The thermal photon rate at next-to-leading order (and a sneak peek at jets)

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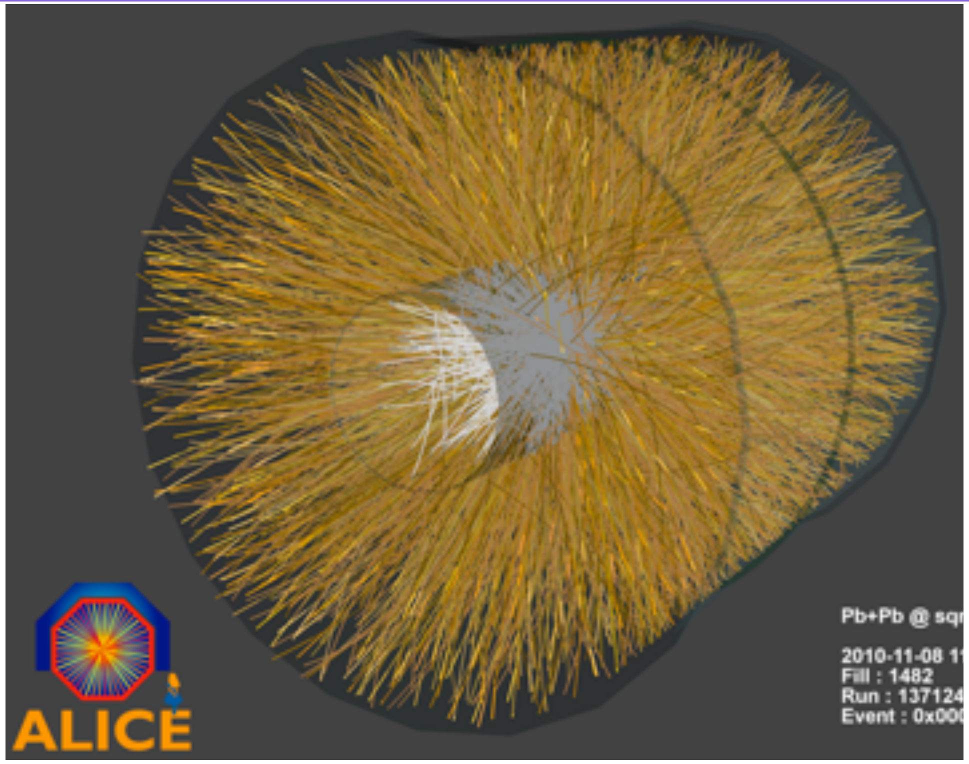
CERN, November 1st 2013

Hard probes

- Heavy-ion collisions: experimental investigation of the deconfined phase of the diagram



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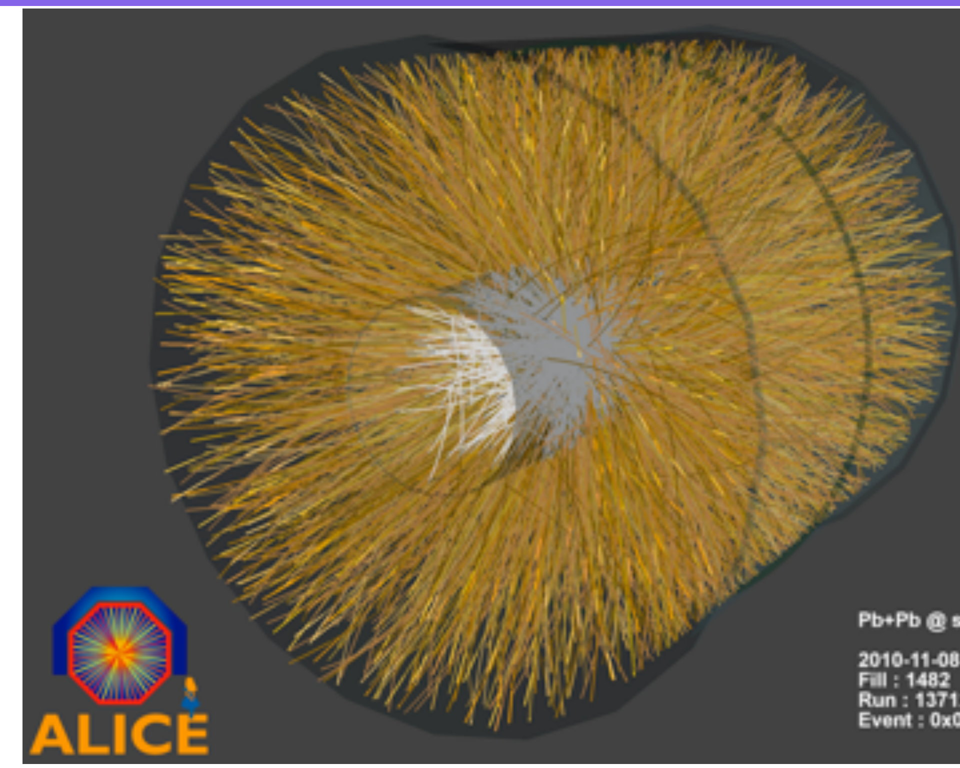

- Characterization of the medium through 2 classes of observables:
  - Bulk properties (hydro, flow, etc...)
  - Hard probes (jets, e/m probes, quarkonia...)

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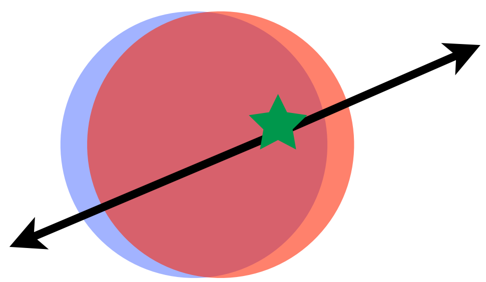
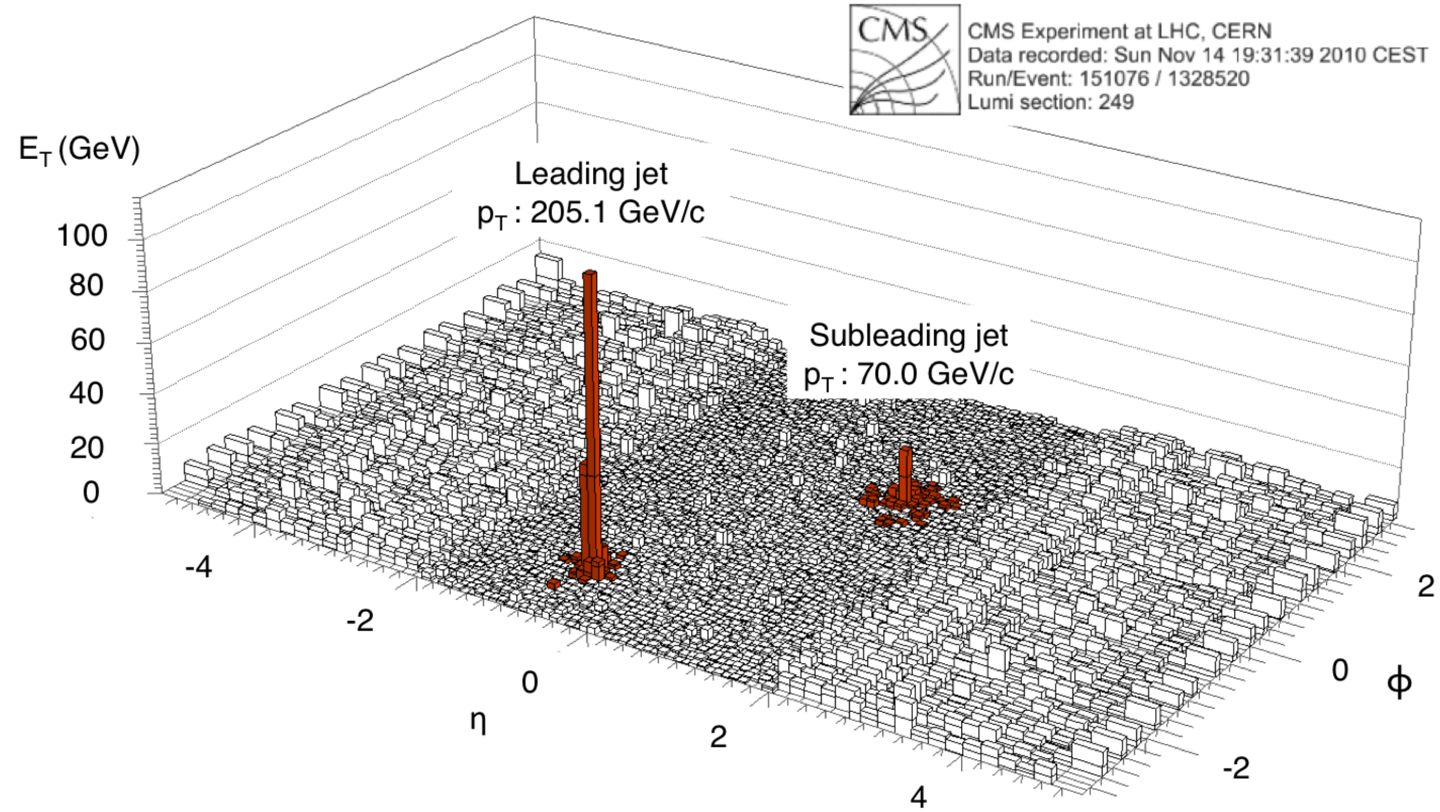
- Hadrons may decay into *decay photons*

Jets in heavy-ion collisions





CMS **PRC84** (2011)

- Jet broadening $\langle k_\perp^2 \rangle \equiv \hat{q} L$
- Jet energy loss/jet quenching

Motivation

- Improve the phenomenological analyses, if not by giving reliable theory error bands
- On the theory side, understand perturbation theory and its convergence better
  - For thermodynamical quantities (p, s, ...) either strict expansion in g, QCD (T) + EQCD (gT) + MQCD (g^2T) (Arnold-Zhai, Braaten Nieto, etc) or non-perturbative solution of EQCD (Kajantie Laine etc)
  - For dynamical quantities? Poor convergence in heavy-quark diffusion coefficient

NLO transport coefficients

- The only transport coefficient known so far at NLO is the *heavy quark momentum diffusion coefficient,* which is defined through the noise-noise correlator in a Langevin formalism. In field theory it can be written as

$$\kappa = \frac{g^2}{3N_c}\int_{-\infty}^{+\infty} dt \, \mathrm{Tr}\langle U(t,-\infty)^\dagger E_i(t) U(t,0) E_i(0) U(0,-\infty)\rangle$$

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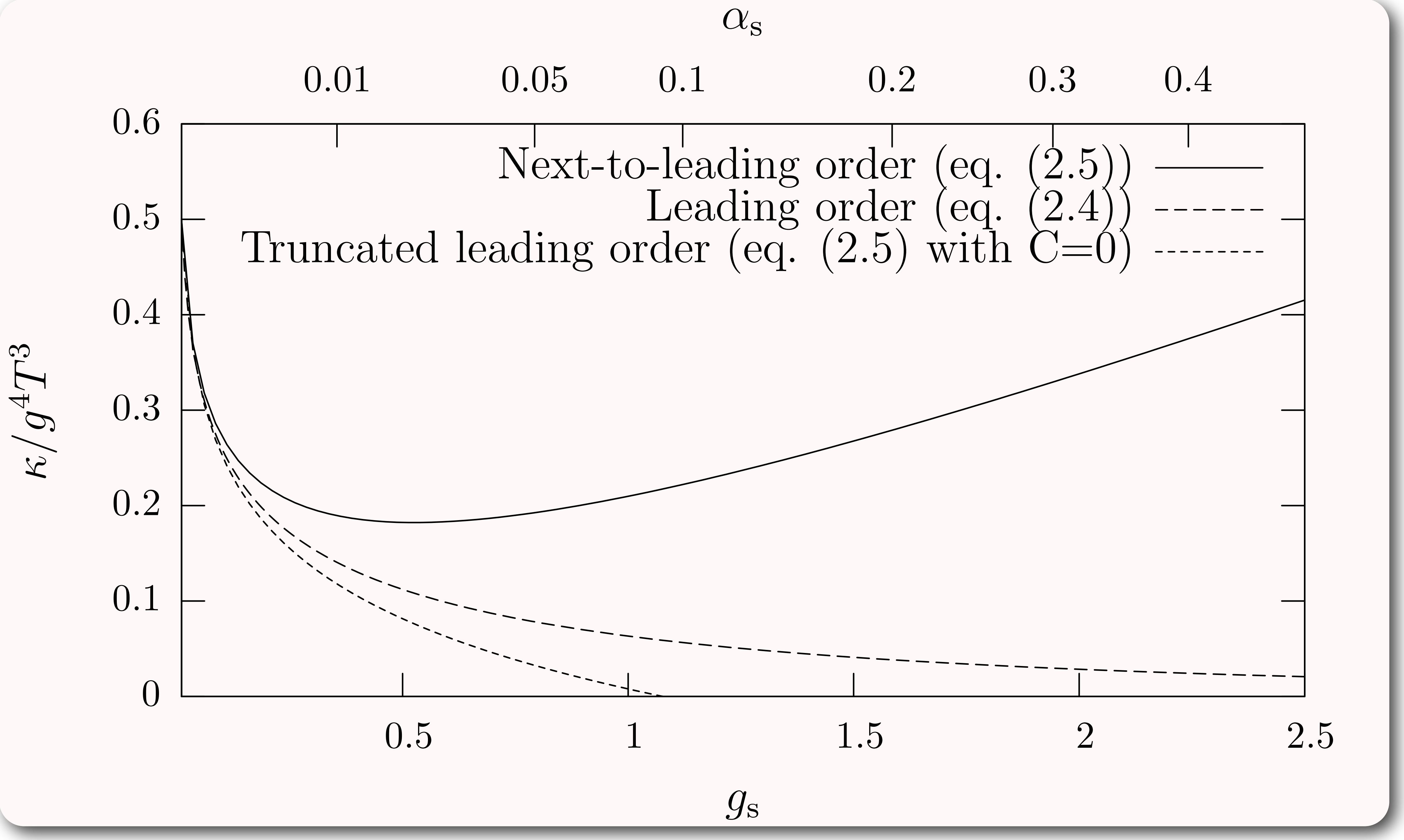
- The NLO computation factors in the coefficient C, which turns out to be sizeable

$$\kappa = \frac{C_H g^4 T^3}{18\pi}\left(\left[N_\mathrm{c}+\frac{N_\mathrm{f}}{2}\right]\left[\ln\frac{2T}{m_D}+\xi\right]+\frac{N_\mathrm{f}\ln 2}{2}+\frac{N_\mathrm{c} m_D}{T}C+\mathcal{O}(g^2)\right) \qquad \xi = \frac{1}{2}-\gamma_E+\frac{\zeta'(2)}{\zeta(2)}$$

Caron-Huot Moore **PRL100, JHEP0802** (2008)

NLO transport coefficients



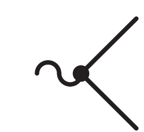
Caron-Huot Moore **PRL100, JHEP0802** (2008)

Overview

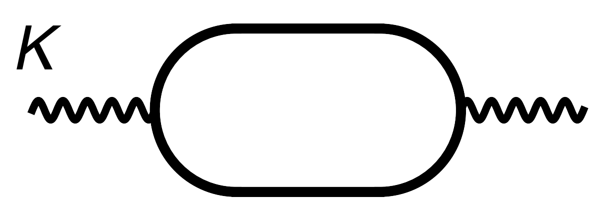
The thermal photon rate

- Wightman current-current correlator (with $k^0 = k \sim T$ *hard*)

$$\frac{d\Gamma}{d^3k} = \frac{e^2}{(2\pi)^3 2k^0} \int d^4Y e^{-iK\cdot Y} \langle J^\mu(Y) J_\mu(0) \rangle \qquad J^\mu = \sum_{q=uds} e_q \bar{q} \gamma^\mu q :$$



- At one loop ($\alpha_{\mathrm{EM}}\, g^0$):



Kinematically impossible to radiate an on-shell photon from on-shell quarks. Need something to kick one of the quarks off-shell

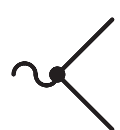
- LO is $\alpha_{\mathrm{EM}}\, g^2$

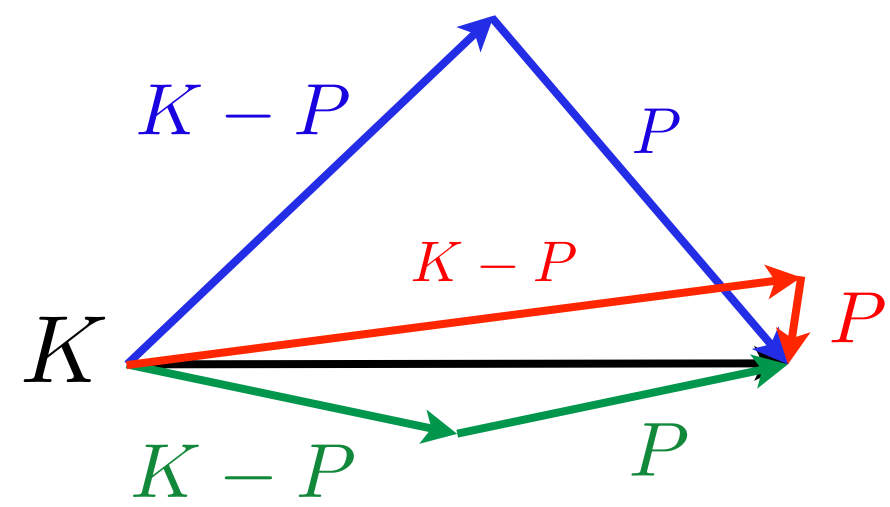
Kinematical regions

- Define a light-cone $K = (k, 0, 0)$

  $P = (p^+, p^-, p_\perp) \qquad p^+ = (p^0 + p^z)/2 \quad p^- = p^0 - p^z$

- Momentum conservation at the current insertion gives three regions

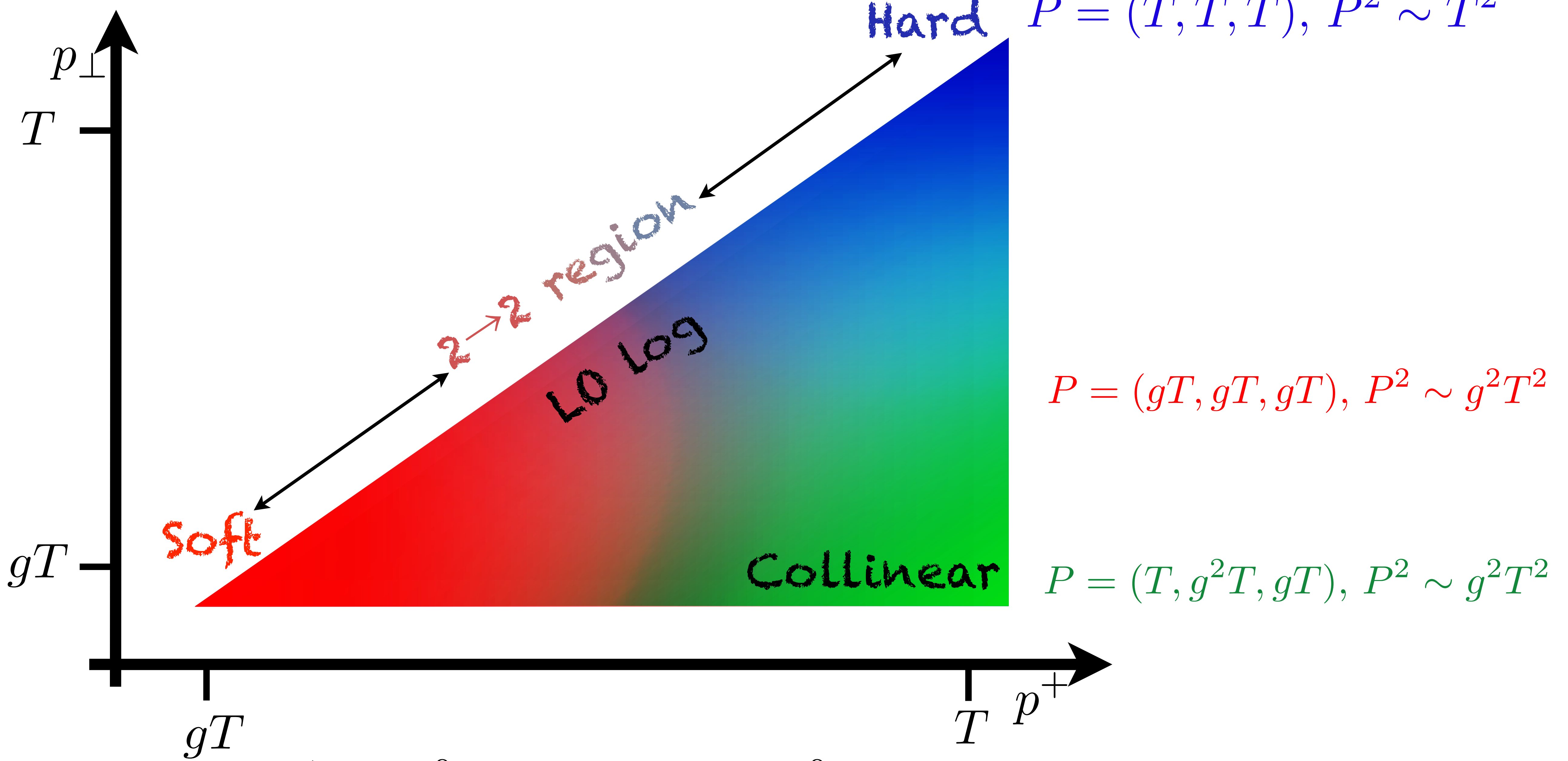
  $J^\mu \;=\; \sum_{q=uds} e_q \bar{q} \gamma^\mu q :$ 

  - Hard off-shell
  - Soft, smaller phase space but enhancement
  - Collinear, both nearly on shell and enhanced

Kinematical regions

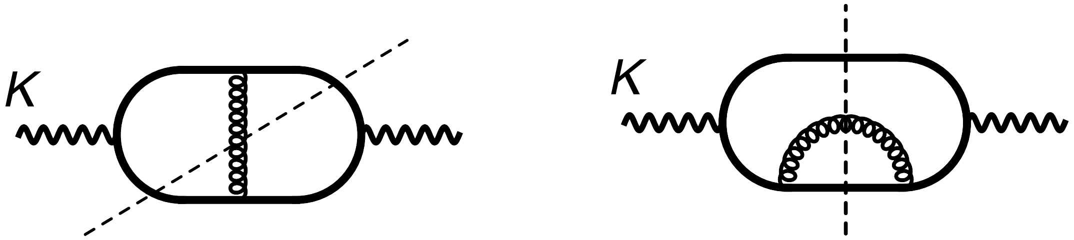
- In the $(p^+, p_\perp)$ plane



$p^+ = (p^0 + p^z)/2 \quad p^- = p^0 - p^z$

The hard region

- Two loop diagrams ($\alpha_{\mathrm{EM}}\, g^2$)



where the cuts correspond to the so-called 2↔2 processes (with their crossings and interferences):

- Perturbation theory works here, α_{s} expansion
- IR divergence (Compton) when t goes to zero

$P \to (T, g^2 T, gT)$

The soft region

- Hard Thermal Loop effective theory Braaten Pisarski

$$\delta\mathcal{L} = -\frac{m_D^2}{2}\mathrm{Tr}\int \frac{d\Omega_v}{4\pi} F^{\mu\alpha}\frac{v_\alpha v_\beta}{(v\cdot D)^2}F^\beta_\mu + i\omega_0^2\,\overline{\psi}\int \frac{d\Omega_v}{4\pi}\frac{\not{v}}{v\cdot D}\,\psi$$

$$v = (1, \mathbf{v}) \quad v^2 = 0$$

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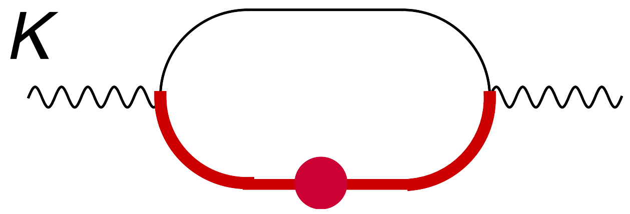
- The Landau cut of the HTL propagator opens up the phase space in this (apparently one-loop) diagram



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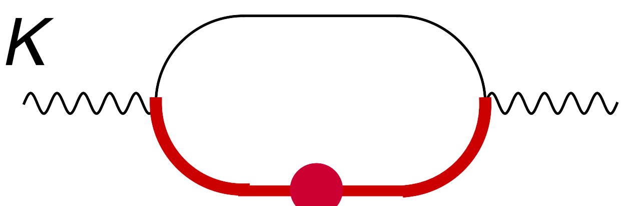


- In the end one obtains the result

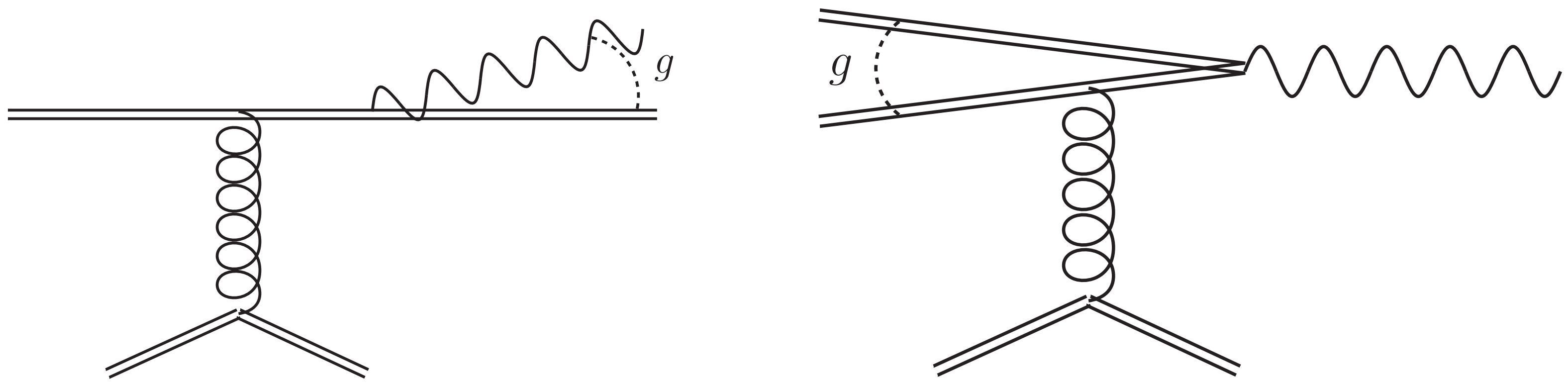
$$\left.\frac{d\Gamma_\gamma}{d^3k}\right|_{2\leftrightarrow 2}\propto e^2g^2\left[\log\frac{T}{m_\infty}+C_{2\leftrightarrow 2}\left(\frac{k}{T}\right)\right]$$

The dependence on the cutoff cancels out

Kapusta Lichard Siebert **PRD44** (1991) Baier Nakkagawa Niegawa Redlich **ZPC53** (1992)

The collinear region

- Collinear enhancement in these diagrams

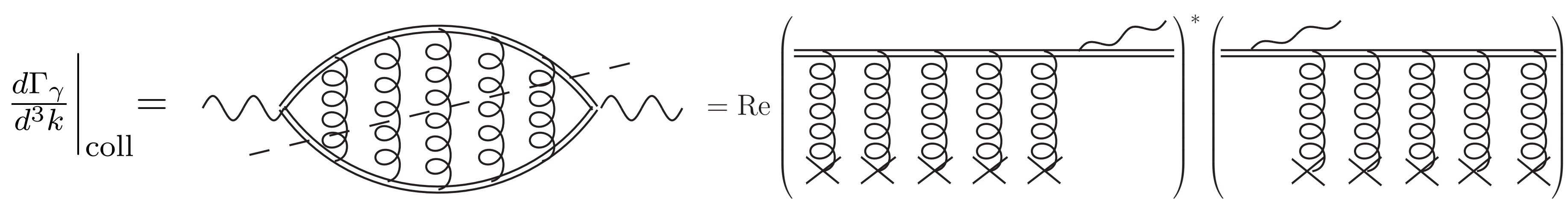


Aurenche Gelis Kobes Petitgirard Zaraket 1998-2000

- Long photon formation time ~$1/(g^2T)$, same as soft scattering rate. Multiple soft scatterings interfere and must be resummed => Landau-Pomeranchuk-Migdal effect (LPM)

The LPM effect

- Introduced by Landau and Pomeranchuk (then Migdal) for QED in the 50's
- Extended to photons in QCD in Baier Dokshitzer Mueller Peigne Schiff **NPB478** (1996) Zakharov 96-98
- Rigorous treatment and diagrammatics in AMY (Arnold Moore Yaffe) **JHEP** 0111, 0112, 0226 (2001-02)
- In the JJ correlator diagrams like



have to be resummed consistently

LPM resummation

- Quark statistical functions × DGLAP splitting × transverse evolution

$$\frac{d\Gamma}{d^3k} = \frac{\alpha}{\pi^2 k}\int \frac{dp^+}{2\pi} n_{\rm F}(k+p^+)[1-n_{\rm F}(p^+)]\frac{(p^+)^2+(p^++k)^2}{2(p^+(p^++k))^2}\lim_{\mathbf{x}_\perp\to 0} 2\mathrm{Re}\boldsymbol{\nabla}_{\mathbf{x}_\perp}\mathbf{f}(x_\perp)$$

$$\begin{array}{ccccc} x^+ & \gg & x_\perp & \gg & x^- \\ 1/g^2T & \gg & 1/gT & \gg & 1/T \end{array}$$

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- Transverse diffusion and Wilson-loop correlators evolve the transverse density **f** *along the spacetime light-cone*

$$-2i\nabla\delta^2(\mathbf{x}_\perp) = \left[\frac{ik}{2p^+(k+p^+)}\left(m_\infty^2 - \nabla^2_{\mathbf{x}_\perp}\right) + \mathcal{C}(x_\perp)\right]\mathbf{f}(\mathbf{x}_\perp)$$

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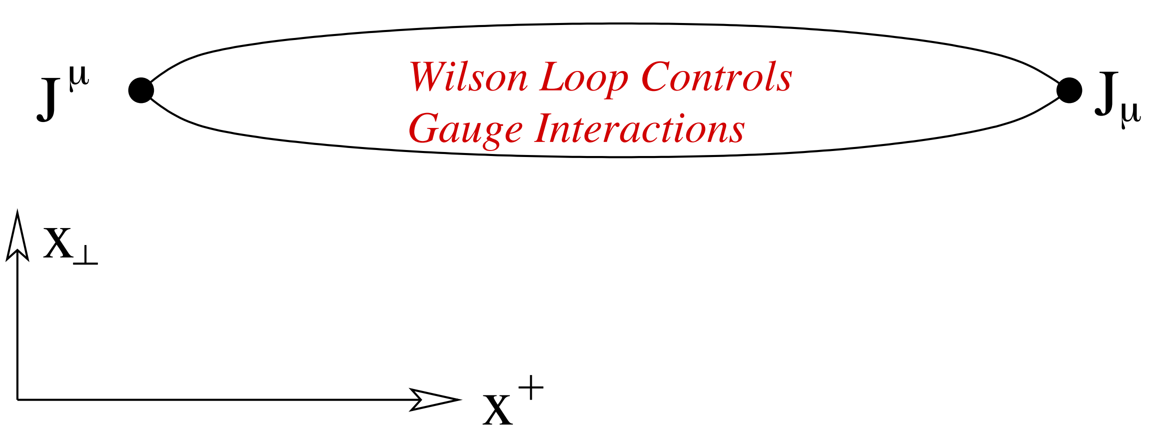
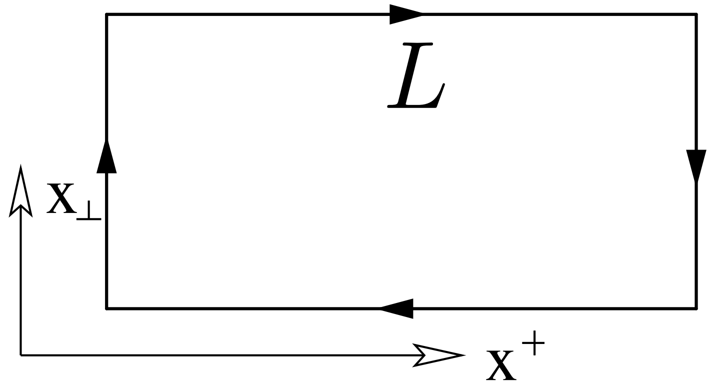


Zakharov 1996-98 AMY 2001-02

LPM resummation



$\propto e^{\mathcal{C}(x_\perp)L}$

BDMPS-Z, Wiedemann, Casalderrey-Solana Salgado, D'Eramo Liu Rajagopal, Benzke Brambilla Escobedo Vairo

- All points at spacelike or lightlike separation, only preexisting correlations
- Soft contribution becomes Euclidean! Caron-Huot **PRD79** (2008)
  - Can be "easily" computed in perturbation theory
  - Possible lattice measurements Laine Rothkopf JHEP1307 (2013) Panero Rummukainen Schäfer **1307.5850**

Euclideanization of light-cone soft physics

- For $t/x_z = 0$: equal time Euclidean correlators.

$$G_{rr}(t=0,\mathbf{x}) = \sum\!\!\!\!\!\!\int_p G_E(\omega_n, p) e^{i\mathbf{p}\cdot\mathbf{x}}$$

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- Soft physics dominated by *n=0* (and *t*-independent) =>EQCD!

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$$G_{rr}(t,\mathbf{x})_{\mathrm{soft}} = T\int d^3p\, e^{i\mathbf{p}\cdot\mathbf{x}}\, G_E(\omega_n=0,\mathbf{p})$$

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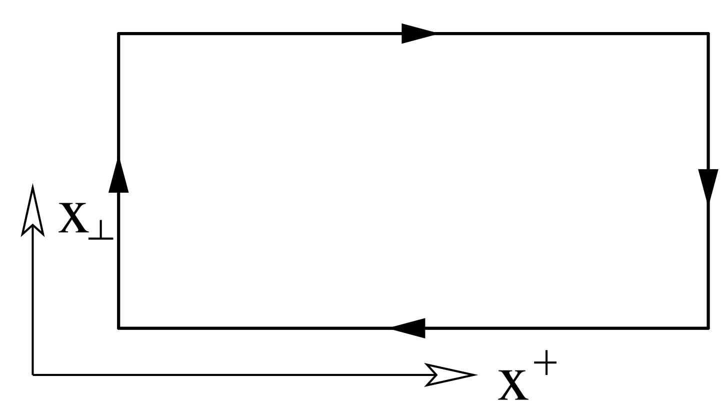
Caron-Huot **PRD79** (2009)

Euclideanization of light-cone soft physics

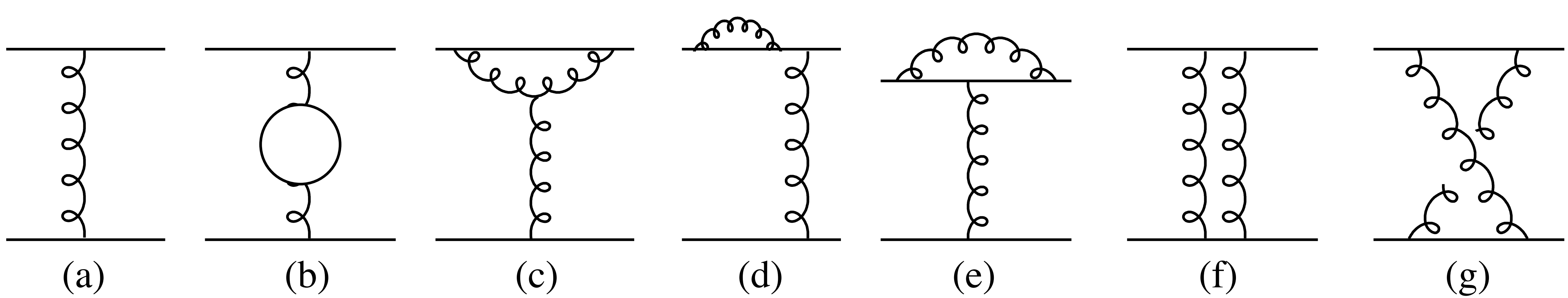


$\propto e^{\mathcal{C}(x_\perp)L}$

- At leading order

$$C(x_\perp) \propto T\int \frac{d^2q_\perp}{(2\pi)^2}\left(1-e^{i\mathbf{x}_\perp\cdot\mathbf{q}_\perp}\right)G_E^{++}(\omega_n=0,q_z=0,q_\perp) = T\int \frac{d^2q_\perp}{(2\pi)^2}\left(1-e^{i\mathbf{x}_\perp\cdot\mathbf{q}_\perp}\right)\left(\frac{1}{q_\perp^2}-\frac{1}{q_\perp^2+m_D^2}\right)$$

- Agrees with the earlier sum rule in Aurenche Gelis Zaraket **JHEP0205** (2002)
- At NLO: Caron-Huot **PRD79** (2009)

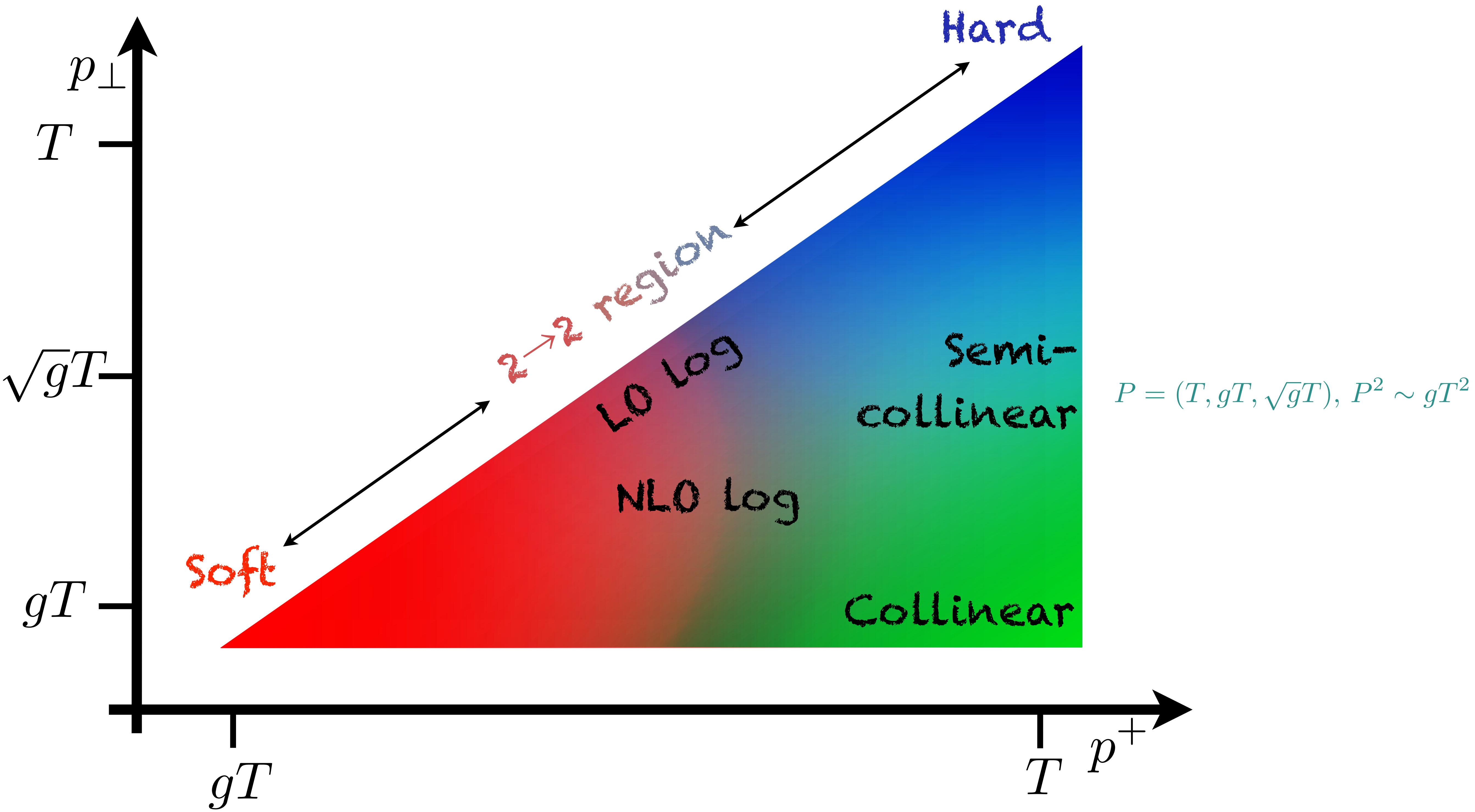


Going to NLO

Sources of NLO corrections

- As usual in thermal field theory, the soft scale gT introduces NLO $O(g)$ corrections
- The soft region and the collinear region both receive $O(g)$ corrections
- There is a new semi-collinear region
- The NLO calculation is still not sensitive to the magnetic scale g^2T.

NLO regions



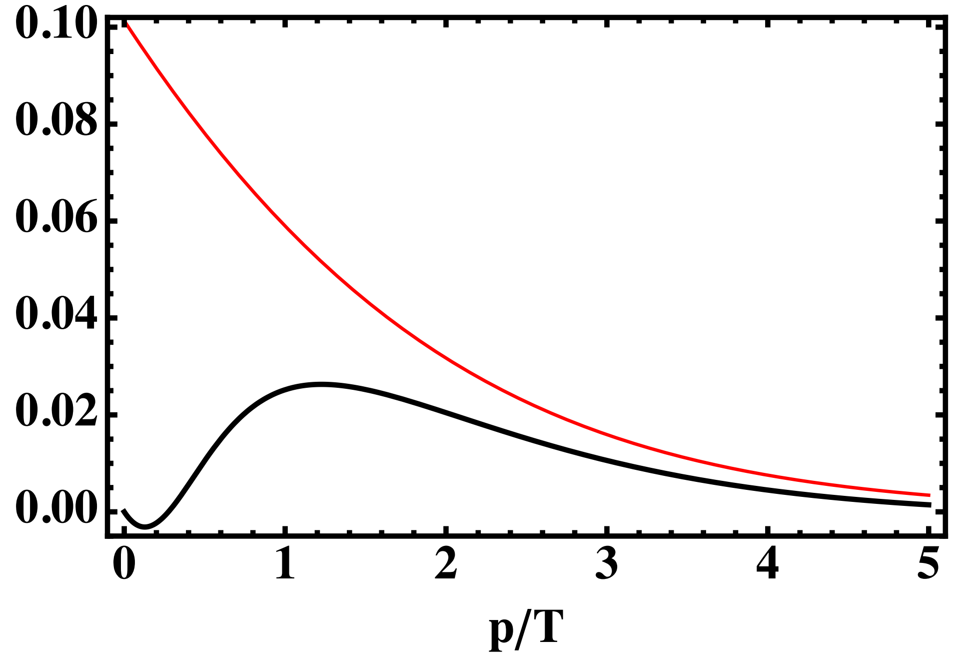
$p_\perp$
T
$\sqrt{g}T$
gT
Hard
2→2 region
Soft
LO log
NLO log
Semi-collinear
Collinear
gT
T
p^+
$P = (T, gT, \sqrt{g}T),\ P^2 \sim gT^2$

NLO asymptotic mass

- The soft contribution is large and handled incorrectly. This part of the integrand needs to be subtracted and replaced by a proper evaluation with HTL

- NLO correction computed in Caron-Huot **PRD79** (2009) with Euclidean techniques

$$\delta m_\infty^2 = 2g^2 C_R T \int \frac{d^3q}{(2\pi)^3} \left(\frac{1}{q^2 + m_D^2} - \frac{1}{q^2} \right) = -g^2 C_R \frac{T m_D}{2\pi}$$



The collinear sector

- The AMY resummation equation is

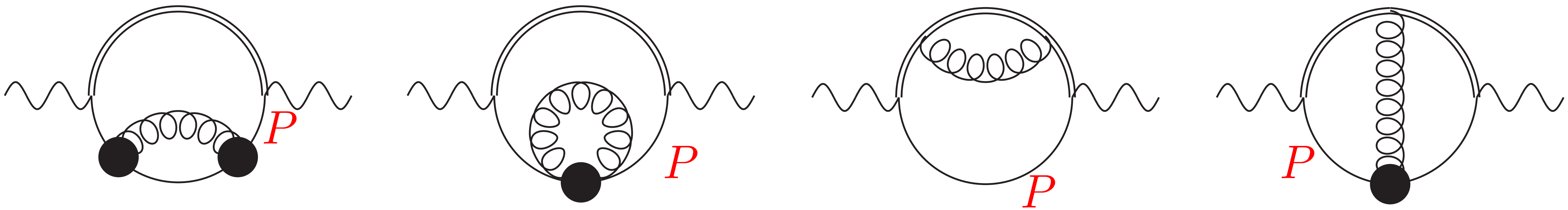
$$\frac{d\Gamma}{d^3k} = \frac{\alpha}{\pi^2 k}\int \frac{dp^+}{2\pi} n_{\rm F}(k+p^+)[1-n_{\rm F}(p^+)] \frac{(p^+)^2+(p^++k)^2}{2(p^+(p^++k))^2} \lim_{\mathbf{x}_\perp\to 0} 2\mathrm{Re}\boldsymbol{\nabla}_{\mathbf{x}_\perp}\mathbf{f}(x_\perp)$$

- Four sources of $O(g)$ corrections
- $p^+ \sim gT$ or $p^+ + k \sim gT$. Mistreated soft limit

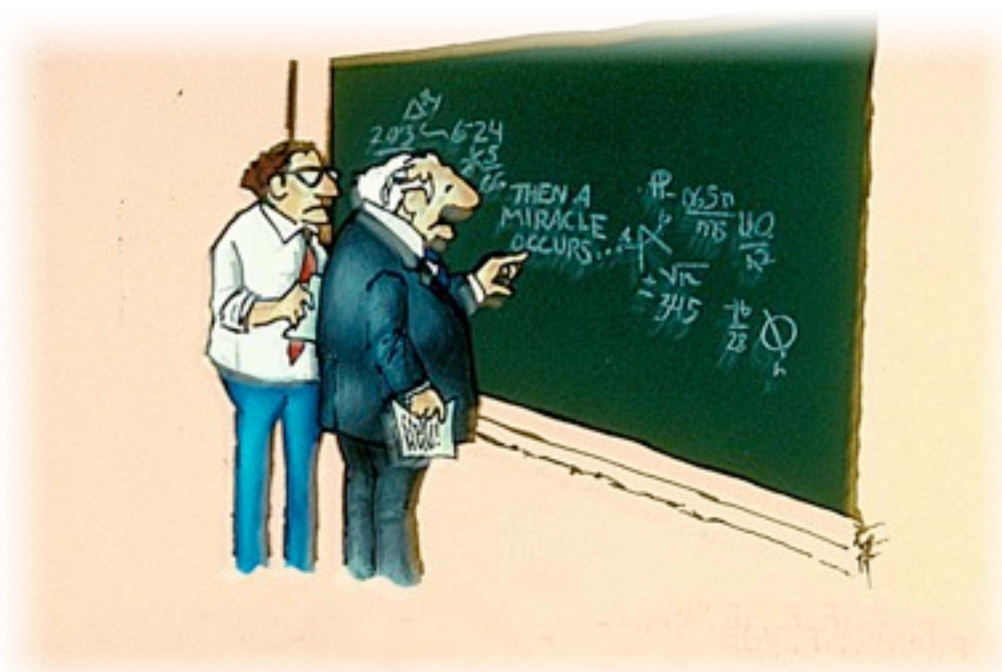
$$\left.\frac{d\Gamma_\gamma}{d^3k}\right|_{\rm soft}^{\rm subtr.} = \frac{\mathcal{A}(k)}{(2\pi)^3}\int_{-\mu^+}^{+\mu^+} dp^+ \frac{8}{T}\int \frac{d^2p_\perp d^2q_\perp}{(2\pi)^4}\frac{m_D^2}{q_\perp^2(q_\perp^2+m_D^2)}\left(\frac{\mathbf{p}_\perp}{p_\perp^2+m_\infty^2} - \frac{\mathbf{p}_\perp+\mathbf{q}_\perp}{(\mathbf{p}_\perp+\mathbf{q}_\perp)^2+m_\infty^2}\right)^2$$

- $p_\perp \sim \sqrt{g}T, p^- \sim gT$. Mistreated semi-collinear limit
- The two inputs in the differential equation, m_∞^2 and $\mathcal{C}(x_\perp)$ receive $O(g)$ corrections.

The NLO soft region

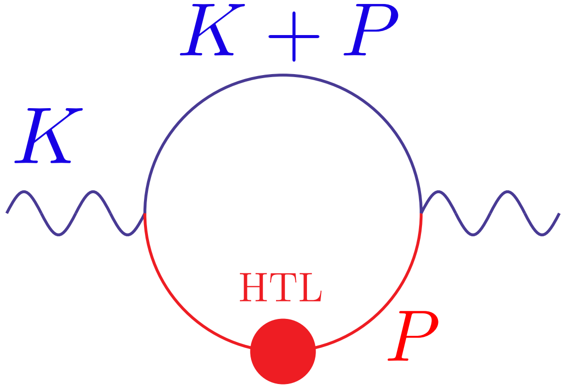


- 4 diagrams with HTL vertices and propagators on the soft line
- Could brute-force them numerically. Or think again about analyticity, light-cones



Fermionic sum rules

- We have found the fermionic analogue of the AGZ sum rule
- The leading-order soft contribution (P fully soft)



$$(2\pi)^3 \frac{d\Gamma_\gamma}{d^3k}_{\text{soft}} \propto \int \frac{dp^+\, d^2p_\perp}{(2\pi)^3} \text{Tr}\left[\gamma^-(S_R(P) - S_A(P))\right]_{p^-=0}$$

where $S(P) = \frac{1}{2}\left[(\gamma^0 - \vec{\gamma}\cdot\hat{p})S^+(P) + (\gamma^0 + \vec{\gamma}\cdot\hat{p})S^-(P)\right]$

$$S_R^\pm(P) = \left.\frac{i}{p^0 \mp \left[p + \frac{\omega_0^2}{p}\left(1 - \frac{p^0 \mp p}{2p}\ln\left(\frac{p^0+p}{p^0-p}\right)\right)\right]}\right|_{p^0=p^0+i\epsilon}$$

Fermionic sum rules

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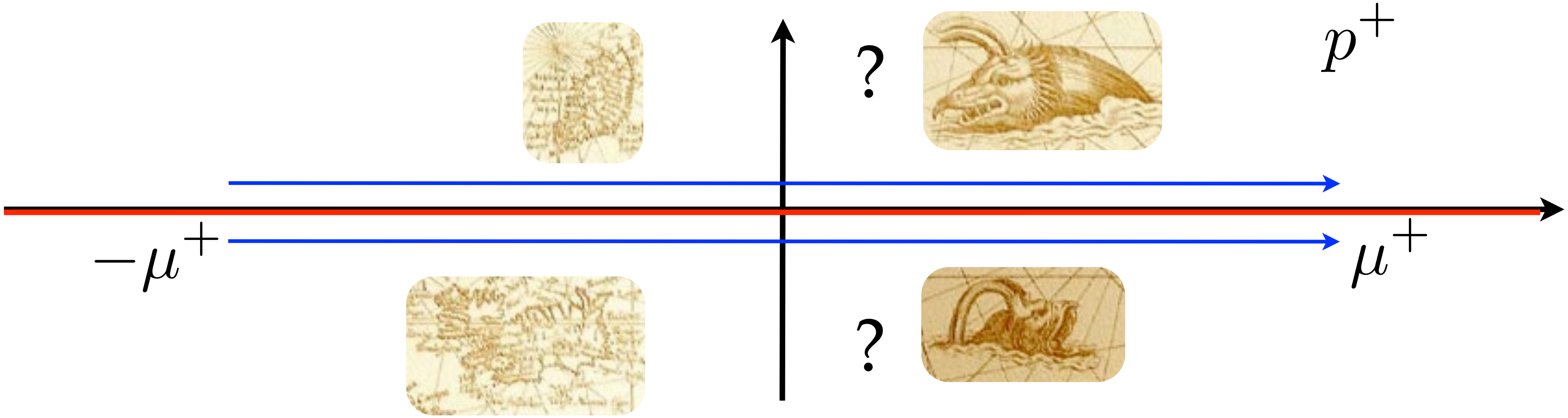

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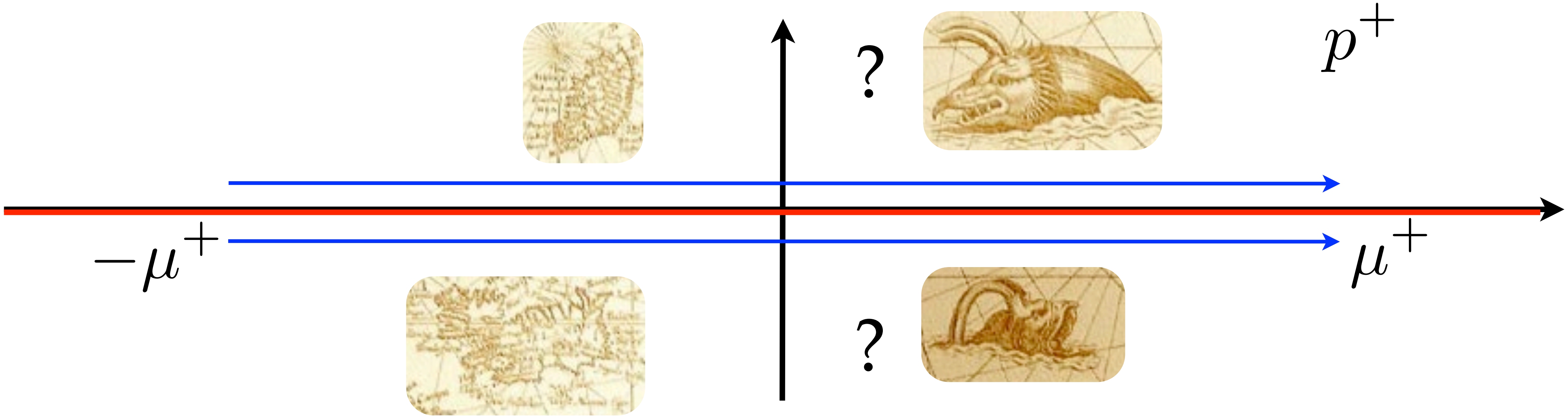

- A retarded propagator is an analytic function of Q in the upper half-plane not just in the frequency, but in any time-like or light-like variable

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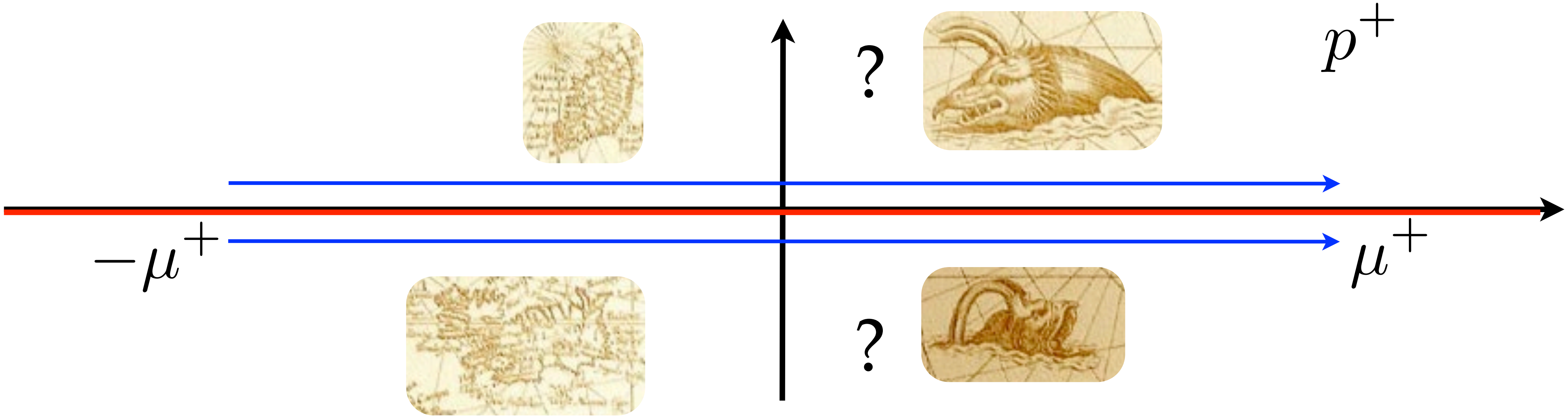

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- Deform the contour away from the real axis

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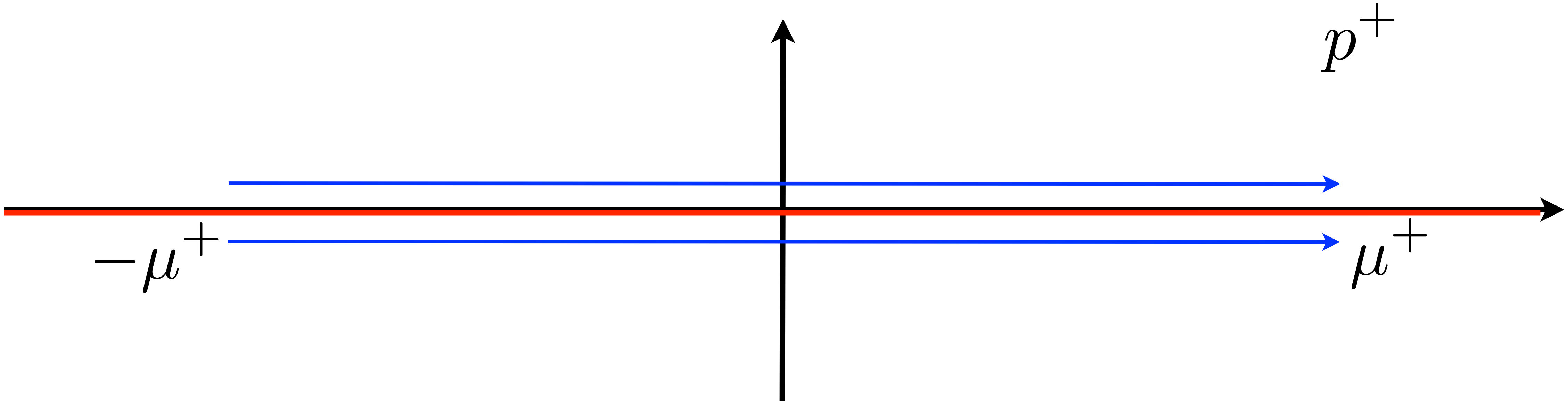

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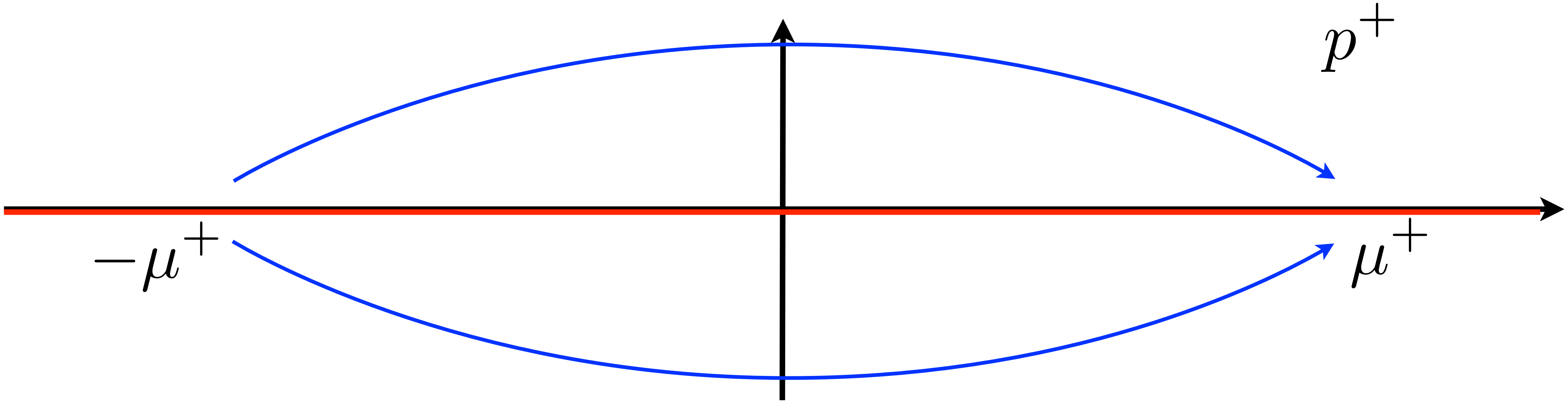

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- Along the arcs at large complex p^+ the integrand has a very simple behavior

$$\mathrm{Tr}\left[\gamma^-(S_R(P) - S_A(P)\right]_{p^-=0} = \frac{i}{p^+}\frac{m_\infty^2}{p_\perp^2 + m_\infty^2} + \mathcal{O}\left(\frac{1}{(p^+)^2}\right)$$

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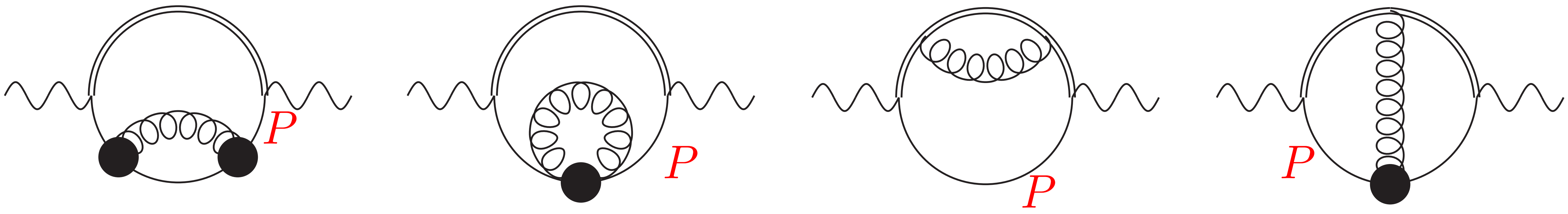
- The $p_\perp$ integral is UV-log divergent, giving the LO UV-divergence that cancels the IR divergence at the hard scale, now analytically

Independently obtained by Besak Bödeker **JCAP1203** (2012)

The NLO soft region



- At NLO one can use the KMS relations and the *ra* basis to write the diagrams in terms of fully retarded and fully advanced functions of P. The hard only depend on p^-.

- The contour deformations are then again possible and the diagrams can be expanded for large complex p^+. On general grounds we expect

$$(2\pi)^3 \frac{d\delta\Gamma_\gamma}{d^3k}\bigg|_{\rm soft} \propto \int \frac{dp^+ d^2p_\perp}{(2\pi)^3}\left[C_0\left(\frac{1}{p^+}\right)^0 + C_1\left(\frac{1}{p^+}\right)^1 + \ldots\right]$$

The soft region

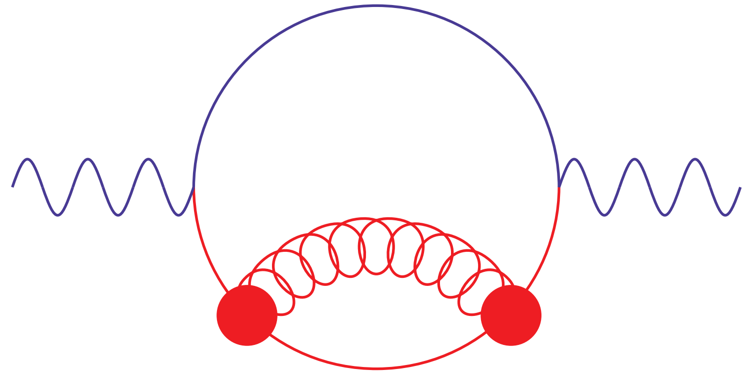
- The $(1/p^+)^0$ term has to be *exactly* the subtraction term we have seen before in the collinear region, to cancel the cutoff dependence. Confirmed by explicit calculation

- At order $1/p^+$ we had the LO result. We can expect

$$\frac{m_\infty^2}{p_\perp^2 + m_\infty^2} \to \frac{m_\infty^2 + \delta m_\infty^2}{p_\perp^2 + m_\infty^2 + \delta m_\infty^2} = \left(\frac{m_\infty^2}{p_\perp^2 + m_\infty^2} + \frac{\delta m_\infty^2 \, p_\perp^2}{(p_\perp^2 + m_\infty^2)^2} + \mathcal{O}(g^2) \right)$$

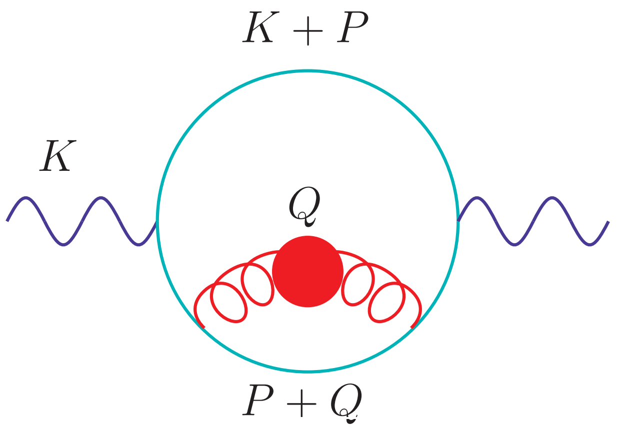
The explicit calculation finds just this contribution.

- The contribution from HTL vertices goes like $(1/p^+)^2$ or smaller on the arcs.



$$\sim \frac{1}{(p^+)^2}$$

The semi-collinear region



P semi-collinear
Q soft

- Kinematical regions ⇒ partly different processes
  - Q timelike ⇒ 2↔2 processes with massive (plasmon) gluon
  - Q spacelike ⇒ 2↔3 processes: wider-angle bremsstrahlung and pair annihilation, no LPM interference

The semi-collinear region

- Subtraction term from the collinear region

$$\frac{d\delta\Gamma_\gamma}{d^3k}\bigg|^{\text{coll subtr.}}_{\text{semi-coll}} = 2\frac{\mathcal{A}(k)}{(2\pi)^3}\int dp^+ \left[\frac{(p^+)^2+(p^++k)^2}{(p^+)^2(p^++k)^2}\right]\frac{n_F(k+p^+)[1-n_F(p^+)]}{n_F(k)}$$
$$\times\frac{1}{g^2C_RT^2}\int\frac{d^2p_\perp}{(2\pi)^2}\frac{4(p^+)^2(p^++k)^2}{k^2p_\perp^4}\int\frac{d^2q_\perp}{(2\pi)^2}\,q_\perp^2\,\mathcal{C}(q_\perp)\,.$$

- Proper evaluation: replace

$$\frac{\hat{q}}{g^2C_R}\equiv\frac{1}{g^2C_R}\int\frac{d^2q_\perp}{(2\pi)^2}q_\perp^2\mathcal{C}(q_\perp)=\int\frac{d^4Q}{(2\pi)^3}\delta(q^-)q_\perp^2G_{rr}^{++}(Q)$$

with

$$\frac{\hat{q}(\delta E)}{g^2C_R}\equiv\int_{-\infty}^{\infty}dx^+\,e^{ix^+\delta E}\,\frac{1}{d_A}\langle v_k^\mu F_\mu{}^\nu(x^+,0,0_\perp)U_A(x^+,0,0_\perp;0,0,0_\perp)v_k^\rho F_{\rho\nu}(0)\rangle,$$

because $\delta E\sim gT$ is no longer negligible

- Another light-cone/Euclidean condensate

Light-cone condensates

- Asymptotic mass Caron-Huot **PRD79** (2009)

$$m_\infty^2 = g^2 C_R (Z_g + Z_f)$$

$$Z_g \equiv \frac{1}{d_A}\left\langle v_\mu F^{\mu\rho} \frac{-1}{(v\cdot D)^2} v_\nu F^\nu_{\ \rho} \right\rangle \qquad v_k = (1,0,0,1)$$

$$= \frac{-1}{d_A}\int_0^\infty dx^+\, x^+ \langle v_{k\,\mu} F_a^{\mu\nu}(x^+,0,0_\perp) U_A^{ab}(x^+,0,0_\perp;0,0,0_\perp) v_{k\,\rho} F^{\rho}_{b\ \nu}(0)\rangle$$

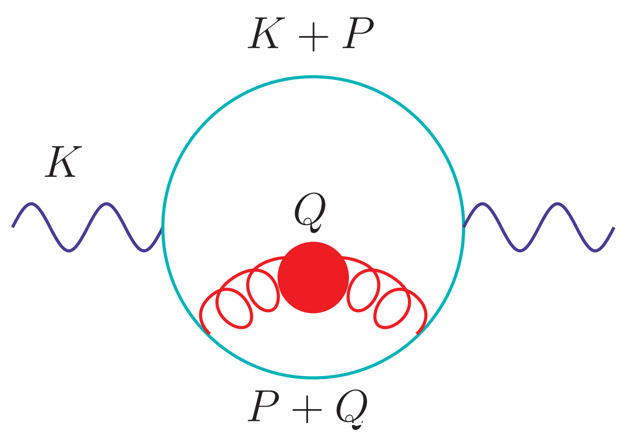
$$Z_f \equiv \frac{1}{2d_R}\left\langle \overline{\psi} \frac{\not v}{v\cdot D} \psi \right\rangle$$

- δE-dependent qhat

$$\frac{\hat{q}(\delta E)}{g^2 C_R} \equiv \int_{-\infty}^{\infty} dx^+\, e^{ix^+\delta E}\, \frac{1}{d_A} \langle v_k^\mu F_\mu^{\ \nu}(x^+,0,0_\perp) U_A(x^+,0,0_\perp;0,0,0_\perp) v_k^\rho F_{\rho\nu}(0)\rangle,$$

For δE→0 the standard definition is recovered

The semi-collinear region



P semi-collinear
Q soft

- Limits and divergences

  ↑ $p_\perp \to \infty \, (\delta E \to \infty)$ subtract the hard limit

  ↓ $\mathrm{p}_\perp \to 0$ subtract the collinear limit $(p_\perp \gg q_\perp)$

  ↙ $\mathrm{p}_\perp \to 0 \wedge p^+ \to 0$ IR log, combines with UV soft log (NLO log)

- Aside from the IR-log, the general behaviour of the P integration can only be obtained numerically.

Results

Summary

- LO rate

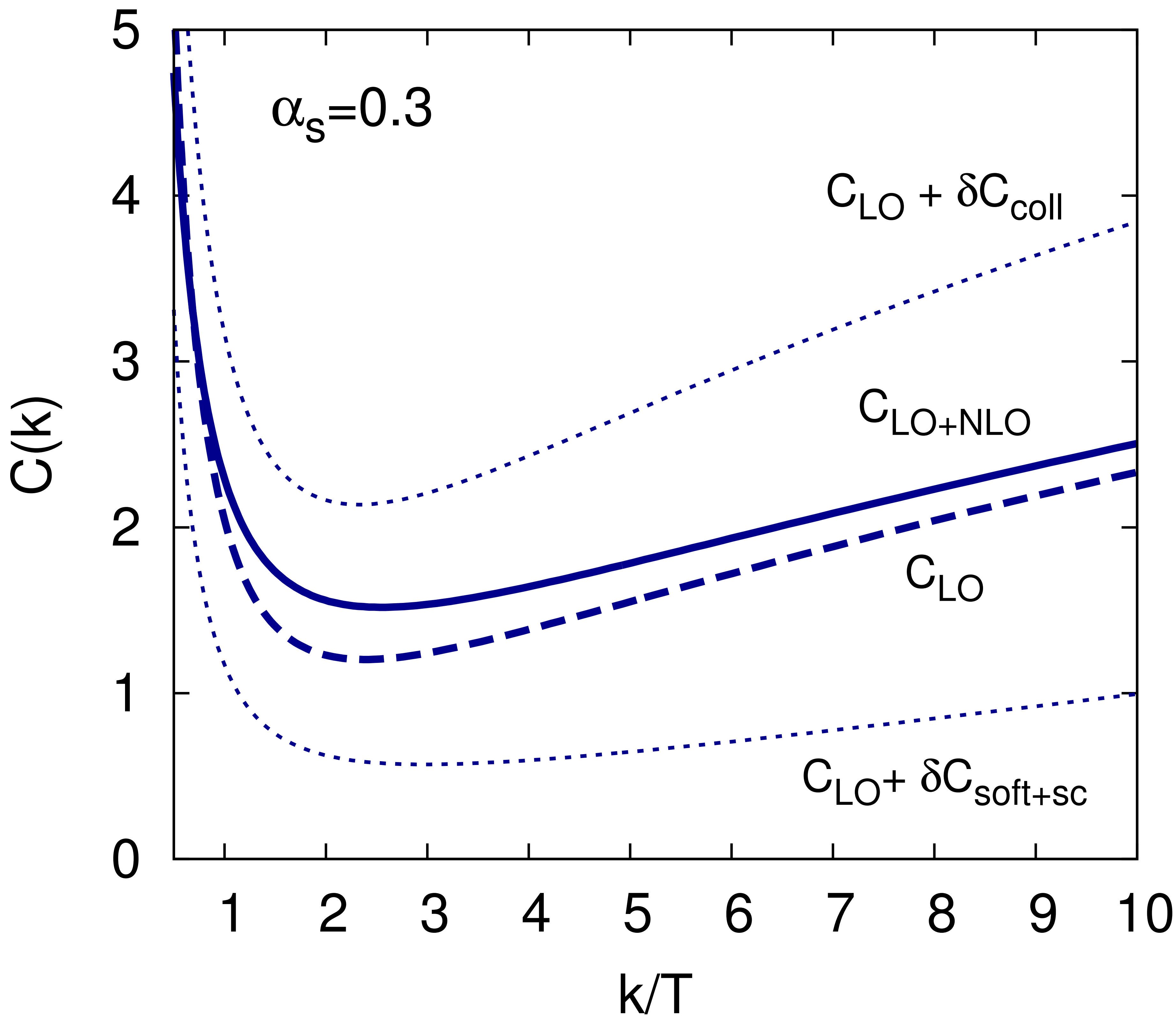
$$(2\pi)^3 \frac{d\Gamma}{d^3k}\bigg|_{\rm LO} = \mathcal{A}(k)\overbrace{\left[\log\frac{T}{m_\infty} + C_{2\to 2}(k) + C_{\rm coll}(k)\right]}^{C_{\rm LO}(k)}$$

$$\mathcal{A}(k) = \alpha_{\rm EM}\, g^2\; C_F T^2 \frac{n_{\rm F}(k)}{2k}\sum_f Q_f^2 d_f$$

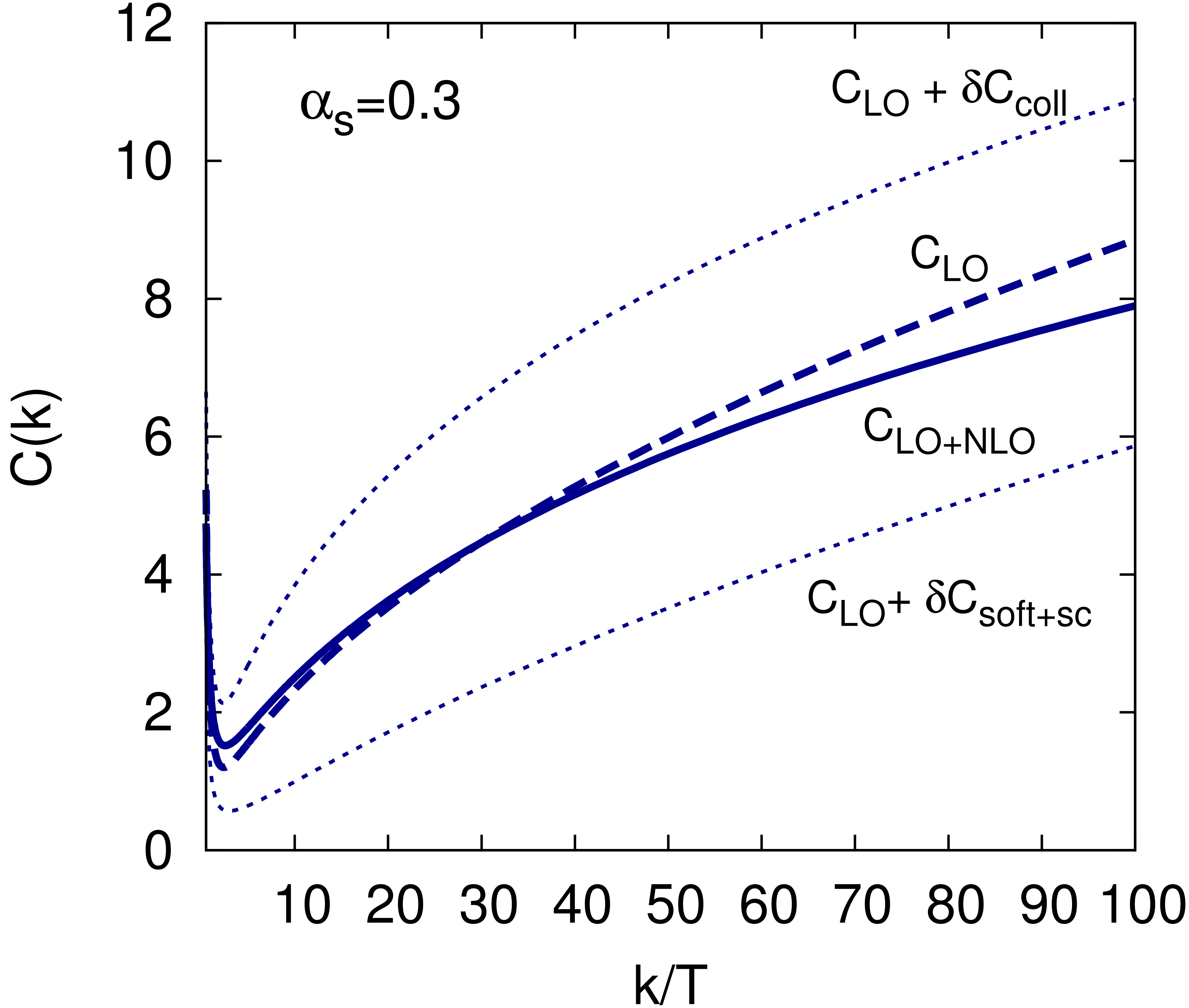
- NLO correction

$$(2\pi)^3 \frac{d\delta\Gamma}{d^3k}\bigg|_{\rm NLO} = \mathcal{A}(k)\overbrace{\left[\underbrace{\frac{\delta m_\infty^2}{m_\infty^2}\log\frac{\sqrt{2Tm_D}}{m_\infty} + \frac{\delta m_\infty^2}{m_\infty^2}C_{\rm soft+sc}(k)}_{\delta C_{\rm soft+sc}(k)} + \underbrace{\frac{\delta m_\infty^2}{m_\infty^2}C_{\rm coll}^{\delta m}(k) + \frac{g^2 C_A T}{m_D}C_{\rm coll}^{\delta \mathcal{C}}(k)}_{\delta C_{\rm coll}(k)}\right]}^{\delta C_{\rm NLO}(k)}$$

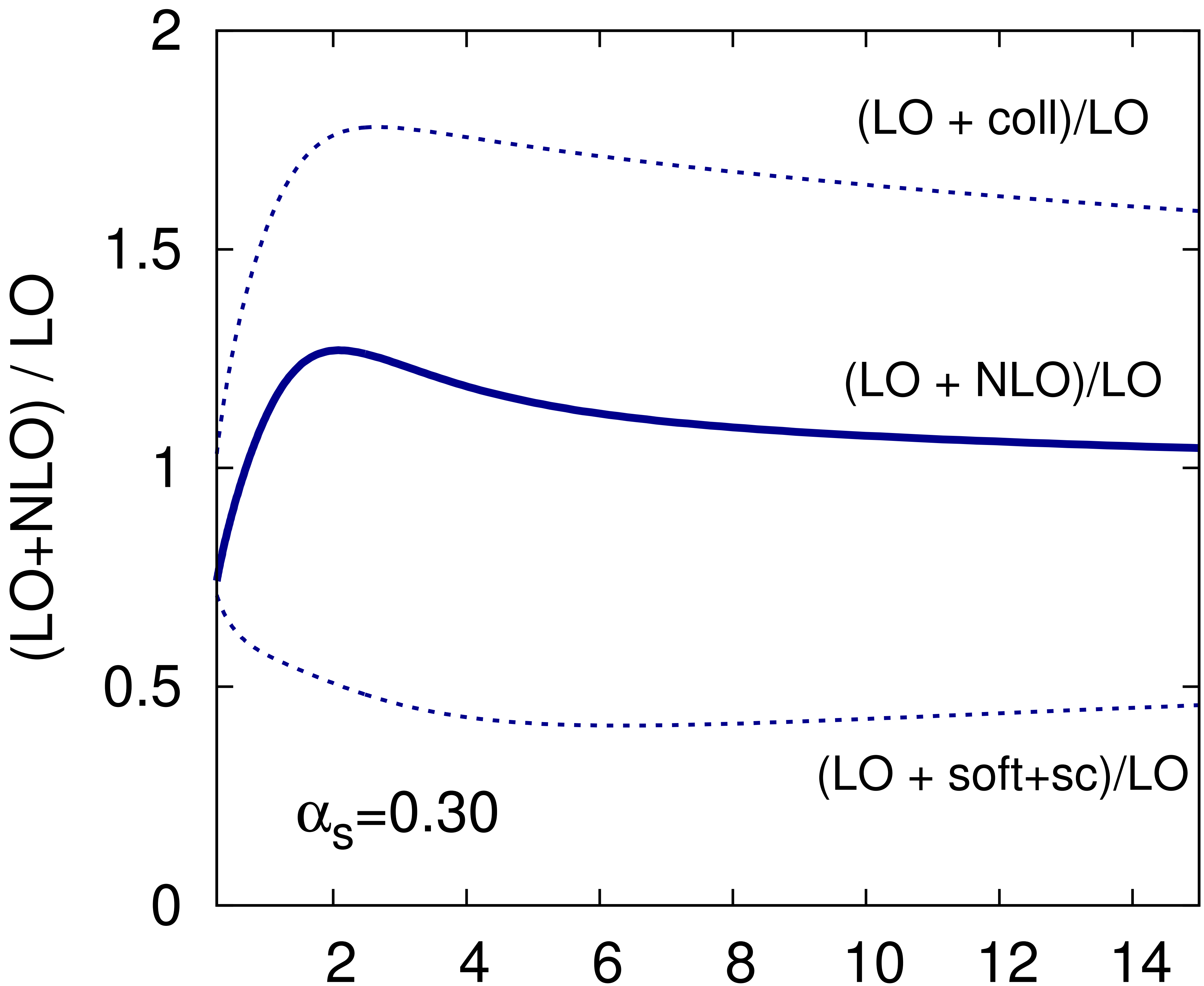
$$(2\pi)^3 \frac{d\delta\Gamma}{d^3k}\bigg|_{\rm NLO} = \mathcal{A}(k)\overbrace{\left[\underbrace{\frac{\delta m_\infty^2}{m_\infty^2}\log\frac{\sqrt{2Tm_D}}{m_\infty} + \frac{\delta m_\infty^2}{m_\infty^2} C_{\rm soft+sc}(k)}_{\delta C_{\rm soft+sc}(k)} + \underbrace{\frac{\delta m_\infty^2}{m_\infty^2}C^{\delta m}_{\rm coll}(k) + \frac{g^2C_AT}{m_D}C^{\delta\mathcal{C}}_{\rm coll}(k)}_{\delta C_{\rm coll}(k)}\right]}^{\delta C_{\rm NLO}(k)}$$



$$(2\pi)^3 \frac{d\delta\Gamma}{d^3k}\bigg|_{\rm NLO} = \mathcal{A}(k)\overbrace{\Bigg[\underbrace{\frac{\delta m_\infty^2}{m_\infty^2}\log\frac{\sqrt{2Tm_D}}{m_\infty} + \frac{\delta m_\infty^2}{m_\infty^2}C_{\rm soft+sc}(k)}_{\delta C_{\rm soft+sc}(k)} + \underbrace{\frac{\delta m_\infty^2}{m_\infty^2}C^{\delta m}_{\rm coll}(k) + \frac{g^2C_AT}{m_D}C^{\delta\mathcal{C}}_{\rm coll}(k)}_{\delta C_{\rm coll}(k)}\Bigg]}^{\delta C_{\rm NLO}(k)}$$



$$(2\pi)^3 \frac{d\delta\Gamma}{d^3k}\bigg|_{\rm NLO} = \mathcal{A}(k)\Bigg[\overbrace{\underbrace{\frac{\delta m_\infty^2}{m_\infty^2}\log\frac{\sqrt{2Tm_D}}{m_\infty} + \frac{\delta m_\infty^2}{m_\infty^2}C_{\rm soft+sc}(k)}_{\delta C_{\rm soft+sc}(k)} + \underbrace{\frac{\delta m_\infty^2}{m_\infty^2}C_{\rm coll}^{\delta m}(k) + \frac{g^2C_AT}{m_D}C_{\rm coll}^{\delta\mathcal{C}}(k)}_{\delta C_{\rm coll}(k)}}^{\delta C_{\rm NLO}(k)}\Bigg]$$



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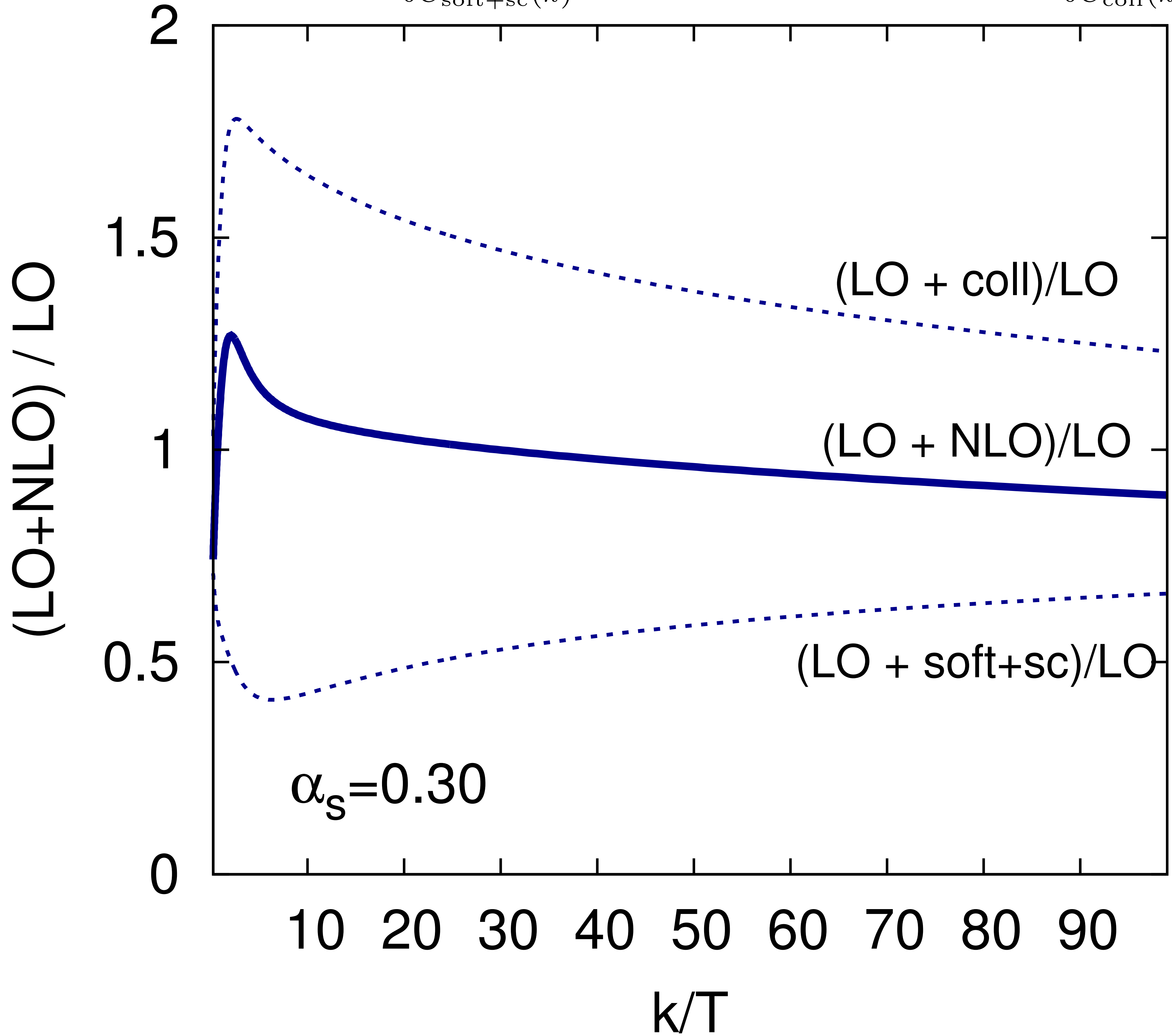
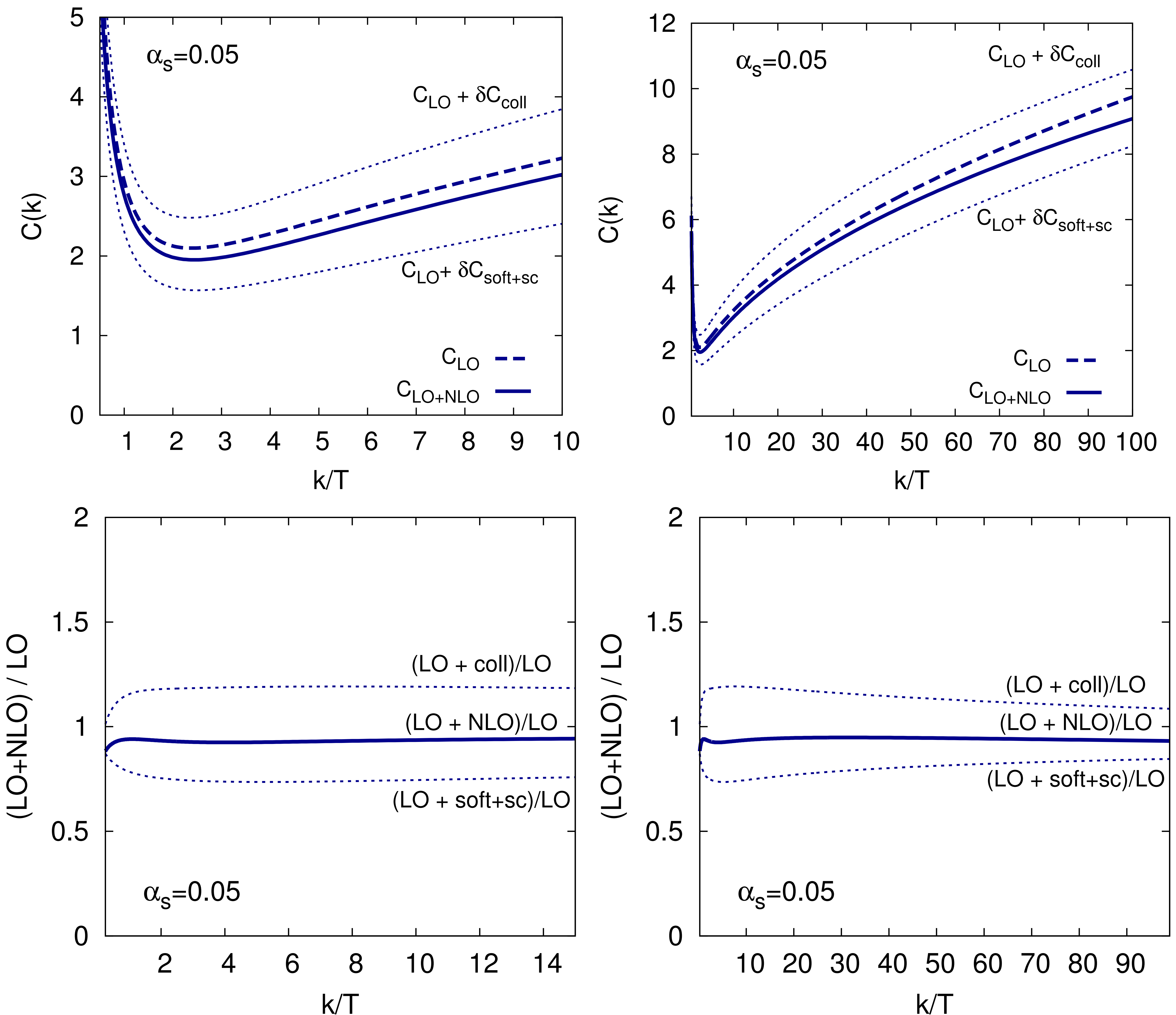




αs=0.05
C_LO + δC_coll
C_LO+ δC_soft+sc
C_LO
C_LO+NLO
C(k)
k/T
(LO + coll)/LO
(LO + NLO)/LO
(LO + soft+sc)/LO
(LO+NLO) / LO

A sneak peek at jets

Jet evolution at LO

- Apply similar technologies to jet evolution and E-loss
- Start from effective Boltzmann-Fokker-Planck approach

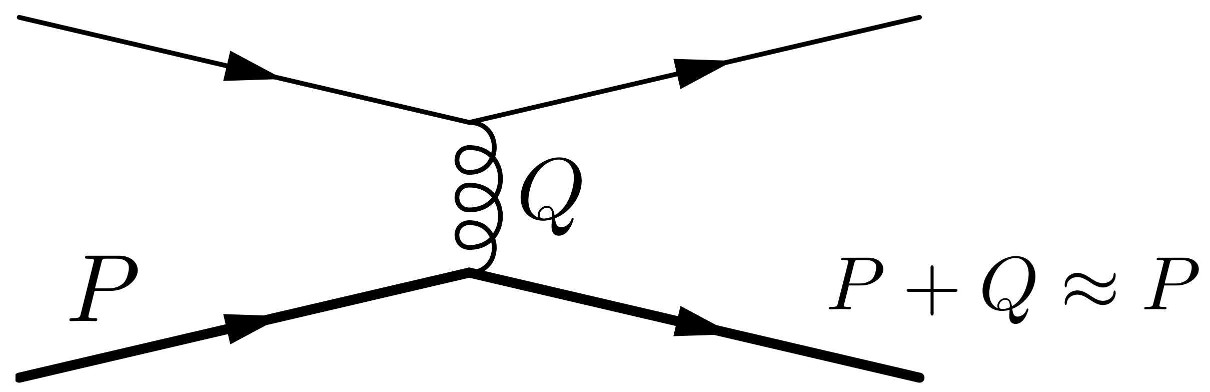
$$\frac{dP(p)}{dt} = \int_{-\infty}^{+\infty} dk \left(P(p+k) \frac{d\Gamma(p+k,k)}{dk} - P(p) \frac{d\Gamma(p,k)}{dk} \right)$$

AMY **JHEP0301** (2003) Jeon Moore **PRC71** (2005)

- 1↔2 and 2↔2 processes in the rates. The former a generalization of the collinear photon emission to gluons. The latter require HTL resummation. In both cases everything but the jet is in equilibrium
- LO rates implemented in MARTINI Schenke Gale Jeon **PRC80** (2009)

Jet evolution at NLO

- Again, need to account for NLO corrections in collinear, semi-collinear and soft regions
- The first two are rather straightforward generalizations of the photon case
- The latter requires some work. In the soft limit 2↔2 exchanges reduce to an energy-loss/momentum diffusion picture



$Q \cdot P \approx 0$ defines new lightcone

Jet evolution at NLO

- Soft limit of the Fokker-Planck equation

$$\frac{dP(p)}{dt} = \int_{-\infty}^{\infty} dq^+ \frac{d\Gamma(p,q^+)}{dq^+} \left(q^+ \frac{dP(p)}{dp^+} + \frac{(q^+)^2}{2} \frac{d^2P(p)}{d(p^+)^2} \right) + \frac{1}{4}\nabla_\perp^2 P(p) \int d^2q_\perp q_\perp^2 \frac{d\Gamma(p,q^+)}{d^2q_\perp}$$

- Energy loss term dE/dt unknown to NLO
- Longitudinal momentum diffusion $\hat{q}_L$ unknown to NLO
- Transverse momentum diffusion $\hat{q}$, known to LO and NLO
- Fluctuation-dissipation $\hat{q}_L = 2T dE/dt$

Longitudinal momentum diffusion

- Field-theoretical lightcone definition

$$\hat{q}_L \equiv \frac{g^2}{d_R}\int_{-\infty}^{+\infty} dx^+ \mathrm{Tr}\left\langle U(-\infty,x^+)F^{+-}(x^+)U(x^+,0)F^{+-}(0)U(0,-\infty)\right\rangle$$

F^{+-}=E^z, longitudinal Lorentz force correlator

- At leading order

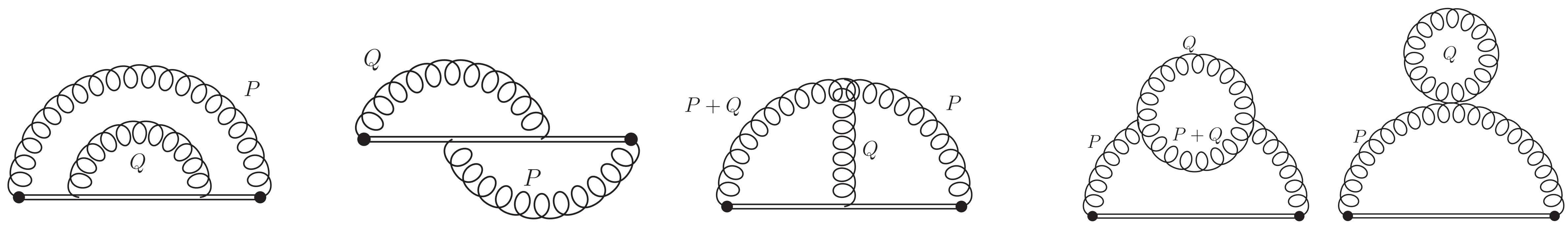
$$\hat{q}_L \propto \int \frac{dq^+d^2q_\perp}{(2\pi)^3}(q^+)^2 G^>_{++}(q^+,q_\perp,0)$$
$$= \int \frac{dq^+d^2q_\perp}{(2\pi)^3} Tq^+(G^R_{++}(q^+,q_\perp,0) - G^A)$$

- Not dominated by zero-mode, but by arcs

$$\hat{q}_L \propto \int \frac{d^2q_\perp}{(2\pi)^2}\frac{Tm_\infty^2}{q_\perp^2+m_\infty^2}$$

Longitudinal momentum diffusion

- At NLO



- Unsurprisingly

$$\hat{q}_L \propto \int \frac{d^2 q_\perp}{(2\pi)^2} \frac{T(m_\infty^2 + \delta m_\infty^2)}{q_\perp^2 + m_\infty^2 + \delta m_\infty^2} = T \int \frac{d^2 q_\perp}{(2\pi)^2} \left[\frac{m_\infty^2}{q_\perp^2 + m_\infty^2} + \frac{\delta m_\infty^2 q_\perp^2}{(q_\perp^2 + m_\infty^2)^2} \right]$$

- Implementation of these results in MARTINI is underway

Conclusions

$$(2\pi)^3 \frac{d\delta\Gamma}{d^3k}\bigg|_{\rm NLO} = \mathcal{A}(k)\Bigg[\overbrace{\underbrace{\frac{\delta m_\infty^2}{m_\infty^2}\log\frac{\sqrt{2Tm_D}}{m_\infty} + \frac{\delta m_\infty^2}{m_\infty^2}C_{\rm soft+sc}(k)}_{\delta C_{\rm soft+sc}(k)} + \underbrace{\frac{\delta m_\infty^2}{m_\infty^2}C_{\rm coll}^{\delta m}(k) + \frac{g^2C_AT}{m_D}C_{\rm coll}^{\delta C}(k)}_{\delta C_{\rm coll}(k)}}^{\delta C_{\rm NLO}(k)}\Bigg]$$

factorization

- The NLO contributio … our terms, with a semicollinear / soft log …
- These four terms com … rge and opposite contributions that lar … ing a relatively small NLO correction. Is the cancellation accidental? At α_s=0.3 the NLO is initially positive, then turns negative and keeps growing at large k/T . At small α_s (α_s=0.05) the correction is always negative
- In the phenomenologically interesting window up to the NLO correction is 10%-20% for αs=0.3

Conclusions

- Apparently complicated dynamical quantities factor into simpler light-cone condensates or operators, which are basically of two kinds
  - Energy-dependent: thermal masses
  - Energy-independent: correlators of the 3D theory
- Application to jet evolution and low invariant-mass dileptons underway. Transport coefficients are harder, but stay tuned