Review of ATLAS & CMS top-quark pair differential cross sections

María Aldaya (CMS, University of Hamburg) Francesco Spanò (ATLAS, Royal Holloway University of London)

for the ATLAS & CMS top-quark pair differential cross section groups



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Bundesministerium für Bildung und Forschung



Outline

- Introduction
- Description of analyses
 - Event selection
 - Kinematic reconstruction
 - Unfolding
- Measurements:
 - ATLAS (I+jets) @ 7 TeV: ATLAS-CONF-2013-099
 - CMS (I+jets, dileptons) @ 7 TeV: Eur. Phys. J. C73 (2013) 2339
 - CMS (I+jets, dileptons) @ 8 TeV: CMS-PAS TOP-12-027, CMS-PAS-TOP-12-028
- First studies at generator level on definition of top quark



SV.=22.9

μτ+Χ

Why measure differentially?

- Precise understanding of top quark distributions is crucial:
 - Precise tests of pQCD for top quark production in different regions of the phase space
 - Theory predictions and models need to be tuned and tested with measurements
 - Extract/use for PDF fits
 - Enhance sensitivity to New Physics
 - Background for Higgs, rare processes and many BSM searches



LHC 2010 – 2012:
 7 TeV: ~ 1 M tt pairs

8 TeV: ~ 5.5 M tt pairs (each ATLAS & CMS)



Entered the era of precision measurements in top-quark-pair production

General analysis strategy

Measure $\sigma(tt)$ as a function of kinematic distributions of top, top pairs, b-jets, leptons, and lepton pairs



Event selection

Lepton+jets:

- Exactly 1 isolated high- p_T lepton (μ or e)
 - CMS: p_T > 30 GeV, $|\eta|$ < 2.1
 - ATLAS: p_T > 25 GeV, $|\eta|$ < 2.5
- Veto additional leptons
- \geq 4 high-p_T jets, \geq 2 b-tagged jets
 - CMS: $p_T > 30$ GeV, $|\eta| < 2.4$
 - ATLAS: p_T > 25 GeV, $|\eta|$ < 2.5
- Additionally: ATLAS: $E_T^{miss} > 30 \text{ GeV}, m_T^W > 35 \text{ GeV}$

Kinematic reconstruction of the $t\bar{t}$ system

Dileptons:

- 2 opp.-sign, high- p_T isolated leptons (ee, $\mu\mu$, μe)
 - $p_T > 20$ GeV, $|\eta| < 2.4$
- QCD veto: m_{\parallel} < 20 GeV (12 GeV for 7 TeV)
- \geq 2 jets (p_T > 30 GeV, |η| < 2.4), \geq 1 b-tagged jets
- ee, $\mu\mu$ channels: $E_T^{miss} > 40 \text{ GeV} (30 \text{ GeV for 7 TeV}),$ $|m_{\parallel} - m_Z| > 15 \text{ GeV}$

Kinematic reconstruction of the $\ensuremath{t\bar{t}}\xspace$ system







Kinematic distributions – *L*+jets (7 TeV, 5 bf-1)

- Pure tī samples after event selection:
 - ~ 80% tī
- Main backgrounds: W+jets(*), tt(dilep), single top, multijet(*)

(*) data-driven normalisation

Reference tt prediction:
 Alpgen+Herwig

Hadronic top: slope observed in data for $p_T > 200 \text{ GeV}$





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Kinematic distributions – dileptons (7 TeV, 5 fb⁻¹)



ytt

400

CMS, 5.0 fb⁻¹ at $\sqrt{s} = 7$ TeV CMS, 5.0 fb⁻¹ at $\sqrt{s} = 7$ TeV 2.5×10³ 10⁵ • Pure tt samples after Events / 0.2 **Dilepton Combined** Data Data **Dilepton Combined** tt Signal tt Signal event selection. tī Other op-quark pairs tt Other 10⁴ Single Top y(tt) Single Top W+Jets W+Jets Z / γ* → ee/μμ ~ 80% tt $Z / \gamma^* \rightarrow ee/\mu\mu$ Ζ/γ* → ττ Z / y* → ττ 10³ Diboson Diboson 1.5 QCD Multijet QCD Multijet N(bjets) Main backgrounds: 10² tt(other), single top, Z+jets (\rightarrow data-driven) 10 0.5 0 2 3 4 5 6 1 Reference tt prediction: N_{b-jets} CMS, 5.0 fb⁻¹ at $\sqrt{s} = 7$ TeV CMS, 5.0 fb⁻¹ at $\sqrt{s} = 7$ TeV MadGraph+Pythia 3<u>×1</u>0⁶ Fop-quark pairs / 10 GeV **Dilepton Combined** Data ZU GeV Data tt Signal **Dilepton Combined** 4.5 tt Signal tī Other Single Top 2.5 tt Other Single Top p_T(tt) p_T(top) W+Jets W+Jets op quarks / Z / γ* → ee/μμ $Z / \gamma^* \rightarrow ee/\mu \overline{\mu}$ 3.5 $Z/\dot{\gamma}^* \rightarrow \tau\tau$ $Z/\dot{\gamma}^* \rightarrow \tau\tau$ Diboson Top p_T spectrum tends to Diboson QCD Multijet QCD Multijet 1.5 lower p_T values in data 2.5 than in simulation 2 1.5⊢ 0.5 1₽ 0.5 0<mark>6</mark> 50 100 150 200 250 300 200 300 350 50 100 150 250 p^{tt} [GeV] pt [GeV]

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IUTLHUWG, 20.11.13

Results: ATLAS & CMS (7 TeV, *l*+jets) – y(tł) 🔀 😤

 CMS: Comparison to MadGraph+Pythia, MC@NLO+Herwig, POWHEG+Pythia, POWHEG+Herwig ATLAS: Comparison to ALPGEN+Herwig, MC@NLO+Herwig, POWHEG+Herwig



 Similar behaviour for dileptons, both at 7 and 8 TeV

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show similar behaviour

Results: ATLAS & CMS (7 TeV, *l*+jets) – y(tł) 🎽 💱

First attempt at direct data comparison: data/NLO prediction (MCFM)



General good agreement between ATLAS & CMS results, within uncertainties

Results: ATLAS & CMS (7 TeV, *l*+jets) – p_T(tt) 🔀 🕸

 CMS: Comparison to MadGraph+Pythia, MC@NLO+Herwig, POWHEG+Pythia, POWHEG+Herwig ATLAS: Comparison to ALPGEN+Herwig, MC@NLO+Herwig, POWHEG+Herwig



- General good agreement between data & simulation, both for ATLAS & CMS
- CMS: Similar behaviour for dileptons, both at 7 and 8 TeV

Results: ATLAS & CMS (7 TeV, *l*+jets) – p_T(tt) 🔀 🐒

First attempt at direct data comparison: data/NLO prediction (MCFM)





Results: ATLAS & CMS (7 TeV, *l*+jets) – m(tł) 🔀 🐒

First attempt at direct data comparison: data/NLO prediction (MCFM)





Results: ATLAS & CMS (7 TeV, *l*+jets) – p_T(top) 🔀 🐒

First attempt at direct data comparison: data/NLO prediction (MCFM)



p_T(top) < 200 GeV:

- Disagreement between ATLAS & CMS data
- ATLAS result in agreement with MCFM

p_T(top) > 200 GeV:

- Good agreement between ATLAS & CMS data
- ATLAS & CMS in disagreement with MCFM

TOPLHCWG meeting

Ttbar differential cross-section

M. Aldaya Martin, L. Bellagamba, M. Goerner, F. Spano' for the differential cross-section analysis group from ATLAS and CMS

17/10/2013

- The idea is to define an efficient process in order to compare the respective measurements and understand possible differences in procedures and/or results

- First discussions among analyzers and conveners involved in the analysis started at TOP2013.

- The first step was the exchange of a list of questions/clarifications between the two experiments. Starting from such lists, a summary set of relevant items has been produced which could represent a draft agenda for a next close meeting between the respective analyzer teams.

Comparison based on 7 TeV results in I+jets for p_T(top), m(tt
), p_T(tt), y(tt
)

ATLAS & CMS to-do list (I)

Definition of the top quark: "top quark at parton level after QCD radiation"

- Which is exactly the top parton used for the unfolding ?
- Check for possible differences between Pythia and Herwig

MC samples and theory predictions (NLO+PS, multi-leg tree-level+PS, MCFM, NLO +NNLL, approx. NNLO):

- Exchange parameters and tunes used for the signal MC samples
- For each signal MC sample used provide a tree with generator-level information to exchange with the other experiment
- For the multi-leg tree-level signal MC check possible differences in the treatment of topologies with large numbers of extra partons.
- Check if the NLO+NNLL and approx. NNLO predictions are treated consistently in both experiments

Selection & Analysis + background uncertainties:

- Check consistency in purity and efficiency definition: in particular is tau+jets considered as signal or background ?
- Provide detailed efficiency tables separated for electron and muon channels.
- Exchange details on the treatment of uncertainties, in particular the background shape uncertainties

ATLAS & CMS to-do list (II)

Unfolding: exchange details of the procedure

- Are corrections (migrations at reco level and correction from reco to parton level) performed in one step?
- Exchange response matrices
- How is the regularisation performed?
- How large is the extrapolation to the full phase space?

Results:

- Provide detailed systematic tables for the different spectra and channels in order to compare the impact of the different sources of systematic uncertainties.
- Provide ratio plots for a direct data/prediction comparison both for reco and unfolded level results.

Suggestions for further checks are welcome !

First study:

- Check definition of top parton after QCD radiation and before decay

- Consistency of the MC samples at generator level used in the ATLAS & CMS differential cross section analyses

MC tt samples: parameters & tunes (7 TeV)



ATLAS

Default samples

Matrix element	Shower & Hadronization	PDF	Tune
MC@NLO v4	Herwig 6.5 + Jimmy 4.31	cteq66 or CT10	AUET1/2
Powheg	Pythia 6	cteq66 (7 TeV)	Perugia 2011 C
Alpgen	Herwig 6.5 + Jimmy 4.31	cteq6ll	AUET2

Additional Powheg+Herwig sample: NLO PDF CT10, AUET2 Herwig 6.5 tune

CMS

Matrix element	Shower & Hadronization	PDF	Tune
MadGraph v5	Pythia 6	cteq6l	Z2 (7 TeV)
Powheg	Pythia 6	ctea6m (7 TeV)	Z2 (7 TeV)
MC@NLO v3.4	Herwig 6 + Jimmy	cteq6m	default tune

• Additional Powheg+Herwig sample: NLO PDF CTEQ6M, AUET2 Herwig 6 tune

All CMS samples overview: p_T(top)

Ratio wrt CMS Powheg+Pythia



Low p_T region mostly similar

- Some differences for p_T > 300 GeV between Pythia (green, red) and Herwig (pink, blue)
- Different behaviour of Powheg+Herwig, especially for p_T < 100 GeV

- Powheg+Herwig provides reasonable description of the data both for ATLAS and CMS
- Similar behaviour in ATLAS & CMS

PYTHIA-SHOWERED samples vs. **HERWIG-SHOWERED** samples



Consistent top quark definition in HERWIG- & PYTHIA-showered samples btw ATLAS & CMS:

- events produced with the same generator and parton shower/hadronization scheme are compatible within the statistical uncertainty of the samples
- **Powheg+Herwig**: different shape over the whole p_T spectrum, both for ATLAS & CMS

PYTHIA-SHOWERED samples vs. **HERWIG-SHOWERED** samples



Consistent top quark definition in HERWIG- & PYTHIA-showered samples btw ATLAS & CMS:

- events produced with the same generator and parton shower/hadronization scheme are compatible within the statistical uncertainty of the samples
- MadGraph+Pythia (Alpgen+Herwig) more central than other Pythia- (Herwig-) showered samples

POWHEG SAMPLES: Pythia vs. Herwig

Ratio wrt CMS Powheg+Herwig



Given a generator, **the relative differences between PYTHIA & HERWIG** are observed by both experiments with sizes that **are consistent** within the statistical uncertainties of the samples

Powheg+Herwig: different shape over the whole p_T spectrum, both for ATLAS & CMS

Summary & outlook

- ATLAS & CMS tt differential cross sections
 - Largely consistent with SM predictions
 - Some tension between ATLAS & CMS at low $p_T(top)$ values
 - → Collaboration between both experiments started to understand differences
- First comparison between ATLAS & CMS
 - ATLAS and CMS have consistent definition of the top quark (top parton after radiation)
 - Compatible behaviour in corresponding sample pairs: same differences between generators and hadronisation schemes
 - Default generators (CMS MadGraph+Pythia, ATLAS Alpgen+Herwig) are similar, consistent within statistical uncertainties
- Powheg+Herwig describes p_T(top) in data better, both for ATLAS & CMS
- → Question for theorists/MC experts:

What is the main difference to other generators where the $p_T(top)$ distribution is different?

- Next steps in ATLAS & CMS comparison:
 - consistency of theory precictions, data/MC comparison, further cross-checks on unfolding, etc

Additional information

Summary of differential $\sigma(t\bar{t})$ measurements

	CMS	ATLAS
channels	e/μ+jets ee/μμ/eμ 7 and 8 TeV	e/μ+jets 7 TeV
simulation default	MadGraph+ <mark>Pythia</mark>	Alpgen+Herwig
simulation cross check	Powheg+Pythia, MC@NLO+Herwig (Powheg+Herwig)	g, Powheg+Herwig, MC@NLO+Herwig
event reconstruction	I+jets: kinematic fit dilepton: ~MWT, v spectrum (MC)	log likelihood fitter
unfolding	regularized unfolding (continuous) regularisation parameter)	SVD unfolding (integer regularisation parameter)
phase space	top/tt: fully extrapolated (status 3) lepton/b-jet: visible particle level	top/tt: fully extrapolated (status 155)
background	Z->II data driven, others: MC	data driven N(QCD&W+jets) shape: MC, others: MC

All CMS samples overview: p_T(tt)

Ratio wrt CMS Powheg+Pythia



- Compatible ATLAS & CMS behaviour in corresponding sample pairs
- Different behaviour of Powheg+Pythia for p_T > 200 GeV

• Vary 4-momenta of leptons.







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Kinematic reconstruction – dileptons





- Underconstrained system (2 neutrinos)
- Constraints:

•
$$p_{x,y}(v_1) + p_{x,y}(v_2) = E_T^{miss}_{x,y}$$

- $m_{top} = m_{antitop} = fixed$
- \bullet Vary m_{top} in 1 GeV steps: 100 300 GeV
- Take solutions with most b-tagged jets
- Choose solution with best reconstructed neutrino energy with respect to simulated spectrum



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Kinematic distributions – dileptons (8 TeV, 12 fb⁻¹)

- Pure tt samples after event selection:
 - ~ 80% tt
- Main backgrounds: tt(other), single top, Z+jets
- Reference tt prediction: MadGraph+Pythia



Unfolding: correcting for detector effects & acceptances



Binning

Chosen to limit migration effects, quantify with:

60 80

• CMS: purity (p^i) & stability (s^i): ≥ 0.4

 $p^{i} = rac{N^{i}_{rec\&gen}}{N^{i}_{rec}}$ $s^{i} = rac{N^{i}_{rec\&gen}}{N^{i}_{gen}}$

• ATLAS: experimental resolution, optimised to minimise uncertainty on final result

Regularisation

 Basic unfolding: simple inversion of response matrix A_{ii}:

$$\mathbf{N}_{i,\mathrm{unf}} = A_{ij}^{-1} \mathbf{N}_{j,\mathrm{meas}}$$

 Regularisation used to remove large statistical fluctuations (SVD)



Phase space (PS)

Correct back to parton or particle level in full or visible phase space (variable dependent) CMS: also visible PS

MadGraph MC@NLO

POWHEG

All top and tf quantities are corrected to parton level after QCD radiation, in full PS

ATLAS & CMS use different criteria to choose the regularisation parameter

More info: $1/\sigma d\sigma/dX @ ATLAS - values (7 TeV)$

- Systematics determined individually for each bin of the measurement
- Normalized cross sections: only shape uncertainties contribute, correlated uncertainties cancel
- Main systematics: JES , signal generator, btagging, ISR/FSR for p_T(tt)

p_{T}^{t} [GeV]	$\frac{1}{\sigma} \frac{\mathrm{d}\sigma}{\mathrm{d}p_{\mathrm{T}}^{l}} \left[10^{-3} \right]$	stat. [%]	syst. [%]		$p_{\rm T}^{t\bar{t}}$ [GeV]	$\frac{1}{\sigma} \frac{d\sigma}{d\sigma^{ii}} \left[10^{-3} \right]$	stat. [%]	syst. [%]
0 to 50	3.4±0.1	±2	± 4		0 to 10	14:2	. 2	+ 10
50 to 100	6.7±0.1	±1	± 1		010 40	14±2	±J	± 10
100 to 150	5.2±0.1	± 2	± 2		40 to 170	3.1 ± 0.4	±2	± 10
150 to 200	2.66±0.08	± 2	± 3		170 to 340	0.25 ± 0.06	± 4	± 20
200 to 250	1.14 ± 0.04	± 2	± 3		340 to 1000	0.010 ± 0.003	± 8	± 30
250 to 350	0.33 ± 0.02	± 3	± 5					
350 to 800	$0.018 {\pm} 0.002$	± 6	± 10		y _{tī}	$\frac{1}{\sigma} \frac{d\sigma}{dy_{t\bar{t}}} \left[10^{-3} \right]$	stat. [%]	syst. [%]
	1 1 [0]			1	-2.5 to -1.0	81±3	± 2	± 3
$m_{t\bar{t}}$ [GeV]	$\frac{1}{\sigma} \frac{\mathrm{d}\sigma}{\mathrm{d}m_{t\bar{t}}} \left[10^{-3} \right]$	stat. [%]	syst. [%]		-1.0 to -0.5	321±9	±1	± 3
250 to 450	2.50 ± 0.08	±1	± 3		-0.5 to 0.0	436±9	+1	+2
450 to 550	2.73 ± 0.07	± 1	± 2		0.0 to 0.5	402+7	. 1	+ 1
45010 550	21/02010/							
550 to 700	1.02 ± 0.04	± 2	± 4		0.010 0.J	425±7	± 1	± 1
550 to 700 700 to 950	1.02 ± 0.04 0.23 ± 0.01	± 2 ± 3	± 4 ± 4		0.5 to 1.0	425±7 321±5	±1 ±1	±1
550 to 700 700 to 950 950 to 2700	1.02 ± 0.04 0.23 ± 0.01 0.0076 ± 0.0005	± 2 ± 3 ± 4	± 4 ± 4 ± 5		0.5 to 1.0 1.0 to 2.5	425±7 321±5 87±5	± 1 ± 1 ± 3	± 1 ± 1 ± 4

More info: $1/\sigma d\sigma/dX @ CMS - values (7 TeV)$

$p_{\mathrm{T}}^{\mathrm{t}}$ bin [GeV]	$1/\sigma \mathrm{d}\sigma/\mathrm{d}p_\mathrm{T}^\mathrm{t}$	stat. [%]	sys. [%]	total [%]
0 to 60	$4.54 \cdot 10^{-3}$	2.5	3.6	4.4
60 to 100	$6.66 \cdot 10^{-3}$	2.4	4.9	5.5
100 to 150	$4.74 \cdot 10^{-3}$	2.4	3.2	4.0
150 to 200	$2.50 \cdot 10^{-3}$	2.6	5.1	5.8
200 to 260	$1.04 \cdot 10^{-3}$	2.9	5.5	6.2
260 to 320	$0.38 \cdot 10^{-3}$	3.7	8.2	9.0
320 to 400	$0.12 \cdot 10^{-3}$	5.8	9. 5	11.1

$p_{\rm T}^{ m t\bar t}$ bin [GeV]	$1/\sigma \mathrm{d}\sigma/\mathrm{d}p_{\mathrm{T}}^{\mathrm{t}\overline{\mathrm{t}}}$	stat. [%]	sys. [%]	total [%]
0 to 20	$1.50 \cdot 10^{-2}$	4.1	11.8	12.5
20 to 45	$1.21 \cdot 10^{-2}$	3.5	7.0	7.8
45 to 75	$0.58 \cdot 10^{-2}$	3.8	9.2	10.0
75 to 120	$0.26 \cdot 10^{-2}$	4.3	14.0	14.6
120 to 190	$0.10 \cdot 10^{-2}$	4.5	7.8	8.9
190 to 300	$0.02 \cdot 10^{-2}$	6.3	18.0	19.1

m ^{tī} bin [GeV]	$1/\sigma \mathrm{d}\sigma/\mathrm{d}m^{\mathrm{t}\mathrm{t}}$	stat. [%]	sys. [%]	total [%]
0 to 345	-	-	-	-
345 to 400	$4.81 \cdot 10^{-3}$	5.2	9.7	11.1
400 to 470	$4.60 \cdot 10^{-3}$	5.0	8.4	9.8
470 to 550	$2.46 \cdot 10^{-3}$	5.2	10.2	11.4
550 to 650	$1.14 \cdot 10^{-3}$	5.6	10.6	12.0
650 to 800	$0.43 \cdot 10^{-3}$	6.2	8.3	10.3
800 to 1100	$0.99 \cdot 10^{-4}$	7.1	20.0	21.2
1100 to 1600	$0.14 \cdot 10^{-4}$	13.5	19.4	23.7

y ^{tī} bin	$1/\sigma d\sigma/dy^{t\bar{t}}$	stat. [%]	sys. [%]	total [%]
-2.5 to -1.3	$0.55 \cdot 10^{-1}$	6.4	10.8	12.5
−1.3 to −0.9	$2.17 \cdot 10^{-1}$	3.4	5.8	6.7
-0.9 to -0.6	$3.12 \cdot 10^{-1}$	3.6	4.4	5.7
-0.6 to -0.3	$4.00 \cdot 10^{-1}$	3.1	3.3	4.5
-0.3 to 0.0	$4.35 \cdot 10^{-1}$	3.1	4.1	5.1
0.0 to 0.3	$4.69 \cdot 10^{-1}$	2.8	3.8	4.8
0.3 to 0.6	$3.94 \cdot 10^{-1}$	3.1	5.9	6.7
0.6 to 0.9	$3.17 \cdot 10^{-1}$	3.4	4.7	5.8
0.9 to 1.3	$2.22 \cdot 10^{-1}$	3.3	5.8	6.6
1.3 to 2.5	$0.50 \cdot 10^{-1}$	6.8	9.7	11.9

More info: $1/\sigma d\sigma/dX @ CMS - syst (7 TeV)$

- Determined individually for each bin of the measurement
- Normalized cross sections: only shape uncertainties contribute, correlated uncertainties cancel

	Source	Method	Systemat	ic uncertainty (%)
			ℓ+jets	dileptons
	Background	vary with 30%-50%	3.5	0.5
	Trigger eff.	p_{T} - η dependent	0.5	1.5
-	Lepton sel.	p_{T} - η dependent	0.5	2.0
Experimental	Jet energy scale	p_{T} - η dependent	1.0	0.5
	Jet energy resolution	$p_{\rm T}$ - η dependent	0.5	0.5
	Pileup	vary $\sigma_{\rm inel.}(\rm pp)\pm8\%$	0.5	0.5
	b tagging	p_{T} - η dependent	1.0	0.5
	Kinematic reco	p_{T} - η dependent	-	0.5
	Q^2	vary factor 0.25-4	2.0	1.0
	ME/PS threshold	vary factor 0.5-2	2.0	1.0
Model	Hadronisation	PYTHIA vs. HERWIG	2.0	2.0
	Top-quark mass	172.5 ± 0.9	0.5	0.5
	PDF choice	PDF4LHC	1.5	1.0

Typical values per bin at 7 TeV

More info: $1/\sigma d\sigma/dX @ CMS - phase space$





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More info: $1/\sigma d\sigma/dX @ CMS - unfolding$

response Matrix $A_{ij} := \frac{N_{rec}^{j \rightarrow i}}{N_{gen}^{j}}$

 transition probability from generator bin j to reconstructed bin i, includes efficiency and acceptance

the corrected and **unfolded event yield** xⁱ is obtained from BG

$$\sum_{i} A_{ij} x^j = N^i_{Sig}$$

this is equivalent to solve the following χ^2 problem:

 $\chi^2_A(\vec{x}) := \left(A\vec{x} - \vec{N}_{Sig}\right)^T COV_{\vec{N}_{Sig}} \left(A\vec{x} - \vec{N}_{Sig}\right) \quad (COV: \text{ covariance matrix})$ $\begin{array}{l} \textbf{regularization is needed to give (non-oszillating) stable results} \\ \rightarrow \text{ add penalty term} \\ \chi^{2}\left(\vec{x}\right) := \chi^{2}_{A}\left(\vec{x}\right) + \tau^{2} \cdot \chi^{2}_{L}\left(\vec{x}\right) \\ \chi^{2}_{L}\left(\vec{x}\right) := \sum_{ij} \frac{x^{i}}{N_{gen}^{i}} L_{ij}^{2} \frac{x^{j}}{N_{gen}^{j}} \\ \end{array}$ (curvature matrix) too few regularization: large negative correlations (oszillating results) too much regularization: large positive correlations (bias) \rightarrow choose unfolding parameter τ such that average squared global correlation ρ is minimal: 70 $\bar{\rho}(\tau) := \frac{1}{n} \sqrt{\sum_{i} \rho_i(\tau)^2} \qquad \rho_i := \max_{(\alpha_1, \dots, \alpha_n)} \rho\left(x^i, \sum_{i \neq i} \alpha_j x^j\right)$ 10-1 1 10 102 10 Parameter t