RF LINACS

Alessandra Lombardi BE/ ABP CERN

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  - Building bloc I (1/2) : Radio Frequency cavity
  - From an RF cavity to an accelerator

- PART 2 (today) :
  - Building bloc II (2/2) : quadrupoles and solenoids
  - Single particle beam dynamics
    - bunching, acceleration
    - transverse and longitudinal focusing
    - synchronous structures
    - DTL drift-kick-drift dynamics
    - slippage in a multicell cavity
  - Collective effects brief examples : space charge and wake fields.

What is a linac

- LINear ACcelerator : single pass device that increases the energy of a charged particle by means of a (radio frequency) electric field.
- Motion equation of a charged particle in an electromagnetic field

$$\frac{d\vec{p}}{dt} = q \cdot \left(\vec{E} + \vec{v} \times \vec{B} \right)$$

$$\begin{aligned}
&\vec{p} = momentum = \gamma m_0 \vec{v} \\
&q, m_0 = charge, mass \\
&\vec{E}, \vec{B} = electric, magnetic field \\
&t = time \\
&\vec{x} = position vector \\
&\vec{v} = \frac{d\vec{x}}{dt} = velocity
\end{aligned}$$

What is a linac-cont'ed

Relativistic or not

$$\frac{d}{dt}\left(\gamma\frac{d\vec{x}}{dt}\right)=\frac{q}{m_0}\cdot\left(\vec{E}+\frac{d\vec{x}}{dt}\times\vec{B}\right)$$

type of focusing

type of particle : charge couples with the field, mass slows the acceleration

type of RF structure

Focusing

- MAGNETIC FOCUSING

(dependent on particle velocity)

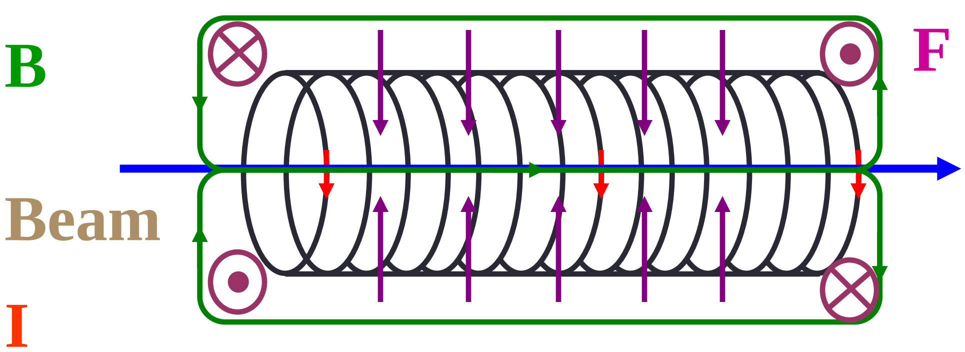
$$\vec{F} = q\vec{v} \times \vec{B}$$

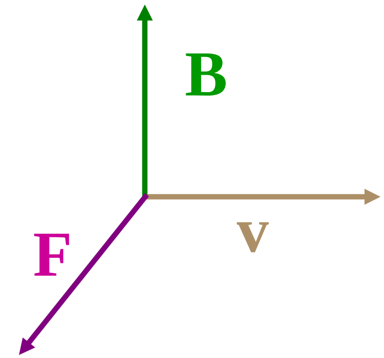
- ELECTRIC FOCUSING

(independent of particle velocity)

$$\vec{F} = q \cdot \vec{E}$$

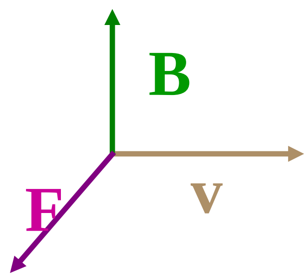
Solenoid





Input : $B = B_\perp$

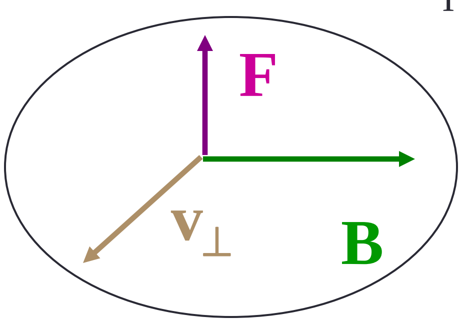
$F \propto v \cdot B$



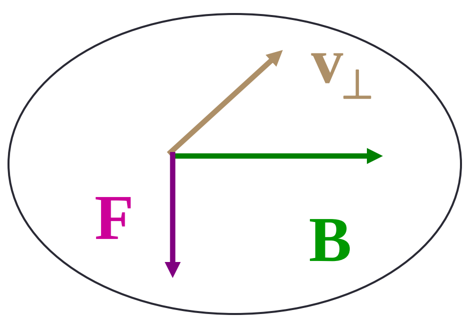
Beam transverse rotation :

$v_\perp \propto v \cdot B \cdot r$

Middle : $B = B_l$



x < 0

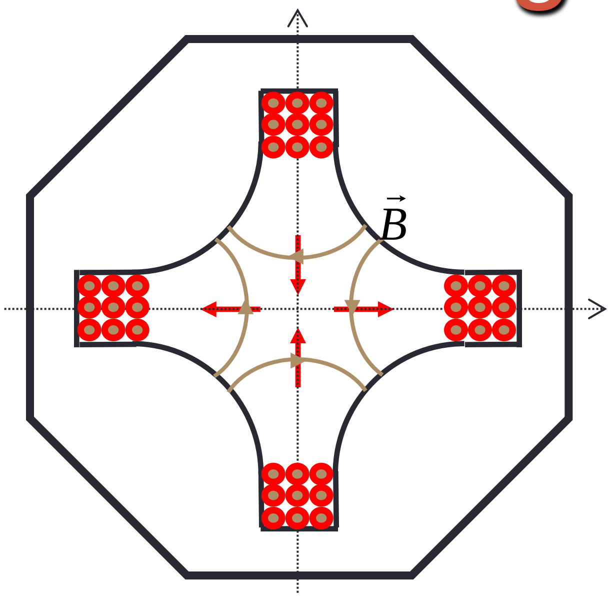


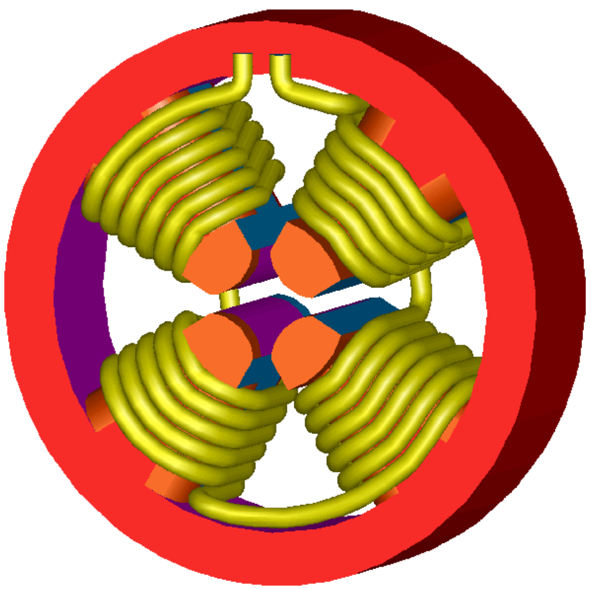
x > 0

$F \propto v_\perp \cdot B \propto v \cdot B^2 \cdot r$

Beam linear focusing
in both planes

Magnetic quadrupole





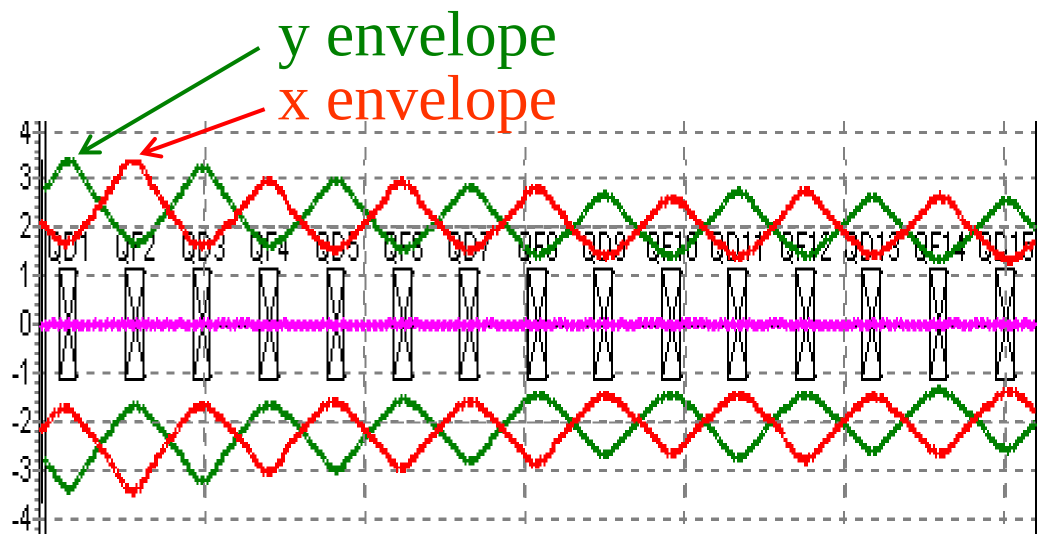
Magnetic field

$$\begin{cases} B_x = G \cdot y \\ B_y = G \cdot x \end{cases}$$

Magnetic force

$$\begin{cases} F_x = -q \cdot v \cdot G \cdot x \\ F_y = q \cdot v \cdot G \cdot y \end{cases}$$

Focusing in one plan, defocusing in the other



sequence of focusing and defocusing quadrupoles

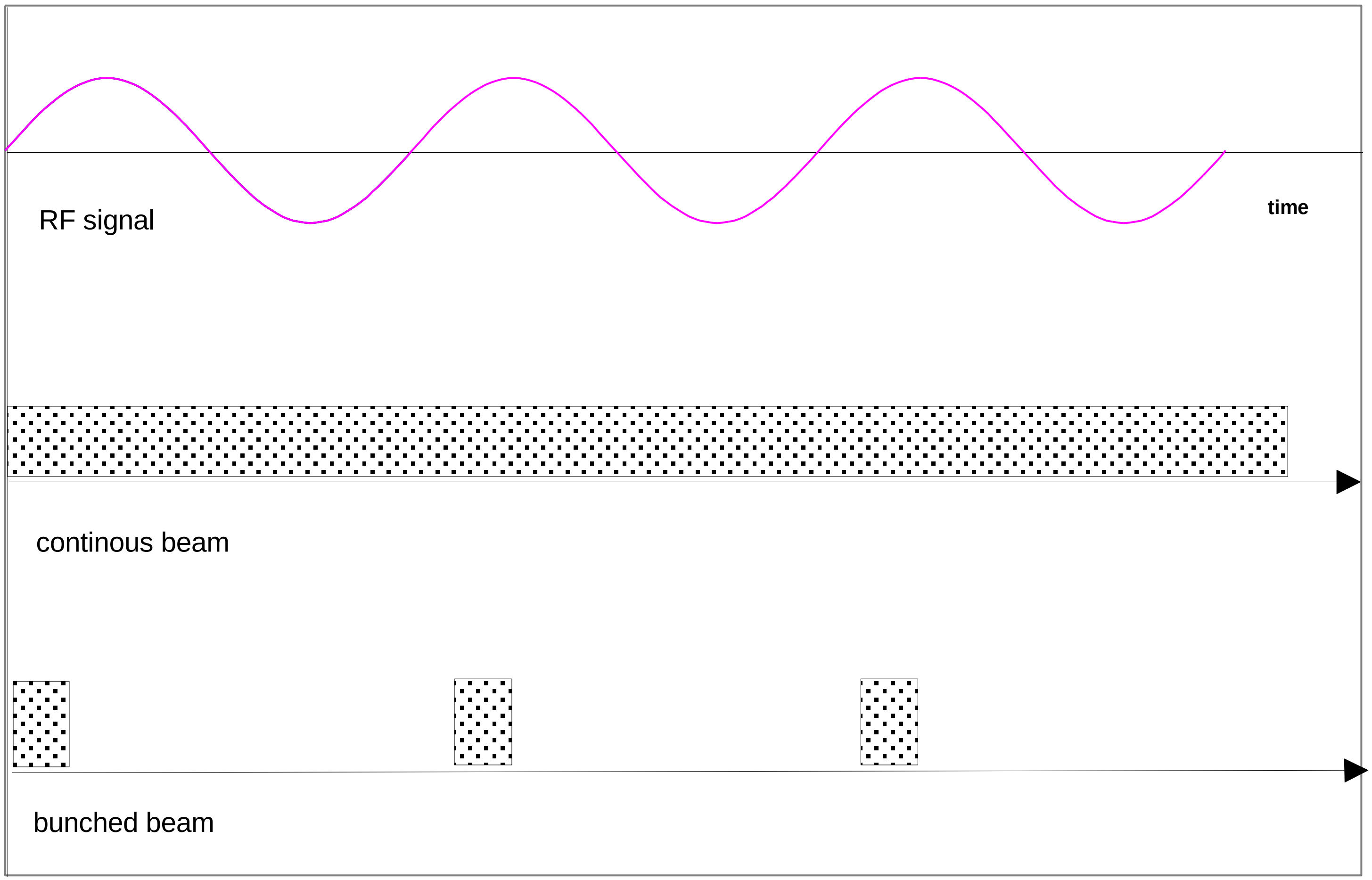
designing an RF LINAC

- cavity design : 1) control the field pattern inside the cavity; 2) minimise the ohmic losses on the walls/maximise the stored energy.

yesterday

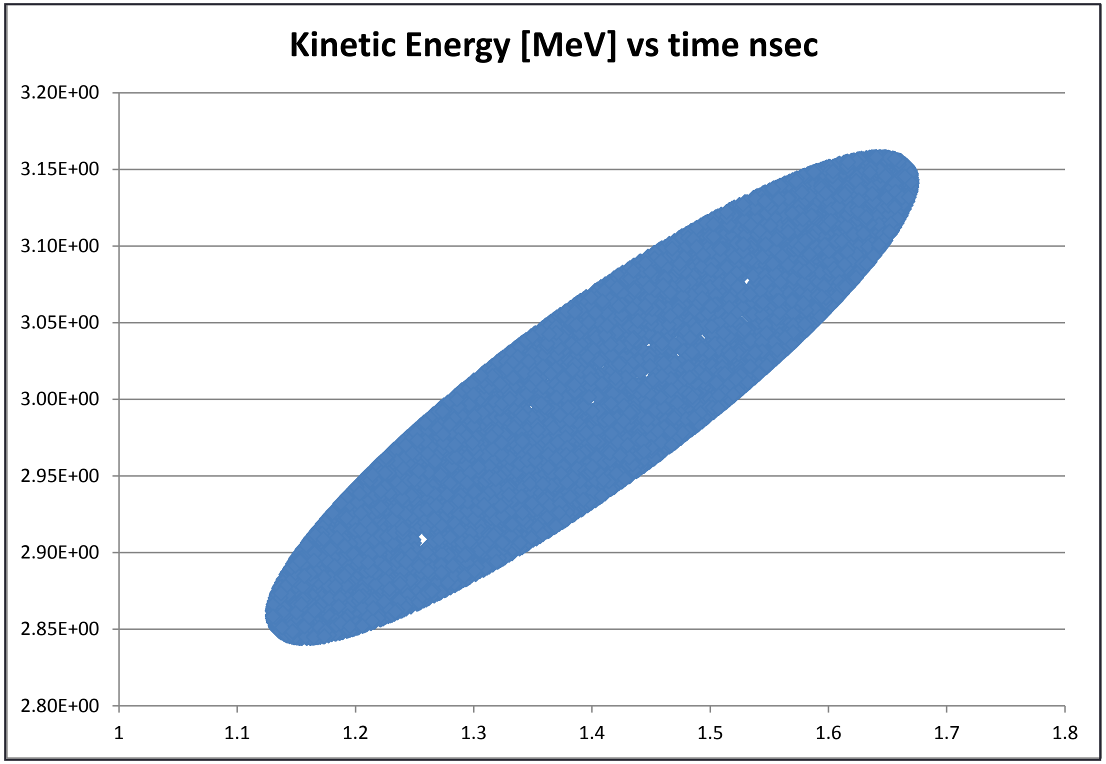
- beam dynamics design : 1) control the timing between the field and the particle, 2) insure that the beam is kept in the smallest possible volume during acceleration

Acceleration-basics



It is not possible to transfer energy to an un-bunched beam

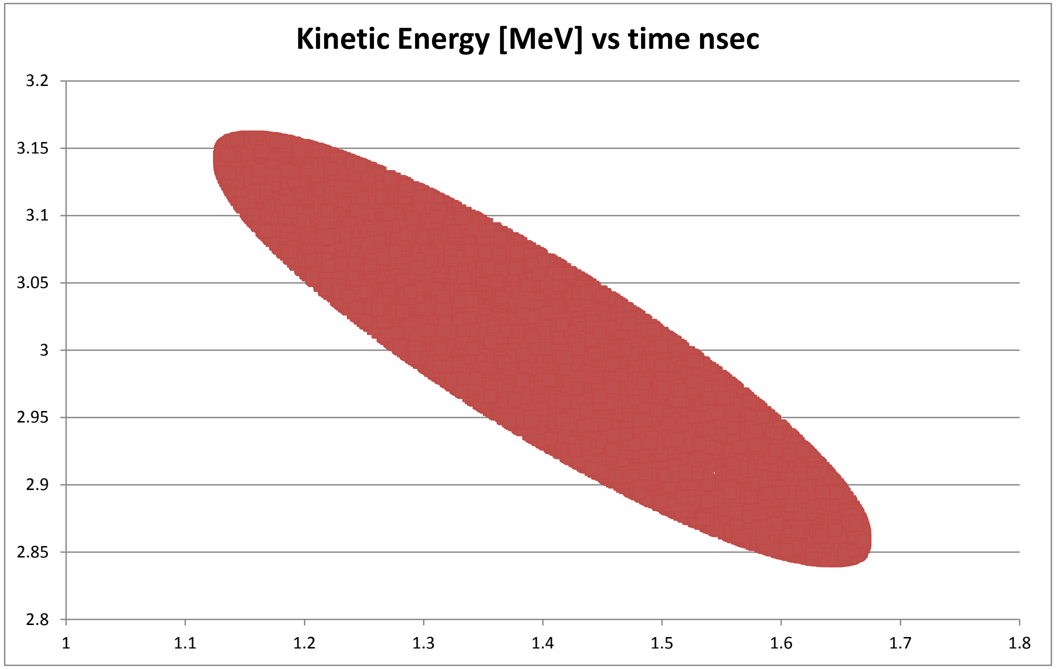
Bunching



earlier later

- Assume we are on the frequency of 352MHz, T = 2.8 nsec
- Q1 : is this beam bunched?
- Q2 : is this beam going to stay bunched ?

Bunching



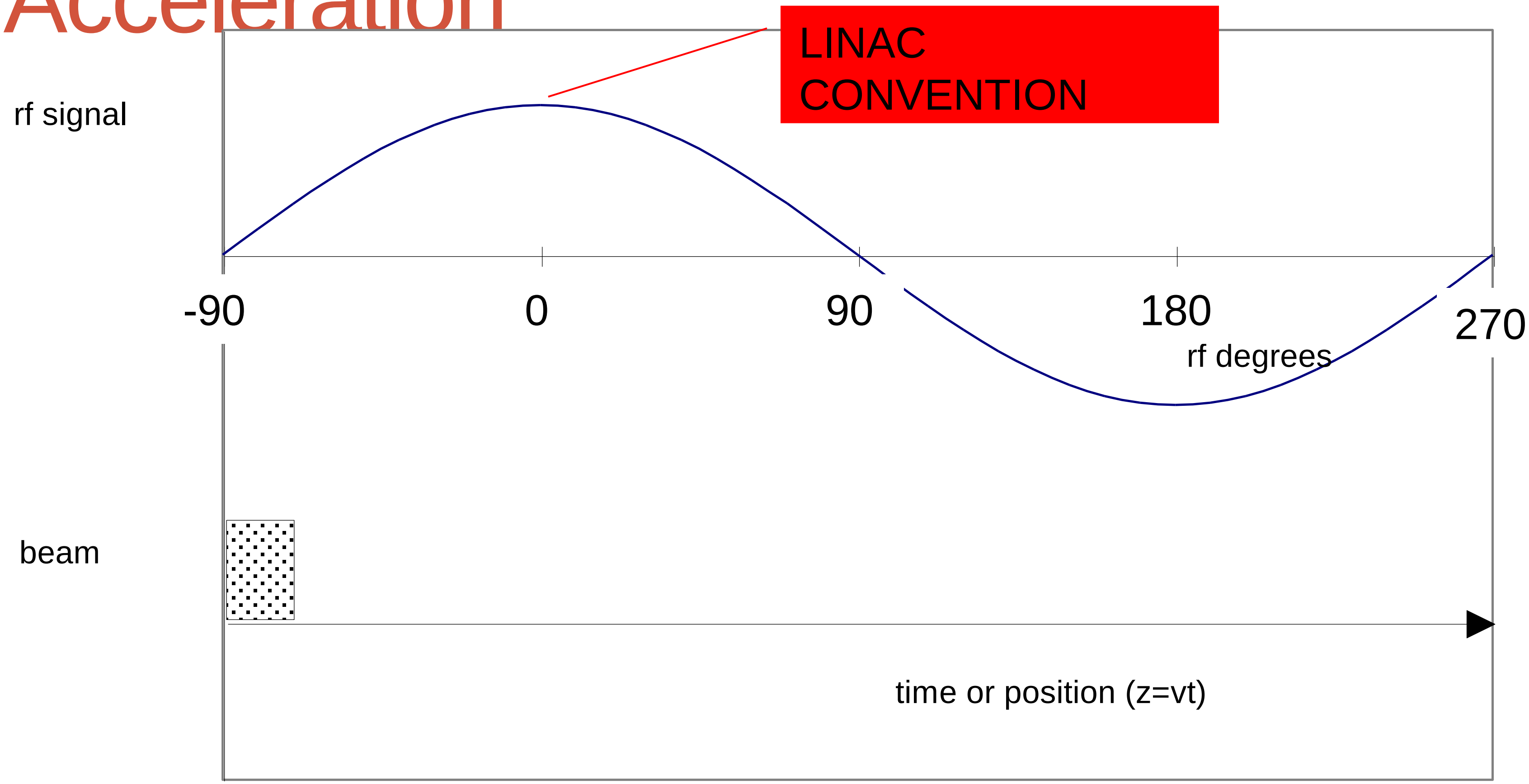
- Assume we are on the frequency of 352MHz, T = 2.8 nsec
- Q3 :how will this proton beam look after say 20 cm?

BUNCHING

- need a structure on the scale of the wavelength to have a net transfer of energy to the beam

- need to bunch a beam and keep it bunched all the way through the acceleration : need to provide LONGITUDINAL FOCUSING

Acceleration



degrees of RF ⇔ length of the bunch(cm) ⇔ duration of the bunch (sec)

$$\Delta\Phi = \Delta z \; 360/\beta\lambda = \Delta t \; 360 * f$$

In one RF period one particle travel a length = $\beta\lambda$

β is the relativistic parameter λ the RF wavelength, f the RF frequency

synchronous particle

- it's the (possibly fictitious) particle that we use to calculate and determine the phase along the accelerator. It is the particle whose velocity is used to determine the synchronicity with the electric field.
- It is generally the particle in the centre (longitudinally) of the bunch of particles to be accelerated

Acceleration

- to describe the motion of a particle in the longitudinal phase space we want to establish a relation between the energy and the phase of the particle during acceleration
- energy gain of the synchronous particle $$\Delta W_s = qE_0 LT\cos(\phi_s)$$
- energy gain of a particle with phase Φ $$\Delta W = qE_0 LT\cos(\phi)$$
- assuming small phase difference ΔΦ =Φ-Φ$_s$

$$\frac{d}{ds}\Delta W = qE_0 T \cdot [\cos(\varphi_s + \Delta\varphi) - \cos\varphi_s]$$

- and for the phase

$$\frac{d}{ds}\Delta\varphi = \omega\left(\frac{dt}{ds} - \frac{dt_s}{ds}\right) = \frac{\omega}{c}\left(\frac{1}{\beta} - \frac{1}{\beta_s}\right) \cong -\frac{\omega}{\beta_s c}\frac{\Delta\beta}{\beta_s} = -\frac{\omega}{mc^3\beta_s^3\gamma_s^3}\Delta W$$

Acceleration-Separatrix

- Equation for the canonically conjugated variables phase and energy with Hamiltonian (total energy of oscillation):

$$\frac{\omega}{mc^3\beta_s^3\gamma_s^3}\left\{\frac{\omega}{2mc^3\beta_s^3\gamma_s^3}(\Delta W)^2+qE_0T\left[\sin(\varphi_s+\Delta\varphi)-\Delta\varphi\cos\varphi_s-\sin\varphi_s\right]\right\}=H$$

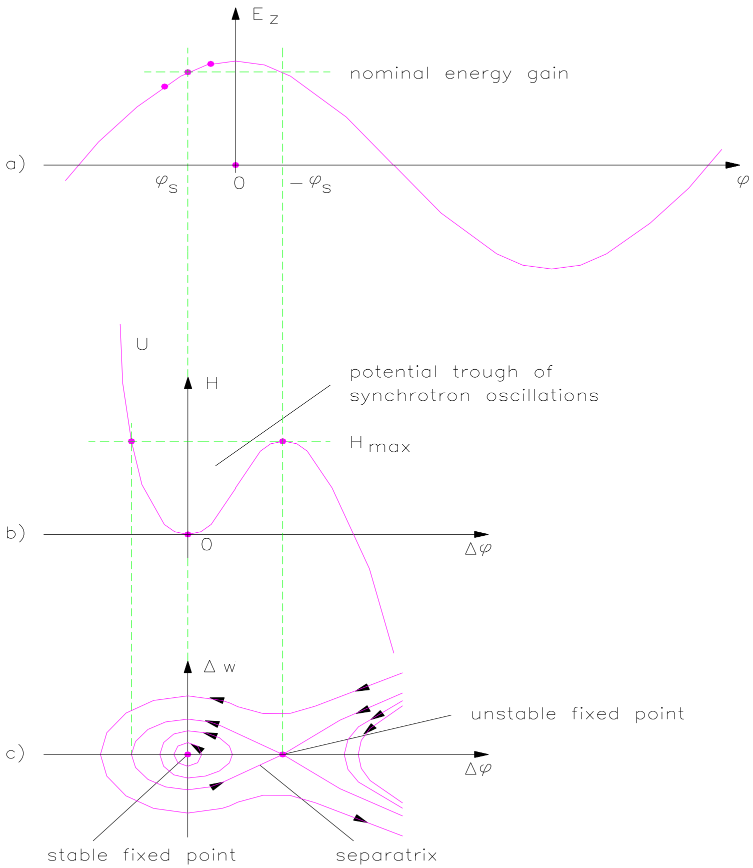
- For each H we have different trajectories in the longitudinal phase space .Equation of the separatrix (the line that separates stable from unstable motion)

$$\frac{\omega}{2mc^3\beta_s^3\gamma_s^3}(\Delta W)^2+qE_0T\left[\sin(\varphi_s+\Delta\varphi)+\sin\varphi_s-(2\varphi_s+\Delta\varphi)\cos\varphi_s\right]=0$$

- Maximum energy excursion of a particle moving along the separatrix

$$\Delta\hat{W}_{\max}=\pm2\left[\frac{qmc^3\beta_s^3\gamma_s^3E_0T(\varphi_s\cos\varphi_s-\sin\varphi_s)}{\omega}\right]^{\frac{1}{2}}$$

Acceleration

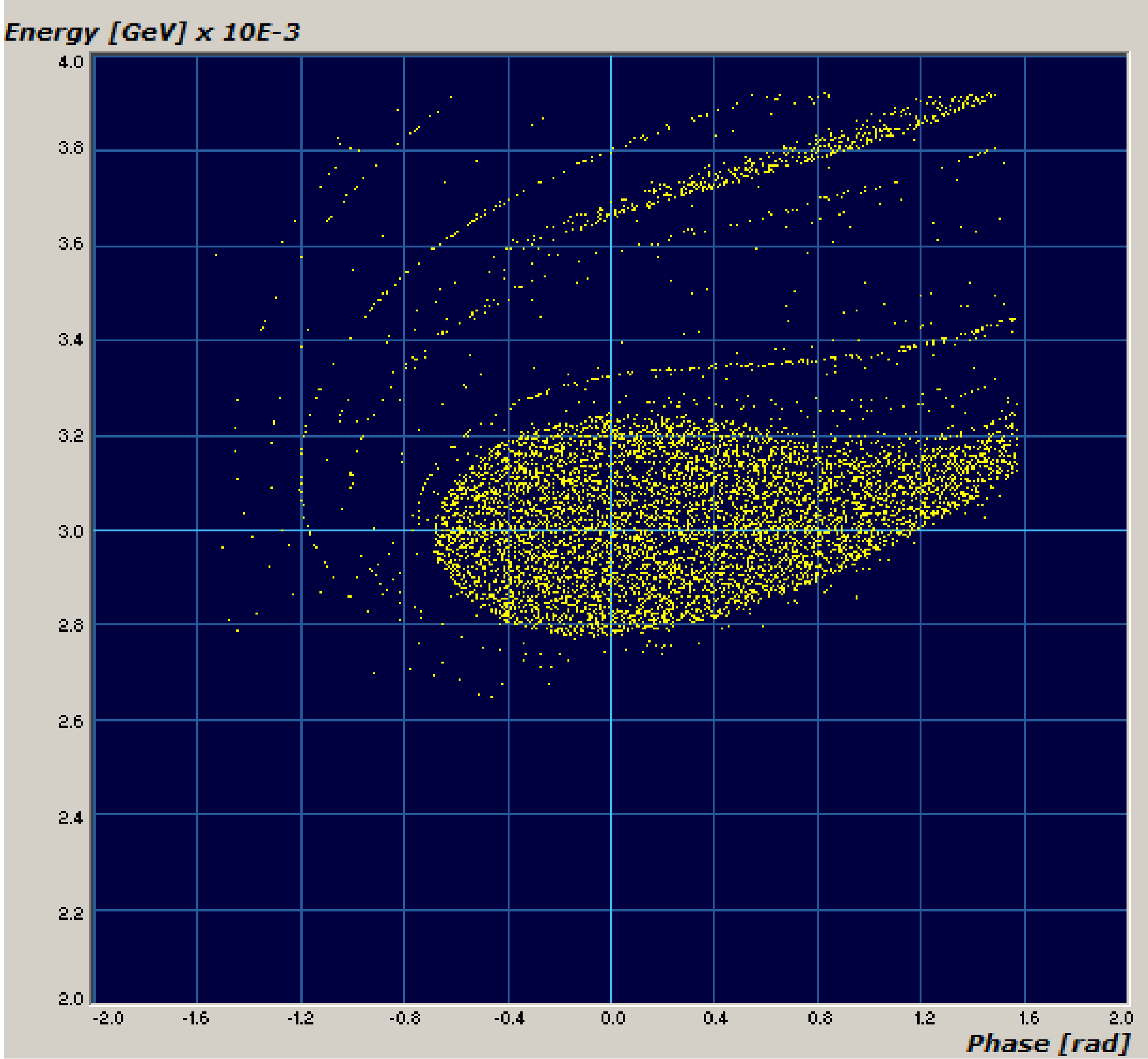


RF electric field as function of phase.

Potential of synchrotron oscillations

Trajectories in the longitudinal phase space each corresponding to a given value of the total energy (stationary bucket)

Longitudinal acceptance



Plot of the longitudinal acceptance of the CERN LINAC4 DTL (352 MHz, 3-50 MeV).
Obtained by plotting the survivors of very big beam in long phase space.

IH beam dynamics-KONUS

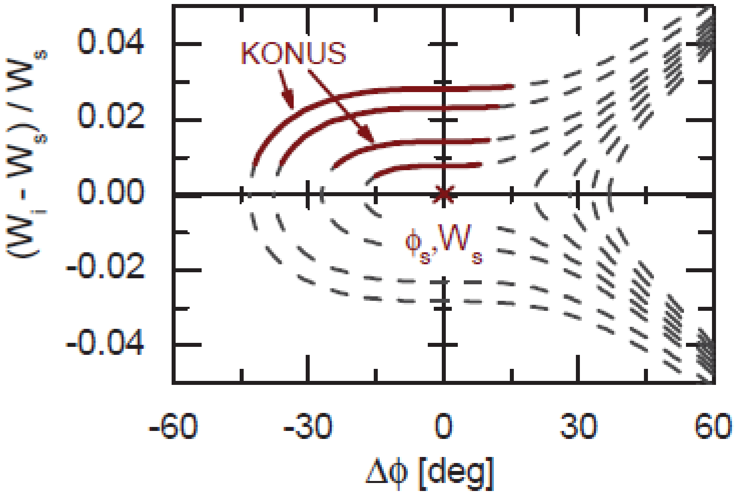


Figure 2: Single particle orbits in $\Delta W/W_s - \Delta\phi$ phase space at $\phi_s = 0°$ with color marking of the area used by KONUS.

Higher accelerating efficiency

Less RF defocusing (see later) – allow for longer accelerating sections w/o transverse focusing

Need re-bunching sections

Exceptions, exceptions…….

Acceleration

- definition of the acceptance : the maximum extension in phase and energy that we can accept in an accelerator :

$$\Delta\varphi \cong 3\varphi_s$$

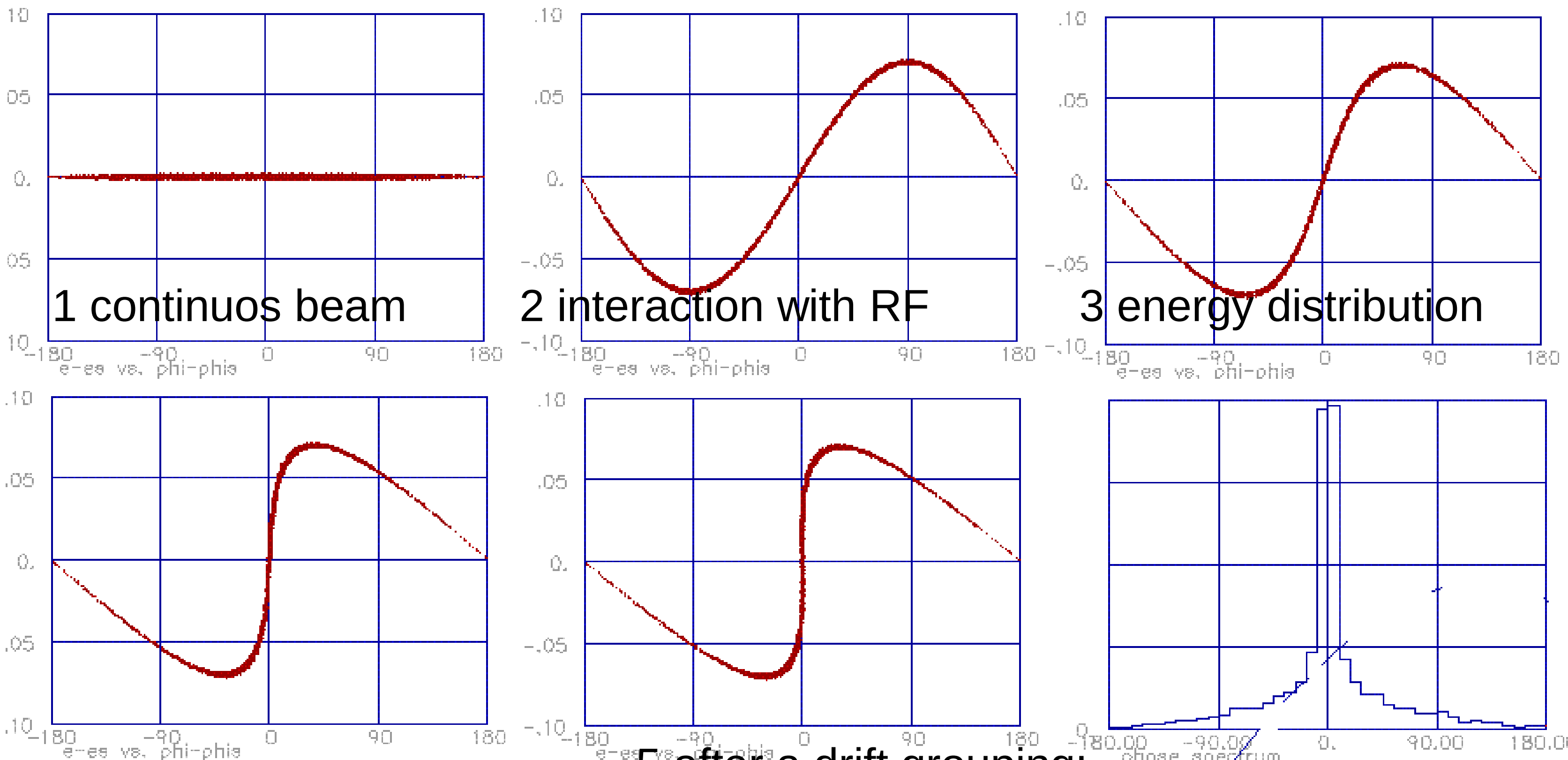
$$\Delta\hat{W}_{\max} = \pm 2\left[\frac{qmc^3\beta_s^3\gamma_s^3 E_0 T(\varphi_s\cos\varphi_s - \sin\varphi_s)}{\omega}\right]^{\frac{1}{2}}$$

bunching

Preparation to acceleration :

- generate a velocity spread inside the beam
- let the beam distribute itself around the particle with the average velocity

Discrete Bunching



4 earlier particles are slowed down, later particles are accelerated

5.after a drift grouping: around the “central” particle

6 distribution of the particles

Adiabatic bunching

- generate the velocity spread continuously with small longitudinal field : bunching over several oscillation in the phase space (up to 100!) allows a better capture around the stable phase : 95% capture vs 50 %

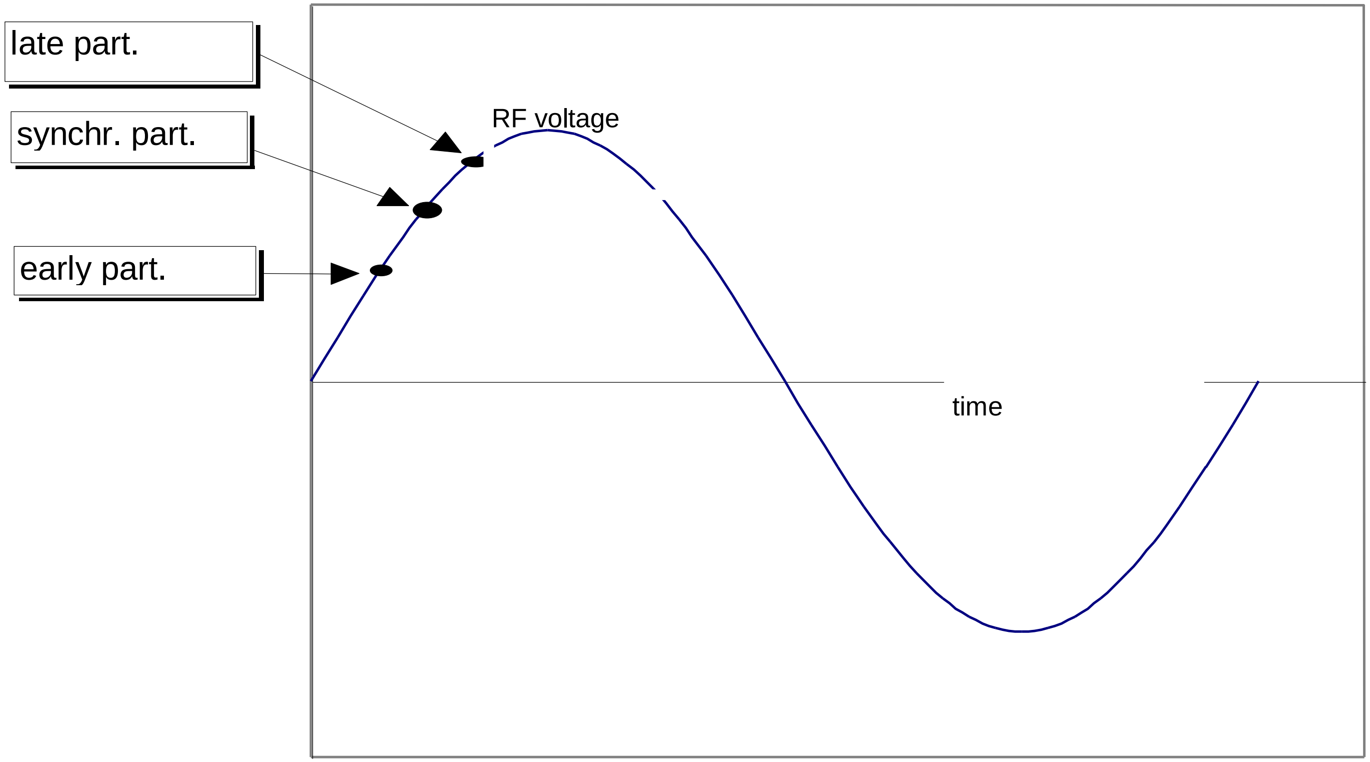
- in an RFQ by slowly increasing the depth of the modulation along the structure it is possible to smoothly bunch the beam and prepare it for acceleration.

movie of the RFQ rfq2.plt

Adiabatic bunching

- Rfq movie

Keep bunching during acceleration



for phase stability we need to accelerate when dEz/dz >0 i.e. on the rising part of the RF wave

Longitudinal phase advance

- if we accelerate on the rising part of the positive RF wave we have a LONGITUDINAL FORCE keeping the beam bunched. The force (harmonic oscillator type) is characterized by the LONGITUDINAL PHASE ADVANCE

$$k_{0l}^2 = \frac{2\pi q E_0 T \sin(-\varphi_s)}{mc^2 \beta_s^3 \gamma^3 \lambda} \left[\frac{1}{m^2} \right]$$

- long equation

$$\frac{d^2 \Delta\varphi}{ds^2} + k_{0l}^2 \left(\Delta\varphi - \frac{\Delta\varphi^2}{2\tan(-\varphi_s)} \right) = 0$$

Longitudinal phase advance

- Per meter

$$k_{0l} = \sqrt{\frac{2\pi q E_0 T \sin(-\varphi_s)}{mc^2 \beta_s^3 \gamma^3 \lambda}} \left[\frac{1}{m}\right]$$

Length of focusing period

L=(Number of RF gaps) βλ

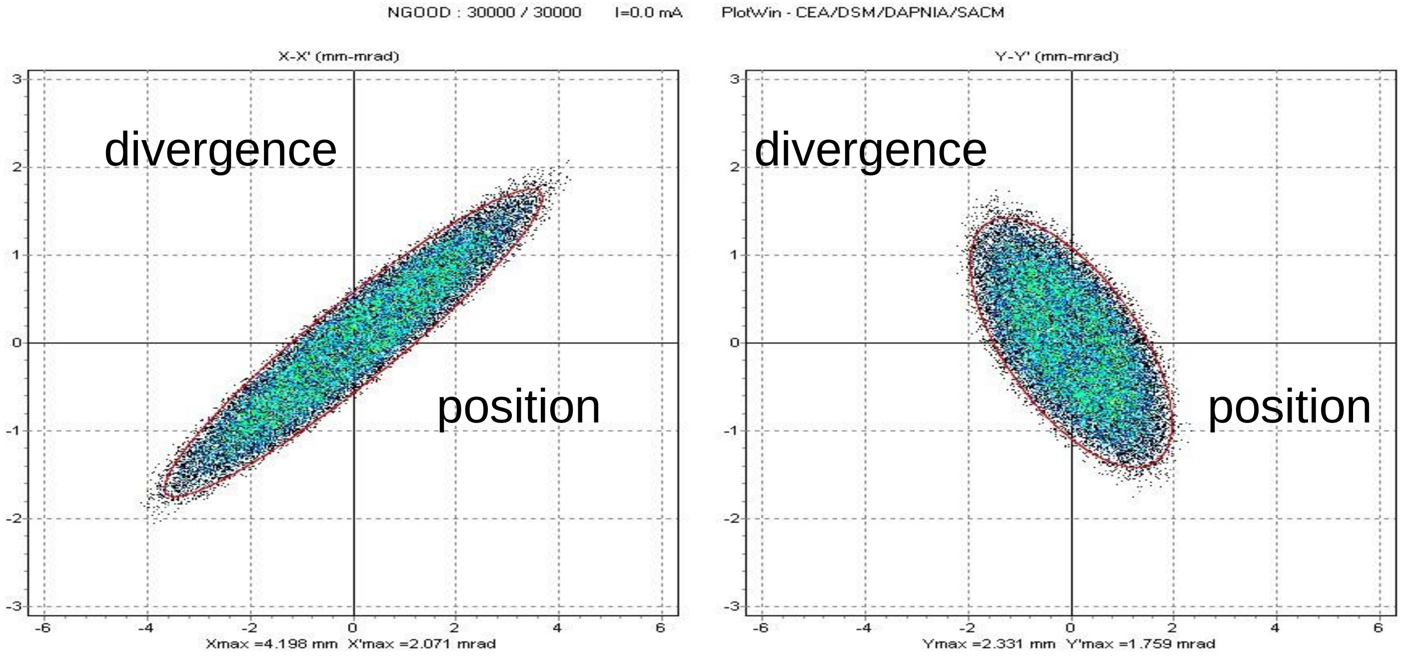
- Per focusing period

$$\sigma_{0l} = \sqrt{\frac{2\pi q E_0 T N^2 \lambda \sin(-\varphi_s)}{mc^2 \beta_s \gamma^3}}$$

- Per RF period

$$\sigma_{0l} = \sqrt{\frac{2\pi q E_0 T \sin(-\varphi_s)}{m \beta_s \gamma^3 \lambda}} \left[\frac{1}{s}\right]$$

Transverse phase space and focusing



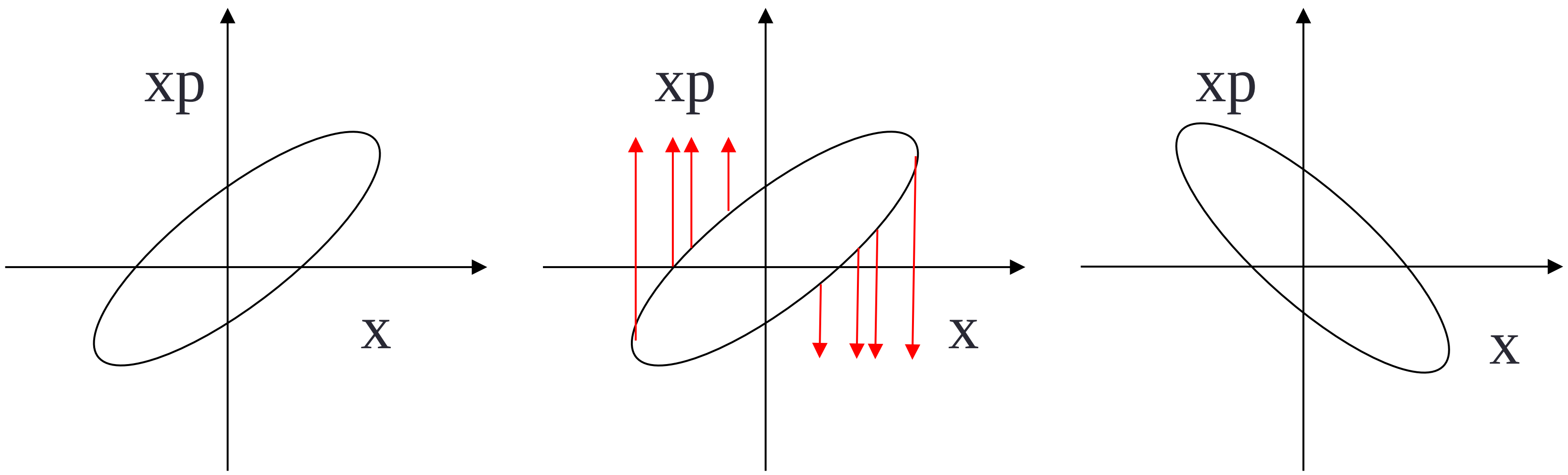
Bet = 6.3660 mm/Pi.mrad
Alp = -2.8807

DEFOCUSED

Bet = 1.7915 mm/Pi.mrad
Alp = 0.8318

FOCUSED

Focusing force



defocused beam

apply force towards the axis proportional to the distance from the axis

focused beam

F(x)=-K x

FODO

- periodic focusing channel : the beam 4D phase space is identical after each period
- Equation of motion in a periodic channel (Hill's equation) has periodic solution :

$$x(z) = \sqrt{\varepsilon_0 \beta(z)} \cdot \cos(\sigma(z))$$

emittance

beta function , has the periodicity of the focusing period

transverse phase advance

$$\sigma(z) = \int_0^z \frac{dz}{\beta(z)}$$

$$\beta(z+l) = \beta(z)$$

review N. Pichoff course

quadrupole focusing

$$\sigma_{0t} = \sqrt{\frac{\theta_0^4}{8\pi^2} + \Delta_{rf}}$$

zero current phase advance per period in a LINAC

$$\theta_0^2 = \frac{qG\lambda^2 N^2 \beta\chi}{m_0 c\gamma}$$

G magnetic quadrupole gradient, [T/m]
N= number of magnets in a period

for $+-$ *(N=2)*

$$\chi = \frac{4}{\pi}\sin\left(\frac{\pi}{2}\Gamma\right)$$

for $++--$ *(N=4)*

$$\chi = \frac{8}{\sqrt{2}\pi}\sin\left(\frac{\pi}{4}\Gamma\right)$$

Γ is the quadrupole filling factor (quadrupole length relative to period length).

RF defocusing

Maxwell equations

$$\nabla \cdot E = 0 \quad \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0$$

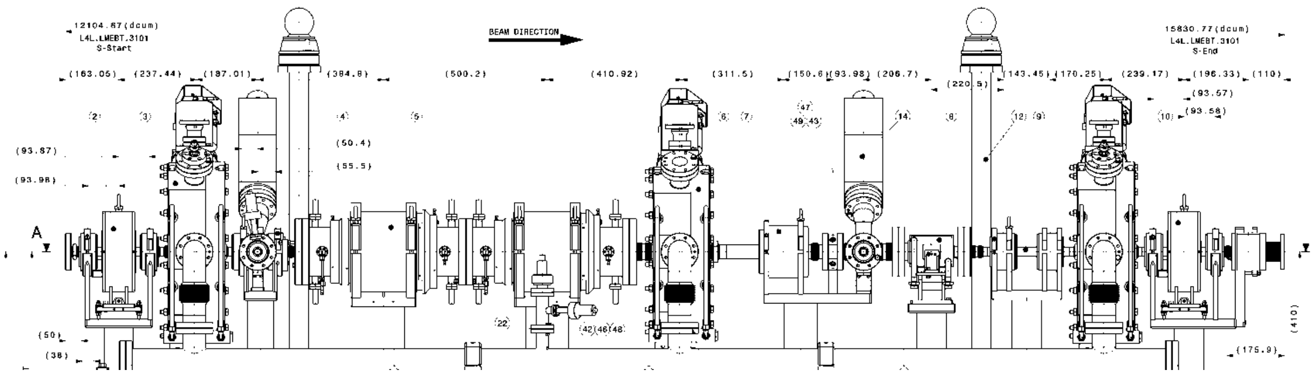
when longitudinal focusing (phase stability) , there is defocusing (depending on the phase) in the transverse planes

$$\Delta_{rf} = \frac{1}{2}\sigma_{0l}^2 = \frac{\pi q \lambda N^2 E_0 T \sin\phi_s}{m_0 c^2 \beta\gamma^3}$$

Number of RF gap in a transverse focusing period

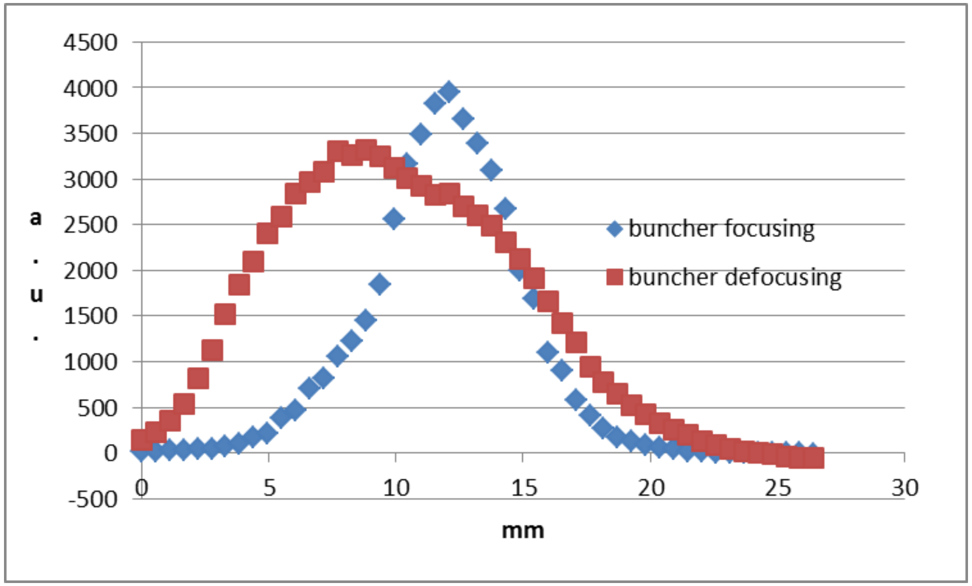
Rf defocusing is MEASURABLE

(not only in text books)

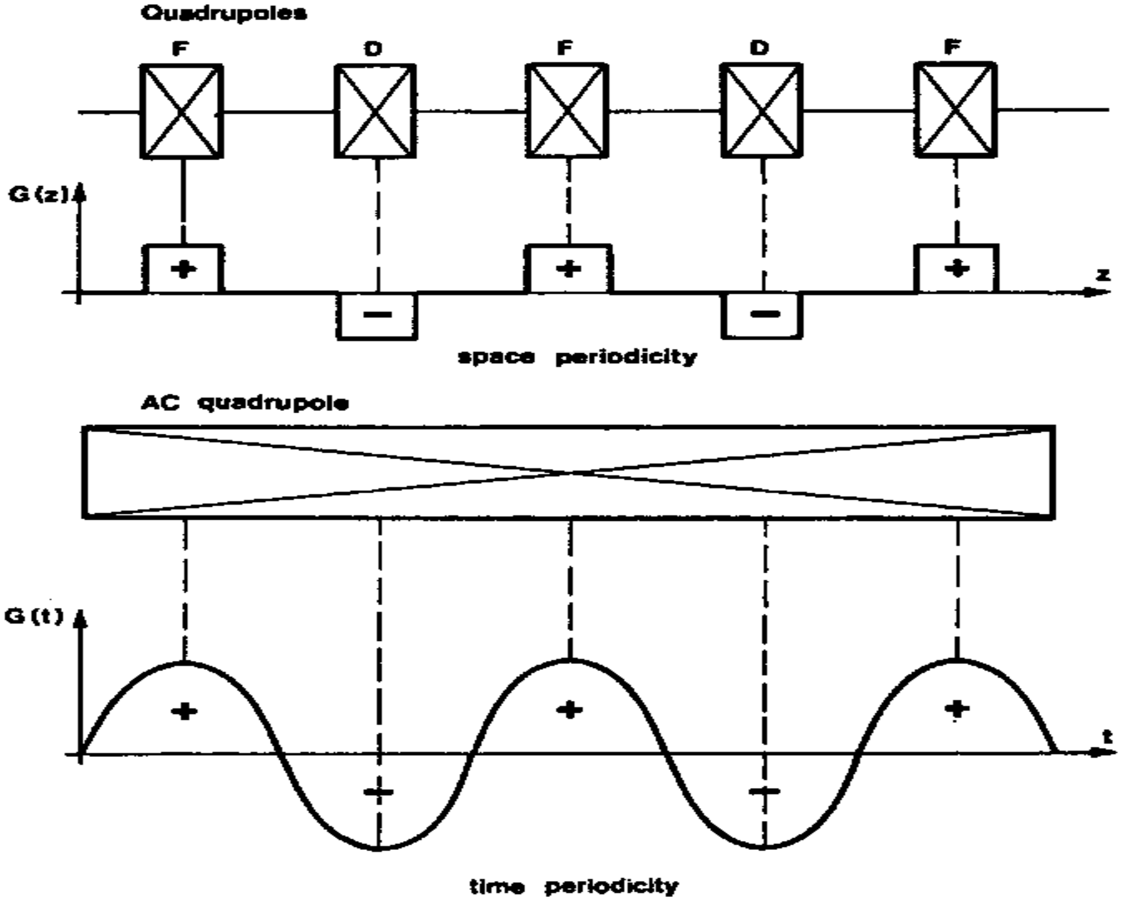


Change the buncher phase and measure the transverse beam profile

Effect of the phase-dependent focusing is visible and it can be used to set the RF phase in absence of longitudinal measurements.



FODO in RFQ vs FODO in DTL



sequence of magnetic quadrupoles

focusing field

RF quadrupole

focusing field

First order rules for designing an accelerator

- Acceleration : choose the correct phase, maintain such a phase thru the process of acceleration
- Focusing : choose the appropriate focusing scheme and make sure it is matched

Synchronous particle and geometrical beta β_g.

- design a linac for one “test” particle. This is called the “synchronous” particle.

- the length of each accelerating element determines the time at which the synchronous particles enters/exits a cavity.
- For a given cavity length there is an optimum velocity (or beta) such that a particle traveling at this velocity goes through the cavity in half an RF period.
- The difference in time of arrival between the synchronous particles and the particle traveling with speed corresponding to the geometrical beta determines the phase difference between two adjacent cavities
- in a synchronous machine the geometrical beta is always equal to the synchronous particle beta and EACH cell is different

Adapting the structure to the velocity of the particle

- Case1 : the geometry of the cavity/structure is continuously changing to adapt to the change of velocity of the “synchronous particle”

- Case2 : the geometry of the cavity/structure is adapted in step to the velocity of the particle. Loss of perfect synchronicity, phase slippage.

- Case3 : the particle velocity is beta=1 and there is no problem of adapting the structure to the speed.

Case1 : $\beta_s = \beta_g$

- The absolute phase φ_i and the velocity $\beta_{i\text{-}1}$ of this particle being known at the entrance of cavity i, its RF phase ϕ_i is calculated to get the wanted synchronous phase ϕ_{si}, $\phi_i = \varphi_i - \phi_{si}$
- the new velocity β_i of the particle can be calculated from, $\Delta W_i = qV_0 T \cdot \cos\phi_{si}$

  ① if the phase difference between cavities i and $i+1$ is given, the distance D_i between them is adjusted to get the wanted synchronous phase ϕ_{si+1} in cavity $i+1$.

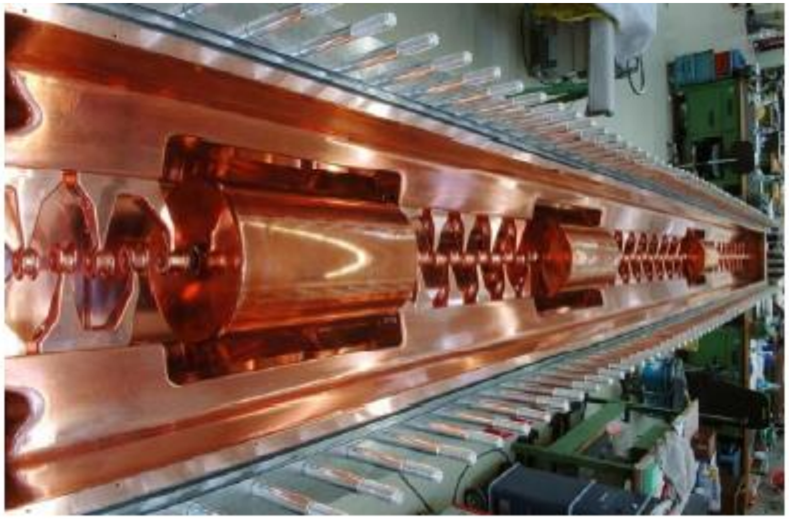
  ②if the distance D_i between cavities i and $i+1$ is set, the RF phase ϕ_i of cavity $i+1$ is calculated to get the wanted synchronous phase ϕ_{si+1} in it.

RF phase	$\phi_{i\text{-}1}$		ϕ_i		ϕ_{i+1}
Particle velocity		$\beta_{si\text{-}1}$		β_{si}	
Distances		$D_{i\text{-}1}$		D_i	
Synchronous phase	$\phi_{si\text{-}1}$		ϕ_{si}		ϕ_{si+1}
Cavity number	$i\text{-}1$		i		$i+1$

Synchronism condition :

$$\phi_{si+1} - \phi_{si} = \omega \cdot \frac{D_i}{\beta_{si}\, c} + \phi_{i+1} - \phi_i + 2\pi n$$

Synchronous structures

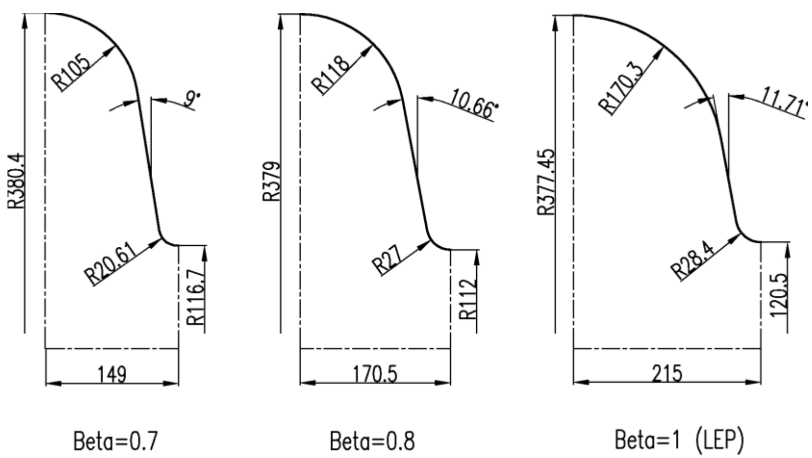


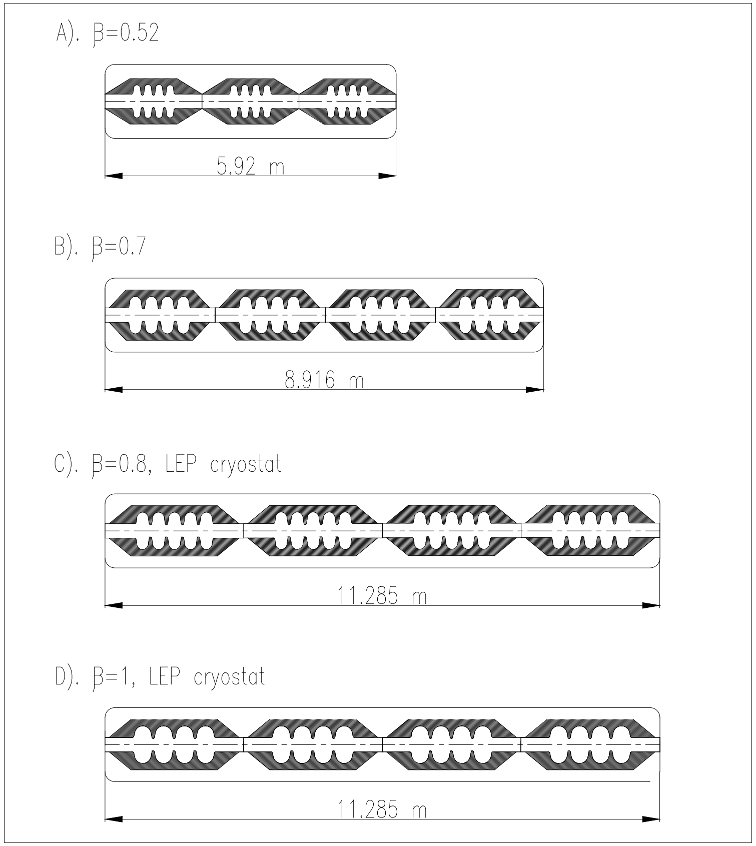
Case 2 : $\beta_s \sim \beta_g$

- for simplifying construction and therefore keeping down the cost, cavities are not individually tailored to the evolution of the beam velocity but they are constructed in blocks of identical cavities (tanks). several tanks are fed by the same RF source.
- This simplification implies a “phase slippage” i.e. a motion of the centre of the beam . The phase slippage is proportional to the number of cavities in a tank and it should be carefully controlled for successful acceleration.

Linacs made of superconducting cavities

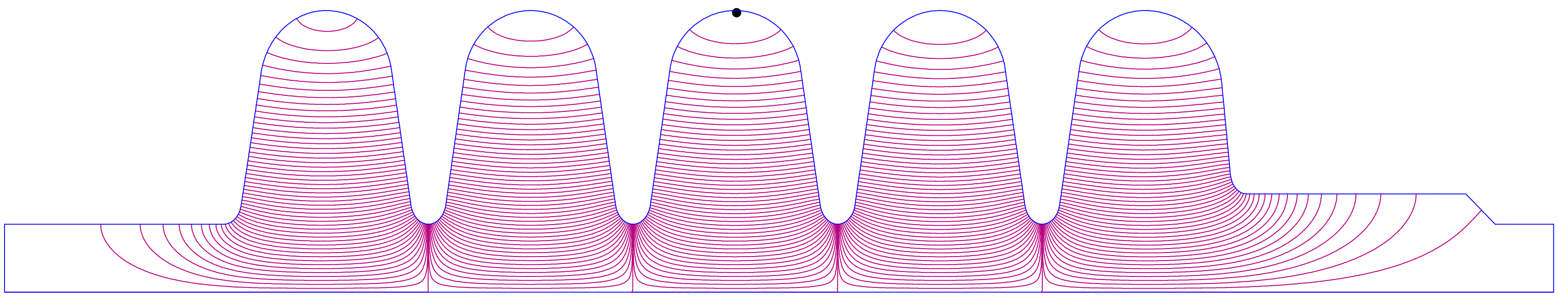
Need to standardise construction of cavities:
only few different types of cavities are made for some β's
more cavities are grouped in cryostats





Example:
CERN design, SC linac 120 - 2200 MeV

phase slippage



Lcavity = $\beta_g \lambda/2$

particle enters the cavity with $\beta_s < \beta_g$. It is accelerated

the particle has not left the cavity when the field has changed sign : it is also a bit decelerated

the particle arrives at the second cavity with a “delay”

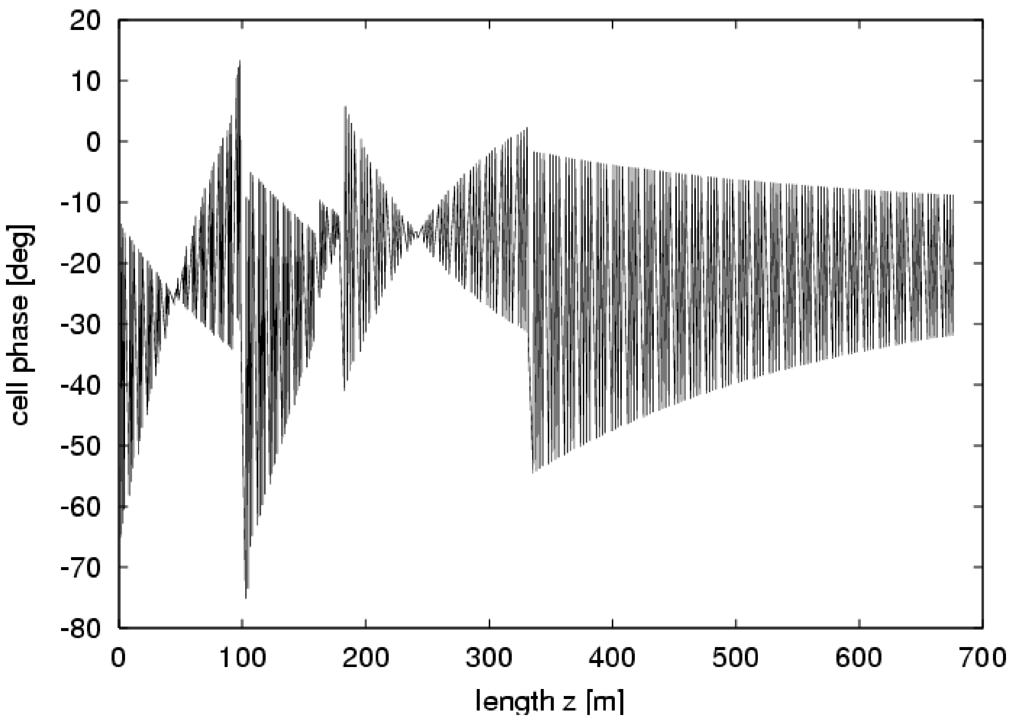
........and so on and so on

we have to optimize the initial phase for minimum phase slippage

for a given velocity there is a maximum number of cavity we can accept in a tank

Phase slippage

In each section, the cell length (βλ/2, π mode!) is correct only for one beta (energy): at all other betas the phase of the beam will differ from the design phase



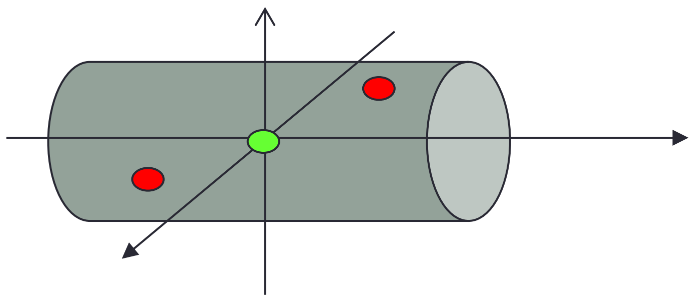
Example of phase slippage:
CERN design for a 352 MHz SC linac
Four sections:
β = 0.52 (120 - 240 MeV)
β = 0.7 (240 - 400 MeV)
β = 0.8 (400 MeV - 1 GeV)
β = 1 (1 - 2.2 GeV)

Phase at the first and last cell of each 4-cell cavity (5-cell at β=0.8)

Space charge

We have to keep into account the space charge forces when determining the transvers and longitudinal focusing.
Part of the focusing goes to counteract the space charge forces.

Assuming an uniformly charged ellipsoid:



Effect is zero on the beam centre:
Contribution of red partciles concel out

$$E_x = \frac{1}{4\pi\varepsilon_0}\frac{3I\lambda}{c\gamma^2}\frac{1-f}{r_x(r_x+r_y)r_z}x$$

$$E_y = \frac{1}{4\pi\varepsilon_0}\frac{3I\lambda}{c\gamma^2}\frac{1-f}{r_y(r_x+r_y)r_z}y$$

$$E_z = \frac{1}{4\pi\varepsilon_0}\frac{3I\lambda}{c}\frac{f}{r_x r_y r_z}z$$

The transverse phase advance per meter becomes:

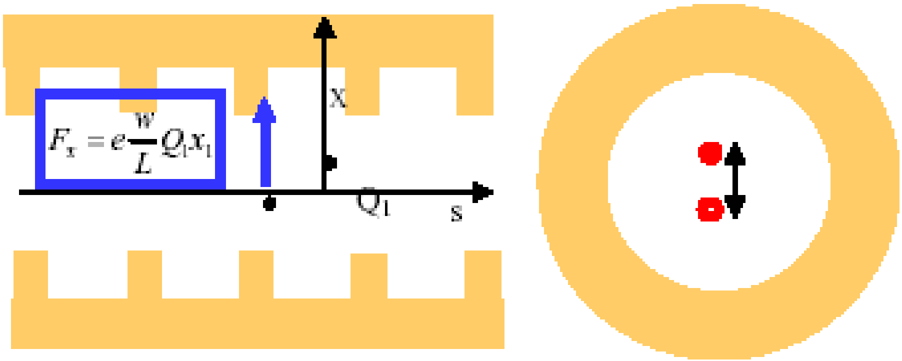
I= beam current
$r_{x,y,z}$=ellipsoid semi-axis
f= form factor
Z_0=free space impedance (377 Ω)

$$k_{ot} = \sqrt{\left(\frac{qGl}{2mc\beta\gamma}\right)^2 - \frac{\pi q E_0 T \sin(-\phi)}{mc^2\lambda(\beta\gamma)^3} - \frac{3Z_0 q I\lambda\,(1-f)}{8\pi mc^2\beta^2\gamma^3 r_x r_y r_z}}$$

instabilities in e-linac

- Phenomenon typical of high energy electrons traveling in very high frequency structures (GHz).
- Electromagnetic waves caused by the charged beam traveling through the structure can heavily interact with the particles that follows.
- The fields left behind the particle are called wake fields.

wake field



a (source) charge *Q1* traveling with a (small) offset *x1* respect to the center of the RF structure perturbs the accelerating field configuration and leaves a wake field behind. A following (test) particle will experience a transverse field proportional to the displacement and to the charge of the source particle:

$$F_x = e\frac{w}{L}Q_1x_1$$

L=period of the structure

W= wake function, depends on the delay between particles and on the RF frequency (very strongly like f^3)

wake field effect

- this force is a dipole kick which can be expressed like :

$$x'' = \frac{eQ_1 w}{\gamma mc^2 L} x_1$$

decreases with energy

wake field effects

- Effect of the head of the bunch on the tail of the bunch (head-tail instabilities)
- In the particular situation of resonance between the lattice (FODO) oscillation of the head and the FODO+wake oscillation of the tail we have BBU (Beam breakUp) causing emittance growth (limit to the luminosity in linear colliders)
- Effect of one bunch on the following.