assuming that the orbit remains the same during the acceleration, how does the RF frequency change between injection and ejection in the PS?

```
(%i24) E_tot_ext : sqrt( E0^2 + P_ext^2 ) $ float(%) *1e-9*'GeV;
       y_ext : E_tot_ext / E0;
       b_ext : sqrt(1-1/ y_ext ^2);
```

(%o25) 26.02 $GeV$

(%o26) 27.73

(%o27) .9993

```
(%i28) v_RF_ext : v_RF_PS_inj * b_ext / b_inj $
       float(% *1e-6*'MHz);
```

(%o29) 3.815 $MHz$

6) Should a phase jump system be implemented in the PS?

```
(%i30) y_tr : 1/sqrt(a_PS);
```

(%o30) 6.086

Since this is between y_inj and y_ext => transition is crossed.

7) With the peak RF voltage in the PS at injection of 25.6kV calculate the PS synchrotron frequency f_s and the synchrotron tune Q_s at injection. Does it verify Q_s << 1 ?

```
(%i31) V_RF_PS : 25600 $
```

```
(%i32) eta_PS  : 1/y_inj^2-a_PS;
```

(%o32) 0.134

phi_s=0 => cos(phi_s)=1

f_s = Omega_s/2pi

```
(%i33) f_s : c/(2*%pi*R_PS)*sqrt(V_RF_PS*eta_PS*h_PS/(2*%pi*E_tot_inj)) $
       float(% * 'Hz);
```

(%o34) 652.1 $Hz$

or

```
(%i35) f_s2 : 1/(2*%pi)*sqrt(v_RF_PS_inj*eta_PS*V_RF_PS/(R_PS*P_inj/c)) $
       float(% * 'Hz);
```

(%o36) 652.1 $Hz$

```
(%i37) Q_s : f_s/(v_RF_PS_inj/h_PS) $
       float(%);
```

(%o38) .001492