

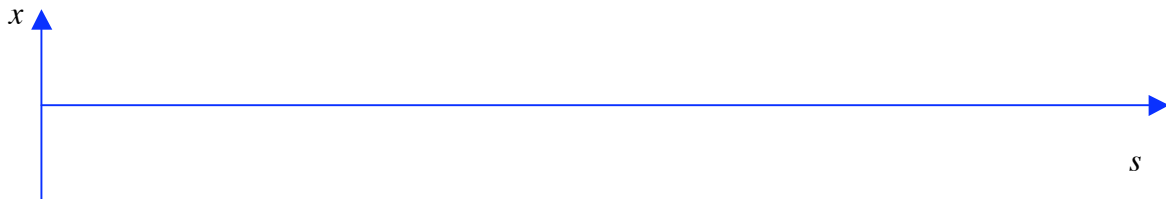
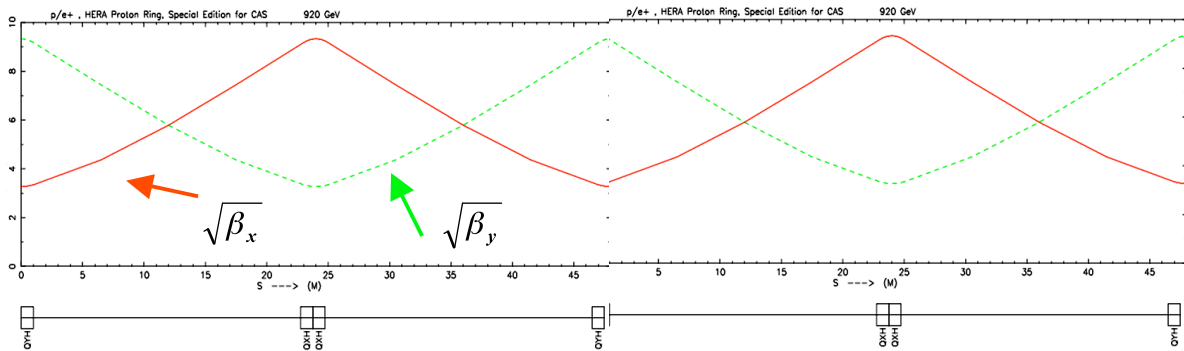
Questions for the Optics Tutorial

CAS Prag Lectures 2014,
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- 1.) *Can you explain in your own words the meaning of ...*
phase advance
beam emittance
 β -function

Concerning the two parameters β -function and beam emittance: they both determine the beam envelope. Can you explain the difference ?

Consider the FODO structure in the figure: The phase advance per cell is 90 degrees and the red curve is the horizontal, the green line the vertical beta function. . . Can you draw in the plot below two particle trajectories, that propagate through the two cells, one starting with $x=0, x'>0$ and the other one starting with $x>0, x'=0$?



*Assume somewhere in the storage ring there is a position where $\alpha = 0$.
 Where would such a situation occur typically?
 How will the phase space ellipse look like?
 Starting from a mini-beta insertion with $\alpha=0$, how does the phase space picture develop in the long drift space ?*

2.) Beam rigidity & particle momentum

... or the stupid problem: after all we have to deal with a relativistic beam

A synchrotron of 25m radius accelerates protons from a kinetic energy of 50 MeV to 1000 MeV. What is the maximum energy of a deuteron beam (Z=1, A=2) that could be accelerated in the machine ?

3.) LHC: particle momentum, geometry of a storage ring and thin lenses

The LHC storage ring at CERN will collide proton beams with a maximum momentum of $p = 7 \text{ TeV}/c$ per beam.

The main parameters of this machine are:

<i>Circumference</i>	$C_0 = 26658.9m$	
<i>particle momentum</i>	$p = 7 \text{ TeV}/c$	
<i>main dipoles</i>	$B = 8.392 \text{ T}$	$l_B = 14.2m$
<i>main quadrupoles</i>	$G = 235 \text{ T/m}$	$l_q = 5.5m$

Calculate the magnetic rigidity of the design beam, the bending radius of the main dipole magnets in the arc and determine the number of dipoles that is needed in the machine.

Calculate the k-strength of the quadrupole magnets and compare its focal length to the length of the magnet. Can this magnet be treated as a thin lens?

How does the matrix for such a (foc.) magnet look like? How would you establish a description of this magnet in thin lens approximation? Compare the matrix elements.

Nota bene: in our notation a foc. magnet has a negative k-value.

4.) Can you explain in your own words the meaning of ...

dispersion

chromaticity

The largest contribution to the chromaticity ξ in a storage ring is – due to the high β -values and strong quadrupole strengths the interaction region.

Would it be a good idea to install sextupole magnets there to compensate ξ locally ?

5.) Apertures and Beam Envelopes:

The LHC magnet structure in the arcs consists of a symmetric FoDo with 90° phase advance per cell and an aperture radius of $r_0 = 20mm$.

a.) Given the value of $\beta_{max} = 500m$ in a QF quadrupole lens, what beam emittance would just touch the vacuum chamber ? (We call this value the “acceptance” of the machine).

b.) If now the typical emittance of a real stored beam at 450 GeV injection energy is $\varepsilon \approx 7 \cdot 10^{-9} \text{ rad m}$, how many σ of beam envelope fit into the vacuum chamber for a $\beta_{max} = 500m$?

c) what will happen if – keeping the beam optics constant – you accelerate the beam to an energy of $E = 7000 \text{ GeV}$?

During luminosity operation at this energy we require at least 14 sigma aperture due to background and quench safety reasons. What is the maximum beta function that can be accepted if the aperture of our mini beta quadrupoles is 20mm?

6.) Question just for the fun of it:

Beam rigidity and – well a little bit of special relativity

When I did that calculation, I myself was very much surprised !!

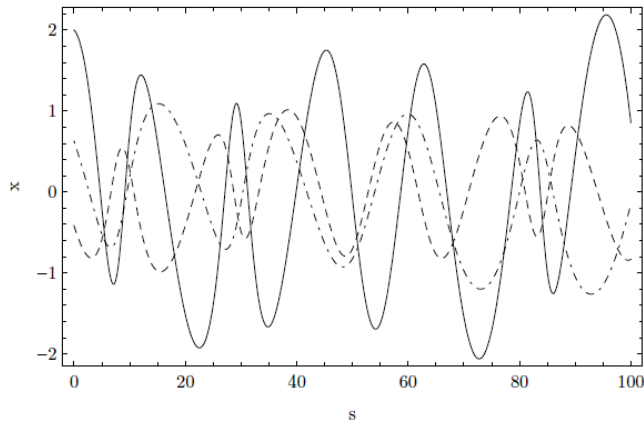
(...solution will be discussed having a good beer in the evening).

Let's build a real cheap storage ring. Just put it to the north pole and use the magnetic field of the earth whose field lines are perpendicular to the surface at that nice place.

Forget about focusing ... that's for nitpickers. What will be the size of the ring for a 10 keV electron beam if the earth magnetic field is about 0.5 Gauß ?

7.) Particle Trajectories:

The following plot represents the trajectories of three particles traveling in the the same non-dispersive transfer line.



Among the three particles only one is significantly off-momentum. Which one (full, dashed or dot-dashed line)? Is its rigidity higher or lower than the on-momentum particles? Justify the answer.