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Accelerator Magnets

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Contents

Theory
- Maxwell's 2 magneto-static equations;

With no currents or steel present:
- Solutions in two dimensions with scalar potential (no currents);
- Cylindrical harmonic in two dimensions (trigonometric formulation);
- Field lines and potential for dipole, quadrupole, sextupole;

Introduction of steel:
- Ideal pole shapes for dipole, quad and sextupole;
- Field harmonics-symmetry constraints and significance;
- Significance and use of contours of constant vector potential;

Contents (cont.)

Three dimensional issues:

- Termination of magnet ends and pole sides;
- The 'Rogowski roll-off'

Introduction of currents:

- Ampere-turns in dipole, quad and sextupole;
- Coil design;
- Coil economic optimisation-capital/running costs;

Practical Issues:

- Backleg and coil geometry- 'C', 'H' and 'window frame' designs;
- FEA techniques - Modern codes- OPERA 2D and 3D;
- Judgement of magnet suitability in design.

Magnets - introduction

Dipoles to bend the beam:

Quadrupoles to focus it:

Sextupoles to correct chromaticity:

We shall establish a formal approach to describing these magnets.

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No currents, no steel:

Maxwell's equations: $\underline{\nabla}.\underline{\mathbf{B}} = 0$;

$\underline{\nabla} \wedge \underline{\mathbf{H}} = \underline{\mathbf{j}}$;

Then we can put: $\underline{\mathbf{B}} = - \underline{\nabla}\phi$

So that: $\underline{\nabla}^2\phi = 0$ (Laplace's equation).

Taking the two dimensional case (ie constant in the z direction) and solving for cylindrical coordinates (r,θ):

$$\phi = (E+F\ \theta)(G+H\ \ln r) + \Sigma_{n=1}^{\infty} (J_n\ r^{n} \cos n\theta + K_n\ r^{n} \sin n\theta + L_n\ r^{-n} \cos n\ \theta + M_n\ r^{-n} \sin n\ \theta\ )$$

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In practical situations:

The scalar potential simplifies to:

$$\phi = \Sigma_n (J_n\ r^{n} \cos n\theta + K_n\ r^{n} \sin n\theta),$$

with n integral and J_n,K_n a function of geometry.

Giving components of flux density:

$$B_r = - \Sigma_n (n\ J_n\ r^{n-1} \cos n\theta + nK_n\ r^{n-1} \sin n\theta)$$
$$B_\theta = - \Sigma_n (-n\ J_n\ r^{n-1} \sin n\theta + nK_n\ r^{n-1} \cos n\theta)$$

Then to convert to Cartesian coordinates:

$x = r \cos \theta$; $y = r \sin \theta$;

and $B_x = - \partial\phi/ \partial x$; $B_y = - \partial\phi/ \partial y$

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Significance

This is an infinite series of cylindrical harmonics; they define the allowed distributions of **B** in 2 dimensions in the absence of currents within the domain of (r,θ).

Distributions not given by above are not physically realisable.

Coefficients J_n, K_n are determined by geometry (remote iron boundaries and current sources).

Note that this formulation can be expressed in terms of complex fields and potentials.

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Dipole field n=1:

Cylindrical:

$\phi = J_1\, r \cos\theta + K_1\, r \sin\theta.$

$B_r = J_1 \cos\theta + K_1 \sin\theta;$

$B_\theta = -J_1 \sin\theta + K_1 \text{ os } \theta;$

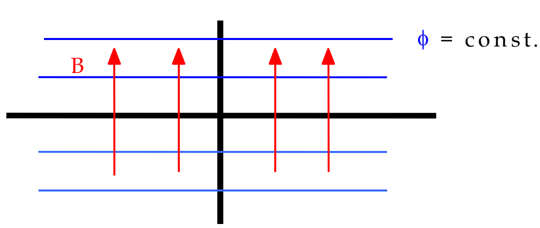
Cartesian:

$\phi = J_1\, x + K_1\, y$

$B_x = -J_1$

$B_y = -K_1$

So, $J_1 = 0$ gives vertical dipole field:



$K_1 = 0$ gives horizontal dipole field.

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Quadrupole field n=2:

Cylindrical:

$\phi = J_2\, r^2 \cos 2\theta + K_2\, r^2 \sin 2\theta;$

$B_r = 2\, J_2\, r\ \cos 2\theta + 2K_2\, r\ \sin 2\theta;$

$B_\theta = -2J_2\, r\ \sin 2\theta + 2K_2\, r\ \cos 2\theta;$

Cartesian:

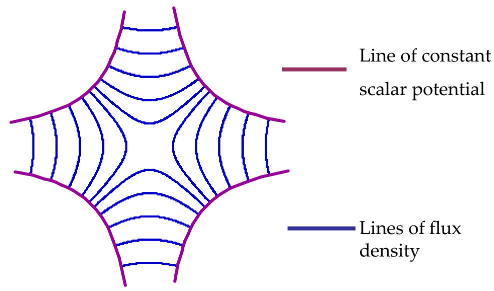
$\phi = J_2\,(x^2 - y^2) + 2K_2\, xy$

$B_x = -2\,(J_2\, x + K_2\, y)$

$B_y = -2\,(-J_2\, y + K_2\, x)$

$J_2 = 0$ gives 'normal' or 'upright' quadrupole field.

$K_2 = 0$ gives 'skew' quad fields (above rotated by $\pi/4$).



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Sextupole field n=3:

Cylindrical;

$\phi = J_3\, r^3 \cos 3\theta + K_3\, r^3 \sin 3\theta;$

$B_r = 3\, J_3 r^2\ \cos 3\theta + 3K_3 r^2\ \sin 3\theta;$

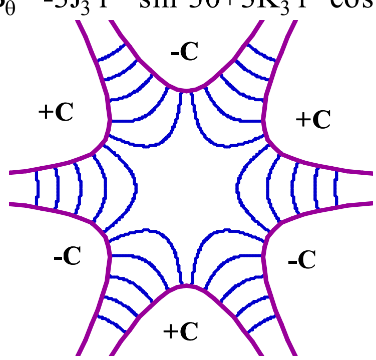
$B_\theta = -3J_3\, r^2\ \sin 3\theta + 3K_3\, r^2 \cos 3\theta;$

Cartesian:

$\phi = J_3\,(x^3 - 3y^2x) + K_3(3yx^2 - y^3)$

$B_x = -3\{J_3\,(x^2 - y^2) + 2K_3 yx\}$

$B_y = -3\{-2\, J_3\, xy + K_3(x^2 - y^2)\}$



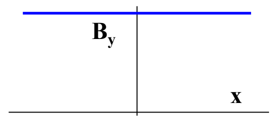
$J_3 = 0$ giving 'upright' sextupole field.

Line of constant scalar potential

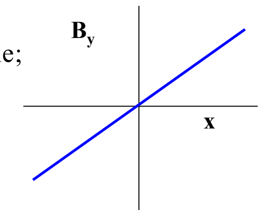
Lines of flux density

Variation of B_y on x axis (upright fields).

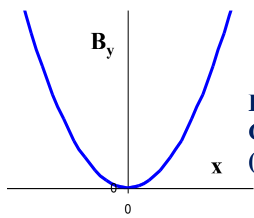
Dipole;
constant field:



Quadrupole;
linear field



Sextupole:
quadratic variation:



$B_y = G_Q\, x$;
G_Q is quadrupole gradient (T/m).

$B_y = G_S\, x^2$;
G_S is sextupole gradient (T/m^2).

Alternative notation:

(used in most lattice programs)

$$B(x) = B\rho \sum_{n=0}^{\infty} \frac{k_n\, x^n}{n!}$$

magnet strengths are specified by the value of k_n;
(normalised to the beam rigidity);

order n of k is different to the 'standard' notation:

dipole is n = 0;
quad is n = 1; etc.

k has units:

k_0 (dipole) m^{-1};
k_1 (quadrupole) m^{-2}; etc.

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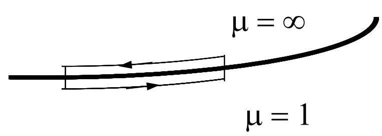
Introducing iron yokes and poles.

What is the ideal pole shape?

- Flux is normal to a ferromagnetic surface with infinite μ:



curl H = 0

therefore $\int$ H.ds = 0;

in steel H = 0;

therefore parallel H air = 0

therefore B is normal to surface.

- Flux is normal to lines of scalar potential, ($\underline{\mathbf{B}} = - \underline{\nabla}\phi$);
- So the lines of scalar potential are the ideal pole shapes!

(but these are infinitely long!)

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Equations of ideal poles

Equations for Ideal (infinite) poles;

($J_n = 0$) for **'upright'** (ie not skew) fields:

Dipole:

$y = \pm g/2$;

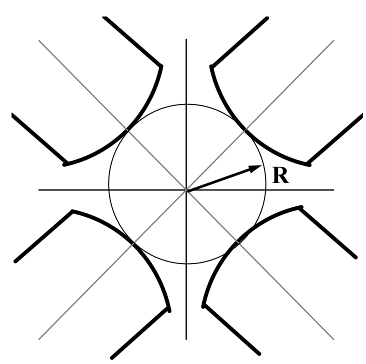
(g is inter-pole gap).

Quadrupole:

$xy = \pm R^2/2$;

Sextupole:

$3x^2y - y^3 = \pm R^3$;



R is the 'inscribed radius' of a multipole magnet.

‘Pole-tip’ Field

The **radial field at pole centre** of a multipole magnet:

$$B_{PT} = G_N R^{(n-1)};$$

Quadrupole: $B_{PT} = G_Q R$; sextupole: $B_{PT} = G_S R^2$; etc;

Has it any significance?

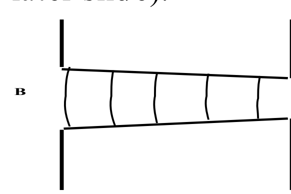
- a quadrupole $R = 50$ mm; $G_Q = 20$ T/ m; $B_{PT} = 1.0$ T;

i) Beam line – round beam r = 40 mm;
pole extends to x = 65 mm; $B_y(65,0) = 1.3$ T; OK ☺;
ii) Synchrotron source – beam ± 50 mm horiz.; ± 10 mm vertical;
pole extends to x = 80 mm; By (80,0)= 1.6 T; perhaps OK ???☺ ;
iii) FFAG – beam ± 65 mm horiz.; ± 8 mm vertical;
Pole extends to x = 105 mm; By (105, 0) = 2.1 T ☹☹☹.

Combined function magnets

'Combined Function Magnets' - often dipole and quadrupole field combined (but see later slide):

A quadrupole magnet with physical centre shifted from magnetic centre.



Characterised by 'field index' n, +ve or -ve depending on direction of gradient; do not confuse with harmonic n!

$$n = -\left(\frac{\rho}{B_0}\right)\left(\frac{\partial B}{\partial x}\right),$$

ρ is radius of curvature of the beam;

B_o is central dipole field

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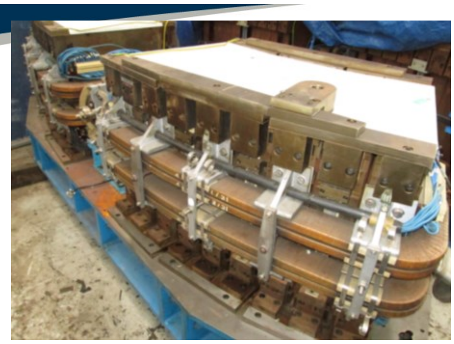
Typical combined dipole/ quadrupole



SRS Booster c.f. dipole

'D' type +ve n.

'F' type -ve n

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NINA Combined function magnets

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Pole for a combined dipole and quad.

Physical and magnetic centres are separated by X_0

Horizontal displacment from true quad centre is x'

Then $B_0 = \left(\frac{\partial B}{\partial x}\right) X_0$

therefore $x' y = \pm R^2 / 2$

As $x' = x + X_0$

Pole equation is $y = \pm \frac{R^2}{2} \frac{n}{\rho} \left(1 - \frac{n x}{\rho}\right)^{-1}$

or $y = \pm g \left(1 - \frac{n x}{\rho}\right)^{-1}$

where g is the **half gap** at the physical centre of the magnet

rewritten as $y = \pm g \left[1 - \frac{x}{B_0}\left(\frac{\partial B}{\partial x}\right)\right]^{-1}$

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Other combined function magnets:

- dipole, quadrupole and sextupole;
- dipole & sextupole (for chromaticity control);
- dipole, skew quad, sextupole, octupole;

Generated by

- pole shapes given by sum of correct scalar potentials

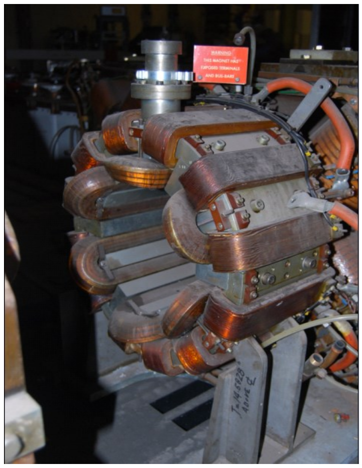
  - amplitudes built into pole geometry – **not variable**!

OR:

- multiple coils mounted on the yoke

  - amplitudes independently varied by coil currents.

The SRS multipole magnet.



Could develop:

- vertical dipole
- horizontal dipole;
- upright quad;
- skew quad;
- sextupole;
- octupole;
- others.

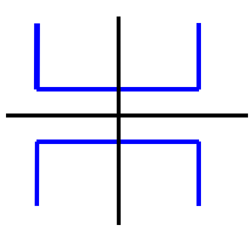
The practical pole in 2D

Practically, poles are finite, **introducing errors**;

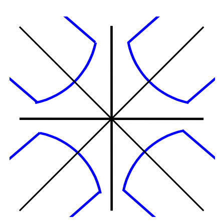
these appear as higher harmonics which degrade the field distribution.

However, the iron geometries have certain symmetries that **restrict** the nature of these errors.

Dipole:



Quadrupole:



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Possible symmetries.

Lines of symmetry:

	Dipole:	Quad
Pole orientation determines whether pole is upright or skew.	$y = 0$;	$x = 0$; $y = 0$
Additional symmetry imposed by pole edges.	$x = 0$;	$y = \pm x$

The additional constraints imposed by the symmetrical pole edges limits the values of n that have non zero coefficients

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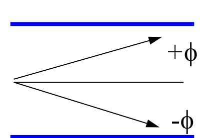
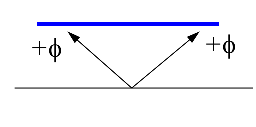
Dipole symmetries

Type	**Symmetry**	**Constraint**
Pole orientation	$\phi(\theta) = -\phi(-\theta)$	all $J_n = 0$;
Pole edges	$\phi(\theta) = \phi(\pi - \theta)$	K_n non-zero only for: $n = 1, 3, 5$, etc;





So, for a fully symmetric dipole, only 6, 10, 14 etc pole errors can be present.

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Quadrupole symmetries

Type	Symmetry	Constraint
Pole orientation	$\phi(\theta) = -\phi(-\theta)$	All $J_n = 0$;
	$\phi(\theta) = -\phi(\pi -\theta)$	$K_n = 0$ all odd n;
Pole edges	$\phi(\theta) = \phi(\pi/ 2 -\theta)$	K_n non-zero only for: n = 2, 6, 10, etc;

So, a fully symmetric quadrupole, only 12, 20, 28 etc pole errors can be present.

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Sextupole symmetries.

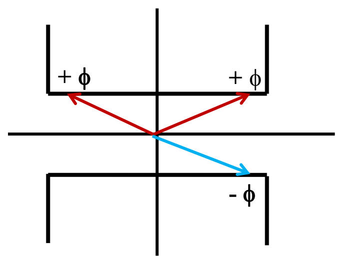
Type	Symmetry	Constraint
Pole orientation	$\phi(\theta) = -\phi(-\theta)$ $\phi(\theta) = -\phi(2\pi/ 3 - \theta)$ $\phi(\theta) = -\phi(4\pi/ 3 - \theta)$	All $J_n = 0$; $K_n = 0$ for all n not multiples of 3;
Pole edges	$\phi(\theta) = \phi(\pi/ 3 - \theta)$	K_n non-zero only for: n = 3, 9, 15, etc.

So, a fully symmetric sextupole, only 18, 30, 42 etc pole errors can be present.

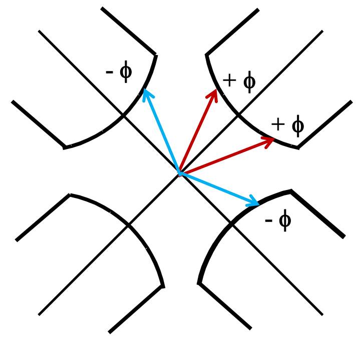
Summary

For perfectly symmetric magnets, the 'allowed' error fields are fully defined by the symmetry of scalar potential ϕ and the trigonometry:

Dipole:



Quadrupole:



Dipole: only dipole, sextupole, 10 pole etc can be present

Sextupole: only sextupole, 18, 30, 42 pole, etc. can be present

Quadrupole: only quadrupole, 12 pole, 20 pole etc can be present

Vector potential in 2 D

We have: $\underline{\mathbf{B}}$ = curl $\underline{\mathbf{A}}$ ($\underline{\mathbf{A}}$ is vector potential);

and div $\underline{\mathbf{A}} = 0$

Expanding: $\underline{\mathbf{B}}$ = curl $\underline{\mathbf{A}}$ =

$$(\partial A_z/\ \partial y - \partial A_y/\ \partial z)\,\mathbf{i} + (\partial A_x/\ \partial z - \partial A_z/\ \partial x)\,\mathbf{j} + (\partial A_y/\ \partial x - \partial A_x/\ \partial y)\,\mathbf{k};$$

where **i, j, k**, are unit vectors in x, y, z.

In 2 dimensions $B_z = 0$; $\partial /\ \partial z = 0$;

So $A_x = A_y = 0$;

and $\underline{\mathbf{B}} = (\partial A_z/\ \partial y)\,\mathbf{i} - (\partial A_z/\ \partial x)\,\mathbf{j}$

$\underline{\mathbf{A}}$ is in the z direction, normal to the 2 D problem.

Note: div $\underline{\mathbf{B}} = \partial^2 A_z/ (\partial x\, \partial y) - \partial^2 A_z/ (\partial x\, \partial y) = 0$;

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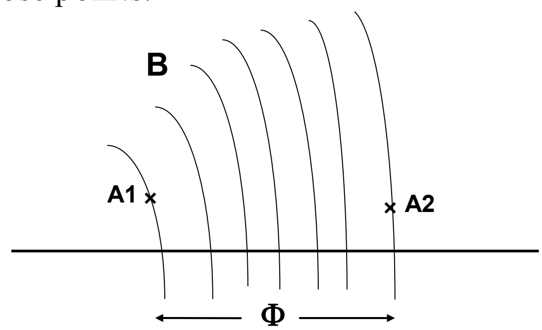
Total flux between two points $\propto \Delta A$

In a **two dimensional problem** the magnetic flux between two points is proportional to the difference between the vector potentials at those points.

$$\Phi \propto (A_2 - A_1)$$



Proof on next slide.

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Proof:

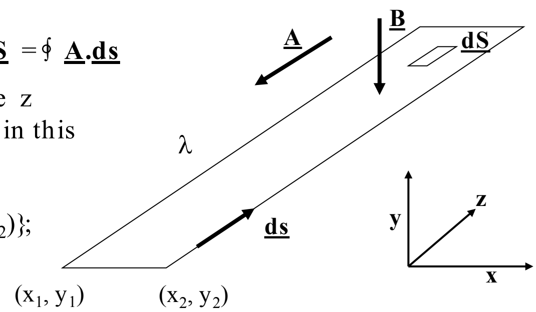
Consider a rectangular closed path, length λ in z direction at (x_1,y_1) and (x_2,y_2); apply Stokes' theorem:

$$\Phi = \iint \underline{\mathbf{B}}.\underline{\mathbf{dS}} = \iint (\text{curl } \underline{\mathbf{A}}).\underline{\mathbf{dS}} = \oint \underline{\mathbf{A}}.\underline{\mathbf{ds}}$$

But A is exclusively in the z direction, and is constant in this direction.

So:

$$\int \mathbf{A}.\mathrm{ds} = \lambda \{A(x_1,y_1) - A(x_2,y_2)\};$$



$$\Phi = \lambda \{A(x_1,y_1) - A(x_2,y_2)\};$$

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Contours of constant A

Therefore:

i) Contours of constant vector potential in 2D give a graphical representation of lines of flux.

ii) These are used in 2D FEA analysis to obtain a graphical image of flux distribution.

iii) The total flux cutting the coil allows the calculation of the inductive voltage per turn in an ac magnet:

$$V = - d\,\Phi/dt;$$

iv) For a sine wave oscillation frequency ω:

$$V_{peak} = \omega\,(\Phi/2)$$

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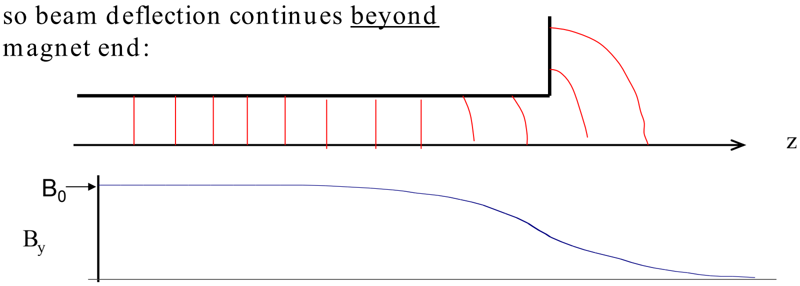
In 3D – pole ends (also pole sides).

Fringe flux will be present at pole ends so beam deflection continues beyond magnet end:



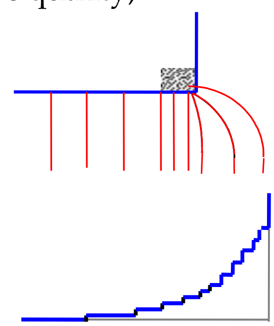
The magnet's strength is given by $\int B_y(z)\,dz$ along the magnet , the integration including the fringe field at each end;

The **'magnetic length'** is defined as **$(1/B_0)(\int B_y(z)\,dz)$** over the same integration path, where B_0 is the field at the azimuthal centre.

End Fields and Geometry.

At high(ish) fields it is necessary to terminate the pole (transverse OR longitudinal) in a controlled way:;

- to prevent saturation in a sharp corner (see diagram);
- to maintain length constant with x, y;
- to define the length (strength) or preserve quality;
- to prevent flux entering normal to lamination (ac).



Longitudinally, the end of the magnet is therefore 'chamfered' to give increasing gap (or inscribed radius) and lower fields as the end is approached.

Classical end or side solution

The **'Rogowski' roll-off**: Equation:

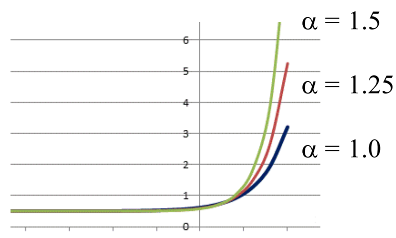
$$y = g/2 + (g/\pi\alpha)\,[\exp(\alpha\pi x/g) - 1];$$

g/2 is dipole half gap;

y = 0 is centre line of gap;

α (~1); parameter to control the roll off;

With $\alpha = 1$, this profile provides the maximum rate of **increase** in gap with a monotonic **decrease** in flux density at the surface;



For a high B_y magnet this avoids any additional induced non-linearity

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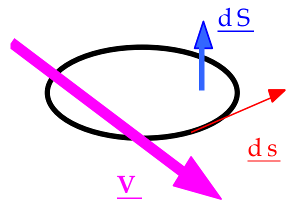
Introduction of currents

Now for $\underline{j} \neq 0$ $\underline{\nabla} \wedge \underline{H} = \underline{j}$;

To expand, use Stoke's Theorum: for any vector $\underline{V}$ and a closed curve s :

$$\int \underline{\mathbf{V}}.\underline{\mathbf{ds}} = \iint \mathbf{curl}\ \underline{\mathbf{V}}.\underline{\mathbf{dS}}$$



Apply this to: **curl H** = $\underline{j}$;

then in a magnetic circuit:

$$\boxed{\int \underline{\mathbf{H}}.\underline{\mathbf{ds}} = N\ I;}$$

N I (Ampere-turns) is total current cutting $\underline{S}$

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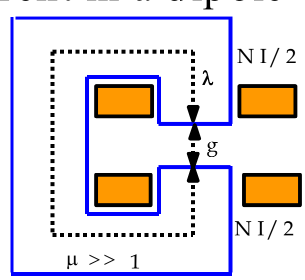
Excitation current in a dipole

B is approx constant round the loop made up of λ and g, (but see below);



But in iron, $\mu >> 1$,
and $H_{iron} = H_{air}/\mu$;
So

$$\boxed{B_{air} = \mu_0\ NI / (g + \lambda/\mu);}$$

g, and λ/μ are the 'reluctance' of the gap and iron.

Approximation ignoring iron reluctance ($\lambda/\mu << g$):

$$\boxed{NI = B\ g / \mu_0}$$

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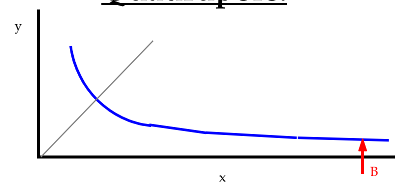
Excitation current in quad & sextupole

For quadrupoles and sextupoles, the required excitation can be calculated by considering fields and gap at large x. For example:

Quadrupole:



Pole equation: $xy = R^2/2$
On x axes $B_Y = G_Q x$;
where G_Q is gradient (T/m).

At large x (to give vertical lines of B):

$$N\,I = (G_Q x)\,(R^2/2x)/\mu_0$$

$$\boxed{N\,I = G_Q\,R^2/2\,\mu_0 \text{ (per pole).}}$$

The same method for a **Sextupole**, (coefficient G_S,), gives:

$$\boxed{N\,I = G_S\,R^3/\,3\mu_0 \text{ (per pole)}}$$

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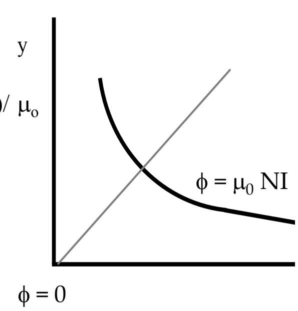
General solution-magnets order n

In air (remote currents!), $\mathbf{B} = \mu_0\,\mathbf{H}$
$\underline{\mathbf{B}} = -\,\underline{\nabla}\phi$

Integrating over a limited path (not circular) in air: $N\,I = (\phi_1 - \phi_2)/\,\mu_o$
ϕ_1, ϕ_2 are the scalar potentials at two points in air.
Define $\phi = 0$ at magnet centre;
then potential at the pole is: $\mu_o\,NI$



Apply the general equations for magnetic field harmonic order n for non-skew magnets (all Jn = 0) giving:

$$N\,I = (1/\,n)\,(1/\,\mu_0)\,\{B_r/\,R^{(n-1)}\}\,R^{\,n}$$

Where:

NI is excitation per pole;
R is the inscribed radius (or half gap in a dipole);
term in brackets {} is magnet gradient G_N in T/ m $^{(n-1)}$.

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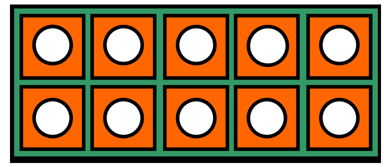
Coil geometry

Standard design is rectangular copper (or aluminium) conductor, with cooling water tube. Insulation is glass cloth and epoxy resin.



Amp-turns (NI) are determined, but total copper area (A_{copper}) and number of turns (N) are two degrees of freedom and need to be decided.

Current density:
$j = NI/ A_{copper}$
Optimum j determined from **economic** criteria.

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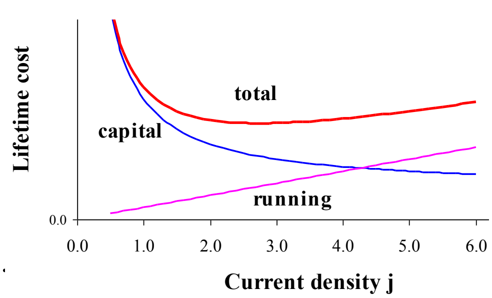
Current density - optimisation

Advantages of low j:
- **lower power loss** – power bill is decreased;
- **lower power loss** – power converter size is decreased;
- **less heat** dissipated into magnet tunnel.

Advantages of high j:
- **smaller coils;**
- **lower capital cost;**
- **smaller magnets.**

Chosen value of j is an optimisation of magnet capital against power costs.



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Number of turns per coil-N

The value of number of turns (N) is chosen to match power supply and interconnection impedances.

Factors determining choice of N:

Large N (low current)	**Small N (high current)**
Small, neat terminals.	Large, bulky terminals
Thin interconnections-hence low cost and flexible.	Thick, expensive connections.
More insulation layers in coil, hence larger coil volume and increased assembly costs.	High percentage of copper in coil volume. More efficient use of space available
High voltage power supply -safety problems.	High current power supply. -greater losses.

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Examples-turns & current

From the Diamond 3 GeV synchrotron source:

Dipole:

N (per magnet):	40;	
I max	1500	A;
Volts (circuit):	500	V.

Quadrupole:

N (per pole)	54;	
I max	200	A;
Volts (per magnet):	25	V.

Sextupole:

N (per pole)	48;	
I max	100	A;
Volts (per magnet)	25	V.

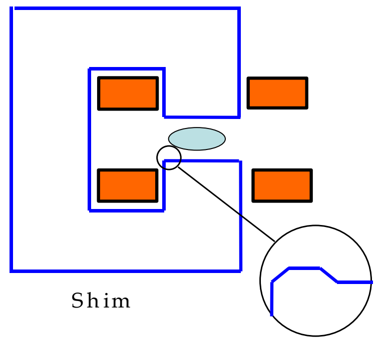
Magnet geometry

Dipoles can be 'C core' 'H core' or 'Window frame'

"C' Core:

Advantages:
- Easy access;
- Classic design;

Disadvantages:
- Pole shims needed;
- Asymmetric (small);
- Less rigid;



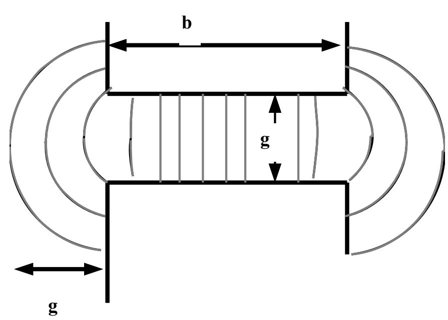
The 'shim' is a small, additional piece of ferro-magnetic material added on each side of the two poles – it compensates for the finite cut-off of the pole, and is optimised to reduce the 6, 10, 14...... pole error harmonics.

Flux in the gap.

Flux in the yoke includes the gap flux and stray flux, which extends (approx) one gap width on either side of the gap.

Thus, to calculate total flux in the back-leg of magnet length λ:

$$\Phi = B_{gap}\,(b + 2g)\,\lambda.$$

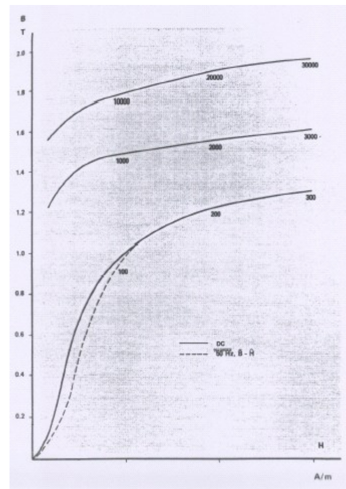


Width of backleg is chosen to limit B_{yoke} and hence maintain high μ.

Steel- B/ H curves

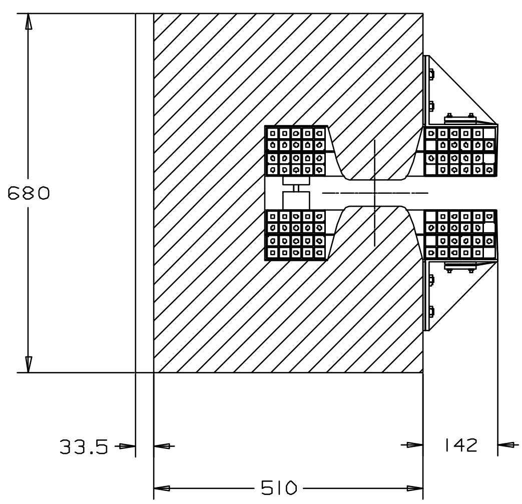
-for typical silicon steel laminations

Note:
- the relative permeability is the <u>gradient</u> of these curves;
- the lower gradient close to the origin - lower permeability;
- the permeability is maximum at between 0.4 and 0.6 T.



Typical 'C' cored Dipole

Cross section of the Diamond storage ring dipole.



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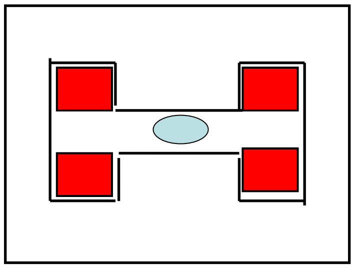
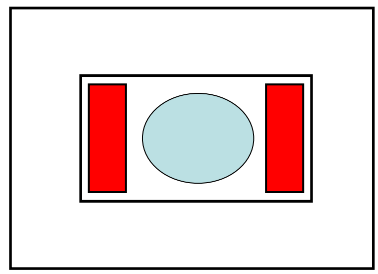
H core and window-frame magnets



"Window Frame'
Advantages:
- High quality field;
- No pole shim;
- Symmetric & rigid;

Disadvantages:
- Major access problems;
- Insulation thickness



'H core':
Advantages:
- Symmetric;
- More rigid;

Disadvantages:
- Still needs shims;
- Access problems.

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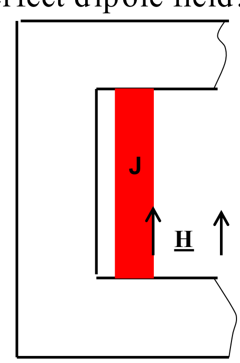
Window frame dipole

Providing the conductor is continuous to the steel 'window frame' surfaces (impossible because coil must be electrically insulated), and the steel has infinite μ, this magnet generates perfect dipole field.



Providing current density J is uniform in conductor:

- **H** is uniform and vertical up outer face of conductor;
- **H** is uniform, vertical and with same value in the middle of the gap;
- $\rightarrow$ perfect dipole field.

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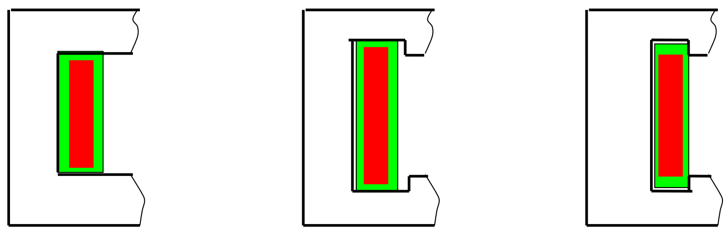
Practical window frame dipole.

Insulation added to coil:



B increases close to coil insulation surface	B decrease close to coil insulation surface	best compromise

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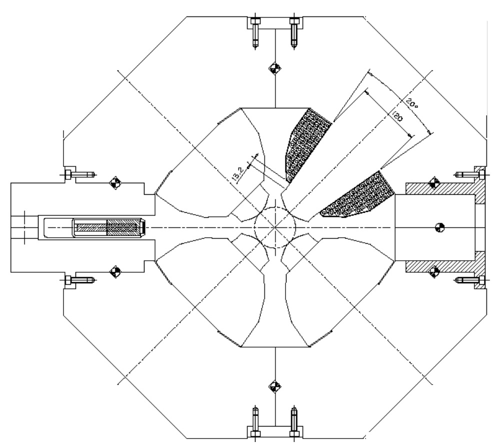
Open-sided Quadrupole

'Diamond' storage ring quadrupole.

The yoke support pieces in the horizontal plane need to provide space for beam-lines and are not ferro-magnetic.

Error harmonics include n = 4 (octupole) a finite permeability error.



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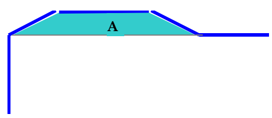
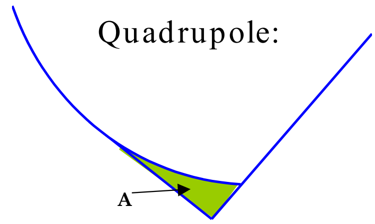
Typical pole designs

To compensate for the non-infinite pole, shims are added at the pole edges. The area and shape of the shims determine the amplitude of error harmonics which will be present.

Dipole:



Quadrupole:



The designer optimises the pole by 'predicting' the field resulting from a given pole geometry and then adjusting it to give the required quality.

When high fields are present, chamfer angles must be small, and tapering of poles may be necessary

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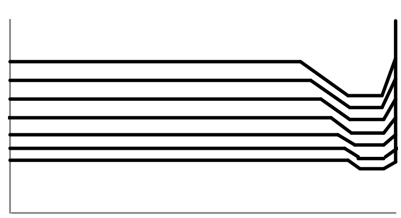
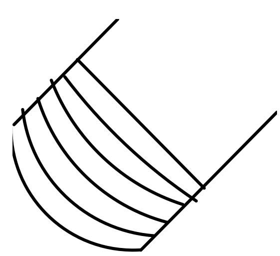
Pole-end correction.

As the gap is increased, the size (area) of the shim is increased, to give *some* control of the field quality at the lower field. This is far from perfect!



Transverse adjustment at end of dipole



Transverse adjustment at end of quadrupole

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Assessing pole design

A first assessment can be made by just examining $B_y(x)$ within the required 'good field' region.

Note that the expansion of $B_y(x)_{\,y=0}$ is a Taylor series:

$$B_y(x) = \sum_{n=1}^{\infty} \{b_n\, x^{(n-1)}\}$$
$$= \underbrace{b_1}_{\text{dipole}} + \underbrace{b_2 x}_{\text{quad}} + \underbrace{b_3 x^2}_{\text{sextupole}} + \dots\dots\dots$$

Also note:

$$\partial B_y(x) / \partial x = b_2 + 2\, b_3 x + \dots\dots.$$

So quad gradient $G \equiv b_2 = \partial B_y(x) / \partial x$ in a quad

But sext. gradient $G_s \equiv b_3 = \tfrac{1}{2}\, \partial^2 B_y(x) / \partial x^2$ in a sext.

So coefficients are not equal to differentials for n = 3 etc.

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Is it 'fit for purpose'?

A simple judgement of field quality is given by plotting:

- **Dipole:** $\{B_y(x) - B_y(0)\}/ B_Y(0)$ $(\Delta B(x)/ B(0))$
- **Quad:** $dB_y(x)/ dx$ $(\Delta g(x)/ g(0))$
- **6poles:** $d^2B_y(x)/ dx^2$ $(\Delta g_2(x)/ g_2(0))$

'Typical' acceptable variation inside 'good field' region:

$$\Delta B(x)/ B(0) \le 0.01\%$$
$$\Delta g(x)/ g(0) \le 0.1\%$$
$$\Delta g_2(x)/ g_2(0) \le 1.0\%$$

Design computer codes.

Computer codes are now used; eg the Vector Fields codes -'OPERA 2D' and 3D.

These have:

- finite elements with variable triangular mesh;
- multiple iterations to simulate steel non-linearity;
- extensive pre and post processors;
- compatibility with many platforms and P.C. o.s.

Technique is iterative:

- calculate flux generated by a defined geometry;
- adjust the geometry until required distribution is achieved.

Design Procedures – OPERA 2D

Pre-processor:

The model is set-up in 2D using a GUI (graphics user's interface) to define 'regions':

- steel regions;
- coils (including current density);
- a 'background' region which defines the physical extent of the model;
- the symmetry constraints on the boundaries;
- the permeability for the steel (or use the pre-programmed curve);
- mesh is generated and data saved.

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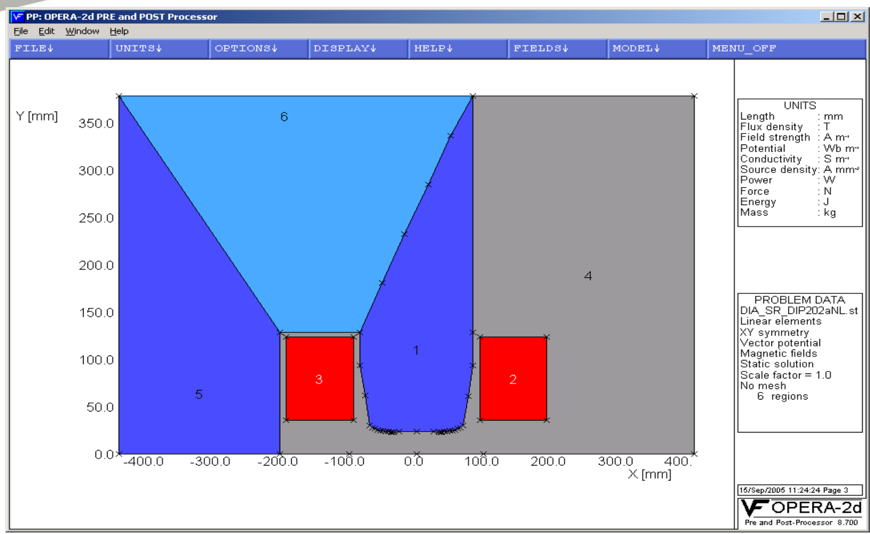
Model of Diamond storage ring dipole



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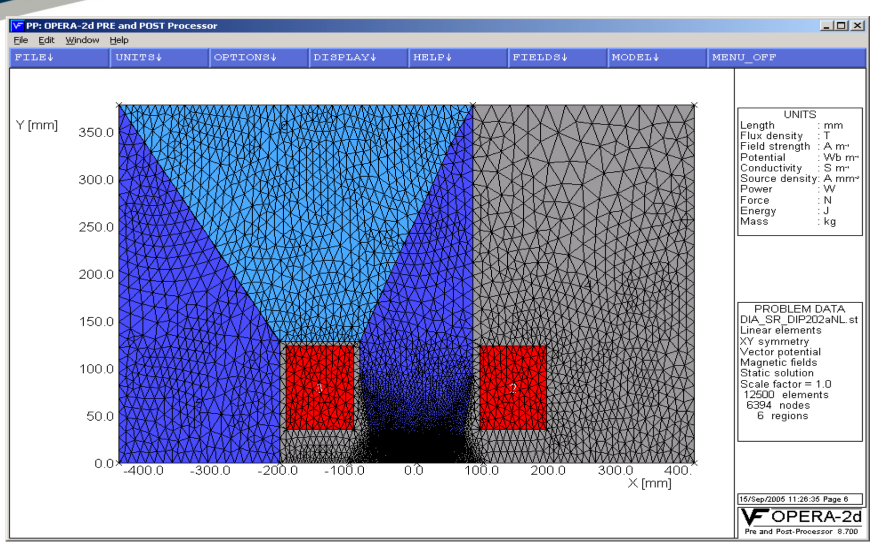
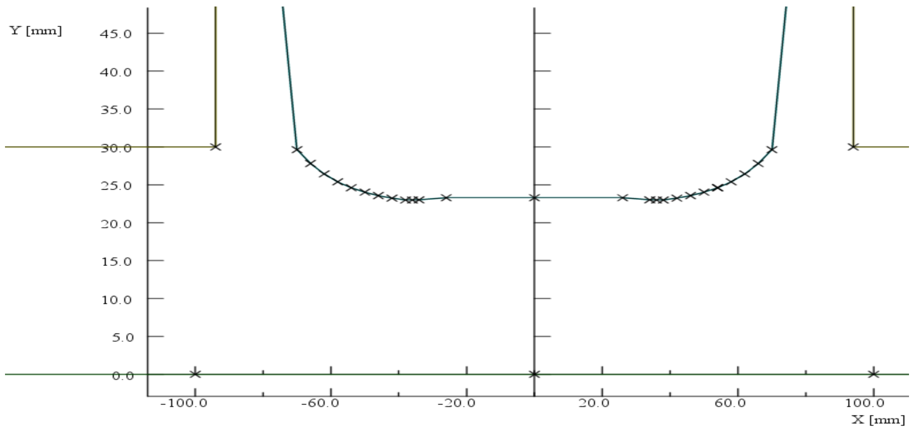
With mesh added

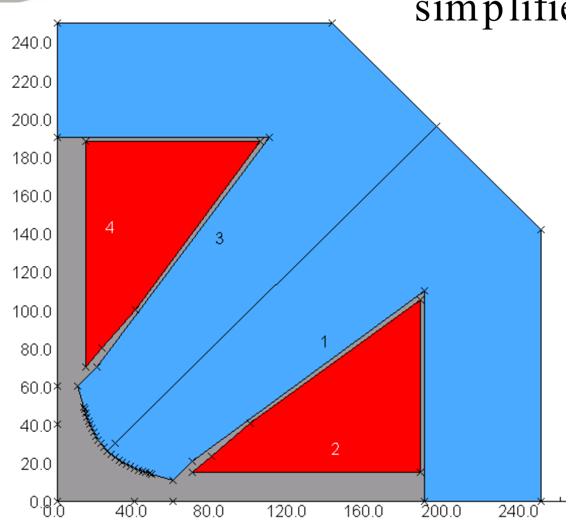


Close-up of pole region.

Pole profile, showing shim and Rogowski side roll-off for Diamond 1.4 T dipole.:



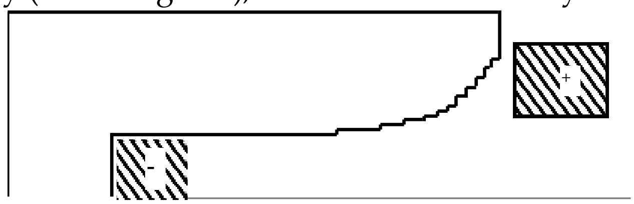
Diamond quadrupole: a simplified model



Note – one eighth of Quadrupole could be used with opposite symmetries defined on horizontal and y = x axis.
But I have often experienced discontinuities at the origin with such 1/ 8 th geometry.

Calculation of end effects using 2D codes

FEA model in longitudinal plane, with correct end geometry (including coil), but 'idealised' return yoke:



This will establish the end distribution; a numerical integration will give the 'B' length.

Provided dBY/dz is not too large, single 'slices' in the transverse plane can be used to calculated the radial distribution as the gap increases. Again, numerical integration will give ∫ B.dl as a function of x.

This technique is less satisfactory with a quadrupole, but end effects are less critical with a quad.

Calculation.

<u>Data Processor:</u>

either:

- linear - which uses a predefined constant permeability for a single calculation, or

- non-linear - which is iterative with steel permeability set according to B in steel calculated on previous iteration.

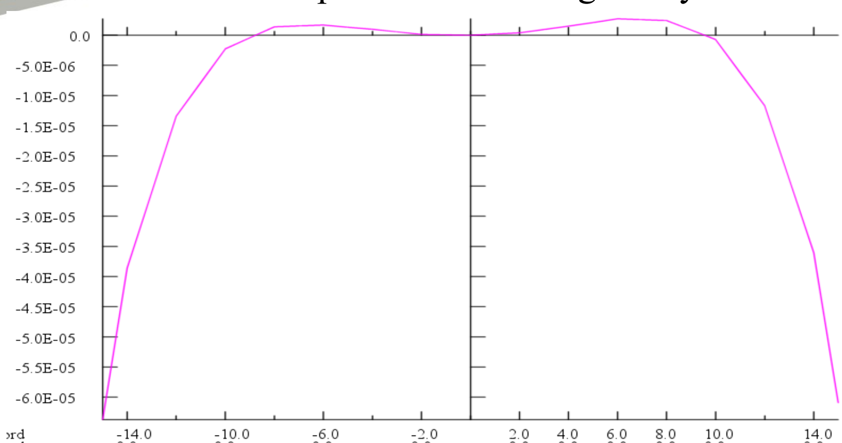
Data Display – OPERA 2D

Post-processor:

uses pre-processor model for many options for displaying field amplitude and quality:

- field lines;
- graphs;
- contours;
- gradients;
- harmonics (from a Fourier analysis around a pre-defined circle).

2 D Dipole field homogeneity on x axis



Diamond s.r. dipole: $\Delta B/B = \{By(x) - B(0,0)\}/B(0,0)$;
typically $\pm$ 1:10^4 within the 'good field region' of $-12mm \leq x \leq +12$ mm..

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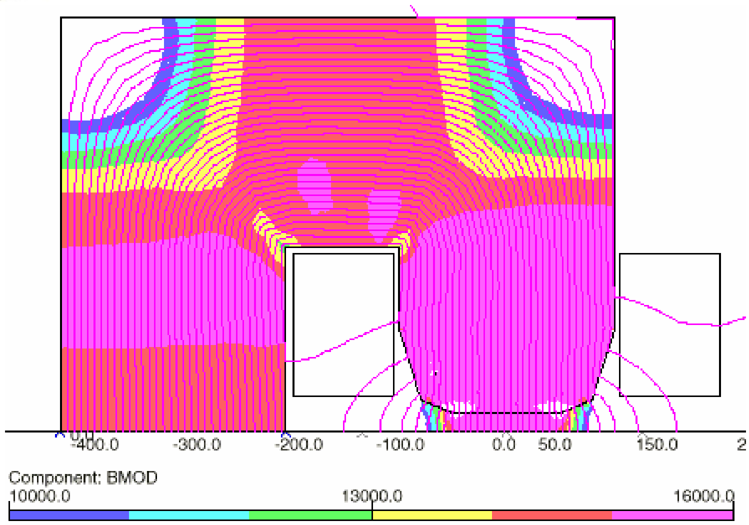
2 D Flux density distribution in a dipole



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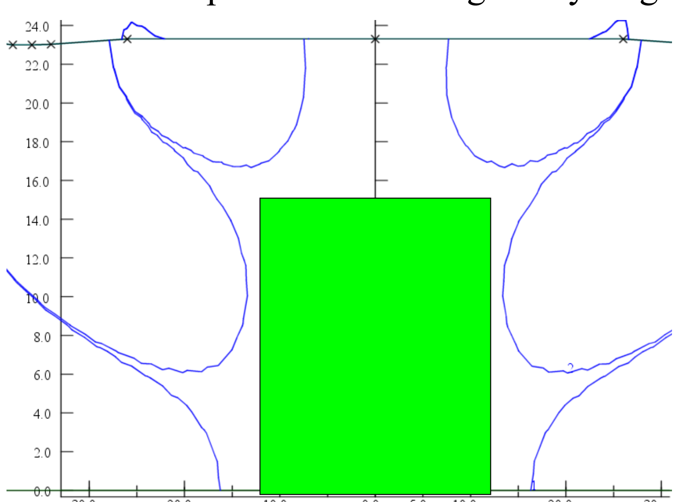
2 D Dipole field homogeneity in gap

Transverse (x,y) plane in Diamond s.r. dipole;

contours are ±0.01%

required good field region:



Harmonics indicate magnet quality

The amplitude and phase of the integrated harmonic components in a magnet provide an assessment:

- when accelerator physicists are calculating beam behaviour in a lattice;
- when designs are judged for suitability;
- when the manufactured magnet is measured;
- to judge acceptability of a manufactured magnet.

Measurement of a magnet after manufacture will be discussed in the section on measurements.

End geometries - dipole

Simpler geometries can be used in some cases.

The Diamond dipoles have a Rogawski roll-off at the ends (as well as Rogawski roll-offs at each side of the pole).

See photographs to follow.

This give small negative sextupole field in the ends which will be compensated by adjustments of the strengths in adjacent sextupole magnets – this is possible because each sextupole will have its own individual power supply.

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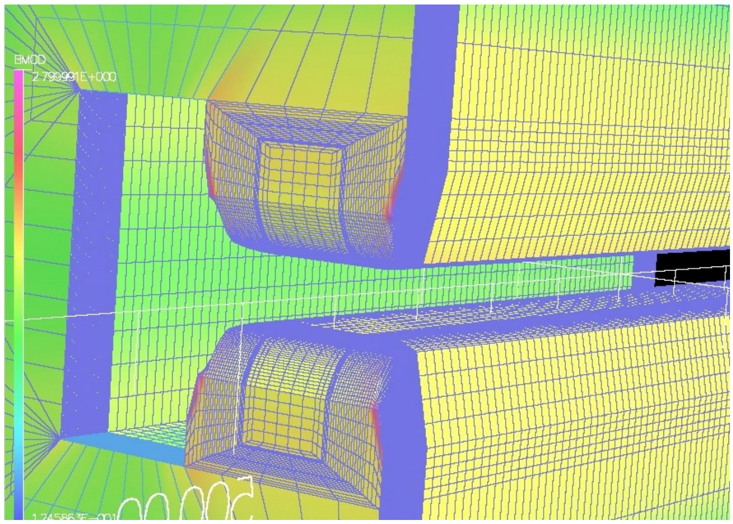
OPERA 3D model of Diamond dipole.



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Diamond dipole poles

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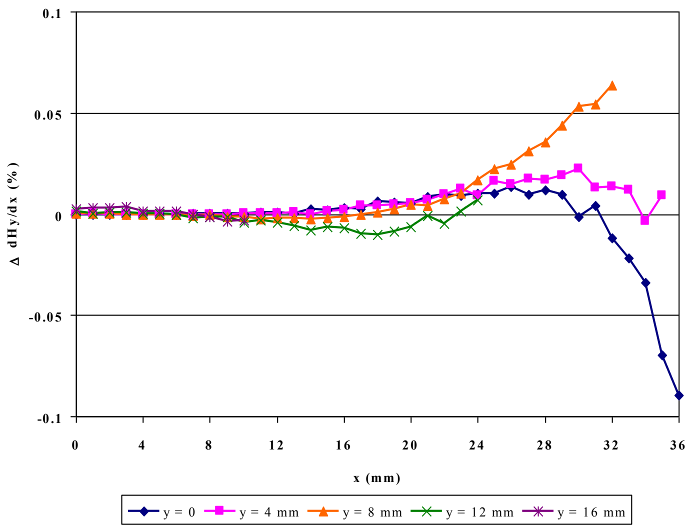
2 D Assessment of quadrupole gradient quality

Diamond WM quadrupole:

graph is percentage variation in dBy/dx vs x at different values of y.

Gradient quality is to be ± 0.1 % or better to x = 36 mm.



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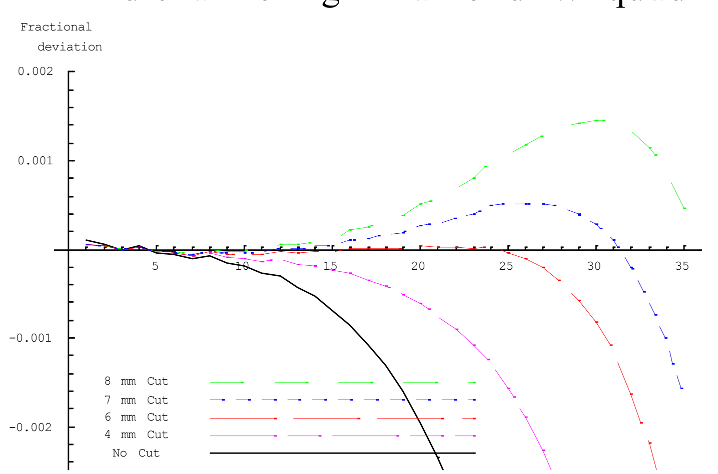
End chamfering - Diamond 'W' quad

Tosca results - different depths 45° end chamfers on $\Delta g/ g_0$ integrated through magnet and end fringe field (0.4 m long WM quad).



Thanks to Chris Bailey (DLS) who performed this working using OPERA 3D.

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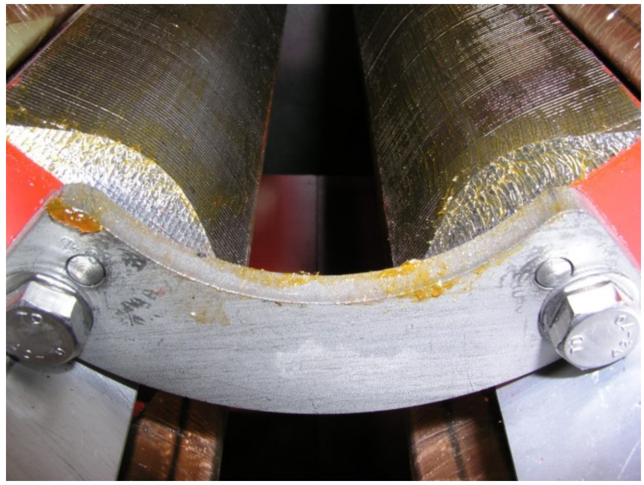
Simplified end geometries - quadrupole

Diamond quadrupoles have an angular cut at the end; depth and angle were adjusted using 3D codes to give optimum integrated gradient.



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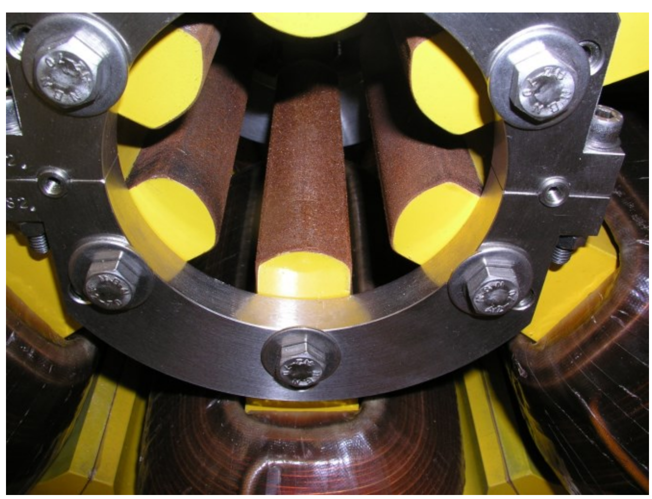
Sextupole ends

It is not usually necessary to chamfer sextupole ends (in a d.c. magnet). Diamond sextupole end:



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'Artistic' Diamond Sextupoles