

# RF Systems I

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The CERN Accelerator School

**Introduction to Accelerator Physics, Prague, Czech Republic, 31 Aug – 12 Sept 2014**

## Definitions & basic concepts

dB

$t$ -domain vs.  $\omega$ -domain

phasors

# Decibel (dB)

- Convenient logarithmic measure of a power ratio.
- A “Bel” (= 10 dB) is defined as a power ratio of  $10^1$ . Consequently, 1 dB is a power ratio of  $10^{0.1} \approx 1.259$ .
- If  $rdB$  denotes the measure in dB, we have:

$$rdB = 10 \text{ dB} \log\left(\frac{P_2}{P_1}\right) = 10 \text{ dB} \log\left(\frac{A_2^2}{A_1^2}\right) = 20 \text{ dB} \log\left(\frac{A_2}{A_1}\right)$$

- $\frac{P_2}{P_1} = \frac{A_2^2}{A_1^2} = 10^{rdB/(10 \text{ dB})}$        $\frac{A_2}{A_1} = 10^{rdB/(20 \text{ dB})}$

$rdB$	-30 dB	-20 dB	-10 dB	-6 dB	-3 dB	0 dB	3 dB	6 dB	10 dB	20 dB	30 dB
$\frac{P_2}{P_1}$	0.001	0.01	0.1	0.25	.50	1	2	3.98	10	100	1000
$\frac{A_2}{A_1}$	0.0316	0.1	0.316	0.50	.71	1	1.41	2	3.16	10	31.6

- Related: dBm (relative to 1 mW), dBc (relative to carrier)

# Time domain – frequency domain (1)

- An arbitrary signal  $g(t)$  can be expressed in  $\omega$ -domain using the **Fourier transform** (FT).

$$g(t) \supseteq G(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(t) e^{j\omega t} dt$$

- The inverse transform (IFT) is also referred to as **Fourier Integral**.

$$G(\omega) \Leftarrow g(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} G(\omega) e^{-j\omega t} d\omega$$

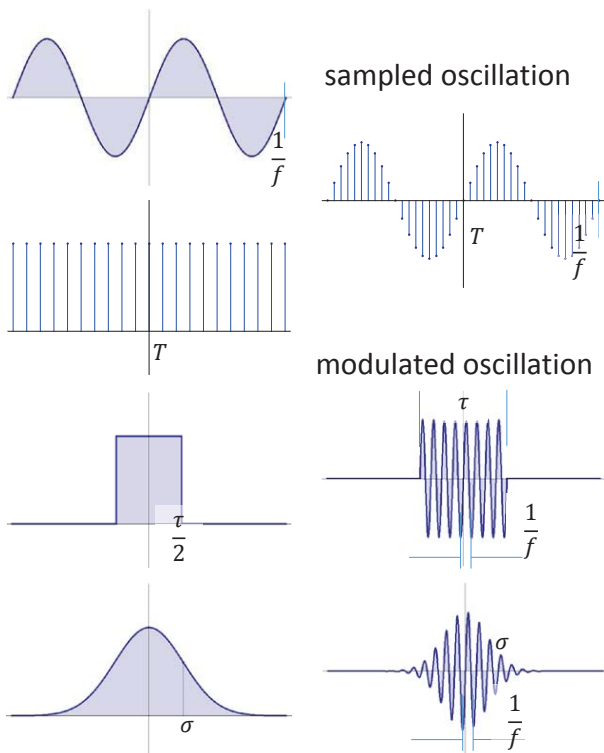
- The advantage of the  $\omega$ -domain description is that linear time-invariant (LTI) systems are much easier described.
- The mathematics of the FT requires the extension of the definition of a *function* to allow for infinite values and non-converging integrals.
- The FT of the signal can be understood at looking at “*what frequency components it’s composed of*”.

# Time domain – frequency domain (2)

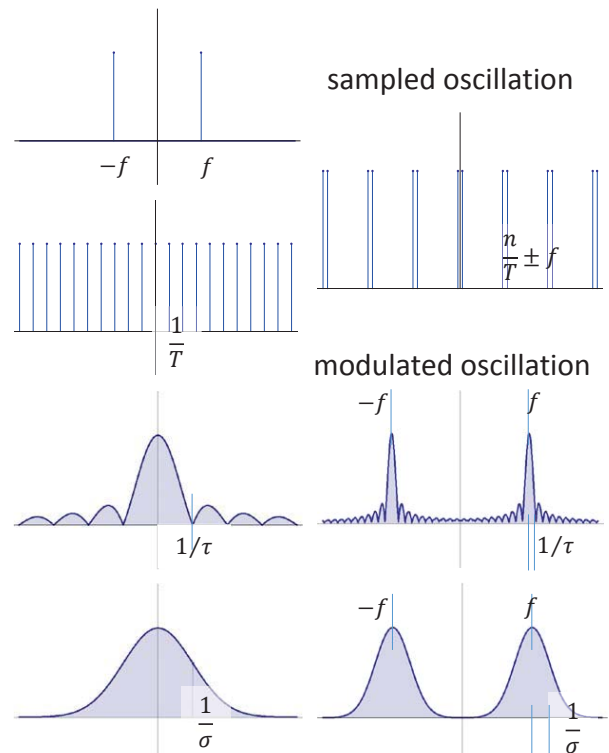
- For  $T$ -periodic signals, the FT becomes the Fourier-Series,  $d\omega$  becomes  $2\pi/T$ ,  $\int$  becomes  $\sum$ .
- The cousin of the FT is the **Laplace transform**, which uses a complex variable (often  $s$ ) instead of  $j\omega$ ; it has generally a better convergence behaviour.
- Numerical implementations of the FT require discretisation in  $t$  (sampling) and in  $\omega$ . There exist very effective algorithms (FFT).
- In digital signal processing, one often uses the related z-Transform, which uses the variable  $z = e^{j\omega\tau}$ , where  $\tau$  is the sampling period. A delay of  $k\tau$  becomes  $z^{-k}$ .

# Time domain – frequency domain (3)

## • Time domain



## • Frequency domain

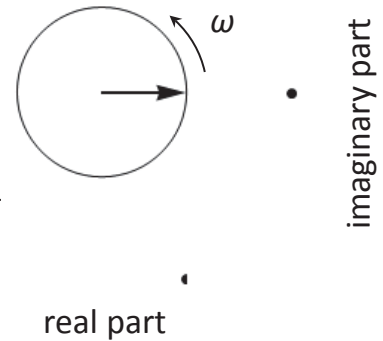


# Fixed frequency oscillation (steady state, CW)

## Definition of phasors

- General:  $A \cos(\omega t - \varphi) = A \cos \omega t \cos \varphi + A \sin \omega t \sin \varphi$
- This can be interpreted as the projection on the real axis of a rotation in the complex plane.

$$\Re\{A(\cos \varphi + j \sin \varphi)e^{j\omega t}\}$$

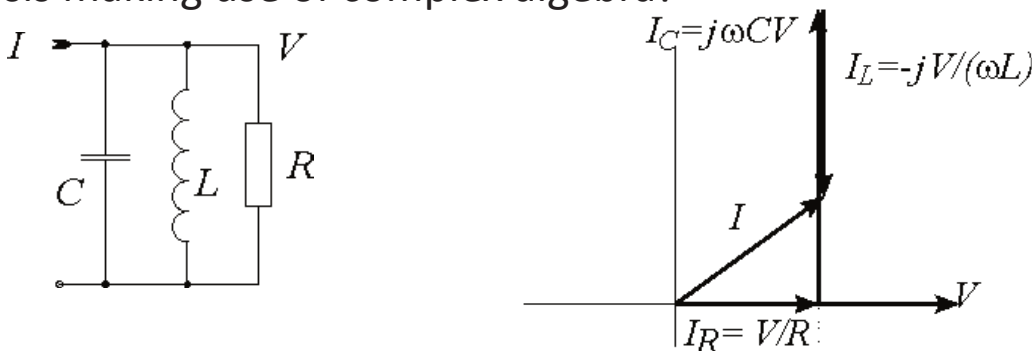


- The complex amplitude  $\tilde{A}$  is called “phasor”;

$$\tilde{A} = A(\cos \varphi + j \sin \varphi)$$

## Calculus with phasors

- Why this seeming “complication”?  
Because things become easier!
- Using  $\frac{d}{dt} \equiv j\omega$ , one may now forget about the rotation with  $\omega$  and the projection on the real axis, and do the complete analysis making use of complex algebra!



$$I = V \left( \frac{1}{R} + j\omega C - \frac{j}{\omega L} \right)$$

# Slowly varying amplitudes

- For band-limited signals, one may conveniently use “slowly varying” phasors and a fixed frequency RF oscillation.
- So-called in-phase ( $I$ ) and quadrature ( $Q$ ) “baseband envelopes” of a modulated RF carrier are the real and imaginary part of a slowly varying phasor.

## On Modulation

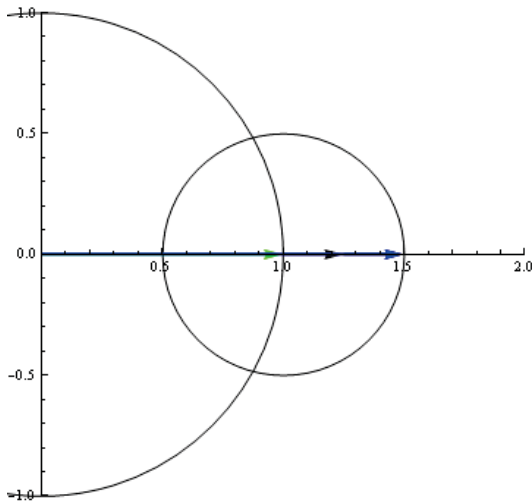
AM

PM

I-Q

# Amplitude modulation

$$(1 + m \cos \varphi) \cdot \cos(\omega_c t) = \Re \left\{ \left( 1 + \frac{m}{2} e^{j\varphi} + \frac{m}{2} e^{-j\varphi} \right) e^{j\omega_c t} \right\}$$

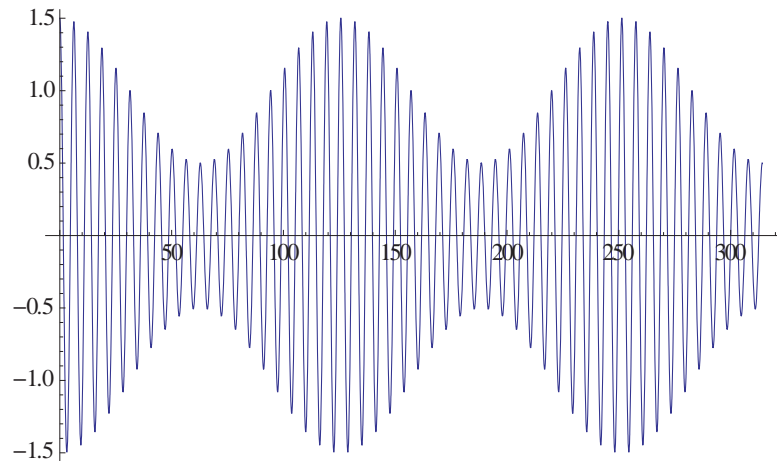


green: carrier  
black: sidebands at  $\pm f_m$   
blue: sum

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$m$ : modulation index or modulation depth

example:  $\varphi = \omega_m t = 0.05 \omega_c t$   
 $m = 0.5$



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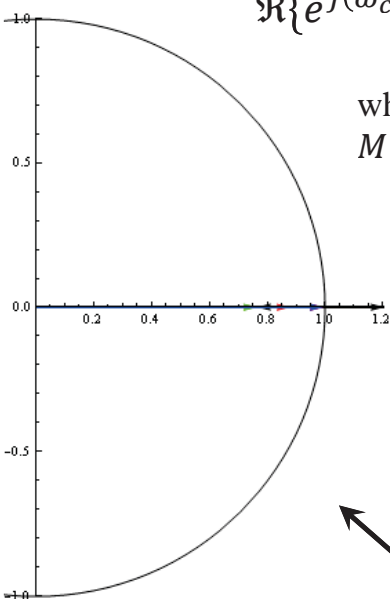
# Phase modulation

$$\Re \{ e^{j(\omega_c t + M \sin \varphi)} \} = \Re \left\{ \sum_{n=-\infty}^{\infty} J_n(M) e^{j(n\varphi + \omega_c t)} \right\}$$

where

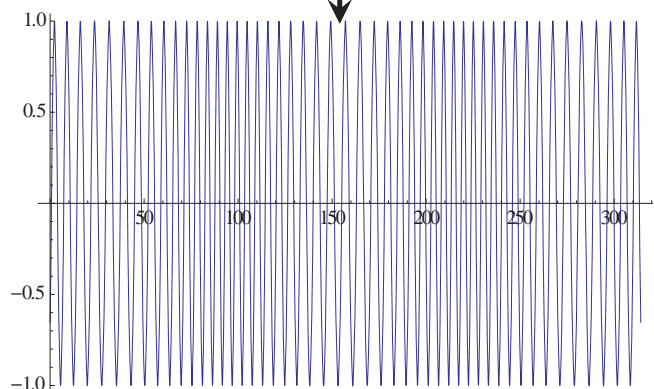
$M$ : modulation index (= max. phase deviation)

example:  $\varphi = \omega_m t = 0.05 \omega_c t$   
 $M = 4$



Green:  $n = 0$  (carrier)  $M = 1$   
black:  $n = 1$  sidebands  
red:  $n = 2$  sidebands  
blue: sum

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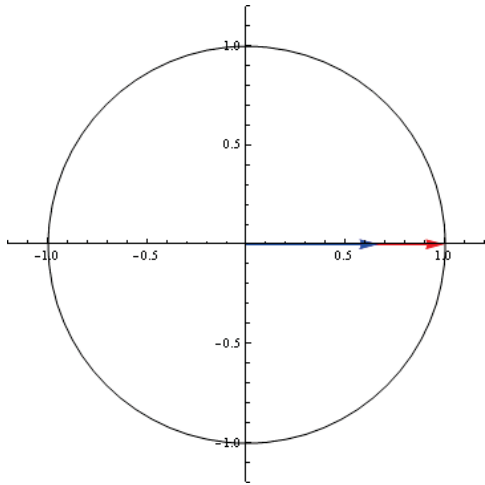


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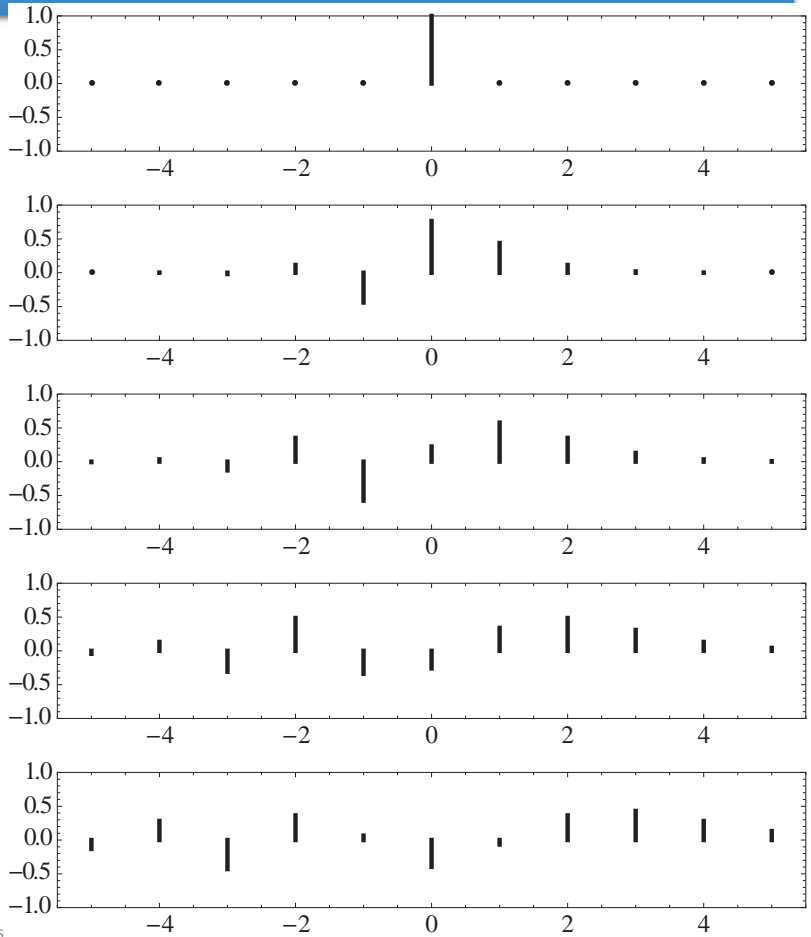
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# Spectrum of phase modulation

Plotted: spectral lines for sinusoidal PM at  $f_m$   
 Abscissa:  $(f - f_c)/f_m$



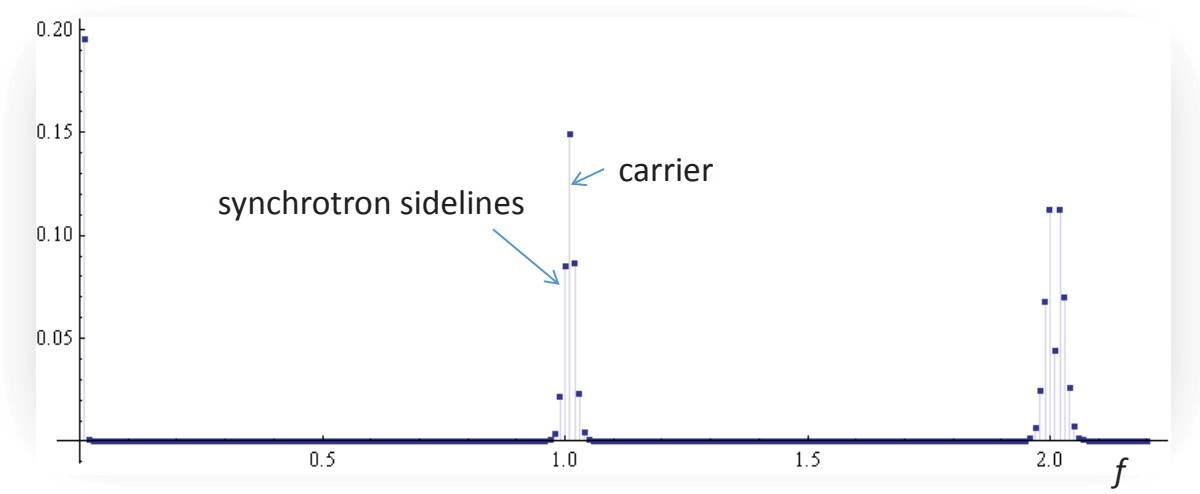
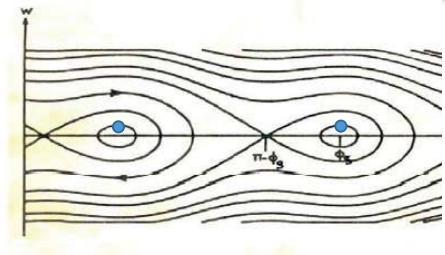
Phase modulation with  $M = \pi$ :  
 red: real phase modulation  
 blue: sum of sidebands  $n \leq 3$



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# Spectrum of a beam with synchrotron oscillation, $M = 1 (= 57^\circ)$

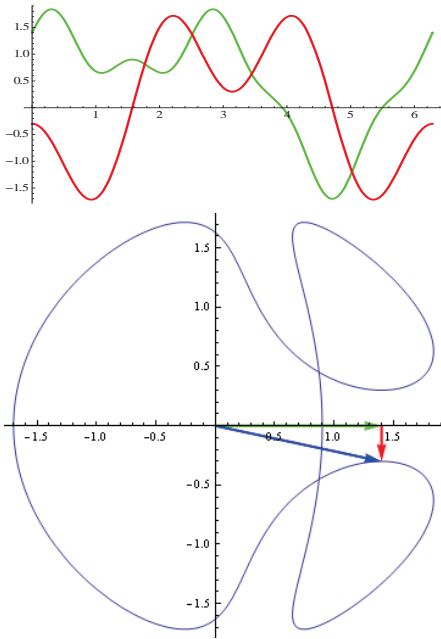


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# Vector (I-Q) modulation



More generally, a modulation can have both amplitude and phase modulating components. They can be described as the in-phase (*I*) and quadrature (*Q*) components in a chosen reference,  $\cos(\omega_r t)$ . In complex notation, the modulated RF is:

$$\Re\{(I(t) + j Q(t))e^{j \omega_r t}\} = \Re\{(I(t) + j Q(t))(\cos(\omega_r t) + j \sin(\omega_r t))\} = I(t) \cos(\omega_r t) - Q(t) \sin(\omega_r t)$$

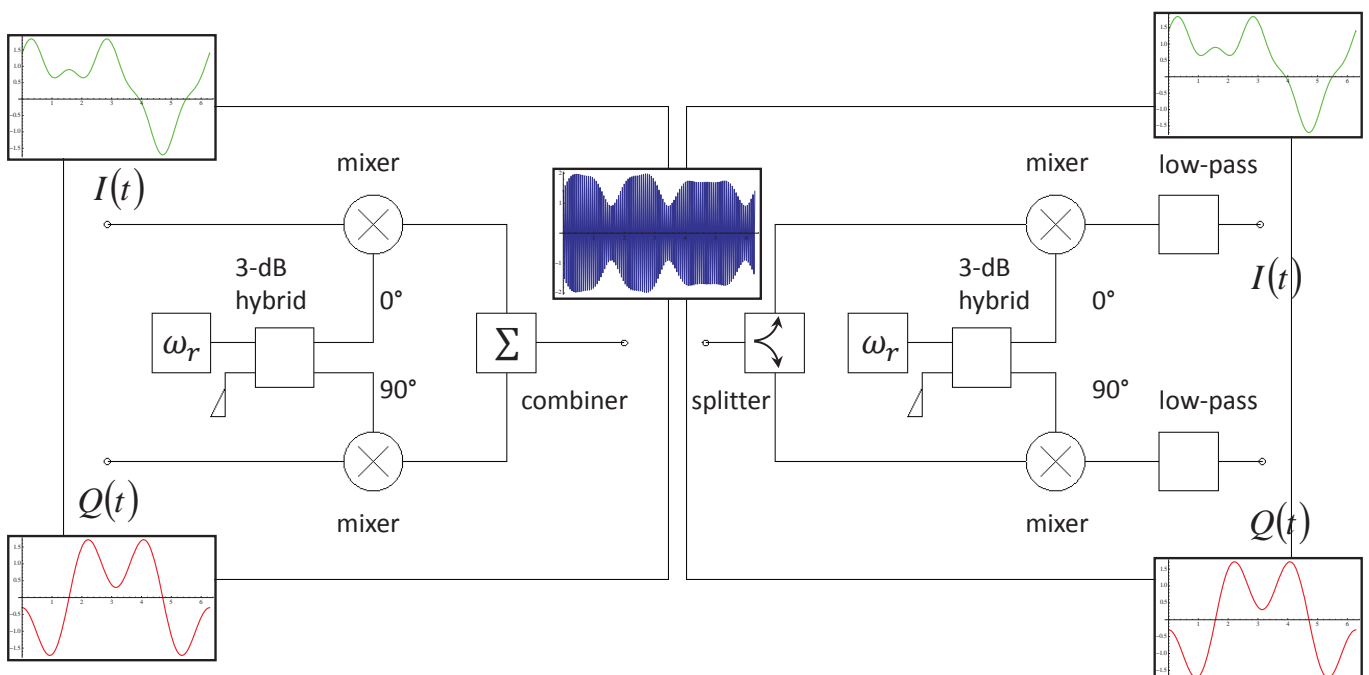
So *I* and *Q* are the Cartesian coordinates in the complex “Phasor” plane, where amplitude and phase are the corresponding polar coordinates.

$$I(t) = A(t) \cos(\varphi)$$

$$Q(t) = A(t) \sin(\varphi)$$

I-Q modulation:  
 green: *I* component  
 red: *Q* component  
 blue: vector-sum

# Vector modulator/demodulator



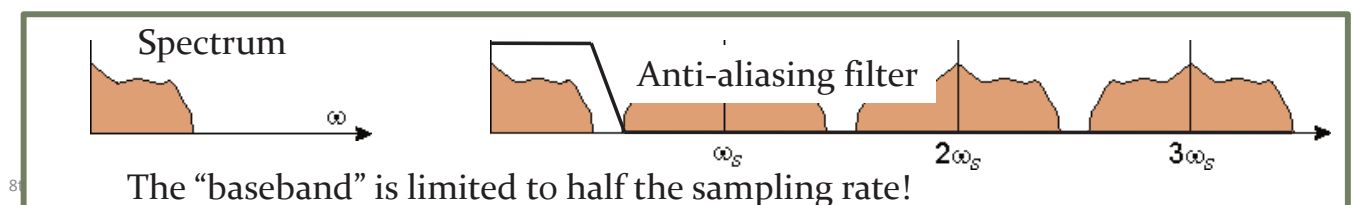
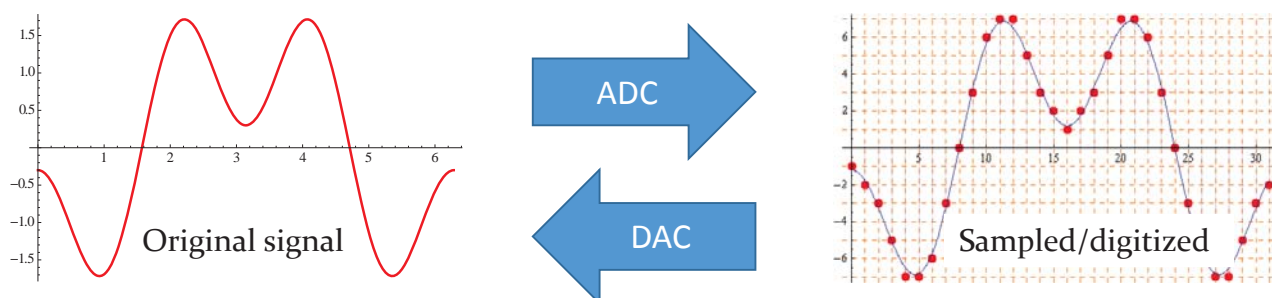


# Digital Signal Processing

Just some basics

## Sampling and quantization

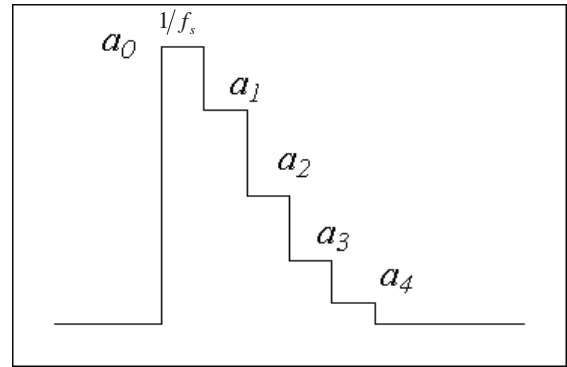
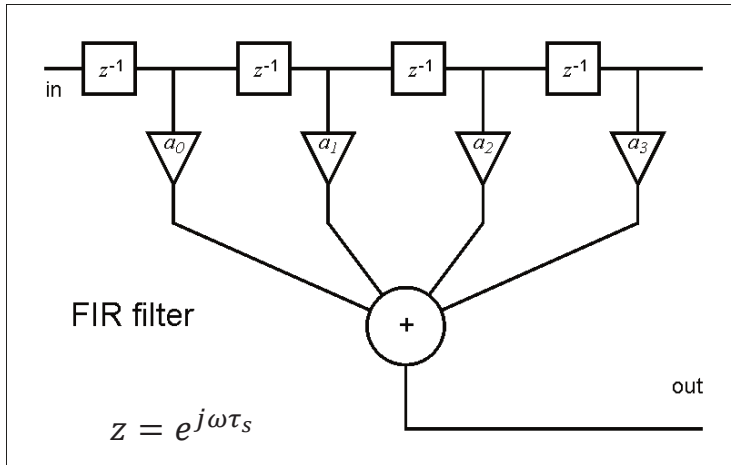
- Digital Signal Processing is very powerful – note recent progress in digital audio, video and communication!
- Concepts and modules developed for a huge market; highly sophisticated modules available “off the shelf”.
- The “slowly varying” phasors are ideal to be sampled and quantized as needed for digital signal processing.
- Sampling (at  $1/\tau_s$ ) and quantization ( $n$  bit data words – here 4 bit):



The “baseband” is limited to half the sampling rate!

# Digital filters (1)

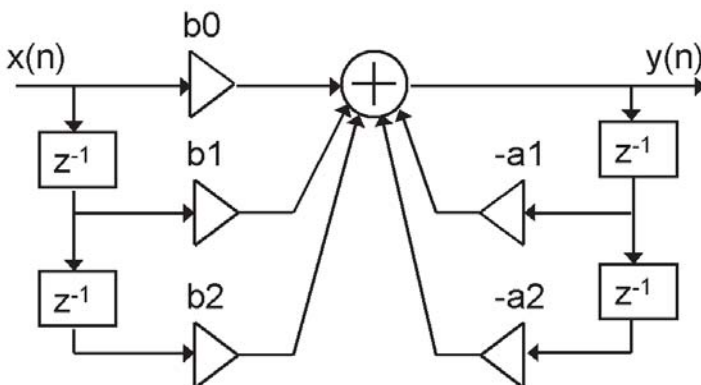
- Once in the digital realm, signal processing becomes “computing”!
- In a “finite impulse response” (FIR) filter, you directly program the coefficients of the impulse response.



Transfer function:  
 $a_0 + a_1z^{-1} + a_2z^{-2} + a_3z^{-3} + a_4z^{-4}$

# Digital filters (2)

- An “infinite impulse response” (IIR) filter has built-in recursion, e.g. like

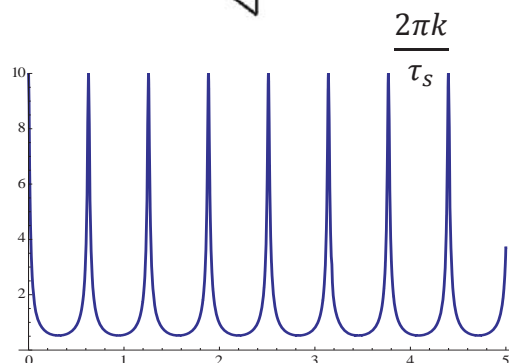


Transfer function:  

$$\frac{b_0 + b_1z^{-1} + b_2z^{-2}}{1 + a_1z^{-1} + a_2z^{-2}}$$

Example:

$$\frac{b_0}{1 + b_k z^{-k}}$$



... is a comb filter.

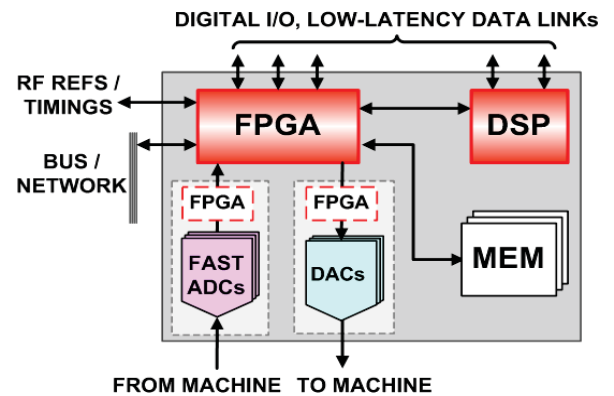
# Digital LLRF building blocks – examples

- General D-LLRF board:

- modular!

FPGA: Field-programmable gate array

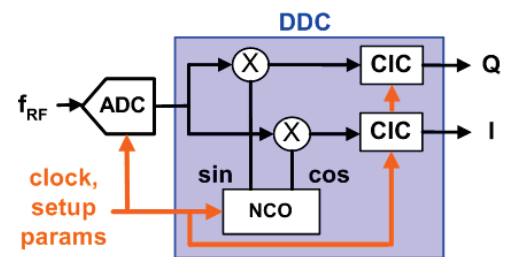
DSP: Digital Signal Processor



- DDC (Digital Down Converter)

- Digital version of the *I-Q* demodulator

CIC: cascaded integrator-comb  
(a special low-pass filter)



## RF system & control loops

e.g.: ... for a synchrotron:

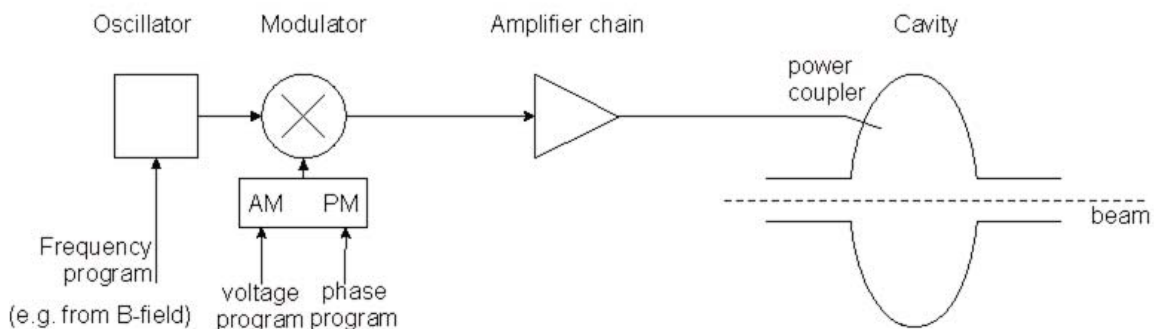
Cavity control loops

Beam control loops

# Minimal RF system (of a synchrotron)

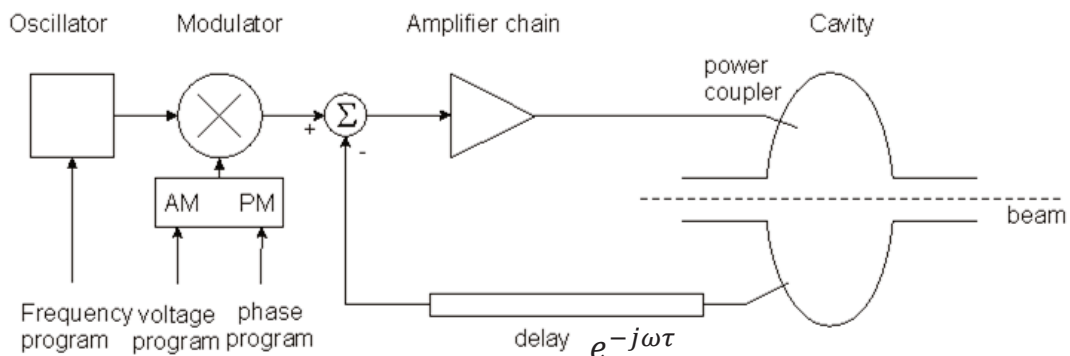
## Low-level RF

## High-Power RF



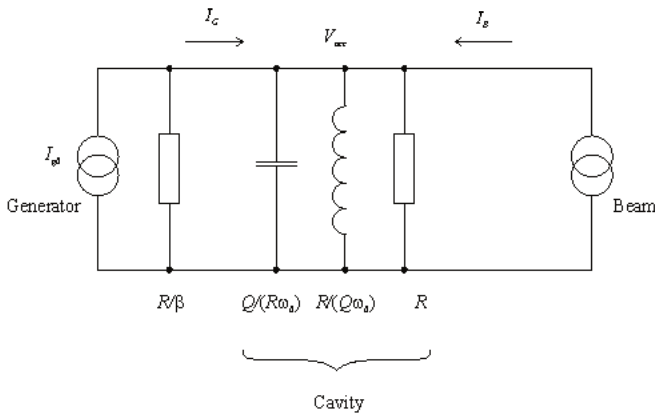
- The frequency has to be controlled to follow the magnetic field such that the beam remains in the centre of the vacuum chamber.
- The voltage has to be controlled to allow for capture at injection, a correct bucket area during acceleration, matching before ejection; phase may have to be controlled for transition crossing and for synchronisation before ejection.

# Fast RF Feed-back loop



- Compares actual RF voltage and phase with desired and corrects.
- Rapidity limited by total group delay (path lengths) (some 100 ns).
- Unstable if loop gain = 1 with total phase shift  $180^\circ$  – design requires to stay away from this point (stability margin)
- The group delay limits the gain·bandwidth product.
- Works also to keep voltage at zero for strong beam loading, i.e. it reduces the beam impedance.

# Fast feedback loop at work



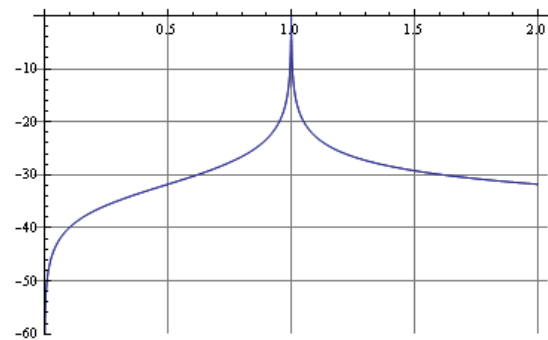
Without feedback,  $V_{acc} = (I_{G0} + I_B) \cdot Z(\omega)$ , where

$$Z(\omega) = \frac{R/(1 + \beta)}{1 + jQ \left( \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} \right)}$$

Detect the gap voltage, feed it back to  $I_{G0}$  such that  $I_{G0} = I_{drive} - G \cdot V_{acc}$ , where  $G$  is the total loop gain (pick-up, cable, amplifier chain ...)

Result:

$$V_{acc} = (I_{drive} + I_B) \cdot \frac{Z(\omega)}{1 + G \cdot Z(\omega)}$$

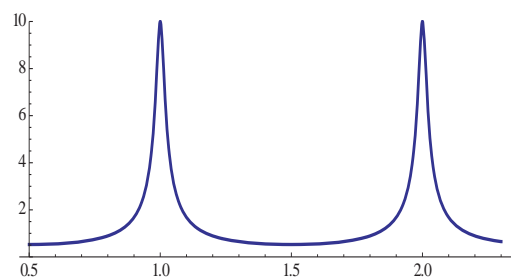
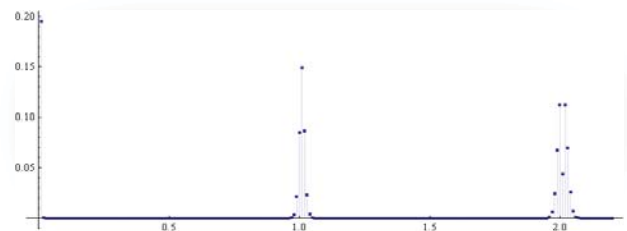


- Gap voltage is stabilised!
- Impedance seen by the beam is reduced by the loop gain!

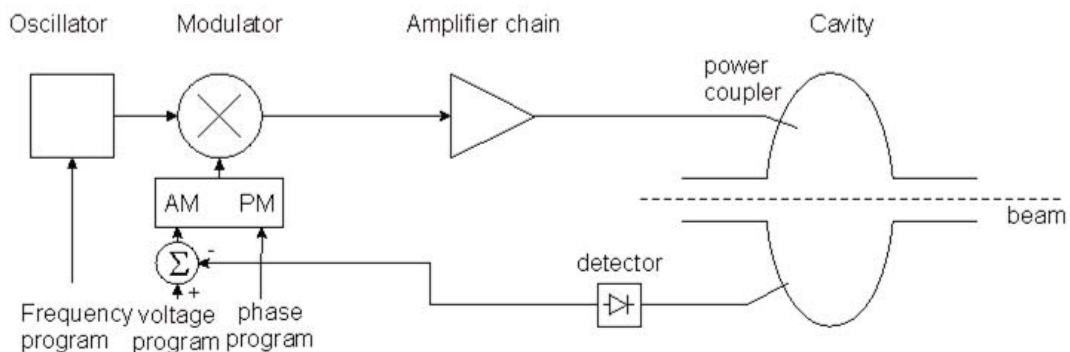
Plot on the right:  $\frac{1+\beta}{R} \left| \frac{Z(\omega)}{1+G \cdot Z(\omega)} \right|$  vs.  $\omega$ , with the loop gain varying from 0 dB to 50 dB.

# 1-turn delay feed-back loop

- The speed of the “fast RF feedback” is limited by the group delay – this is typically a significant fraction of the revolution period.
- How to lower the impedance over many harmonics of the revolution frequency?
- Remember: the beam spectrum is limited to relatively narrow bands around the multiples of the revolution frequency!
- Only in these narrow bands the loop gain must be high!
- Install a comb filter! ... and extend the group delay to exactly 1 turn – in this case the loop will have the desired effect and remain stable!

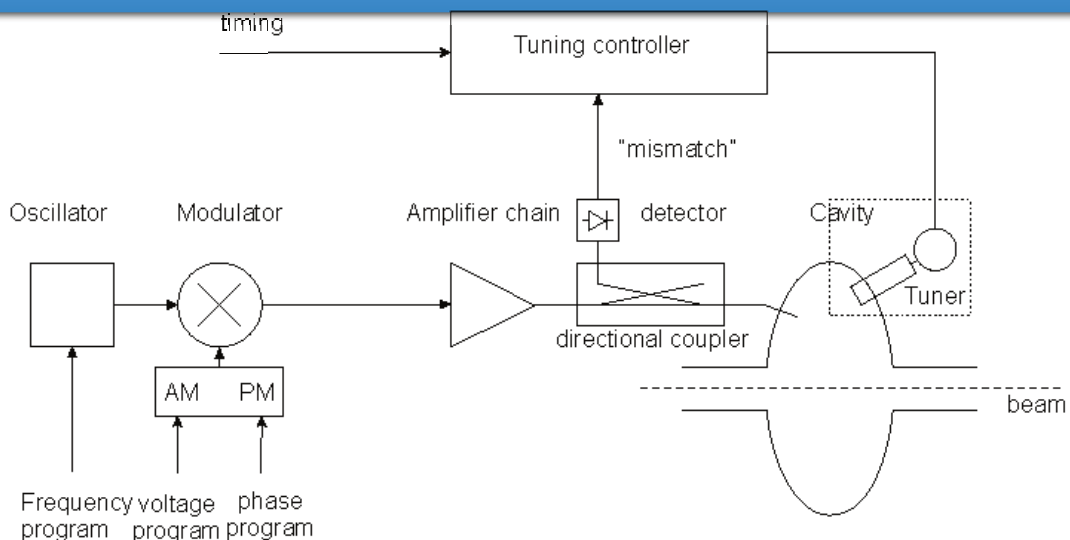


# Field amplitude control loop (AVC)



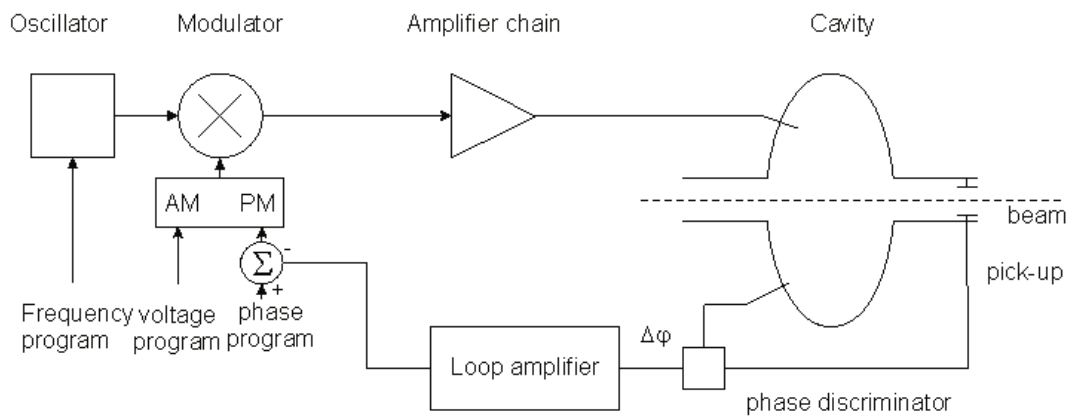
- Compares the detected cavity voltage to the voltage program. The error signal serves to correct the amplitude

# Tuning loop

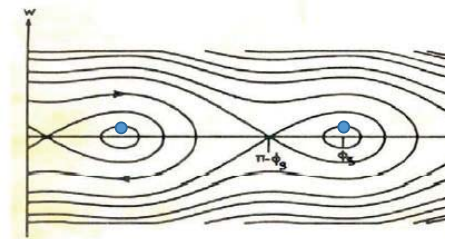


- Tunes the resonance frequency of the cavity  $f_r$  to minimize the mismatch of the PA.
- In the presence of beam loading, the optimum  $f_r$  may be  $f_r \neq f$ .
- In an ion ring accelerator, the tuning range might be > octave!
- For fixed  $f$  systems, tuners are needed to compensate for slow drifts.
- Examples for tuners:
  - controlled power supply driving ferrite bias (varying  $\mu$ ),
  - stepping motor driven plunger,
  - motorized variable capacitor, ...

# Beam phase loop



- Longitudinal motion:  $\frac{d^2(\Delta\phi)}{dt^2} + \Omega_s^2(\Delta\phi)^2 = 0$ .
- Loop amplifier transfer function designed to damp synchrotron oscillation.  
Modified equation:  $\frac{d^2(\Delta\phi)}{dt^2} + \alpha \frac{d(\Delta\phi)}{dt} + \Omega_s^2(\Delta\phi)^2 = 0$



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# Other loops

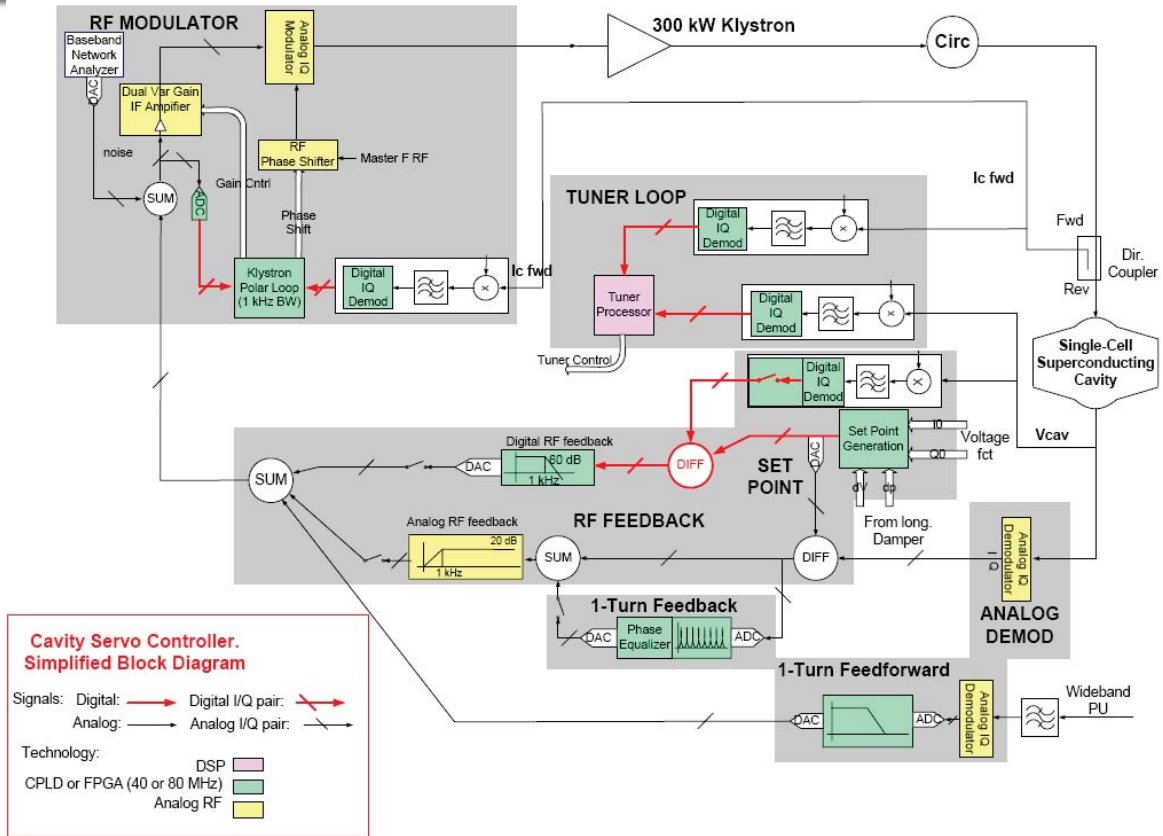
- Radial loop:
  - Detect average radial position of the beam,
  - Compare to a programmed radial position,
  - Error signal controls the frequency.
- Synchronisation loop (e.g. before ejection):
  - 1<sup>st</sup> step: Synchronize  $f$  to an external frequency (will also act on radial position!).
  - 2<sup>nd</sup> step: phase loop brings bunches to correct position.
- ...

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# A real implementation: LHC LLRF



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## Fields in a waveguide

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# Homogeneous plane wave

$$\vec{E} \propto \vec{u}_y \cos(\omega t - \vec{k} \cdot \vec{r})$$

$$\vec{B} \propto \vec{u}_x \cos(\omega t - \vec{k} \cdot \vec{r})$$

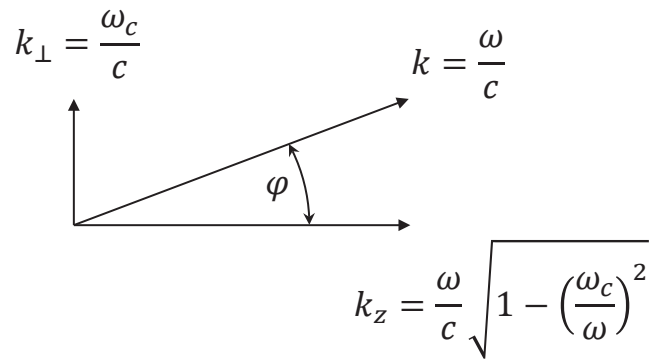
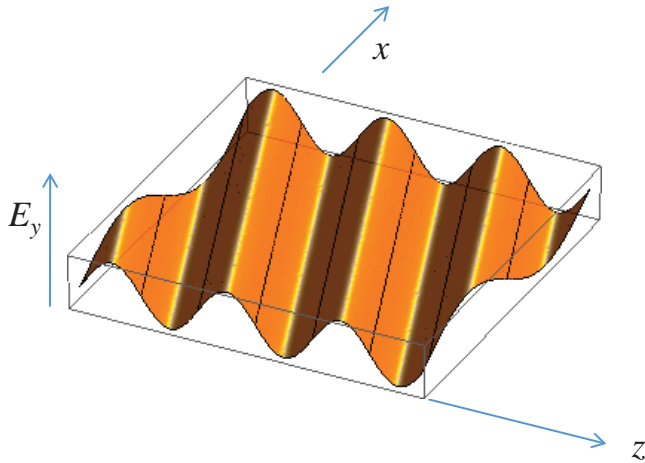
$$\vec{k} \cdot \vec{r} = \frac{\omega}{c} (z \cos \varphi + x \sin \varphi)$$

**Wave vector  $\vec{k}$ :**

the direction of  $\vec{k}$  is the direction of propagation,

the length of  $\vec{k}$  is the phase shift per unit length.

$\vec{k}$  behaves like a vector.



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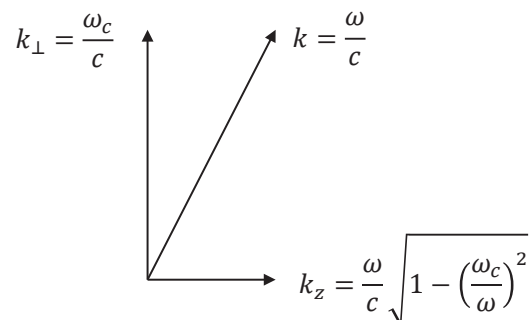
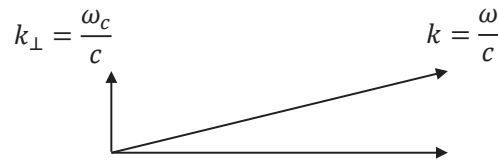
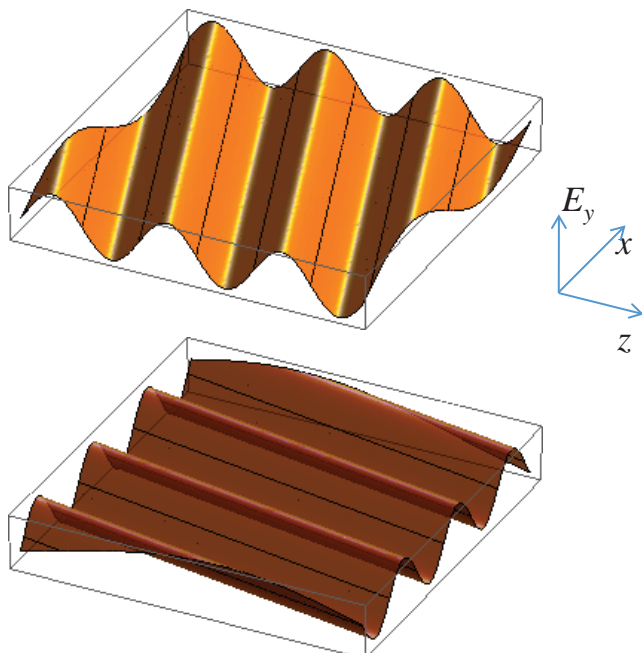
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# Wave length, phase velocity

- The components of  $\vec{k}$  are related to the wavelength in the direction of that

component as  $\lambda_z = \frac{2\pi}{k_z}$  etc. , to the phase velocity as  $v_{\varphi,z} = \frac{\omega}{k_z} = f\lambda_z$ .

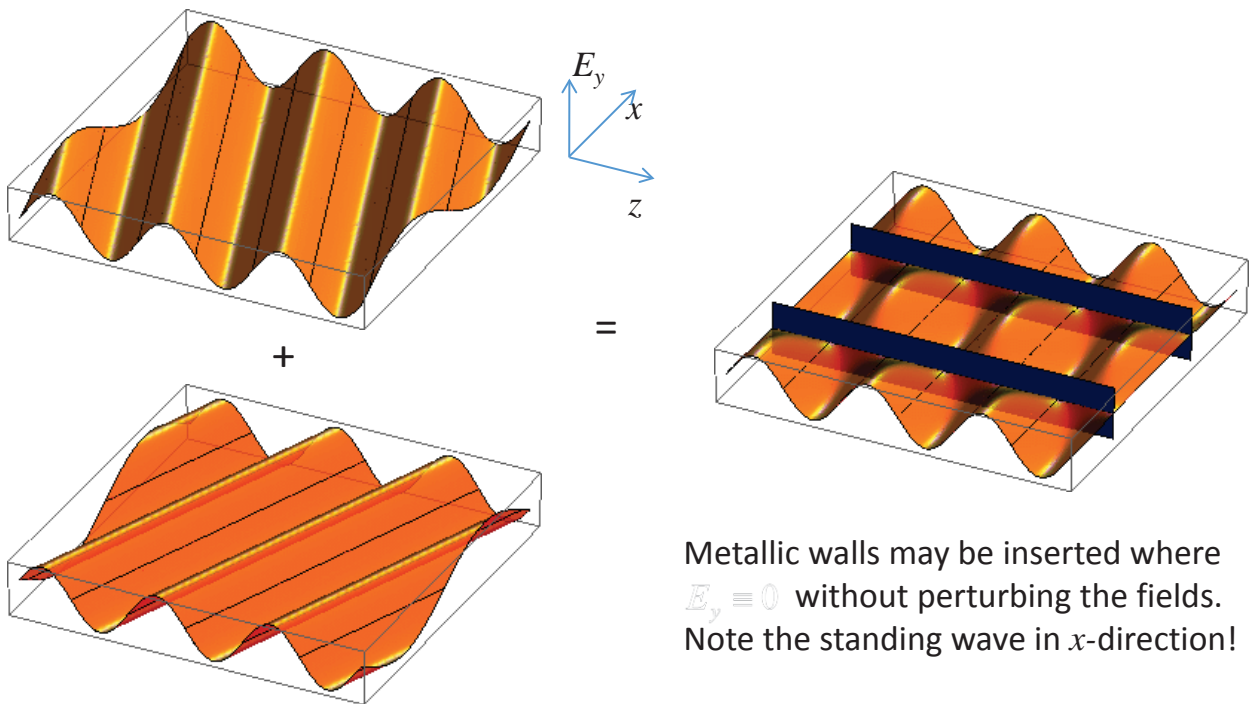


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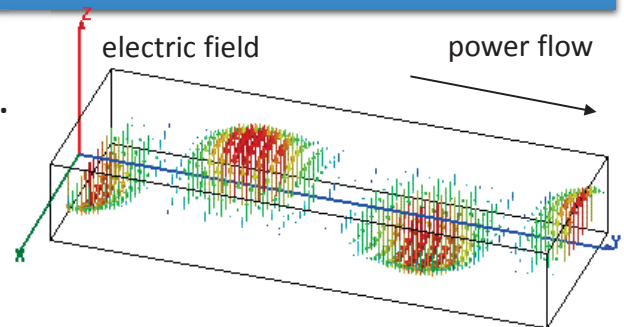
# Superposition of 2 homogeneous plane waves



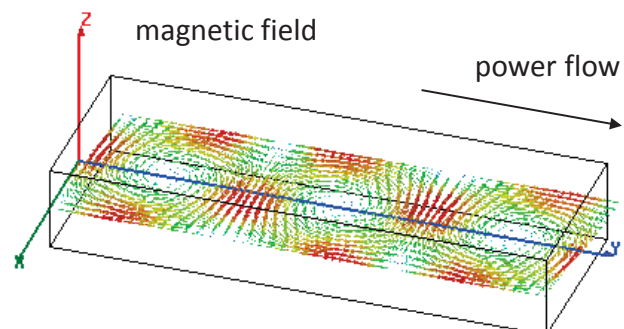
This way one gets a hollow rectangular waveguide!

# Rectangular waveguide

Fundamental ( $TE_{10}$  or  $H_{10}$ ) mode in a standard rectangular waveguide. E.g. forward wave



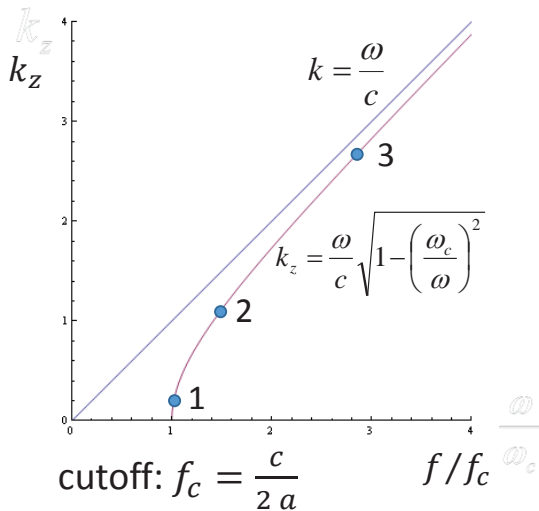
$$\text{power flow: } \frac{1}{2} \text{Re} \left\{ \iint \vec{E} \times \vec{H}^* dA \right\}$$



# Waveguide dispersion

What happens with different waveguide dimensions (different width  $a$ )?

The "guided wavelength"  $\lambda_g$  varies from  $\infty$  at  $f_c$  to  $\lambda$  at very high frequencies.

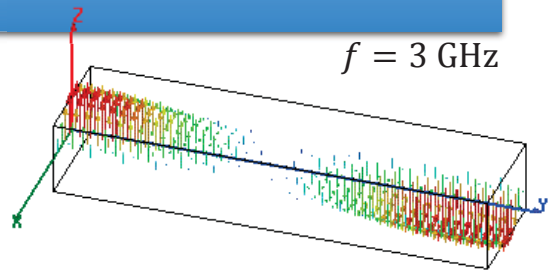


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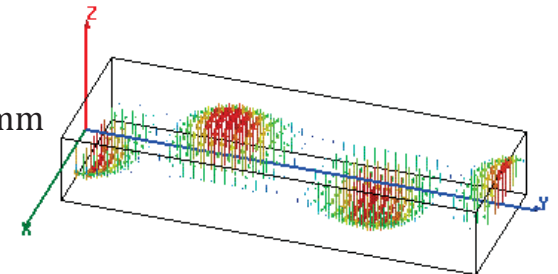
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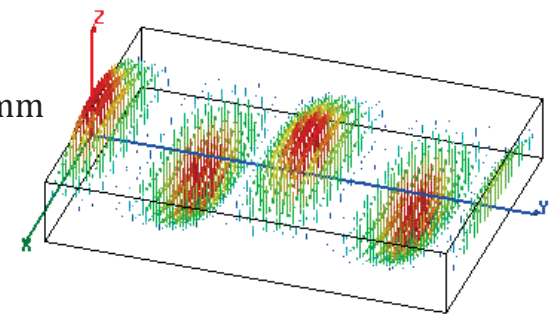
- 1:  
 $a = 52 \text{ mm}$   
 $\frac{f}{f_c} = 1.04$



- 2:  
 $a = 72.14 \text{ mm}$   
 $\frac{f}{f_c} = 1.44$



- 3:  
 $a = 144.3 \text{ mm}$   
 $\frac{f}{f_c} = 2.88$



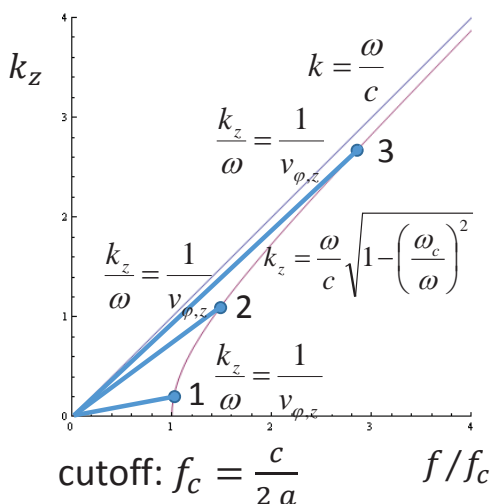
# Phase velocity $v_{\phi,z}$

The phase velocity is the speed with which the crest or a zero-crossing travels in  $z$ -direction.

Note in the animations on the right that, at constant  $f$ , it is  $v_{\phi,z} \propto \lambda_g$ .

Note that at  $f = f_c$ ,  $v_{\phi,z} = \infty$ !

With  $f \rightarrow \infty$ ,  $v_{\phi,z} \rightarrow c$ !

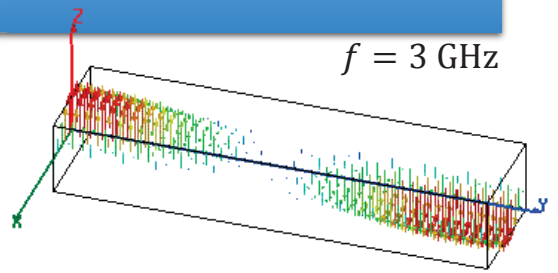


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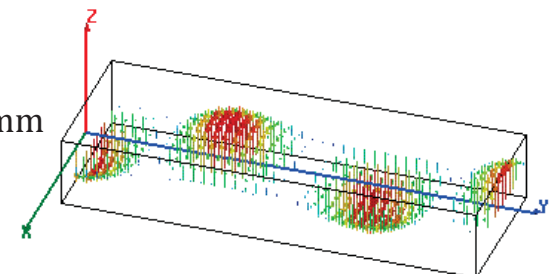
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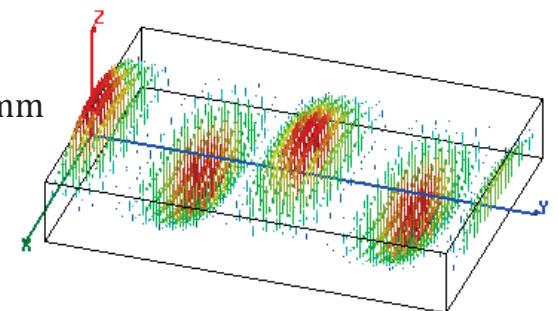
- 1:  
 $a = 52 \text{ mm}$   
 $\frac{f}{f_c} = 1.04$



- 2:  
 $a = 72.14 \text{ mm}$   
 $\frac{f}{f_c} = 1.44$

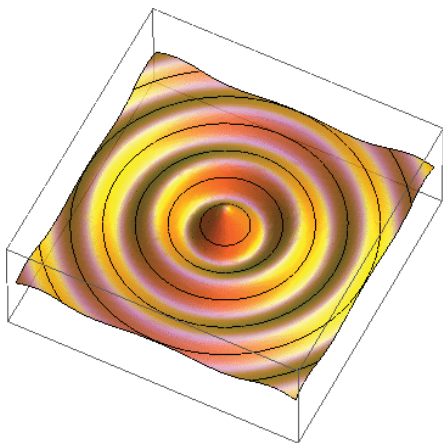


- 3:  
 $a = 144.3 \text{ mm}$   
 $\frac{f}{f_c} = 2.88$

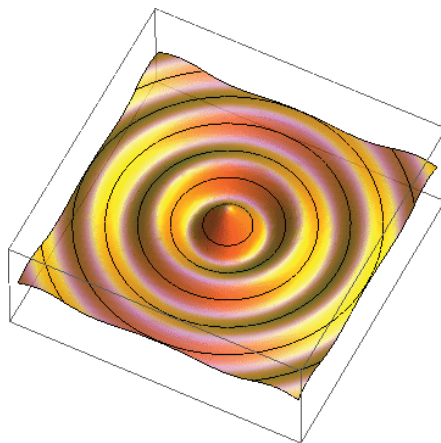


# Radial waves

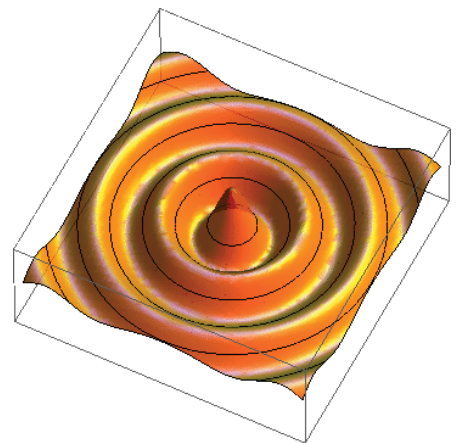
- Also radial waves may be interpreted as superpositions of plane waves.
- The superposition of an outward and an inward radial wave can result in the field of a round hollow waveguide.



$$E_z \propto H_n^{(2)}(k_\rho \rho) \cos(n\varphi)$$



$$E_z \propto H_n^{(1)}(k_\rho \rho) \cos(n\varphi)$$

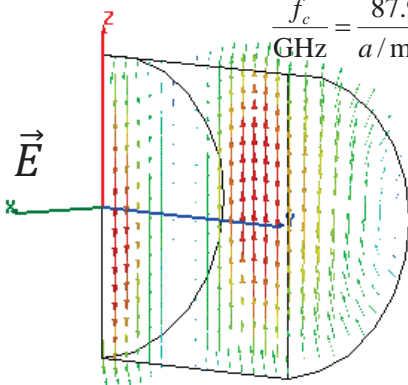


$$E_z \propto J_n(k_\rho \rho) \cos(n\varphi)$$

# Round waveguide modes

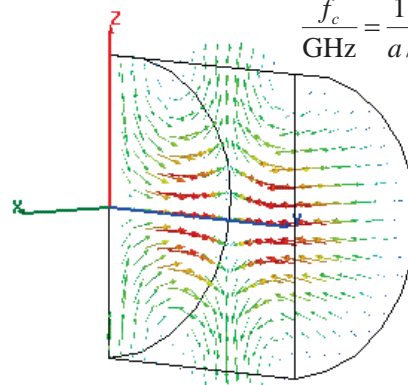
TE<sub>11</sub> – fundamental

$$\frac{f_c}{\text{GHz}} = \frac{87.9}{a/\text{mm}}$$



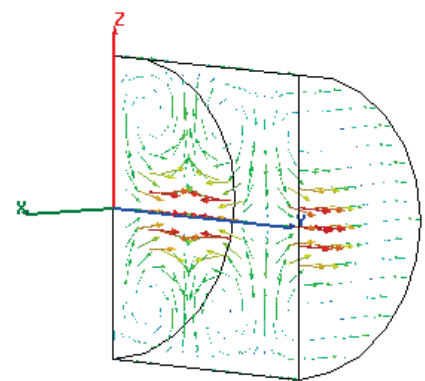
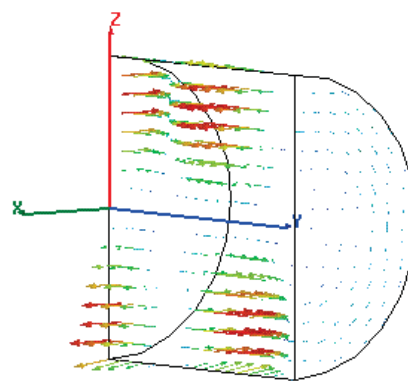
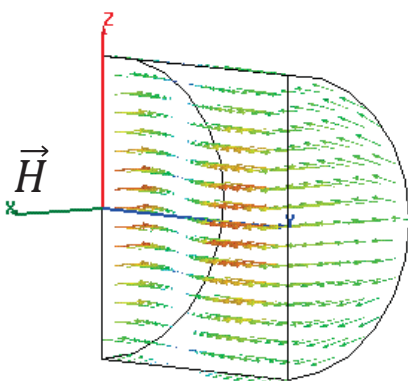
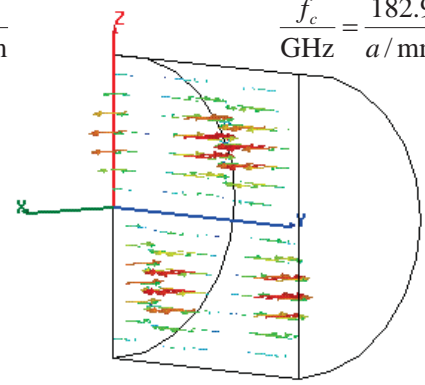
TM<sub>01</sub> – axial field

$$\frac{f_c}{\text{GHz}} = \frac{114.8}{a/\text{mm}}$$



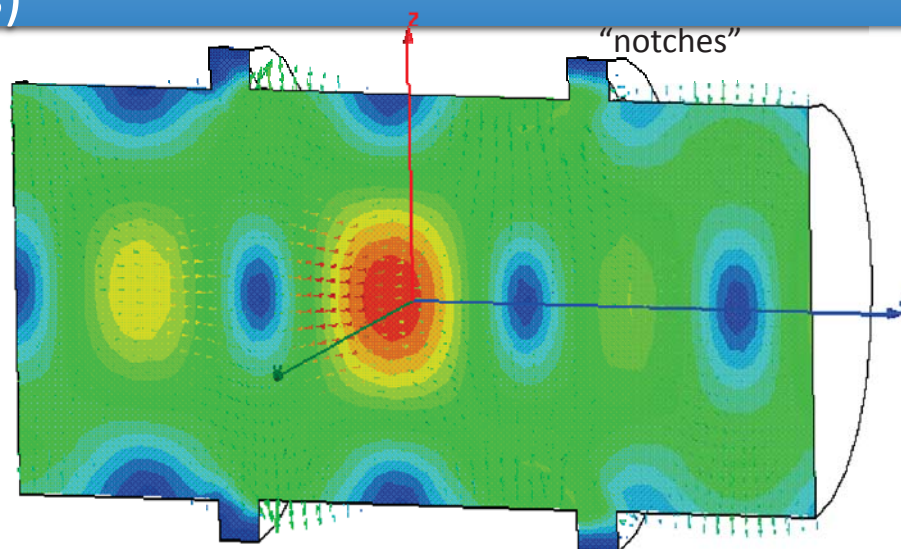
TE<sub>01</sub> – low loss

$$\frac{f_c}{\text{GHz}} = \frac{182.9}{a/\text{mm}}$$



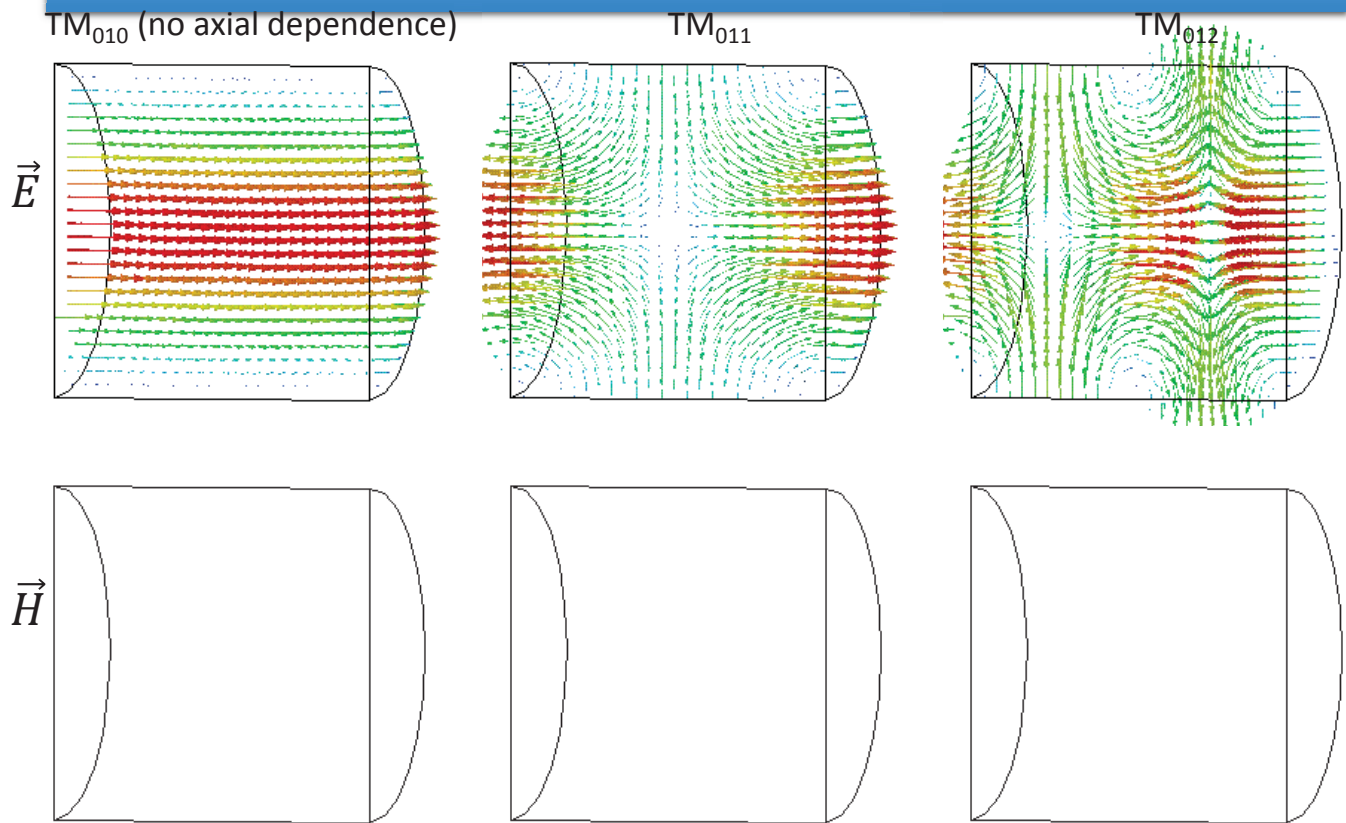
# From waveguide to cavity

## Waveguide perturbed by discontinuities (notches)



Reflections from notches lead to a superimposed standing wave pattern.  
"Trapped mode"

# Short-circuited waveguide

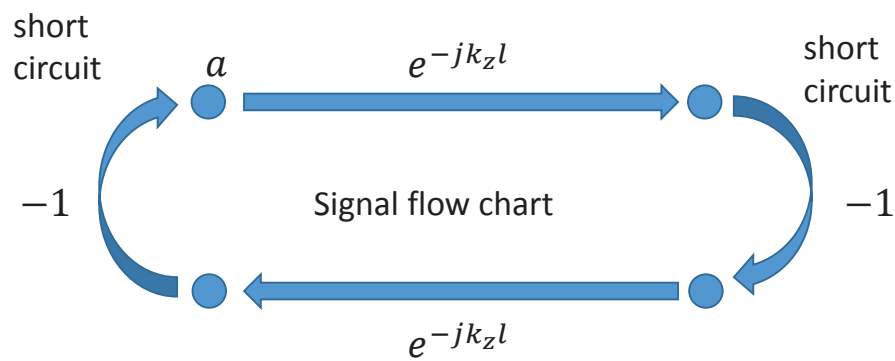


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# Single waveguide mode between two shorts



Eigenvalue equation for field amplitude  $a$ :

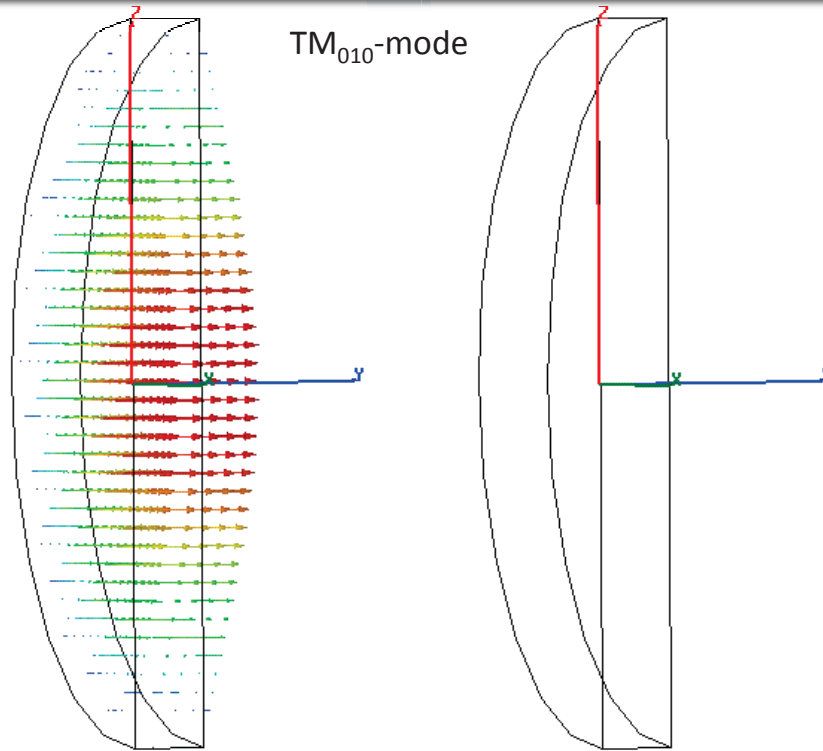
$$a = a e^{-jk_z 2l}$$

Non-vanishing solutions exist for  $2k_z l = 2\pi m$ .

With  $k_z = \frac{\omega}{c} \sqrt{1 - \left(\frac{\omega}{\omega_c}\right)^2}$ , this becomes  $f_0^2 = f_c^2 + \left(c \frac{m}{2l}\right)^2$ .

# Simple pillbox cavity

(only 1/2 shown)



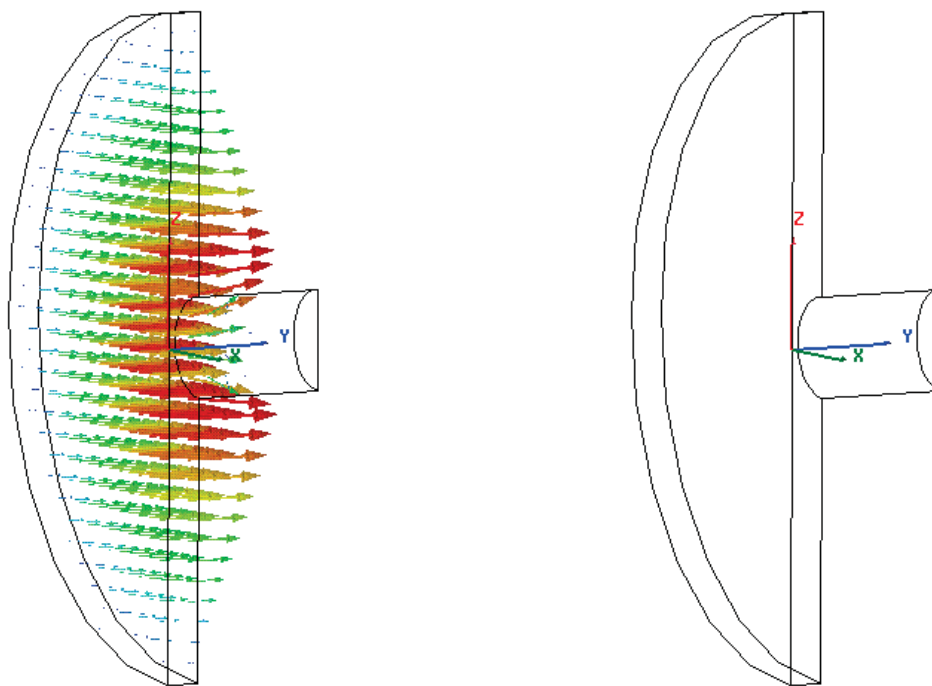
electric field (purely axial)

magnetic field (purely azimuthal)

# Pillbox with beam pipe

$TM_{010}$ -mode (only 1/4 shown)

One needs a hole for the beam pipe – circular waveguide below cutoff



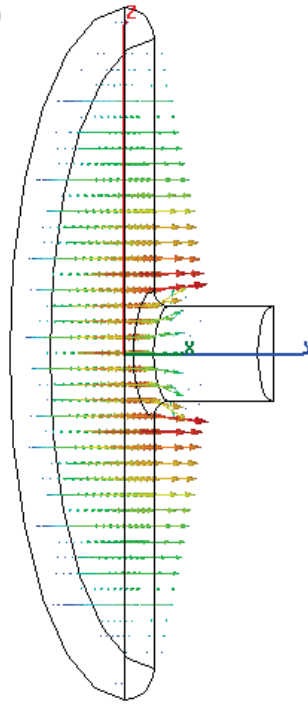
electric field

magnetic field

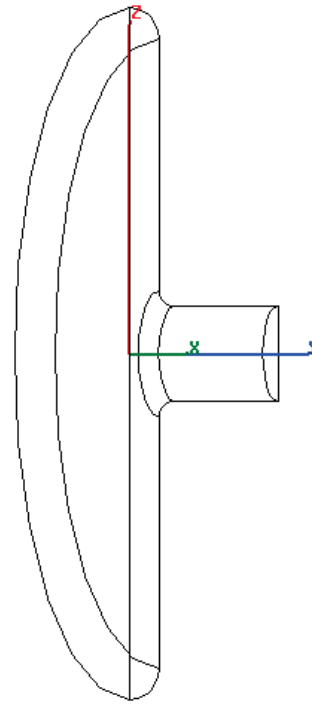
# A more practical pillbox cavity

TM<sub>010</sub>-mode (only 1/4 shown)

Round of sharp edges  
(field enhancement!)



electric field



magnetic field

# Some real “pillbox” cavities

CERN PS 200 MHz cavities





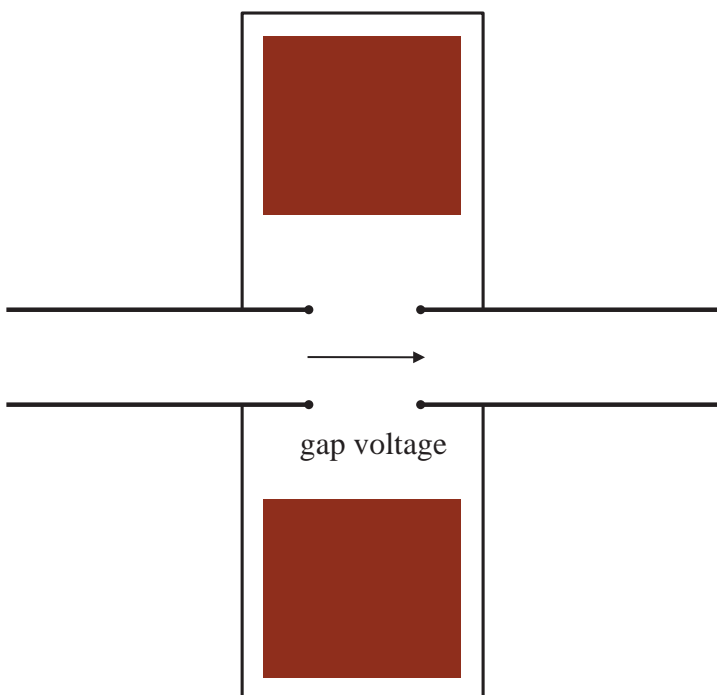
# Accelerating gap

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## Accelerating Gap



- We want a voltage across the gap!
- It cannot be DC, since we want the beam tube on ground potential.
- Use  $\oint \vec{E} \cdot d\vec{s} = - \iint \frac{d\vec{B}}{dt} \cdot d\vec{A}$
- The “shield” imposes a
  - upper limit of the voltage pulse duration or – equivalently –
  - a lower limit to the usable frequency.
- The limit can be extended with a material which acts as “open circuit”!
- Materials typically used:
  - ferrites (depending on  $f$ -range)
  - magnetic alloys (MA) like Metglas®, Finemet®, Vitrovac®...
- resonantly driven with RF (ferrite loaded cavities) – or with pulses (induction cell).

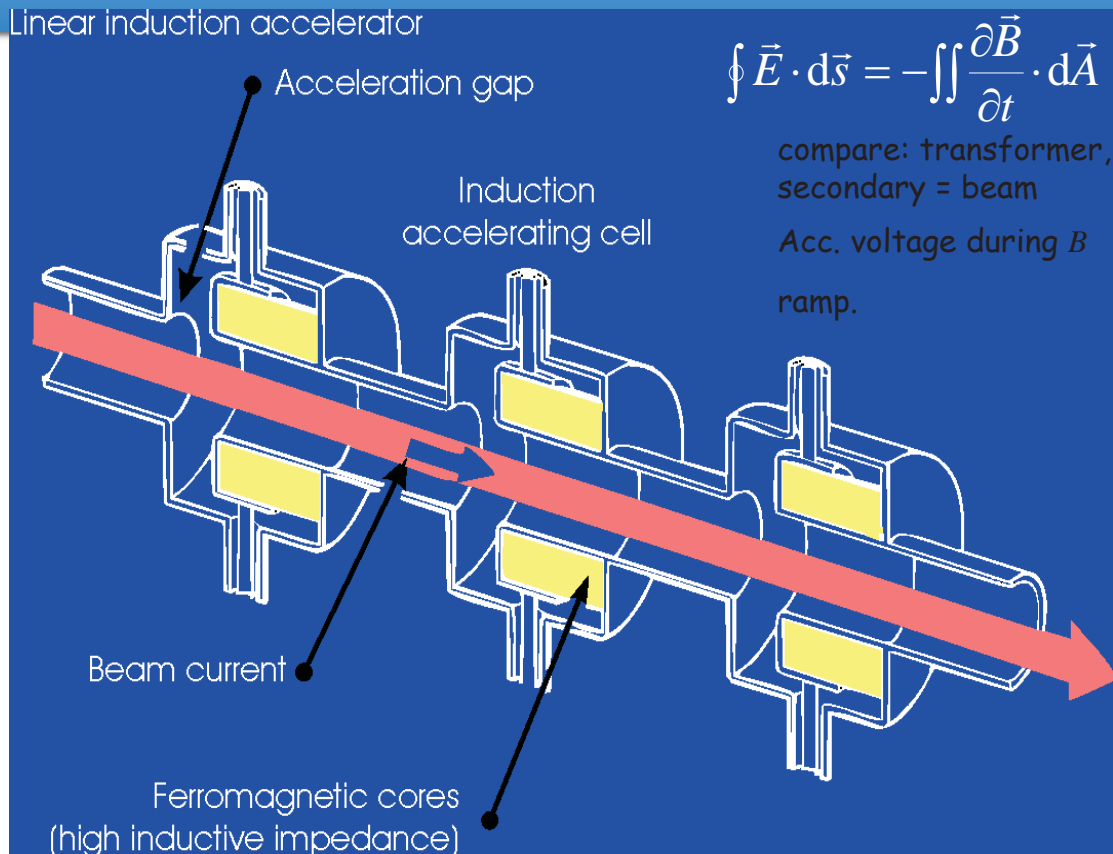
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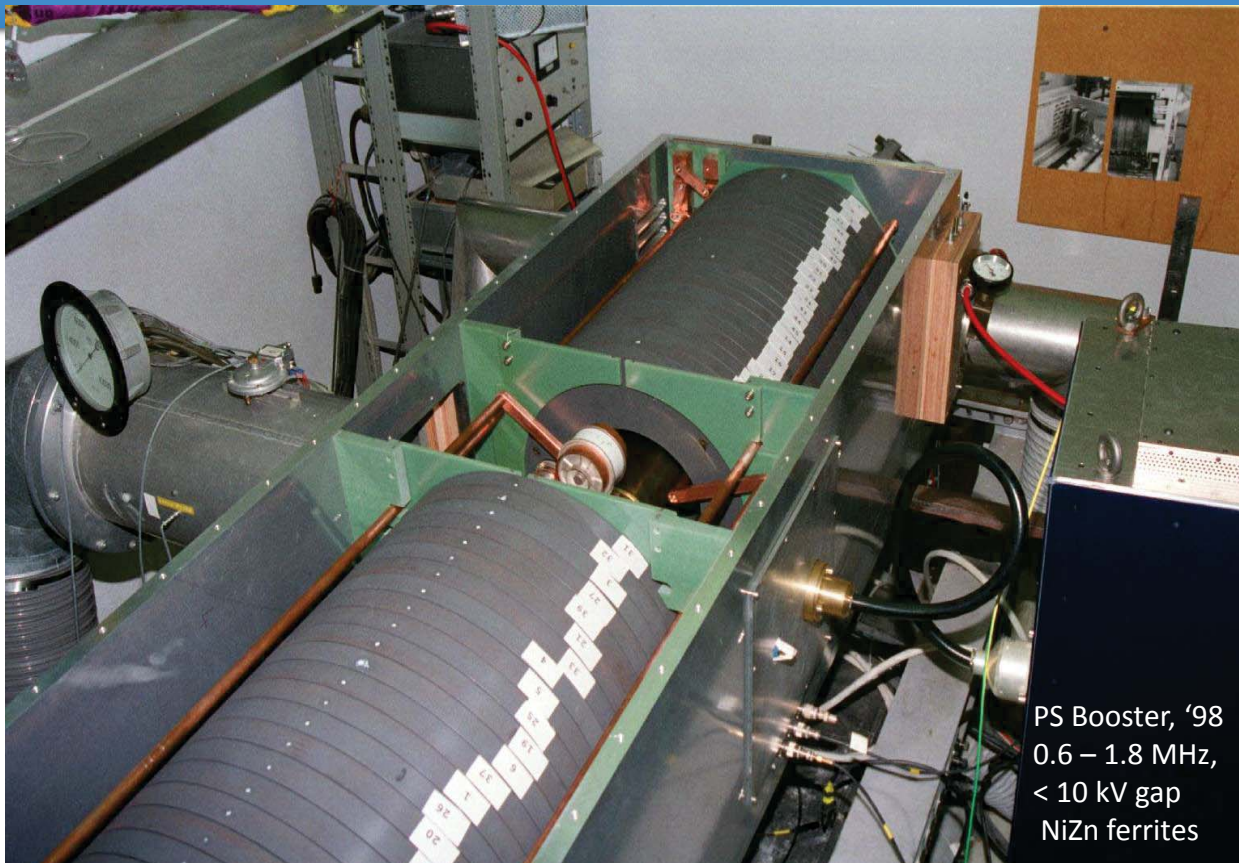
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# Linear induction accelerator

Linear induction accelerator



# Ferrite cavity



# Gap of PS cavity (prototype)



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# End of RF Systems I

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