## **Cyclotrons**

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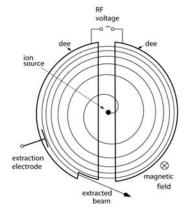
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## Cyclotrons - Outline

- the classical cyclotron
   history of the cyclotron, basic concepts and scalings, classification of cyclotron-like accelerators
- separated sector cyclotrons
   focusing in Thomas-cyclotrons, spiral angle, classical extraction: pattern/stepwidth, transv./long. space charge
- cyclotron subsystems
   extraction schemes, RF resonators, magnets, vacuum issues, instrumentation
- applications and examples of existing cyclotrons TRIUMF, RIKEN SRC, PSI Ring, PSI medical cyclotron
- discussion
   Classification of circular accelerators
  - Pro's and Con's of cyclotrons for different applications



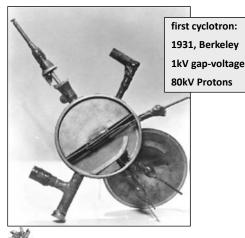
## The Classical Cyclotron

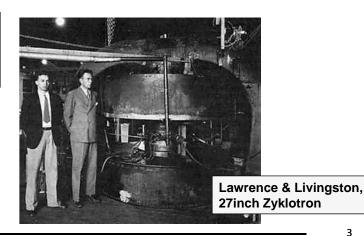


two capacitive electrodes "Dees", two gaps per turn internal ion source homogenous B field constant revolution time (for low energy,  $\gamma \sim 1$ )

#### powerful concept:

- → simplicity, compactness
- continuous injection/extraction
- → multiple usage of accelerating voltage





## cyclotron frequency and K value

• cyclotron frequency (homogeneous) B-field:

$$\omega_c = \frac{eB}{\gamma m_0}$$

• cyclotron K-value:

charge-to-mass ratios:

ightarrow K is the **kinetic energy reach** for protons **from bending strength** in non-relativistic approximation:  $K = \frac{e^2}{2m_0}(B\rho)^2$ 

 $\rightarrow$  K can be used to rescale the energy reach of protons to other

$$\frac{E_k}{A} = K\left(\frac{Z}{A}\right)^2$$

→ K in [MeV] is often used for naming cyclotrons examples: K-130 cyclotron / Jyväskylä cyclone C230 / IBA



## classical cyclotron - isochronicity and scalings

continuous acceleration  $\rightarrow$  revolution time must stay constant, though  $E_k$ , R vary

magnetic rigidity:

$$BR = \frac{p}{e} = \beta \gamma \frac{m_0 c}{e}$$

orbit radius from isochronicity:

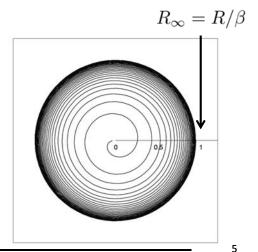
$$R = \frac{c}{\omega_c} \beta = R_{\infty} \beta$$
$$= \frac{c}{\omega_c} \sqrt{1 - \gamma^{-2}}$$

deduced scaling of B:

$$R \propto \beta; BR \propto \beta \gamma \longrightarrow B(R) \propto \gamma(R)$$

thus, to keep the isochronous condition, B must be raised in proportion to  $\gamma(R)$ ; this contradicts the focusing requirements (discussed later)

radius increment per turn decreases with increasing energy → extraction becomes more and more difficult at higher energies





### field index

the field index describes the (normalized) radial slope of the bending field:

$$k = \frac{R}{B} \frac{dB}{dR} \qquad \text{from}$$

$$= \frac{\beta}{\gamma} \frac{d\gamma}{d\beta}$$

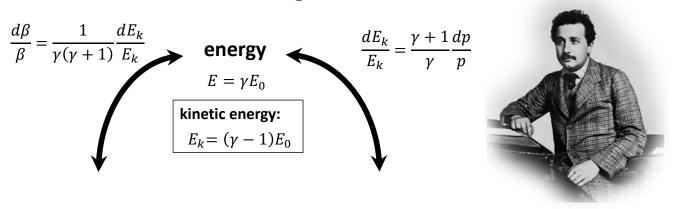
from isochronous condition:





## cyclotrons work at intermediate relativistic energies

A.Einstein 1879-1955



velocity

momentum

 $p = \beta \gamma m_0 c$ 

 $v = \beta c$ 



PSI cyclotron for protons

E <sub>k</sub> [MeV]	γ	β	p [MeV/c]
590	1.63	0.79	1207

revolution time:

$$\tau = \frac{2\pi R}{\beta c}$$

$$\frac{dp}{p} = \gamma^2 \frac{d\beta}{\beta}$$

olution time:  $\tau = \frac{2\pi R}{\beta c} \qquad \frac{dp}{p} = \gamma^2 \frac{d\beta}{\beta} \qquad \text{bending strength:} \\ BR = \beta \gamma \frac{m_0 c}{e}$ 

## equation of motion in a classical cyclotron

centrifugal force mv<sup>2</sup>/r

focusing: consider small deviations x from beam orbit R (r = R+x):

$$\ddot{x} + \frac{q}{m}vB_z(R+x) - \frac{v^2}{R+x} = 0,$$

$$\ddot{x} + \frac{q}{m}v\left(B_z(R) + \frac{\mathrm{d}B_z}{\mathrm{d}R}x\right) - \frac{v^2}{R}\left(1 - \frac{x}{R}\right) = 0,$$

$$\ddot{x} + \omega_c^2(1+k)x = 0.$$

using:  $\omega_c = qB_z/m \approx v/R$ ,  $r\dot{\varphi} \approx v$ ,  $k = \frac{R}{R} \frac{dB}{dR}$ 

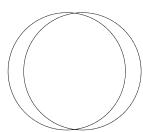


## betatron tunes in cyclotrons

thus in radial plane:

$$\begin{array}{rcl} \omega_r & = & \omega_c \sqrt{1+k} = \omega_c \nu_r \\ \nu_r & = & \sqrt{1+k} \\ & \approx & \gamma \end{array}$$

note: simple case for k = 0:  $v_r = 1$ (one circular orbit oscillates w.r.t the other)



using Maxwell to relate  $B_7$  and  $B_R$ :

$$rot \vec{B} = \frac{dB_R}{dz} - \frac{dB_z}{dR} = 0$$

in vertical plane:

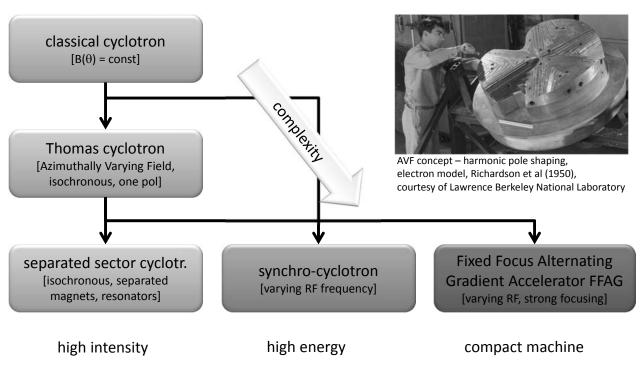
$$u_z = \sqrt{-k}$$
 k<0 to obtain

vertical focus.

thus: in classical cyclotron k < 0 required; however this violates isochronous condition  $k = \gamma^2 - 1 > 0$ 



## classification of cyclotron like accelerators



- next: sector cyclotrons
  - AVF vs. separated sector cyclotron
  - focusing in sector cyclotrons
  - extraction: pattern/stepwidth
  - transv./long. space charge

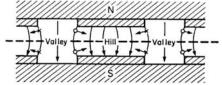


## focusing in sector cyclotrons

hill / valley variation of magnetic field (Thomas focusing) makes it possible to design cyclotrons for higher energies

Flutter factor: 
$$F^2 = \frac{\overline{B_z^2} - \overline{B_z}^2}{\overline{B_z}^2}$$

with flutter and additional spiral angle of bending field:



[illustration of focusing at edges]

 $\nu_z^2 = -\frac{R}{B_z} \frac{dB_z}{dR} + F^2 (1 + 2\tan^2 \delta)$ 

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## Azimuthally Varying Field vs. Separated Sector Cyclotrons



PSI Ring cyclotron

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- AVF = single pole with shaping
- often spiral poles used
- internal source possible
- D-type RF electrodes, rel. low energy gain
- compact, cost effective
- depicted Varian cyclotron: 80% extraction efficiency; not suited for high power
- modular layout, larger cyclotrons possible, sector magnets, box resonators, stronger focusing, injection/extraction in straight sections
- external injection required, i.e. preaccelerator
- box-resonators (high voltage gain)
- high extraction efficiency possible:

e.g. PSI:  $99.98\% = (1 - 2 \cdot 10^{-4})$ 



## derivation of turn separation in a cyclotron

starting point: bending strength for p

- → compute total log.differential
- $\rightarrow$  use field index  $k = R/B \cdot dB/dR$

$$BR = \sqrt{\gamma^2 - 1} \frac{m_0 c}{e}$$

$$\frac{dB}{B} + \frac{dR}{R} = \frac{\gamma d\gamma}{\gamma^2 - 1}$$

$$\frac{dR}{d\gamma} = \frac{\gamma R}{\gamma^2 - 1} \frac{1}{1 + k}$$

radius change per turn

$$\frac{dR}{dn_t} = \frac{dR}{d\gamma} \frac{d\gamma}{dn_t}$$
 [ $U_t$  = energy gain per turn] 
$$= \frac{U_t}{m_0 c^2} \frac{\gamma R}{(\gamma^2 - 1)(1 + k)}$$
 isochronicity not conserved (last turns)

 $= \frac{U_t}{m_0c^2}\frac{R}{(\gamma^2-1)\gamma} \hspace{1cm} \text{isochronicity conserved (general scaling)}$ 



## turn separation - discussion

for clean extraction a large stepwidth (turn separation) is of utmost importance; in the PSI Ring most efforts were directed towards maximizing the turn separation

general scaling at extraction:

$$\Delta R(R_{\rm extr}) = \frac{U_t}{m_0 c^2} \frac{R_{\rm extr}}{(\gamma^2-1)\gamma} \begin{bmatrix} \text{desirable:} \\ \bullet & \text{limited energy (< 16)} \\ \bullet & \text{large radius } R_{\rm extr} \\ \bullet & \text{high energy gain } U_t \end{bmatrix}$$

limited energy (< 1GeV)</li>

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scaling during acceleration:

$$\frac{dR}{dn_t} \approx \frac{U_t}{m_0 c^2} \frac{R}{\beta^2} \to \Delta R(R) \propto \frac{1}{R}$$

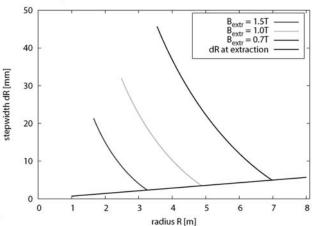
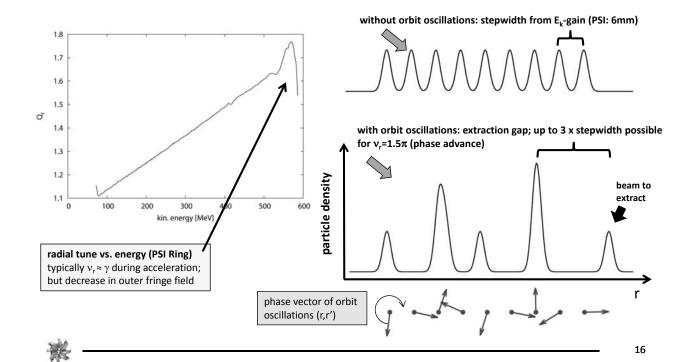


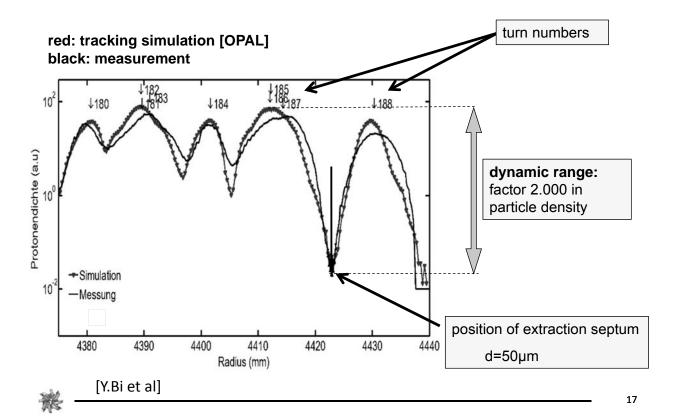
illustration: stepwidth vs. radius in cyclotrons of different sizes; 100MeV inj  $\rightarrow$  800MeV extr

### extraction with off-center orbits

betatron oscillations around the "closed orbit" can be used to increase the radial stepwidth by a factor 3!



## extraction profile measured at PSI Ring Cyclotron



## longitudinal space charge

#### sector model (W.Joho, 1981):

- → accumulated energy spread transforms into transverse tails
- consider rotating uniform sectors of charge (overlapping turns)
- · test particle "sees" only fraction of sector due to shielding of vacuum chamber with gap height 2w

two factors are proportional to the number of turns:

- 1) the charge density in the sector
- 2) the time span the force acts

$$\Delta U_{sc} = \frac{8}{3} e I_p Z_0 \ln \left( 4 \frac{w}{a} \right) \cdot \frac{n_{\max}^2}{\beta_{\max}} \approx 2.800\Omega \cdot e I_p \cdot \frac{n_{\max}^2}{\beta_{\max}}$$

derivation see: High Intensity Aspects of Cyclotrons, ECPM-2012, PSI

#### in addition:

- in addition: 3) the inverse of turn separation at extraction:  $\frac{1}{\Delta R_{\rm extr}}$   $\propto$   $n_{\rm max}$ 
  - $\blacktriangleright$  thus the attainable current at constant losses scales as  $n_{\text{max}}^{-3}$

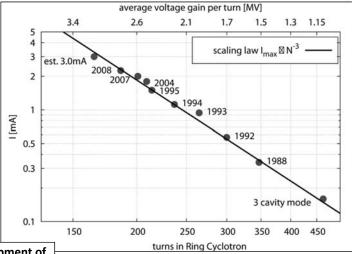


## longitudinal space charge; evidence for third power law

- at PSI the maximum attainable current indeed scales with the third power of the turn number
- maximum energy gain per turn is of utmost importance in this type of high

intensity cyclotron

ightarrow with constant losses at the extraction electrode the maximum attainable current indeed scales as:  $I_{
m max} \propto {n_t}^{-3}$ 



historical development of current and turn numbers in PSI Ring Cyclotron

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## transverse space charge

## with overlapping turns use current sheet model!

vertical force from space charge: 
$$F_y=\frac{n_v e^2}{\epsilon_0 \gamma^2} \cdot y, \ n_v=\frac{N}{(2\pi)^{\frac{3}{2}} \sigma_y D_f R \Delta R}$$
 [constant charge density, D<sub>f</sub> = I<sub>avg</sub>/I<sub>peak</sub>]

focusing force: 
$$F_y = -\gamma m_0 \omega_c^2 \nu_{y0}^2 \cdot y$$

thus, eqn. of motion: 
$$\ddot{y}+\left(\omega_c^2\nu_{y0}^2-\frac{n_ve^2}{\epsilon_0m_0\gamma^3}\right)y=0$$

## → equating space charge and focusing force delivers an **intensity limit for loss of focusing**!

tune shift from forces: 
$$\Delta\nu_y \quad \approx \quad -n_v \frac{2\pi r_p R^2}{\beta^2 \gamma^3 \nu_{y0}}$$
 
$$\approx \quad -\sqrt{2\pi} \frac{r_p R}{e\beta c \nu_{y0} \sigma_z} \frac{m_0 c^2}{U_t} \; I_{\rm avg}$$



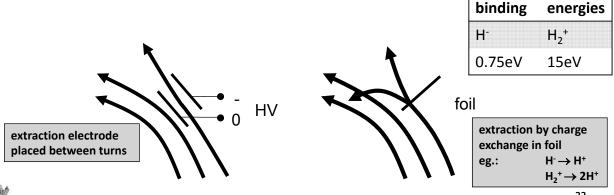
## next: cyclotron subsystems

- extraction schemes
- RF systems/power efficiency
- cyclotron magnets
- comments on vacuum
- specific instrumentation

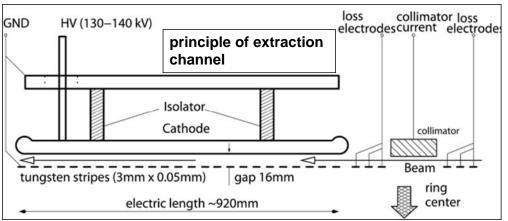


## injection/extraction schemes

- deflecting element should affect just one turn, not neighboured turn  $\rightarrow$ critical, cause of losses
- often used: electrostatic deflectors with thin electrodes
- alternative: charge exchange, stripping foil; accelerate H- or H<sub>2</sub>+ to extract protons (problem: significant probability for unwanted loss of electron; Lorentz dissociation: B-field low, scattering: vacuum 10<sup>-8</sup>mbar)



## injection/extraction with electrostatic elements



## parameters extraction chan.:

 $E_k = 590 MeV$ 

E = 8.8 MV/m

 $\theta$  = 8.2 mrad

 $\rho$  = 115 m

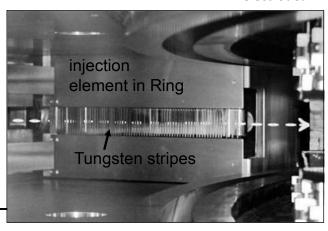
U = 144 kV

major loss mechanism is scattering in 50μm electrode!

#### electrostatic rigidity:

$$E\rho = \frac{\gamma + 1}{\gamma} \frac{E_k}{q}$$

$$\theta = \frac{qlE}{E_k} \frac{\gamma}{\gamma + 1}$$



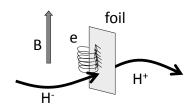
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#### extraction foil

- thin foil, for example carbon, removes the electron(s) with high probability
- new charge state of ion brings it on a new trajectory → separation from circulating beam
- lifetime of foil is critical due to heating, rad.damage; conversion efficiencies, e.g. generation of neutrals, must be considered carefully

stripped electrons deposit energy in the foil



How much power is carried by the electrons?

$$E_e = \frac{m_e}{m_p} E_p = 5.4 \cdot 10^{-4} E_k^p$$

 $\rightarrow$  1/2000 of beam power

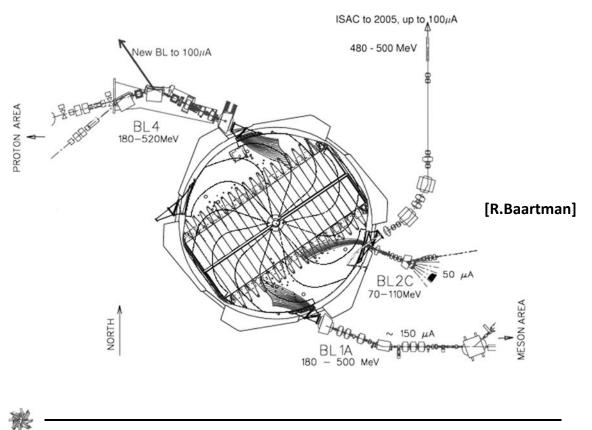
Bending radius of electrons?

$$\rho^e = \frac{m_e}{m_p} \rho^p$$

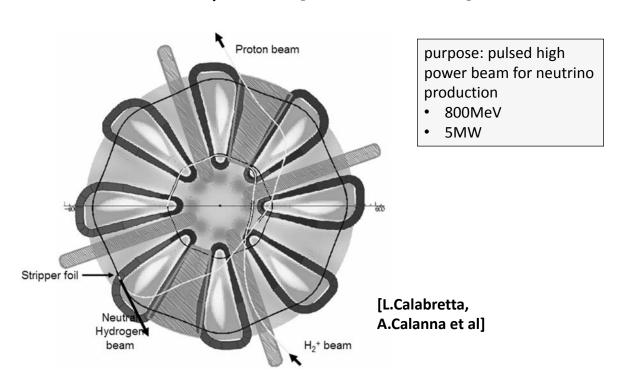
 $\rightarrow$  typically mm



## example: multiple H- stripping extraction at TRIUMF



# example: H<sub>2</sub><sup>+</sup> stripping extraction in planned Daedalus cyclotron [neutrino source]



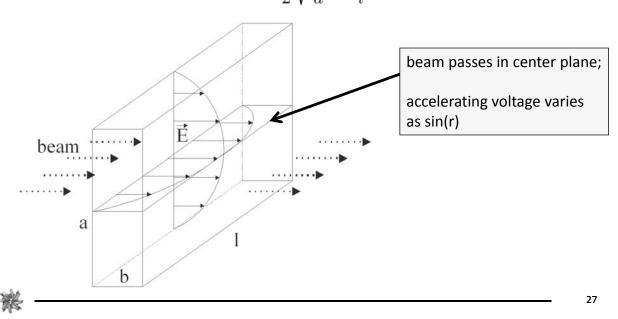


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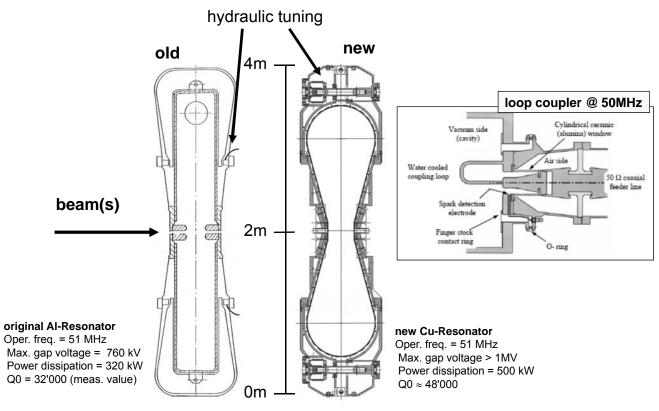
## components: cyclotron resonators

cyclotron resonators are basically box resonators



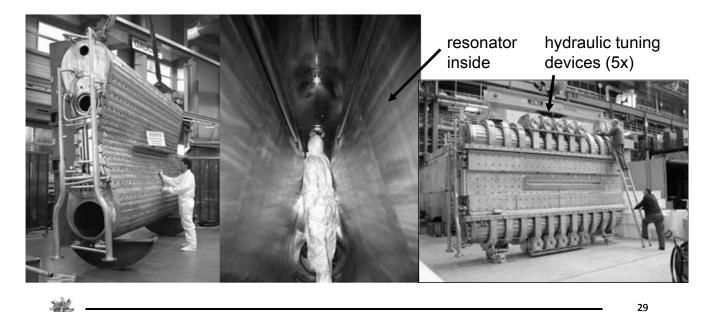


## cross sections of PSI resonators



## copper resonator in operation at PSI's Ring cyclotron

- f = 50.6MHz;  $Q_0 = 4.8 \cdot 10^4$ ;  $U_{max} = 1.2MV$  (presently 0.85MV)
- transfer of up to 400kW power to the beam per cavity
- Wall Plug to Beam Efficiency (RF Systems): 32%



## components: sector magnets

 cyclotron magnets typically cover a wide radial range → magnets are heavy and bulky, thus costly

#### **PSI** sector magnet

iron weight: 250 tons coil weight: 28 tons orbit radius: 2.1...4.5 m spiral angle: 35 deg

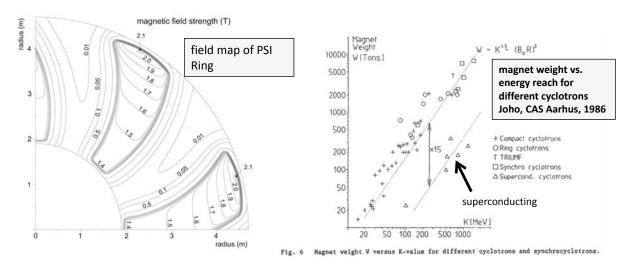






### components: sector magnets

- focusing and isochronicity need to be precisely controlled →
  sophisticated pole shaping including spiral bounds, many trim coil circuits
- modern cyclotrons use superconducting magnets; but for high intensity compactness is generally disadvantageous



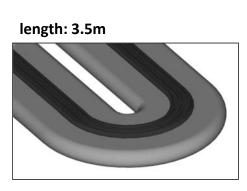


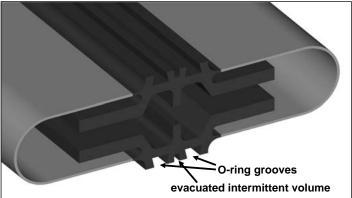
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## cyclotron vacuum system

- vacuum chamber with large radial width → difficult to achieve precisely matching sealing surfaces → noticeable leak rates must be accepted
- · use cryo pumps with high pumping speed and capacity
- $\approx 10^{-6}$ mbar for p,  $\approx 10^{-8}$ mbar for ions (instability! e.g. AGOR at KVI)
- · design criterion is easy access and fast mountability (activation)

#### example: inflatable seals installed between resonators; length: 3.5m

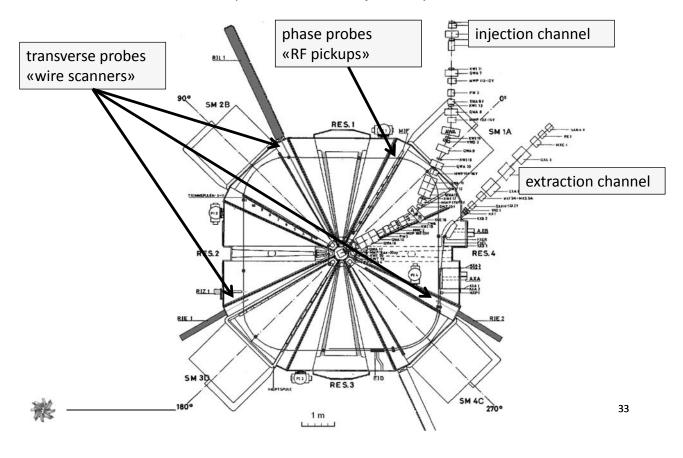




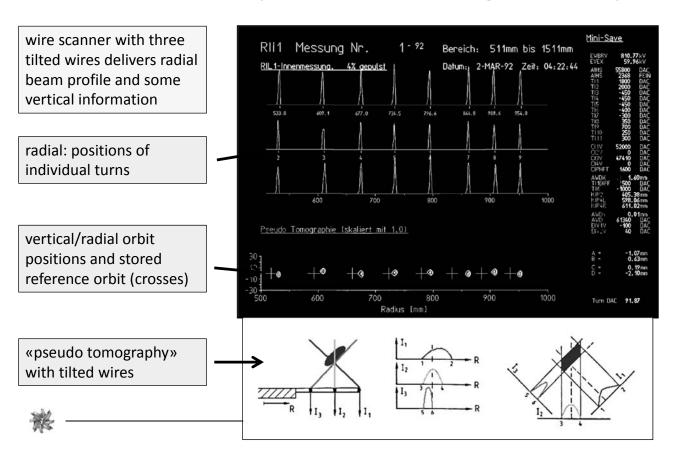


## cyclotron instrumentation

example: PSI 72MeV injector cyclotron



## instrumentation: radial probe for turn counting / orbit analysis



## next: cyclotron examples

• TRIUMF, RIKEN SRC, PSI-HIPA, PSI-Comet

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## comparison of cyclotrons

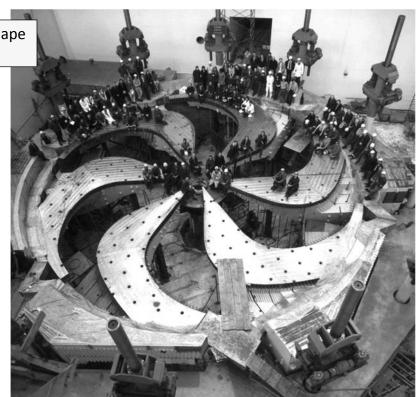
	TRIUMF	RIKEN SRC (supercond.)	PSI Ring	PSI medical (supercond.)
particles	H- → p	ions	р	р
K [MeV]	520	2600	592	250
magnets (poles)	(6)	6	8	(4)
peak field strength [T]	0.6	3.8	2.1	3.8
R <sub>inj</sub> /R <sub>extr</sub> [m]	0.25/3.87.9	3.6/5.4	2.4/4.5	-/0.8
P <sub>max</sub> [kW]	110	1 (86Kr)	1300	0.25
extraction efficiency (tot. transmission)	0.9995 (0.70)	(0.63)	0.9998	0.80
extraction method	stripping foil	electrostatic deflector	electrostatic deflector	electrostatic deflector
comment	variable energy	ions, flexible	high intensity	compact



## cyclotron examples: TRIUMF / Vancouver

photo: iron poles with spiral shape ( $\delta_{\text{max}}\text{=}70\text{deg})$ 

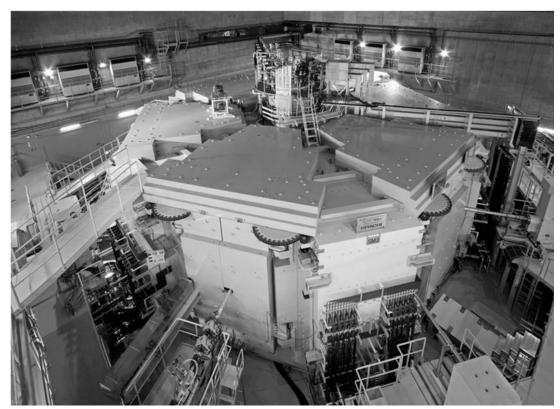
- p, 520MeV, up to 110kW beam power
- diameter: 18m (largest n.c. cyclotron worldwide)
- extraction by stripping H⁻
   → variable energy;
   multiple extraction points
   possible





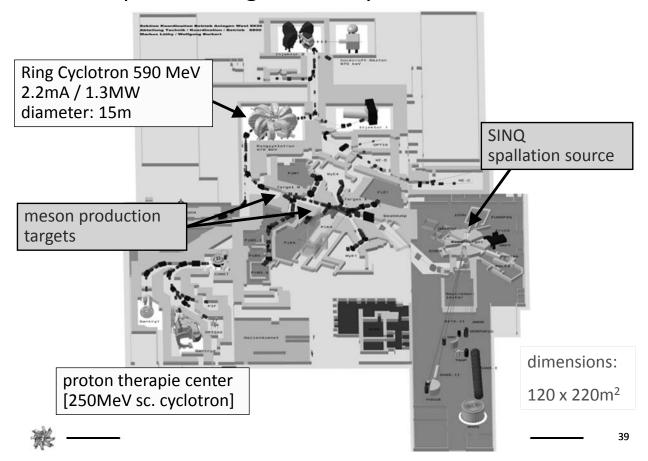
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## RIKEN SRC in the vault

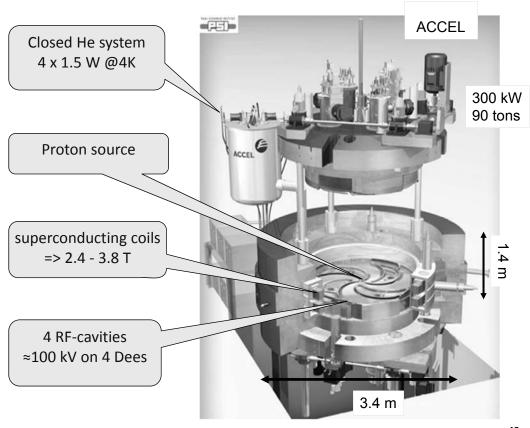




## examples: PSI High Intensity Proton Accelerator

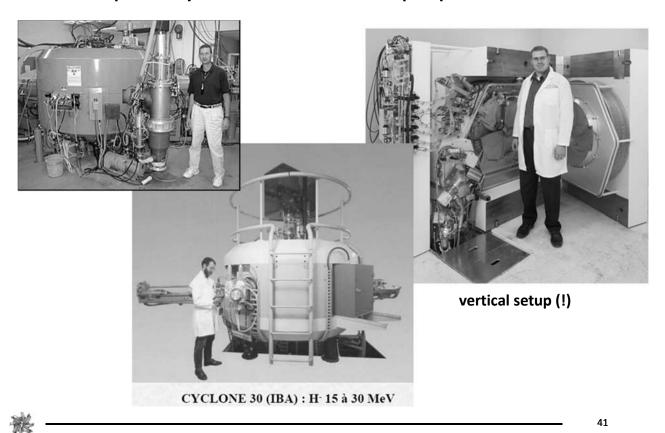


## 250 MeV proton cyclotron (ACCEL/Varian)





## compact cyclotrons for Isotope production





## classification of circular accelerators

	bending radius	bending field vs. time	bending field vs. radius	RF frequency vs. time	operation mode (pulsed/CW)	
betatron	<b>→</b>	<i>&gt;&gt;</i>	7		ш	induction
microtron	<i>&gt;&gt;</i>	<b>→</b>	<b>→</b>	<b>→</b>		varying <i>h</i>
classical cyclotron	<i>&gt;&gt;</i>	<b>→</b>	<b>→</b>	<b>→</b>		simple, but limited E <sub>k</sub>
isochronous cyclotron	<i>&gt;&gt;</i>	<b>→</b>	<i>&gt;&gt;</i>	<b>→</b>		suited for high power!
synchro- cyclotron	<i>&gt;&gt;</i>	<b>→</b>	<u></u>	1	ш_	higher E <sub>k</sub> , but low P
FFAG	>	<b>→</b>	<i>&gt;&gt;</i>	<i>&gt;</i>	ш_	strong focusing!
a.g. synchrotron	<b>→</b>	<i>&gt;&gt;</i>		<del>}</del>	ш	high E <sub>k</sub> , strong focus



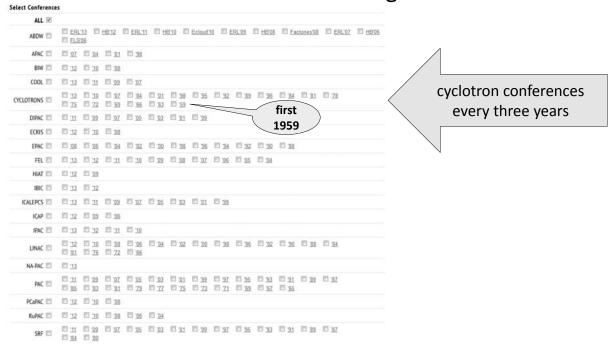
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# pro and contra cyclotron

limitations of cyclotrons	typical utilization of cyclotrons
<ul> <li>energy limitation ≈1GeV due to relativistic effects</li> <li>relatively weak focusing is critical for space charge effects (10mA?)</li> <li>tuning is difficult; field shape; many turns; limited diagnostics</li> <li>wide vacuum vessel (radius variation)</li> </ul>	<ul> <li>medical applications ≤250MeV; intensity range well covered</li> <li>isotope production → several 10MeV</li> <li>acceleration of heavy ions (e.g. RIKEN)</li> <li>very high intensity proton beams (PSI:1.4MW, TRIUMF: 100kW, ADS Concepts )</li> </ul>



# cyclotron conferences – a valuable source of knowledge



old cyclotron conferences are digitized for JACOW (effort of M.Craddock!) cyclotrons 2016: organized by PSI in Zürich



## some literature w.r.t. cyclotrons

comprehensive overview on cyclotrons	L.M.Onishchenko, Cyclotrons: A Survey, Physics of Particles and Nuclei 39, 950 (2008) <a href="http://www.springerlink.com/content/k61mg262vng17411/fulltext.pdf">http://www.springerlink.com/content/k61mg262vng17411/fulltext.pdf</a>	
scaling of PSI concept to 10MW	Th.Stammbach et al, The feasibility of high power cyclotrons, Nuclear Instruments and Methods in Physics Research B 113 (1996) 1-7	
space charge effects and scalings	W.Joho, High Intensity Problems in Cyclotrons, Proc. 5th intl. Conf. on Cyclotrons and their Applications, Caen, 337-347 (1981) <a href="http://accelconf.web.cern.ch/AccelConf/c81/papers/ei-03.pdf">http://accelconf.web.cern.ch/AccelConf/c81/papers/ei-03.pdf</a>	
long. space charge; comparison to analytical result	E.Pozdeyev, A fast code for simulation of the longitudinal space charge effect in isochronous cyclotrons, cyclotrons (2001) <a href="http://accelconf.web.cern.ch/AccelConf/c01/cyc2001/paper/P4-11.pdf">http://accelconf.web.cern.ch/AccelConf/c01/cyc2001/paper/P4-11.pdf</a>	
H <sub>2</sub> <sup>+</sup> concept for high power	L.Calabretta et al, A multi megawatt cyclotron complex to search for cp violation in the neutrino sector, cyclotrons (2010); upcoming NIM paper! <a href="http://accelconf.web.cern.ch/AccelConf/Cyclotrons2010/papers/tua1cio01.pdf">http://accelconf.web.cern.ch/AccelConf/Cyclotrons2010/papers/tua1cio01.pdf</a>	
OPAL simulations; documentation	J.Yang, A. Adelmann, et al. Phys. Rev. STAB Vol. 13 Issue 6 (2010) http://amas.web.psi.ch	
cyclotrons 2013 conference Vancouver	http://accelconf.web.cern.ch/AccelConf/CYCLOTRONS2013/ conference summary: http://accelconf.web.cern.ch/AccelConf/CYCLOTRONS2013/talks/fr2pb03_talk.pdf	



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