Preface

The purpose of this tutorial is to think about a Conceptual Design Study (CDS) for a proposed new machine.
A few performance parameters are given and the basic building blocks for this machine with a feasible and realistic layout and related parameters should be defined. The necessary knowledge is acquired from the lectures as the school proceeds. Of course a formal, unique solution to this exercise does <u>not</u> exist. There will be many alternative solutions for such a design. It is rather expected to discuss the arising issues with other participants, tutors and lecturers during the school to arrive at a proposal.
The main objectives are to touch upon the strategies how the different steps and exercises are attacked and how the material of the lectures is put into practice. Some steps in the design build upon previous steps.
During the final tutorial session just before the end of the school the outcome and possible difficulties and other issues can be discussed in the small tutorial groups together with a tutor.

General comments:

- Please choose "reasonable" parameters as input for the thoughts and computations. Take into account implications for engineering (geometry and symmetry).
- Not for all systems the details have to be worked out, an educated estimate is sufficient (and even encouraged). A concept for further required studies should be established.

  Organize yourself within small groups to work on the specific topics.

Outline

The design should follow different stages, divided into topics. Several of the topics can be worked out independently (not all of them), while others depend on previous deliberations.

1. General considerations
   - Layout and choice of the machine

2. Basic systems, beam dynamics and components:
   - Bending, layout and dipoles
   - Focusing, Quadrupoles
   - RF system (frequency, frequency swing, ...)

3. Machine performance:
   - Luminosity
   - Synchrotron radiation
   - Multiparticle effects

4. Overall layout:
   - Injector chain

5. Operational considerations:
   - Instrumentation and diagnostics
   - Control of machine parameters

1 Topic 1a (Lattice I):

1.1 Problem:

Consider a machine for proton-proton collisions (collider) :

- Circumference C = 60 km, Centre of Mass Energy 40 TeV
- Energy at Injection 4 TeV
- Maximum β in the arc $\approx$ 300 m in both planes
- Length of available dipoles 10 m

Find a consistent set of parameters assuming it is made only of FODO cells, using only the given constraints.
Ignore the need for insertions at this stage.
First assume that the focusing and defocusing quadrupoles have the same strengths.

- Parameters to estimate (recommendation $\Rightarrow$ in this order): m
  - A first estimate for the strengths of dipoles
  - Maximum and minimum β in the arc
  - Phase advance per cell
  - Length and number of cells
  - Tunes of the machine
  - Number of dipoles per cell
  - Optional: chromaticity, momentum compaction (α_c) and average $\bar{\beta}$

Remark: please use reasonable numbers, i.e. ring should have an integer number of cells, not a fraction of a cell etc., take into account possible periodicity to make your life simple (e.g. 4, 6, 8, ..).

Reminder:
For a FODO cell with phase advance ϕ and a cell length L we have for the maximum and minimum β within the cell (see lecture on transverse dynamics):

$$\hat{\beta} \;=\; L \;\cdot\; \frac{1 + sin(\phi/2)}{sin(\phi)} \;=\; L_{1/2} \;\cdot\; \frac{1 + sin(\phi/2)}{sin(\phi/2) \cdot cos(\phi/2)} \tag{1}$$

and

$$\check{\beta} \;=\; L \;\cdot\; \frac{1 - sin(\phi/2)}{sin(\phi)} \;=\; L_{1/2} \;\cdot\; \frac{1 - sin(\phi/2)}{sin(\phi/2) \cdot cos(\phi/2)} \tag{2}$$

1.2 Possible Solution:

Since the centre of mass energy is 40 TeV, the top energy of the accelerator should be 20 TeV.

1. First step (rough estimate for dipoles):

Fill factor with dipoles typically $\approx$ 70%, i.e. 42 km of dipoles.
Bending radius $\rho = 42000/2\pi = 6684.5\ m$
Field: $\frac{B}{p} = \frac{B \cdot c}{E} = \frac{1}{\rho} \Rightarrow B \approx 10.5$ T

2. Arc cells

a) quadrupoles:

For a FODO cell with phase advance ϕ and a cell length L we have for the maximum and minimum β (see lecture on transverse dynamics):

$$\hat{\beta} = L \cdot \frac{1 + sin(\phi/2)}{sin(\phi)} = L_{1/2} \cdot \frac{1 + sin(\phi/2)}{sin(\phi/2) \cdot cos(\phi/2)} \tag{3}$$

and

$$\check{\beta} = L \cdot \frac{1 - sin(\phi/2)}{sin(\phi)} = L_{1/2} \cdot \frac{1 - sin(\phi/2)}{sin(\phi/2) \cdot cos(\phi/2)} \tag{4}$$

Smallest $\hat{\beta}$ for phase advance $\approx 90^o$
$\Longrightarrow \hat{\beta}_{min} \approx 1.7071 \cdot L$
$\Longrightarrow L = 175.74\ m$

We need 341.41 cells (!), i.e. should be 342 !
We choose 360 cells to get a periodicity of 6 and 8.
final cell length L = 166.67 m
Gives:

$\hat{\beta} = 284.5$ m, $\check{\beta} \approx 48.0$ m.
Tunes:
Given the phase advance per cell of 90^o, the tune is
$Q =$ number of cells $\cdot \mu/2\pi = 360 \cdot 0.25 = 90.00$.

For discussion: is this a good value ? If not, what should be done ?

b) dipoles:

Fill factor $\approx$ 70% we put 120 m of dipoles in a cell $\Longrightarrow$ 12 dipoles per cell
We have 12*360 = 4320 dipoles in the machine.

To get 2π deflection angle = $2\pi/4320 = 0.0014544$ rad

$B = \frac{0.0014544 \cdot 3.33564 \cdot 20000}{10} = 9.703$ T (close to estimate)
(Reminder: $3.33564 = \frac{10^9}{c}$, conversion from eV to GeV/c)

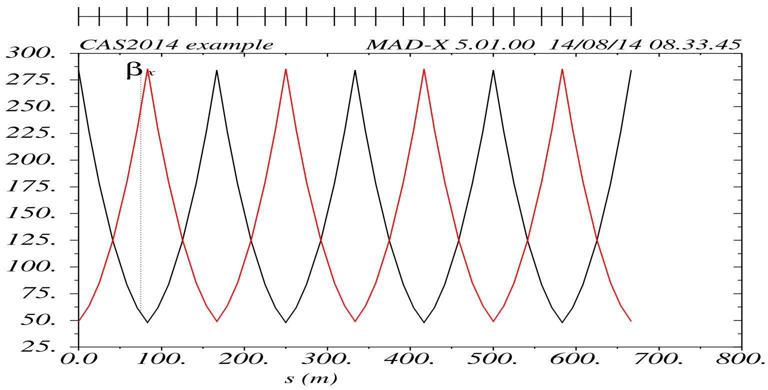
c) optional:

Chromaticities:
A good estimate for the chromaticities of a FODO lattice is Q' ≈ 1.3· Q. They are always negative and for our case ≈ -117.

Momentum compaction:
A good estimate for the momentum compaction of a FODO lattice is given by $\alpha \approx 1/Q^2 = 0.0001234$.

Average $\bar{\beta}$:
An estimate for the average β is: $\bar{\beta} \approx$ R/Q = 106 m.

For demonstration, rough implementation in MADX:



- Tunes: 90.0, 90.0
- chromaticities: -114.5, -114.5
- momentum compaction: 0.000141
- γ_{tr}: 84.20

if wanted, the demonstration and all input files are available for tutors and students.

2 Topic 1b (Lattice II):

2.1 Problem:

For discussion, calculation not needed:

(a): In exercise 1a we have assumed that the strengths of focusing and defocusing quadrupoles are the same.

- What happens if they are different, e.g. defocusing quadrupoles are 0.5% weaker and the focusing quadrupoles 0.5% stronger ?
- Is it possible ?
- Can you think of a reason (or reasons) to do that ?

Please discuss with colleagues, lecturers and tutors, try an estimate.

(b): How can you compute and control the beam emittance with the optical parameters ?

Consider two cases:

1. With protons like in our case
2. What changes for electrons ?

2.2 Guidelines for discussion:

(a) Unequal quadrupole strength:

(a1) yes, it is of course possible

(a2) changes in both planes, little change of maximum β, tunes change by about 1%, in opposite directions

(a3) e.g. avoid coupling

b) Beam emittance: measuring beam size relying on known β-function via $\sigma^2 = \epsilon \cdot \beta$

Do not forget to use non-normalized emittance.

(b) Emittance control:

1. You cannot. It is given by the beam itself.

2. Synchrotron radiation determines horizontal beam size, phase advance and insertions devices can change radiation and damping . Vertical emittance via coupling or change of damping partition. (lecture on electron dynamics).

3 Topic 2 (RF system):

3.1 Problem:

Consider the previous pp collider and design a RF system:

- Assume an acceleration from 4 to 20 TeV in 10 minutes
- r.m.s. bunch length $\sigma_s = 0.06$ m
- use exact speed of light for calculations: 299792458 m/s

Find a consistent and realistic set of parameters to fulfill the given constraints.

- Parameters to find:
  - RF frequency
  - Harmonic number
  - Frequency change during acceleration (hint: look at lecture on relativity), how to realize it ?
  - Energy gain per turn per proton during acceleration
  - Minimum RF voltage

3.2 Possible Solution:

1. First step (basic parameters):

The r.m.s. bunch length is given as 0.06 m and the full length of a bunch has to be accommodated within a RF bucket, i.e. the RF wavelength must be chosen long enough. A good guess would be a wavelength of $\lambda = 0.60$ m. (again to make the calculations simpler).
This immediately gives a harmonic number of h = 100000.
For the RF frequency we get:
$\nu \;=\; c/\lambda = 499.654$ MHz.

2. Frequency change:

During acceleration the velocity, i.e. β_r changes and may require a change of the RF frequency.

We have:

$\beta_r(4 \text{ TeV}) = 0.999999945$

and

$\beta_r(20 \text{ TeV}) = 0.999999998$

we have for the change of β_r, i.e. relative change of velocity
$\Delta\beta_r = 0.000000054$

There is no need for any change of the RF frequency ...

3. Energy gain per turn:

10 minutes = 600 s

Number of turns during acceleration is $600{\cdot}5000 = 3{\cdot}\ 10^6$ turns.

Relative energy change is 5.

Energy gain during acceleration $\Delta E = 16$ TeV.

$\Longrightarrow$ 16 TeV/$3{\cdot}\ 10^6 \;=\;$ 5.3 MeV per turn.
It requires a RF voltage 10 - 15 MV.

4 Topic 3 (luminosity):

4.1 Problem:

Hint: For this exercise please use the simplified luminosity formula introduced in the lecture "Introduction to Accelerators". The intricate subtleties, technical details and problems are discussed in a dedicated lecture towards the end of the school.
As a reminder, the formula is:

$$L = \frac{kN_1N_2f}{4\pi\sigma_x\sigma_y}$$

with:
k: number of colliding bunches
N_1, N_2: particles per bunch
f: revolution frequency
σ_x, σ_y: horizontal and vertical beam sizes

Consider a pp collider with the following constraints:

- Minimum distance between bunches 15 m (to allow space for experiments)
- Maximum beam power 500 MJ
- Number of collisions per bunch crossing (μ) not larger than 4
- Hints:
    - define luminosity per bunch
    - total cross section at 40 TeV is $\approx$ 80 mbarn ($\approx 80\ 10^{-27}$ cm^2)

Find a consistent set of parameters to obtain the required luminosity within the given constraints.

- Parameters to find:
    - Number of bunches
    - Luminosity
    - Number of protons per bunch
    - Possible parameters for β^* and ϵ_n (reasonable guesses, there are many possible solutions)

4.2 Possible Solution:

1. First step (basic parameters):

Revolution frequency f is c/C = 300000km/60km = 5000 Hz

Maximum number of bunches n_b = C/spacing = 60km/15m = 4000

Given the harmonic number h = 100000 from a previous calculation, every 25th bucket is filled with a bunch.

Number of collisions per second is: Luminosity · cross section.
Inelastic cross section for proton-proton collisions at 20TeV ≈ 80 mbarn
(1 mbarn = 10^{-27} cm^2)

$N_{coll}/s \;=\; L \cdot\; \sigma_c \;=\; L \cdot 80 \text{ mbarn}$

$$\mu = \frac{L \cdot 80 \text{ mbarn}}{f \cdot n_b} \;=\; 4 \quad \Longrightarrow\; L \;=\; 10^{33} cm^{-2} s^{-1}$$

2. Second step (protons per bunch):

Maximum beam energy 500 MJ:
1 eV = $1.602 \cdot 10^{-19}$ J $\Longrightarrow$ total number of protons per beam N_{tot}:

$500 \cdot 10^6 = 20 \cdot 10^{12} \cdot N_{tot} \cdot 1.602 \cdot 10^{-19} \;\;\Longrightarrow\;\; N_{tot} \;=\; 1.56 \cdot 10^{14}$ p

For 4000 bunches per beam: $N_{tot}/4000 = N_b \;=\; 3.9 \cdot 10^{10}$ p

3. Third step (beam size:
We know already N_b, f, n_b, and the required luminosity. Unknown is the necessary beam size σ.

$$L = 10^{33} = \frac{N_b^2 \cdot f \cdot n_b}{4\pi \cdot \sigma^2} \Longrightarrow \sigma = \sqrt{\frac{N_b^2 \cdot f \cdot n_b}{4\pi \cdot L}} = 15.6 \ \mu\text{m}$$

$$\sigma = \sqrt{\epsilon_n \cdot \beta^* / \gamma}$$

$$\gamma = 20 \cdot 10^3 / 0.938 = 21322$$

The condition for ϵ_n and β^* is therefore: $\epsilon_n \cdot \ \beta^* = 5.19 \cdot 10^{-6}$

Possible options (combinations):

$\epsilon_n = 5.2 \ \mu m$ and $\beta^* = 1$ m
$\epsilon_n = 2.6 \ \mu m$ and $\beta^* = 2$ m
.....

5 Topic 4 (Geometrical aperture):

5.1 Problem:

Consider the previous pp collider and re-assess the parameters:

- Maximum beam size in the arc: r.m.s. 0.5 mm

Find a consistent set of parameters to obtain the required constraints.

- Parameters to find:
  - Maximum emittance ϵ_n
  - Related β^*

These constraints will complete the set of parameters needed for the previous exercise.

5.2 Possible Solution:

1. First deliberation:
the largest beam size we have at injection energy, the calculation needs to be done for 4 TeV only.

$$\sigma = \sqrt{\epsilon \cdot \beta_{max}} = \sqrt{\frac{\epsilon_n \cdot \beta_{max}}{\gamma}}$$

The value for β_{max} we know as 300 m (by construction !) and the relativistic $\gamma(4 \text{ TeV}) = 4264.4$

Solving the equation for ϵ_n we get a maximum allowed emittance of
$\epsilon_n = 3.55 \cdot 10^{-6}$ m = 3.55 μm.

Assuming no emittance growth during acceleration and using the previous findings, we need a $\beta^* = 1.46$ m to get the required luminosity of $L = 10^{33} cm^{-2} s^{-1}$. For practical purpose and aesthetic reasons we would choose $\beta^* = 1.5$ m. Discuss whether this adjustment is relevant.

6 Topic 5 (Synchrotron radiation):

6.1 Problem:

Consider the previous pp collider and evaluate effect of synchrotron radiation:

- Parameters to find:

  Energy loss per turn per proton
  Power loss per proton (in W)
  Power loss for full beam (in W)

6.2 Possible Solution:

1. Synchrotron radiation loss:

The synchrotron radiation loss is largest at top energy of 20 TeV, only necessary to compute it at top energy (and scale if needed).

$\Delta\ E_{turn} \ = \ C_{\gamma} \cdot \frac{E^4\ [GeV^4]}{\rho\ [m]}$
Here ρ is again the bending radius and for E we use 20 TeV.

$C_{\gamma}(electrons) \ = \ 8.8575 \cdot 10^{-5} \ \ \frac{[m]}{[GeV^3]}$

$C_{\gamma}(protons) \ = \ 7.79 \cdot 10^{-18} \ \ \frac{[m]}{[GeV^3]}$

We get for protons in our machine: 186 keV/turn

2. Power loss per proton:

Energy loss per proton per turn is 186 keV.

Energy loss per second: 5000· 186 keV.

Power loss: 5000· 186 keV· $1.602{\cdot}10^{-19} = 1.5{\cdot}10^{-10}$ W
Power loss per beam: $1.5{\cdot}10^{-10} \cdot 1.56{\cdot}10^{14}$ =23.4 kW

7 Topic 6 (Collective effects):

Collective effects may limit the machine performance. Discuss and/or estimate effects in the collider at top energy:

(a) Is the direct space charge an issue ? What is the effect on the beam ? Can it also be useful ?

(b) The impedances of the machine can be complex. What are the effects of the real and the imaginary part ? Can the imaginary part be measured ? If yes, what can be a possibility ?

(c) To stabilize the beam, chromaticity might have to be controlled, what should be the sign of the chromaticity:

  (c1) At injection energy ?

  (c2) At top energy ?

  (c3) When do we have to change the sign ?

7.1 Possible Solution:

(a) At high energy the direct space charge is very small ($\propto \gamma^{-3}$), effect is a (defocusing) tune spread. May help for Landau damping.

(b) Real part produces heating and growth, imaginary part a tune shift. Computation needs the complex impedances. Can derive specifications for maximum values for impedances.

(c) From the tune one can estimate the transition energy, will be around $\gamma_{tr} \approx 60$. In all cases the machine operates above transition, the chromaticity should be positive. Running below transition would require negative chromaticity. The natural chromaticity might be sufficient for small machines.

8 Topic 7 (Accelerator chain):

The protons cannot be accelerated from rest energy (particle source !) to the final energy in a single accelerator.
Please discuss the following issues, where possible, make rough estimates:

- What are the limitations to the energy range of a proton accelerator ?
- How would a possible chain look like ?
- Propose a chain with energy range and accelerator type
- What are implications for the parameters for the collider ?

8.1 Possible Solution:

(a) Discuss the effects of beam size and aperture. Difference between non-relativistic and ultra-relativistic beams.

(b) Assumptions on the energy range of the accelerators, typical maximum relative energy change is ≈ 20 (rule of thumb).
Typical chain could be:

Source
500 MeV/c (kinetic energy) LINAC
10 GeV/c Low energy Injector(synchrotron)
200 GeV/c Medium energy Injector(synchrotron)
4000 GeV/c High energy Injector(synchrotron)

For discussion: emittances (long. and trans.), RF frequency, bunch spacing, cycle time, ... Is it possible to avoid transition crossing, if yes, what should be done ?

9 Topic 8 (Operational considerations):

The operational needs to be considered from the first design and defines some of the specifications.

1. Machine control:

Discuss how some critical parameters can be controlled.

- (a) Orbit:
  - How can it be measured ?
  - Which elements can be used to control it ?
  - How many of these elements would you suggest ?
- (b) Tune:
  - How can it be measured ?
  - How can the horizontal tune be changed (vertical should be kept as it is) ?
- (c) Chromaticity:
  - How can it be measured ?
  - Which elements can be used to control it ?
  - How many of these elements would you suggest ?

2. Machine optimization:

A requirement from experiments: 10 events per run for a process with a cross-section of 0.385 pbarn.

Assuming previous parameters and an exponential decay of the luminosity with a lifetime (1/e) of $\tau = 15$ hours, what is the required length T_r of a run ?
This is required as input for the design of auxiliary systems, such as cooling, vacuum, machine protection ...

9.1 Possible Solution:

1. Machine control:
(a) Measurement can be done with beam position monitors, ideally one per quadrupole.
The correction requires correction dipoles, should be close to quadrupoles.
For 360 cells we would need 720 orbit correctors.

(b) Options are analysis of excited coherent oscillations, PLL, Schottky signals.
Change of tune in one plane requires changes of both quadrupole elements.

(c) Observe tune change with energy change (e.g. RF frequency change).
Correction with sextupoles. It is a global parameter and can be corrected with very few elements. To avoid side effects (from the non-linear field) they should be distributed. Ideally one per quadrupole, i.e. 720.

2. Machine optimization:

Number of events per second = $1.0\ 10^{33}\ \mathrm{cm}^{-2}\mathrm{s}^{-1} \cdot 0.385$ pbarn
$= 1.0\ 10^{33}\ \mathrm{cm}^{-2}\mathrm{s}^{-1} \cdot 0.385\ 10^{-36}\ \mathrm{cm}^2$

Total number of events: $0.385\ \ 10^{-36}\ \mathrm{cm}^{-2}\ \cdot\ \int_{run} L\ \mathrm{d}t$
($\int_{run} L\ \mathrm{d}t = \mathrm{L}_{int}$, integrated luminosity during a run, measured in cm^{-2})

we need: $10 = 0.385\ \ 10^{-36}\ \cdot\ \int_0^{T_r} \mathrm{L}\ \mathrm{d}t$

It follows that $\int_0^{T_r} \mathrm{L}\ \mathrm{d}t = 2.6\ 10^{37}\ \mathrm{cm}^{-2}$

We assume: $L(t) = L_0 \mathrm{e}^{\frac{-t}{\tau}}$ with $\tau = 15$ hours = 54000 s.

From $\int \mathrm{L}\ \mathrm{d}t = L_0 \int_0^{T_r} \mathrm{e}^{\frac{-t}{\tau}} \mathrm{d}t = 2.6\ 10^{37}\ \mathrm{cm}^2$

it follows after integrations that $T_r = 9.85$ hours, 10 hours for any practical purpose.