“MT2” observables : where to ?

or

Something about kinematic variables.

ZPW2014

”ZPW2014 Monte Carlo Simulation” and “2nd Mini-Workshop on Advances in Matrix Element Methods”

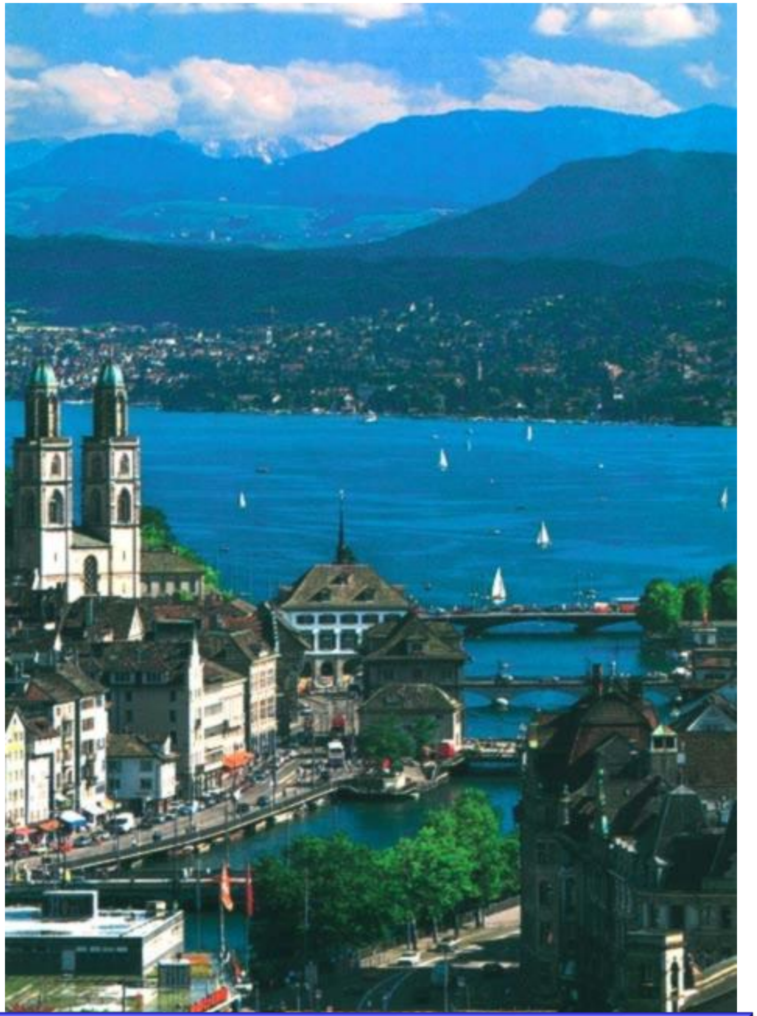
January 2014

Christopher Lester
University of Cambridge

Why am I here?

Kyle Cranmer:

- "we'd like to compare and contrast with searches based on kinematic variables ... with those of matrix element methods"



Discover in Zurich talk title on indico (not supplied by me)

- "MT2 observables :

  where to?"

Not necessarily compatible aims!

My interpretation ...

- Talk about why appropriate kinematic variables assist matrix element methods
  - Robust
  - Simple (often ~1D)
  - Make sensible use of physics

PROBLEM ...

- I don’t work with Matrix Element Methods ...
- ... so I guess what you do.

Pinnacle of aspiration ...

Land of milk (chocolate) and honey !

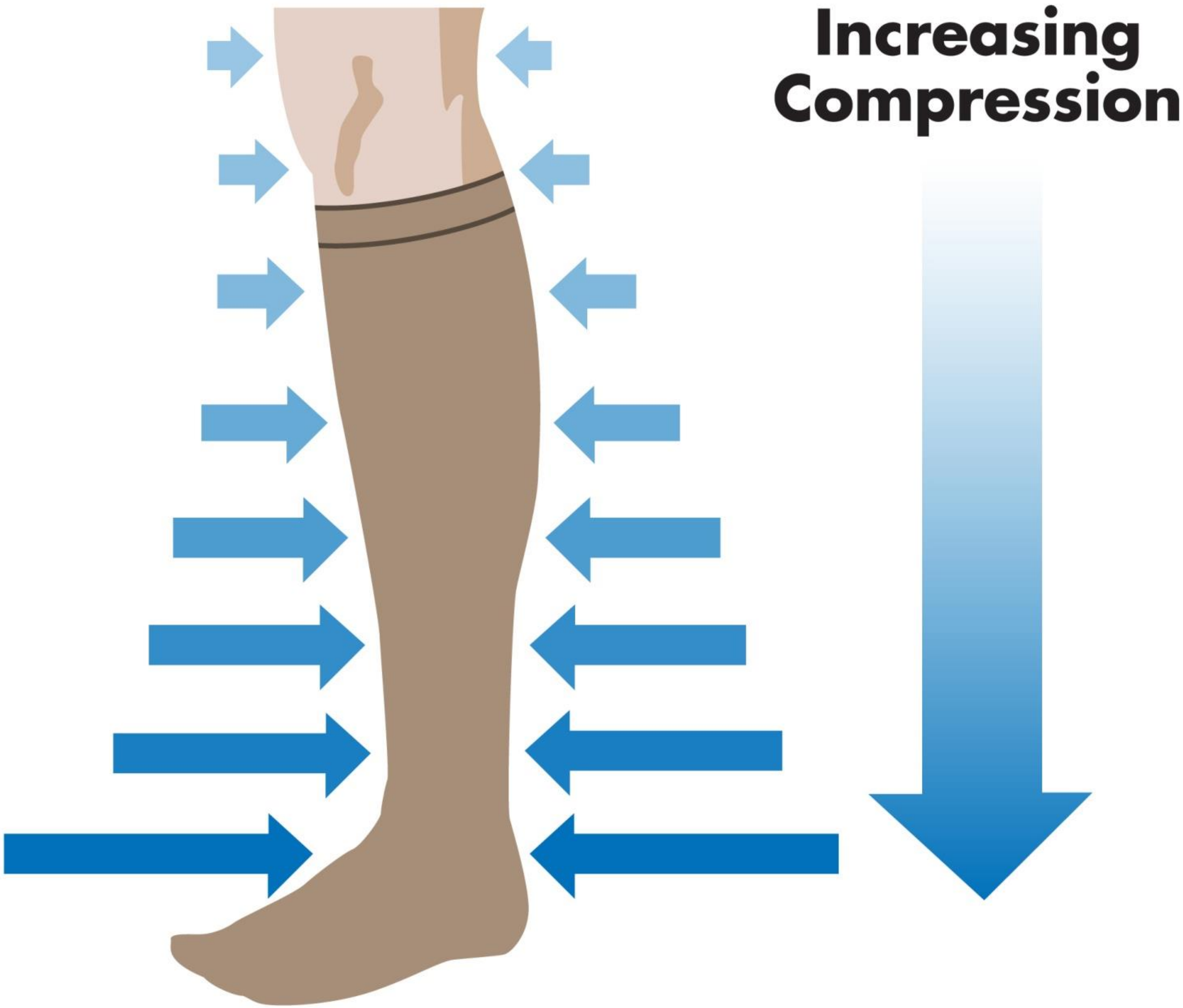
Neyman-Pearson Lemma ?

PROBLEM ...

- I discover yesterday that:
  - EITHER I'm more of an idealist than the MEM people
  - OR MEM people are more **sensible / pragmatic** than I expected!

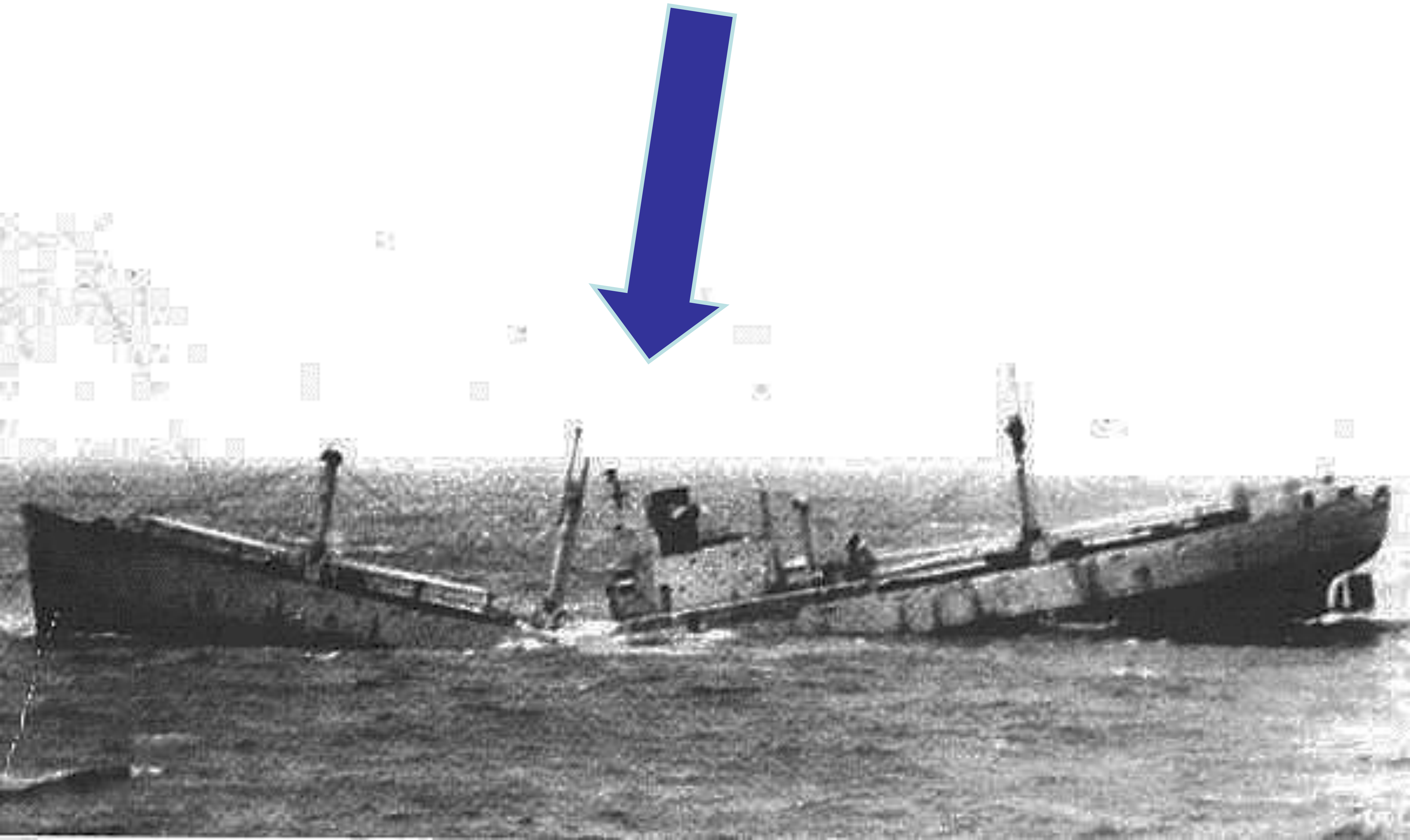
- Seems to be very little difference between what we are doing!
  - (computing time and integrals excepted)

Too sensible ?



Or just well prepared?

This is what you did to my talk !



You are:

- Making event hypotheses

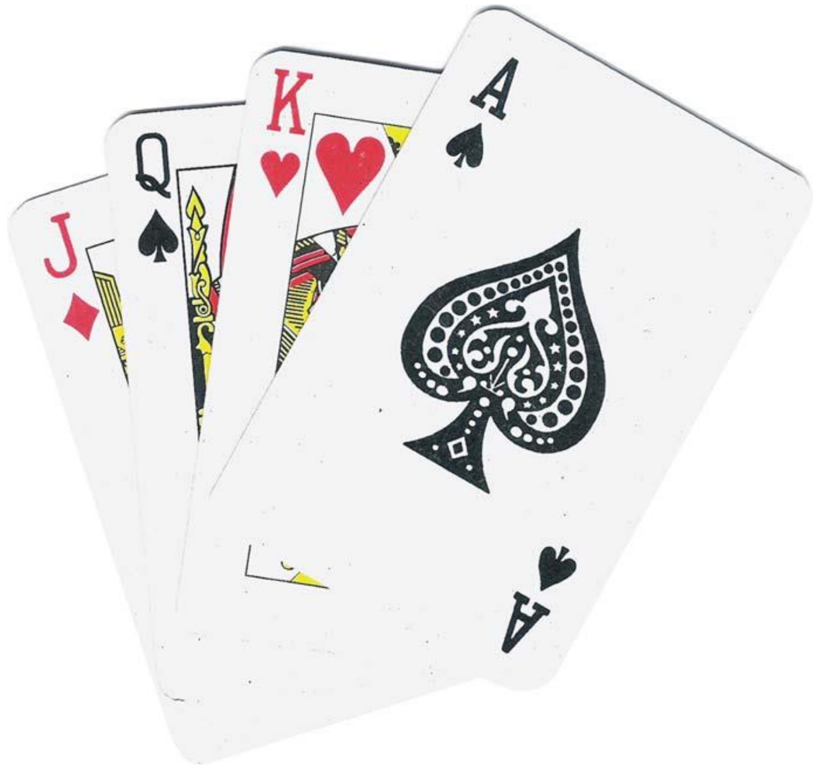
You are:

- Making event hypotheses
- Putting particles on mass shell (mostly)
- Worrying about unobserved momenta
- Making approximations
- Trying to make good use of physics
- Want to understand/believe the result

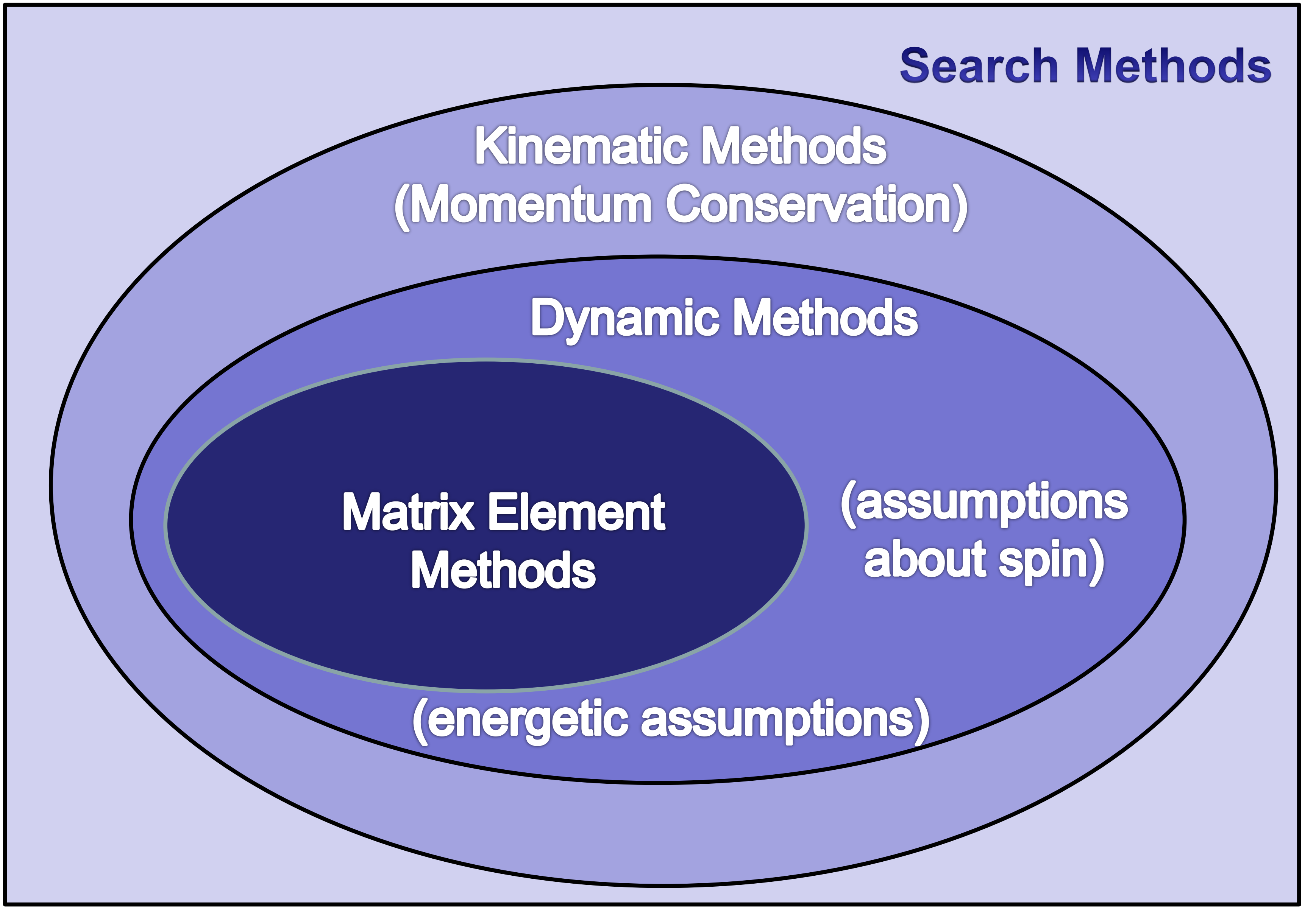
Kinematic variables are ...

- Doing all the same things, but ..
- Aiming not quite as high,
- Hoping for something more robust or at least debuggable

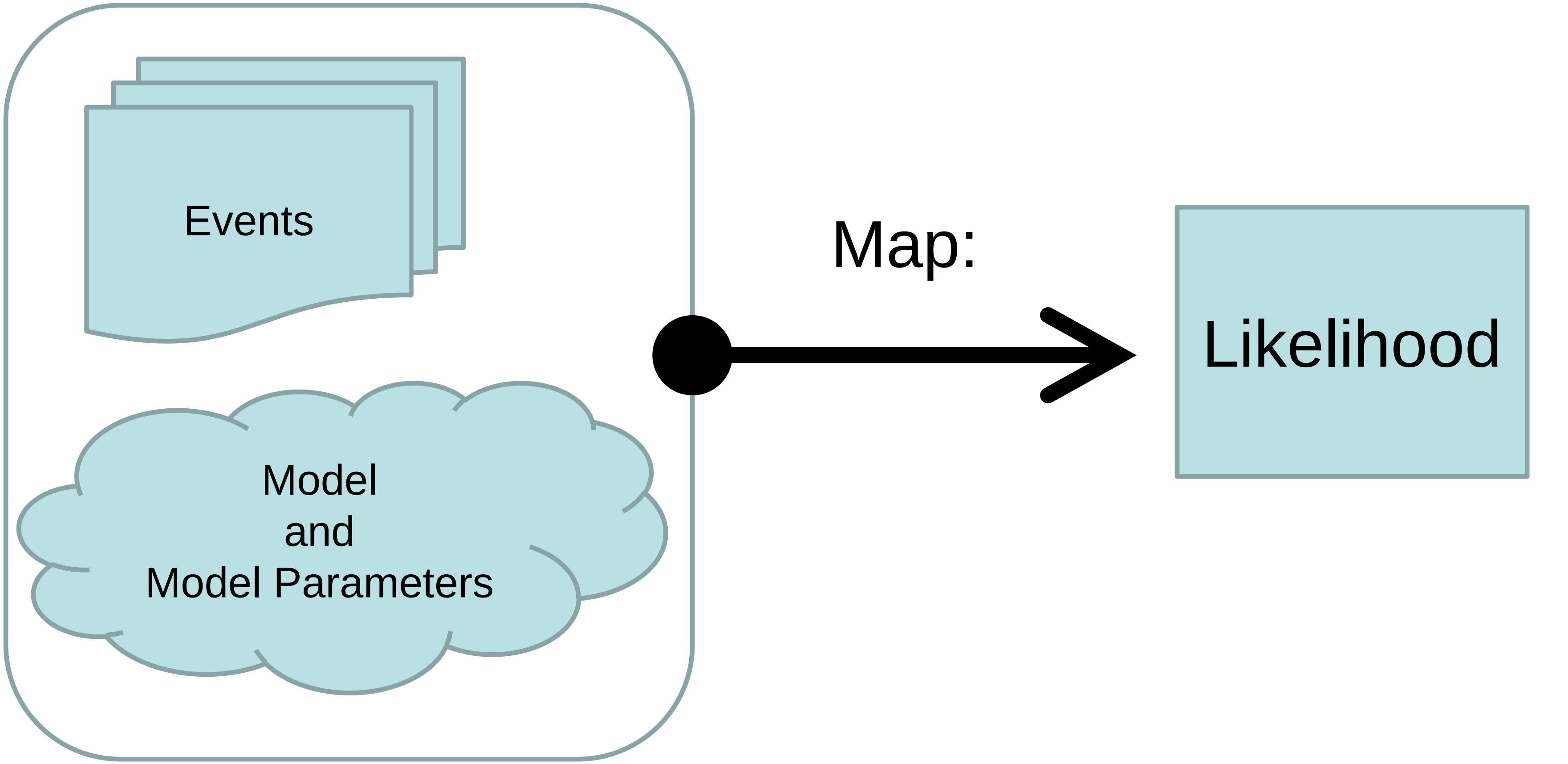
Lay all my cards on the table



Terminology / definitions



Idealised Matrix Element Method

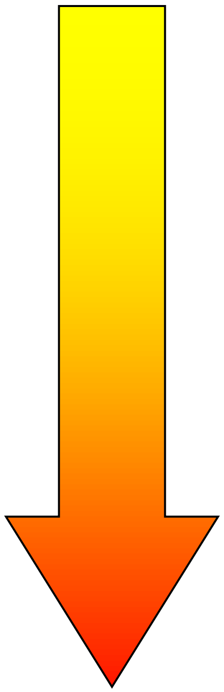


What is the event space?

- Just a dijet invariant mass ?
  - Have we thown away too much info?
- A small set of inputs for a BDT ?
  - Events that don’t match these criteria?
- All particles and momenta?
  - Fakes?
- Candidates?
  - Perhaps – but can’t do sub-jet analysis, say.
- Energy deposits?
  - Can’t do out-of-time events, long lived, ...
- 0111011010111000010101010?
  - Ouch!

Few

things



Many

things

- The space on which the “Matrix Element Map” acts is not well defined !

- Level depends on taste/preference/interest.

- Not all MEMs are equal.

- Why the focus on particles and momenta?

$$dSigma = BLAH\ |M|^2\ d\ PhaseSpace$$

Bad news

- A fit to a 1D plot of a dilepton invariant mass distribution with and without a signal MC is, in effect, a MEM !!

Good news

- BDTs only work because they have the right variables in them – turning them into MEMs.

“we get the BDT variables from the literature”

Has anyone in the room fed this to a BDT :

- $v1 = p1x$,
- $v2 = p1y$,
- $v3 = p1z$,
- $v4 = p2x$,
- $v5 = p2y$,
- $v6 = p2z$

... and got back anything like this:

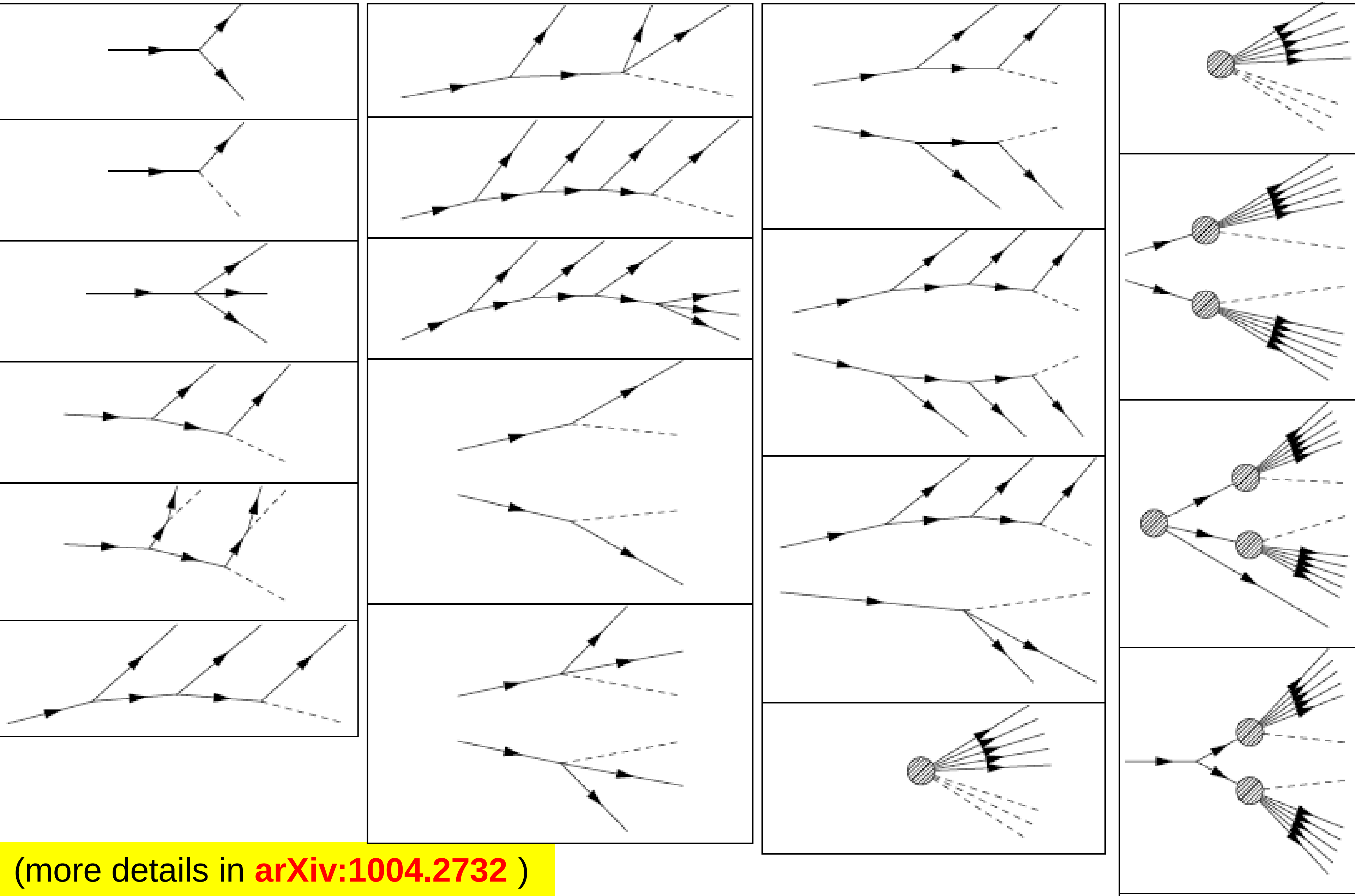
$$(\mathrm{sqrt}(v1^2 + v2^2+v3^2) + \mathrm{sqrt}(v4^2 + v5^2 + v6^2))^2 - (v1 + v4)^2 - (v2+v5)^2 - (v3-v6)^2$$

?

Sensible event variables

- Capture important information in event
- Make good use of physics hypothesis

(all kinematic variables start with a hypothesis)



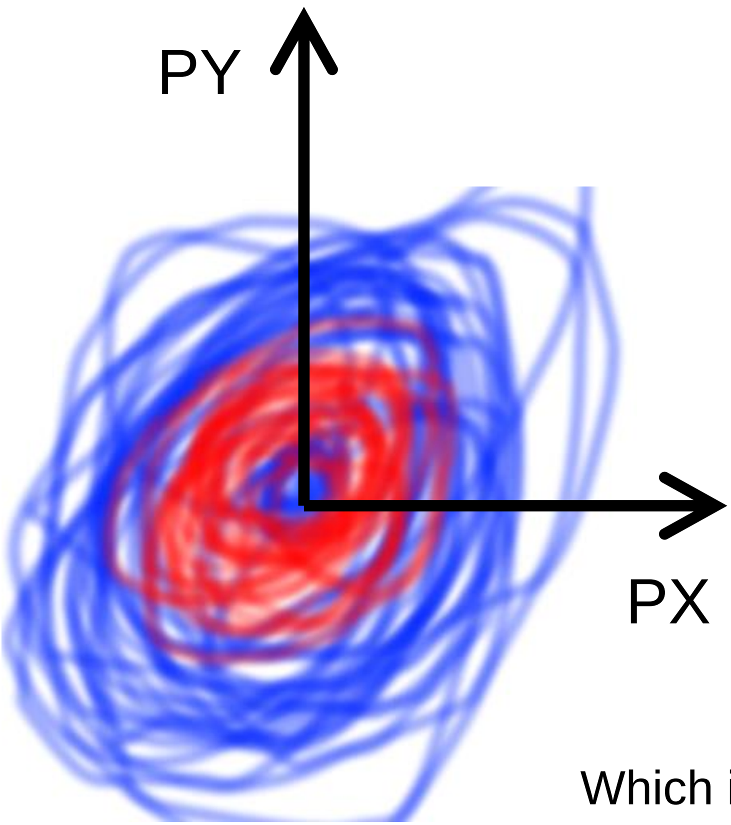
(more details in **arXiv:1004.2732**)

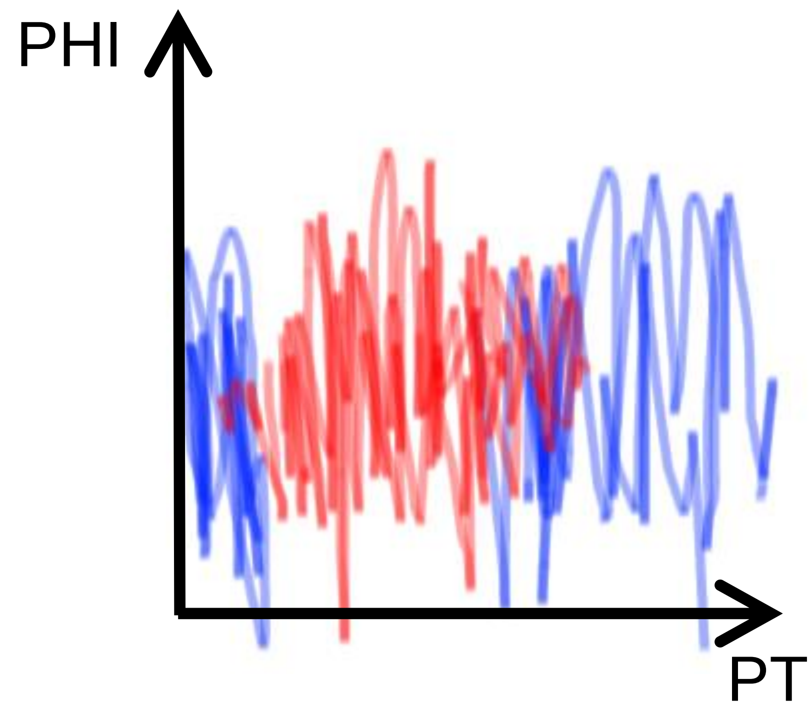
Sensible event variables

- Capture important information in event
- Make good use of physics hypothesis
- Make us insensitive to irrelevant information
  - This is a process of DE-CORRELATION
    (dumb BDTs struggle with strange shapes)
    or FACTORISATION

Obvious example of factorisation

- Absolute origin of azimuthal angle phi is (almost) always irrelevant at the LHC





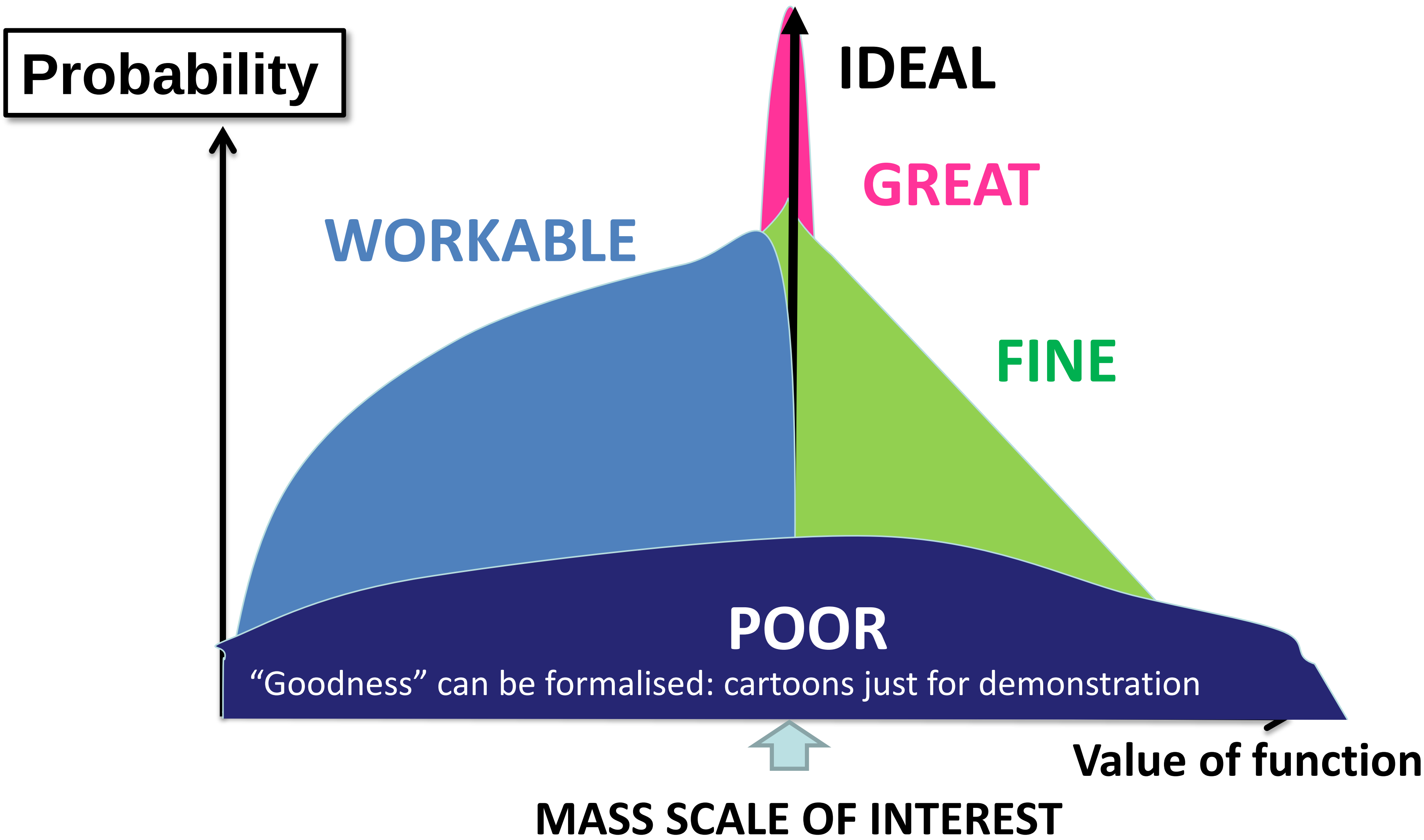
Which is easier to work with in a BDT?

Sensible event variables

- Capture important information in event
- Make good use of physics hypothesis
- Make us insensitive to irrelevant information
  - This is a process of DE-CORRELATION
    (dumb BDTs struggle with strange shapes)
    or FACTORISATION
- Have beneficial properties for both signal and background (VERY HARD – many supposedly useful kinematic variables fail this requirement)

Good vs poor variables

(For this slide, am indebted to Alan Barr, Merton Col)

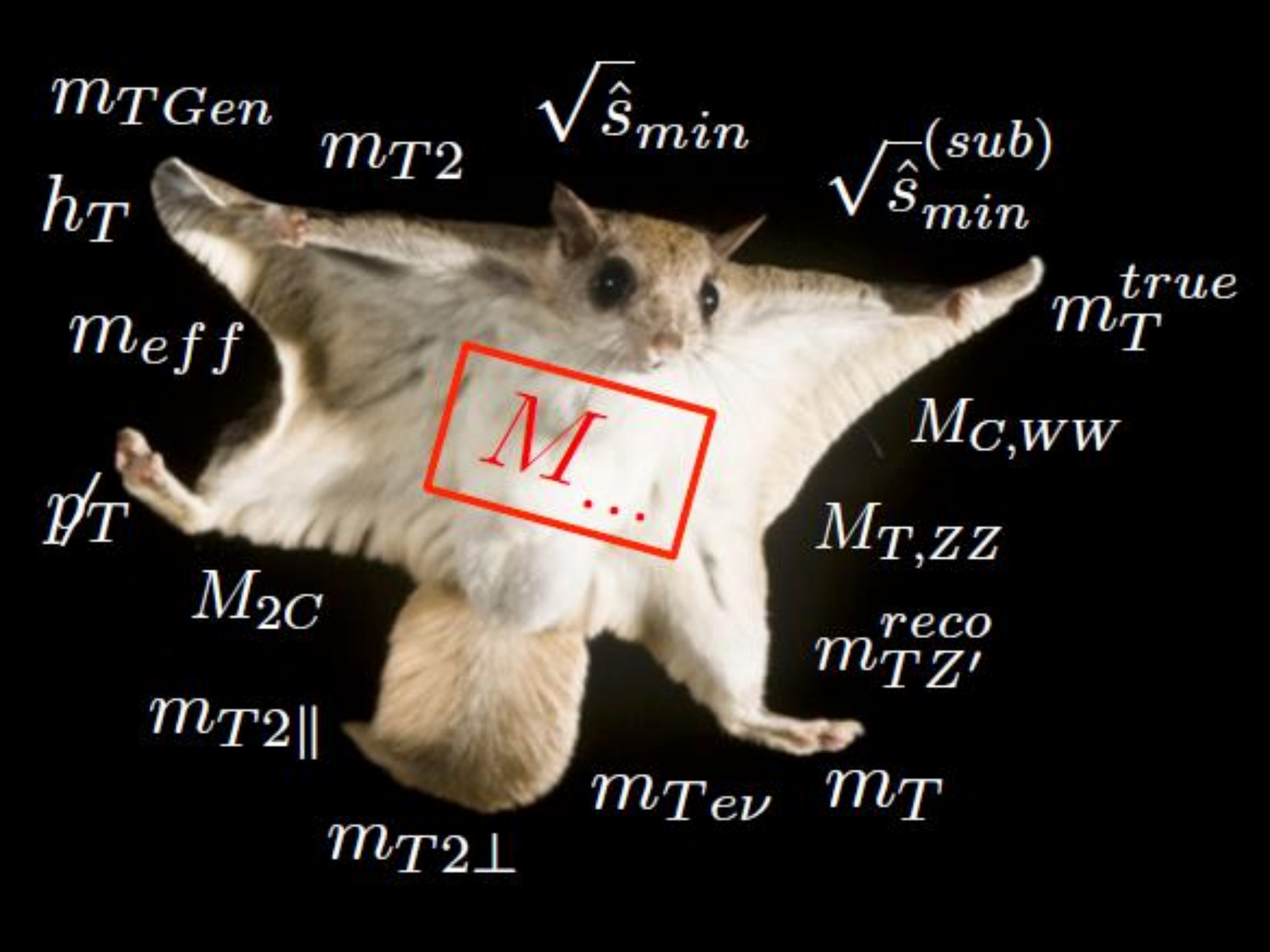


Aren’t there 100s of confusingly named kinematic variables?

MT2, MCT, MCTPERP, ...

Aren’t they all very complicated ?

No. Many are very similar to each other, differing only in small details as the following picture illustrates


m_{TGen}
m_{T2}
$\sqrt{\hat{s}}_{min}$
$\sqrt{\hat{s}}_{min}^{(sub)}$
h_T
m_T^{true}
m_{eff}
$M_{C,WW}$
$M_{...}$
$\not{p}_T$
$M_{T,ZZ}$
M_{2C}
$m_{TZ'}^{reco}$
$m_{T2\|}$
$m_{Te\nu}$
m_T
$m_{T2\perp}$

They look very different,
but actually they are very similar.

All answer a simple question which uses the event hypothesis.

On blackboard?

M

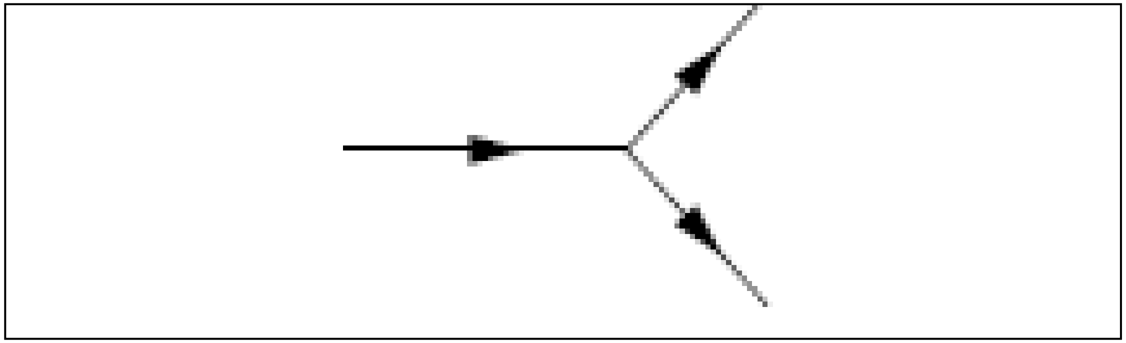
MT

MT2

Mass Peak

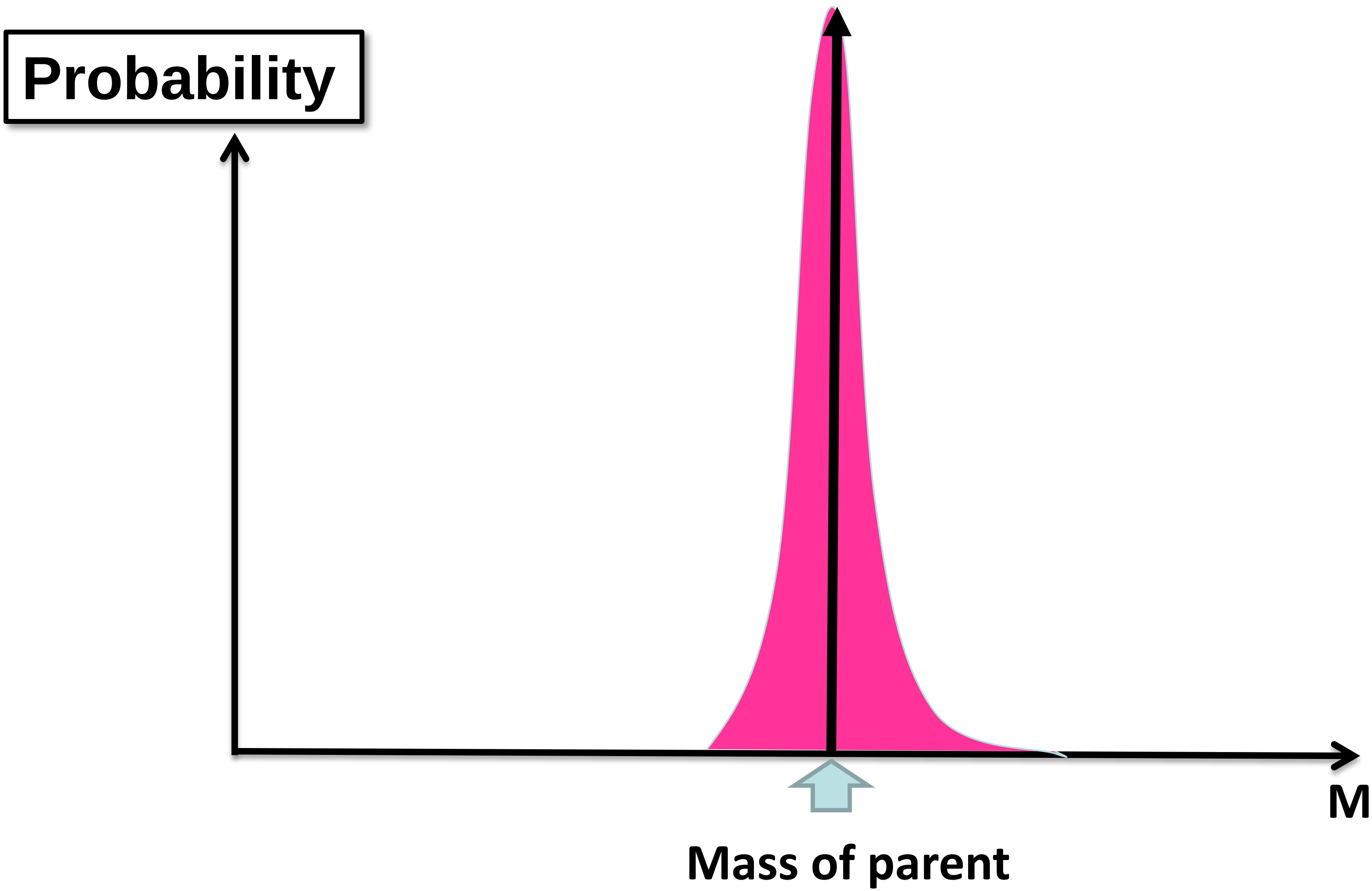
M:

- Assuming this:



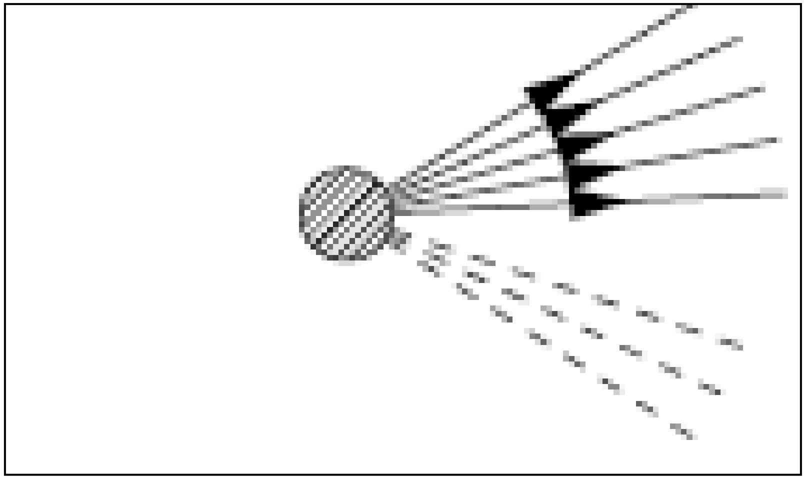
  - –How heavy was parent?

M



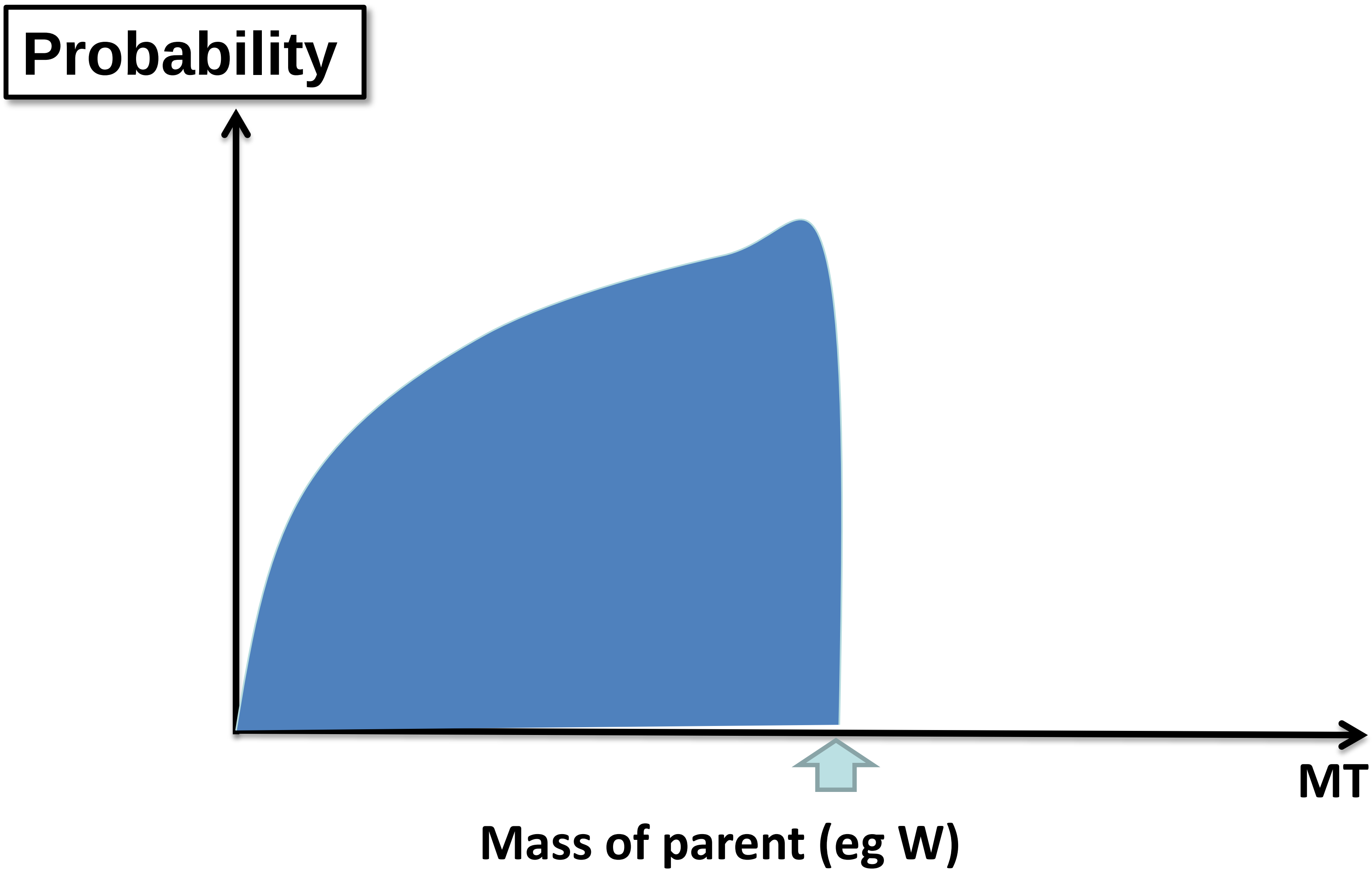
MT(chi):

- Assuming this:



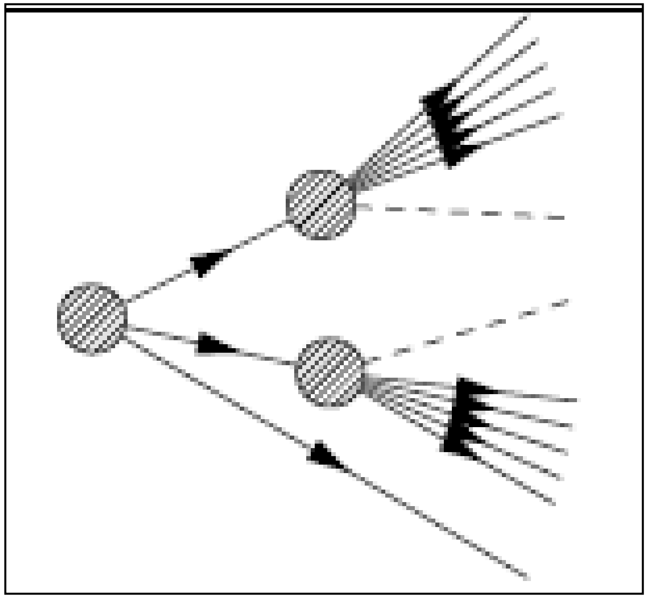
- And assuming that invisibles have rest-masses totalling a value chi (that need not be correct)
- And assuming there are no other invisible particles in the event, then:
  - What is LARGEST mass that parent could have had?

MT
(e.g. W → l ν)



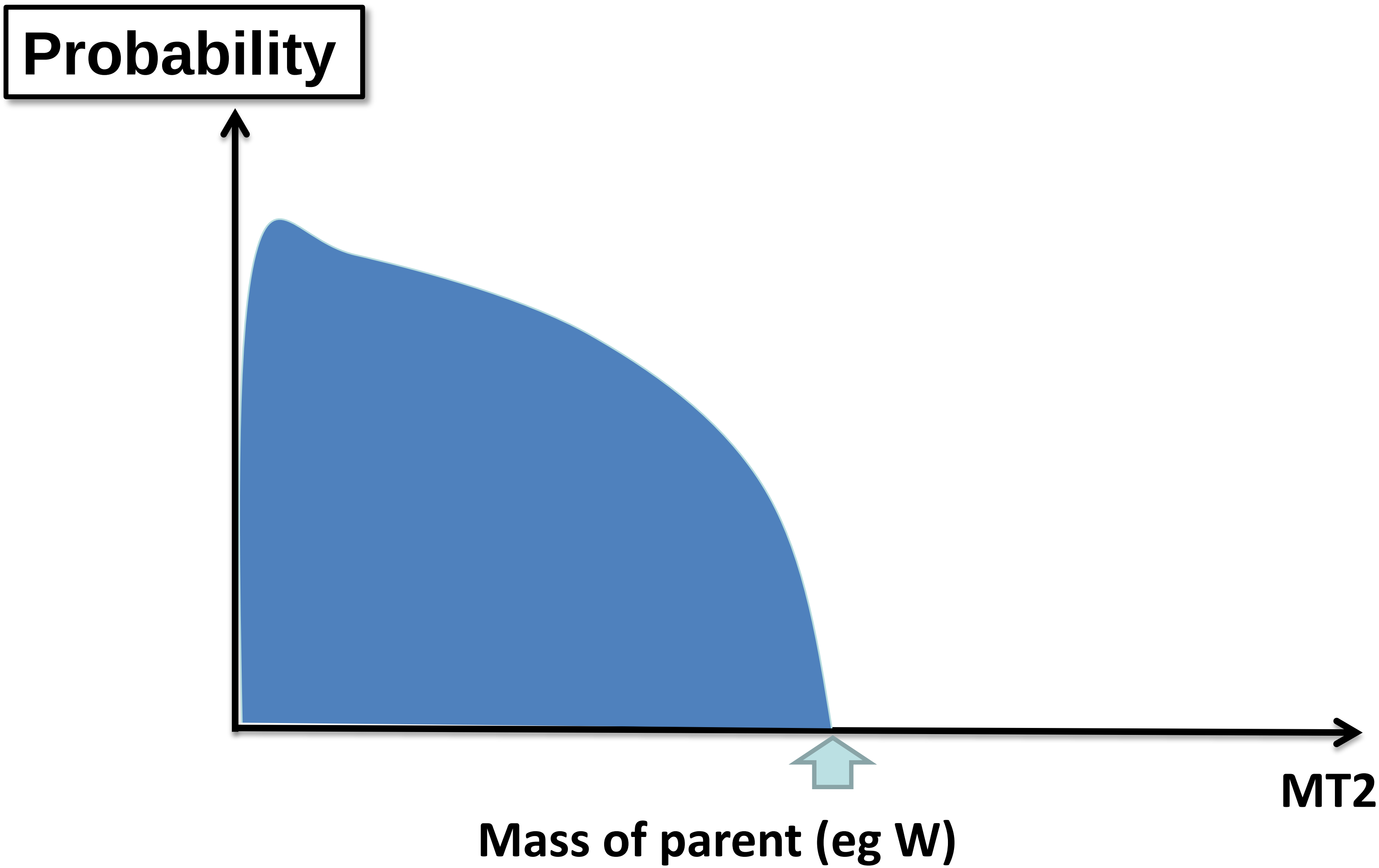
MT2(chi):

- Assuming pair production like this:



- And assuming that invisibles produced by each parent (individually) have a total mass chi (which need not be correct)
- And assuming there are no other invisible particles in the event, then:
  – What is LARGEST mass that parent could have had?

MT2
(e.g. sq sq →q chi q chi)



Why the emphasis on bounds?

- One-sidedness good for pushing backgrounds away from signals
- One-sidedness good for creating discontinuous features such as endpoints, which are themselves easy to see.
- (Sensible) bounds can be saturated and so have straightforward interpretation.

Why might endpoints be good?

- High contrast endpoint is robust: its position cannot be not modified by
  - ANY background shape uncertainty
  - Non-uniform acceptance
- More of a problem if edge leading to endpoint is smeared out. Much harder to find “low contrast” endpoints.
- Each event separate and easier to understand as 1D variable is meaningful.

Formalising an old idea … kinematic boundaries, creases, edges, cusps etc

Phase Space

Observable Momentum

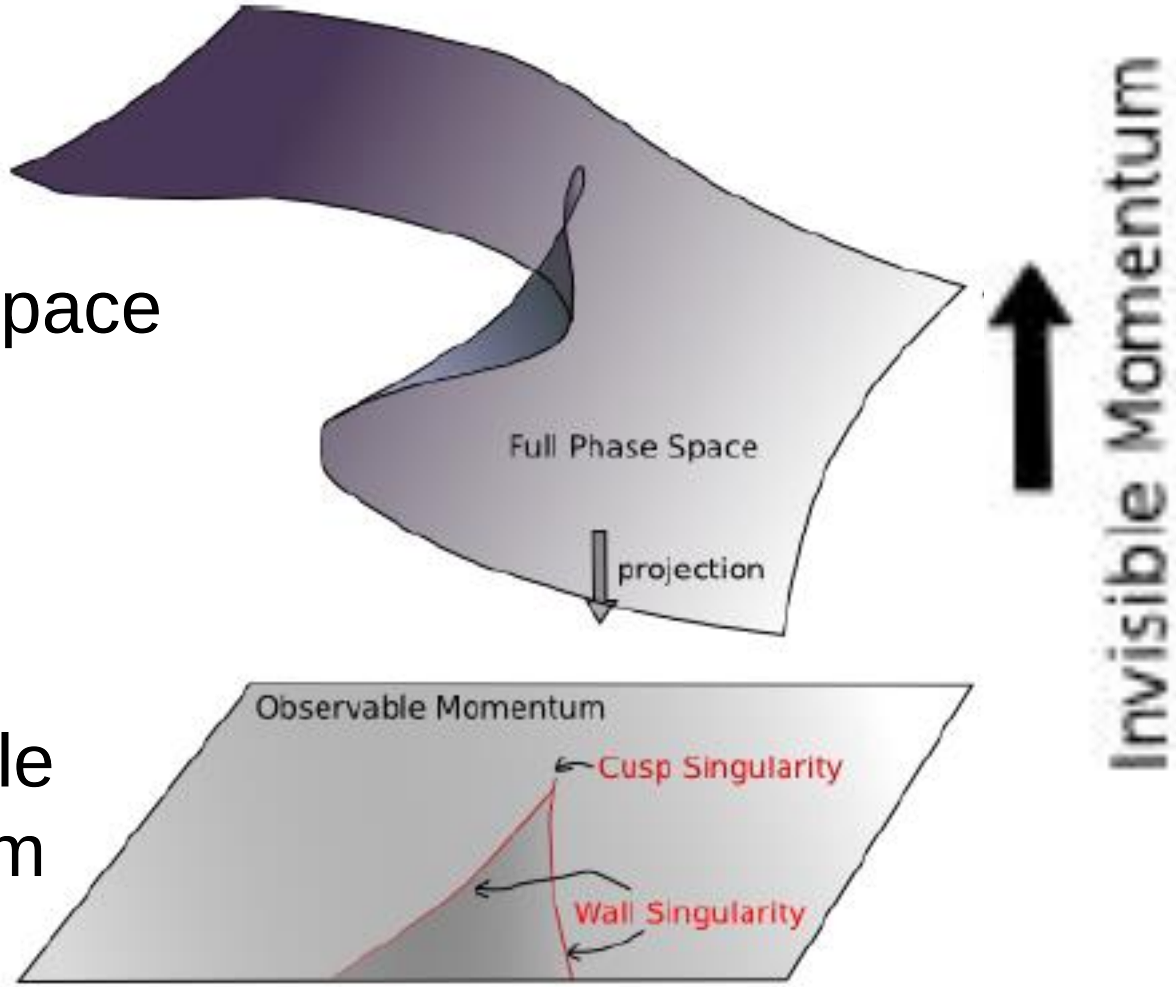
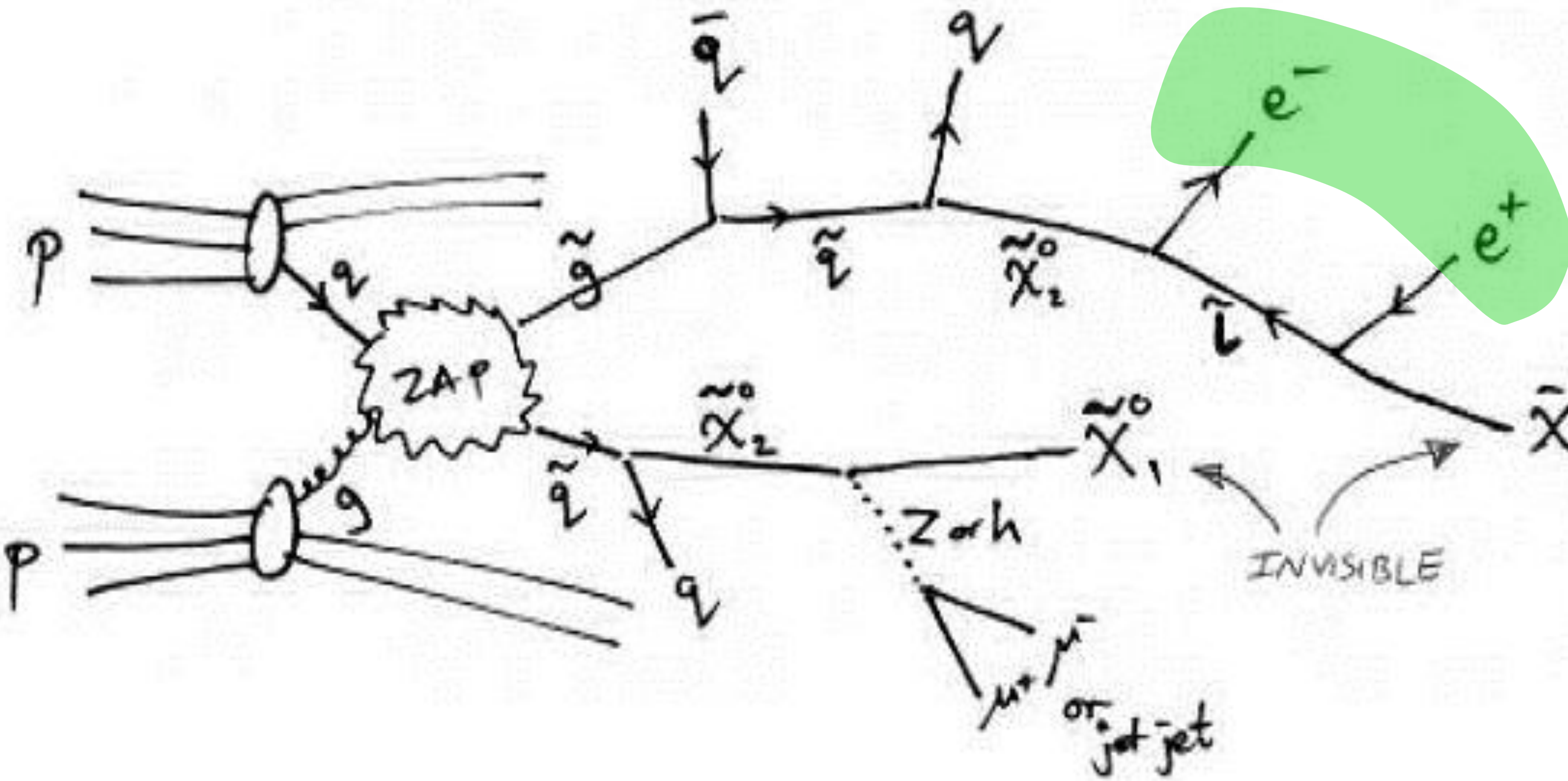


FIG. 1: A schematic diagram describing the relation between the full phase space and the projected observable phase space.

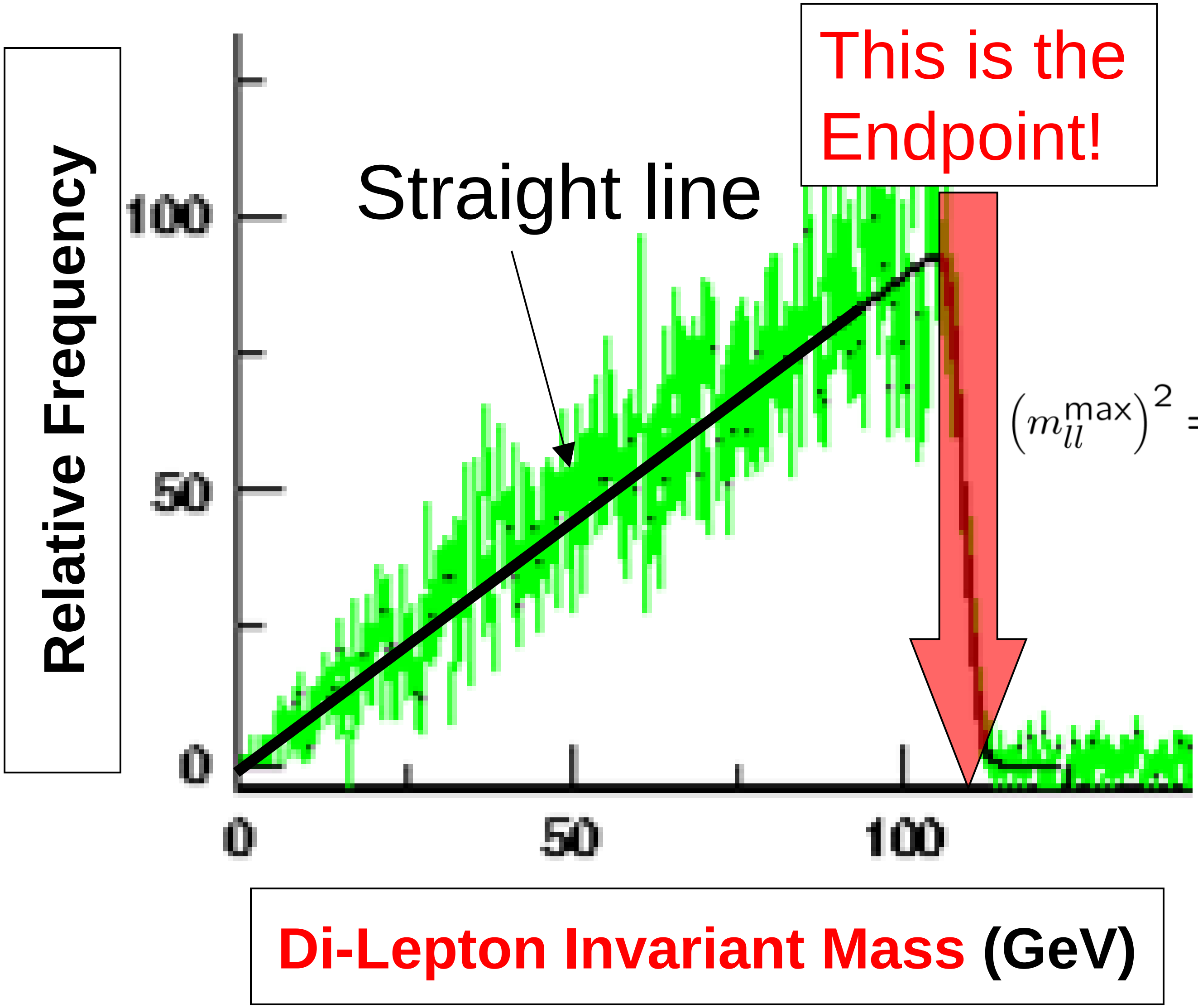
I.W.Kim: "Algebraic singularity method of mass measurement with missing energy" arXiv:0910.1149v2 [hep-ph]

Example simplest kinematic endpoint

- Consider M_{LL}



Dilepton invariant mass distribution



$$\left(m_{ll}^{\max}\right)^2 = \frac{\left(m^2_{\tilde{\chi}_2^0} - m^2_{\tilde{l}_R}\right)\left(m^2_{\tilde{l}_R} - m^2_{\tilde{\chi}_1^0}\right)}{m^2_{\tilde{l}_R}}$$

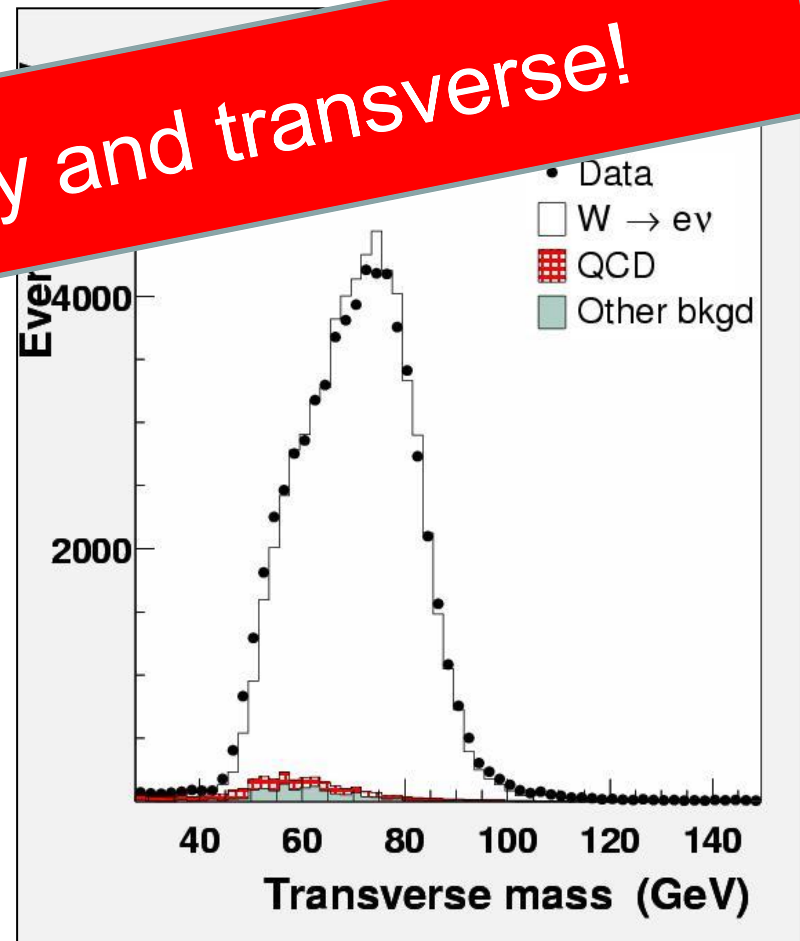
Contrast “OLD” and “NEW” definitions of transverse mass:

Most people think this is transverse mass:

$$m_T^2 = m_e^2 + m_\nu^2 + 2(e_e e_\nu - \mathbf{p}_{\mathbf{T}e} \cdot \mathbf{p}_{\mathbf{T}\nu})$$

$$\left(e = \sqrt{m^2 + p_T^2}\right)$$

Looks confusing, arbitrary and transverse!

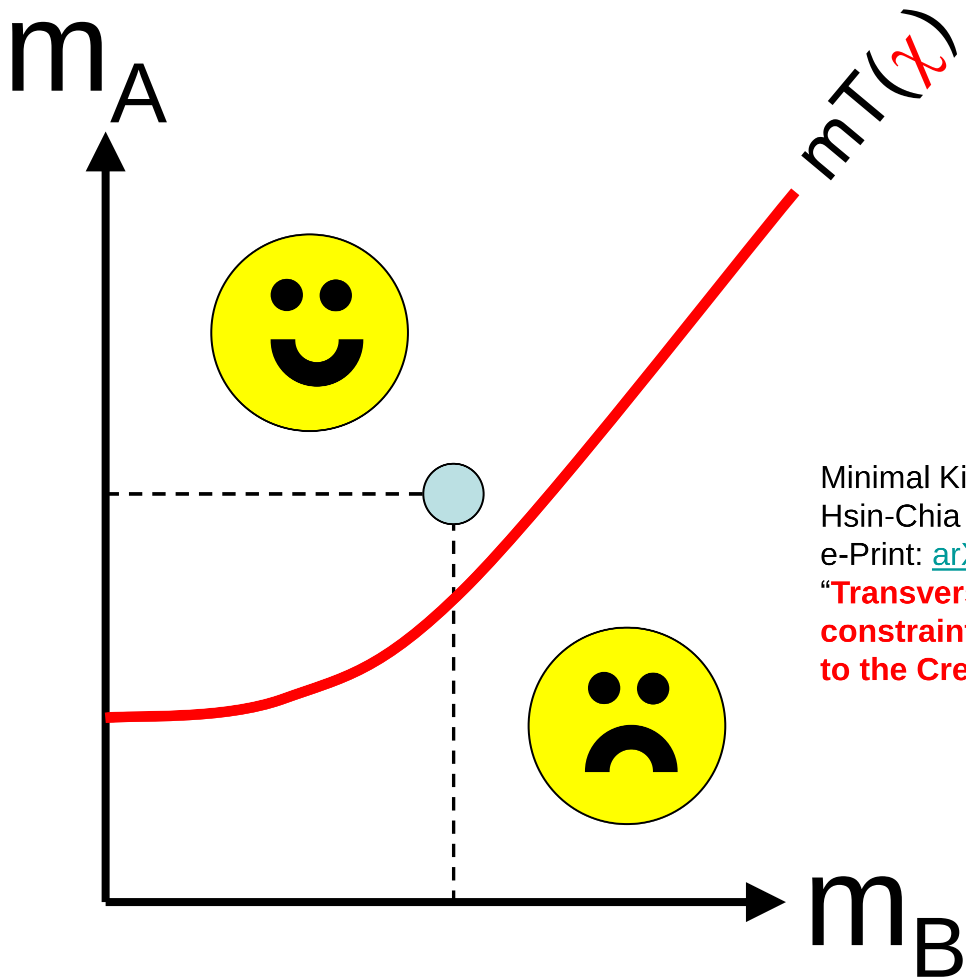


!! NOT THIS !!

$$m_T = \sqrt{2\left|\vec{P}_{Te}\right|\left|\vec{P}_{T\nu}\right|(1-\cos\vartheta)}$$

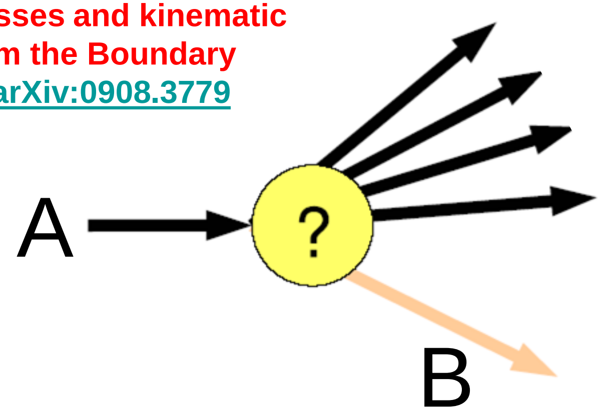
!! This is **NOT** the transverse mass !!

This is the nice modern statement:



The transverse mass curve is the boundary of the region of (mother,daughter) mass-space consistent with the observed event!

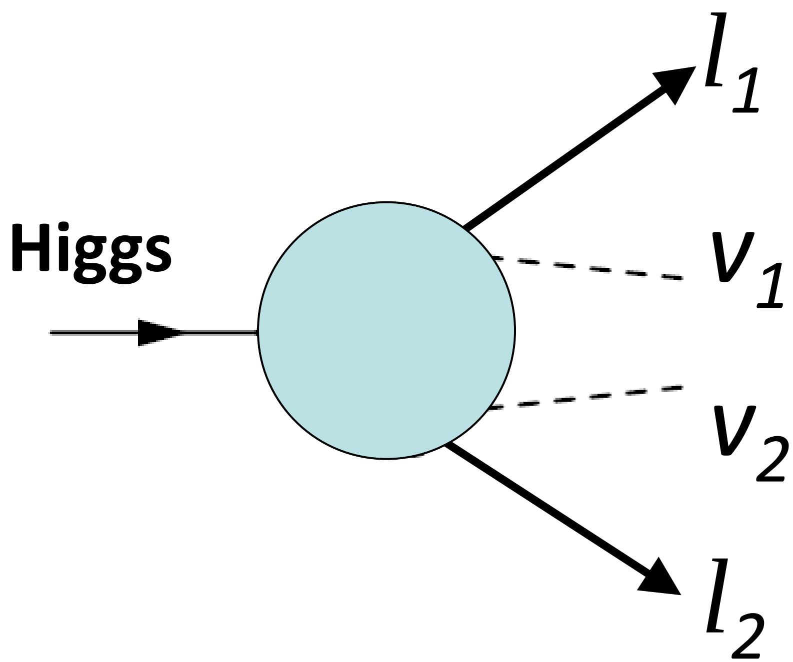
Minimal Kinematic Constraints and m(T2),
Hsin-Chia Cheng and Zhenyu Han (UCD)
e-Print: arXiv:0810.5178 [hep-ph] and
"**Transverse masses and kinematic constraints, from the Boundary to the Crease**" **arXiv:0908.3779**



Does it ever work?

Accidental successes ...

Higgs →WW* → lνlν



$$Q_1^\mu Q_{1\mu} = 0,$$
$$Q_2^\mu Q_{2\mu} = 0,$$

$$\vec{q}_{1\mathrm{T}} + \vec{q}_{2\mathrm{T}} = \vec{\not{p}}_{\mathrm{T}}.$$

Written up in http://arxiv.org/abs/1106.2322

Higgs →WW* → lνlν

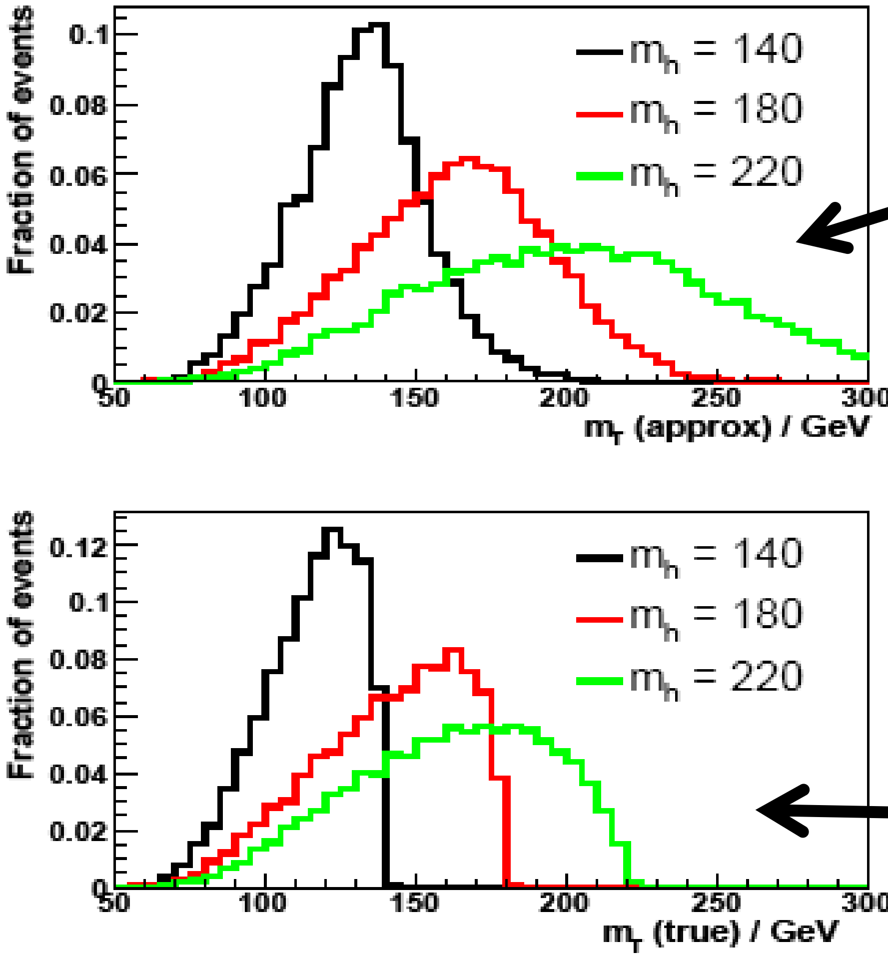


FIG. 1: Signal-only distributions of m_T^{approx} (top) and m_T^{true} (bottom) for various values of m_h (in GeV). No cuts on $\Delta\phi_{\ell\ell}^{\max}$ and $p_{T\,WW}^{\min}$ have been applied.

Previous variable (not a bound)

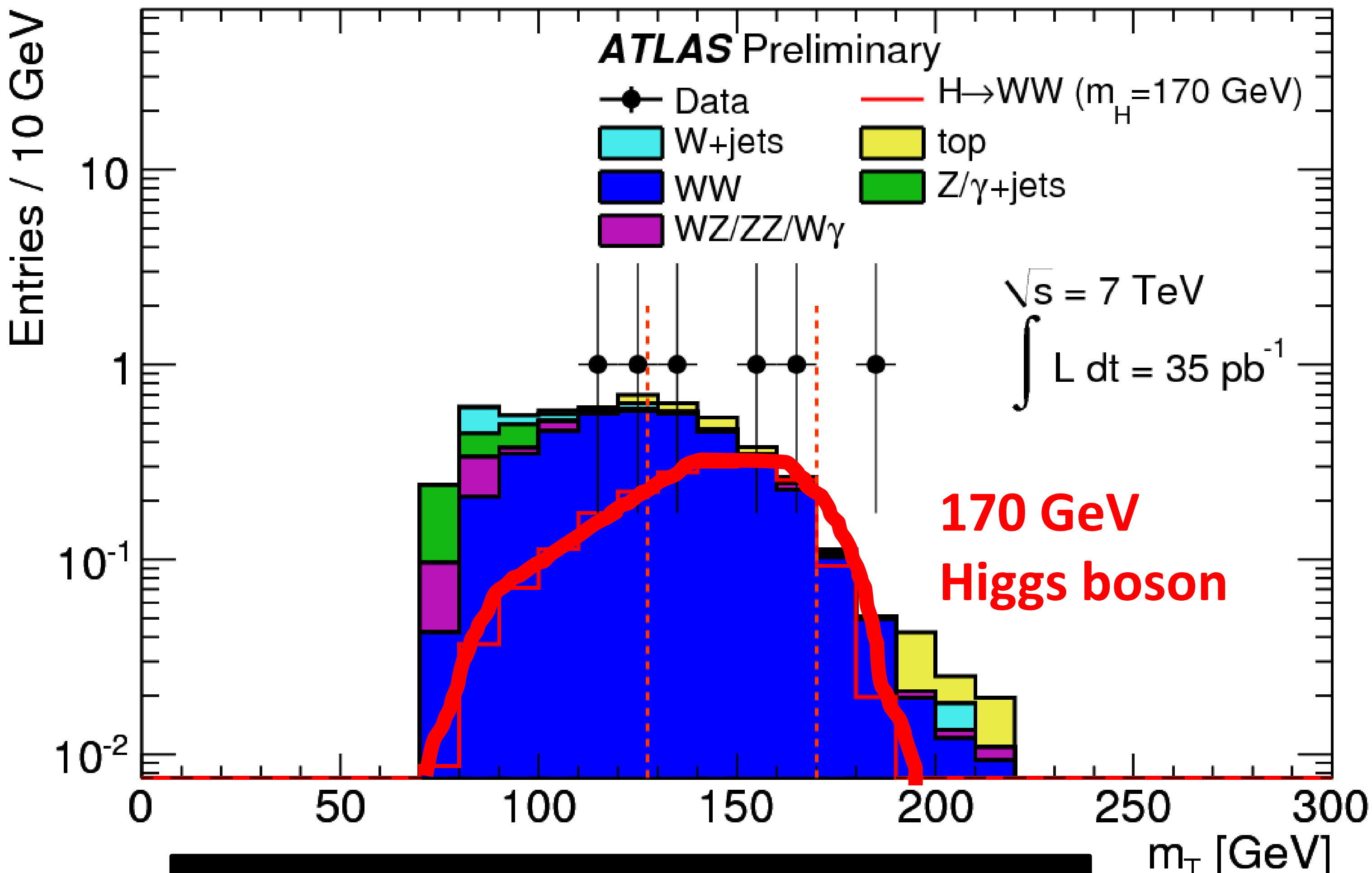
Why are endpoints often more robust than shapes?

Proper bound var MTTrue = M1T

Written up in http://arxiv.org/abs/1106.2322

ATLAS-CONF-2011-005

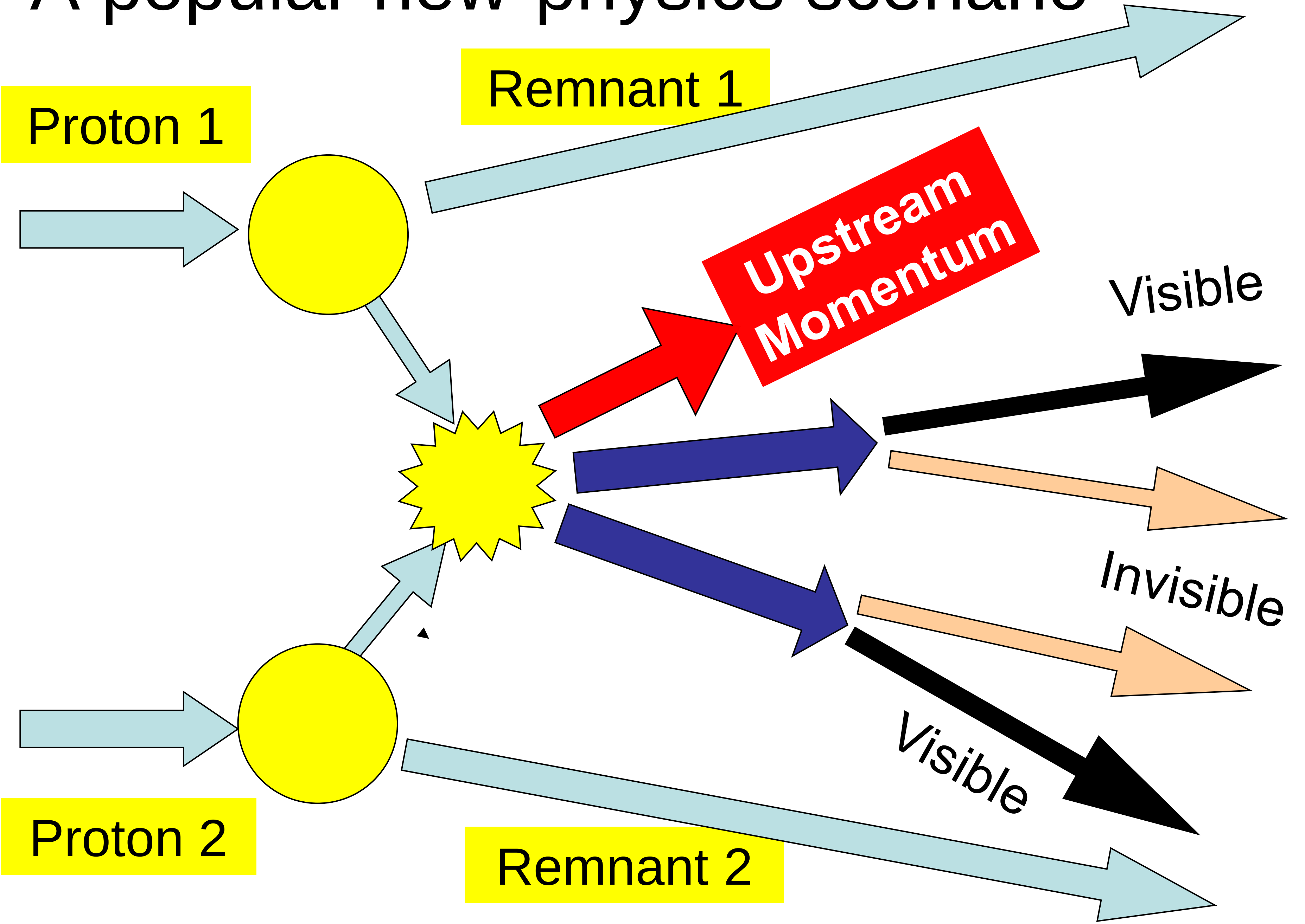
Against the 2010 LHC data…



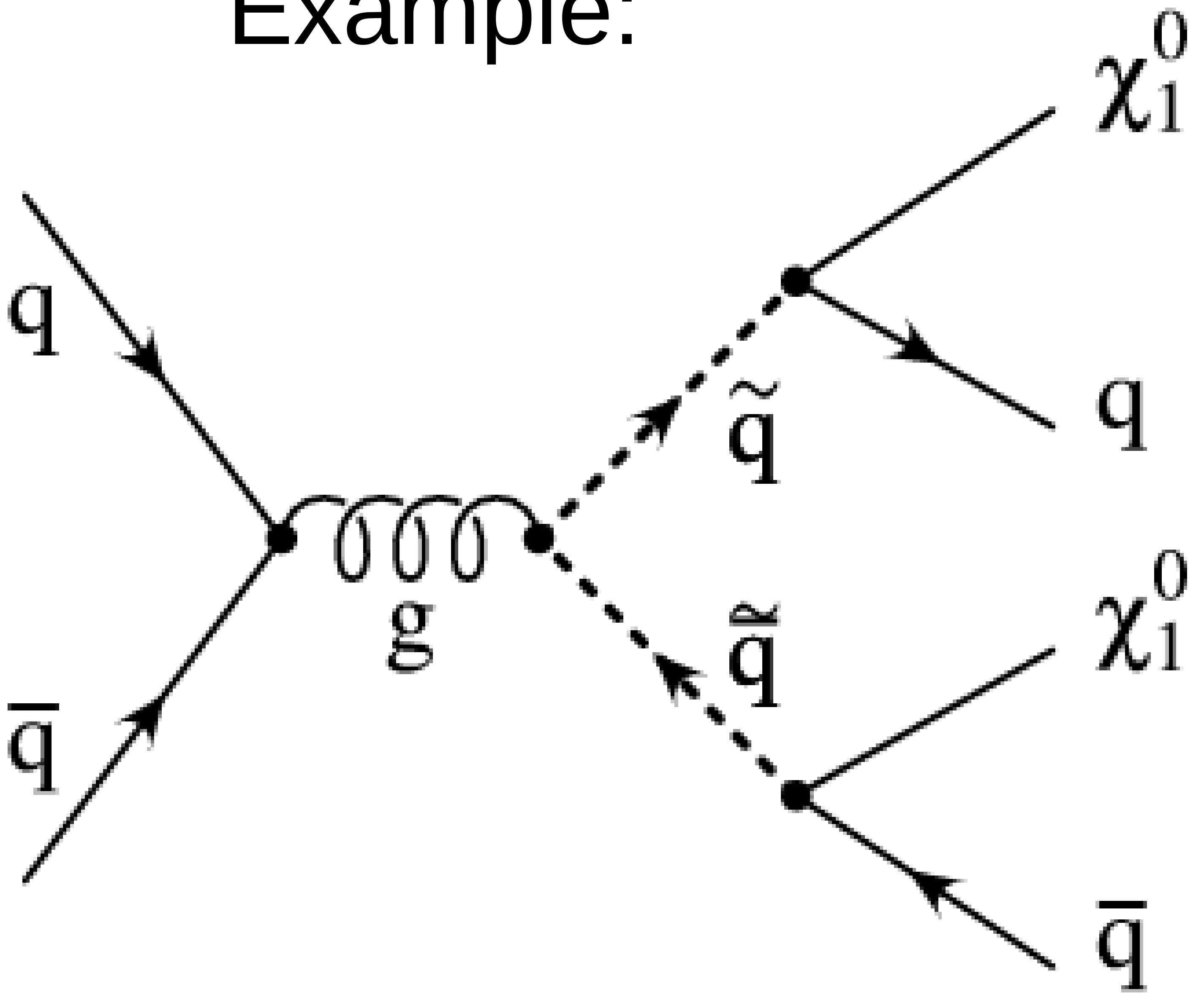
Big improvement in LHC Higgs Sea

More frightening example

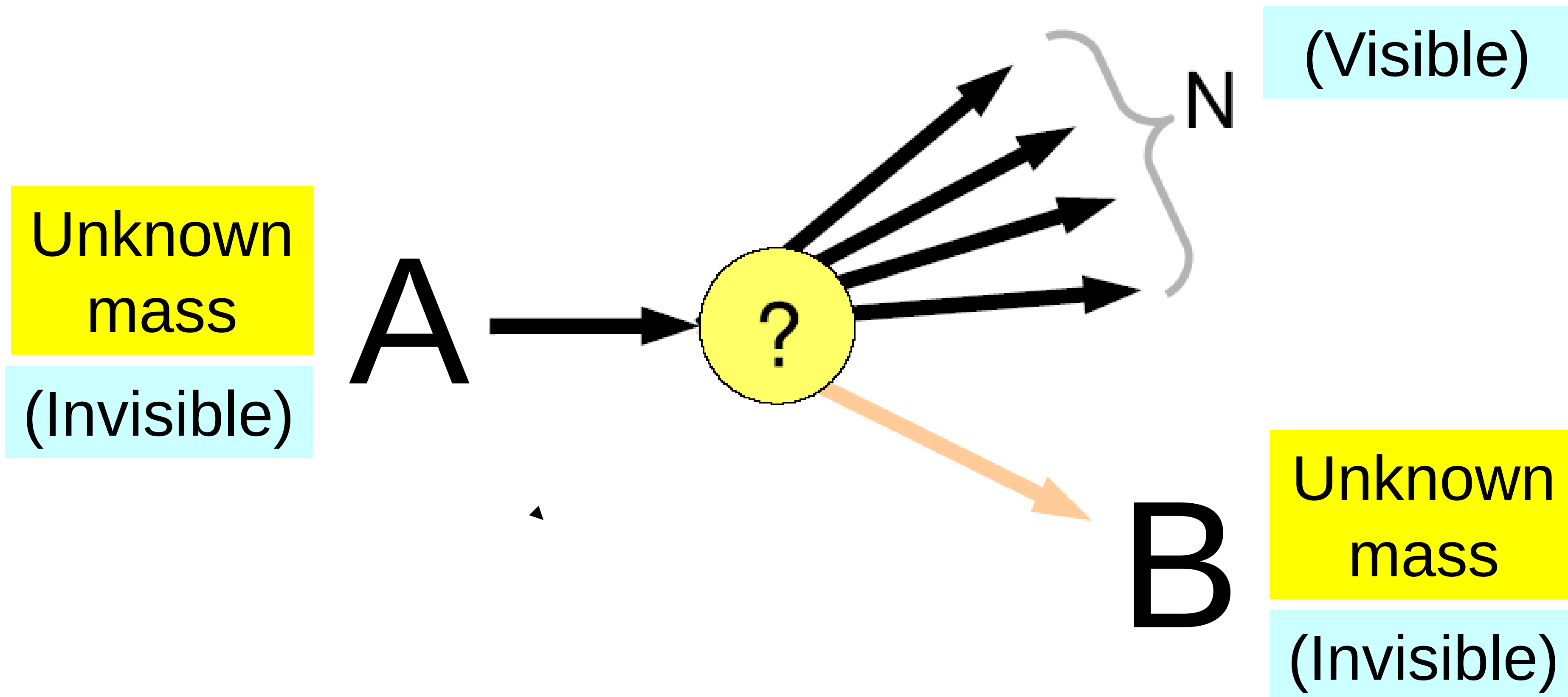
A popular new-physics scenario



Example:



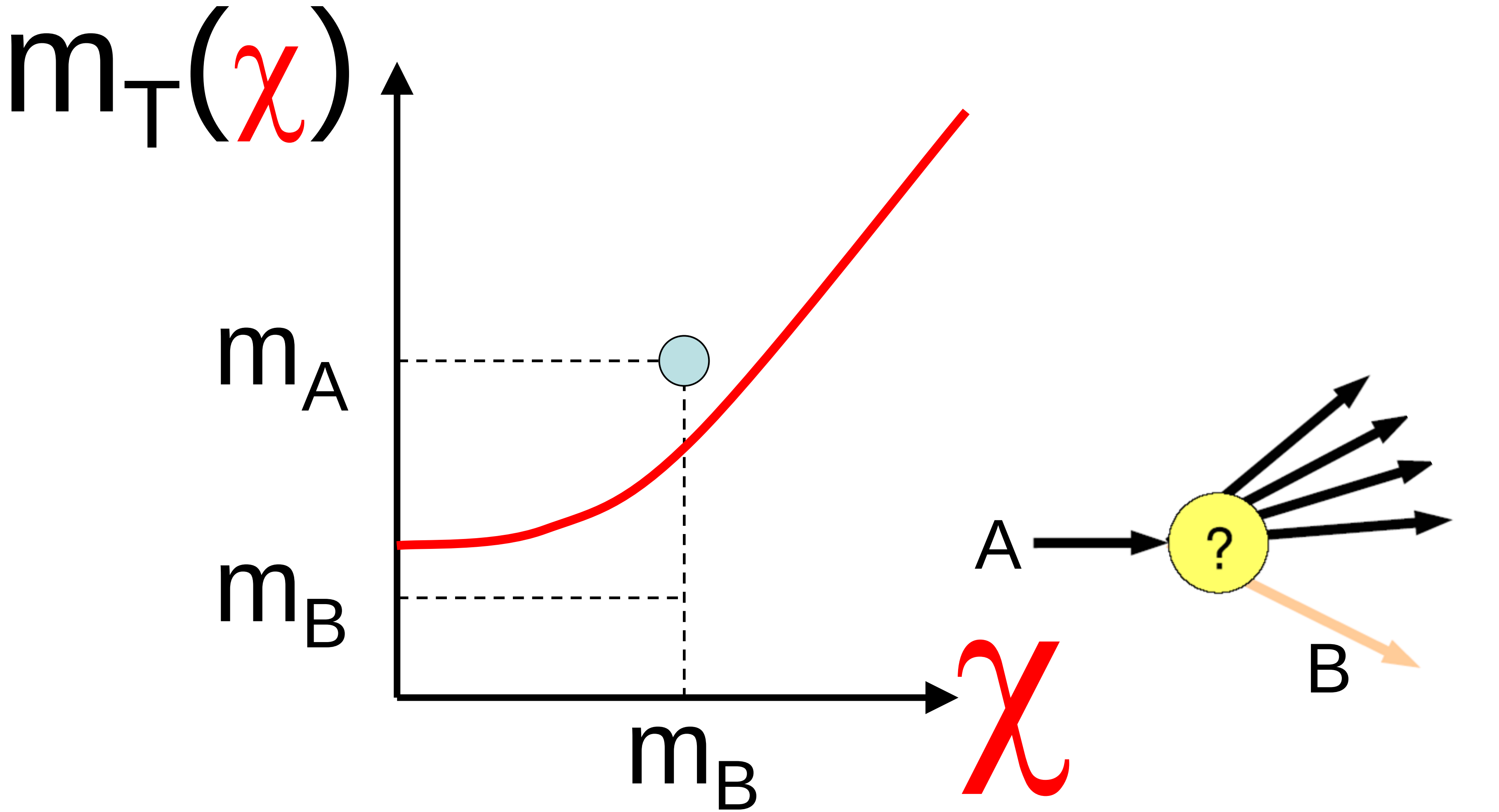
We have two copies of this:



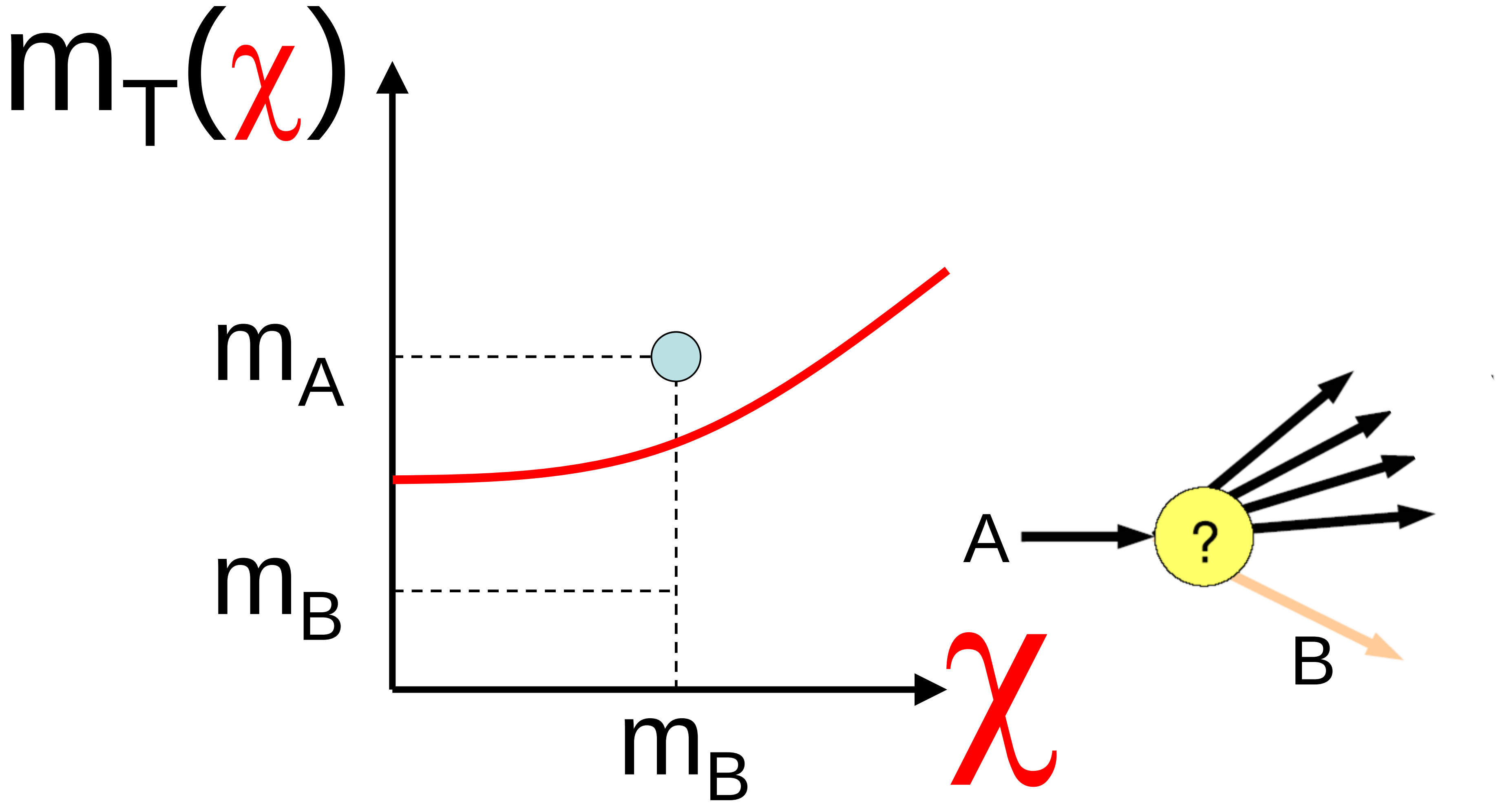
One copy could be just as relevant!

so just consider transverse mass initially on one “side” only.

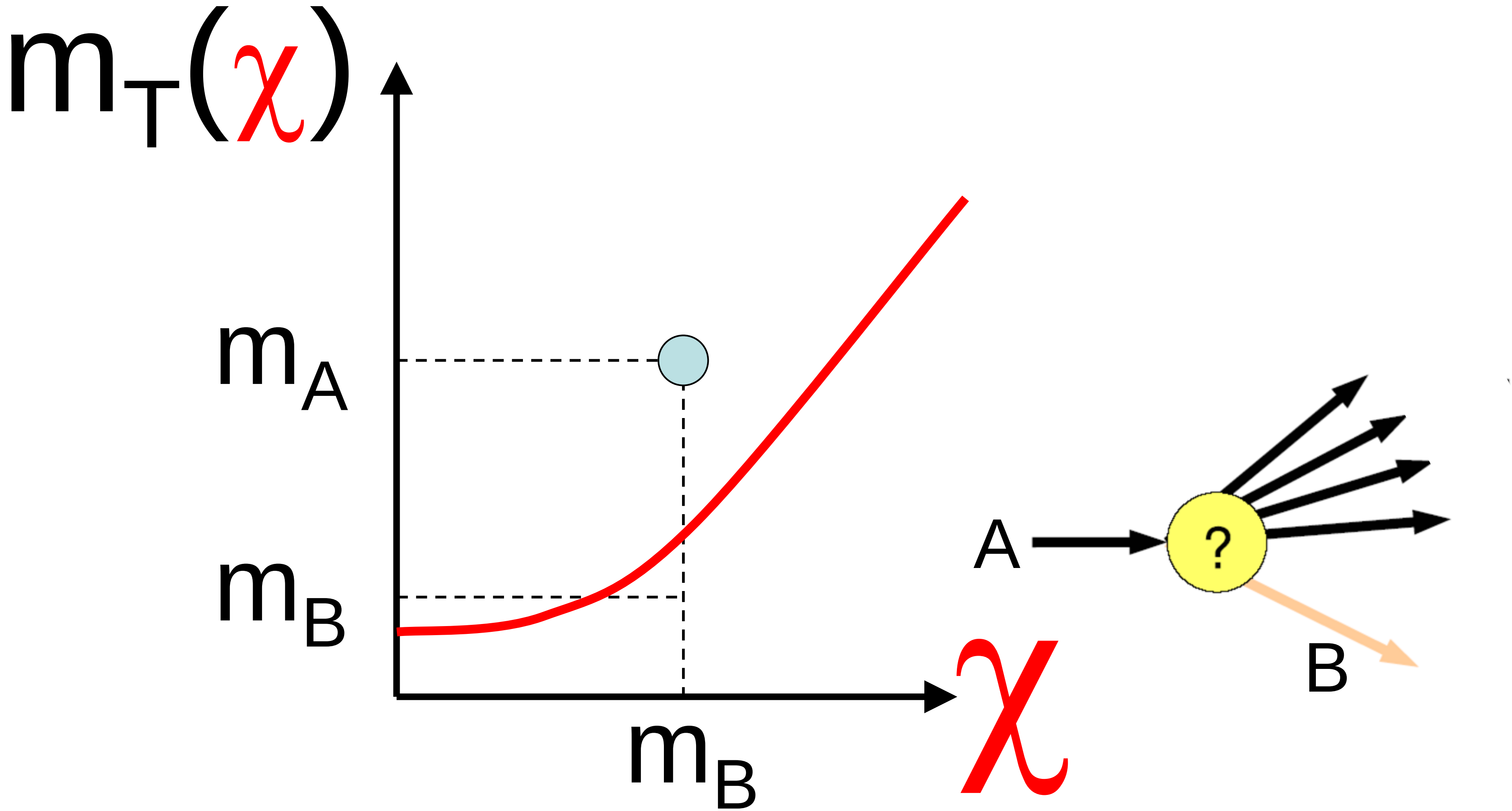
Event 1 of 8

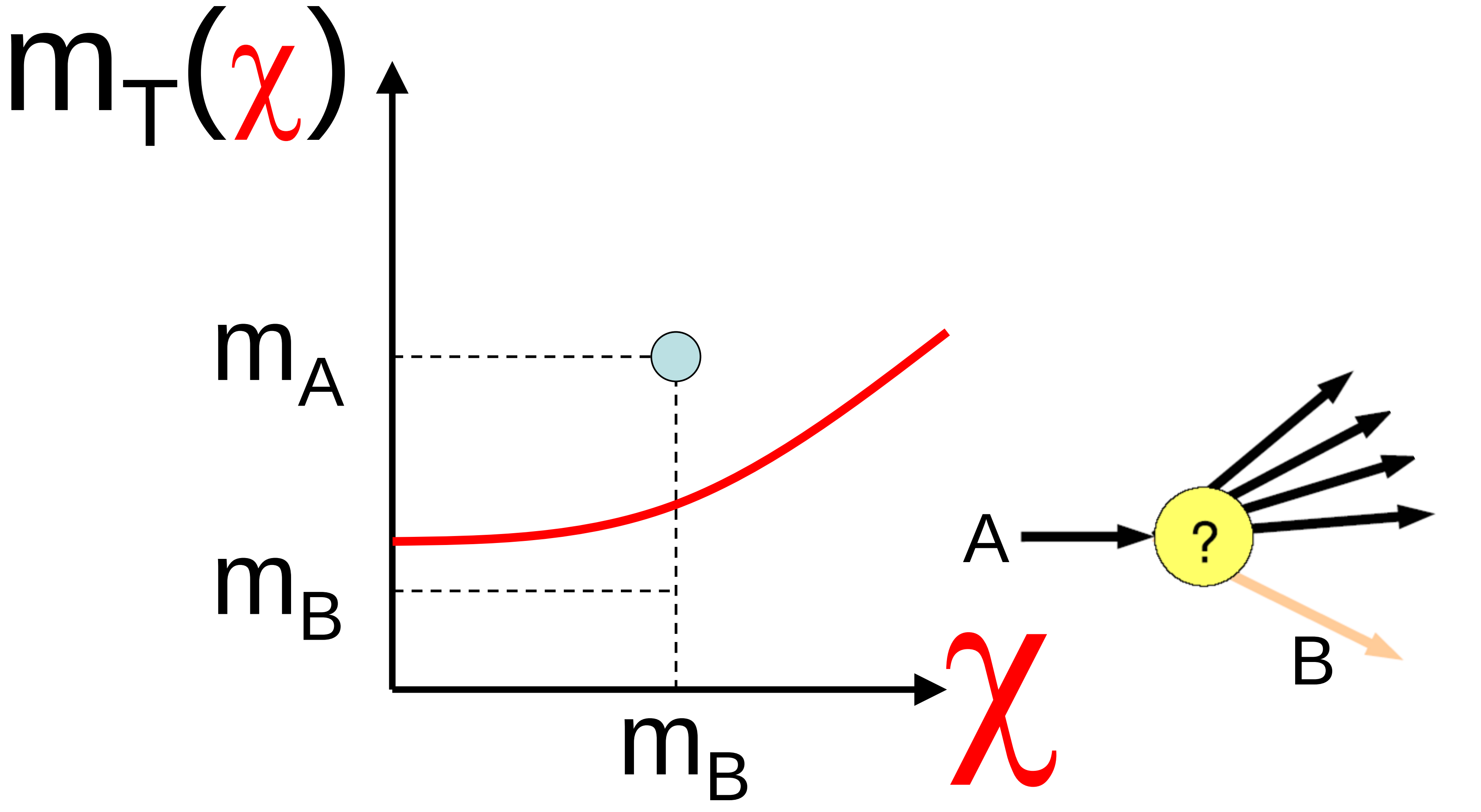


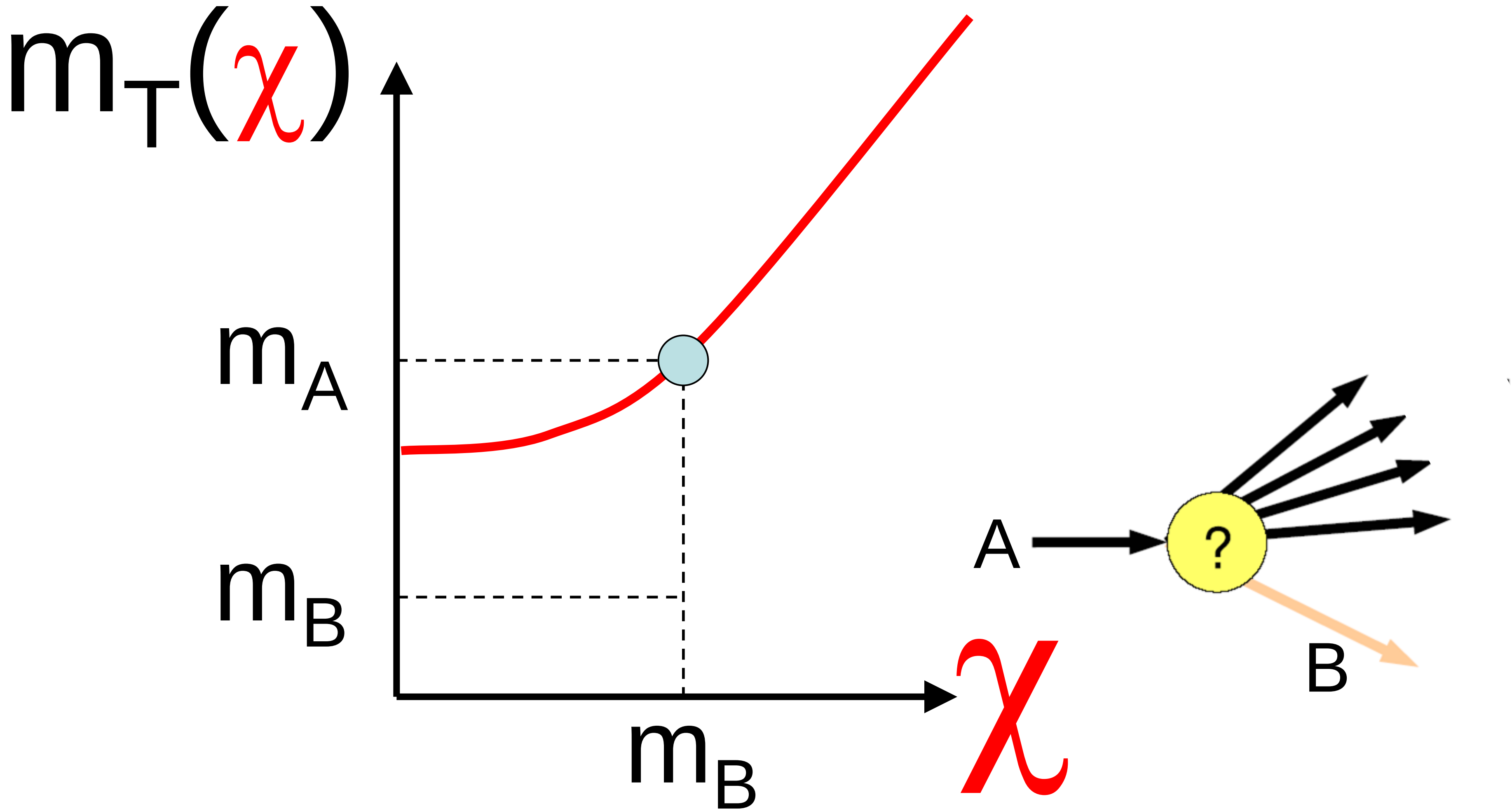
Event 2 of 8



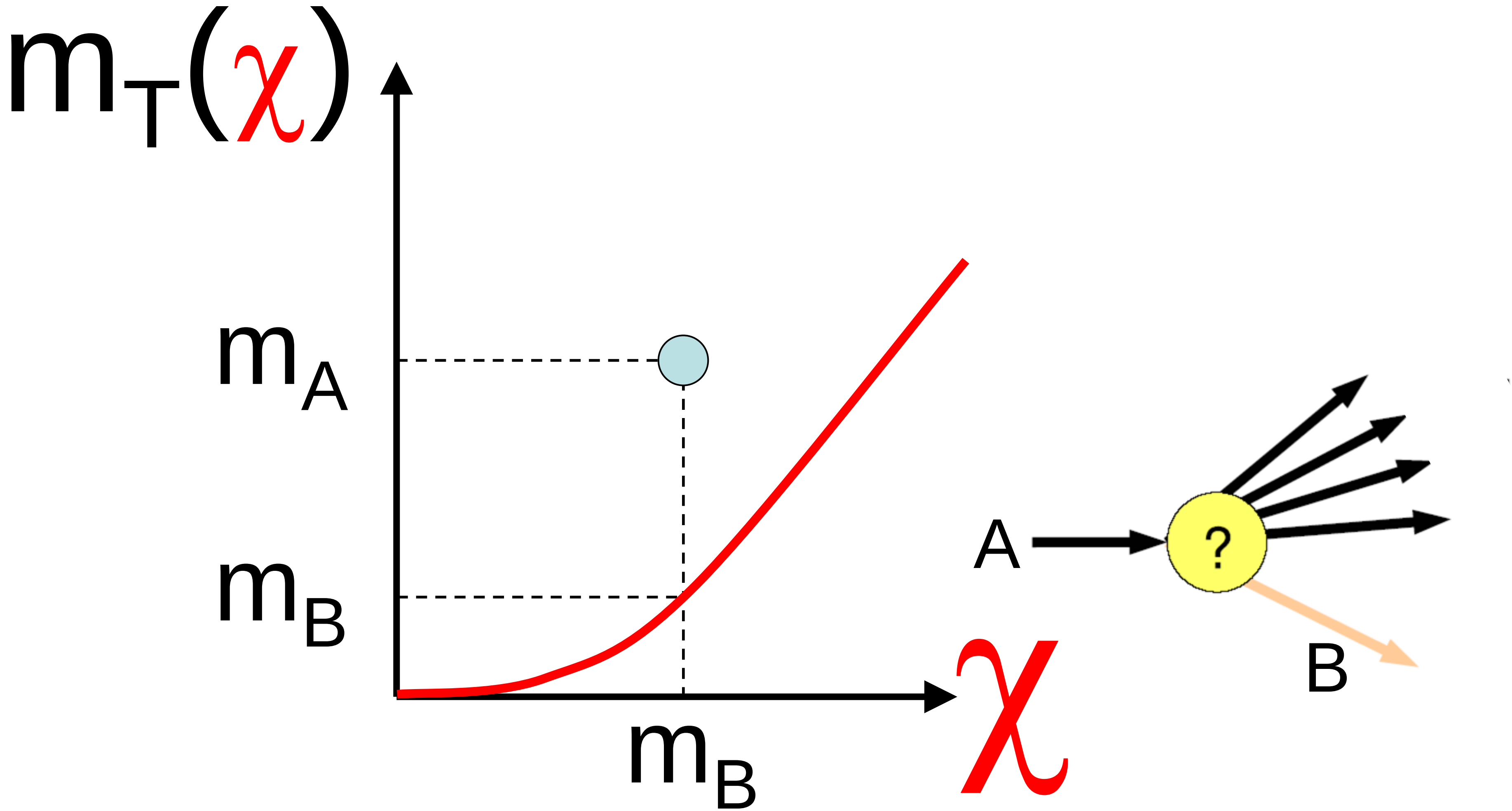
Event 3 of 8



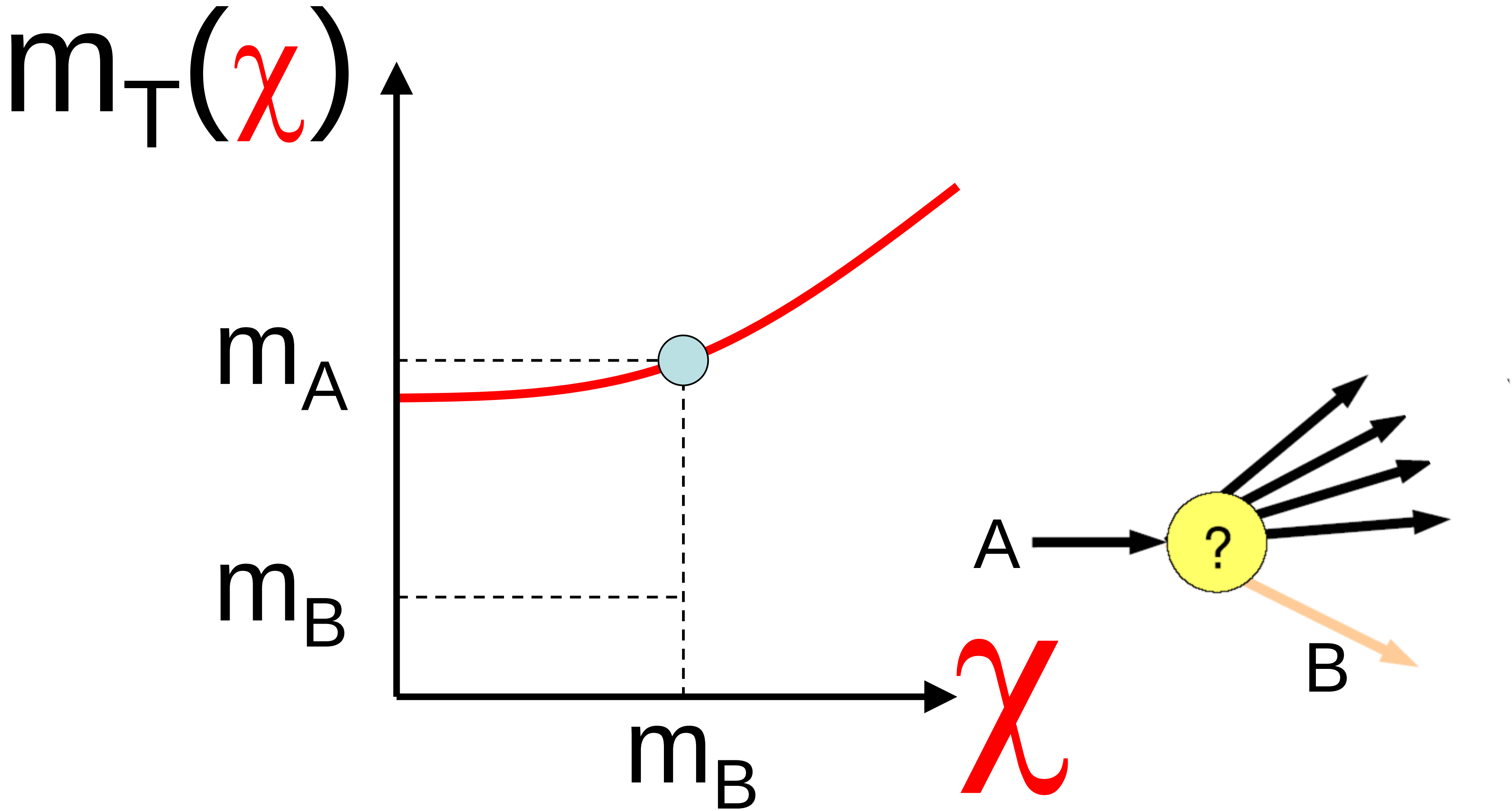

$m_T(\chi)$
m_A
m_B
m_B
χ
A
?
B


$m_T(\chi)$
m_A
m_B
m_B
χ
A
?
B

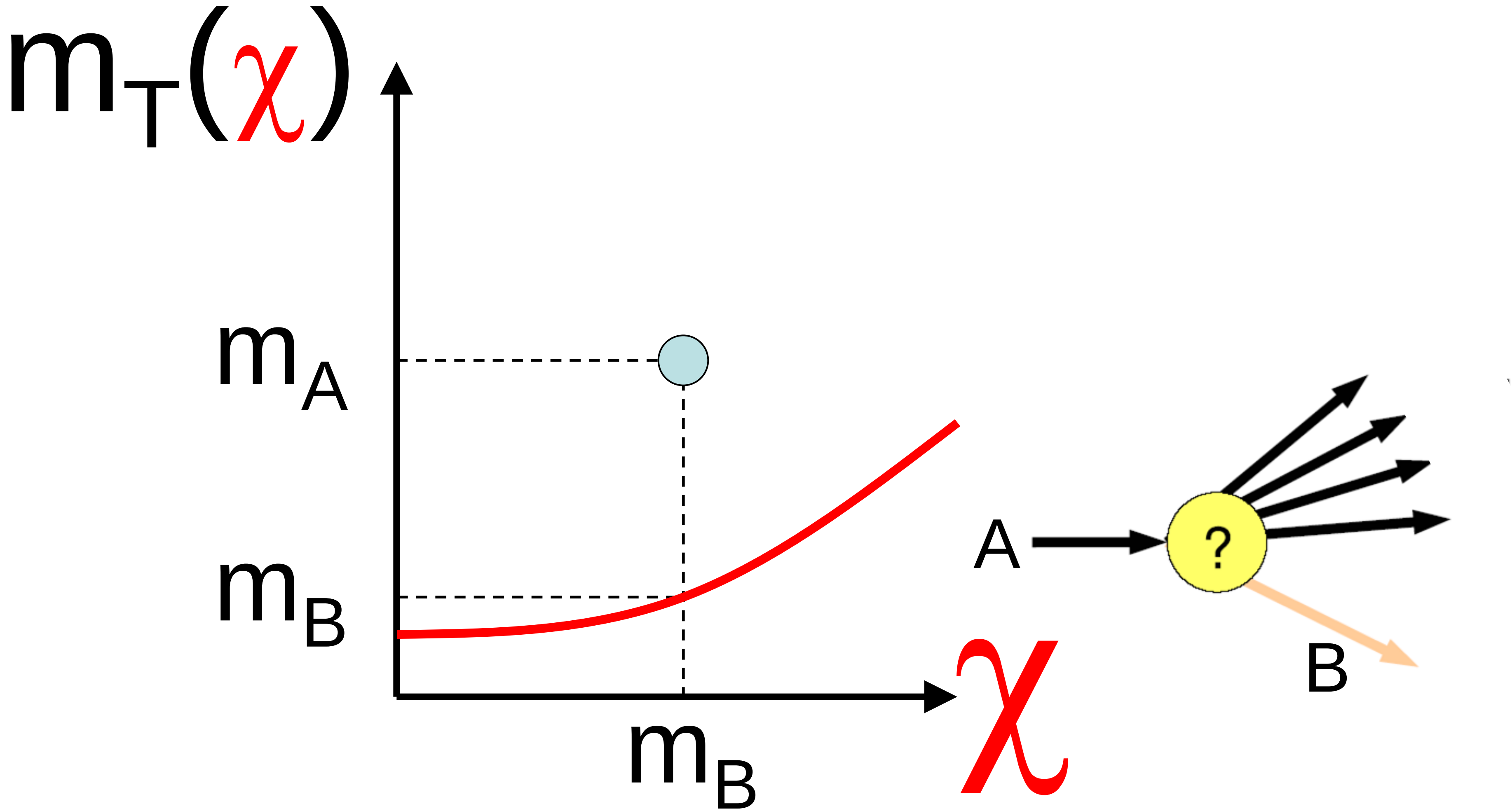
Event 6 of 8



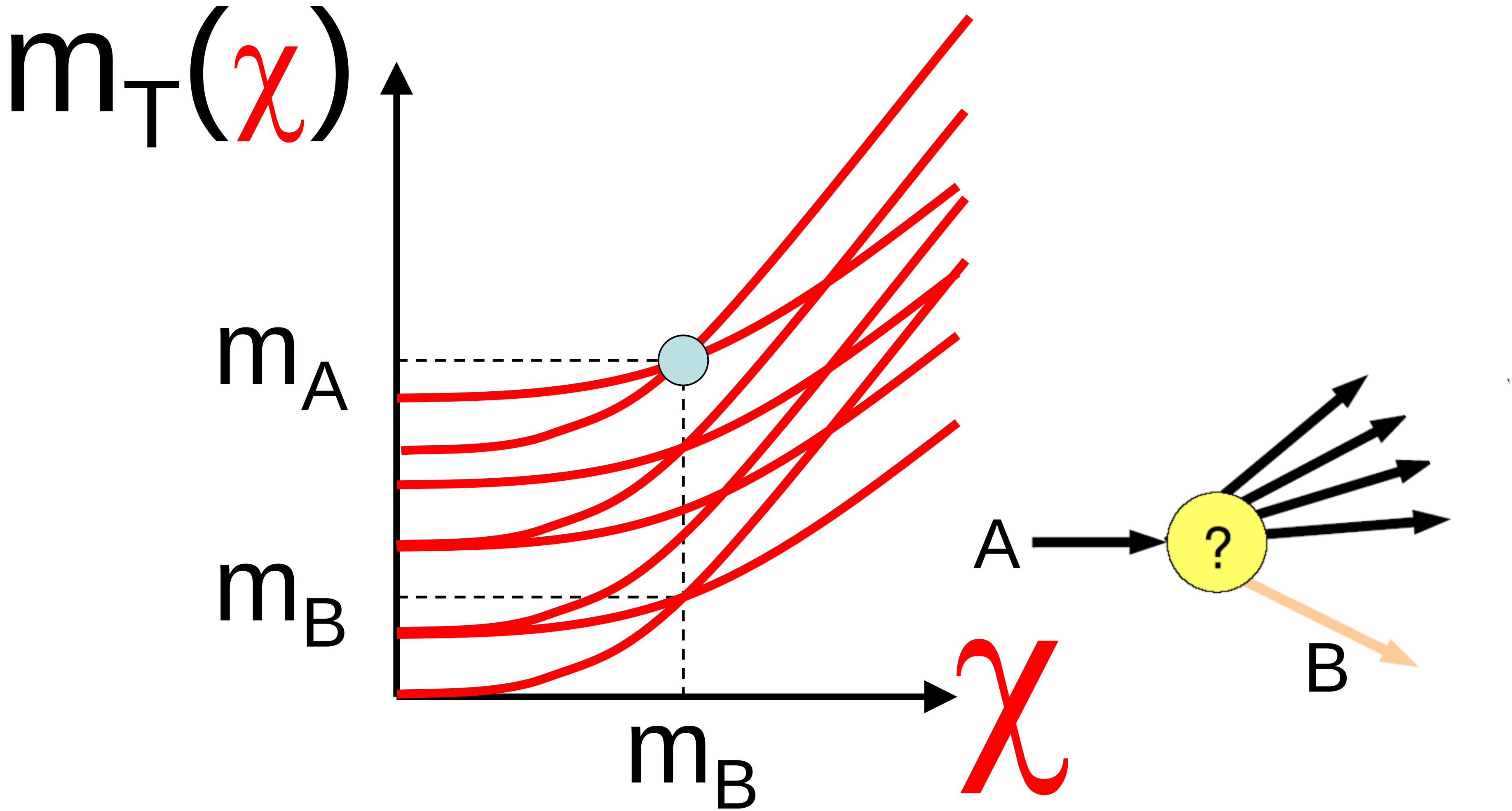
Event 7 of 8



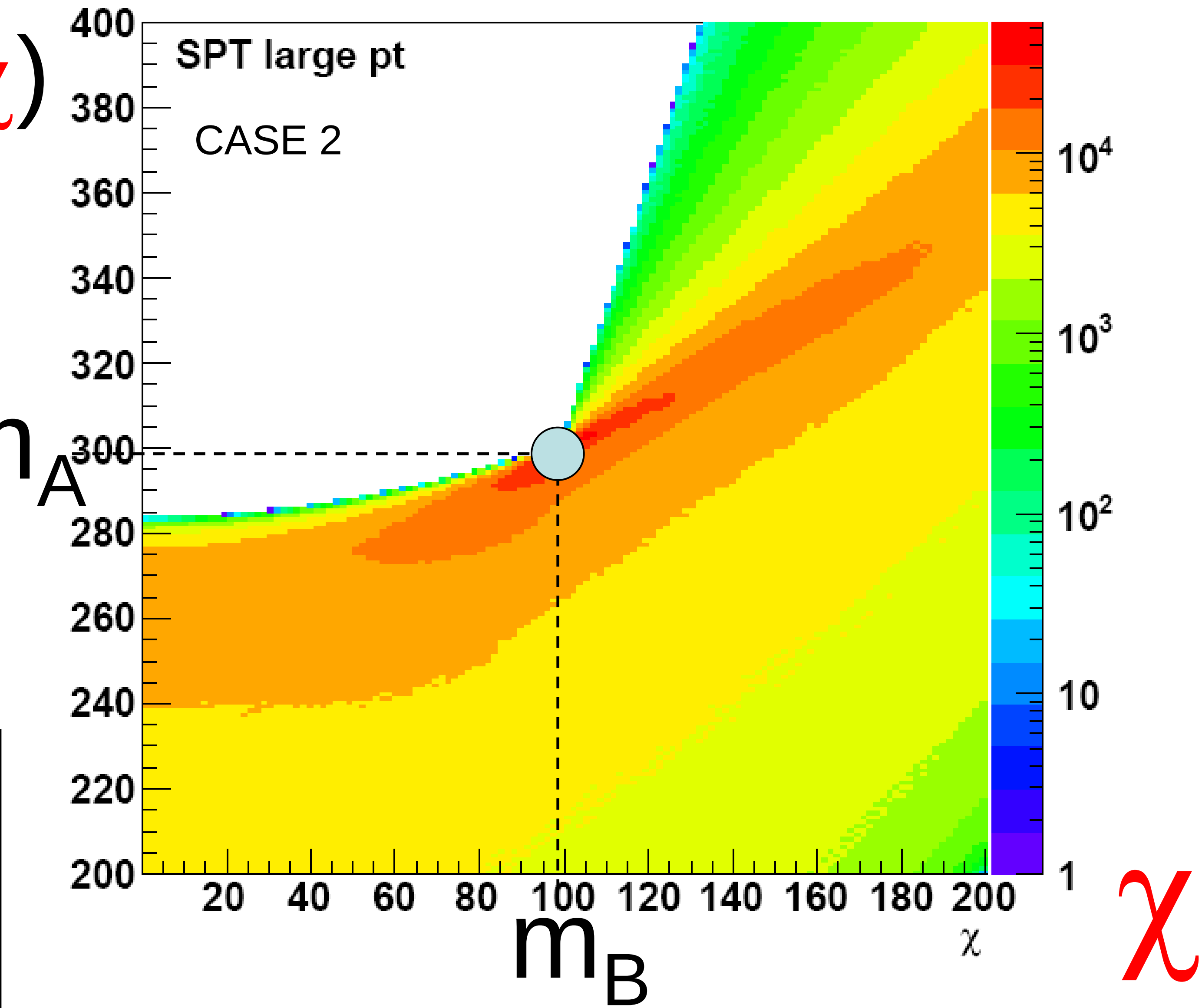
Event 8 of 8



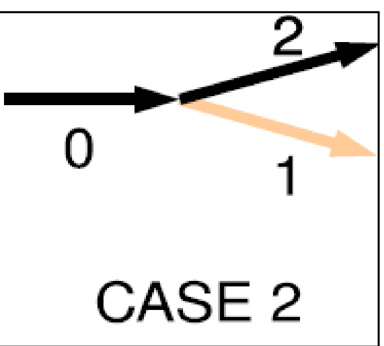
Overlay all 8 events



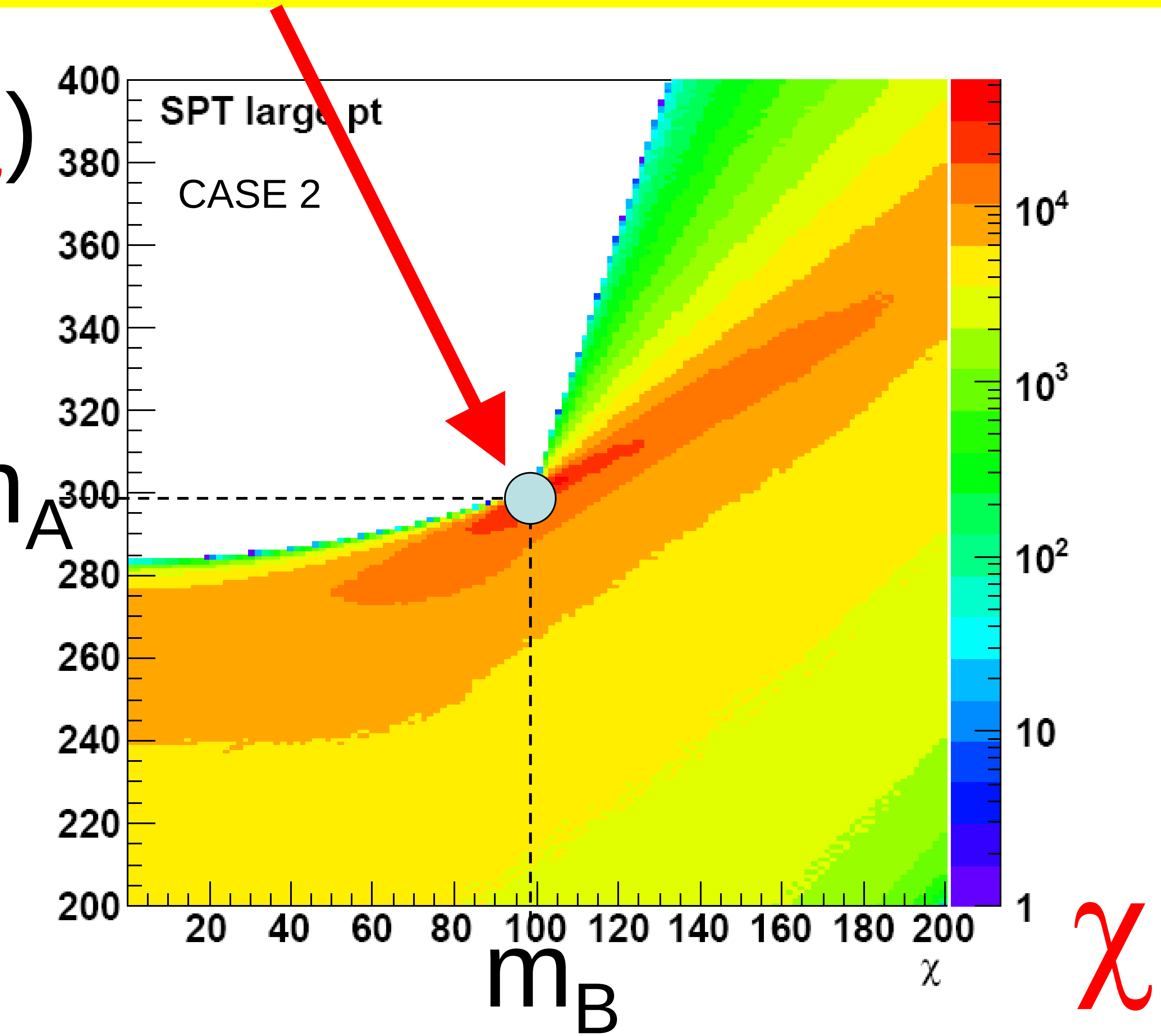
Overlay many events



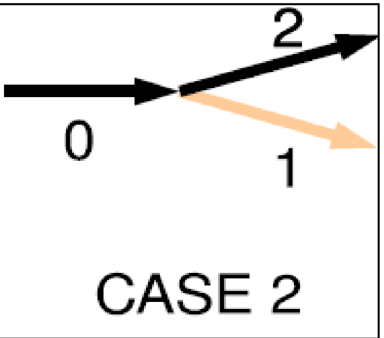
arXiv: 0711.4008



Here is a transverse mass "KINK"



arXiv: 0711.4008

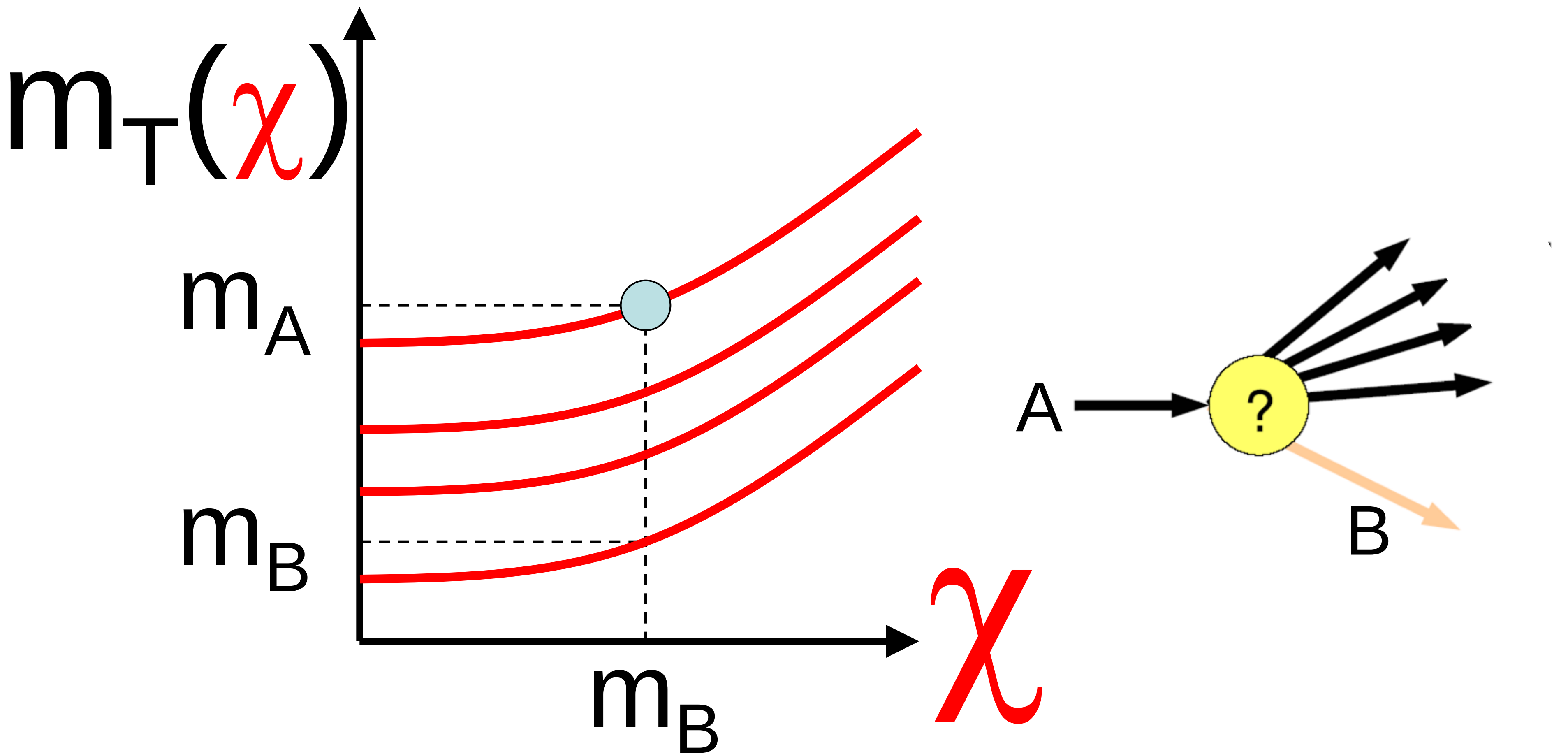


What causes the kink?

- Two entirely independent things can cause the kink:
  - (1) Variability in the "visible mass"
  - (2) Recoil of the "interesting things" against Upstream Transverse Momentum
- Which is the dominant cause depends on the particular situation … let us look at each separately:

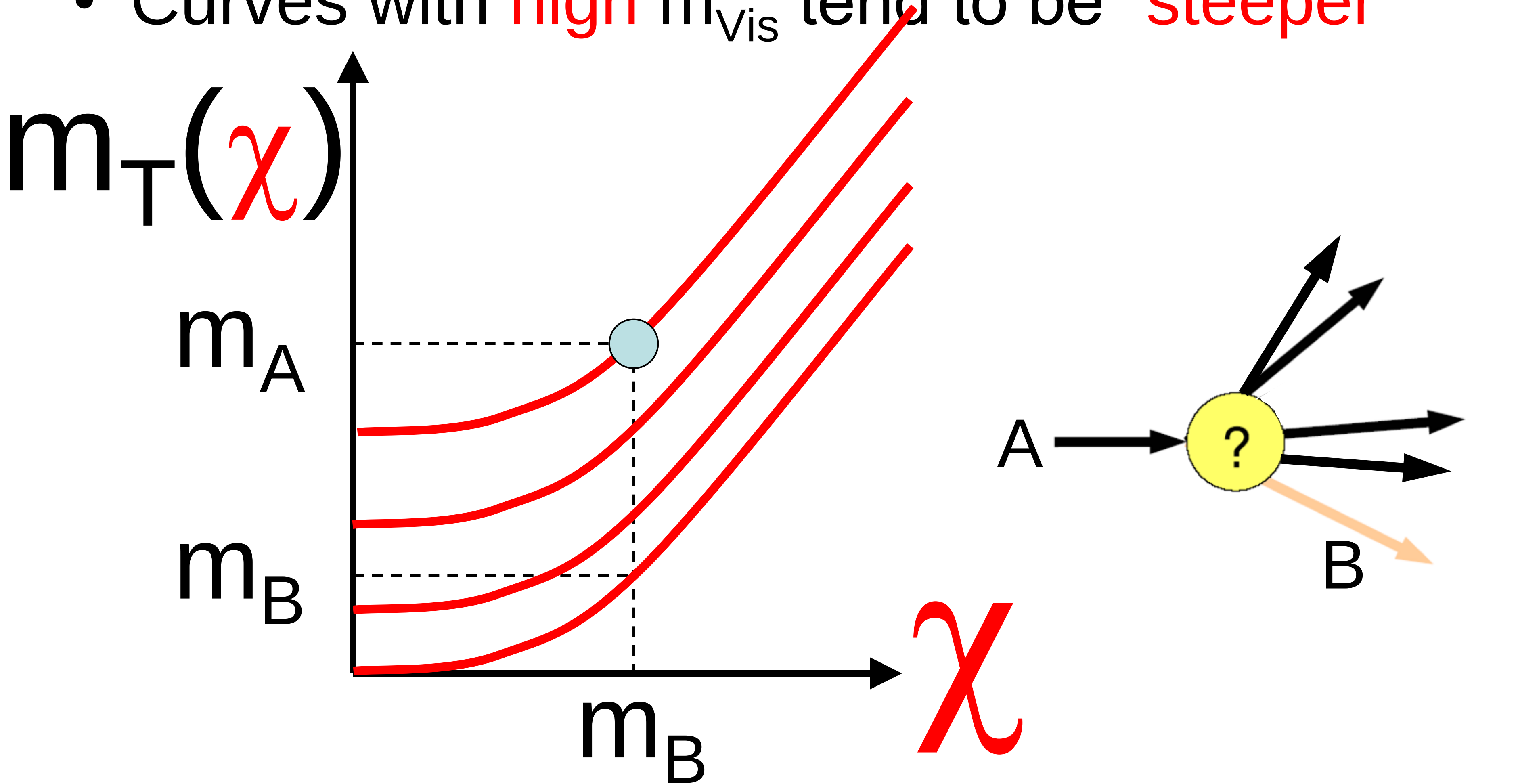
Kink cause 1: Variability in visible mass

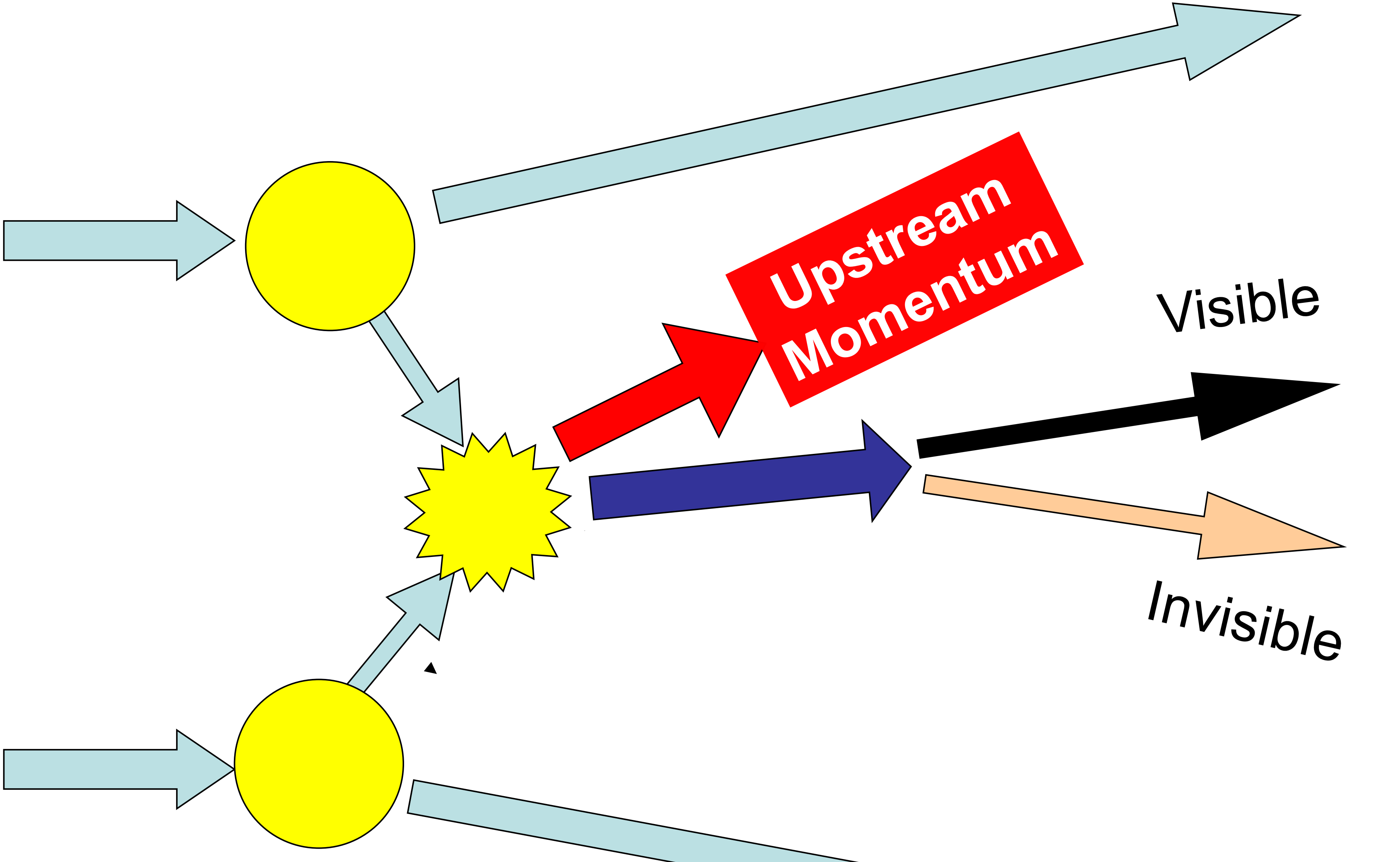
- m_{Vis} can change from event to event
- Gradient of $m_T(\chi)$ curve depends on m_{Vis}
- Curves with low m_{Vis} tend to be "flatter"



Kink cause 1: Variability in visible mass

- m_{Vis} can change from event to event
- Gradient of $m_T(\chi)$ curve depends on m_{Vis}
- Curves with high m_{Vis} tend to be "steeper"

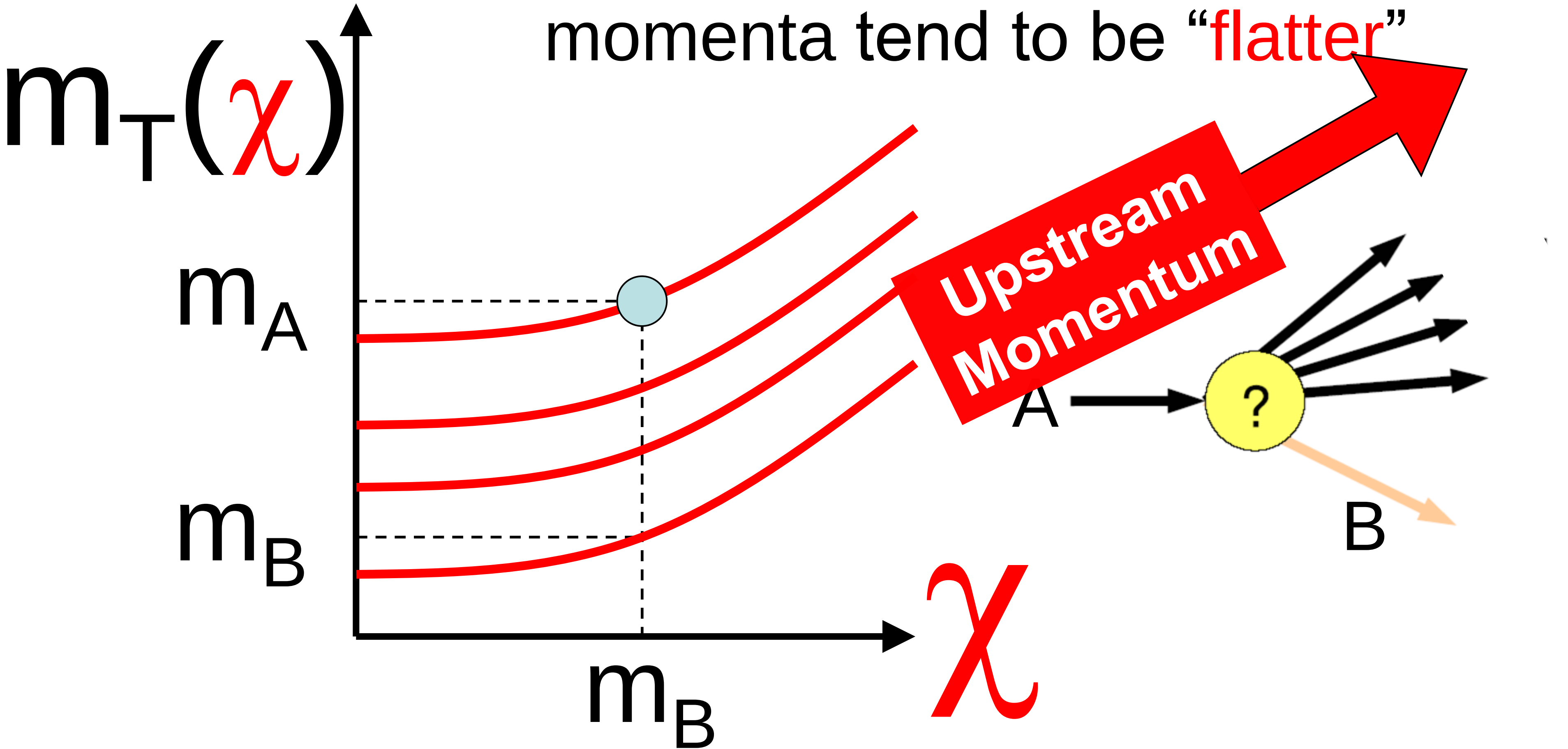




Kink cause 2 :
Recoil against Upstream Momentum

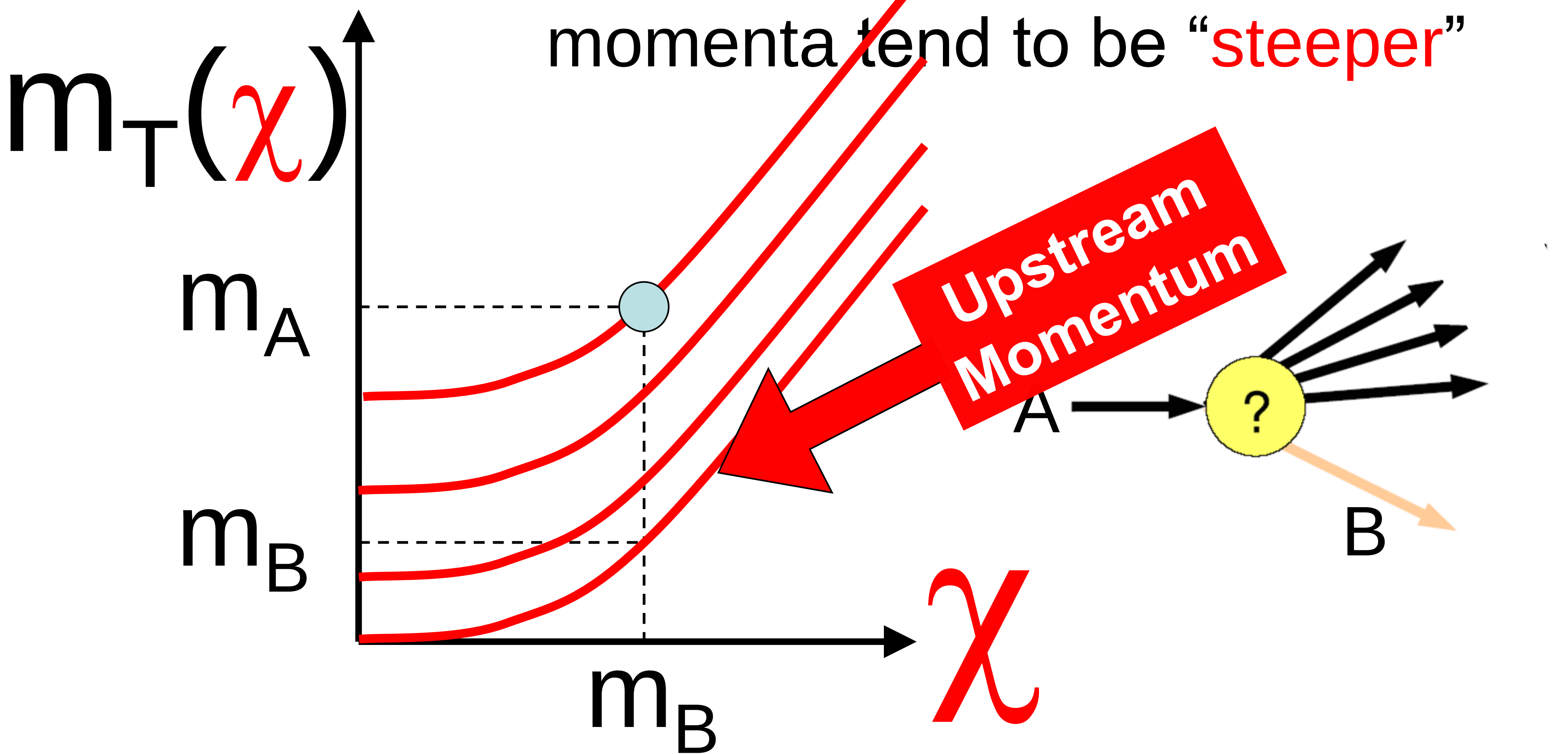
Kink cause 2: Recoil against UTM

- UTM can change from event to event
- Gradient of $m_T(\chi)$ curve depends on UTM
- Curves with UTM parallel to visible momenta tend to be "flatter"

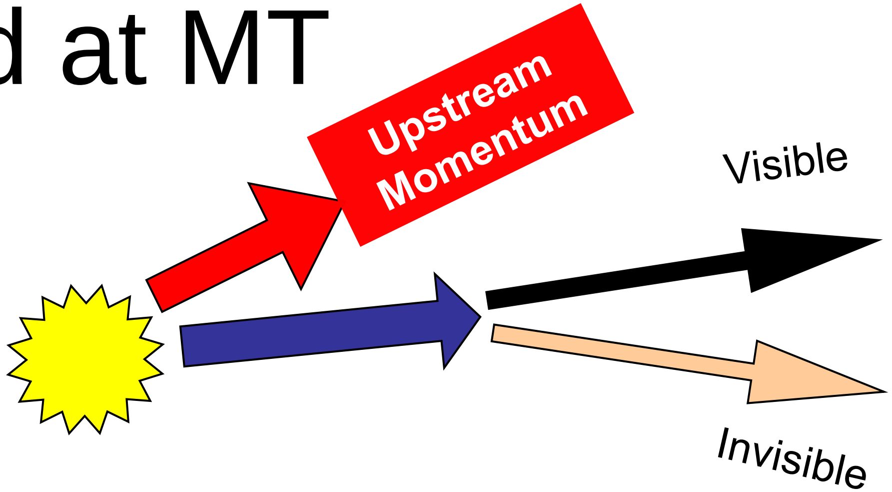


Kink cause 2: Recoil against UTM

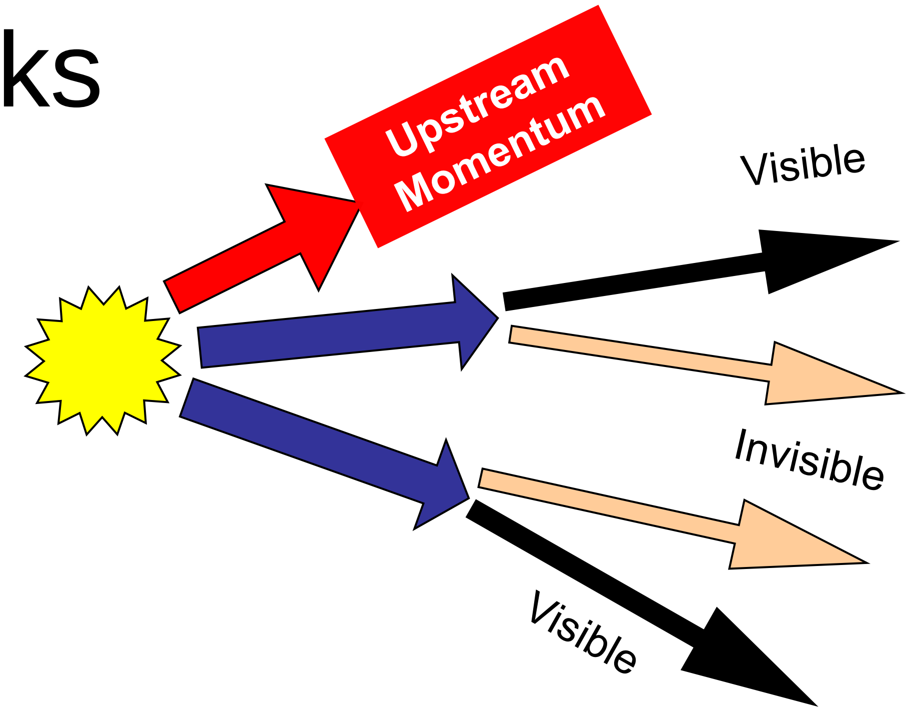
- UTM can change from event to event
- Gradient of $m_T(\chi)$ curve depends on UTM
- Curves with UTM opposite to visible momenta tend to be "steeper"



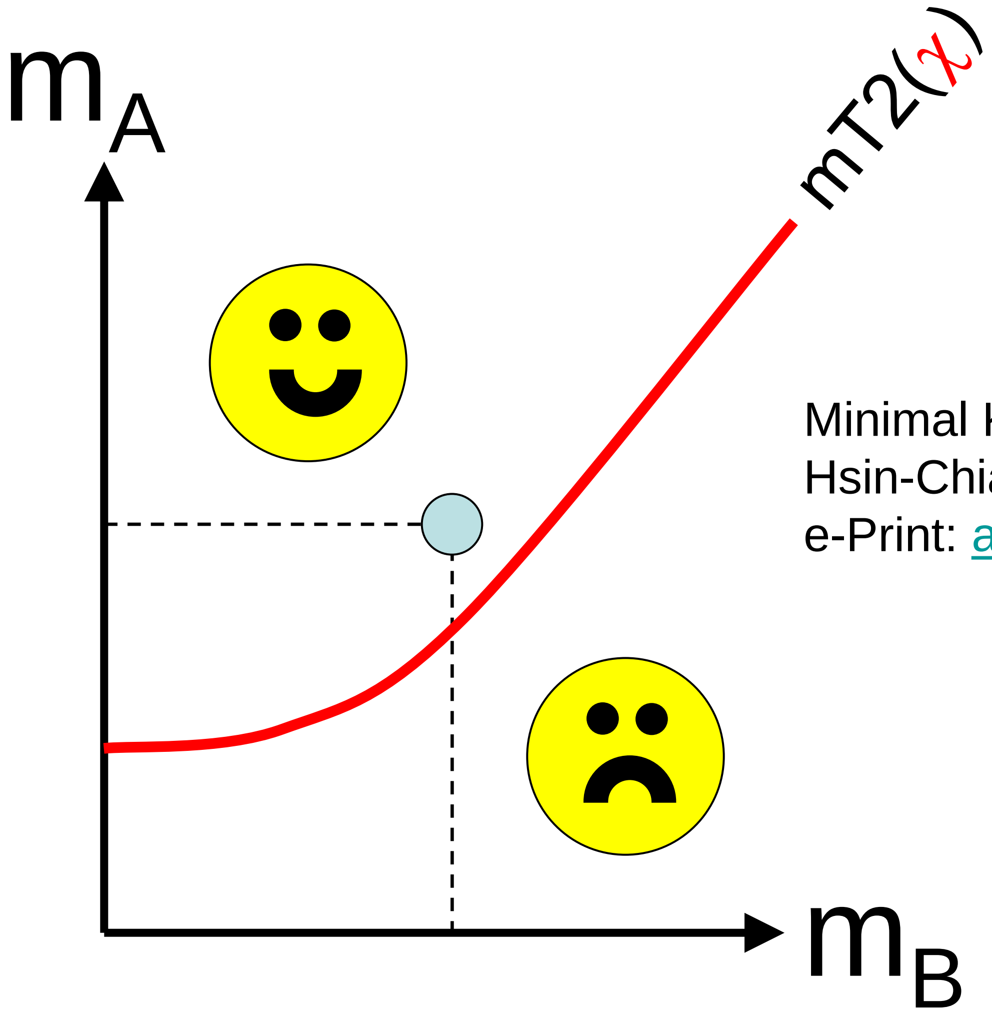
We looked at MT for this:



But everything works just the same for MT2 in events with pair decays:



MT2 (like MT) is also a mass-space boundary



The MT2(chi) curve is the **boundary** of the region of (mother, daughter) mass-space consistent with the observed event!

Minimal Kinematic Constraints and m(T2), Hsin-Chia Cheng and Zhenyu Han (UCD) e-Print: arXiv:0810.5178 [hep-ph]

MT2 and MT behave in exactly the same way as each other, and consequently they share the same kink structure.

Somewhat surprisingly, MT and MT2 kink-based methods are the only(*) methods that have been found which can in principle determine the mass of the invisible particles in short chains! (see arXiv:0810.5576)

(*) There is evidence (Alwall) that Matrix Element methods can do so too, though at the cost of model dependence and very large amounts of CPU.

Health warning!

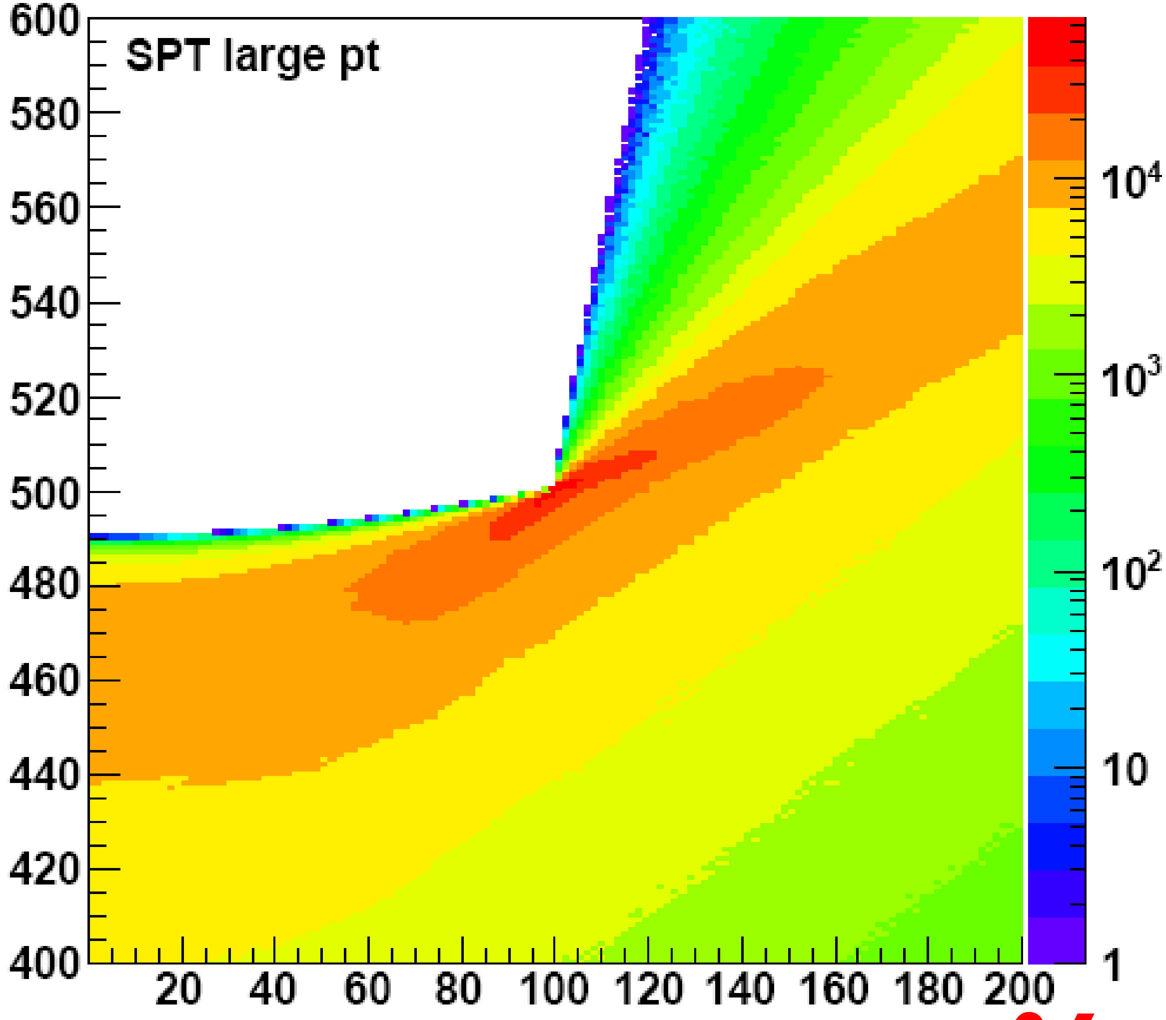
(for those of you interested in LHC dark matter constraints)

Rather worryingly, M_T kinks are at present the only known kinematic methods which (at least in principle) allow determination of the mass of the invisible particle in short chains at hadron colliders!

[We will see a dynamical method that works for single three+ body decays shortly. Likelihood methods can determine masses in pair decays too, though at cost of model dependence and CPU. See Alwall.]

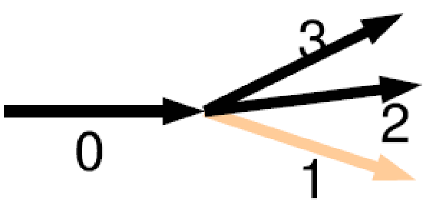
That last statement should worry you!

$m_T(\chi)$



χ

arXiv: 0711.4008



CASE 3v

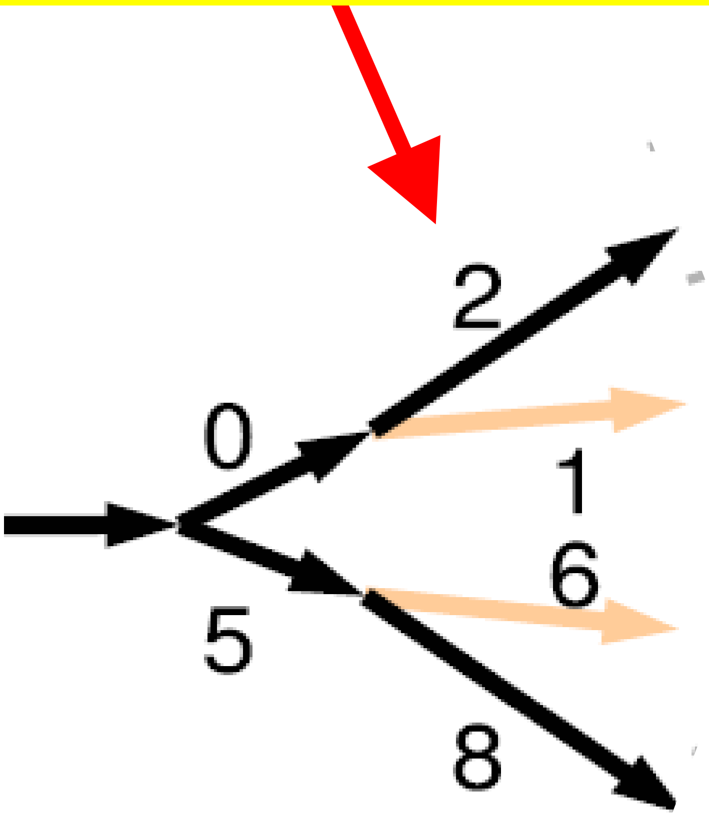
Weighing Wimps with Kinks at Colliders arXiv: 0711.4008

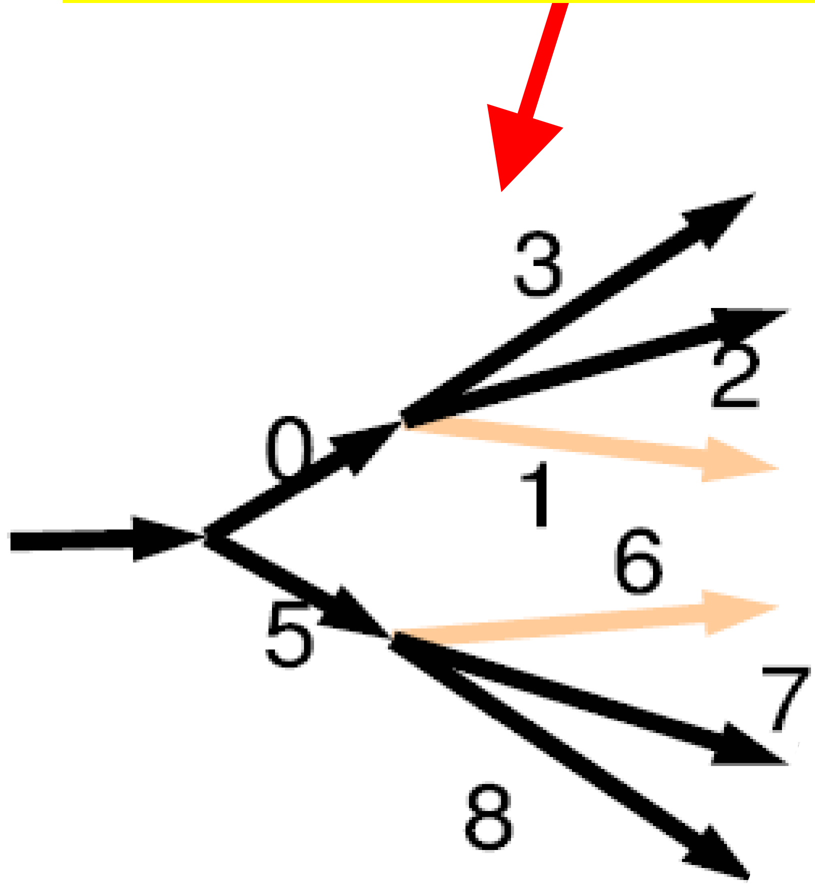
Spot the kink

Are kinks observable ?

Expect KINK only from UTM Recoil (perhaps only from ISR!)

Expect stronger KINK due to both UTM recoil, AND variability in the visible masses.





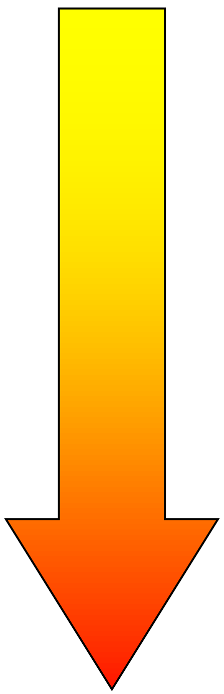
arXiv: 0711.4008

Winding down

Types of Technique

Few

assumptions



Many

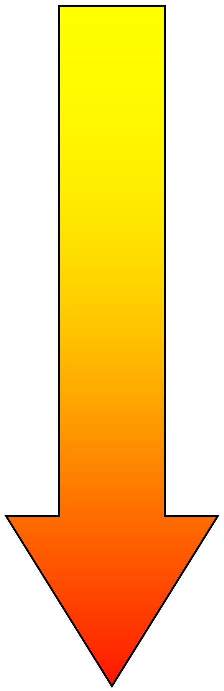
assumptions

- Missing transverse momentum
- M_eff, H_T
- s Hat Min
- M_T
- M_TGEN
- M_T2 / M_CT
- M_T2 (with “kinks”)
- M_T2 / M_CT (parallel / perp)
- M_T2 / M_CT (“sub-system”)
- RAZOR
- “Polynomial” constraints
- Multi-event polynomial constraints
- Whole dataset variables
- Cross section
- Max Likelihood / Matrix Element

Types of Technique

Vague

conclusions



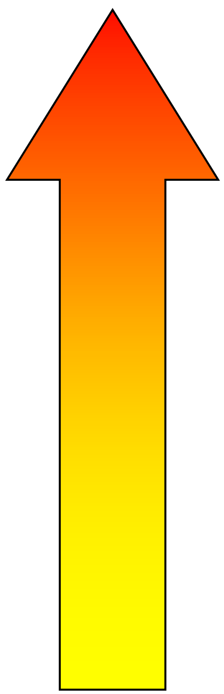
Specific

conclusions

- Missing transverse momentum
- M_eff, H_T
- s Hat Min
- M_T
- M_TGEN
- M_T2 / M_CT
- M_T2 (with “kinks”)
- M_T2 / M_CT (parallel / perp)
- M_T2 / M_CT (“sub-system”)
- RAZOR
- “Polynomial” constraints
- Multi-event polynomial constraints
- Whole dataset variables
- Cross section
- Max Likelihood / Matrix Element

Types of Technique

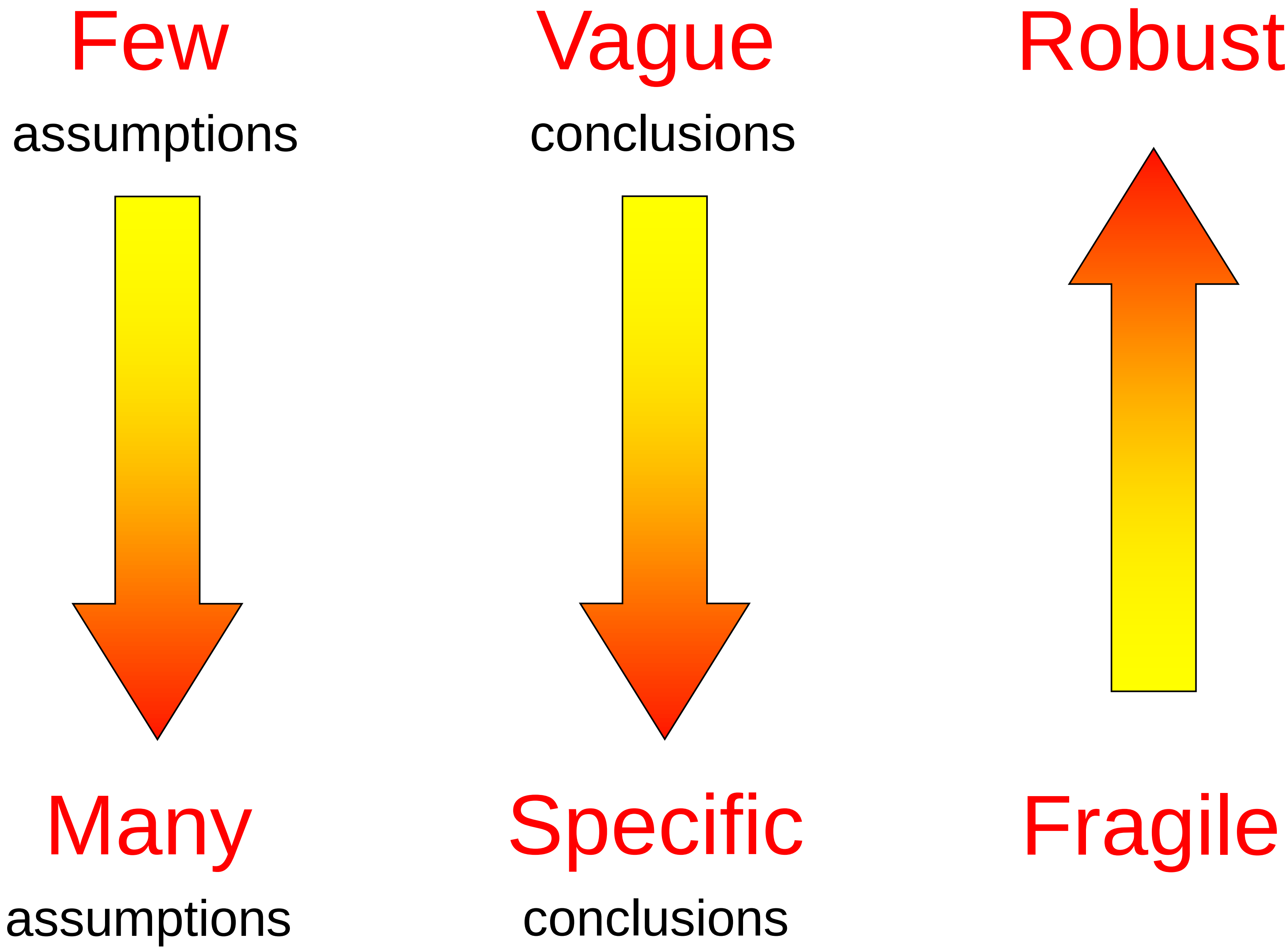
Robust



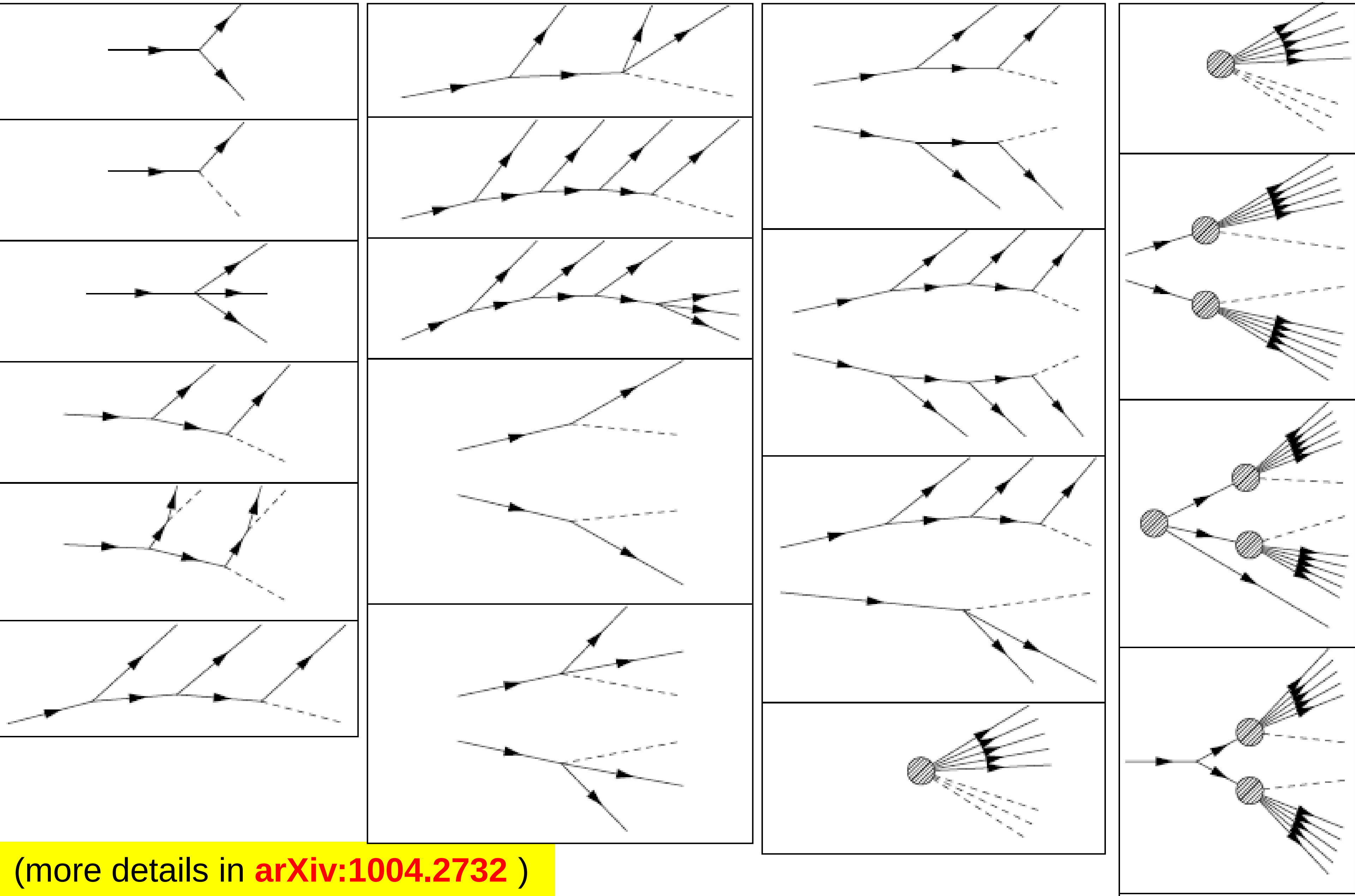
Fragile

- Missing transverse momentum
- M_eff, H_T
- s Hat Min
- M_T
- M_TGEN
- M_T2 / M_CT
- M_T2 (with “kinks”)
- M_T2 / M_CT (parallel / perp)
- M_T2 / M_CT (“sub-system”)
- RAZOR
- “Polynomial” constraints
- Multi-event polynomial constraints
- Whole dataset variables
- Cross section
- Max Likelihood / Matrix Element

The balance of benefits

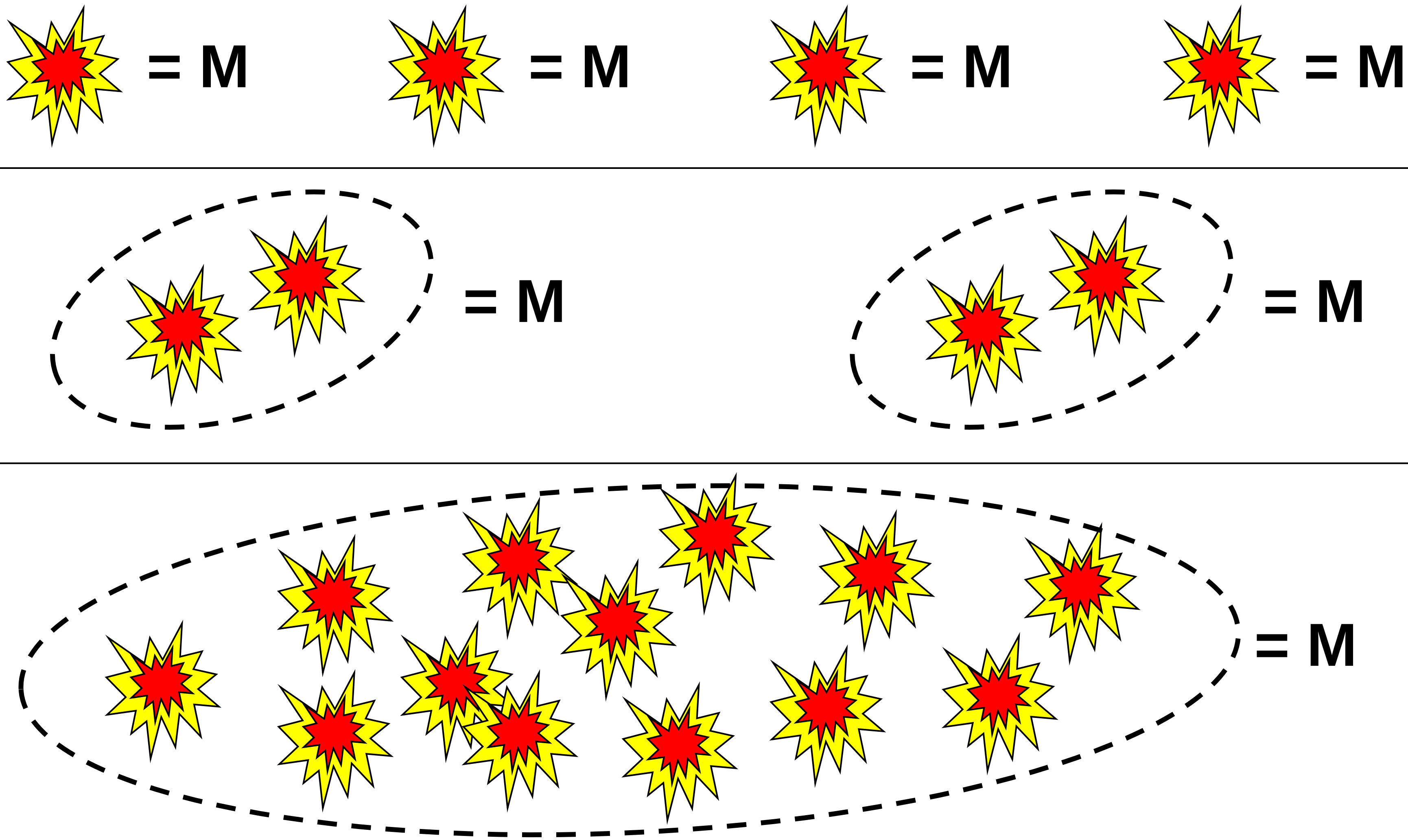


The number of hypotheses is virtually unlimited ...



(more details in **arXiv:1004.2732**)

… and from few events required, to many events required ….



“MT2 type” variables : where next ?

“Addition type sums” : 1+1=2, 2+1=3, 3+5=8

Where next?

A very odd question!

Is there a Poster Boy MEM Analysis?

- What about pair production of slepton to lepton neutralino with the two sparticles nearly degenerate (less than MW apart) ?
- MT2 useless here as cannot suppress W pair BG. A dynamic variable (allowed to use spins) may do ok, though. Under investigated? Good opportunity to beat BDT to answer – or prove that it’s impossible.
- Worrying: Johan Alwall IPMU 2008 MEM study produced same endpoint-kink structure as MT2. Perhaps no spin correlations?

I’ve not seen as poster boy ME analysis yet.

Kinematics – usually OK.
Dynamics – sometimes needed.
BDT power illusory.
Kyn/Dyn vars fast!
Simpler than people think.

Let’s stop here!

Extras if time …

Experiments therefore use additional selection variables that are sensitive to these differences:

Variable	Used by
• MT2	(ATLAS &CMS)
• Razor	(CMS)
• Meff or HT	(ATLAS & CMS)
• Alpha_T	(CMS)
• pTmiss/Sqrt(HT)	(ATLAS)

Experiments therefore use additional selection variables that are sensitive to these differences:

Variable	Motivated by	
• MT2	Signals of BSM pair production	(general)
• Razor		(at threshold)

- Meff or HT
- Alpha_T
- pTmiss/Sqrt(HT)

Experiments therefore use additional selection variables that are sensitive to these differences:

Variable	Motivated by
• MT2	
• Razor	
• Meff or HT	Very little! (yet it works well)
• Alpha_T	
• pTmiss/Sqrt(HT)	

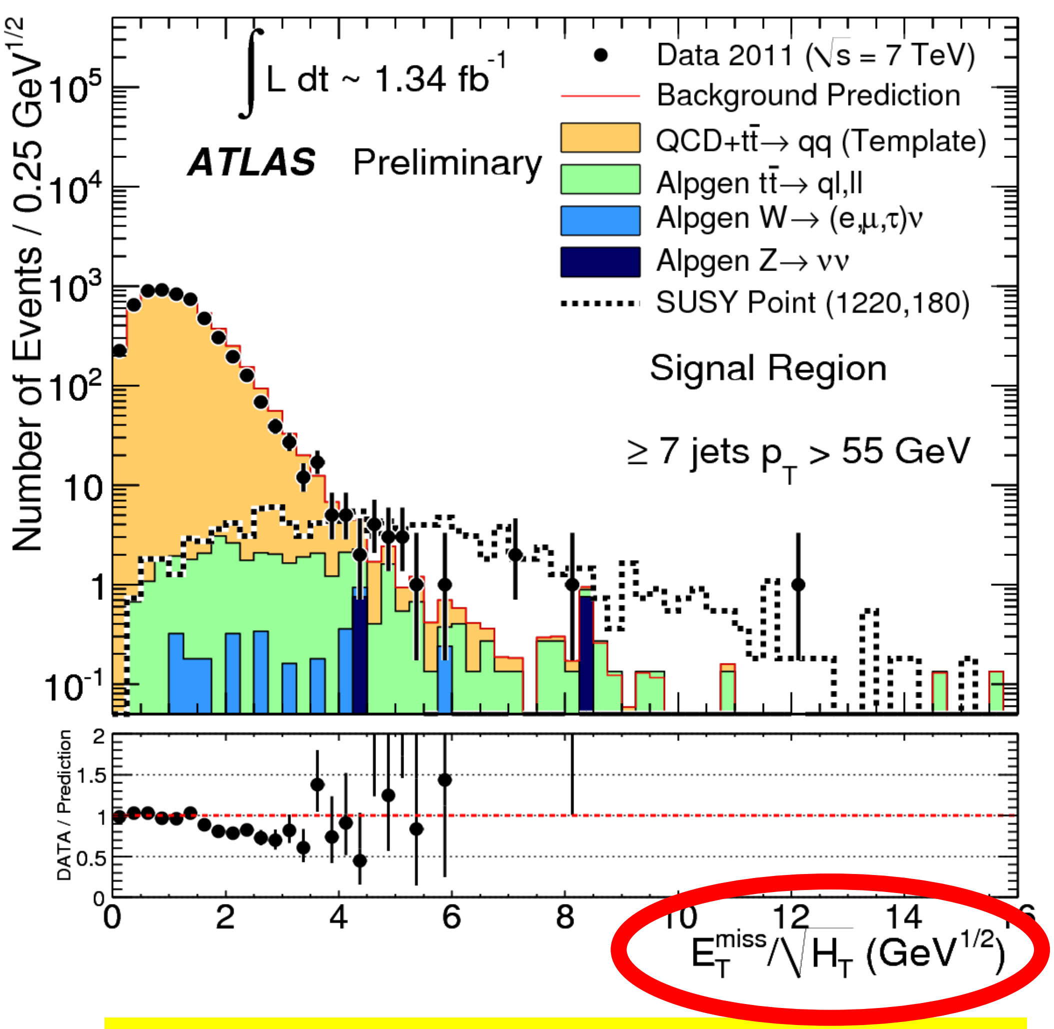
Experiments therefore use additional selection variables that are sensitive to these differences:

Variable	Motivated by
• MT2	
• Razor	
• Meff or HT	
• Alpha_T	QCD rejection
• pTmiss/Sqrt(HT)	

Experiments therefore use additional selection variables that are sensitive to these differences:

Variable	Motivated by
• MT2	
• Razor	
• Meff or HT	
• Alpha_T	
• pTmiss/Sqrt(HT)	Understanding of detector – ability to derive backgrounds from data

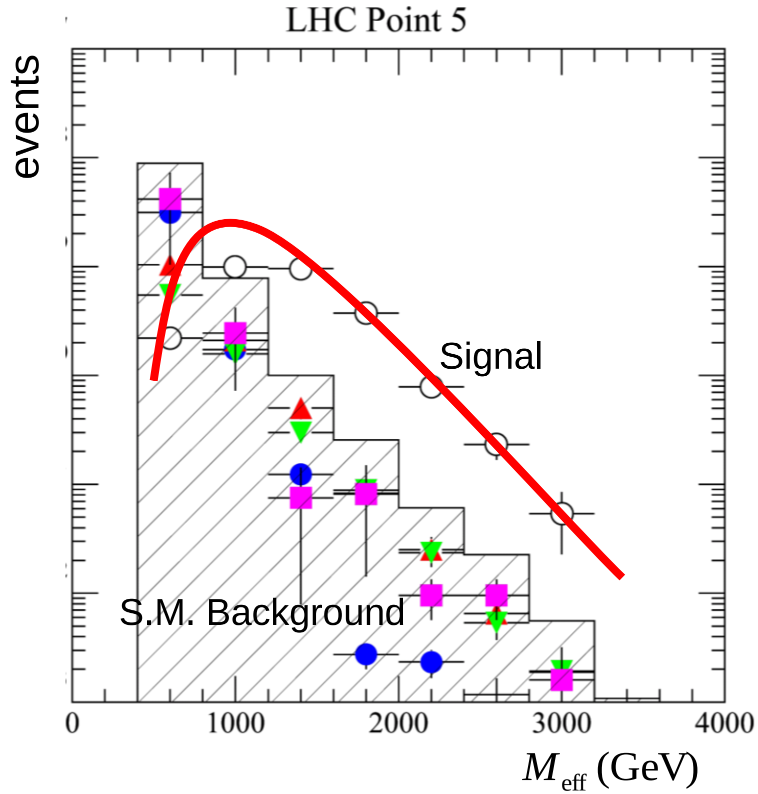
Careful choice of variable allows astounding search for pTmiss excess in 7 & 8 jet events!



Personally, I find this the most exciting development of the summer!

Meff (similarly HT)

Definition: $$M_{\text{eff}} = \mathbf{p}_{\mathrm{T}}^{\text{missing}} + \sum_i \left| \mathbf{p}_{\mathrm{T}}^{\text{jet}_i} \right|$$



Significance Variables

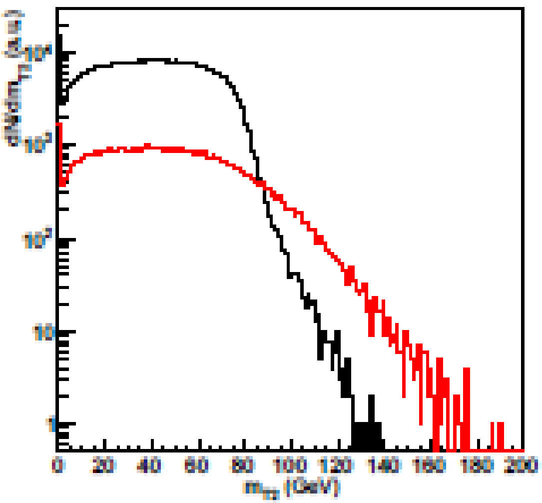
Benjamin Nachman[a] **Christopher G. Lester**[b]

[a] *DAMTP, CMS, University of Cambridge, Wilberforce Road, Cambridge, CB3 0HA, U.K.*

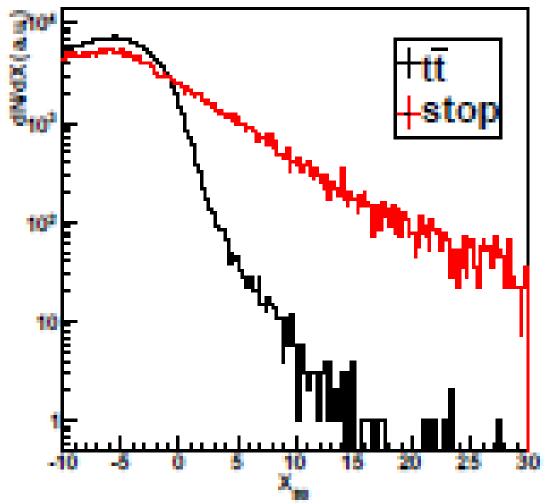
[b] *Cavendish Laboratory, Department of Physics, JJ Thomson Avenue, Cambridge, CB3 0HE, U.K.*

E-mail: bnachman@cern.ch, Lester@hep.phy.cam.ac.uk

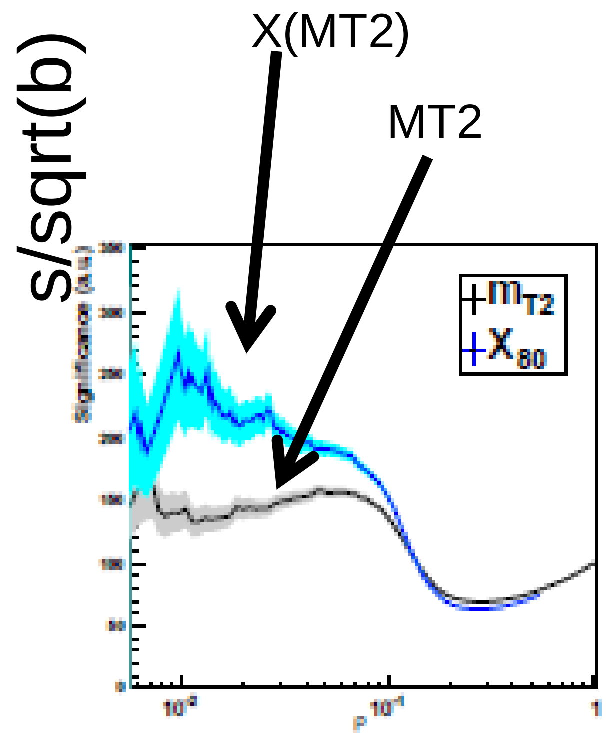
Abstract: Many particle physics analyses which need to discriminate some background process from a signal ignore event-by-event resolutions of kinematic variables. Adding this information, as is done for missing momentum significance, can only improve the power of existing techniques. We therefore propose the use of significance variables which combine kinematic information with event-by-event resolutions. We begin by giving some explicit examples of constructing optimal significance variables. Then, we consider three applications: new heavy gauge bosons, Higgs to $\tau\tau$, and direct stop squark pair production. We find that significance variables can provide additional discriminating power over the original kinematic variables: $\sim 20\%$ improvement over m_T in the case of $H \to \tau\tau$ case, and $\sim 30\%$ impovement over m_{T2} in the case of the direct stop search.



MT2



X(MT2)



Signal Efficiency

MT2 distribution

X(MT2) distribution

s/sqrt(b)

for different signal efficiencies

$$X(MT2) = \frac{MT2 - SCALE}{mT2\ Uncertainty}$$

Why are we adding transverse momenta?

- Why not multiply?
  (or add logs)?
  [serious motivation in arXiv:1103.5682 !]

$$M_{happy} = \left(\prod_{i=1}^{n} \mathbf{p}_T^i \right)^{\frac{1}{n}}$$

- Serious proposal to use $\mathrm{Meff}^2 - (u_T)^2$ in arXiv:1105.2977
- Why are the signs the same? Why equal weights? Silly?
- How many years would it take ATLAS/CMS to discover the invariant mass for Z -> a b ?

$$\mathrm{M}^2 = \left(\sqrt{m_a^2 + a_x^2 + a_y^2 + a_z^2} + \sqrt{m_b^2 + b_x^2 + b_y^2 + b_z^2} \right)^2 - \left(a_x + b_x\right)^2 - \left(a_y + b_y\right)^2 - \left(a_z + b_z\right)^2$$

Other MT2 related variables (1/3)

- MCT ("Contralinear-Transverse Mass") (arXiv:0802.2879)
  - Is equivalent to MT2 in the special case that there is no missing momentum (and that the visible particles are massless).
  - Proposes an interesting multi-stage method for measuring additional masses
  - Can be calculated fast enough to use in ATLAS trigger.

Other MT2 related variables (2/3)

- MTGEN ("MT for GENeral number of final state particles") (arXiv:0708.1028)
  - Used when
    - each "side" of the event decays to MANY visible particles (and one invisible particle) and
    - it is not possible to determine which decay product is from which side … all possibilities are tried
- Inclusive or Hemispheric MT2 (Nojirir + Shimizu) (arXiv:0802.2412)
  - Similar to MTGEN but based on an assignment of decay product to sides via hemisphere algorithm.
  - Guaranteed to be >= MTGEN

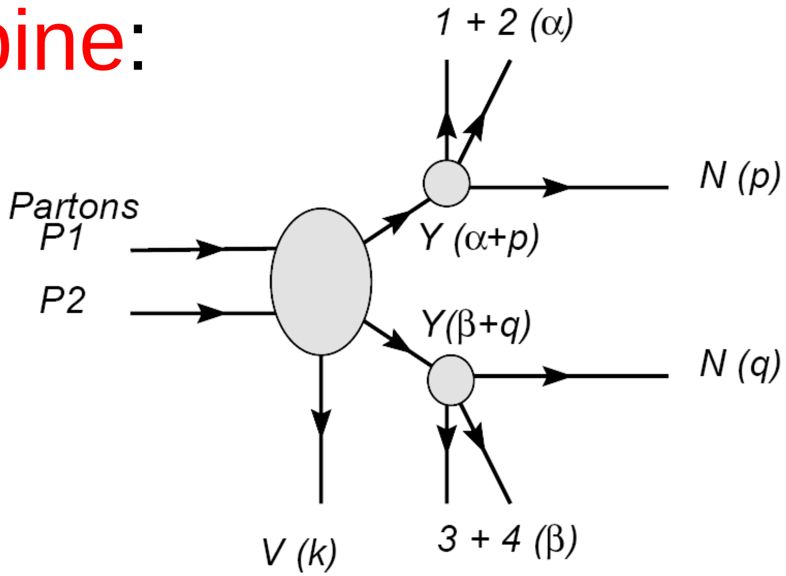
Other MT2 related variables (3/3)

- M2C ("MT2 Constrained") arXiv:0712.0943 (wait for v3 ... there are some problems with the v1 and v2 drafts)
- M2CUB ("MT2 Constrained Upper Bound") arXiv:0806.3224

- There is a sense in which these two variables are really two sides of the same coin.
  - if we could re-write history we might name them more symmetrically
  - I will call them m_{Small} and m_{Big} in this talk.

m_{Small} and m_{Big}

- Basic idea is to combine:
  - MT2
- with
  - a di-lepton invariant mass endpoint measurement (or similar) providing:



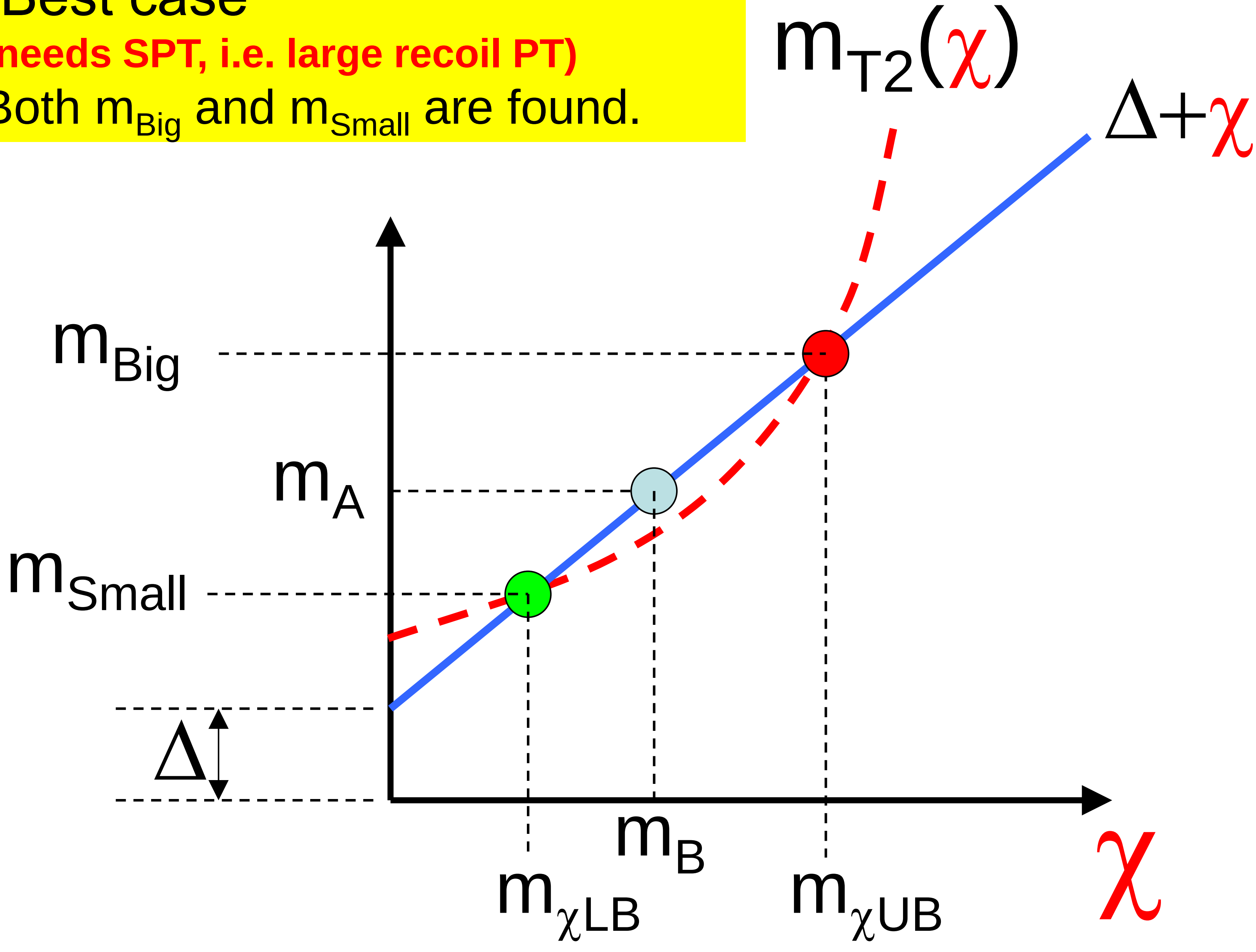
$$\Delta = M_A - M_B$$

(or M_Y-M_N in the notation of their figure above)

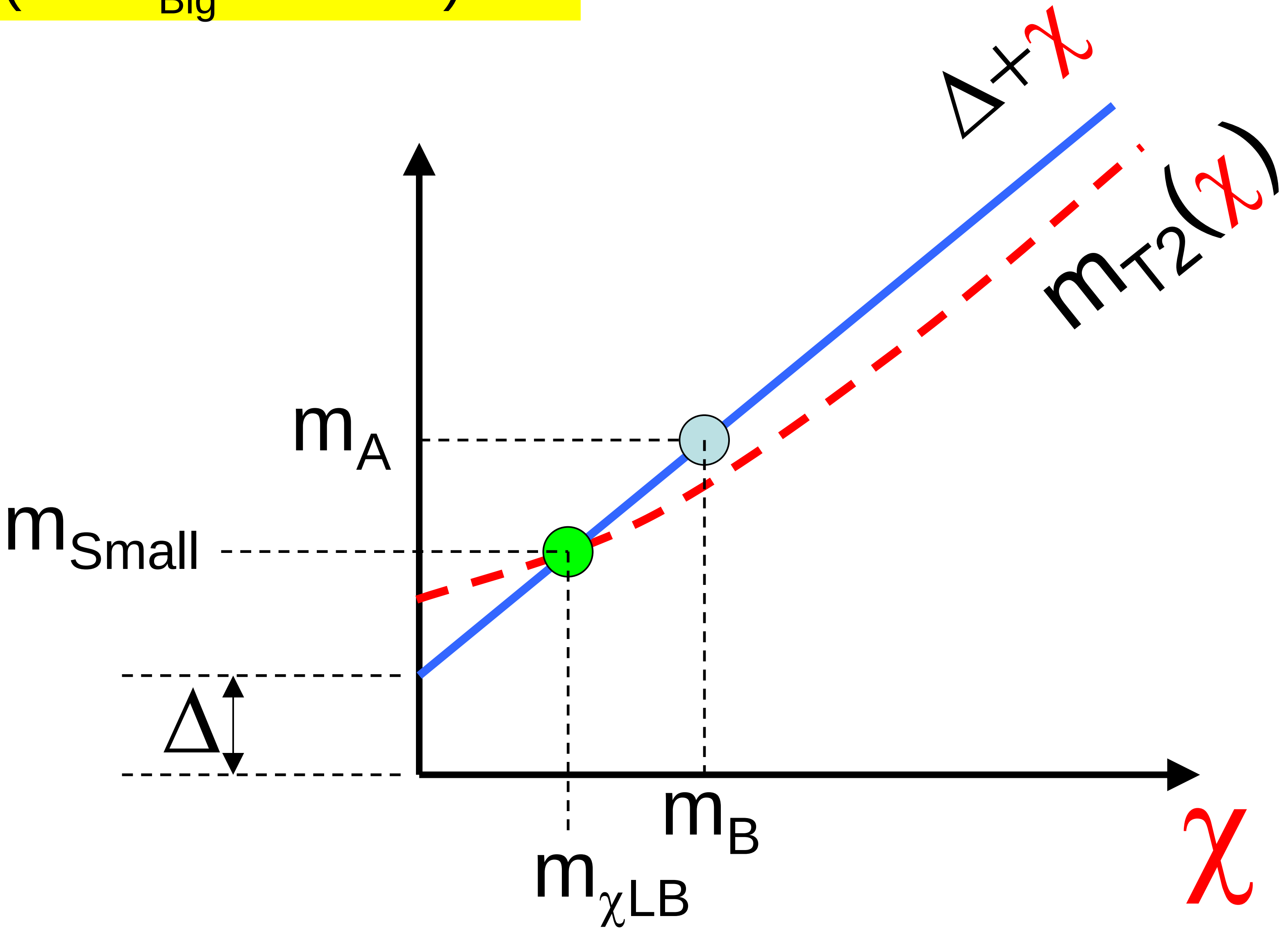
“Best case”

(needs SPT, i.e. large recoil PT)

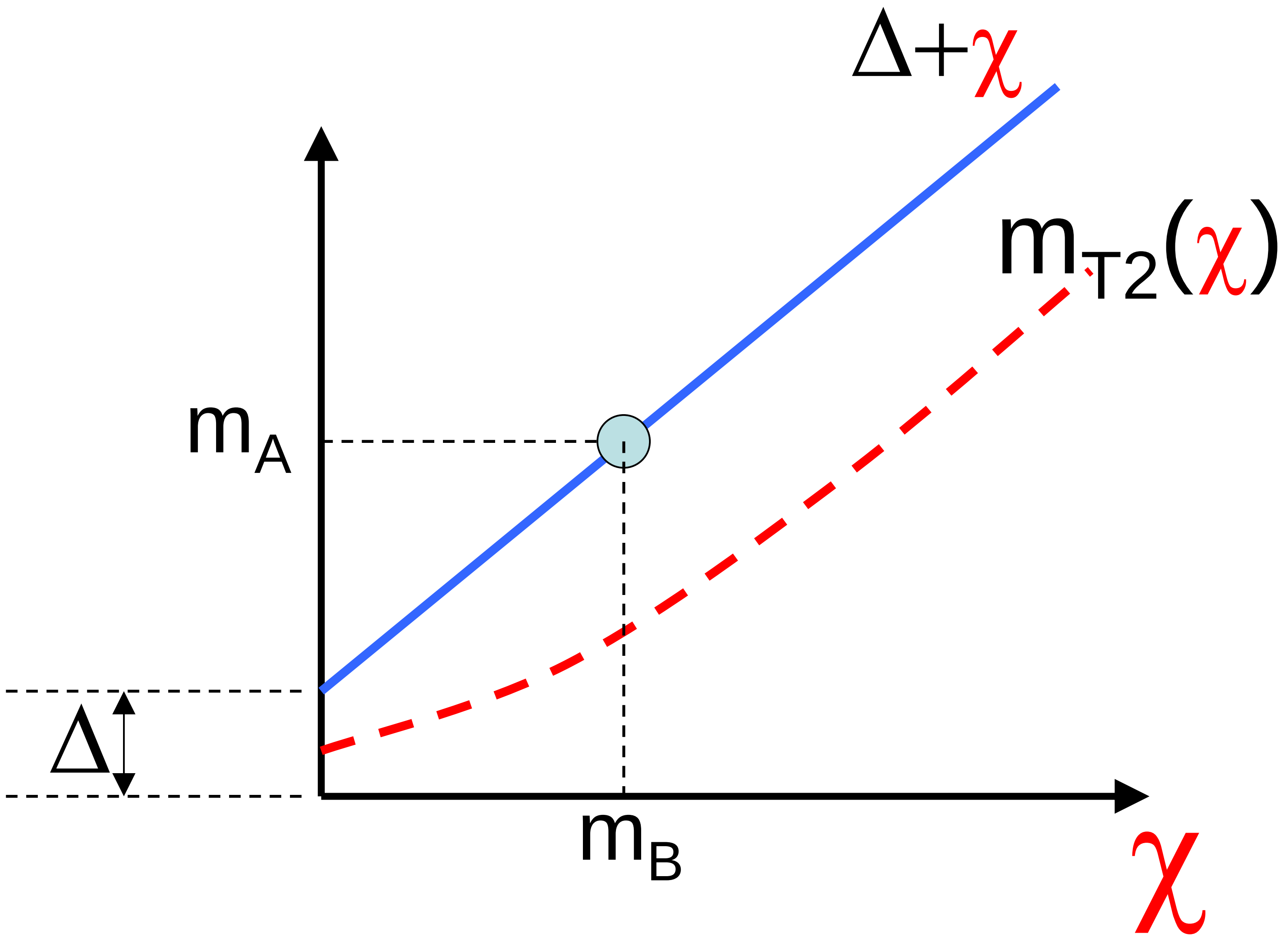
Both m_{Big} and m_{Small} are found.



“Typical ZPT case”
(no m_{Big} is found)



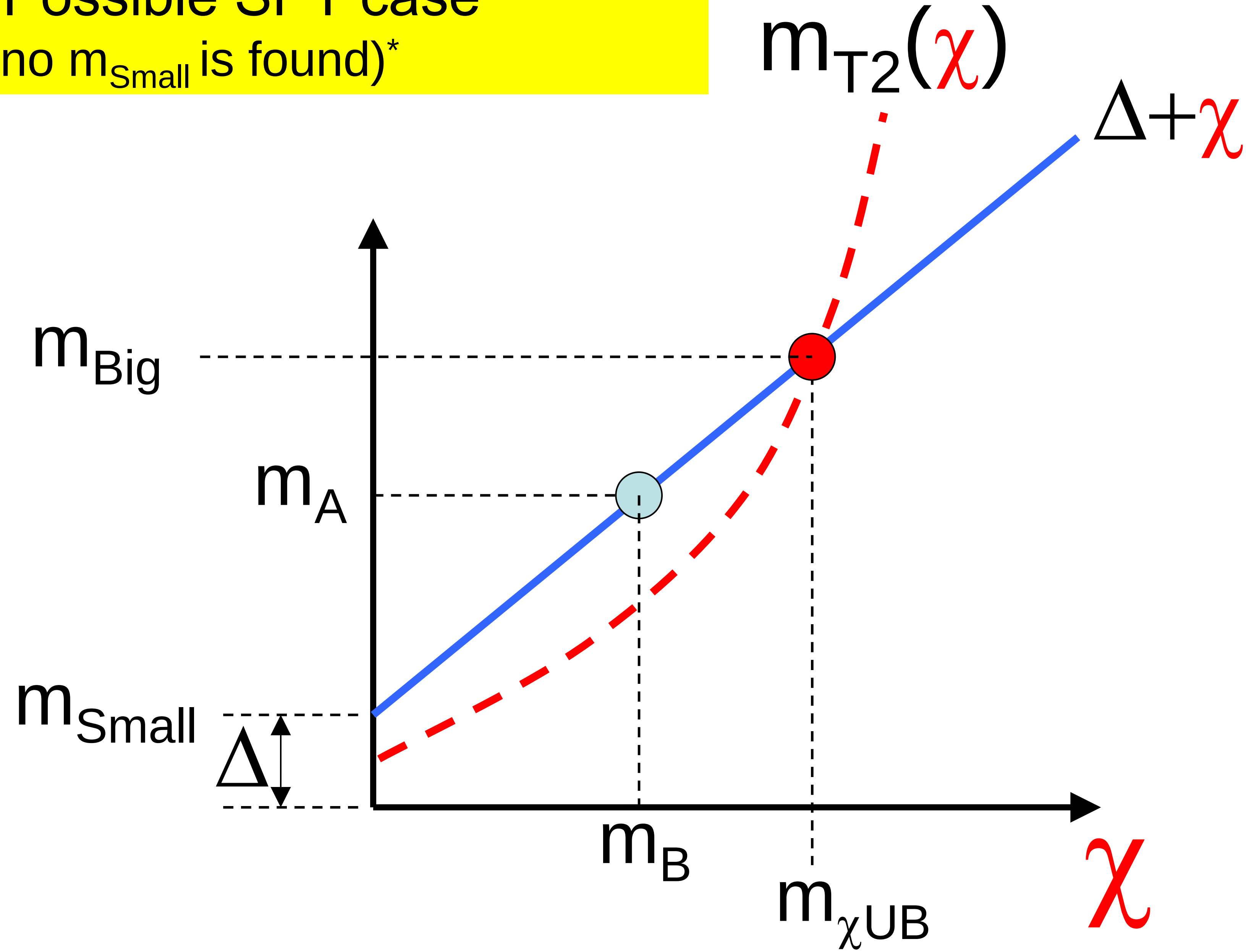
"Possible ZPT case"
(neither m_{Big} nor m_{Small} is found)*



* Except for conventional definition of m_{Small} to be Δ in this case.

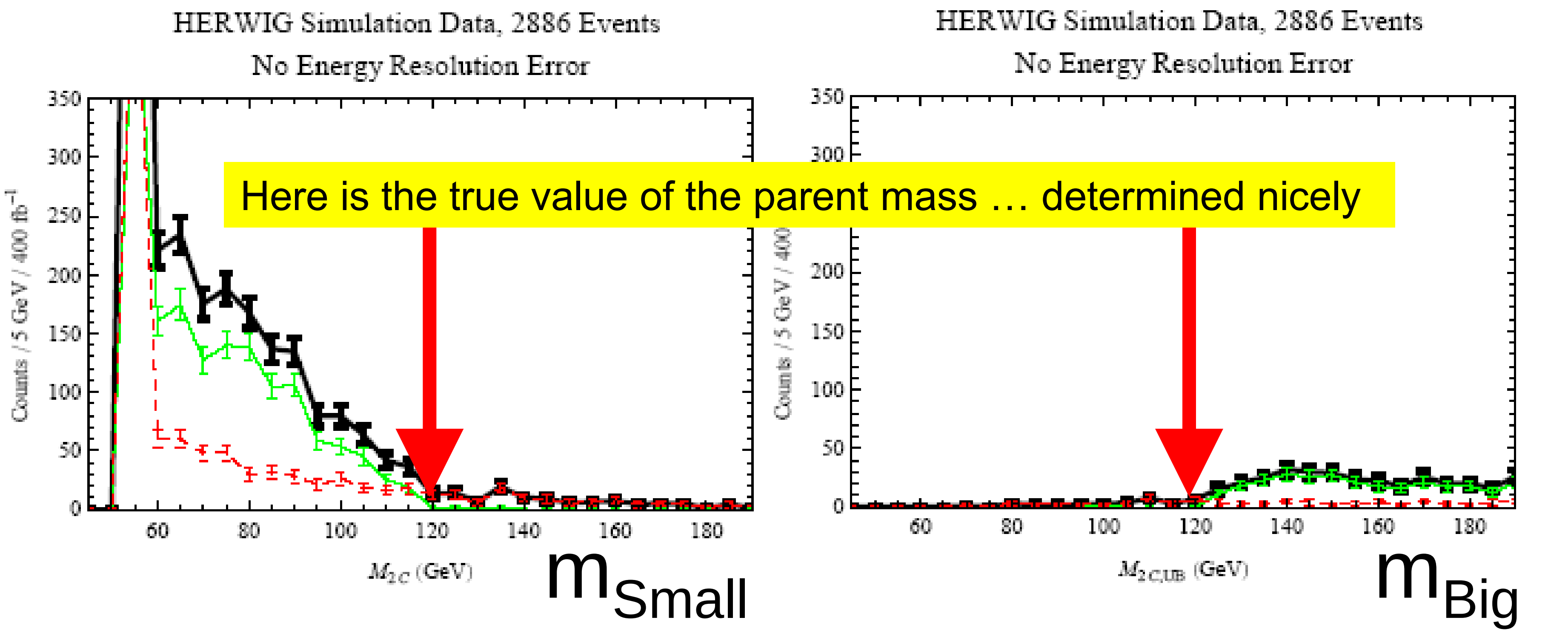
“Possible SPT case”

(no m_{Small} is found)*



* Except for conventional definition of m_{Small} to be Δ in this case.

What m_{Small} and m_{Big} look like,
and how they determine the parent mass

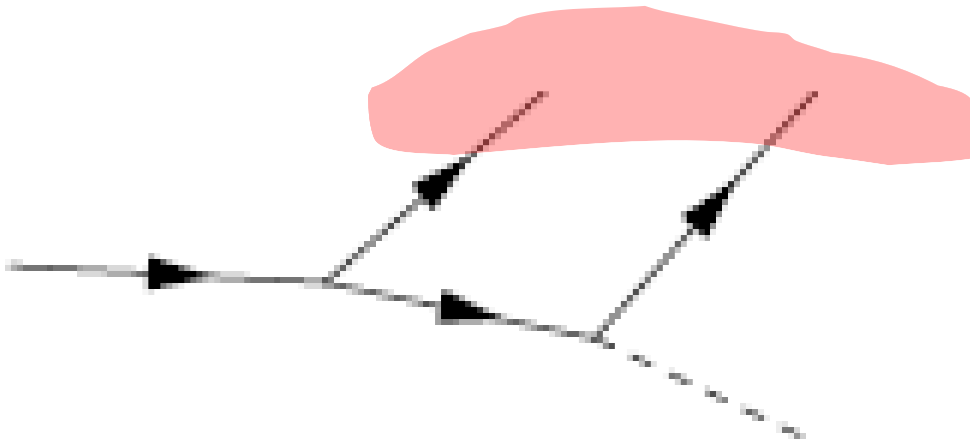


arXiv:0806.3224

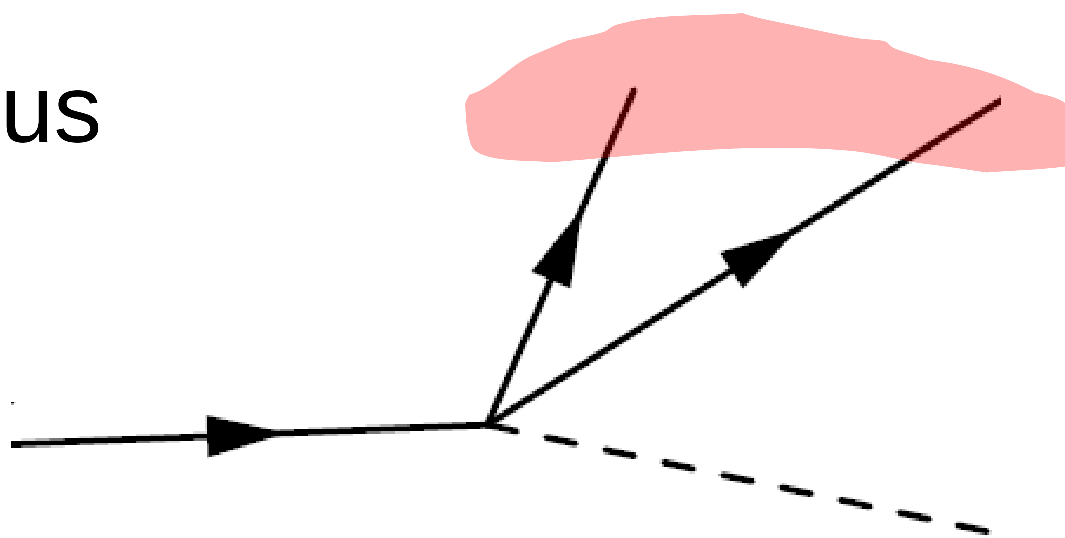
Outcome:

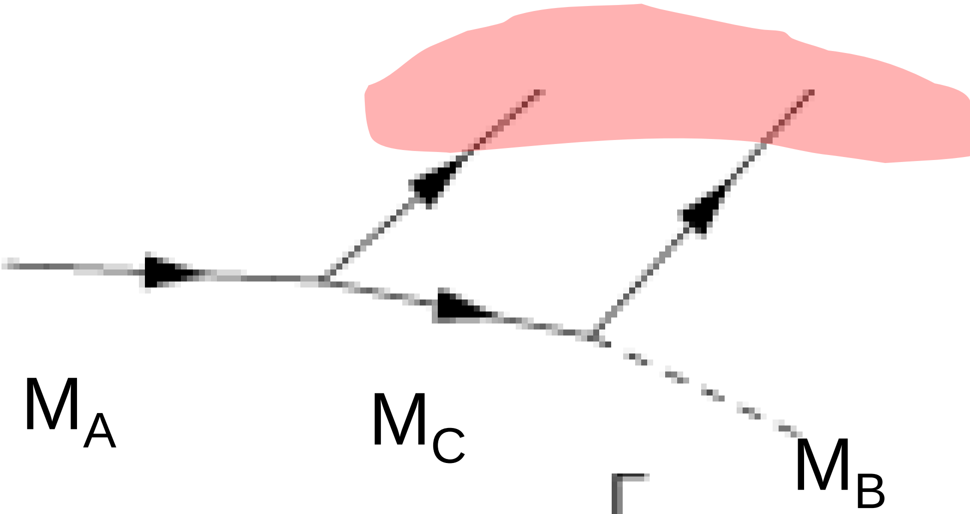
- m_{Big} provides the first potentially-useful event-by-event upper bound for m_A
  - (and a corresponding event-by-event upper bound for m_B called $m_{\chi UB}$)
- m_{Small} provides a new kind of event-by-event lower bound for m_A which incorporates consistency information with the dilepton edge
- m_{Big} is always reliant on SPT (large recoil of interesting system against "up-stream momentum") – cannot ignore recoil here!

Compare shapes of invariant mass distributions for the highlighted pairs of visible massless momenta:



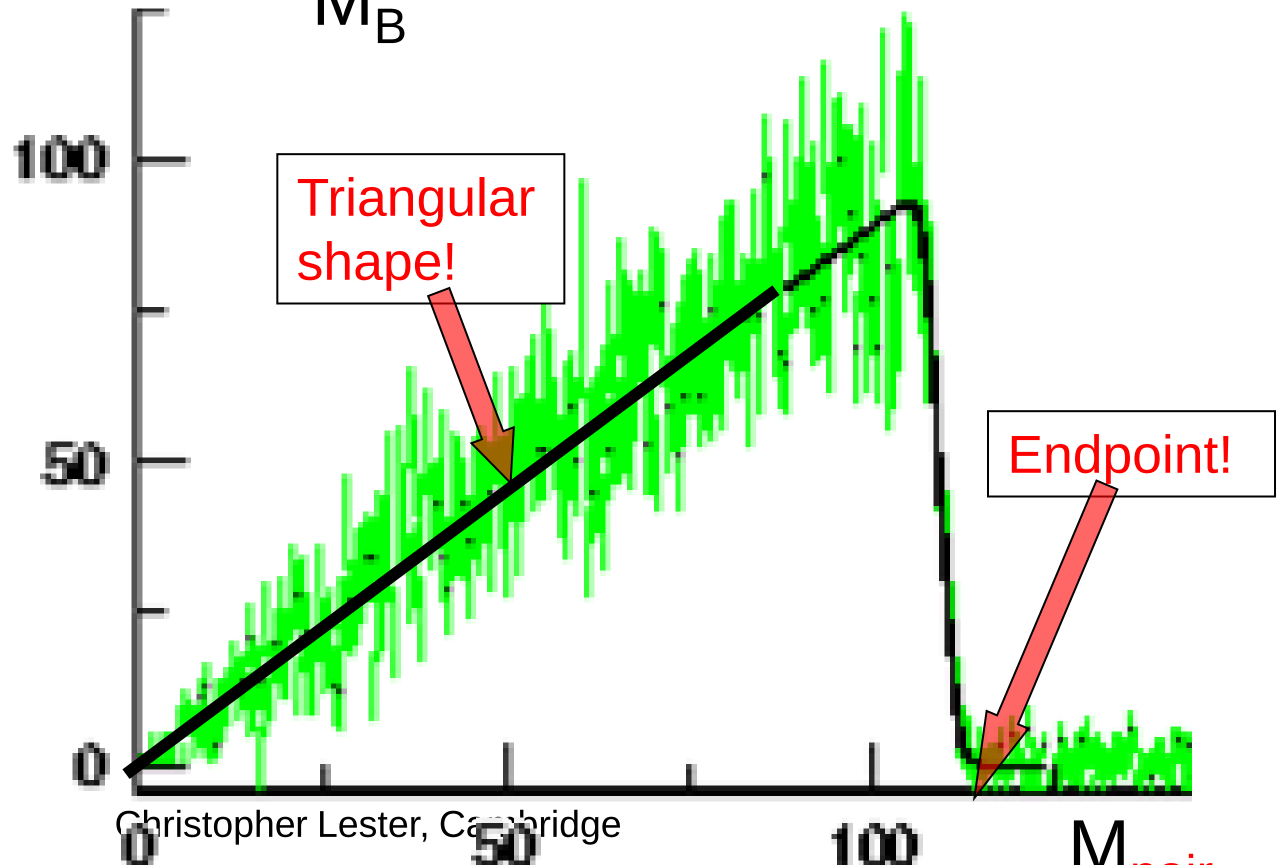
versus



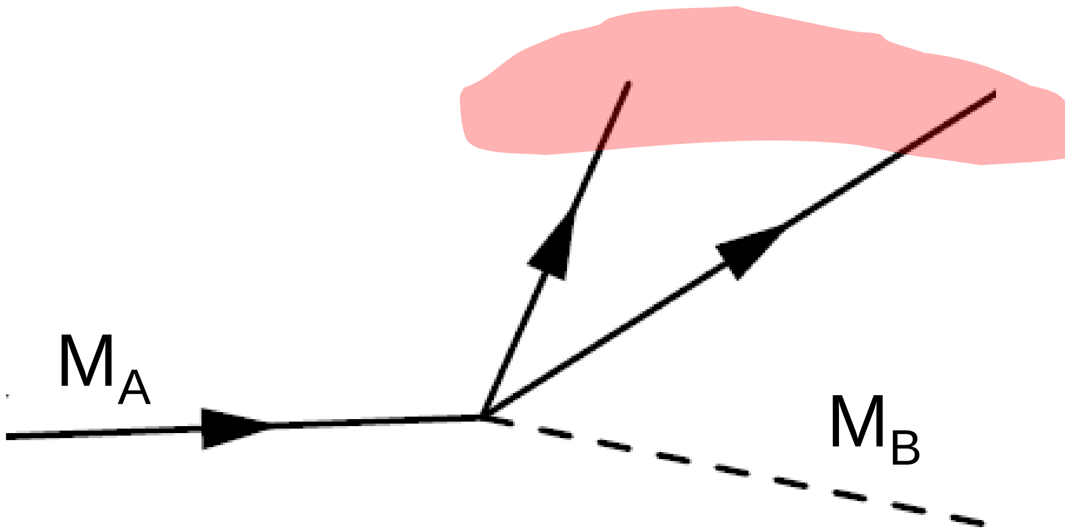


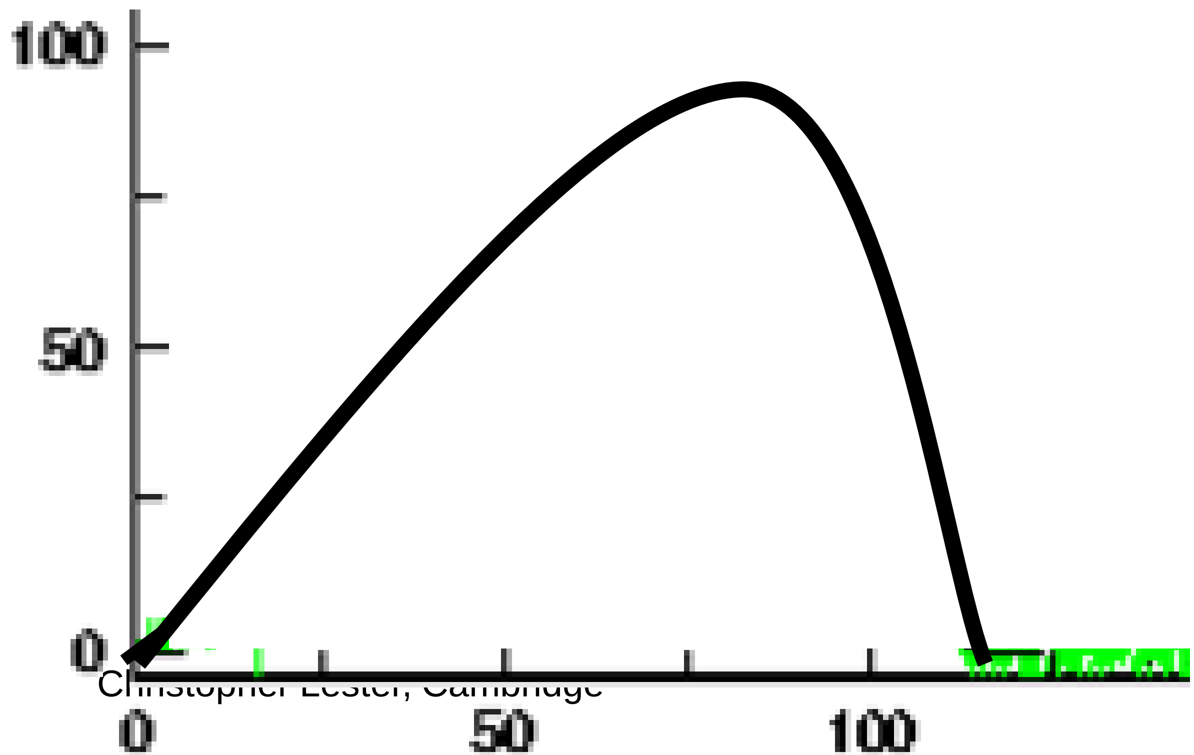
Only "measurement"
(but not only inference!)

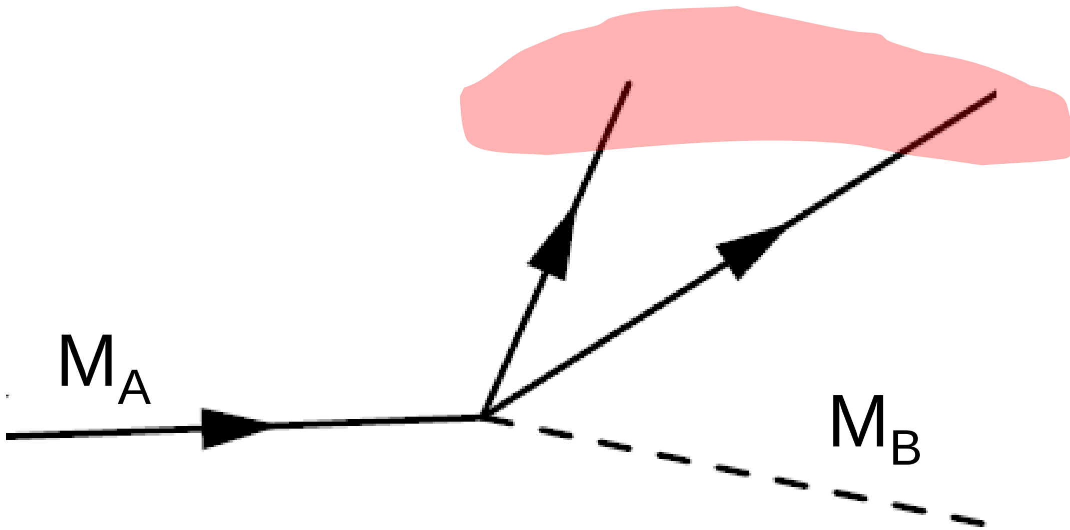
$$\left(m_{ll}^{\max}\right)^2 = \frac{\left(m^2_{\tilde{\chi}^0_2} - m^2_{\tilde{l}_R}\right)\left(m^2_{\tilde{l}_R} - m^2_{\tilde{\chi}^0_1}\right)}{m^2_{\tilde{l}_R}}$$

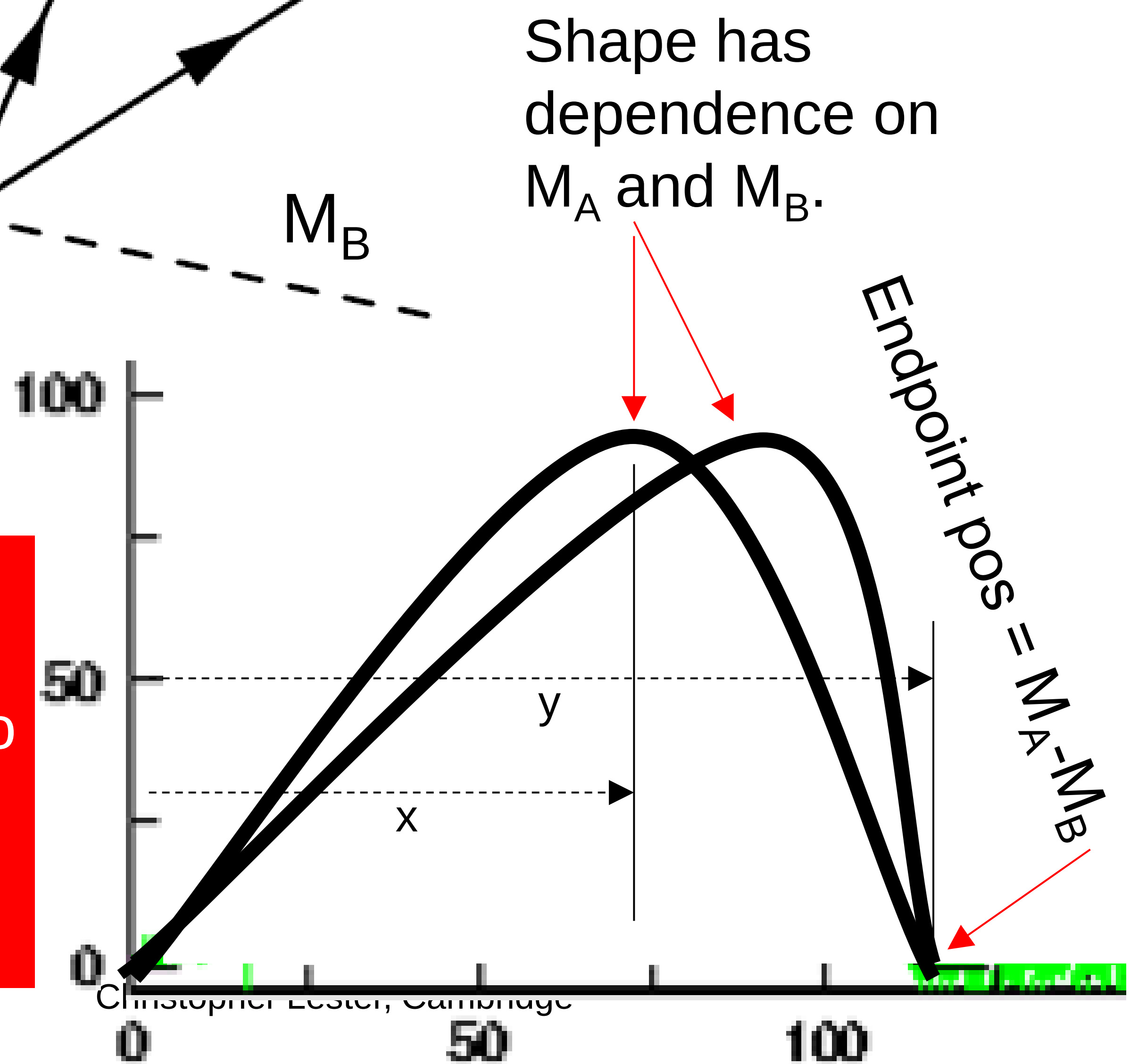


One piece of information (the endpoint position) is not sufficient to determine M_A, M_B and M_C .


M_A
M_B

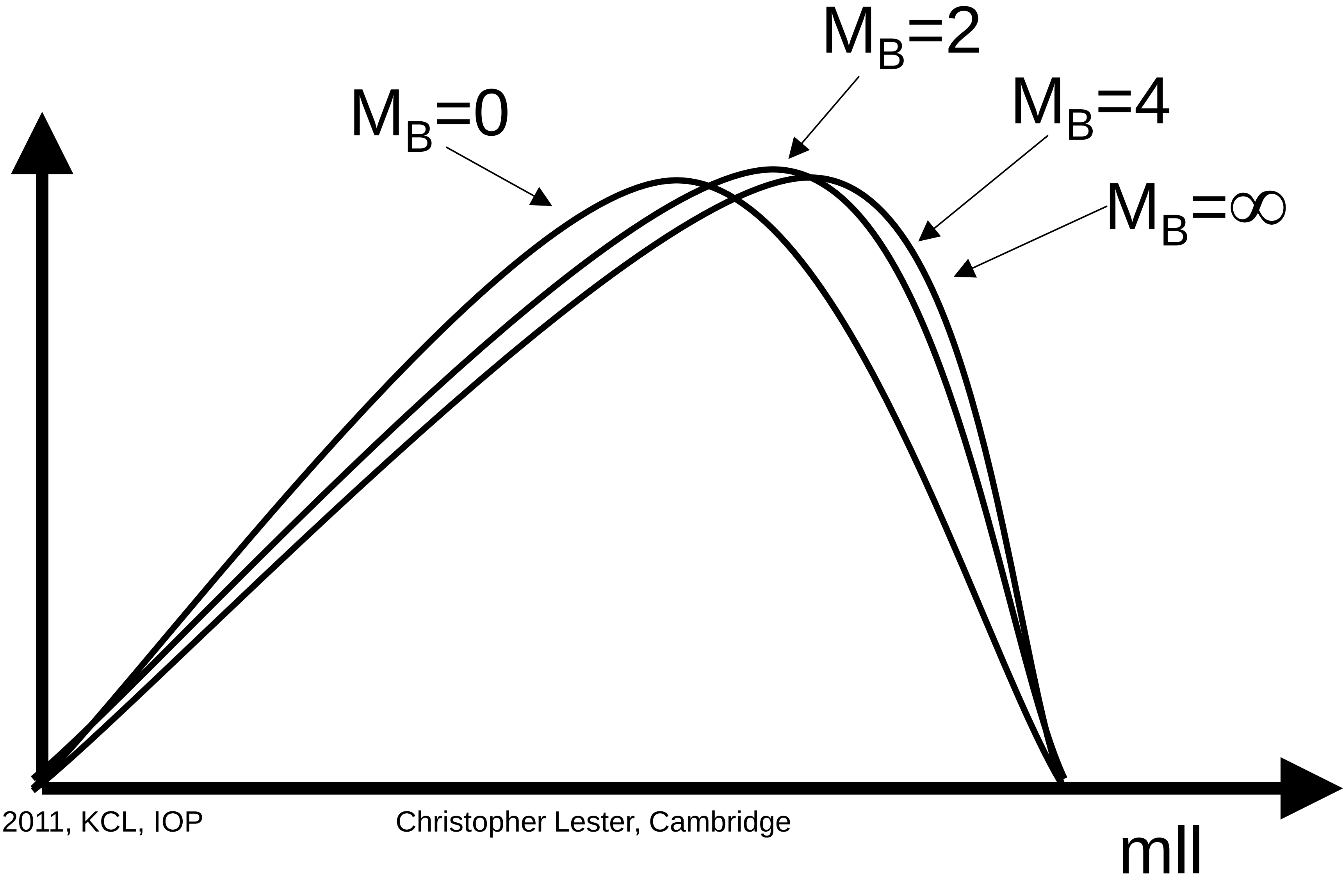

100
50
0
0
50
100





Do we have enough information from shape alone to find M_A and M_B in this three body decay perhaps?

At fixed M_A-M_B in principle one has

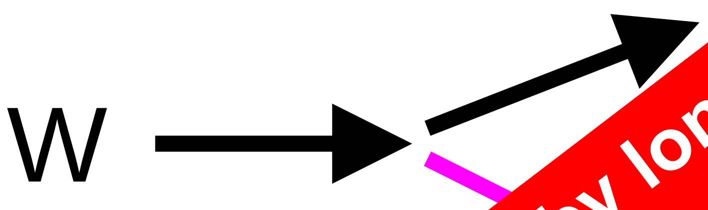


In practice:

- Real experimental fears concerning efficiency and acceptance corrections …
- Huge errors in the fit give very poor sensitivity to absolute mass scale.
- This is why endpoints, edges and resonances are good, but shapes less so
- So no magic DM bullet here, despite good constraints on mass differences.

An old Lorentz variant solution: the (full!) W transverse ma

$$m_T^2 = m_e^2 + m_\nu^2 + 2(\ldots \mathbf{p}_{Te} \cdot \mathbf{p}_{T\nu})$$



$$e_e = \sqrt{m_e^2 + p_{Te}^2}$$

$$e_\nu = \sqrt{m_\nu^2 + p_{T\nu}^2}$$

Not affected by longitudinal momenta …

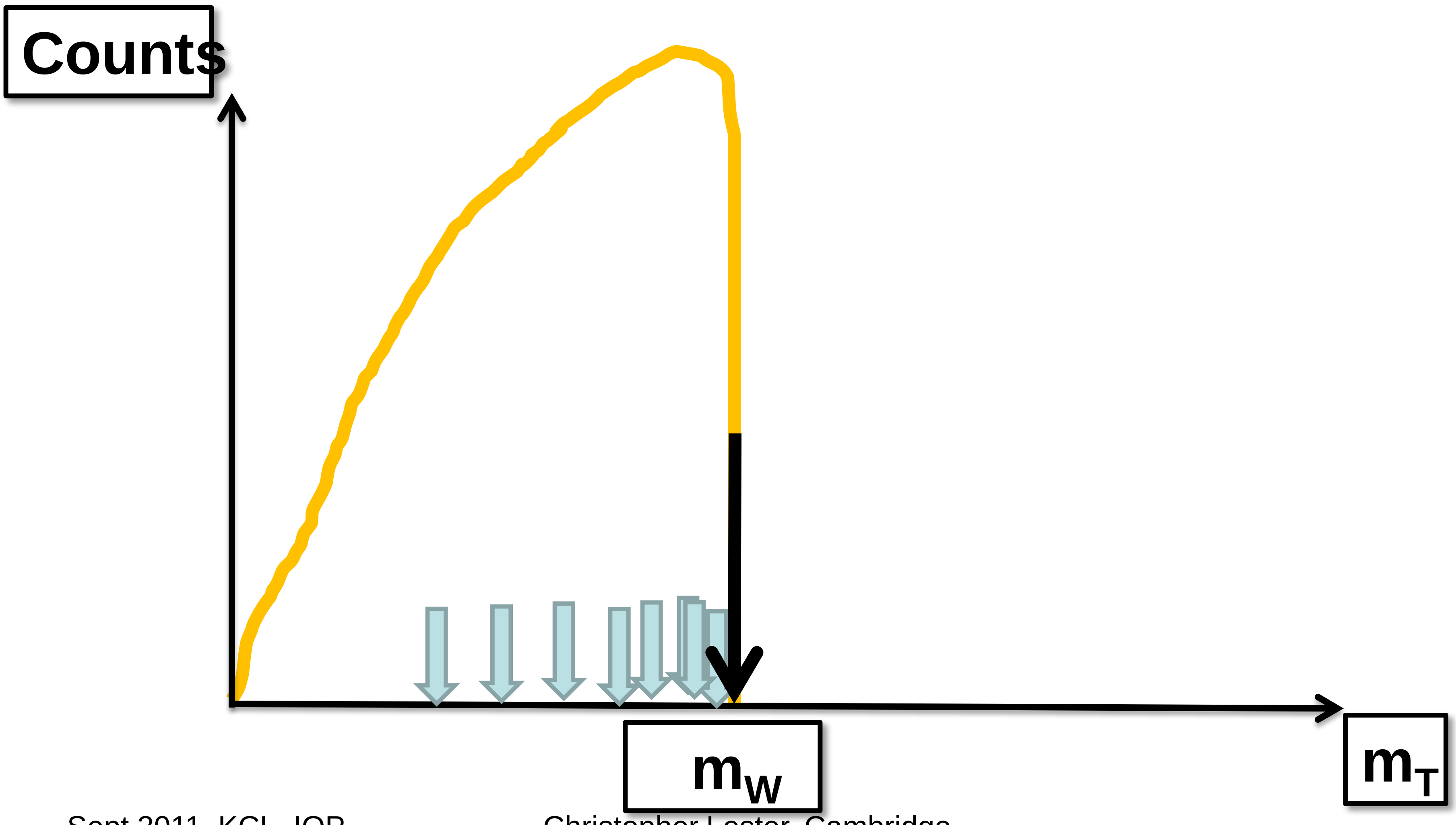
… but does vary with transverse boosts.

THIS !!

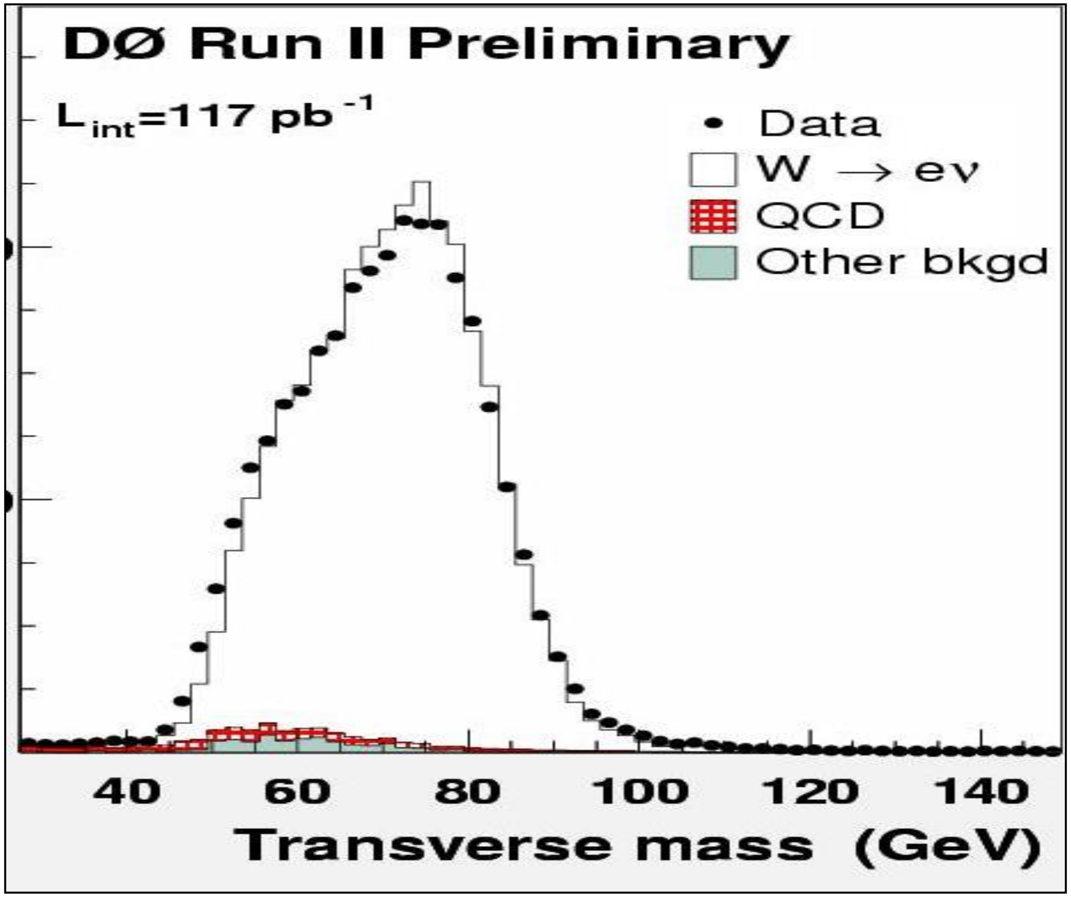
$$\ldots \left|\vec{P}_{Te}\right|\left|\vec{P}_{T\nu}\right|(1-\cos\vartheta)$$

This is **NOT** the transverse mass !!

Motivation: measure parent mass from upper endpoint (bound)



e.g. used to measure W mass



But is mT boost variance at odds with upper bound?
And what is connection to DM ?

Even though mT values vary
with transverse W boosts

the ENDPOINT of the mT
distribution is fixed

provided you get the right DM mass.

What causes the kink?

- Two entirely independent things can cause the kink:
  - (1) Variability in the "visible mass"
  - (2) Recoil of the "interesting things" against Upstream Transverse Momentum
- Which is the dominant cause depends on the particular situation … let us look at each separately:

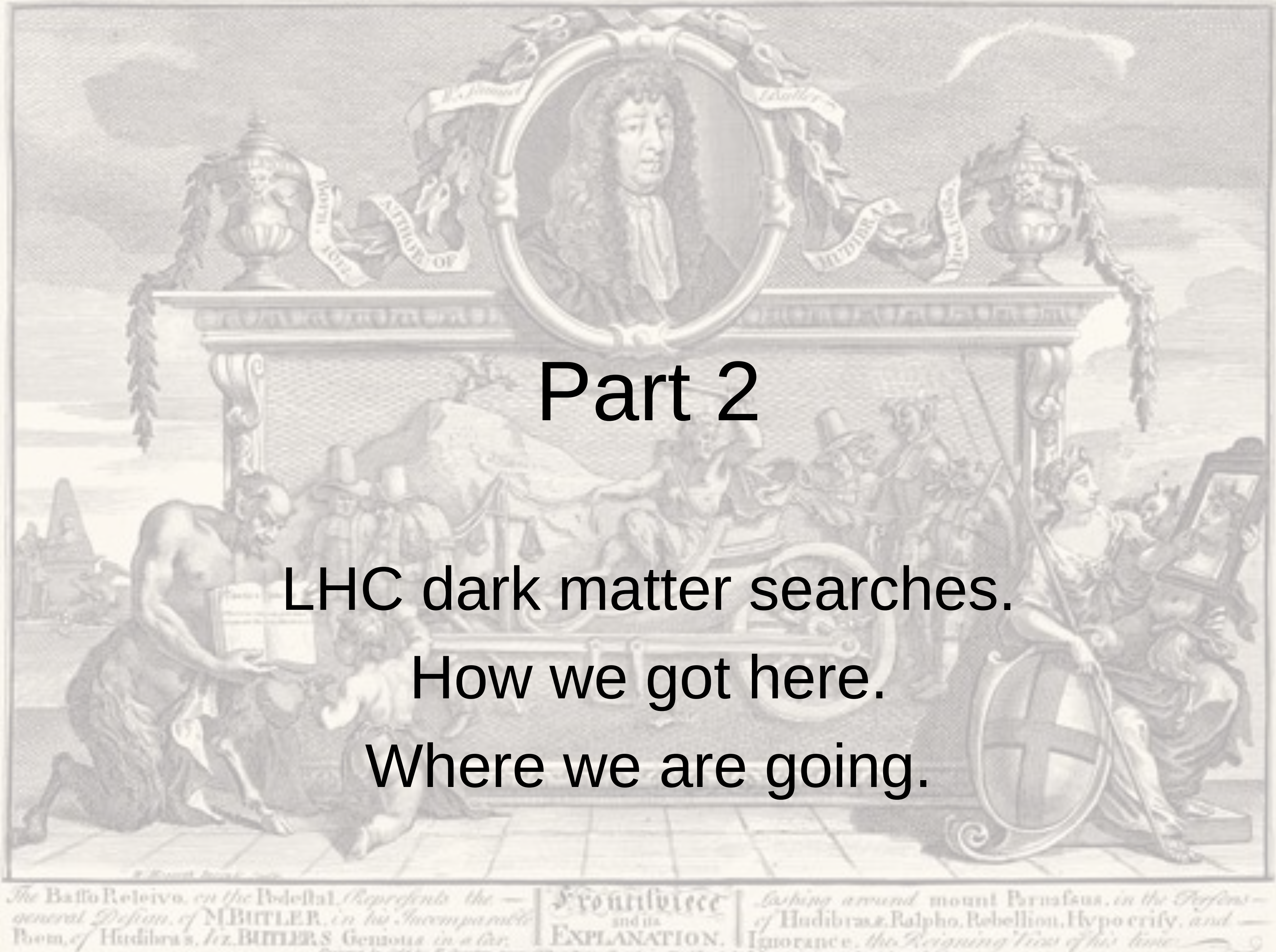
All short chain methods are variations on the above.

Other methods for longer chains -- but is it now realistic to expect the LHC can gather enough data to make long chain studies viable without strong production?

Can we now conclude that:

"The LHC will never be able to claim that it has seen dark matter!"

?



Part 2

LHC dark matter searches.

How we got here.

Where we are going.

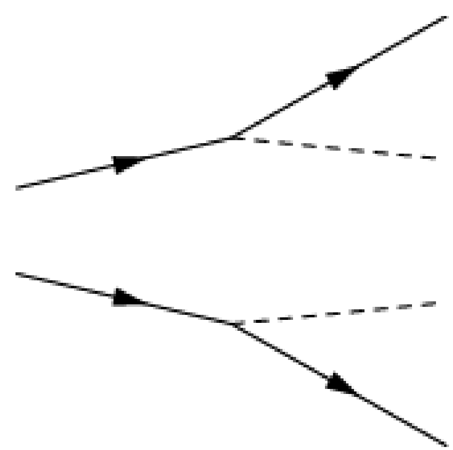
Nothing above suggests that the LHC is incapable of putting severe constraints on DM !

- Still expect DM to lead to excess of pTmiss in some channel
  - (even if excess of pTmiss is not necessarily indicative of DM)
- … so experiments look for events having pTmiss with almost any imaginable combination of:
  - “leptons”, “untagged jets”, “b-jets”, “photons” etc.
  - All credit here to the students shovelling coal into the furnaces!
- But how are these events selected, and what is then done with them?

Could just require existence of "some number" of these objects

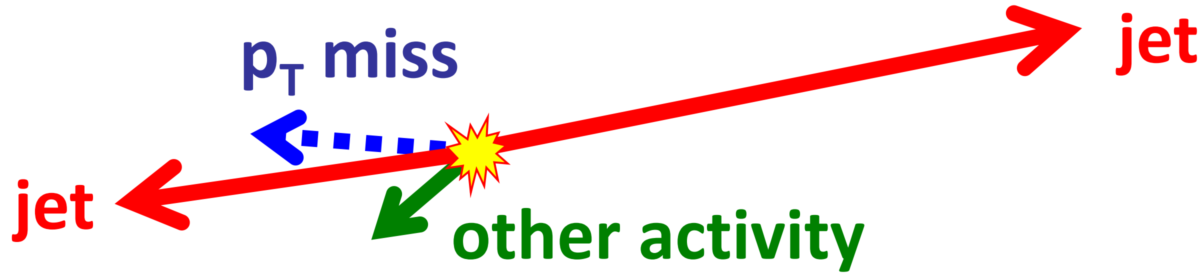
- But that alone is not enough.

- A search for di-squark production with invisible DM candidate neutralino (right) will fail if the only requirement is an excess in events with "two jets and pTmiss" – there is too much of this from QCD, Z+jets, W+jets, ttbar etc.

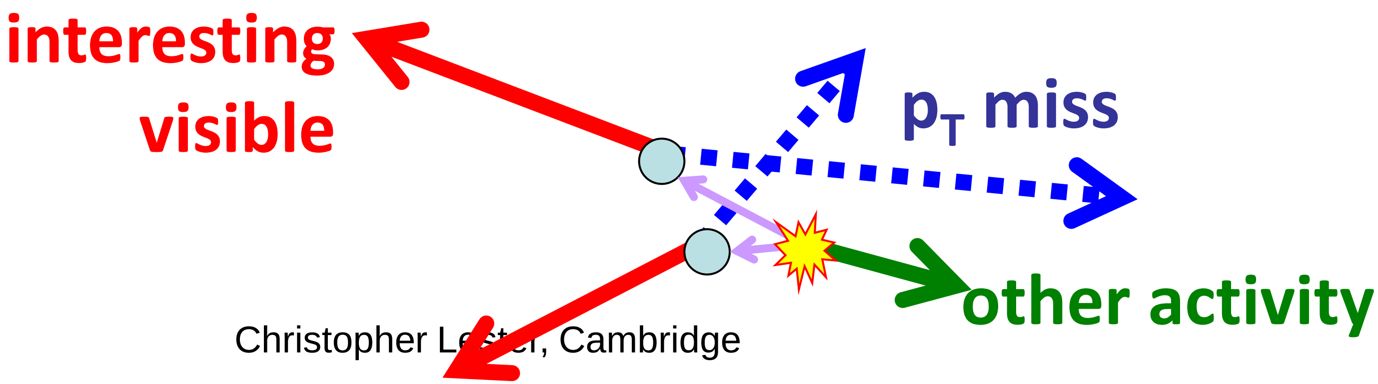


But kinematics of signals and (some) backgrounds can differ:

- e.g. QCD is mostly back to back:



- SUSY / DM models need not be back to back:



Yet Meff has excellent reach!

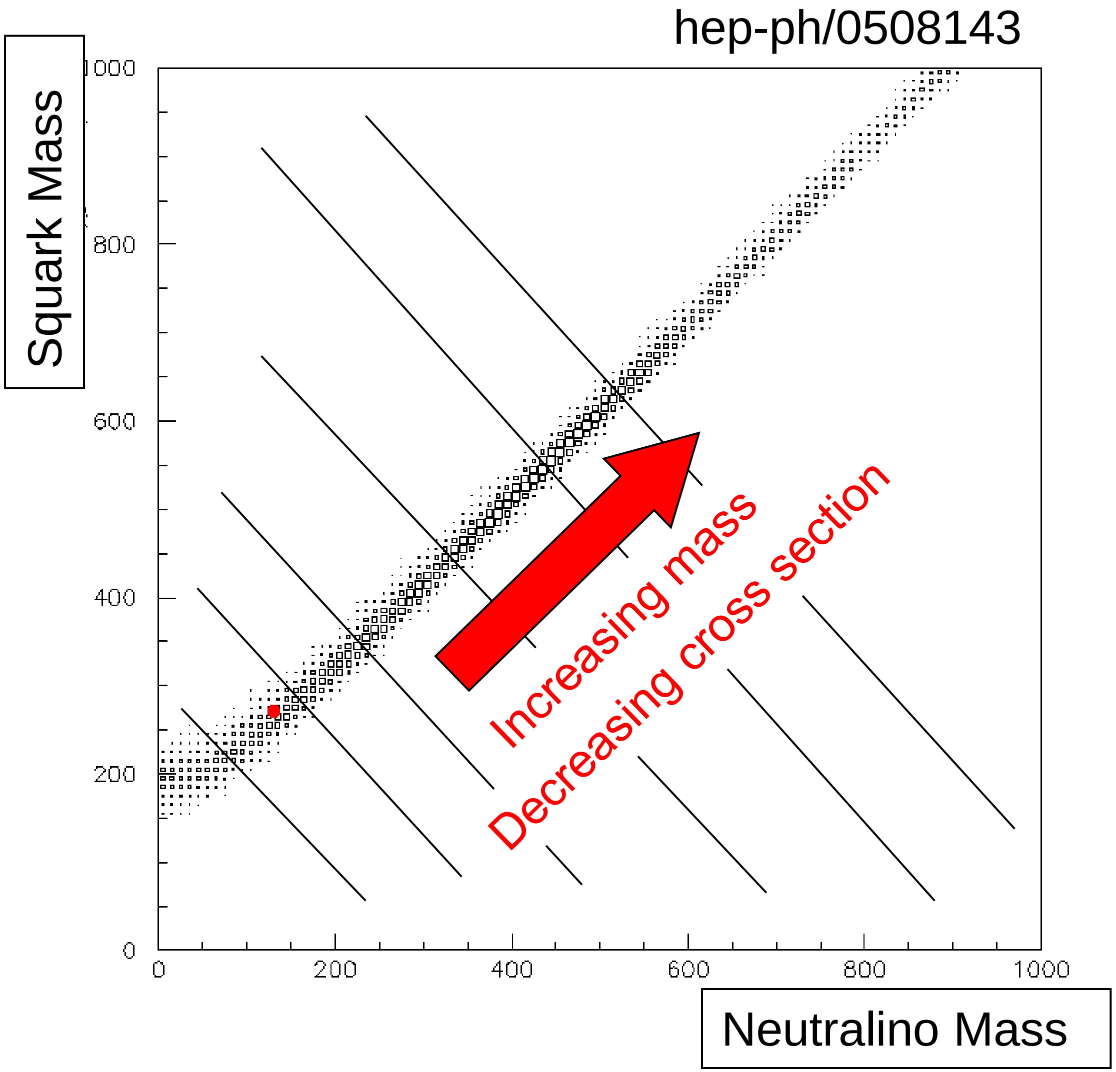
What we are likely to see happen:

- The extremes of the "reach" plots will be driven by BG and detector-motivated (rather than signal-specific) variables, cuts are hard, event numbers small, and BG uncertainties are very important.
- Increasingly strong limits on BSM cross sections for points in the "interior" of the excluded regions, where the signals are themselves well behaved, and S/sqrt(B) increases with sqrt(luminosity*time).

Conclusions

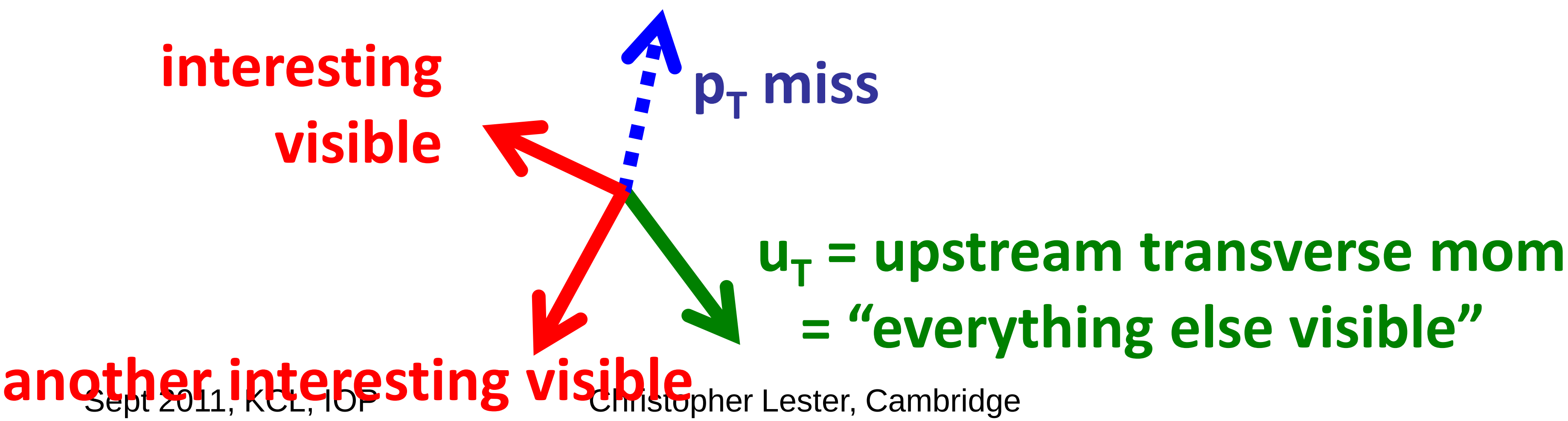
- We may already have already begun to witness the end of the LHC as machine that will be able to claim it has seen Dark Matter. But:
  - the LHC still has a huge potential to constrain models with massive invisible particles
  - we are going to see many more improvements at 7 TeV, both in
    - new channels (from new ideas, new BG estimation techniques)
    - stronger limits on BSM cross sections, and
    - (to a lesser extent) increased reach in existing searches
  - we can expect big increases in reach at 14 TeV when the energy frontier expands

Cross section information is orthogonal to mass differences



Missing transverse momentum

$$\vec{\mathbf{p}}_T^{\,miss} = -\sum_i \vec{\mathbf{p}}_T^{\,i^{th}visible}$$



Events have missing energy too, and it's not missing momentum

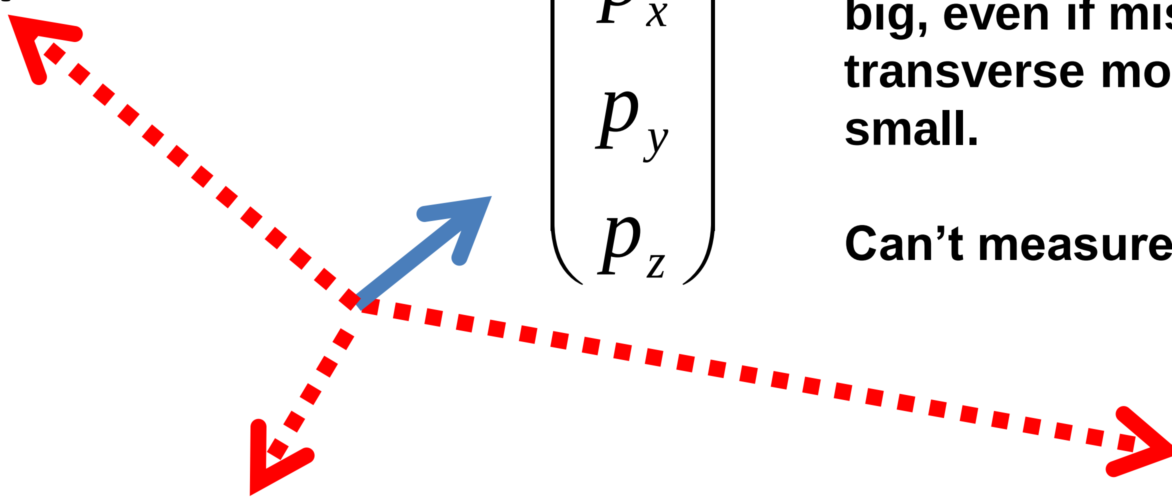
$$\begin{pmatrix} E \\ p_x \\ p_y \\ p_z \end{pmatrix}$$

Total 4-momentum of invisbiles.

Missing energy could be big, even if missing transverse momentum is small.

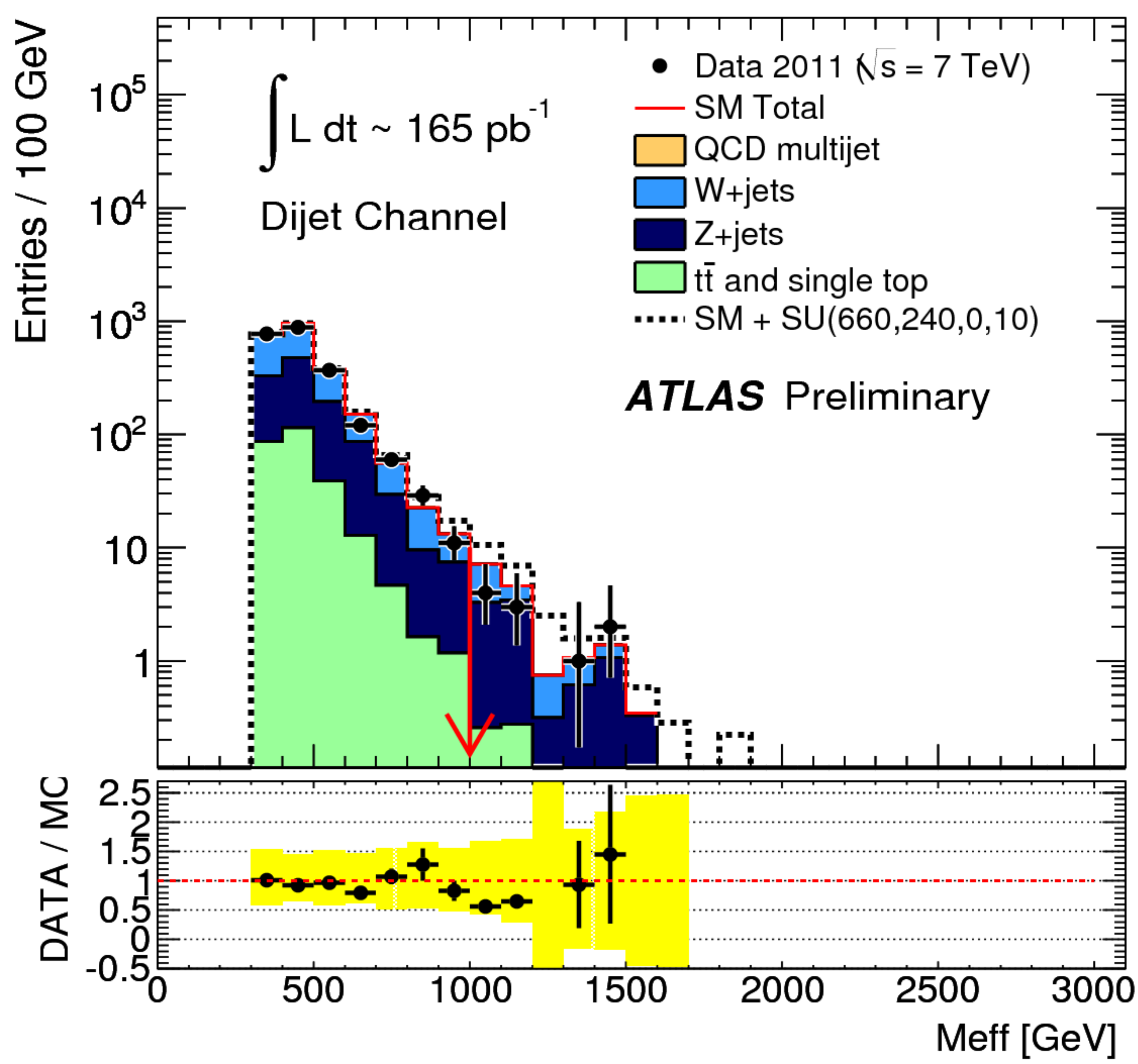
Can't measure E or pz

invisible particle

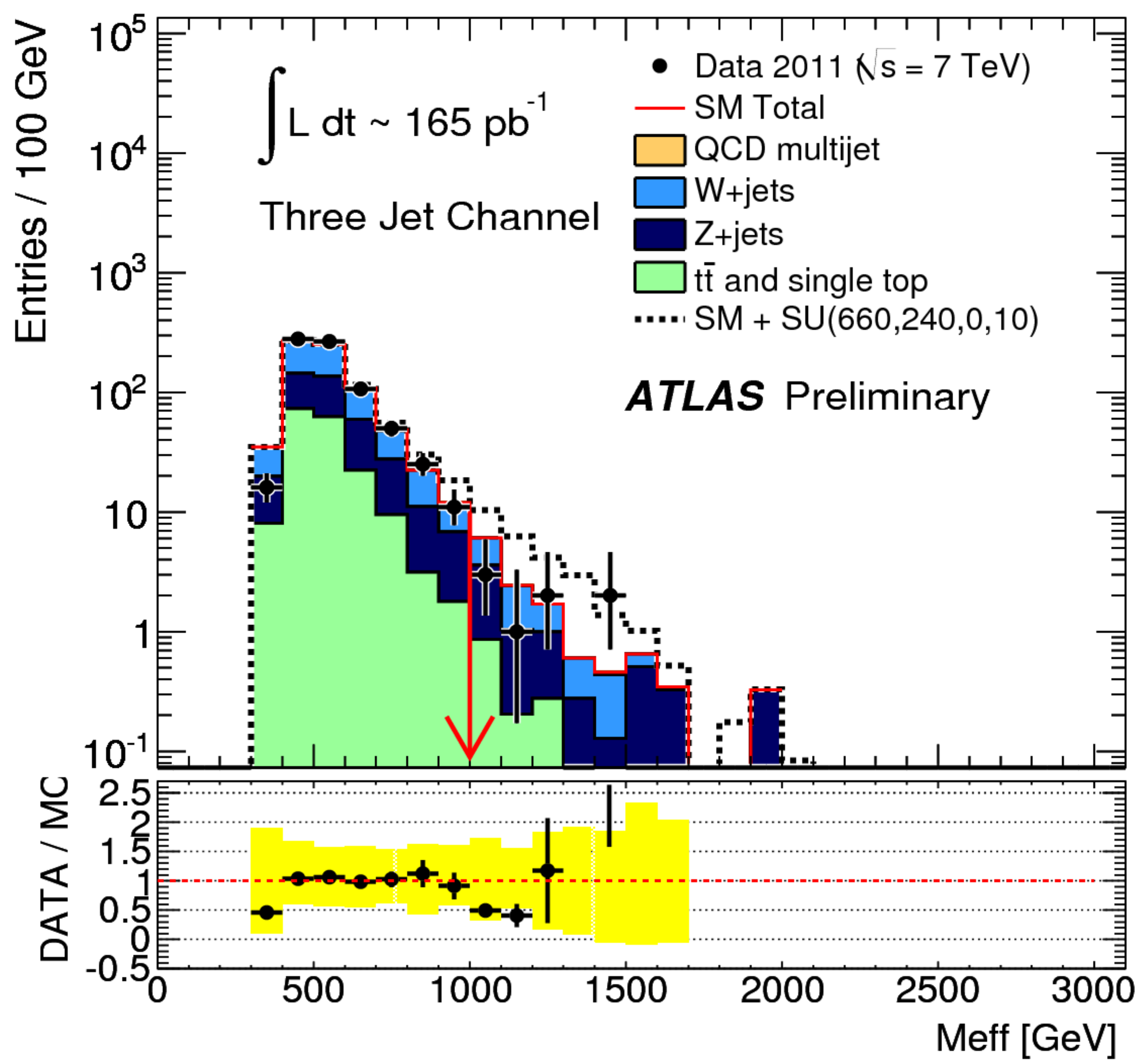


invisible particle

invisible particle

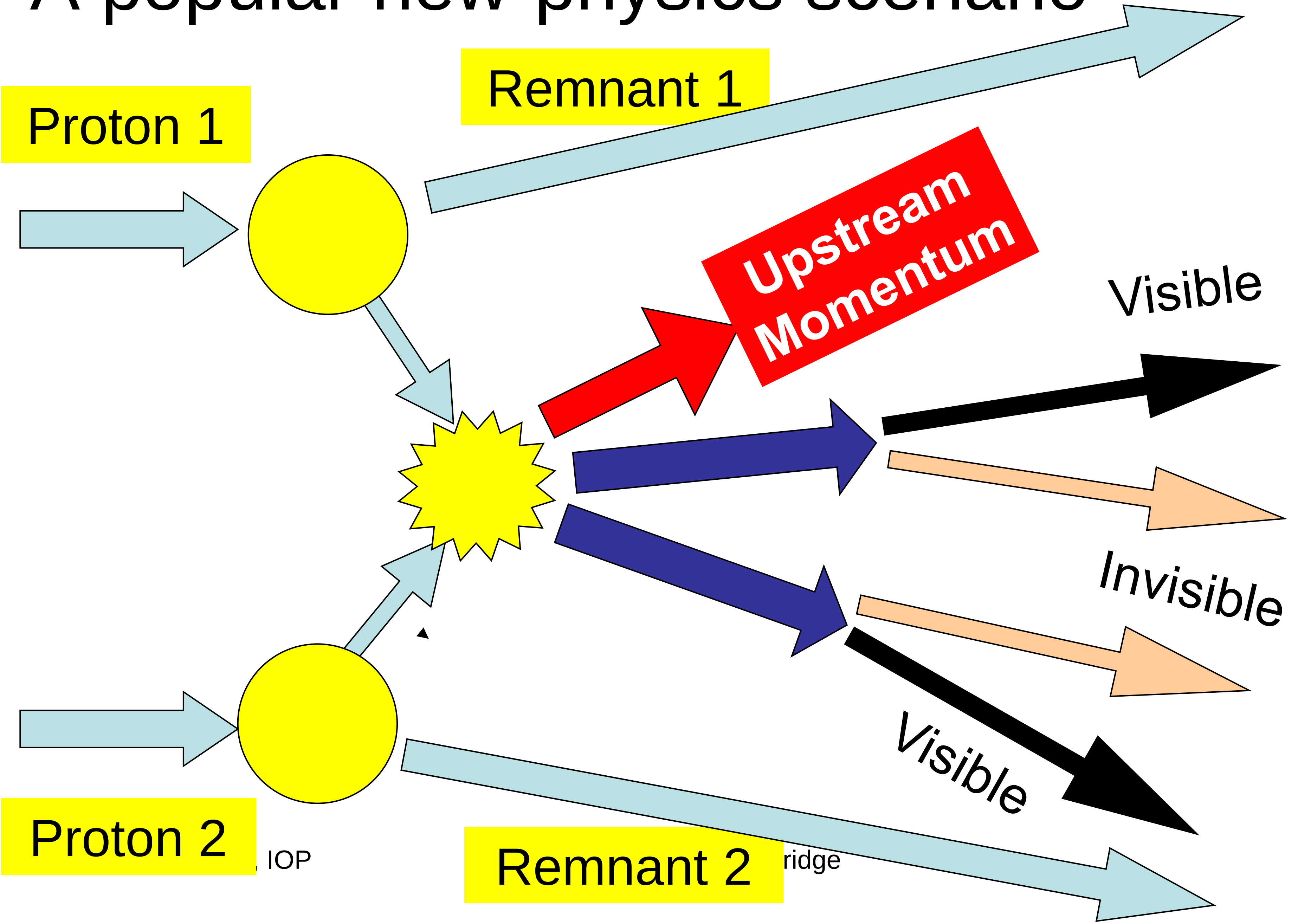


Latest ATLAS 0-lepton, jets, missing transverse momentum data.

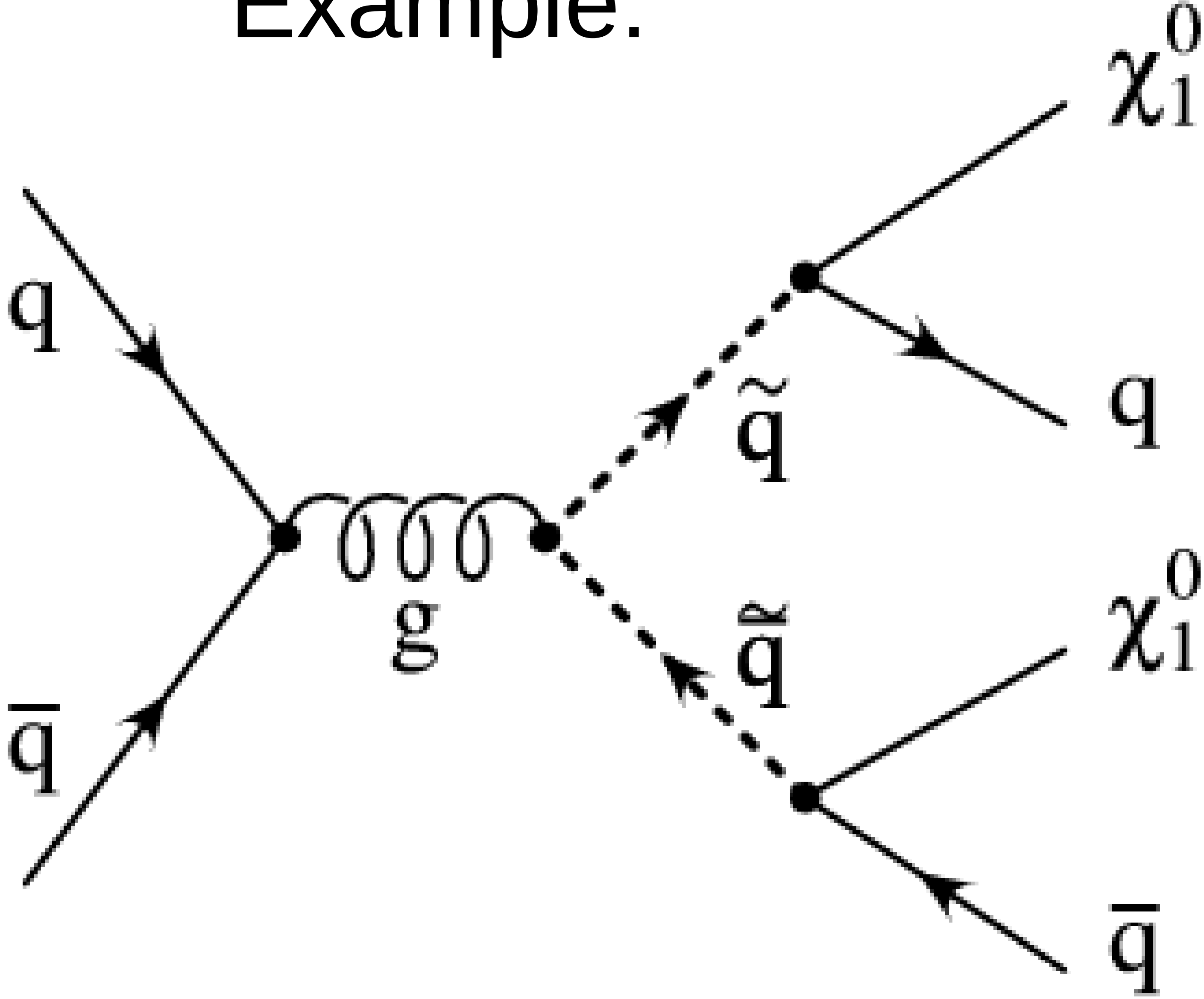


Latest ATLAS 0-lepton, jets, missing transverse momentum data.

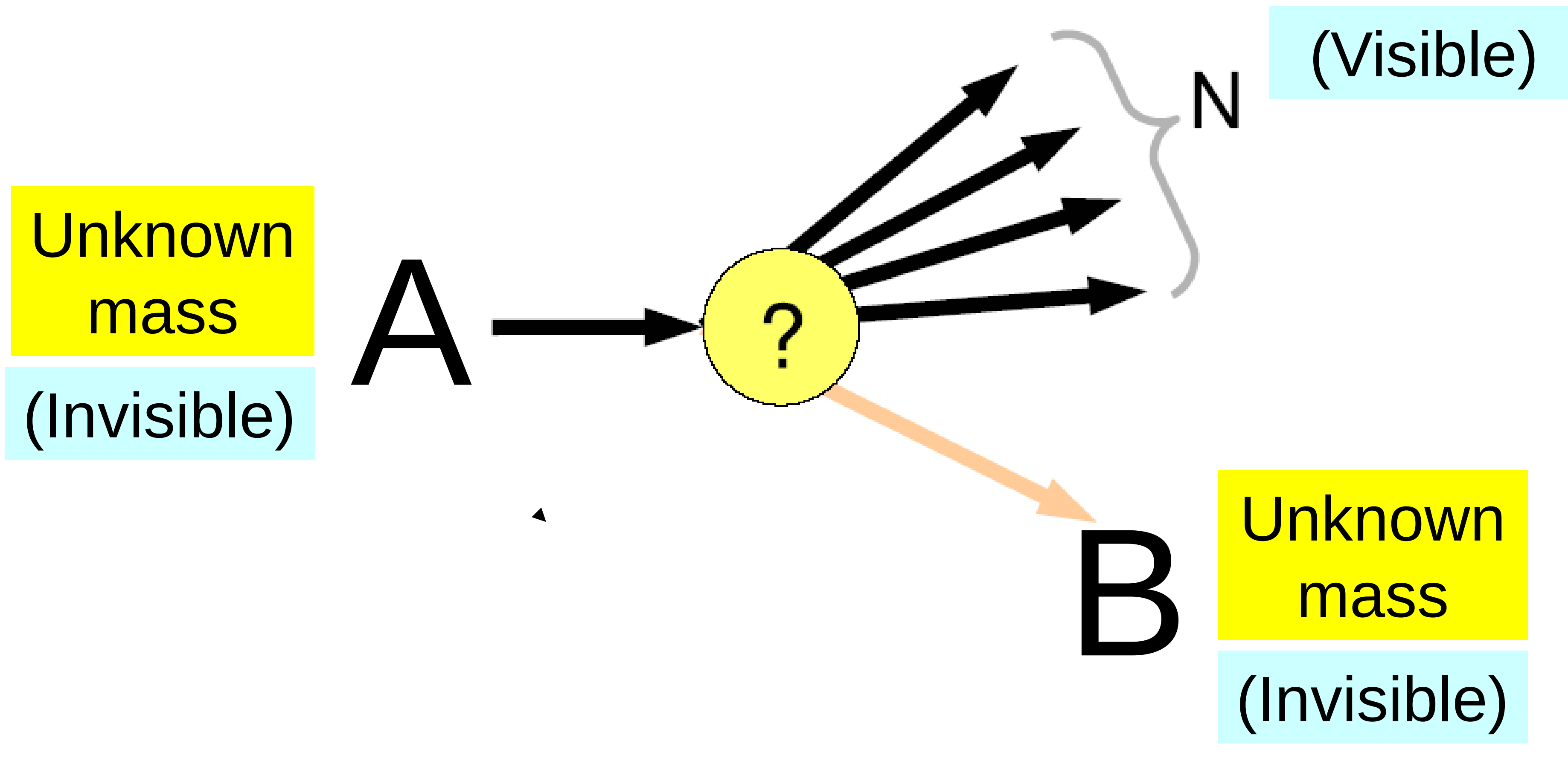
A popular new-physics scenario



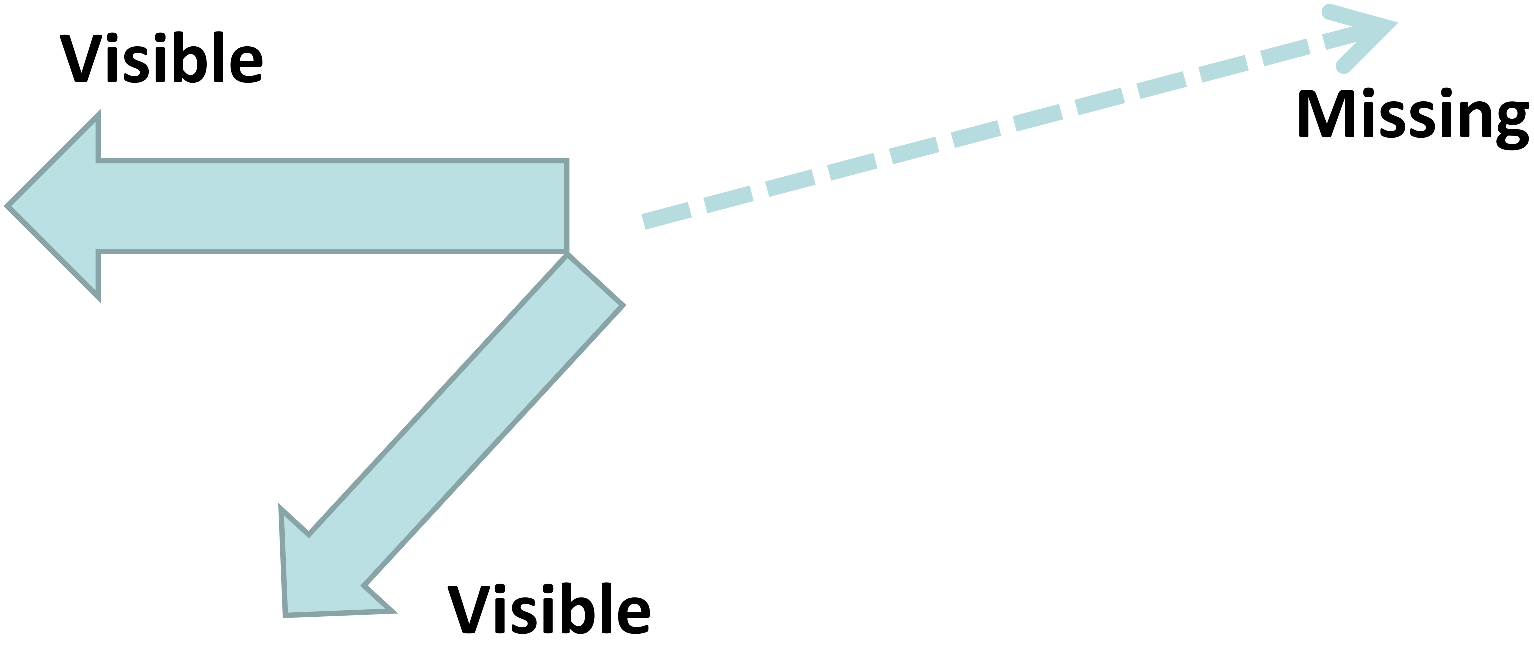
Example:

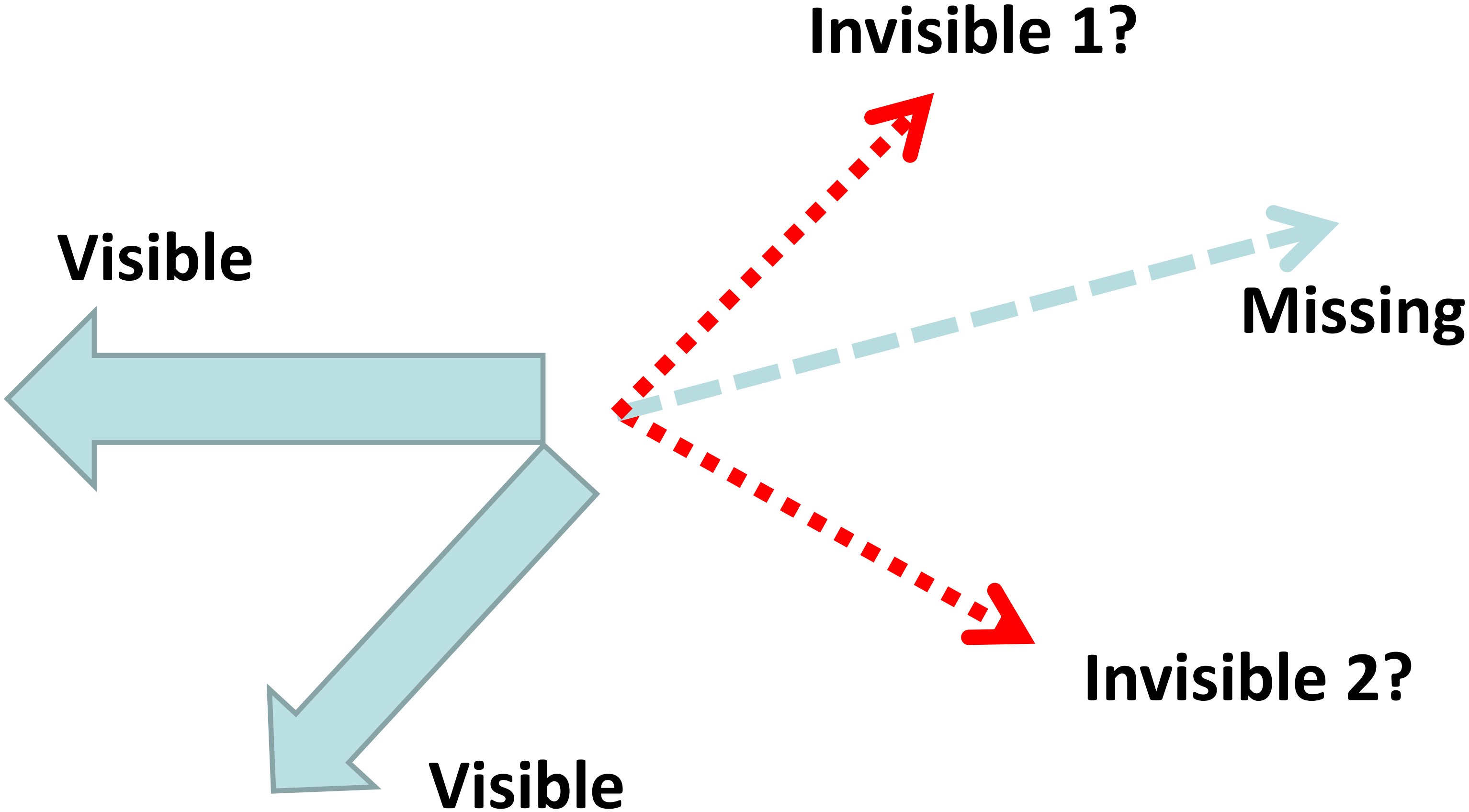


We have two copies of this:

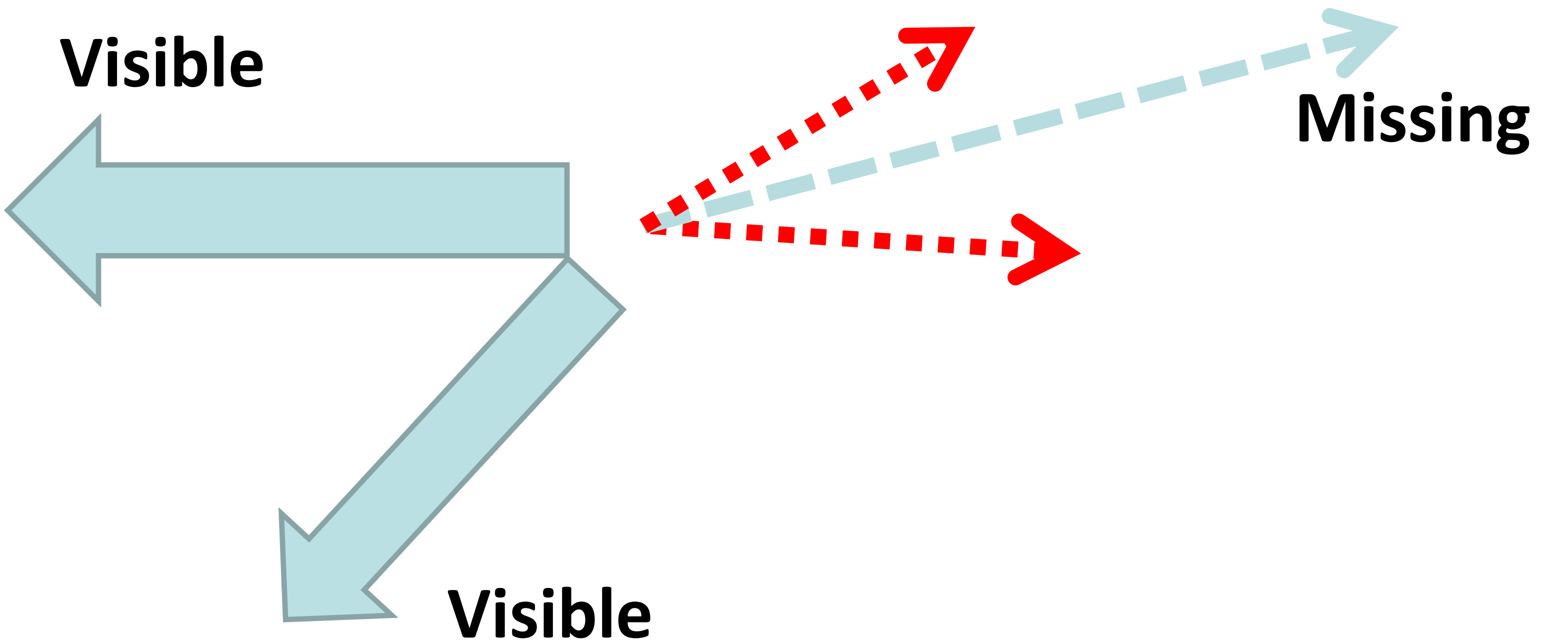


But don’t know p_T of B this time! ☹

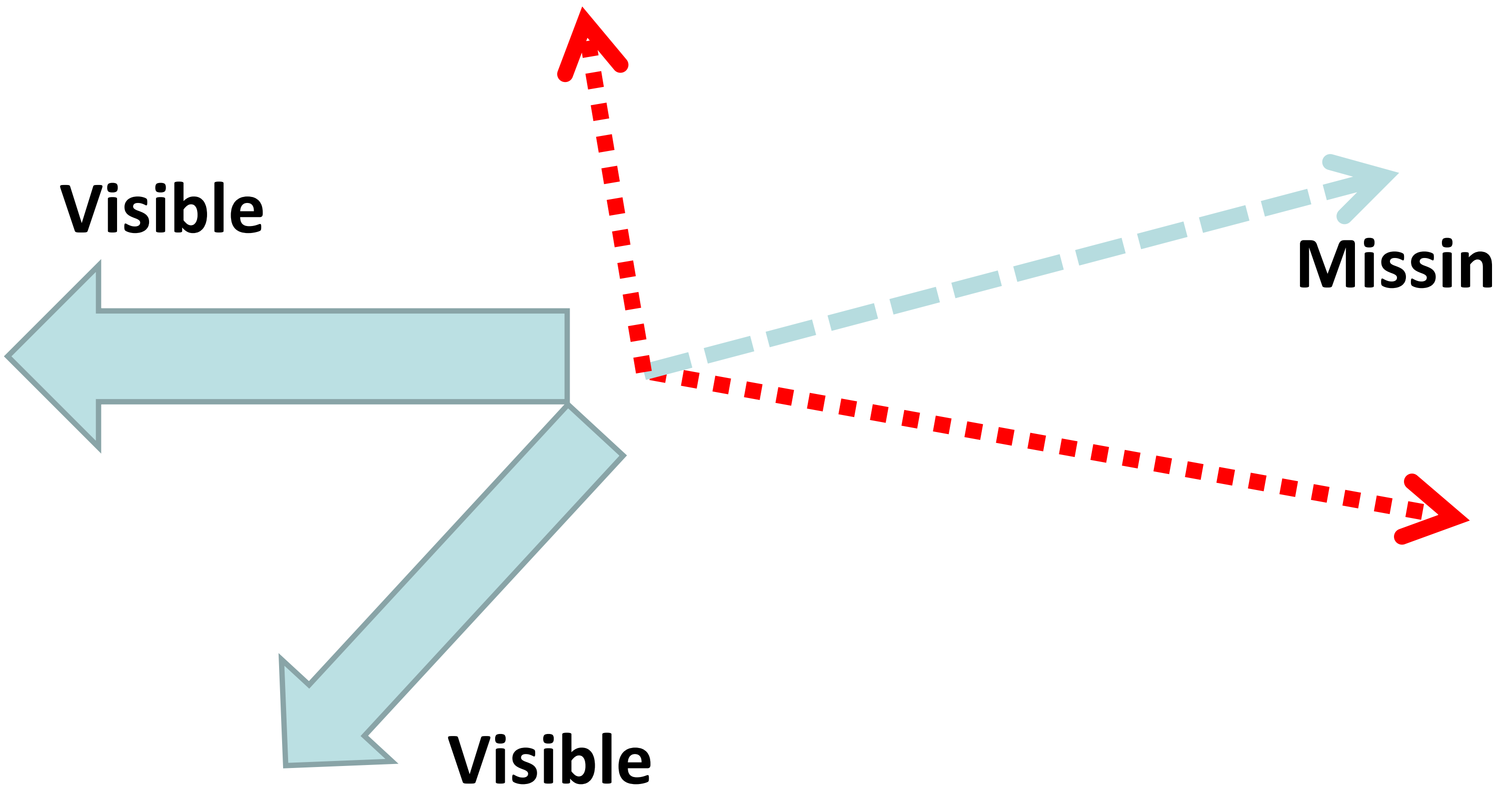

Visible
Missing
Visible



a possible “splitting”

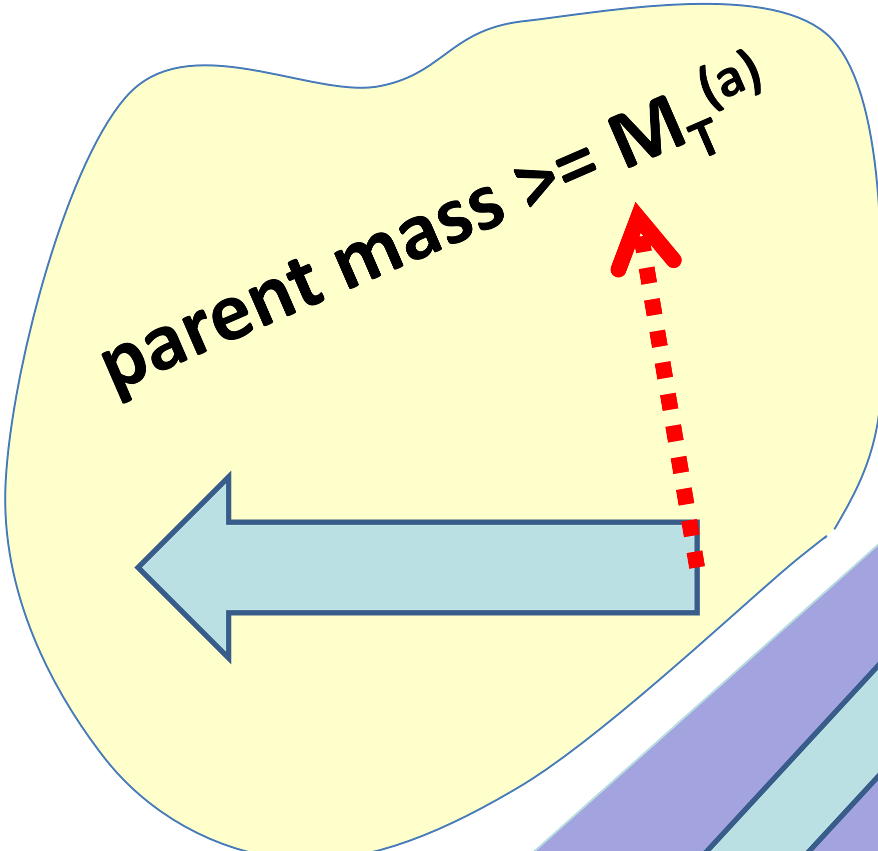


another possible “splitting”



another possible "splitting"

If **this** splitting is “correct”:

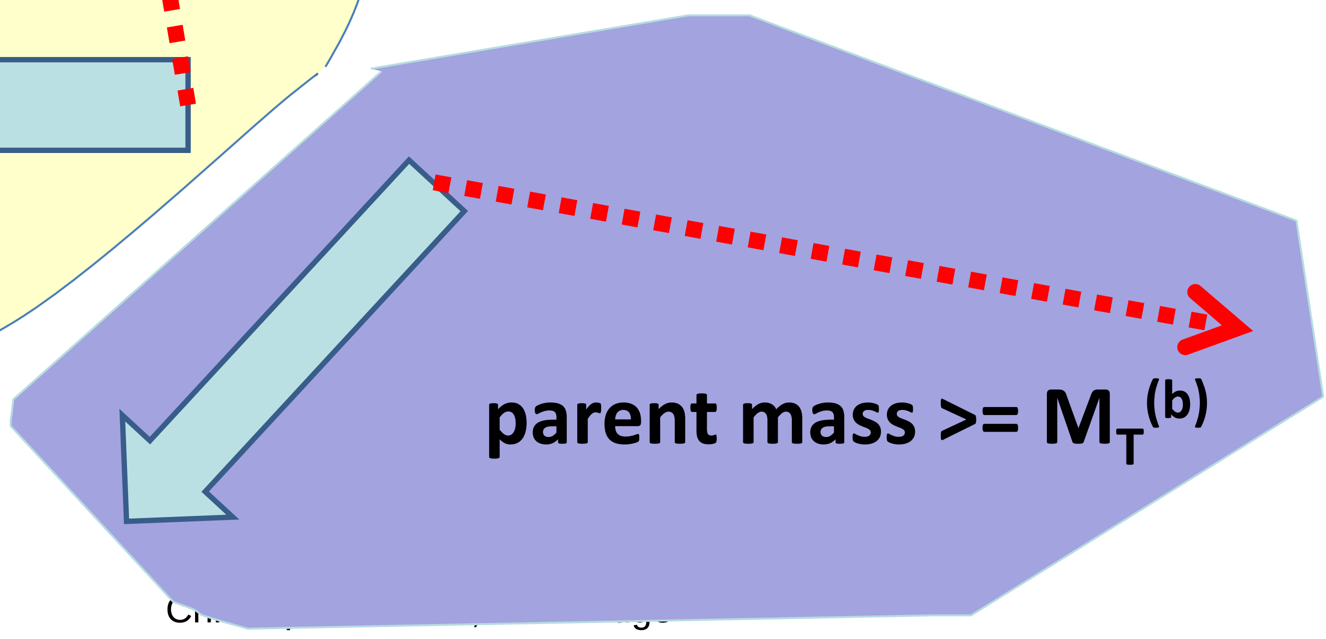


Therefore:

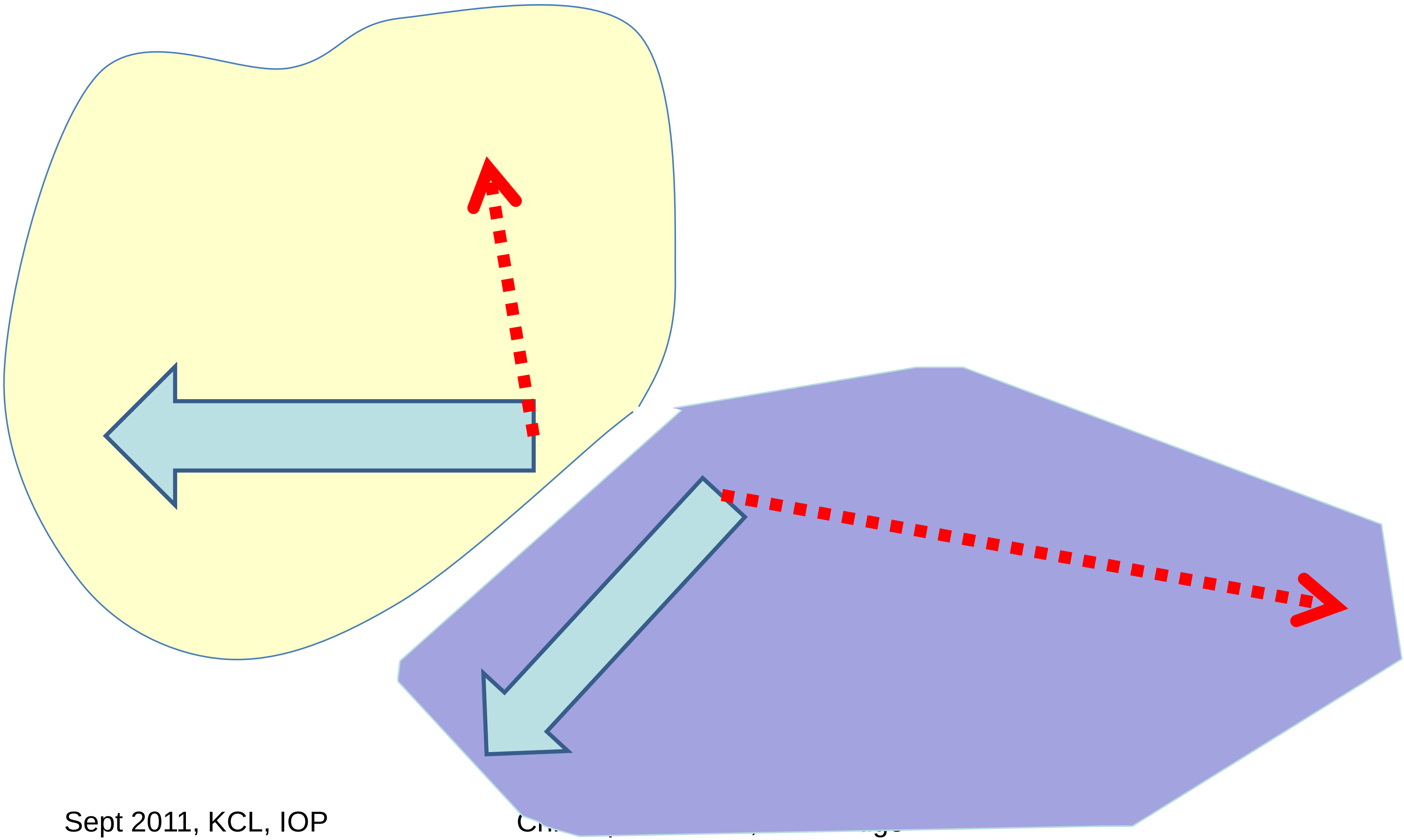
parent mass

>=

Max[M_T(a), M_T(b)]



But this splitting might be wrong!



But can say that:

$$\text{parent mass} \geq \underset{\text{over all splittings of ptmiss}}{\mathrm{Min}} \left\{ \mathrm{Max}[\, M_T(a), M_T(b) \,] \right\}$$

This is m_{T2} the "Stransverse Mass"

$$m_{T2}(v_1, v_2, \not{p}_T, m_i^{(1)}, m_i^{(2)}) \equiv \min_{\sum \mathbf{q}_T = \not{\mathbf{p}}_T} \left\{ \max\left(m_T^{(1)}, m_T^{(2)} \right) \right\}$$

The most conservative partition consistent with the constraint

Take the better of the two lower bounds

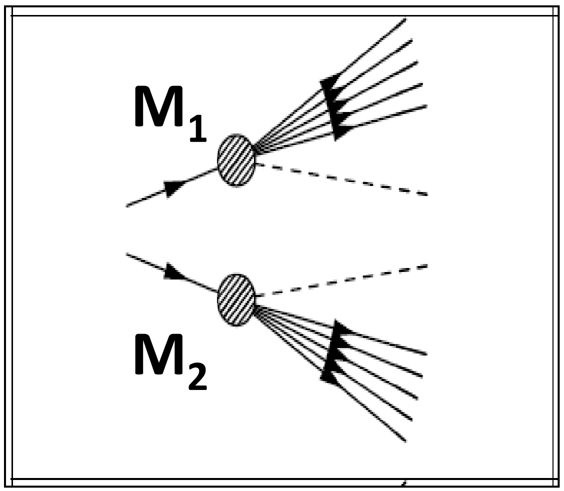
It is the <u>generalisation of transverse mass to pair production</u>.
Clear how to generalise it to any other types of production.

[Received six comments about "mis-spelling" of transverse in ATLAS editorial board!]

Note MT2 def is part of the four-step procedure:

[(1) select topology, (2) parent mass, (3) constraints, (4) find maximal lower bound]

described earlier.



Note, other approaches: MCT, Rogan, etc.

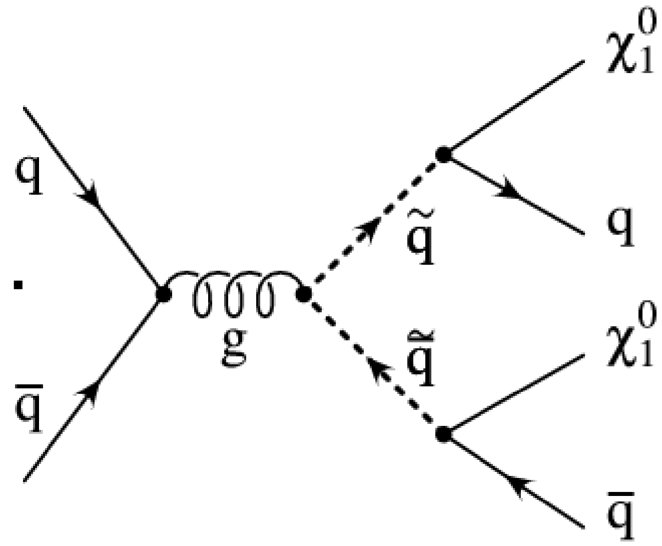
CONSTRAINTS

$$M_1 = M_2 \quad + \quad \sum_{i=1}^{N_{\mathcal{I}}} \vec{q}_{iT} = \vec{\not{p}}_T \equiv -\vec{u}_T - \sum_{i=1}^{N_\nu} \vec{p}_{iT}$$

Momentum conservation in transverse plane

In other words:

- If your event is signal …



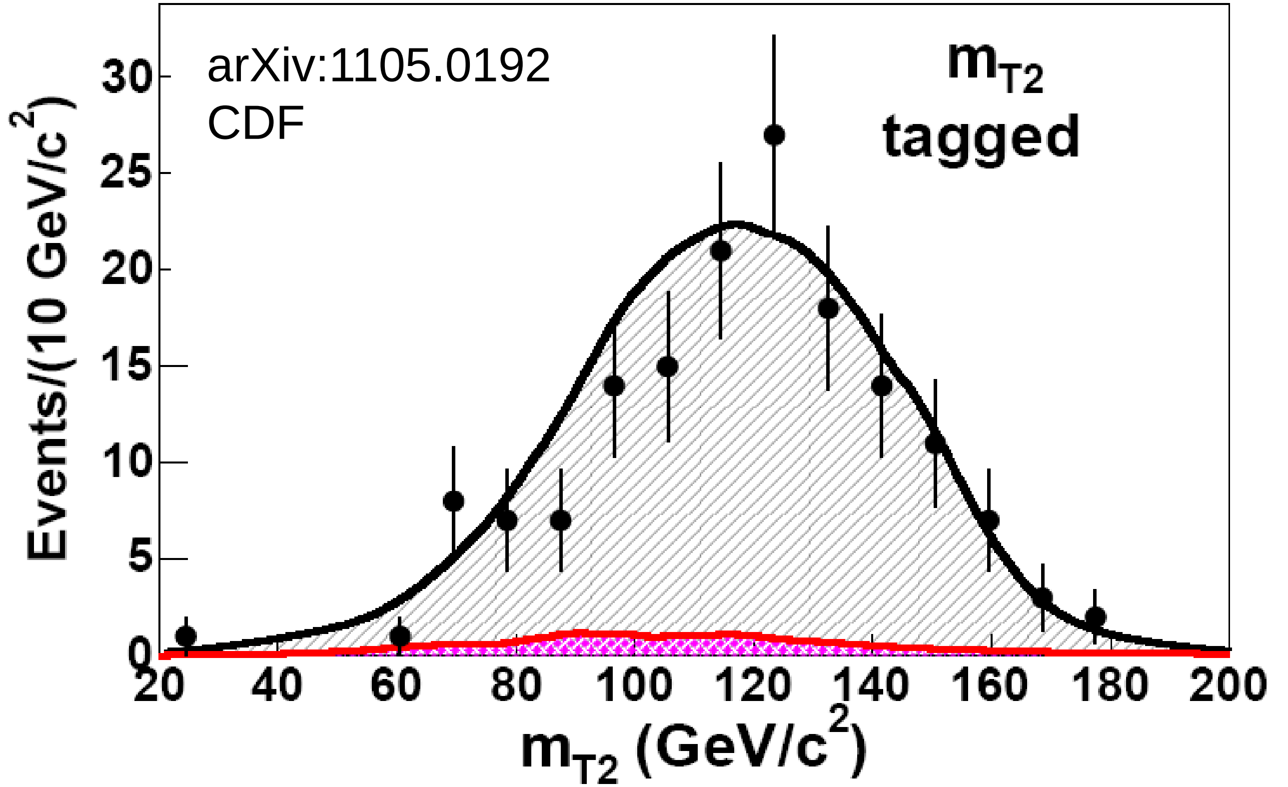
and if MT2 is “350 GeV” …

then the squark mass is >= 350 GeV.

Indeed, can show MT2 is, by construction, the best possible lower bound on the squark mass.

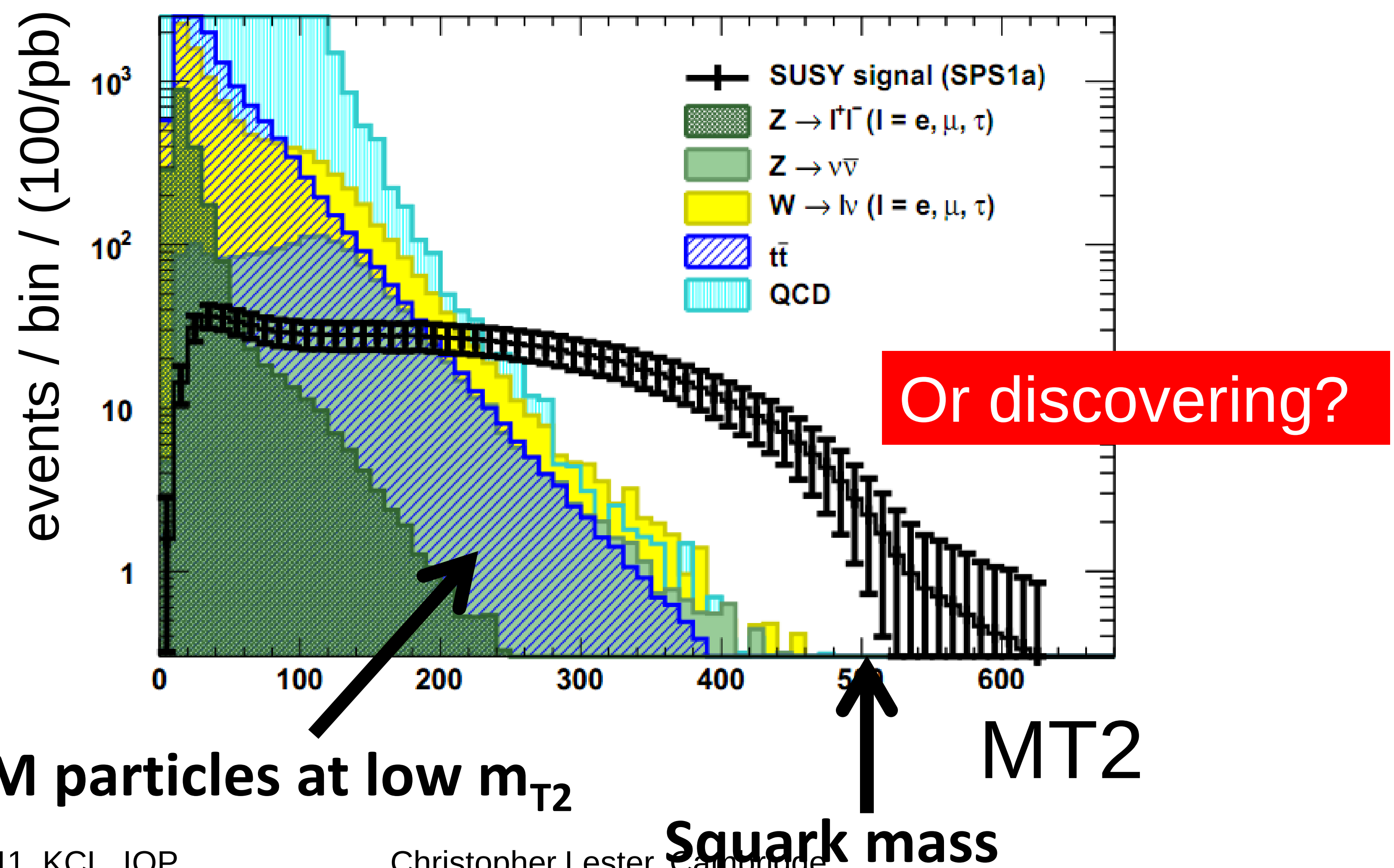
MT2 example in real data …..

- “Top Quark Mass Measurement using mT2 in the Dilepton Channel at CDF” (arXiv:0911.2956 and arXiv:1105.0192) reports that they “achieve **the single most precise measurement of m_{top} in [the dilepton] channel to date**”. Also under study by ATLAS.



Top-quark physics is an important testing ground for mT2 methods, both at the LHC and at the Tevatron. If it can’t work there, its not going to work elsewhere.

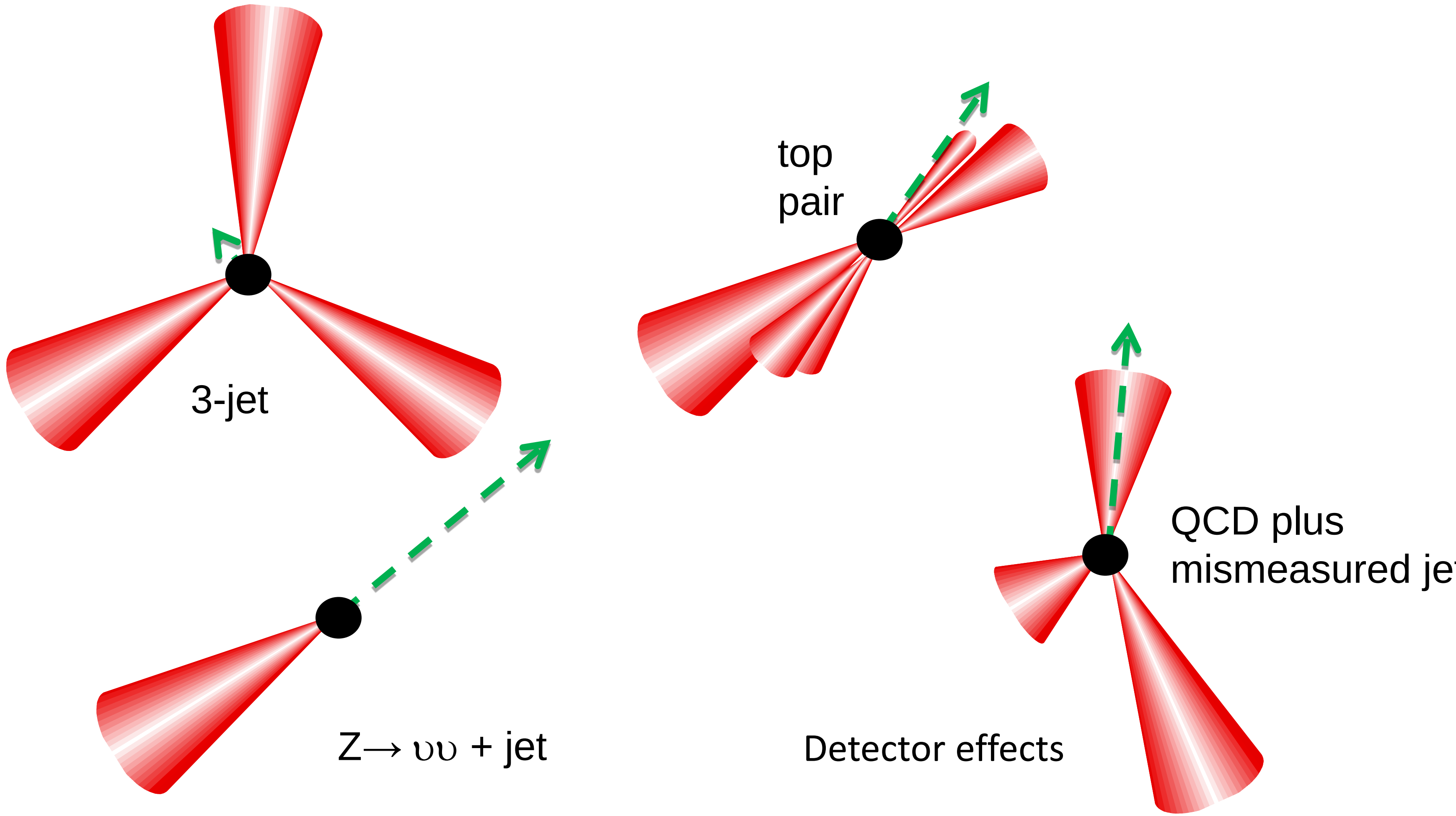
Example MT2 distribution …
… ?weighing? 500 GeV squarks



arXiv:0907.2713

… works because MT2 for all BGs is provably low … due to small QCD mass scale

arXiv:0907.2713



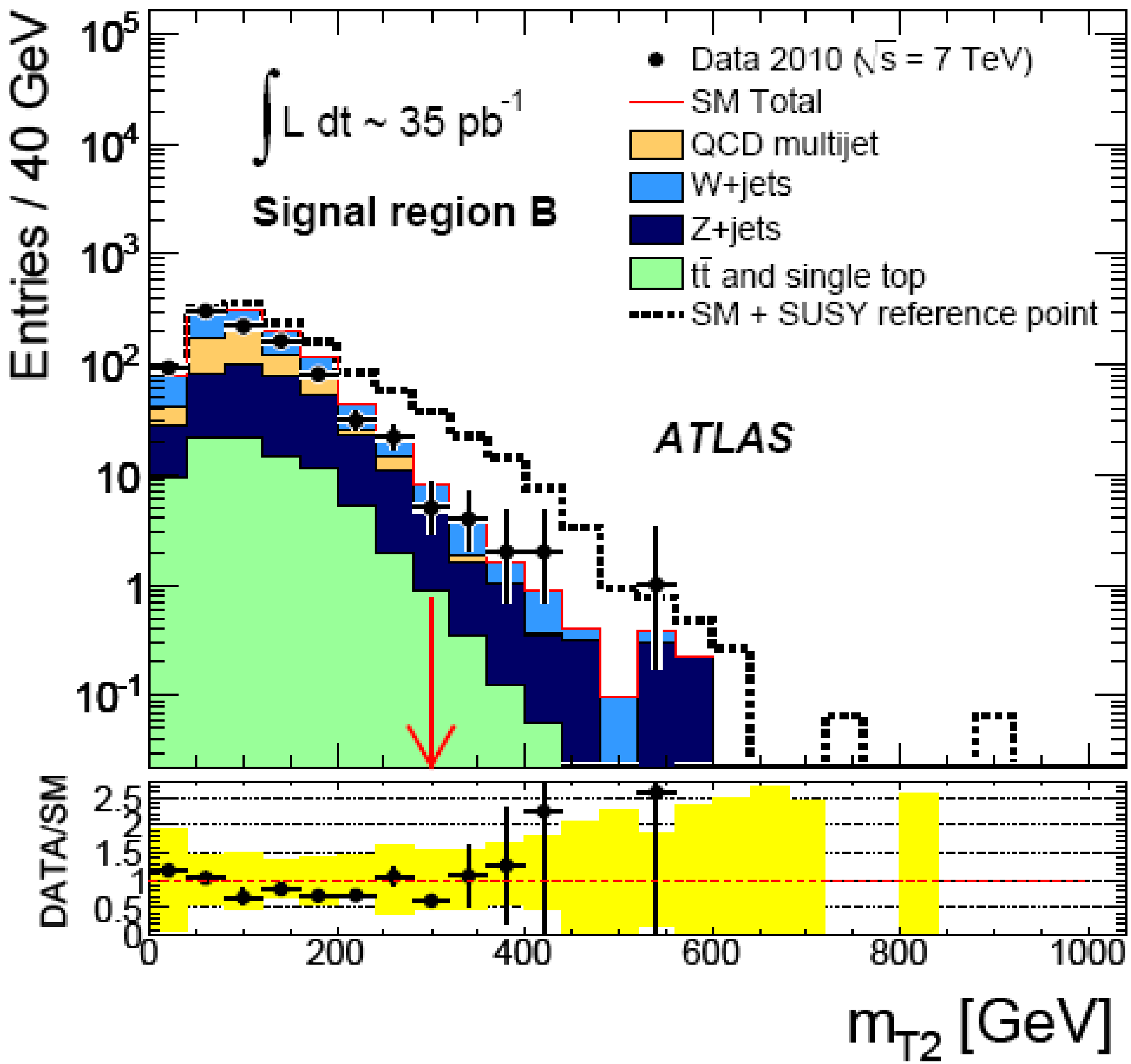
All these have m_{T2} either < m_{top} or → $m_<$

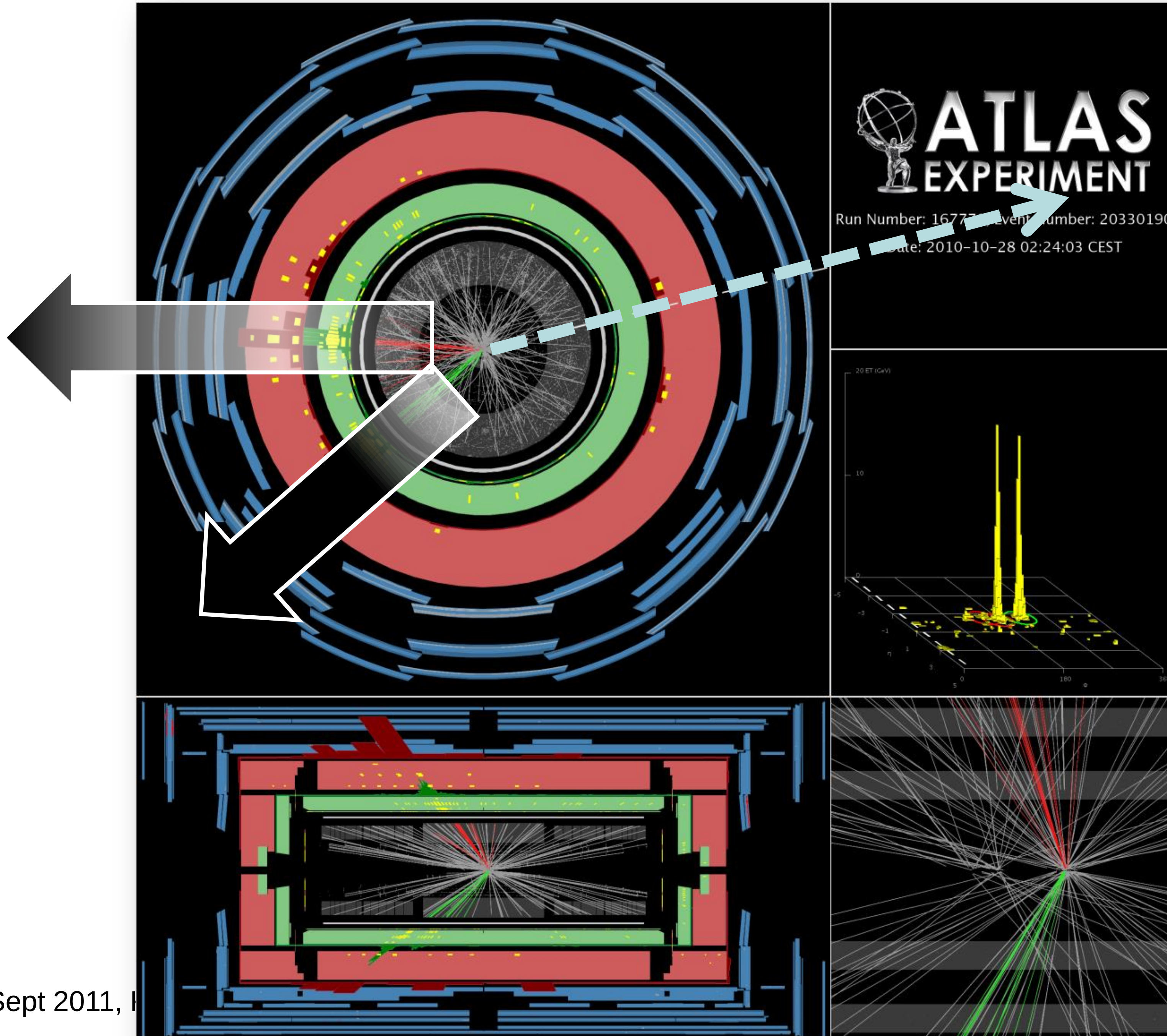
arXiv:0907.2713

So good for low multiplicity pair production signal discovery – disleptons?

Process	$m_{T2}(v_1, v_2, \not{p}_T, 0, 0)$	Comments
QCD di-jet $\to$ hadrons	$= \max m_j$ by Lemmas 1,4	
QCD multi jets $\to$ hadrons	$= \max m_j$ by Lemma 4	
$t\bar{t}$ production	$= \max m_j$ by Lemma 4	fully hadronic decays
	$\le m_t$ by Lemmas 1,7	any leptonic decays
Single top / tW	$= \max m_j$ by Lemma 4	fully hadronic decays
	$\le m_t$ by Lemmas 2,7	any leptonic decays
Multi jets: "fake" $\not{p}_T$	$= \max m_j$ by Lemma 5	single mismeasured jet[a]
	$= \max m_j$ by Lemm[illegible]	two mismeasured jets[a]
Multi jets: "real" $\not{p}_T$	$= \max m_j$ by Lemma 5	single jet with leptonic b decay[a]
	$= \max m_j$ by Lemma 6	two jets with leptonic b decays[a]
$Z \to \nu\bar{\nu}$	$= 0$ by Lemma 3	
$Z\,j \to \nu\bar{\nu}\,j$	= [illegible] Lemma 3	one ISR jet[a]
$W \to \ell\nu$ [b]	[illegible] by Lemma 3	
$W\,j \to \ell\nu\,j$ [b]	$\le m_W$ by Lemma 2	one ISR jet[a]
$WW \to \ell\nu\ell\nu$ [b]	$\le m_W$ by Lemma 1	
$ZZ \to \nu\bar{\nu}\nu\bar{\nu}$	$= 0$ by Lemma 3	also $= m_j$ for one ISR jet[a]
$LQ\,\overline{LQ} \to q\nu\bar{q}\bar{\nu}$	$\le m_{LQ}$	} i.e. can take large values
$\tilde{q}\,\bar{\tilde{q}} \to q\tilde{\chi}_1^0\,\bar{q}\tilde{\chi}_1^0$	$\le m_{\tilde{q}}$	
$q_1, \bar{q}_1 \to q\gamma_1, \bar{q}\gamma_1$	$\le m_{q_1}$	

Putting it to work for discovery




ATLAS
EXPERIMENT
2010-10-28 02:24:03 CEST

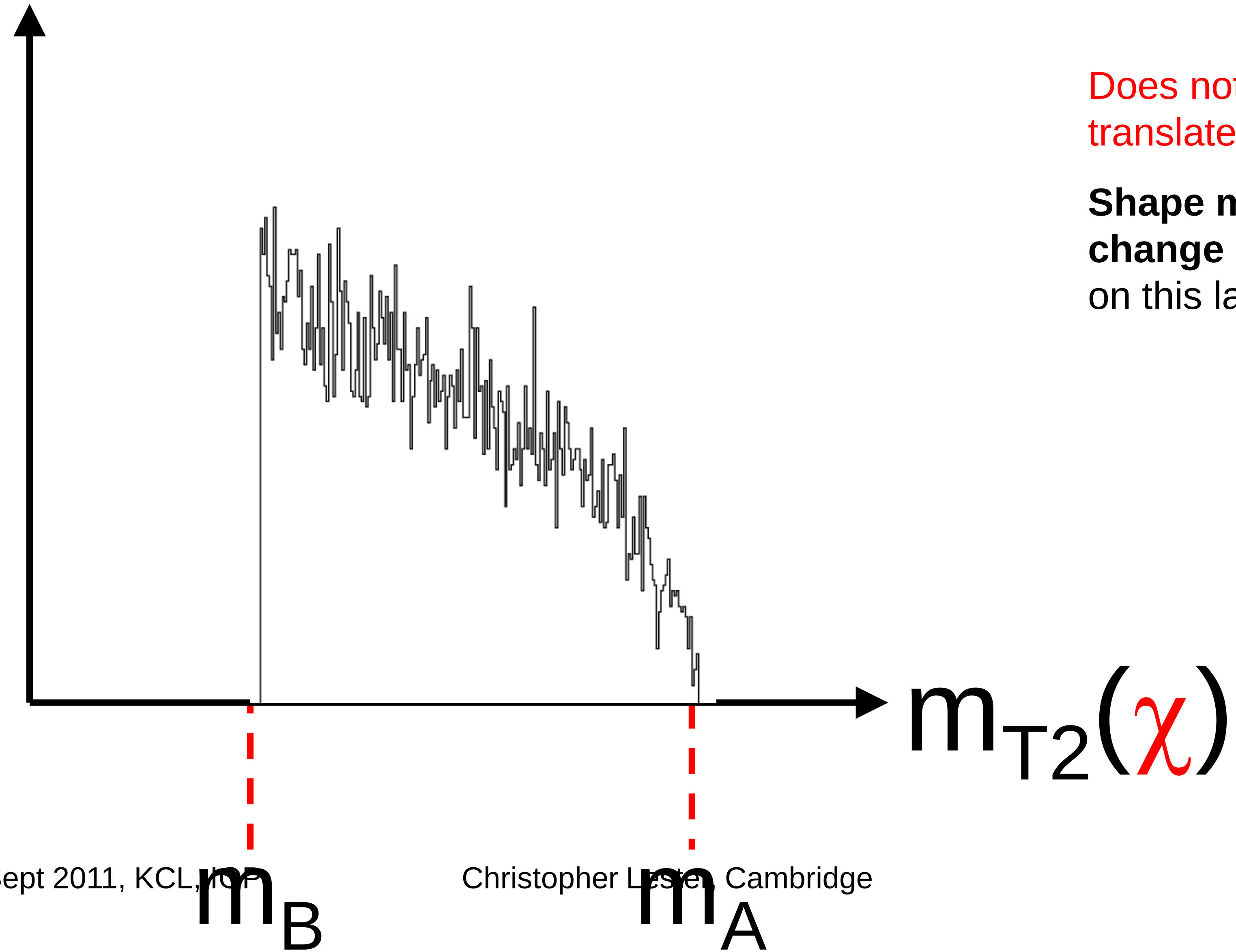
Health warning!

But note: high multiplicity environment already proving to be a challenge for mT2 (post 35/pb) and di-squark search in most recent data is being conducted with Meff. Problem is diagnosing the di-jet system.

Have dodged question of mass of invisible daughters.

What if we don't know their masses?

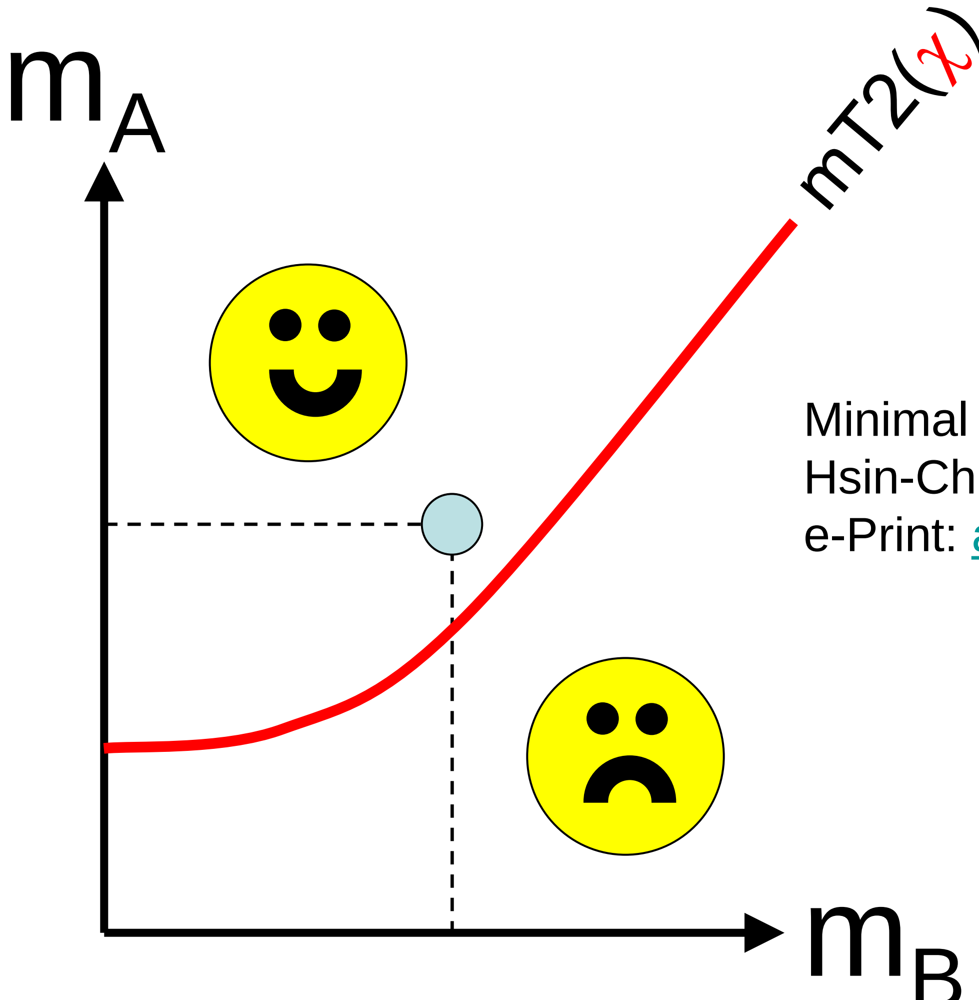
Varying "χ" … to first order



Does not just translate …

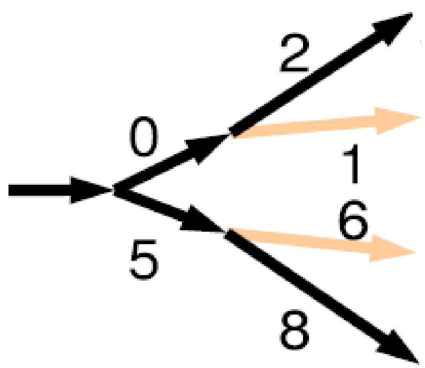
Shape may also change … more on this later.

MT2 inherits mass-space boundary from MT



The MT2(chi) curve is the **boundary of** the region of (mother, daughter) mass-space consistent with the observed event!

Minimal Kinematic Constraints and m(T2),
Hsin-Chia Cheng and Zhenyu Han (UCD)
e-Print: arXiv:0810.5178 [hep-ph]



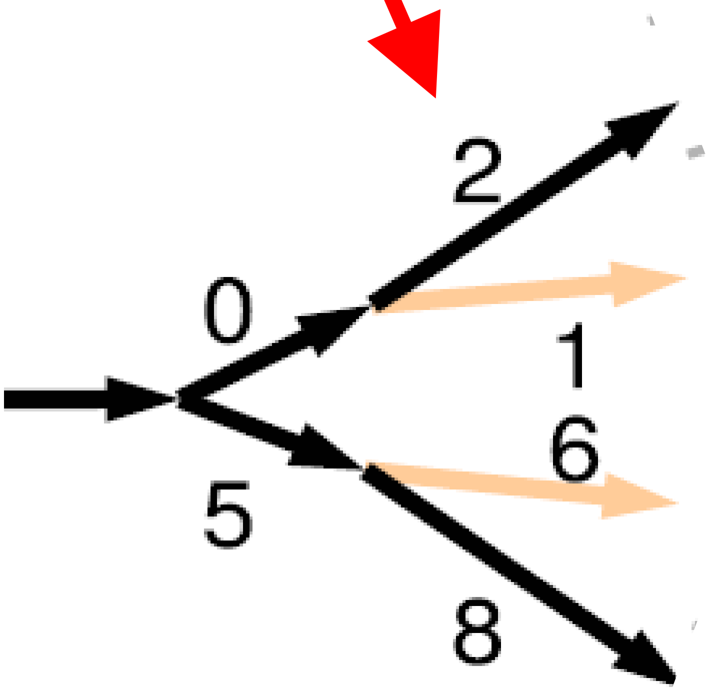
MT2 is defined in terms of MT

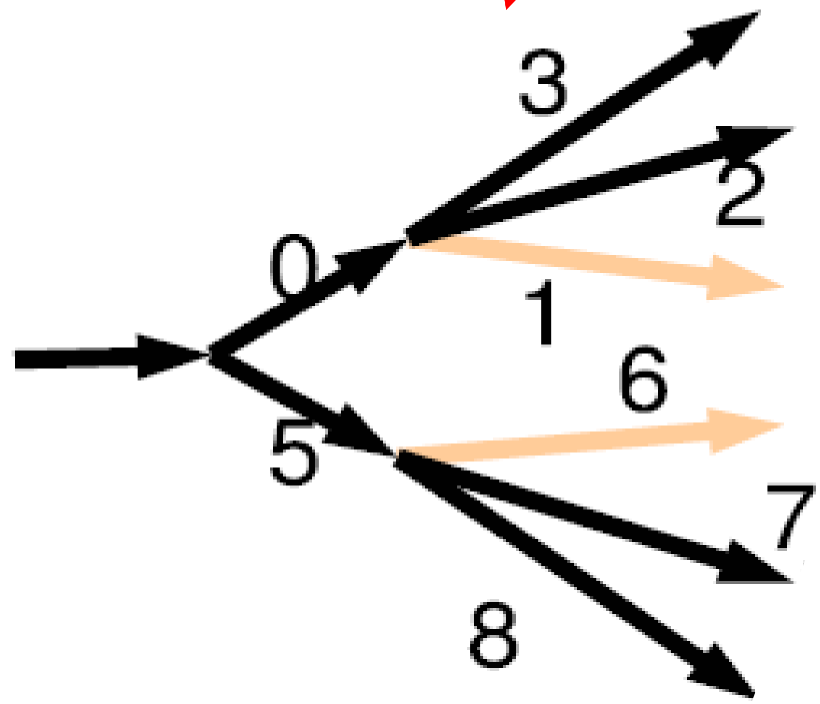
- Consequently, MT2 inherits the “kink structure” of MT and can (in principle) be used to:
  - EASILY measure the parent-daughter mass difference,
  - might PERHAPS measure the absolute mass scale using utm boosts kinks or variable visible mass kinks (HARD)

Are MT2 kinks observable ?

Expect KINK only from UTM Recoil (perhaps only from ISR!)

Expect stronger KINK due to both UTM recoil, AND variability in the visible masses.

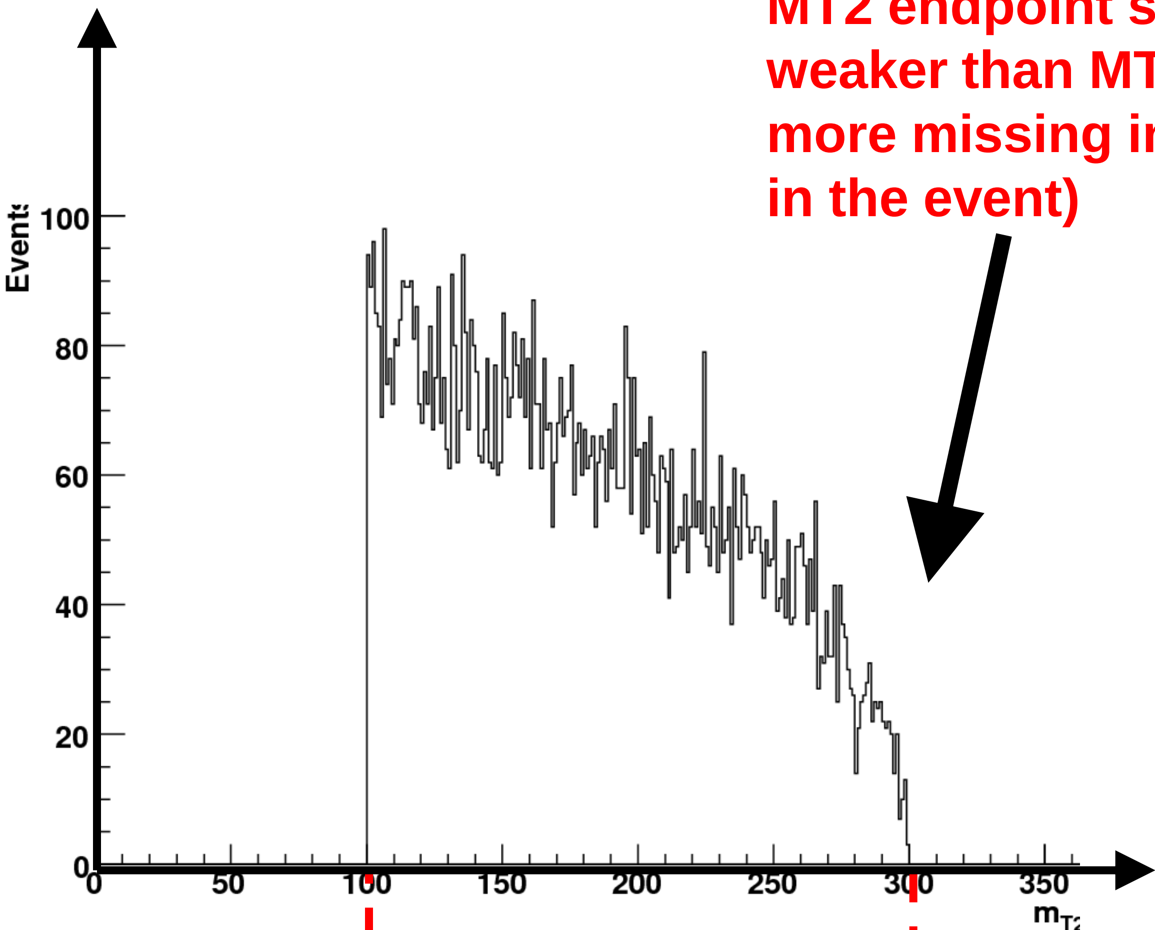


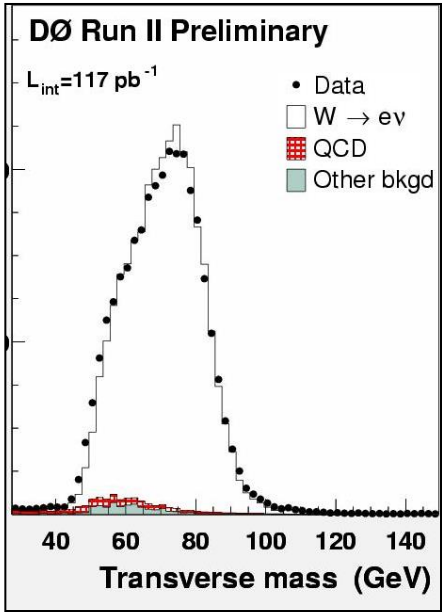


arXiv: 0711.4008

Perhaps: MT2's endpoint structure is weaker than MT's.

MT2 endpoint structure is weaker than MT (due to more missing information in the event)





$m_{T2}(m_B)$

m_B m_A

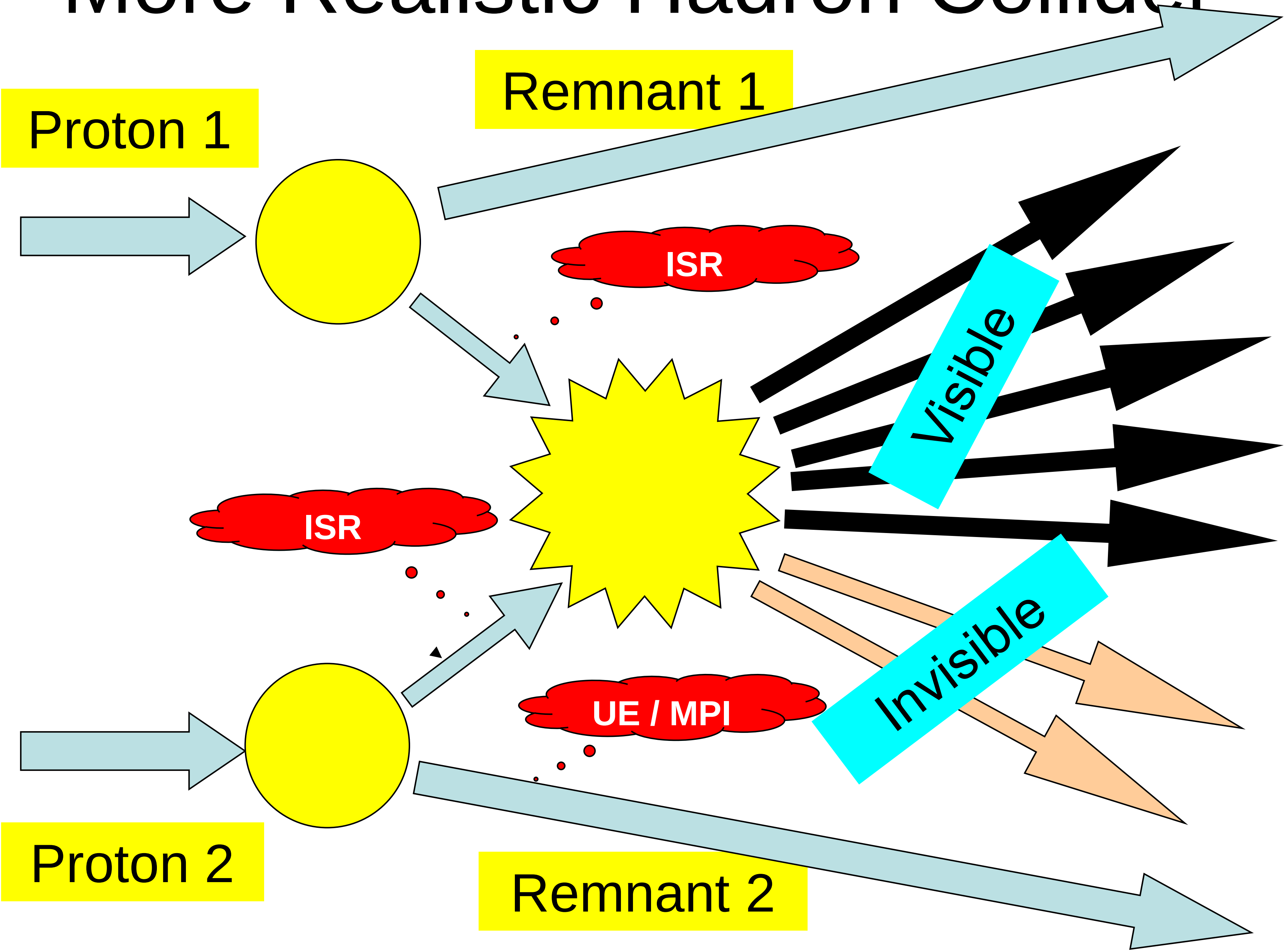
Caveat Mensor!

(for those of you interested in LHC dark matter constraints)

Disappointingly, M_{T2} kinks, are the only known kinematic methods which (at least in principle) allow determination of the mass of the invisible daughters of pair produced particles in short chains.

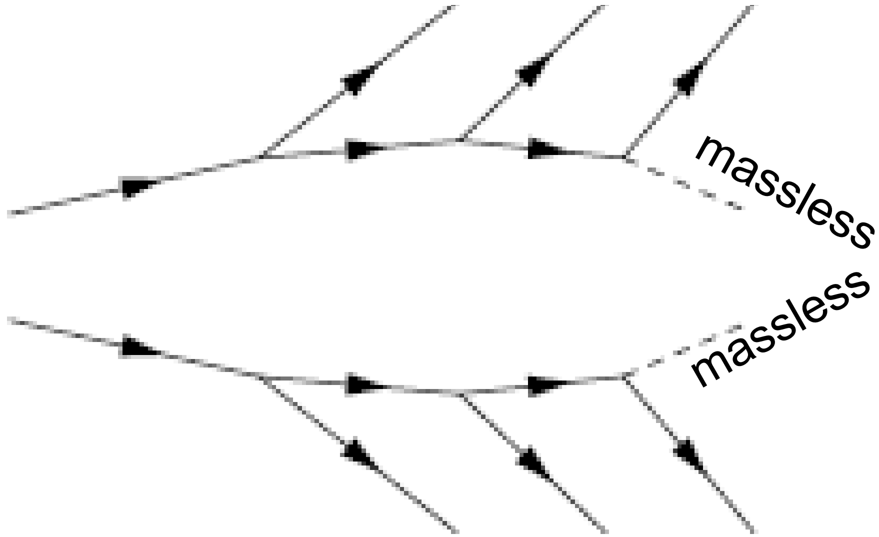
[We will see a dynamical method that works for three+ body decays shortly. Likelihood methods can determine masses in pair decays too, though at cost of model dependence and CPU. See Alwall.]

More Realistic Hadron Collider



“Just”-constrained events

(and over-constrained events)

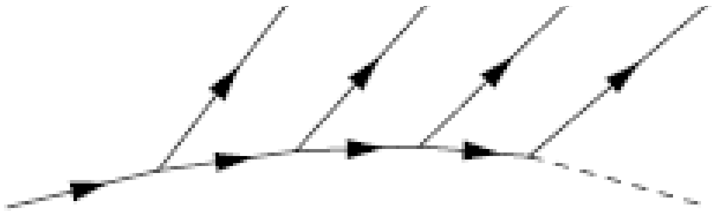


Left: case considered in hep-ph/9812233

- Even if there are invisible decay products, events can often be fully reconstructed if decay chains are long enough.
- (mass-shell constraints must be >= unknown momenta)

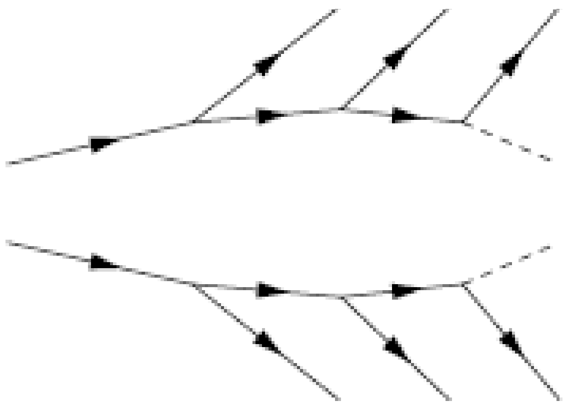
Small collections of under-constrained events can be over-constrained!

- For example (hep-ph/0312317) quintuples of events of the form:

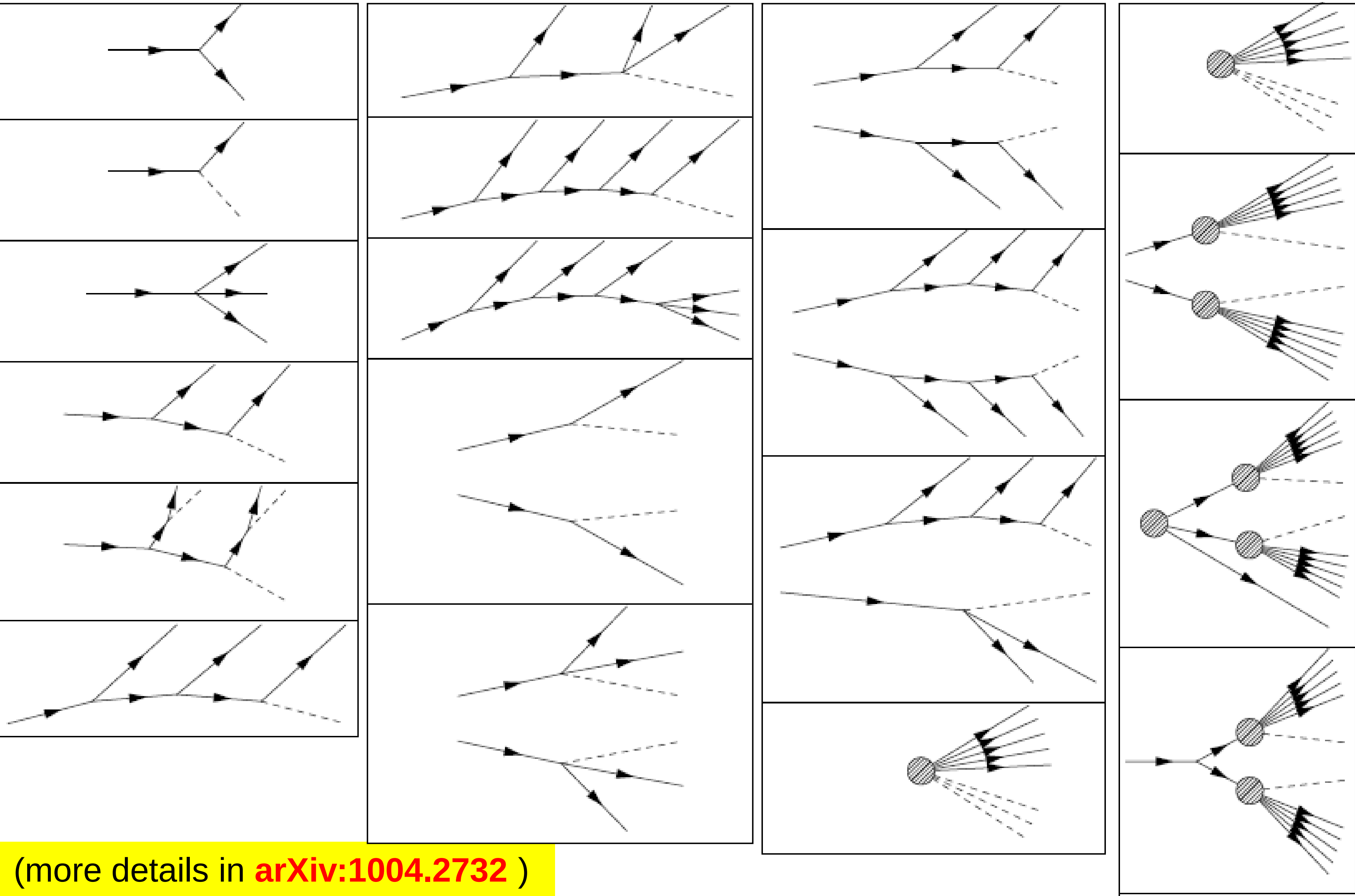


are exactly constrained

- similarly pairs of events
 of the form:
 (arXiv:0905.1344)
 are exactly constrained.



Once again – needed an index.



(more details in **arXiv:1004.2732**)

Not time to talk about many things

- Parallel and perpendicular MT2 and MCT
- Subsystem MT2 and MCT methods
- Solution counting methods (eg arXiv:0707.0030)
- Hybrid Variables
- Phase space boundaries (arXiv:0903.4371)
- Cusps and Singularity Variables (Ian-Woo Kim)
- and many more!

In 30 minutes I have only scratched the surface of the variables that have been discussed. Even the recent review of mass measurement methods arXiv:1004.2732 makes only a small dent in 70+ pages. However it provides at least an index …