Synchrotron radiation in LHC: spectrum and dynamics

The Large Hadron Collider (LHC) will collide 7 TeV protons in the same tunnel (circumference 27 km) where the Large Electron Project (LEP) operated with 100 GeV electrons. Assuming for simplicity the same bending radius in dipole magnets of 2900 m, calculate the following synchrotron radiation parameters for the three cases: for protons at the injection energy of 450 GeV, at the design energy of 7 TeV and for the electrons at 100 GeV:

- typical (‘critical’) photon energy of emitted synchrotron radiation
- energy loss per turn
- the damping times for energy and transverse oscillations

The proton mass is 0.938 GeV and the electron mass is 0.511 MeV

Future Circular Collider (FCC)

New superconducting materials may allow in the future to build 20 Tesla dipole magnets. Assuming 10 km bending radius (about 100 km long tunnel, 2/3 filled with dipoles)

- To what maximum energy will FCC be able to accelerate protons?
- What will be the typical photon energy emitted by protons and what will be the energy loss per turn?
- Estimate the damping time and the equilibrium emittance of the protons stored at the top energy

Synchrotron light source that never was: LEP at 5 GeV

The equilibrium electron beam emittance at 100 GeV at LEP was about 0.06 mm·mrad.

If we tried to operate LEP at 5 GeV, how small an emittance could one hope to achieve using the same lattice?

What would be the damping time at this energy (a few milliseconds at 100 GeV)?

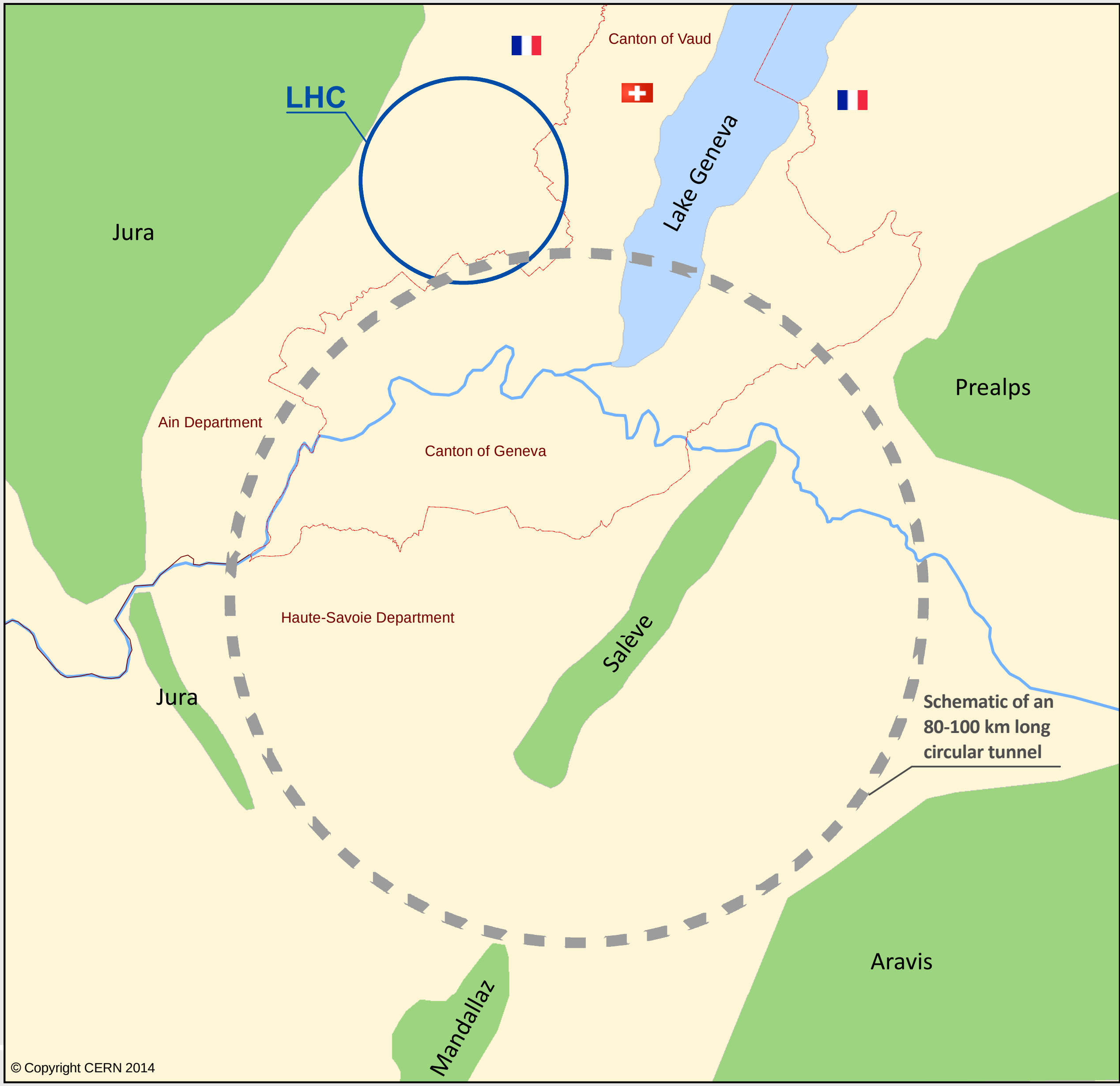
Find the length of wiggler magnets you would need to install in the straight sections of the ring (assume wiggler field of 2 Tesla), in order to reduce the damping times at 5 GeV by a factor of 1000.

High Energy options: 100 km tunnel?

50 TeV c.o.m.
with 8.3 T
LHC type dipoles

100 TeV c.o.m.
with 16 T

120 TeV c.o.m
with 20 T



Synchrotron radiation power

Power emitted is proportional to:

$$P \propto E^2 B^2$$

$$P_\gamma = \frac{cC_\gamma}{2\pi} \cdot \frac{E^4}{\rho^2}$$

$$P_\gamma = \frac{2}{3}\alpha\hbar c^2 \cdot \frac{\gamma^4}{\rho^2}$$

$$C_\gamma = \frac{4\pi}{3}\frac{r_e}{(m_e c^2)^3} = 8.858 \cdot 10^{-5}\left[\frac{\mathrm{m}}{\mathrm{GeV}^3}\right]$$

$$\alpha = \frac{1}{137}$$

$$\hbar c = 197\ \mathrm{Mev} \cdot \mathrm{fm}$$

Energy loss per turn:

$$U_0 = C_\gamma \cdot \frac{E^4}{\rho}$$

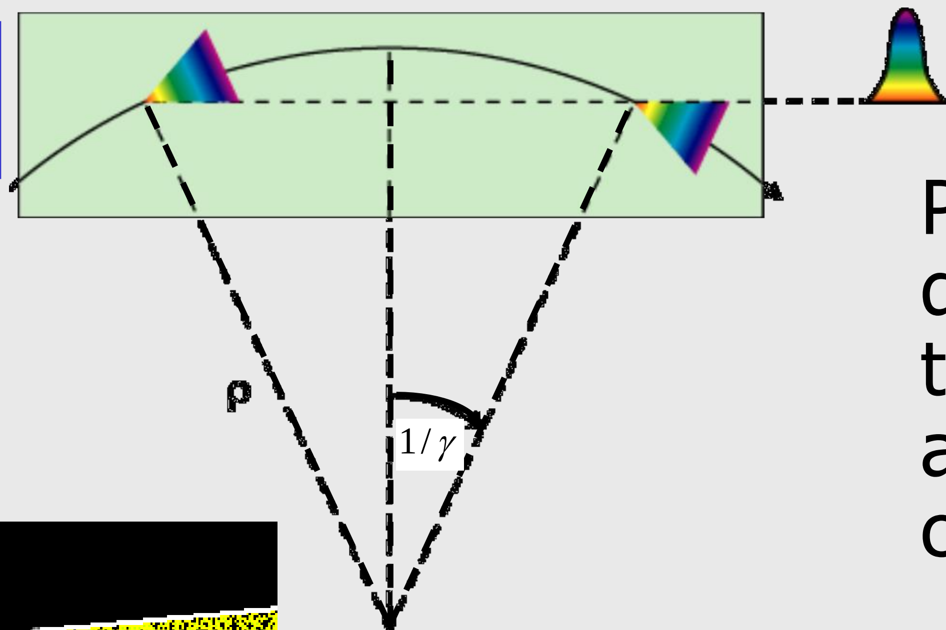
$$\omega_c = \frac{3}{2}\frac{c\gamma^3}{\rho}$$

$$U_0 = \frac{4\pi}{3}\alpha\hbar c\frac{\gamma^4}{\rho}$$

Typical frequency of synchrotron light

Due to extreme collimation of light observer sees only a small portion of electron trajectory **(a few mm)**

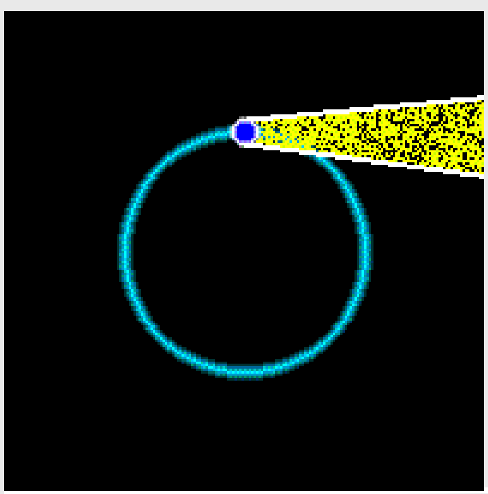
$$l \sim \frac{2\rho}{\gamma}$$



Pulse length: difference in times it takes an electron and a photon to cover this distance

$$\Delta t \sim \frac{l}{\beta c} - \frac{l}{c} = \frac{l}{\beta c}(1-\beta)$$

$$\Delta t \sim \frac{2\rho}{\gamma c} \cdot \frac{1}{2\gamma^2}$$



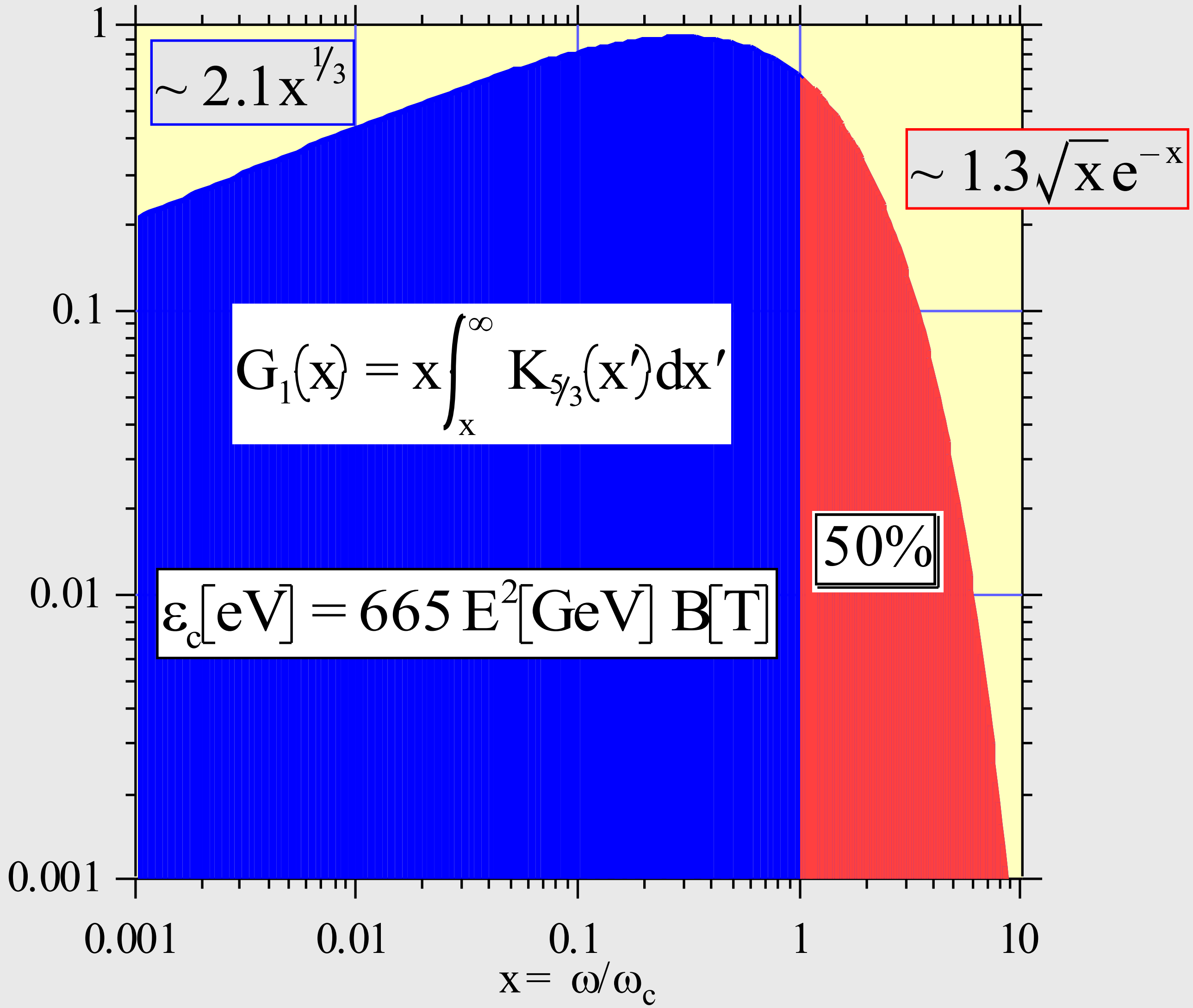
$$\omega \sim \frac{1}{\Delta t} \sim \gamma^3 \omega_0$$

$$\frac{dP}{d\omega} = \frac{P_{tot}}{\omega_c} S\left(\frac{\omega}{\omega_c}\right)$$

$$S(x) = \frac{9\sqrt{3}}{8\pi} x \int_x^{\infty} K_{5/3}(x')dx' \qquad \int_0^{\infty} S(x')dx' = 1$$

$$P_{tot} = \frac{2}{3} \hbar c^2 \alpha \frac{\gamma^4}{\rho^2}$$

$$\omega_c = \frac{3}{2} \frac{c\gamma^3}{\rho}$$



Synchrotron radiation in LHC

The critical photon energy is:

$$\varepsilon_c = \tfrac{3}{2}\hbar c \frac{\gamma^3}{\rho}$$

	450 GeV	7000 GeV	100 GeV electrons
Relativistic γ	480	7500	200‘000
ε_c	0.01 eV	42 eV	0.8 MeV

- The energy lost per turn
  - in LHC at 7 TeV is about 7 keV, while
  - at LEP it was 3 GeV
- The damping times will be on the order of 1 day at LHC and a few milliseconds at LEP.

FCC compared to LHC

The magnetic rigidity at 20 Tesla and radius of 10'000 meters reaches 200'000, the beam energy will be then 60 TeV

$$B\rho[T \cdot m] = \frac{1}{0.2997925} E[GeV]$$

The critical photon energy is proportional to the cube of energy and inversely proportional to the bending radius

$$\varepsilon_c = 7.7\, keV$$

Energy loss per turn will be almost 1500 times higher, or 10 MeV

Damping times will be about 2 seconds

Synchrotron light source: LEP at 5 GeV

The equilibrium emittance scales as square of the storage ring energy, so at 5 GeV the emittance could be 400 times smaller, i.e. 0.15 nm·rad!

The damping times on the other hand scale as the cube of ring energy and would become very long indeed: 24 and 48 seconds.

With wigglers in LEP at 5 GeV, the total radiation loss per turn:

$$U_{tot} = U_d + U_w = P_{\gamma d} \cdot \frac{L_d}{c} + P_{\gamma w} \cdot \frac{L_w}{c}$$

where

$$P_\gamma \propto E^2 B^2$$

then

$$U_{tot} = U_d \left(1 + \frac{B_w^2 L_w}{B_d^2 L_d} \right) = 1000 \cdot U_d$$

and the required length is

$$L_w = 999 \cdot L_d = 999 \cdot 2\pi \cdot 2900 \cdot \left(\frac{0.00575}{2} \right)^2 \approx 150\,\text{m}$$