



# FFAG Accelerators

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Many thanks to Dr. S. Machida for his advice and previous lecture materials

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## Motivation

- Many challenges for future hadron accelerators:

High power

Neutrons, muons, ADS

Reliable

Medical, ADS

Flexible

Is industry limited by  
existing technology?

Rapid  
acceleration

Muon beams  
Unstable nuclei

Cheap

Hadron accelerators aren't  
known for being cheap

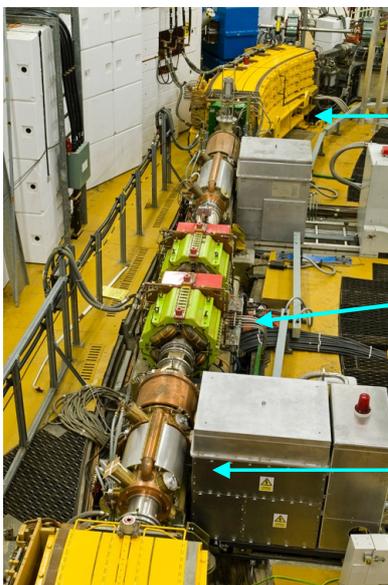
# 'Fixed Field Alternating Gradient' Accelerators

- Are FFAGs like a synchrotron or cyclotron?
  - EMMA non-scaling FFAG
- Fixed field magnets
- Beam dynamics
- Scaling FFAGs
- Advanced FFAG types and optics

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## Is an FFAG like a synchrotron? (1)

*“Particles should be constrained to move in a circle of constant radius thus enabling the use of an annular ring of magnetic field ... which would be varied in such a way that the radius of curvature remains constant as the particles gain energy through successive accelerations” - Marcus Oliphant, 1943*



dipole magnets

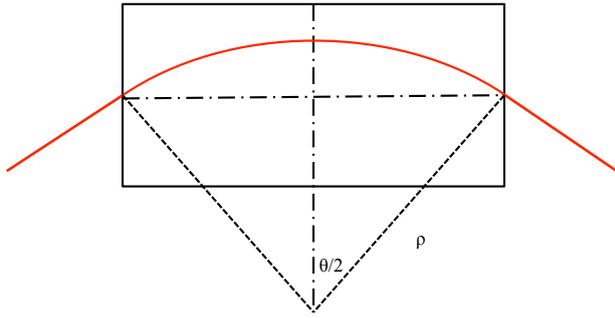
quadrupole magnets

rf cavity

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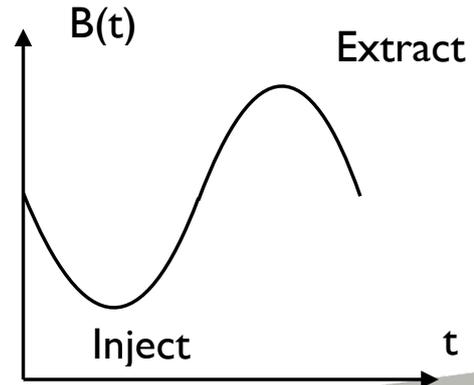
Image courtesy of ISIS, STFC

## Is an FFAG like a synchrotron? (2)



$$\sin(\theta/2) = \frac{B(t)L}{2(B(t)\rho)}$$

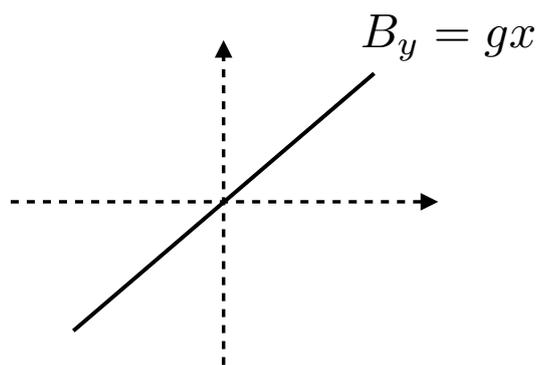
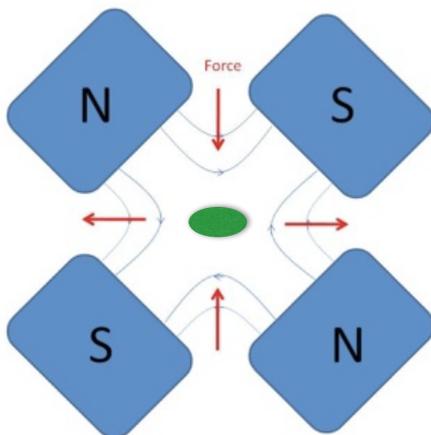
$$\theta \approx \frac{B(t)L}{p(t)/q}$$



What happens if I don't ramp the B field with E?



## Is an FFAG like a synchrotron? (3)



Do we also ramp the quadrupoles in a synchrotron?

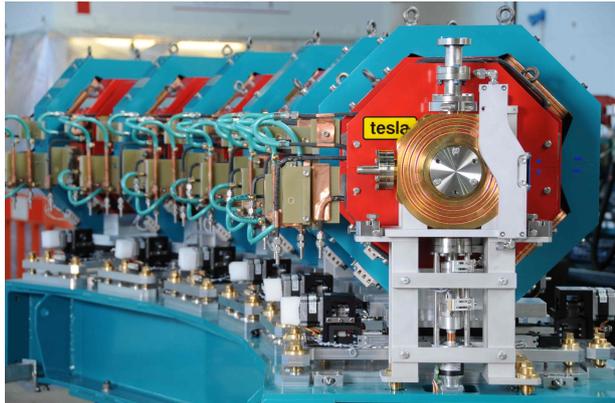
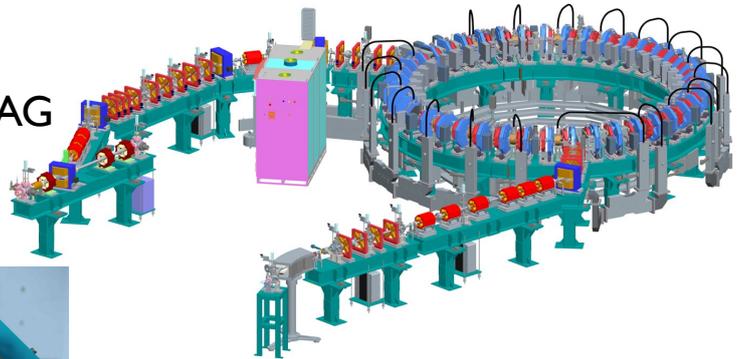
$$k = \frac{g}{p/q}$$

'normalised gradient' of quad

$$\frac{1}{f} = \frac{L(dB(t)/dx)}{p(t)/q}$$

# The 'EMMA' accelerator

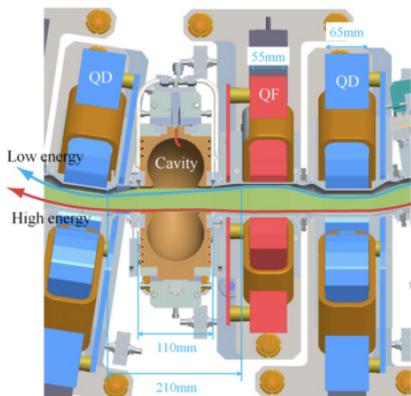
42 Quadrupole doublets  
 10-20 MeV e-  
 Demonstrates 'non-scaling' FFAG



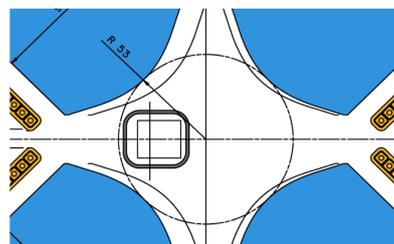
'Electron Model for Many Applications' = EMMA  
 Built and commissioned at  
 STFC Daresbury Laboratory, UK

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EMMA doesn't ramp the B field with time  
 'Fixed Field Alternating Gradient' = FFAG



Quadrupole with radial offset  
 creates bending component



Note: this is just like a 'combined function' magnet

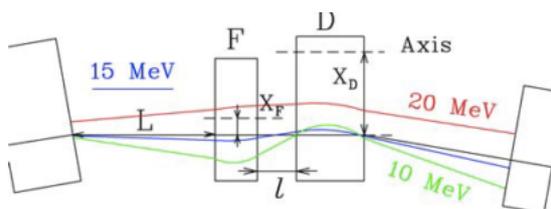


Figure 2: Orbits in a quadrupole doublet cell.

M. Craddock, PAC'07

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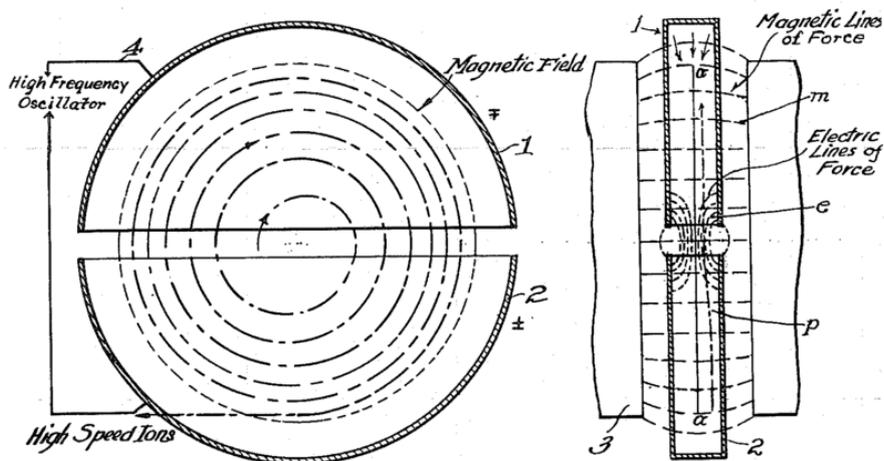
# Fixed-field magnets have advantages

- Simple power supplies and no synchronisation issues
- You can accelerate very quickly (as fast as your RF allows...)
  - in EMMA and in muon FFAGs this is ~10 turns
- Higher repetition rate, so higher average current.

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## Is an FFAG like a cyclotron? (1)

It has fixed field magnets too



The particles spiral outward as they gain energy

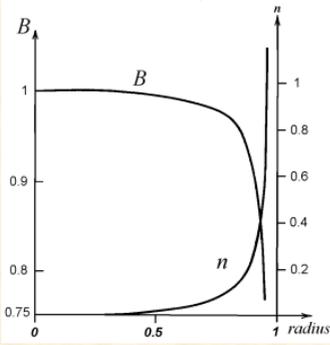
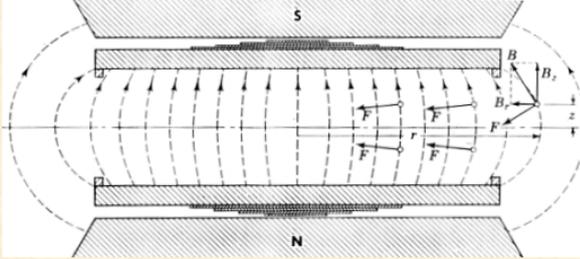
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# Is an FFAG like a cyclotron? (2)

**Weak focusing**

Simultaneous radial and axial focusing : **Weak focusing**

$$0 \leq n \approx - \frac{\partial B_z}{\partial x} \leq 1$$

slightly decreasing field  


Horizontal focusing  $n < 1$  means :

- $0 < n < 1$   $B_z$  can slightly decrease
- $n < 0$   $B_z$  can increase as much as wanted

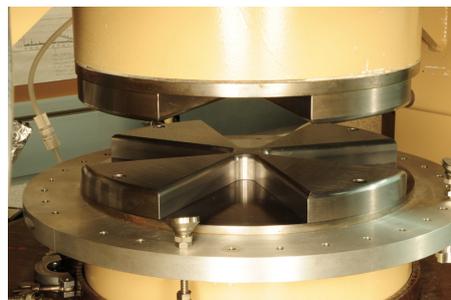
Vertical focusing  $n > 0$  means :

- $B_z$  should decrease with the radius

F. Chautard 18

Slide source: F. Chautard, 2012 CAS

# Is an FFAG like a cyclotron? (3)

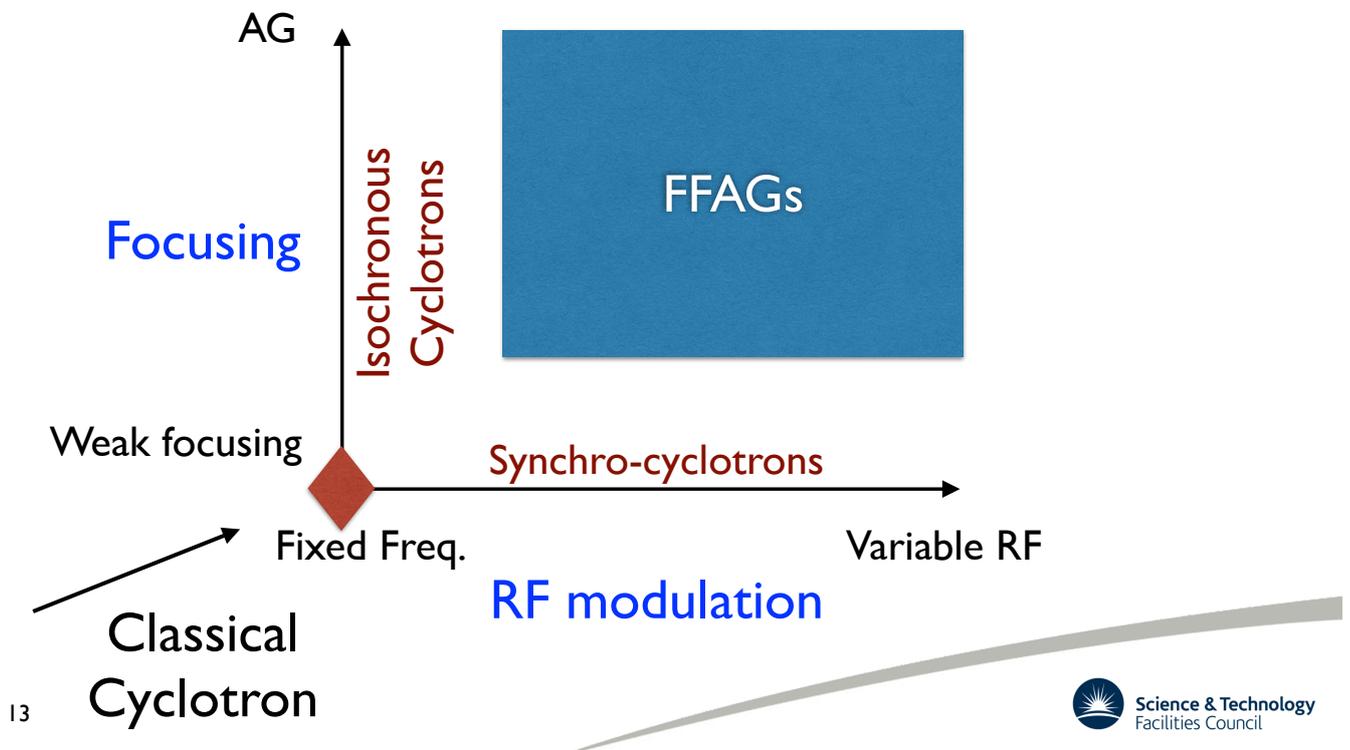


## What about the AVF cyclotron?

You may have heard of 'flutter' in an AVF cyclotron  
An FFAG has a flutter so large that the field reverses sign between  
'hills' and 'valleys'.

In the AVF cyclotron the weak focusing is still important, but in the  
FFAG the dynamics is controlled by the strong focusing

# The circular fixed-field accelerator family



- So an FFAG is like a synchrotron but with fixed-field magnets
- OR like a cyclotron with a field gradient and strong focusing, (and variable RF frequency\*\*)
- But that's not all there is to it...

\*\*FFAGs do not always have variable RF frequency...

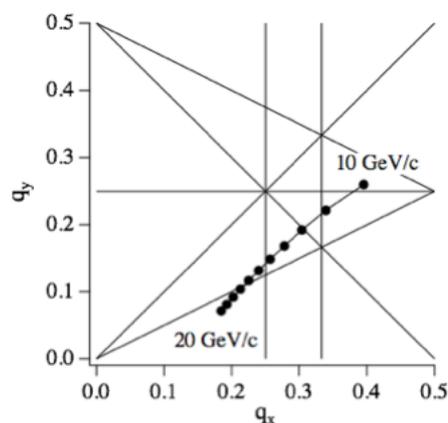
# Circular Accelerators

	Cyclotron	Synchrotron	FFAG
Revolution time	Constant	Variable (except relativistic)	Variable
Orbit radius	Variable	Constant	Variable
Transverse focusing	Variable	Constant	Variable

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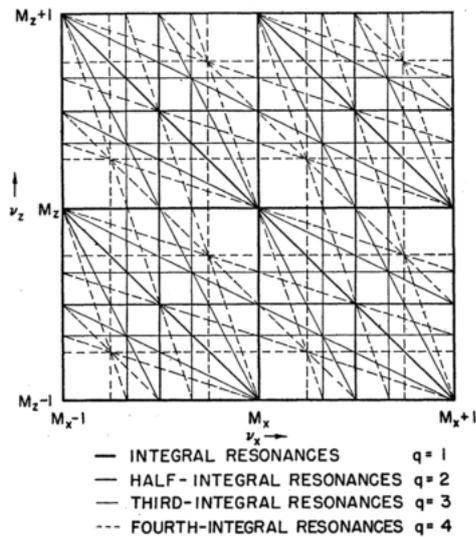
## What does variable focusing mean?

- In a synchrotron the tune is fixed away from resonance lines
- But in an FFAG, the betatron tunes can vary...



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# Resonance crossing



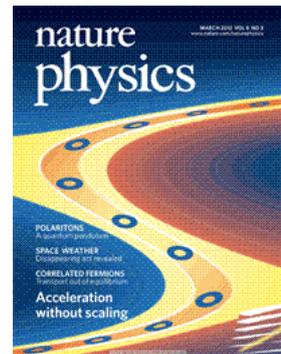
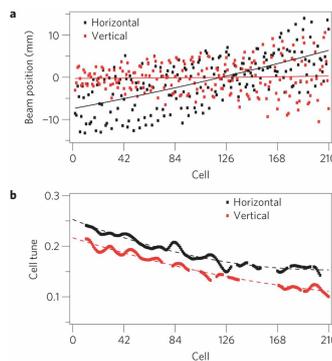
$$n\nu_x + m\nu_y = 0, 1, 2, \dots$$

- There are many resonance lines in tune space
- Normally, particles would be lost on resonance, but if the resonance is weak and the crossing is fast the beam can survive.

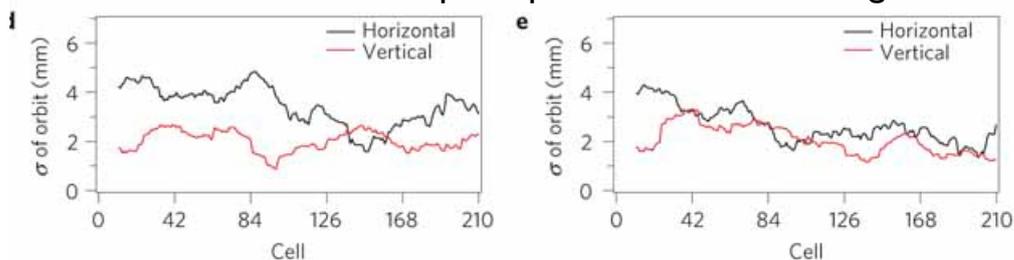
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# Results from EMMA

Orbit and  
tune shift  
with  
momentum



No beam 'blowup' despite resonance crossing

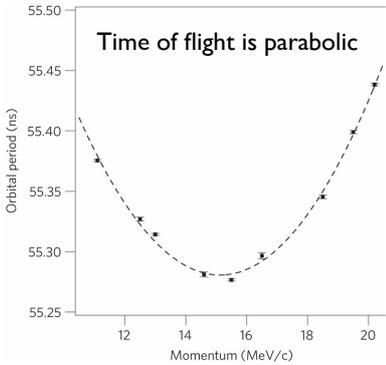


S. Machida et. al., Nature Physics 8, 243–247 (2012)

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# EMMA - longitudinal

Can you have an FFAG with fixed RF frequency?

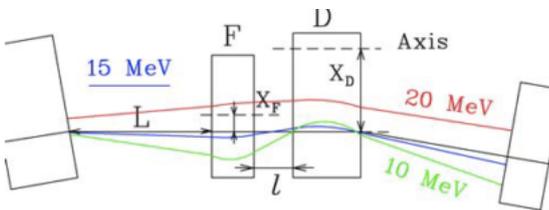
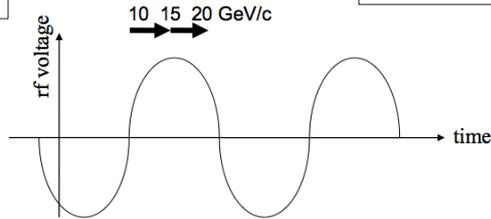


- Suppose we choose rf frequency that is synchronized with revolution frequency at the center.

In the first half of a cycle, a particle lags behind the rf.

At the center momentum, a particle is synchronized with rf.

In the second half, a particle lags again.



total time lag is less than a half of rf cycle, a beam has energy gain.

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Figure 2: Orbits in a quadrupole doublet cell.

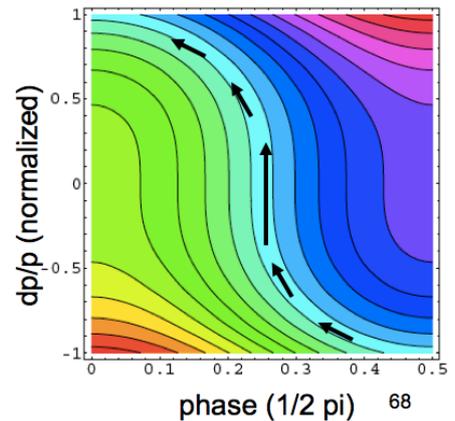
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# EMMA - longitudinal

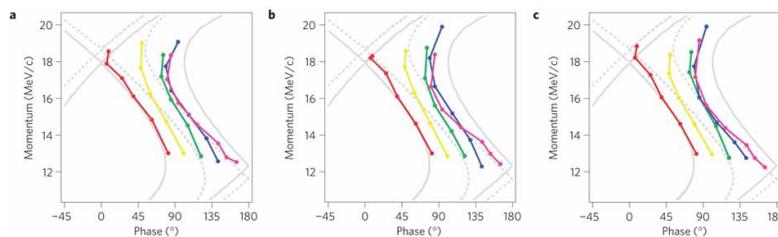
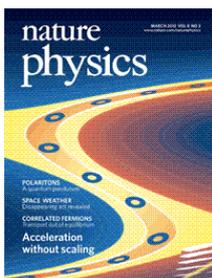
If the RF voltage is sufficient, we can accelerate over the whole energy range

Similar to acceleration in a cyclotron but with imperfect isochronicity

This is called 'serpentine' acceleration and was demonstrated in EMMA



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S. Machida et. al., Nature Physics 8, 243–247 (2012)

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- Electrons & muons are easy to accelerate quickly, but for hadrons it's harder...
- If resonance crossing could be harmful for hadron FFAGs, what can we do to fix it?
- In a synchrotron, off-momentum tune variations = chromaticity
- Can we have stable tunes in an FFAG?

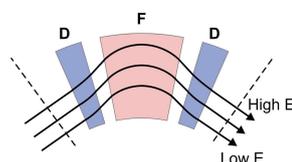
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## Scaling FFAG

- In fact, the first FFAGs had constant tunes and were designed not to cross resonances, we call them 'scaling' FFAGs

The orbits are made 'similar'

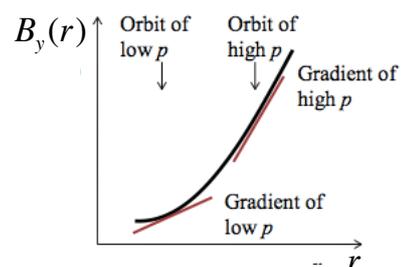
$$\left. \frac{\partial}{\partial p} \left( \frac{\rho}{\rho_0} \right) \right|_{\theta=const.} = 0$$



- $\rho_0$  Average bending radius
- $\rho$  Local bending radius
- $\theta$  Generalised azimuth

The 'field index' is constant

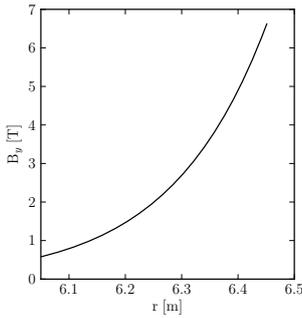
$$\left. \frac{\partial k}{\partial p} \right|_{\theta=const.} = 0 \quad k = \frac{r}{B} \left( \frac{\partial B}{\partial r} \right)$$



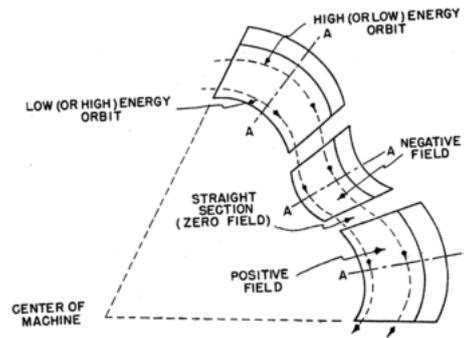
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# Scaling FFAG

- If the field profile is of this form, the 'cardinal conditions' are satisfied.
- We call this type of FFAG a 'Scaling' type.
- Alternating magnets have opposite bending fields



$$B_y = B_0 \left( \frac{r}{r_0} \right)^k F(\theta)$$



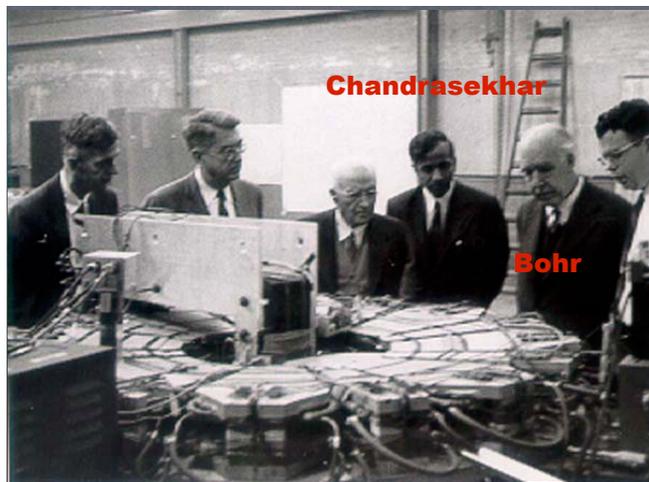
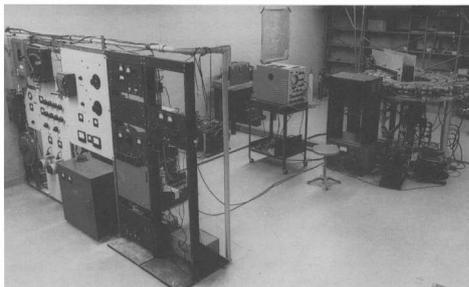
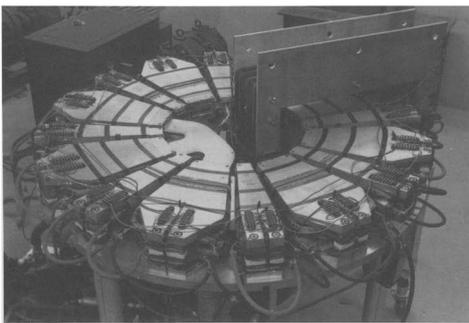
- Note that this field profile does NOT satisfy  $\omega = \frac{eB}{my} \neq const.$  isochronicity (see M. Seidel's cyclotron lecture)

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## The FFAG is not so new...

1956



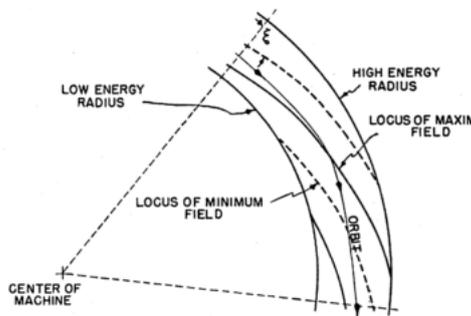
L. W. Jones, AIP Conference Proceedings, 237, 1 (1991)

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# Scaling FFAG types

- There is also another way the field can satisfy this condition



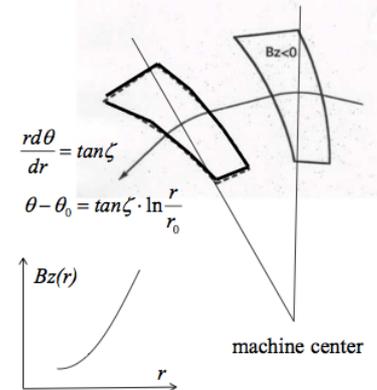
$$B(r, \theta) = B_0 \left( \frac{r}{r_0} \right)^k F(\vartheta)$$

$$F(\vartheta) = F\left(\theta - \tan \zeta \cdot \ln \frac{r}{r_0}\right)$$

Spiral sector type

Spiral angle gives strong edge focusing.

$$\therefore \Delta p_z = \frac{e}{v_x} \int_{-\infty}^{\infty} (-v_y B_x) dx = -e B_{z0} (\tan \zeta) z$$



S. Machida, CAS 2012

Image source: K. Symon, D. Kerst, L. Jones, L. Laslett, and K. Terwilliger, "Fixed-Field Alternating-Gradient Particle Accelerators," Phys. Rev., vol. 103, no. 6, pp. 1837–1859, Sep. 1956.

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## Recent Scaling FFAGs

- In the late 90's and in 2000's, the FFAG idea was re-awakened in Japan,
- Particular focus on hadron FFAGs of scaling type



3-stage FFAG for ADSR studies

2.5 MeV spiral (ion beta) FFAG with induction cores

25 MeV radial (booster) FFAG with RF

150 MeV radial (main) FFAG with RF

Proof of Principle machine finished in 1999 at KEK, demonstrated 1kHz rep. rate



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# Circular Accelerators

	Cyclotron	Synchrotron	Non-scaling FFAG	Scaling FFAG
Revolution time	Constant	Variable (except relativistic)	Variable (small)	Variable
Orbit radius	Variable	Constant	Variable (small)	Variable
Transverse focusing	Variable	Constant	Variable	Constant

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## A quick summary...

- 'Scaling' type is a very specific type of FFAG. Anything else is the 'non-scaling' type.
- EMMA is a linear non-scaling FFAG, which again is quite specific.
- ...Are there any other possibilities?

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# Advanced FFAG optics (1)

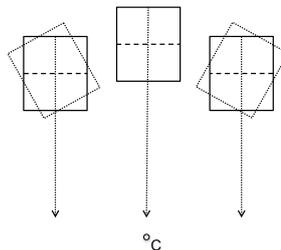
*“There are other variations of these designs which preserve betatron oscillation stability, hold  $v_x$  and  $v_y$  constant, but do not retain the property of similar of equilibrium orbits.”*

*“The magnet edges of focusing and defocusing sectors can be made non-radial, and the fields in positive- and negative- field magnets made different functions of radius”*

- K. Symon, D. Kerst, L. Jones, L. Laslett, and K. Terwilliger, “Fixed-Field Alternating-Gradient Particle Accelerators,” Phys. Rev., vol. 103, no. 6, pp. 1837–1859, Sep. 1956.

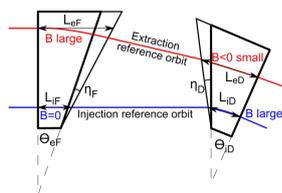
Tune-stable non-scaling FFAG designs have been developed

$$B_z = B_{z0} \left( \frac{r_0 + r}{r_0} \right)^k = B_{z0} \left( 1 + \sum_{n=1}^{\infty} \frac{1}{n!} \frac{k(k-1)\dots(k-n+1)}{r_0^n} r^n \right)$$

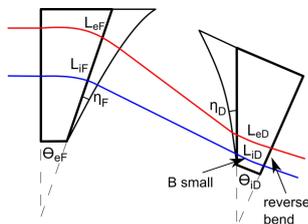


Rectangular magnets,  
Simplified field profile  
Higher stability region  
(S. Machida, S. Sheehy)

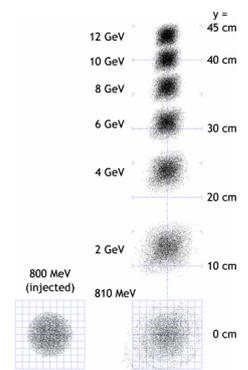
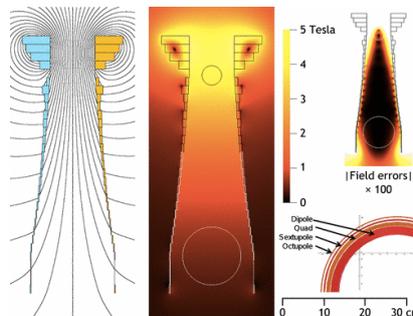
# Advanced FFAG optics (2)



Radial designs with edge profiles  
(C. Johnstone)



Vertical orbit excursion FFAG  
(S. Brooks)



# Current status of FFAG designs

- A whole spectrum of designs have emerged in the last 5-10 years

Potential applications include:

- Accelerator Driven Subcritical Reactor
- Boron Neutron Capture Therapy
- Proton/carbon therapy
- Accelerator-based Neutron Source
- Emittance/Energy Recovery with Internal Target (ERIT)
- e-RHIC injector
- Muon or neutrino factory source
- + many more...



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## Summary

- FFAGs are just a generalisation of synchrotrons or cyclotrons
- Two main types 'scaling' and 'non-scaling'
  - Scaling: specific optics and orbit requirements put a strict requirement on the field profile
  - Non-scaling: removes these restrictions, very general type
- FFAGs may be suitable for many future applications
- In my view, the next big challenge is demonstrating high power operation

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# Reading List

CERN Courier, “Rebirth of the FFAG”, 2004. <http://cerncourier.com/cws/article/cern/29119>

K. Symon, D. Kerst, L. Jones, L. Laslett, and K. Terwilliger, “Fixed-Field Alternating-Gradient Particle Accelerators,” Phys. Rev., vol. 103, no. 6, pp. 1837–1859, Sep. 1956.

S. Machida, “Acceleration in the linear non-scaling fixed-field alternating-gradient accelerator EMMA,” Nat. Phys., vol. 8, no. 3, pp. 243–247, Jan. 2012.

Proceedings of the FFAG workshops