Strings in Compact Cosmological Spaces

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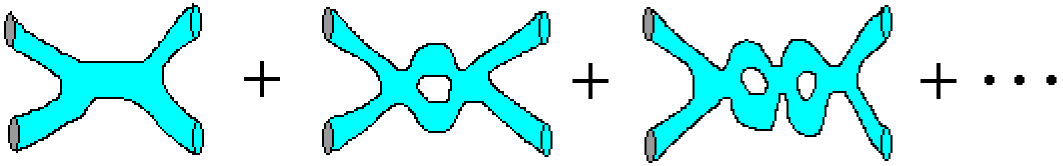
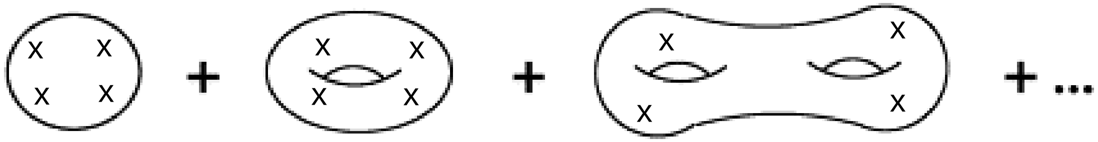
Perturbative string theory

Computes S-matrix using conformal field theory.



Asymptotic string states → vertex operators.



Formalism requires asymptotic regions.

http://web.physics.ucsb.edu/~strings/superstrings/basics.htm; http://dx.doi.org/10.1016/S0370-1573(01)00013-8

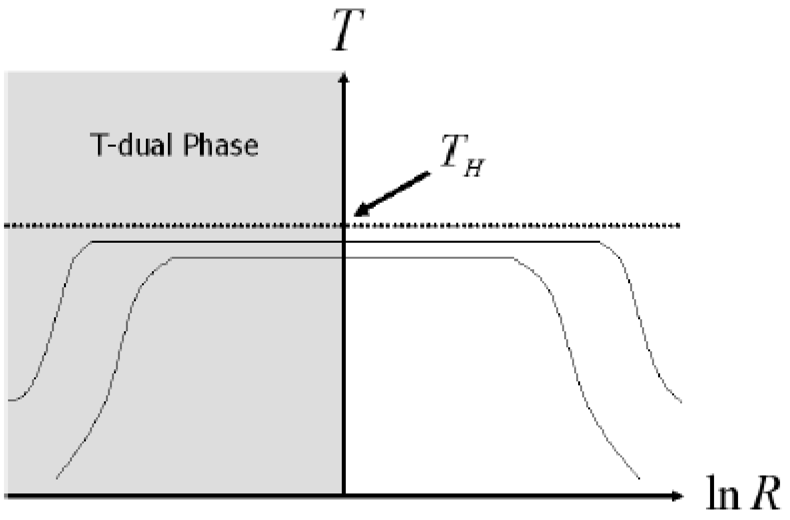
Strings in compact space?

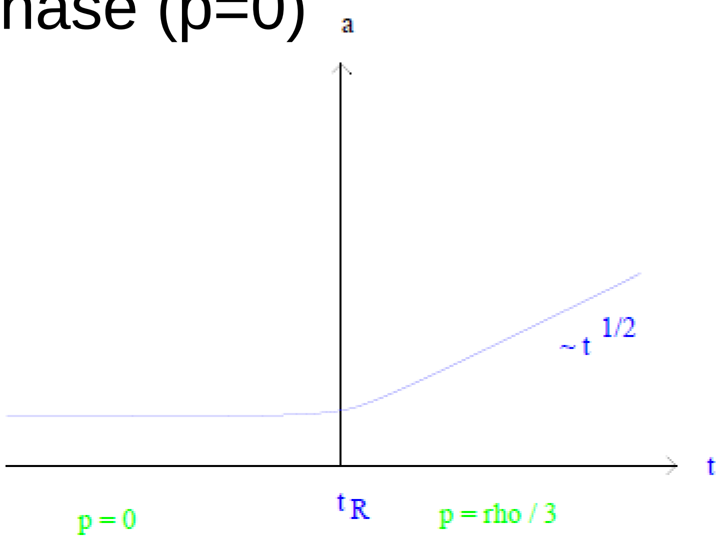
- Consider strings in compact space, for concreteness torus.
- No asymptotic regions → no S-matrix.
- Can we compute how strings interact in such a space?
- Infrared divergences invalidate conventional string perturbation theory.

Goal: propose natural observables and argue that IR divergences are generically absent (and otherwise well-understood).

String gas cosmology

- Compact space, e.g. torus.
- Gas of strings with winding modes (p<0) and momentum modes (p>0).
- Initially in quasi-static Hagedorn phase (p=0)





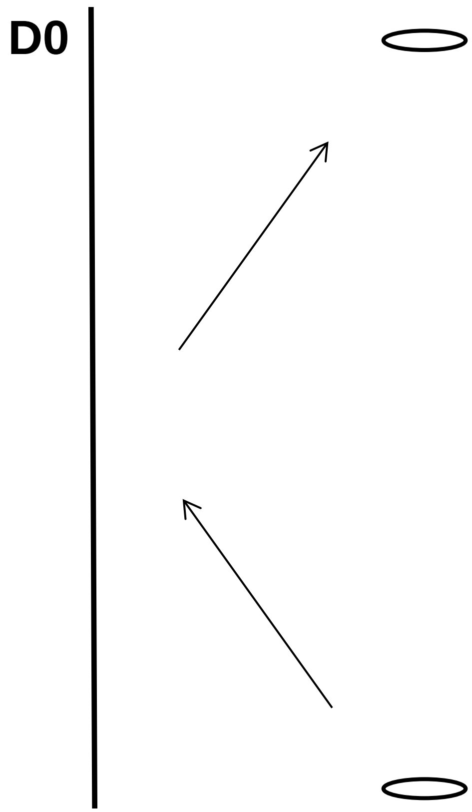
- Winding mode annihilation → radiation domination
- Problem: equations governing Hagedorn phase unknown!

Figures from R. Brandenberger, 1105.3247

Analogy: D0-brane recoil

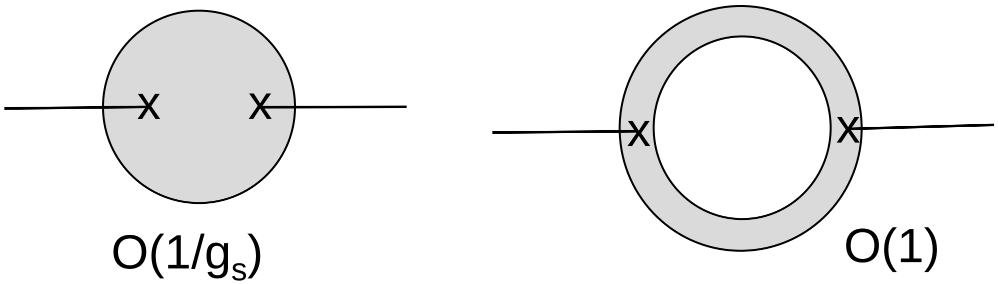
- D0-brane: massive, localized object in string theory.
- Standard CFT treatment: D0-brane introduced as locus where open strings can end, e.g. $\vec{X} = 0$.
- String loop corrections → D0 is dynamical object with mass $1/g_s \ell_s$. (Consider $g_s \ll 1$.)

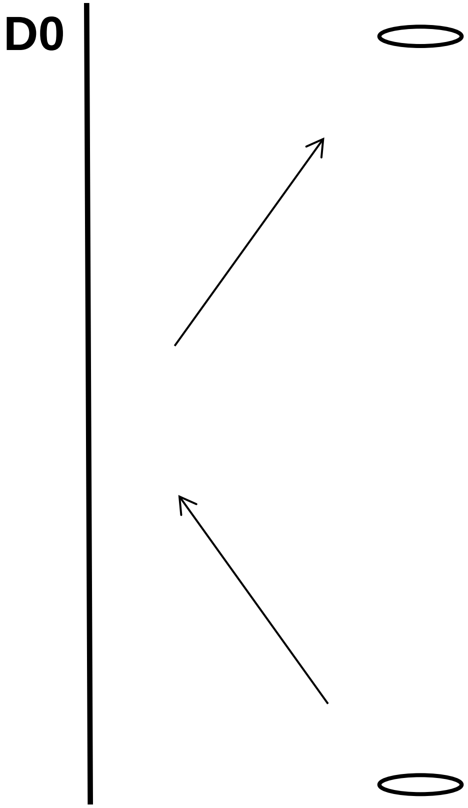
  [Polchinski 1995]
- What happens when strings (with string scale momenta) are scattered off D0?



Analogy: D0-brane recoil

- Closed strings (with string scale momenta) scattering off D0-brane
- Tree-level (disk): well-behaved. Static D0 (with mass $1/g_s$ in string units) can absorb arbitrary string scale momentum.
- One-loop (annulus): divergence!



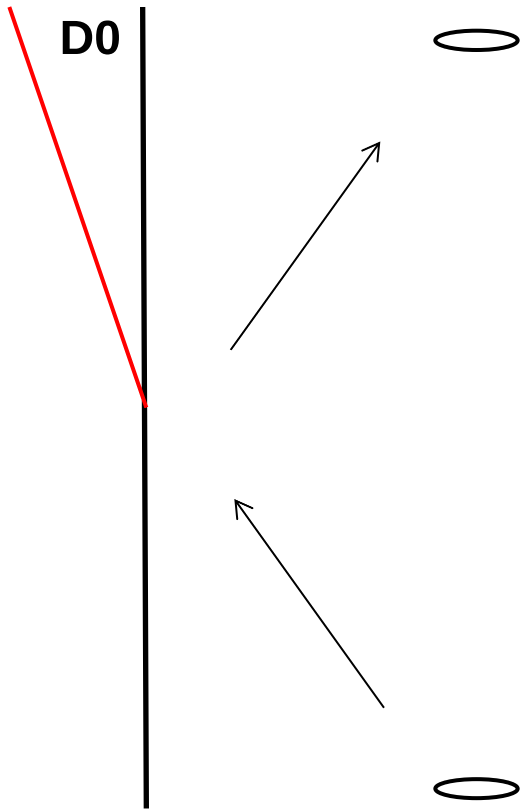


D0-brane recoil: intuition

- IR divergences signal that one is asking the wrong question. For instance, one may be expanding around the wrong background.

- Indeed, for late times, worldline of recoiling D0 deviates strongly from that of a static D0.

- Problem: how to implement this in string theory?



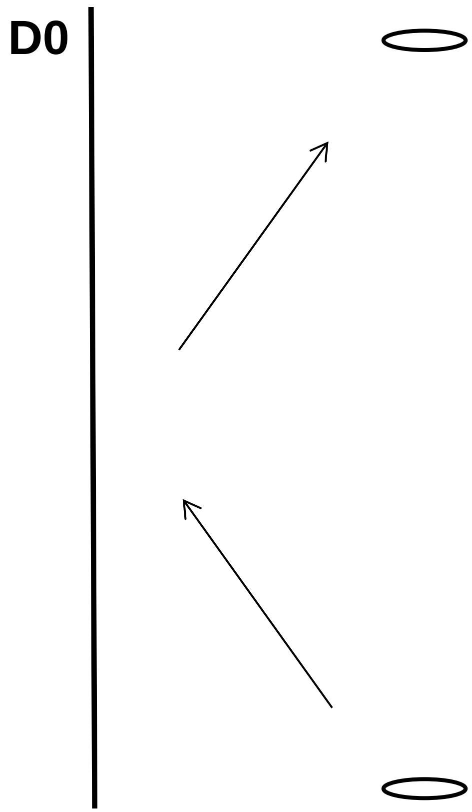
D0-brane recoil: overview

- One-loop (annulus) divergence from plumbing fixture construction.

- How to cure the divergence? Classical modification of D0-brane trajectory is unsatisfactory.

- Worldline formalism: explicitly quantize D0-brane trajectories.

- Resulting disk divergence cancels annulus divergence.

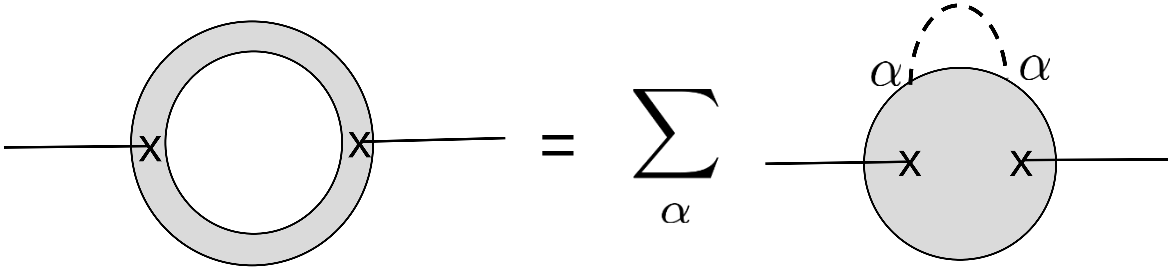


D0 recoil: plumbing fixture formula

$$\left\langle V^{(1)} \dots V^{(n)} \right\rangle_{annulus} = \sum_\alpha \int \frac{dq}{q} q^{h_\alpha - 1} \int d\theta d\theta' \left\langle V_\alpha(\theta) V_\alpha(\theta') V^{(1)} \dots V^{(n)} \right\rangle_{disk}$$

$V_\alpha(\theta)$: complete set of local operators

q : gluing parameter $\longleftrightarrow$ annular modulus



Focus on $q \approx 0$ region: annulus develops long, thin strip. Dominated by smallest h_α (but ignore tachyon):

$$V^i(\theta) = \partial_n X^i(\theta) \exp\left[i\omega X^0(\theta)\right], \quad h = 1 + \alpha' \omega^2$$

D0 recoil: annulus divergence

$$\left\langle V^{(1)} \cdots V^{(n)} \right\rangle_{annulus} = \sum_\alpha \int \frac{dq}{q}\, q^{h_\alpha - 1} \int d\theta d\theta' \left\langle V_\alpha(\theta) V_\alpha(\theta') V^{(1)} \cdots V^{(n)} \right\rangle_{disk}$$

$$V^i(\theta) = \partial_n X^i(\theta) \exp\left[i\omega X^0(\theta)\right], \quad h = 1 + \alpha' \omega^2$$

$$\left\langle V^{(1)} \cdots V^{(n)} \right\rangle_{annulus}^{(div)} \sim \int_\epsilon^1 dq \int_{-\infty}^{\infty} d\omega\, q^{-1+\alpha'\omega^2} \int d\theta d\theta' \left\langle V^i(\theta,\omega) V^i(\theta',\omega) V^{(1)} \cdots V^{(n)} \right\rangle_{disk}$$

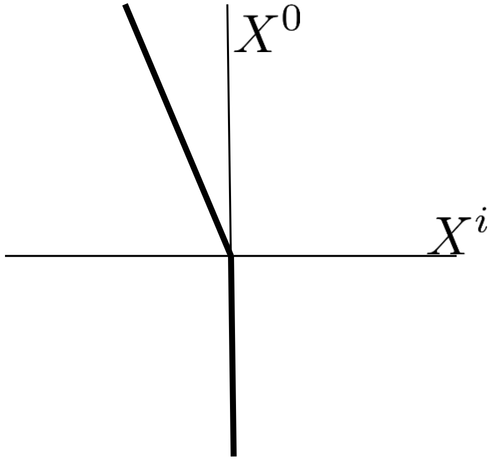
$$\sim P^2 \left\langle V^{(1)} \cdots V^{(n)} \right\rangle_{disk} \int_\epsilon^1 dq \int_{-\infty}^{\infty} d\omega\, q^{-1+\alpha'\omega^2} \sim P^2 \left\langle V^{(1)} \cdots V^{(n)} \right\rangle_{disk} \int_\epsilon^1 \frac{dq}{q\,(-\log q)^{1/2}}$$

$$\sim P^2 \left\langle V^{(1)} \cdots V^{(n)} \right\rangle_{disk} \sqrt{|\log \epsilon|}$$

D0 recoil: modified D0 trajectory?

Deform CFT with local operator $V_{PT} \sim \int d\theta \, \partial_n X^i(\theta) \, \frac{P^i}{M_{D0}} \, X^0(\theta) \, \Theta\left(X^0(\theta)\right)$

Disk with $\underset{\substack{\uparrow \\ O(g_s)}}{V_{PT}}$ inserted cancels annulus divergence.



[Periwal, Tafjord 1996]

Unsatisfactory:

- Why abruptly start moving at time $X^0 = 0$? (The closed strings are in momentum eigenstates.)
- Finite part of amplitude depends on that time.

[BC, Evnin, Nakamura 2006]

D0 recoil: worldline formalism

Dynamical worldline for D0:

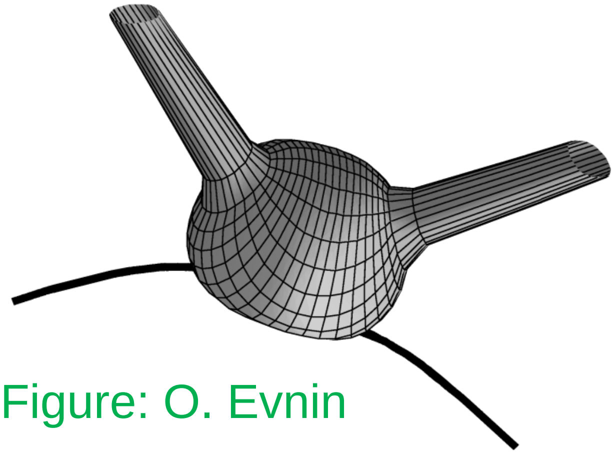


Figure: O. Evnin

$$\begin{array}{ccc} \text{D0 worldline (proper time } t) & \xrightarrow{f^\mu(t)} & \text{spacetime} \\ \uparrow t(\theta) & & \uparrow X(\sigma) \\ \text{open string boundary } (\theta) & \subset & \text{open string world-sheet } (\sigma_0, \sigma_1) \end{array}$$

Path integral over $f^\mu(t),\ t(\theta),\ X(\sigma)$ with constraint $X^\mu(\theta) = f^\mu(t(\theta))$.

Amplitude for D0 to move from x_1 to x_2 while absorbing/emitting closed strings:

$$G(x_1, x_2 | k_n) = \sum (g_s)^\chi \int \mathcal{D}f\, \mathcal{D}t\, \mathcal{D}X\, \delta\left(X^\mu(\theta) - f^\mu(t(\theta))\right) e^{-S_D(f) - S_{st}(X)} \prod \left\{ g_s V^{(n)}(k_n) \right\}$$

[Hirano, Kazama 1996]

D0 recoil: worldline formalism

$$G(x_1, x_2|\, k_n) = \sum (g_s)^\chi \int \mathcal{D}f\, \mathcal{D}t\, \mathcal{D}X\, \delta\left(X^\mu(\theta) - f^\mu(t(\theta))\right) e^{-S_D(f) - S_{st}(X)} \prod \left\{ g_s V^{(n)}(k_n) \right\}$$

- sum over topologies (not necessarily connected, but without disconnected vacuum parts)
- χ : Euler number
- $V^{(n)}$: integrated vertex operators
- Euclidean spacetime
- integration over moduli suppressed (work with disk)
- conformal Killing volume suppressed

Reduction formula → scattering amplitude:

$$\langle p_1 | p_2 \rangle_{k_n} = \lim_{p_1^2, p_2^2 \to -M^2} \left(p_1^2 + M^2\right) \left(p_2^2 + M^2\right) \int dx_1 dx_2 e^{ip_1 x_1} e^{ip_2 x_2} G(x_1, x_2 |\, k_1, \cdots, k_m)$$

D0 recoil: worldline formalism

- Techniques to compute path integral perturbatively were developed in [Evnin 2005] and applied to the recoil problem in [BC, Evnin, Nakamura 2006].
- Result: disk divergence automatically cancels annulus divergence, including numerical factors (given natural identification of cutoffs).
- Equivalent with deforming D0 CFT by a bilocal recoil operator

$$V_{bilocal} = \frac{1}{2M} \int d\theta\, d\theta'\, G_{kick}\left(X^0(\theta) - X^0(\theta')\right) : \partial_n X^i(\theta) \partial_{n'} X^i(\theta') :$$

with $G_{kick}(t) = \frac{|t|}{2}$.

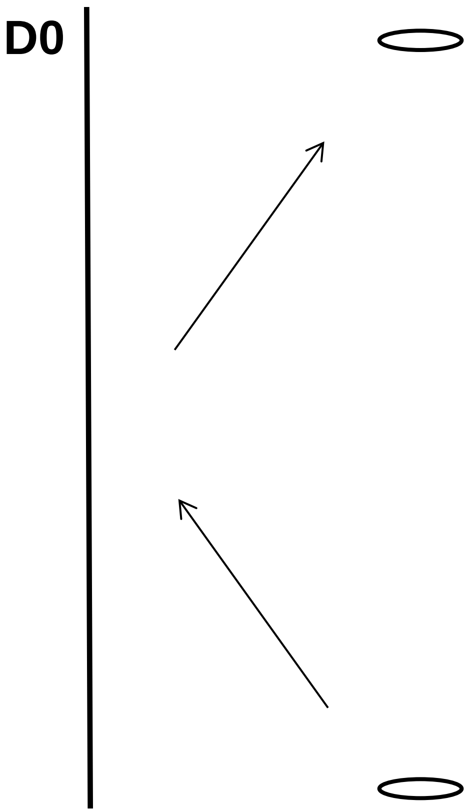
- Intuition: $\partial_{n'} X^i(\theta')$ is momentum density

→ displacement $\int d\theta'\, G_{kick}\left(X^0(\theta) - X^0(\theta')\right)\, \partial_{n'} X^i(\theta')$

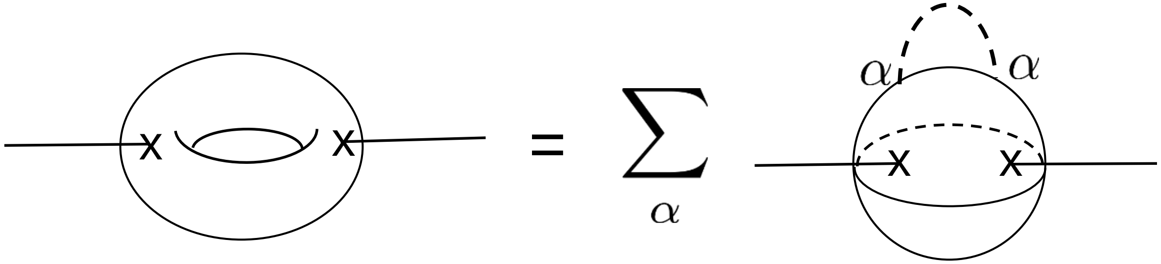
$\partial_n X^i(\theta)$ actually displaces D0 worldline.

D0-brane recoil: summary

- One-loop (annulus) divergence from plumbing fixture construction.
- A classical modification of the D0-brane trajectory is unsatisfactory.
- Worldline formalism: explicitly quantize D0-brane trajectories.
- Resulting disk divergence cancels annulus divergence.



Closed strings in compact space: divergence on torus worldsheet



$$\int d\omega \int_{\epsilon<|q|<1} \frac{d^2q}{q\bar{q}} (q\bar{q})^{\alpha'\omega^2/4} \sim \int_\epsilon^1 \frac{d\kappa}{\kappa(-\log\kappa)^{1/2}} \sim \sqrt{|\log\epsilon|}$$

$$(\kappa = (q\bar{q})^{1/2})$$

Quantizing the zero modes

Explicitly quantize the zero modes:

$$ds^2 = -G_{00}(t)dt^2 + 2G_{0i}(t)dtdx^i + G_{ij}(t)dx^i dx^j, \qquad x^i \in [0, 2\pi)$$

$$B_{\mu\nu} = B_{\mu\nu}(t), \qquad \Phi = \Phi(t)$$

Gauge choice: G_{00} constant, $G_{0i} = 0$.

These are the degrees of freedom of minisuperspace models, which are often considered in quantum cosmology.

Minisuperspace models

- Time reparametrization invariance → Hamiltonian constraint.
- Define amplitude for universe to have spatial sections $G^{(1)}$ and $G^{(2)}$. Integrate over the duration T of the transition:

$$\langle G^{(2)} | G^{(1)} \rangle = \int_{-\infty}^{\infty} dT \int_{\substack{G_{ij}(0)=G_{ij}^{(1)} \\ G_{ij}(T)=G_{ij}^{(2)}}} \mathcal{D}G_{ij}(t)\, e^{-S_{\mathrm{ms}}}$$

- We will quantize the string zero modes in a similar way. But how to turn this into a string amplitude?

Strings in a compact cosmological space (simplified)

$$\langle G^{(2)}|G^{(1)}\rangle_V = \int_{-\infty}^{\infty} dT \int_{\substack{G_{ij}(0)=G_{ij}^{(1)} \\ G_{ij}(T)=G_{ij}^{(2)}}} \mathcal{D}G_{ij}(t)\, e^{-S_{\rm ms}} \int \mathcal{D}X e^{-S_X} \prod \{g_s V^{(n)}\}$$

$$S_X = \frac{1}{4\pi\alpha'} \int d^2\sigma g^{1/2} \left[\left(g^{ab} G_{\mu\nu}(X) + i\epsilon^{ab} B_{\mu\nu}(X)\right) \partial_a X^\mu \partial_b X^\nu + \alpha' R^{(2)} \Phi(X) \right]$$

S_{ms} : minisuperspace action (dilaton-gravity restricted to homogeneous configurations, plus higher derivative corrections).

Will focus on choices for $G^{(1)}$ and $G^{(2)}$ for which integrals over metrics and over T can be treated semiclassically, and for which the saddle point metric is weakly curved and the saddle point duration large (compared to string scale).

Strings in a compact cosmological space (simplified)

$$\langle G^{(2)}|G^{(1)}\rangle_V = \int_{-\infty}^{\infty} dT \int_{\substack{G_{ij}(0)=G^{(1)}_{ij}\\ G_{ij}(T)=G^{(2)}_{ij}}} \mathcal{D}G_{ij}(t)\, e^{-S_{\rm ms}} \int \mathcal{D}X e^{-S_X} \prod\{g_s V^{(n)}\}$$

$$G_{ij}(t) = \bar{G}_{ij}(t) + \gamma_{ij}(t), \qquad \left.\frac{\delta S_{\rm ms}}{\delta G_{ij}}\right|_{G=\bar{G}} = 0, \qquad \bar{G}(0) = G^{(1)}, \qquad \bar{G}(T) = G^{(2)}$$

Work perturbatively in small fluctuations $\gamma_{ij}(t)$ and in fluctuations of T around saddle point value $\bar{T}$ (with $\bar{G}_{ij}$ weakly curved and $\bar{T}$ large, e.g. compared to the string scale).

External string states are allowed to have stringy energies and momenta, so we do consider amplitudes beyond those of low-energy field theory.

Torus amplitude (simplified)

Plumbing fixture:

$$\langle G^{(2)}|G^{(1)}\rangle_V^{(\mathrm{torus})} \sim \frac{e^{-\bar{S}_{\mathrm{ms}}}}{\det^{1/2}[\delta_G^2 S_{\mathrm{msGauss}}]} \int \mathcal{D}X e^{-\bar{S}_X} \sum_{ab} \int \frac{dq\, d\bar{q}}{q\bar{q}} q^{h_a-1} \bar{q}^{\tilde{h}_a-1} \mathcal{G}^{ab} V_a V_b \prod \{g_s V^{(n)}\}$$

Contribution from small q (cutoff at $|q| = \epsilon$):

$$\langle G^{(2)}|G^{(1)}\rangle_V^{(\mathrm{torus;div})} \sim \frac{e^{-\bar{S}_{\mathrm{ms}}}}{\det^{1/2}[\delta_G^2 S_{\mathrm{msGauss}}]} \int \mathcal{D}X e^{-\bar{S}_X} \sum_{ab} \frac{\varepsilon^{2(h_a-1)} - 1}{h_a - 1} \mathcal{G}^{ab} V_a V_b \prod \{g_s V^{(n)}\}$$

Finite for $G^{(1)}$ and $G^{(2)}$ such that $\bar{T}$ is finite (even without cutoff)!

We introduce cutoff anyway to be able to explore what happens for tuned $G^{(1)}$ and $G^{(2)}$ for which $\bar{T} \to \infty$.

Sphere amplitude (simplified)

$$\langle G^{(2)}|G^{(1)}\rangle_V = e^{-\bar{S}_{\rm ms}} \int \mathcal{D}\gamma_{ij}(t) e^{-S_{\rm msGauss}(\gamma)} \int \mathcal{D}X e^{-\bar{S}_X - \delta S_X} \prod \{g_s V^{(n)}\}$$

$$\langle G^{(2)}|G^{(1)}\rangle_V^{(\rm sphere;NLO)} \sim e^{-\bar{S}_{\rm ms}} \int \mathcal{D}X e^{-\bar{S}_X} \prod \{g_s V^{(n)}\}$$

$$\times \left(\int d^2\sigma_1 d^2\sigma_1 \Gamma_{ijkl}(X^0(\sigma_1), X^0(\sigma_2)) \partial X^i(\sigma_1)) \bar{\partial} X^j(\sigma_1) \partial X^k(\sigma_2)) \bar{\partial} X^l(\sigma_2) \right)$$

$$\Gamma_{ijkl}(t_1, t_2) = \sum_a \frac{\Gamma^a_{ij}(t_1)\Gamma^a_{kl}(t_2)}{h_a - 1} \qquad \mathcal{V}^a \equiv \Gamma^a_{ij}(X^0)\partial X^i \bar{\partial} X^j = \delta^{h_a - 1} V^a$$

(Dirichlet Green function)

$$\langle G^{(2)}|G^{(1)}\rangle_V^{(\rm sphere;NLO)} \sim e^{-\bar{S}_{\rm ms}} \int \mathcal{D}X e^{-\bar{S}_X} \prod \{g_s V^{(n)}\} \sum_a \int d\sigma_1 d\sigma_2 \frac{\delta^{2(h_a-1)}}{h_a - 1} V^a(\sigma_1) V^a(\sigma_2)$$

Sphere + torus: IR divergences?

- We are focusing on amplitudes allowing a semiclassical treatment ($G^{(1)}$ and $G^{(2)}$ such that saddle point metric is weakly curved and saddle point duration $\bar{T}$ large).
- Generic such amplitudes are IR finite (no zero modes on finite time interval with Dirichlet boundary conditions).
- How about amplitudes tuned s.t. $\bar{T} \to \infty$? Cutoff-dependent contributions have the right structure to cancel each other. A cutoff-independent divergence remains; can be understood as a consequence of considering Dirichlet boundary conditions. Cf. Green function of $(d/dt)^2$ (free particle) with Dirichlet boundary conditions at t=0 and t=T diverges as T → ∞. The divergence is unrelated to string worldsheets. (For a free particle this remaining divergence disappears upon applying a "reduction formula".)

Worldsheet with holes (more precise)

Off-shell amplitudes in Minkowski space:

$$G(\ell_1, \cdots, \ell_n) = \left(\prod_{i=1}^{n} \int_{Diff(S^1)} d\Sigma_i \right) \int [d\tau] \left(\det{}' P_1^\dagger P_1 \right)^{1/2} (\det{}_{(\mathrm{Dir})}[-\nabla^2])^{-13} e^{-S_{cl}}$$

[Cohen, Moore, Nelson, Polchinski 1986]

Amplitude for strings in compact cosmological space:

$$\langle G^{(2)}, \ell^{(2)} | G^{(1)}, \ell^{(1)} \rangle = \int_{-\infty}^{\infty} dT \int_{\substack{G_{ij}(0)=G_{ij}^{(1)} \\ G_{ij}(T)=G_{ij}^{(2)}}} \mathcal{D}G_{ij}(t)\, e^{iS_{\mathrm{ms}}} \left(\prod_{\ell_m^{(1)} \ell_n^{(2)}} \int_{Diff(S^1)} \right) \int_{\ell_m^{(1)} \ell_n^{(2)}} \mathcal{D}X e^{-S_X}$$

Summary

- Asymptotic regions are important for conventional perturbative string theory, as they allow the definition of an S-matrix.

- Extrapolating conventional perturbative string theory to compact spaces leads to IR divergences on a torus worldsheet.

- These divergences are similar to annulus divergences related to D0-brane recoil. For D0-branes, a worldline formalism has been developed in which the annulus divergence is canceled by a sphere divergence.

- We have defined amplitudes for the universe to transition from one spatial slice to another while absorbing and emitting closed strings. Generic such amplitudes that can be treated semiclassically are IR finite. Finetuned amplitudes have a cutoff-independent IR divergence, which can be understood as due to Dirichlet boundary conditions on the metric.