



Towards a Measurement of Spin Correlations in $t\bar{t}$ Events at the LHC using Matrix Element Method

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The Top Quark

- The most massive particle known to date ($m_t \sim 173 \text{ GeV}$)
 - ◆ very short lifetime

$$\tau_t = \frac{1}{\Gamma_t} \sim 0.5 \times 10^{-24} \text{ s} < \frac{1}{\Lambda_{QCD}} < \frac{m_t}{\Lambda_{QCD}^2} \sim 3 \times 10^{-21} \text{ s} \ll \tau_b \sim 10^{-12} \text{ s}$$
$$\tau_t < \tau(\text{hadronization}) < \tau(\text{spin-decorrelation}) \ll \tau_b$$

No hadronic bound states \rightarrow bare quark properties are accessible (mass, V_{tb} , charge, ...).

spin effects propagate to decay products.

Measurements test

- \rightarrow top quark being bare
- \rightarrow pQCD in $t\bar{t}$ production
- \rightarrow existence of
 - $\rightarrow t \rightarrow H^+ b$
 - \rightarrow production through heavy particles
 - \rightarrow heavy higgs $\rightarrow t\bar{t}$

$t\bar{t}$ Spin Correlations

- The spin correlation strength A is defined as

$$A = \frac{(N_{\uparrow\uparrow} + N_{\downarrow\downarrow}) - (N_{\uparrow\downarrow} + N_{\downarrow\uparrow})}{(N_{\uparrow\uparrow} + N_{\downarrow\downarrow}) + (N_{\uparrow\downarrow} + N_{\downarrow\uparrow})}$$

- *Not a free parameter of the SM Lagrangian*, but depends on:
 - ◆ SM couplings, production mode, collision energy, basis of the spin quantization axis, ...
- A is proportional to the fraction of events f with SM spin correlations

$$A_{basis}^{meas} = A_{basis}^{SM} f \quad \text{where} \quad f = \frac{N_{SM}}{N_{SM} + N_{non-SM}}$$

- At Tevatron: beam axis basis ($A=0.37$ in lepton+jets)
- At LHC: helicity basis ($A=0.16$ in lepton+jets)

ttbar Spin Correlations

D0 : $f = 0.85 \pm 0.29 (stat \oplus sys)$

[MEM, PRL 108, 032004 (2012)]

ATLAS :

$f = 1.19 \pm 0.09 (stat) \pm 0.15 (sys) \leftarrow \Delta\phi$

[Dilepton, ATLAS-CONF-2013-101]

$f = 0.87 \pm 0.11 (stat) \pm 0.12 (sys) \leftarrow S\text{-ratio of on-shell MEs from fusion of like-helicity gluons} \leftarrow \text{not MEM}$

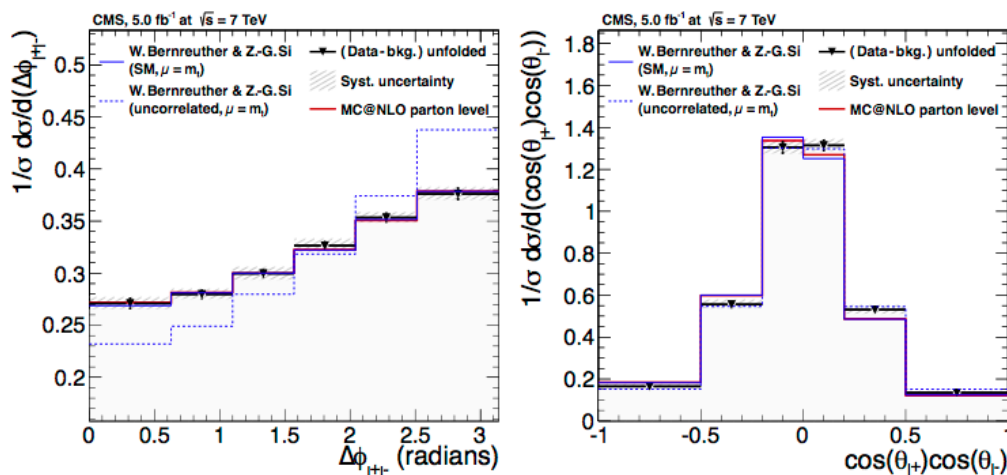
$f = 0.75 \pm 0.19 (stat) \pm 0.25 (sys) \leftarrow \cos(\theta_+) \cos(\theta_-)$ helicity basis; direct extraction of A.

$f = 0.83 \pm 0.14 (stat) \pm 0.17 (sys) \leftarrow \cos(\theta_+) \cos(\theta_-)$ maximal basis; defined event by event.

CMS : $f = 0.74 \pm 0.08 (stat) \pm 0.24 (sys) \leftarrow \Delta\phi$

P. Uwer, PLB 609, 271 (2005)

[Dilepton, CMS-PAS-TOP-12-004]



$$\frac{1}{\sigma} \frac{d\sigma}{d \cos \theta_+ d \cos \theta_-} = \frac{1}{4} (1 + A \alpha_+ \alpha_- \cos \theta_+ \cos \theta_-)$$

All measurements are consistent with SM, i.e., $f=1$.

[Dilepton, CMS-PAS-TOP-13-003, arXiv:1311.3924v1]

$t\bar{t}$ Spin Correlations

- Angles between the decay products are the most sensitive variables to the spin correlation (depending on the final state particles).
- However, we have the potential to do better by exploiting the full event information.
 - ◆ Use matrix element method to construct templates based on event likelihoods

$$P(x_i|H) = \frac{1}{\sigma_{obs}} \int f_{PDF}(q_1) f_{PDF}(q_2) dq_1 dq_2 \frac{(2\pi)^4 |M(y,H)|^2}{q_1 q_2 s} W(x,y) d\Phi_6$$

q_1 and q_2 : the initial parton kinematics

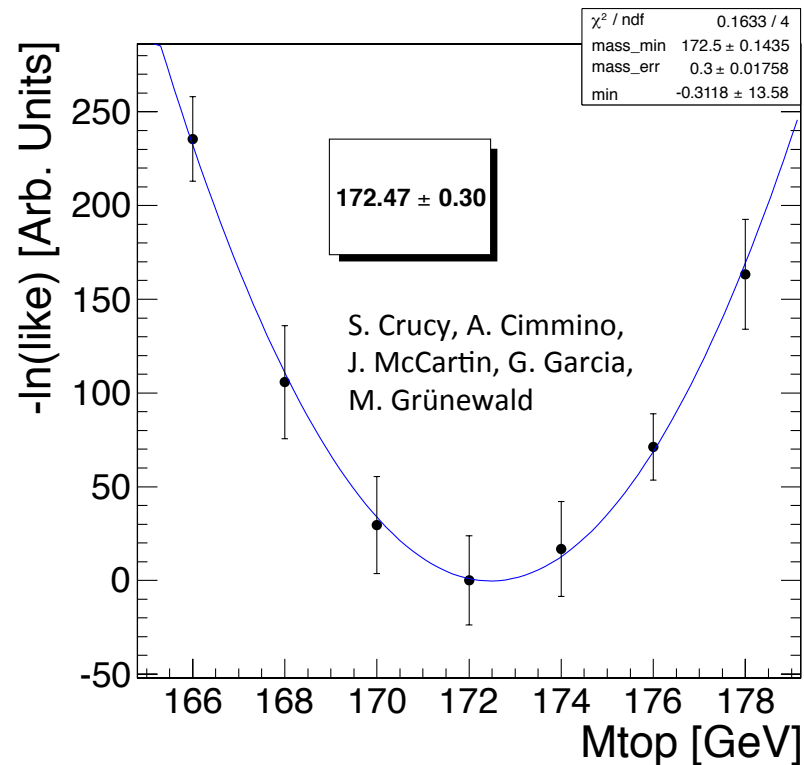
$W(x,y)$: Transfer Function that maps the reconstructed kinematics x to the parton level kinematics, y .

H : hypothesis under consideration

Calculate the likelihood/event for each hypothesis:
for the SM and the non-SM hypotheses.

First Steps in the Measurement: Strategy

- As opposed to top quark mass measurement: our hypotheses are discrete (i.e. SM & spin-uncorrelated SM)
 - ◆ We can not vary a parameter in the matrix element to obtain specific values of f (or A).
 - ◆ Instead we calculate the likelihoods for SM and non-SM cases separately.
 - ◆ SM and non-SM events can be mixed to obtain fractional f values for pseudo-experiments.



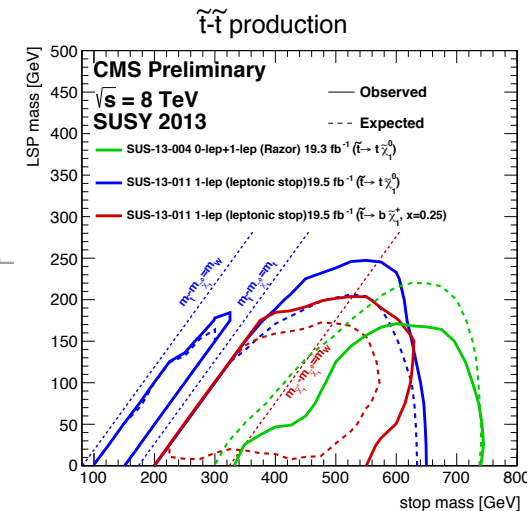
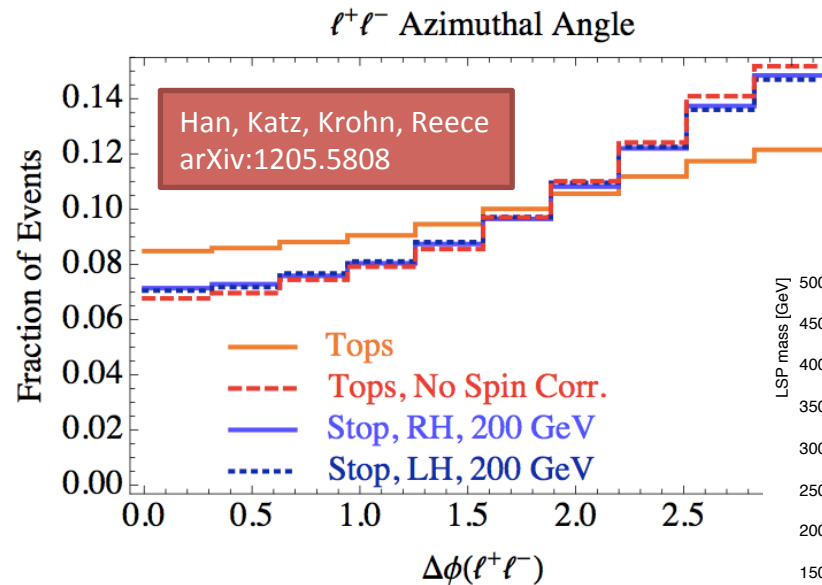
In the M_{top} measurement with MEM, likelihood is a continuous function of M_{top} .

ttbar Spin Correlations: non-SM Model

- f can be extracted by performing a template fit to the distribution of a variable that discriminates between SM and non-SM events.

Spin-uncorrelated SM: calculated by manually imposing spherical decay for the top quarks.
 → An “artificial model”, but e.g. SUSY with $m(\text{stop}) \sim m(\text{top})$ predicts un-correlated spins as in “spin-uncorrelated SM”.

$$f = \frac{N_{SM}}{N_{SM} + N_{non-SM}}$$



Effective theory: e.g. top chromo-moments (Bernreuther & Si, arXiv:1305.2066)

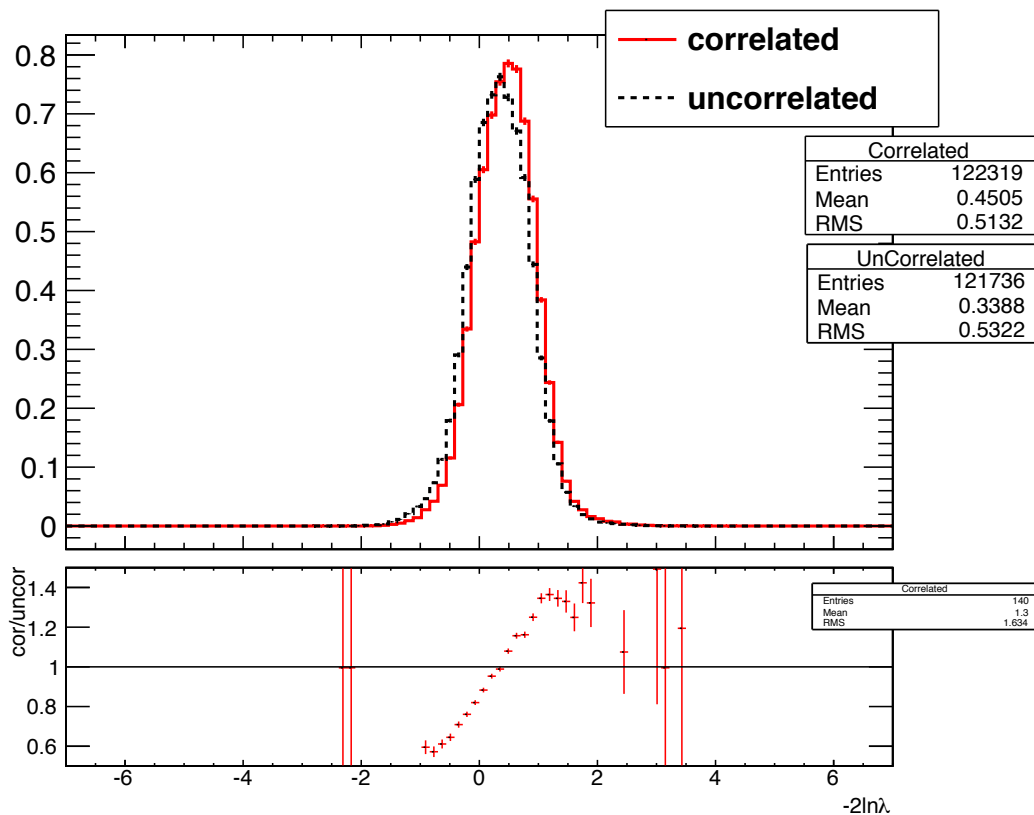
See also P. Artoisenet's talk

First Steps in the Measurement: Strategy

- With the event likelihoods (one for each hypothesis), we form a discriminating variable

$$-2\ln\lambda = 2\ln L_{H=C} - 2\ln L_{H=U}$$

normalised entries



$$-2\ln\lambda = -2\ln \frac{P(H_{non-SM})}{P(H_{SM})}$$

maximum discriminating variable
(Neyman and Pearson
Phil. Trans. R. Soc. Ser. A 231
(1933) 289)

- A template fit to this variable will be performed.
- Likelihoods calculated using MadWeight.

Templates at Gen Level with acceptance cuts.

First Steps in the Measurement: Matrix Elements

$$P(x_i|H) = \frac{1}{\sigma_{obs}} \int f_{PDF}(q_1) f_{PDF}(q_2) dq_1 dq_2 \frac{(2\pi)^4 |M(y,H)|^2}{q_1 q_2^S} W(x,y) d\Phi_6$$

- For the likelihood calculation, we need the MEs describing the $t\bar{t}$ process for **SM and non-SM** cases valid for both **on- and off-shell** top quarks.
 - ◆ SM ME: is known from the SM Lagrangian and implemented as default in MadWeight and MadGraph.
 - ◆ non-SM:
 - *Spin-uncorrelated ME*: calculated by manually imposing spherical decay for the top quarks.
 - *Effective theory with top chromo-moments*
- The spin-uncorrelated ME had to be manually implemented in **MadWeight** and **MadGraph5** (by replacing the SM ME by the spin-uncor. one in the fortran code).

The matrix elements were kindly provided to us by *Werner Bernreuther*.

First Steps in the Measurement: Matrix Elements

- The calculation are performed, taking into account only the following leading order Feynman diagrams for both SM and spin-uncorrelated cases.

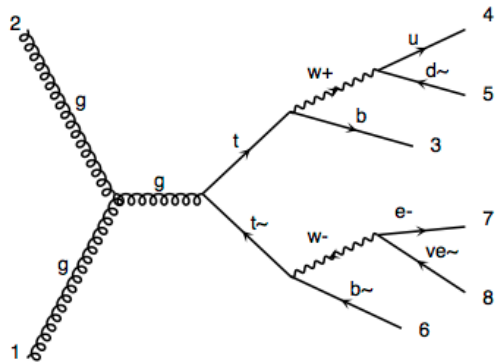


diagram 1 QCD=2, QED=4

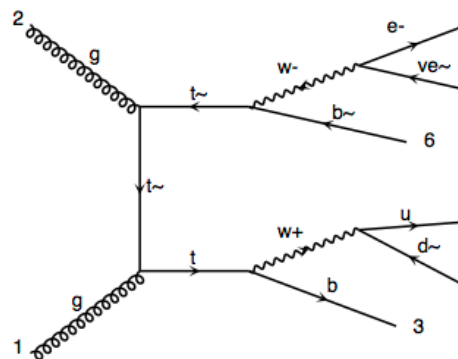


diagram 2 QCD=2, QED=4

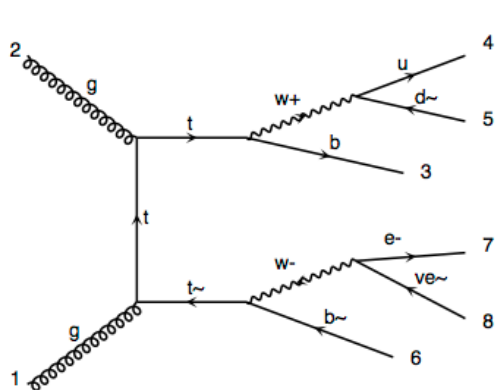


diagram 3 QCD=2, QED=4

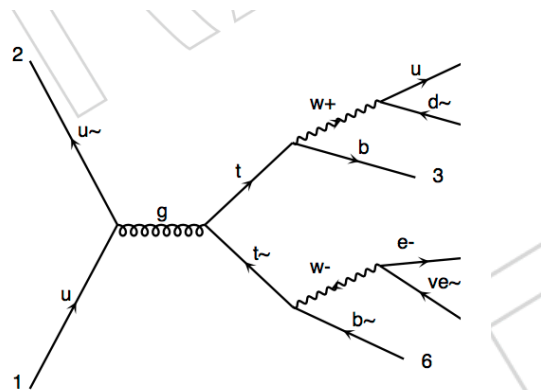
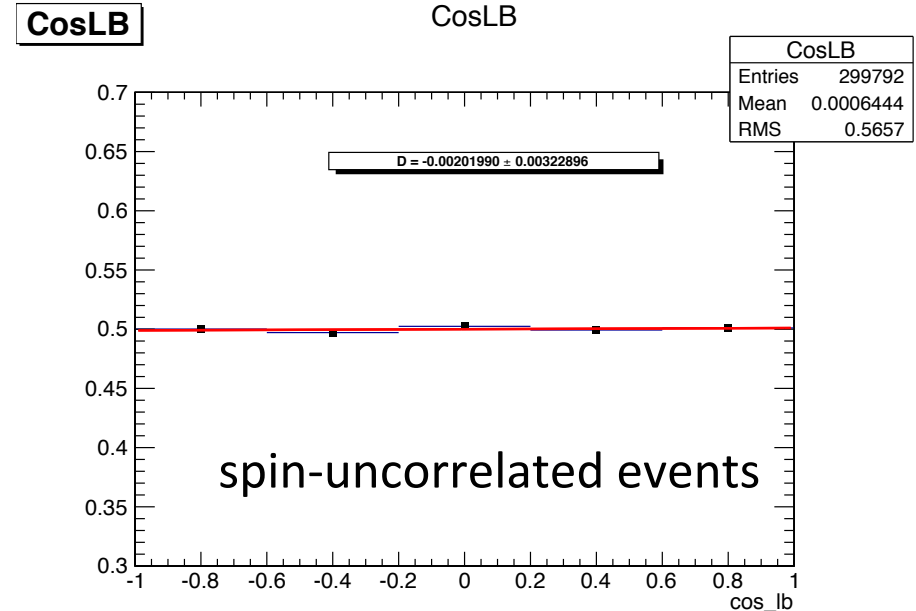
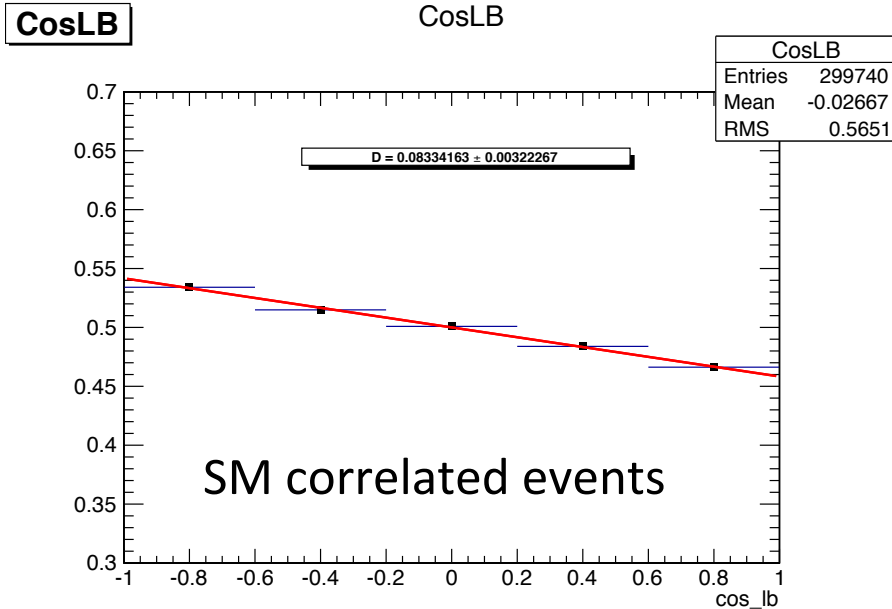


diagram 1 QCD=2, QED=4

Method will be calibrated using MC@NLO (or PowHeg).

First Steps in the Measurement: Matrix Elements

- The implementation of matrix elements was tested by generating events using the default and spin-uncorrelated MEs implemented in MadGraph and checking the angular distributions and cross-sections.



$$\frac{1}{\sigma} \frac{d\sigma}{d \cos \varphi_{lb}} = \frac{1}{2} (1 - D_{lb} \cos \varphi_{lb})$$

The Measurement and Closure Tests

- With the ability to generate events in both hypotheses, we can validate the feasibility of this measurement.
- The measurement is two-fold:
 - ◆ First discriminate between SM and spin-uncorrelated hypotheses → hypothesis testing
 - ◆ Extract the spin correlation strength A by measuring the fraction of events f with SM spin correlations.

Closure Tests: Hypothesis Testing

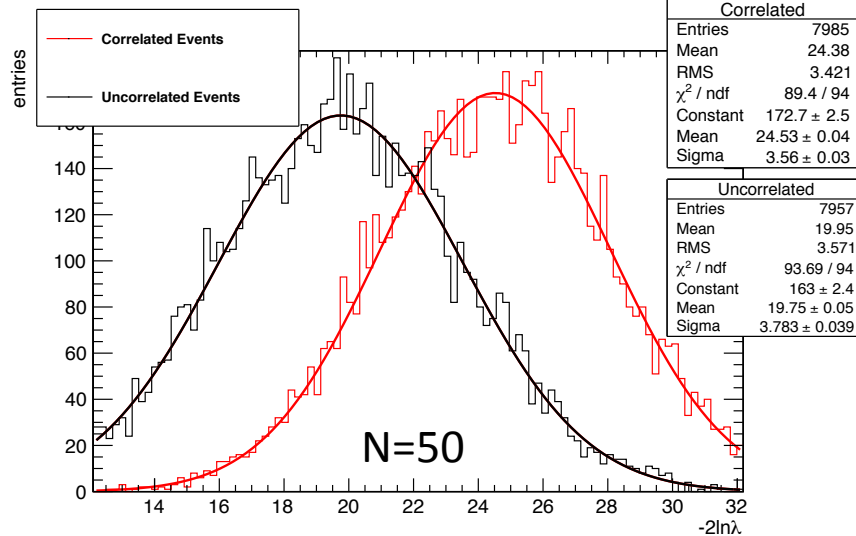
- The sample likelihood can be calculated from the event likelihoods: $-2\ln\lambda_{\text{event}}$

$$-2\ln\lambda_{\text{sample}} = -2\sum\ln\lambda_{\text{event}}$$

- Generate two event pools (one SM and one spin-uncorrelated), with each event processed under both hypotheses.
- Split event pools in pseudo-data of sample size N and calculate $-2\ln\lambda_{\text{sample}}$.

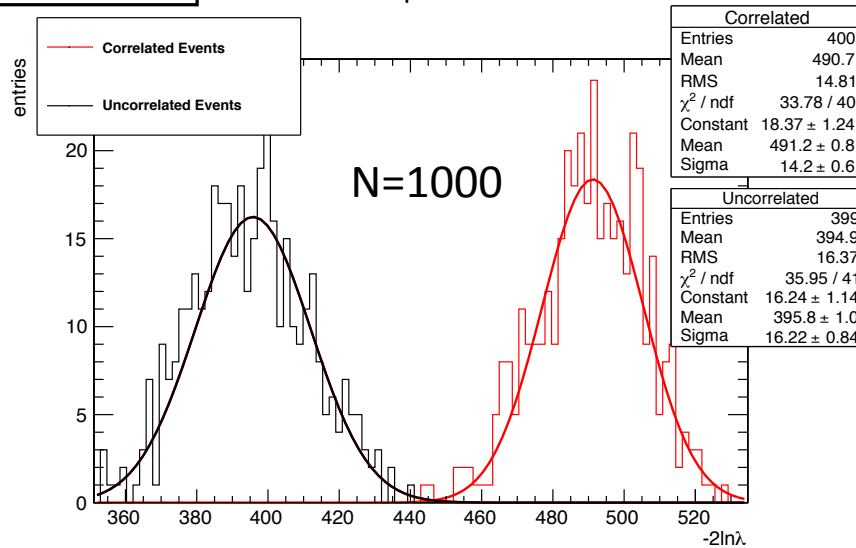
Closure Tests: Hypothesis Testing

Sample $-2\ln\lambda$



- Pseudo-experiments give a Gaussian distribution for $-2\ln\lambda_{\text{sample}}$
- The separating power increases with sample size N and given by

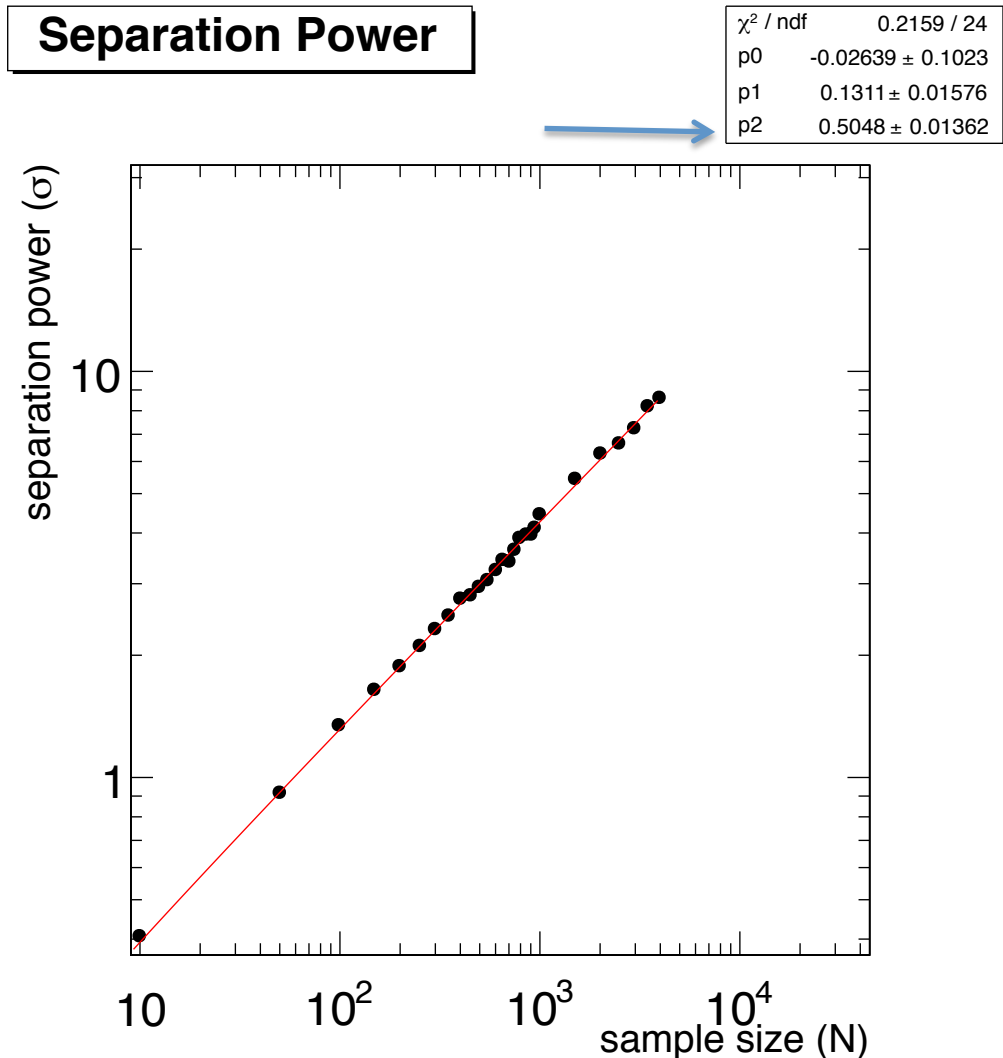
Sample $-2\ln\lambda$



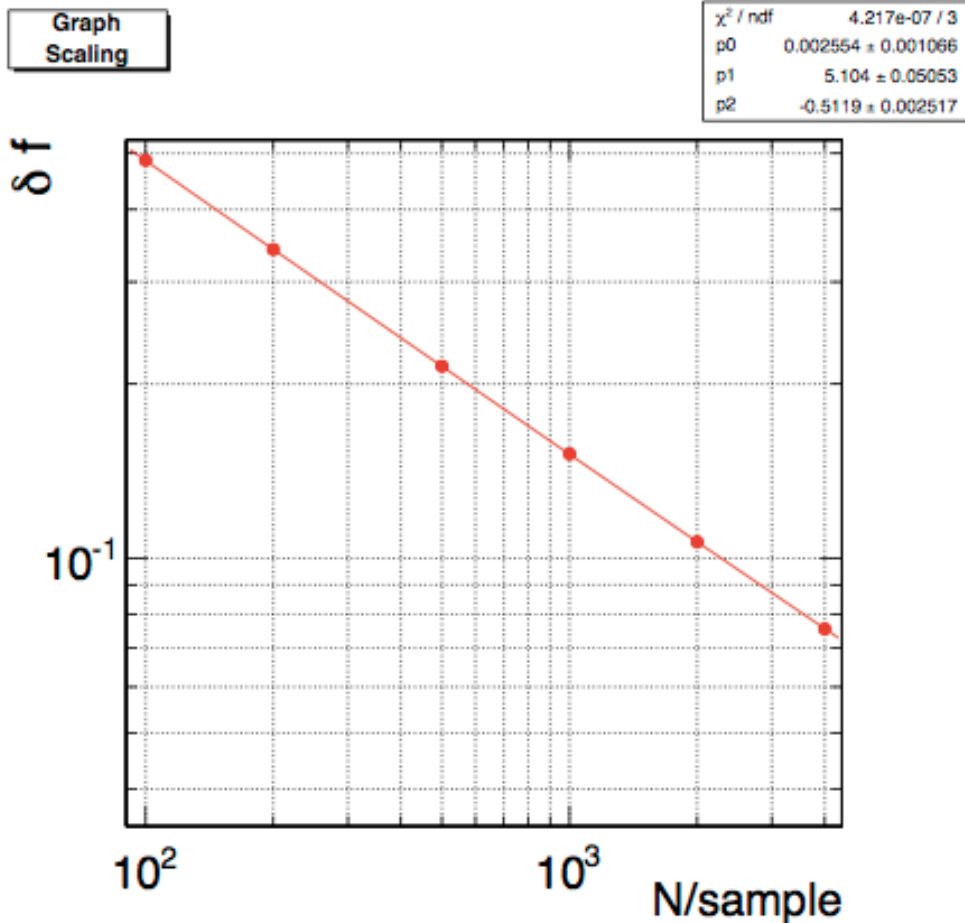
$$\sigma = \frac{\mu_1 - \mu_2}{\sqrt{\sigma_1^2 + \sigma_2^2}}$$

Closure Tests: Hypothesis Testing

- The separating power scales with the square root of the sample size, as expected.



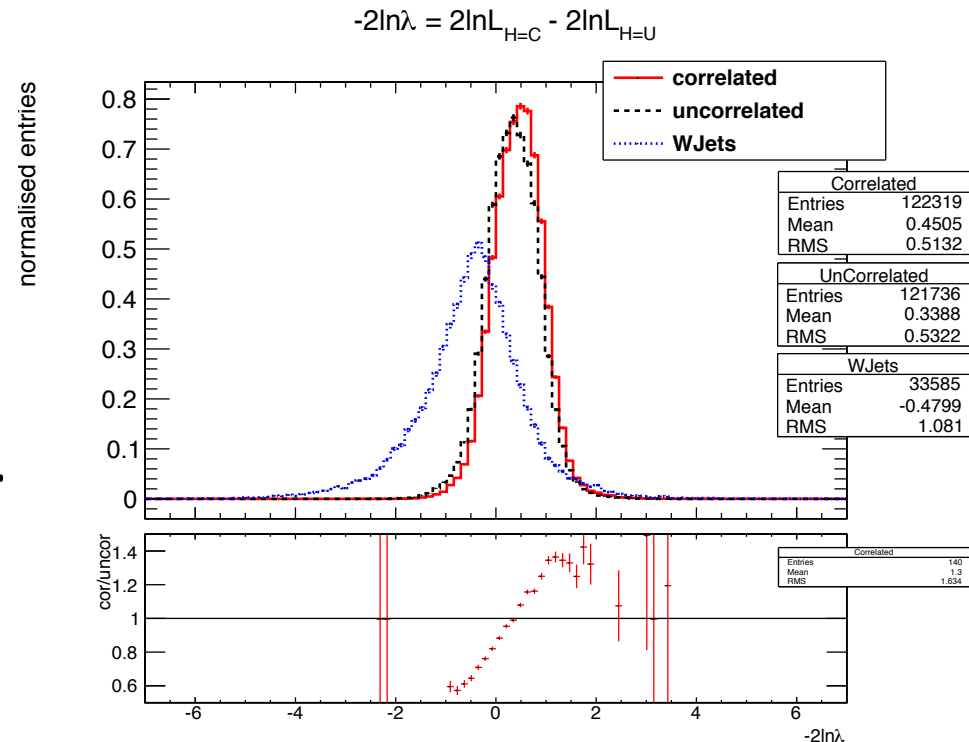
Error Scaling



- The mean fit error from pseudo-experiments vs. pseudo-experiment sample size.
- The error scales with $1/\sqrt{N}$, as expected.
- $\sim 100\text{k}$ high purity $t\bar{t}$ +jets events in 8 TeV CMS data (considering only up to 5 jets).
 - ◆ $\sim 1\%$ statistical precision at the generator level assuming an ideal detector.

Closure Tests: Spin Correlation Fraction

- $-2\ln\lambda_{\text{evt}}$: distribution with the most discriminating power.
- One template for SM events, one template for spin-uncorrelated events, (and template(s) for backgrounds).
 - ◆ Background likelihoods calculated using $t\bar{t}$ MEs on background events.



Template fit with maximum likelihood method using the fit model

$$\mathbf{m} = N_{t\bar{t}} \left[f^{SM} T_{SM} + (1 - f^{SM}) T_{spin-uncor} \right] + N_{bkg} T_{bkg}$$

simultaneously extract f^{SM} , number of $t\bar{t}$ ($N_{t\bar{t}}$) and background (N_{bkg}) events

Closure Tests: Spin Correlation Fraction

- Closure tests

- ◆ With varying spin correlation fractions.
- ◆ Effects of backgrounds, acceptance cuts
- ◆ Transfer functions, ...

$$\frac{p_T(j) > 30 \text{ GeV}, |\eta(j)| < 2.4}{p_T(e) > 30 \text{ GeV}, |\eta(e)| < 2.5}$$

Double Gaussian for light jet and b jets.
 Parametrization vs Energy and eta of the parton.

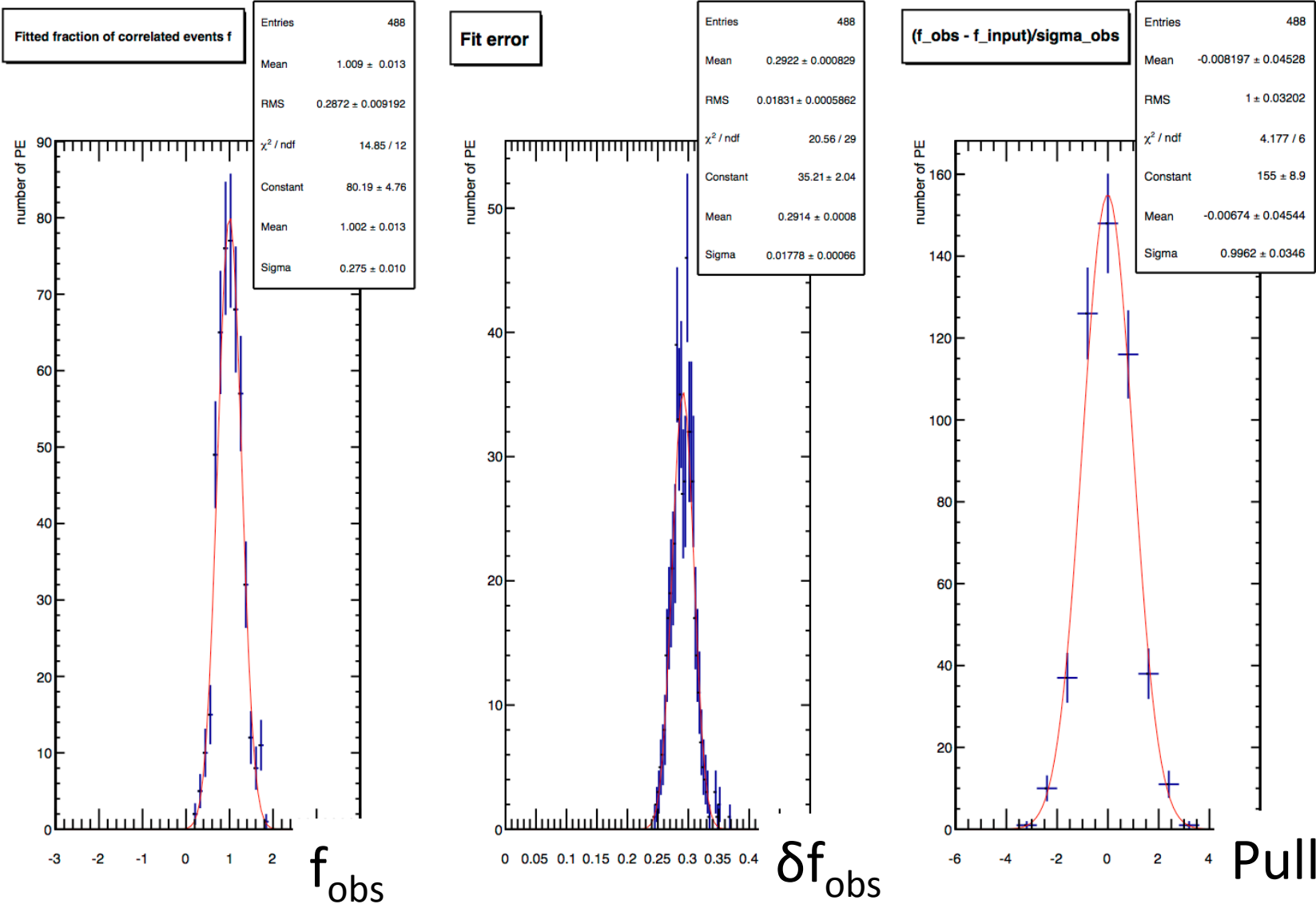
Closure test	N_{evts}^{corr}	N_{evts}^{uncorr}	N_{evts}^{bkg}
Gen. level / No background / No smearing / No acc. cuts	392318	390174	0
Gen. level / No background / No smearing / acc. cuts	121395	121736	0
Gen. level / With background / No smearing / acc. cuts	121395	121736	33585
Gen. level / No background / Smearing / acc. cuts	219100	221400	0
Gen. level / With background / Smearing / acc. cuts	219100	221400	46670

- Very CPU intensive, each event needs to be processed twice, 50 ev ~1h CPU time.
 - ◆ However, with the help of CERN computing resources, we have a reasonably fast turnaround time.

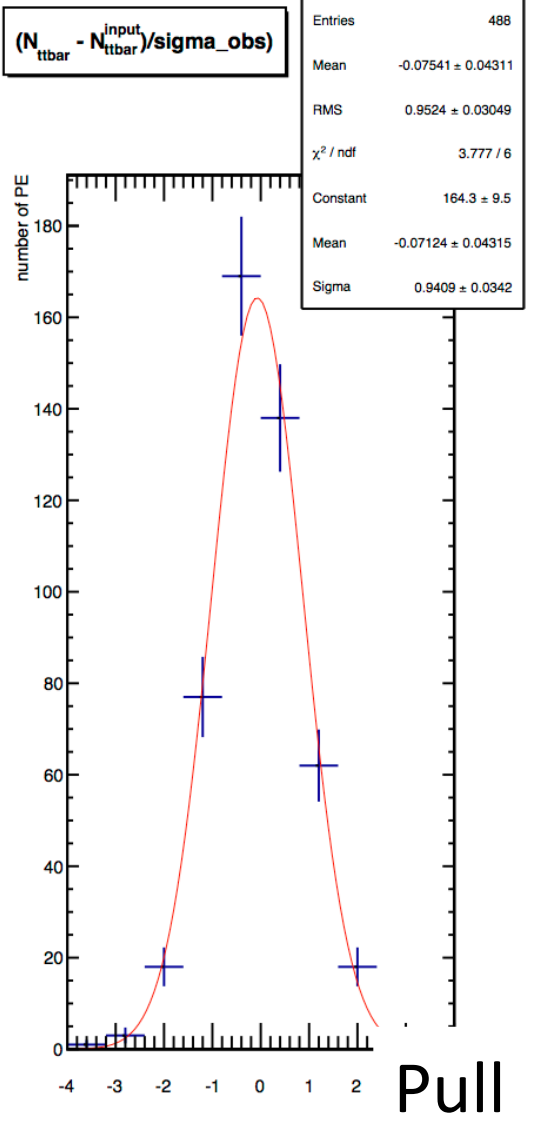
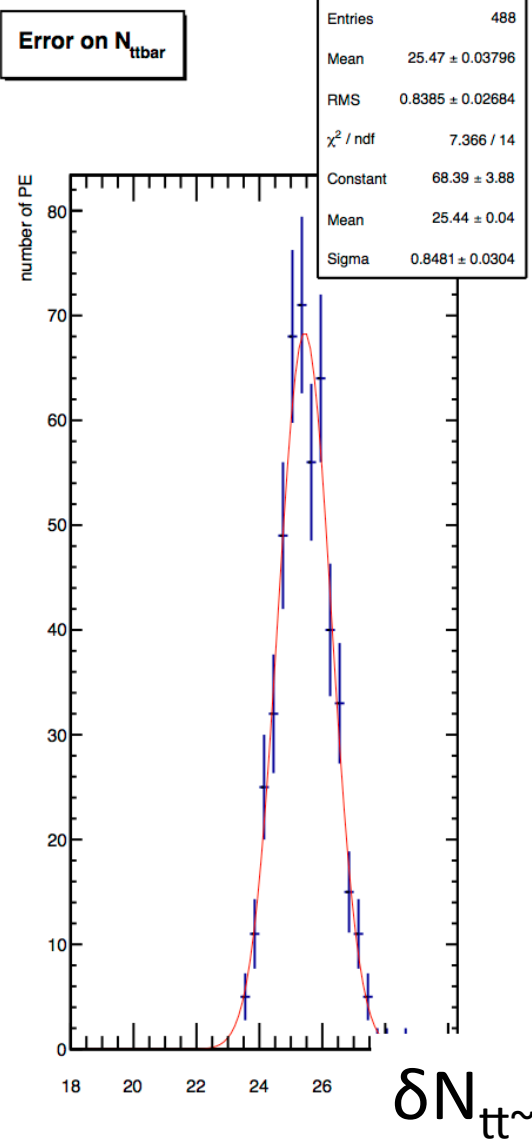
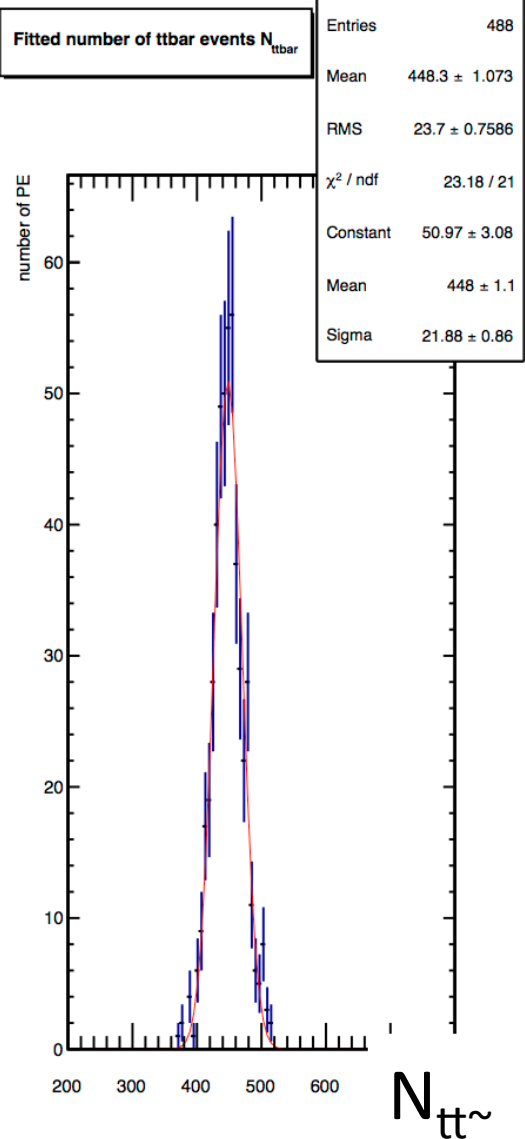
Closure Tests: Spin Correlation Fraction

- Generate pseudo-experiments of 500 events (Poisson fluctuated).
 - ◆ 10% Wjets, 90% SM events.
 - ◆ --> Expect the distributions to be a Gaussian centered around $f=1$, $N_{t\bar{t}}=450$ and $N_{\text{bkg}}=50$.
- Fits are performed with the RooFit package.
 - ◆ Template histograms are normalized and converted to PDFs.

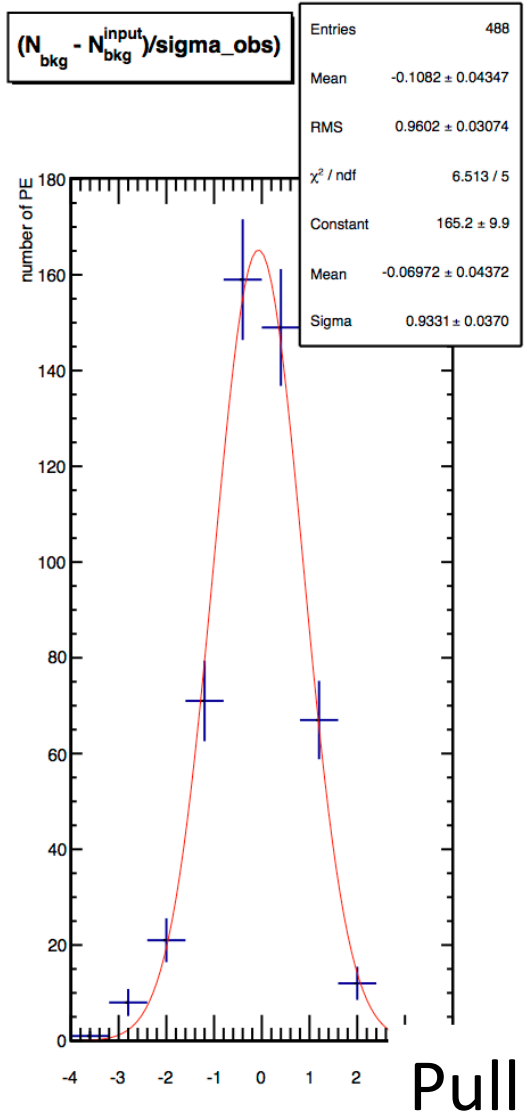
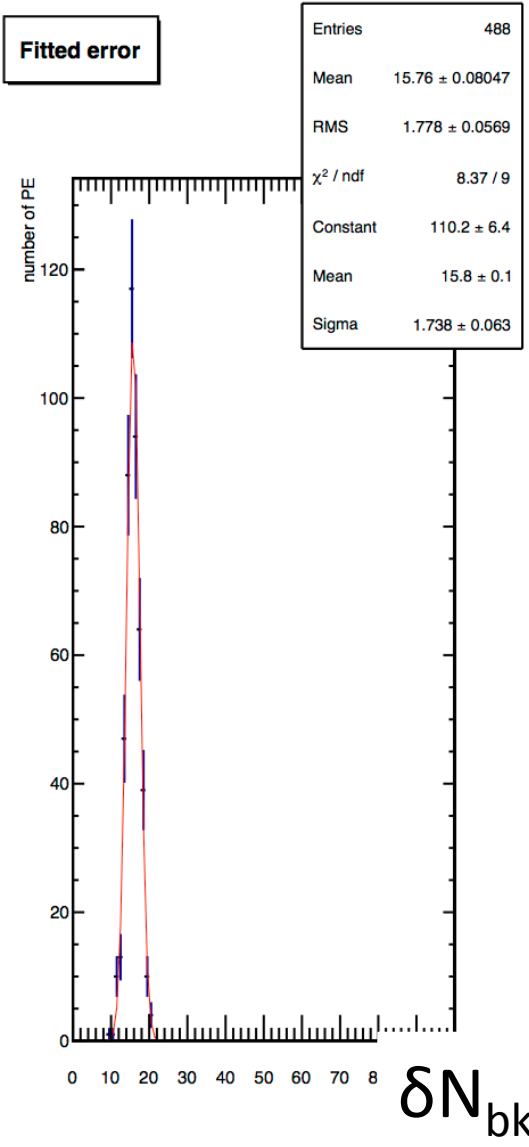
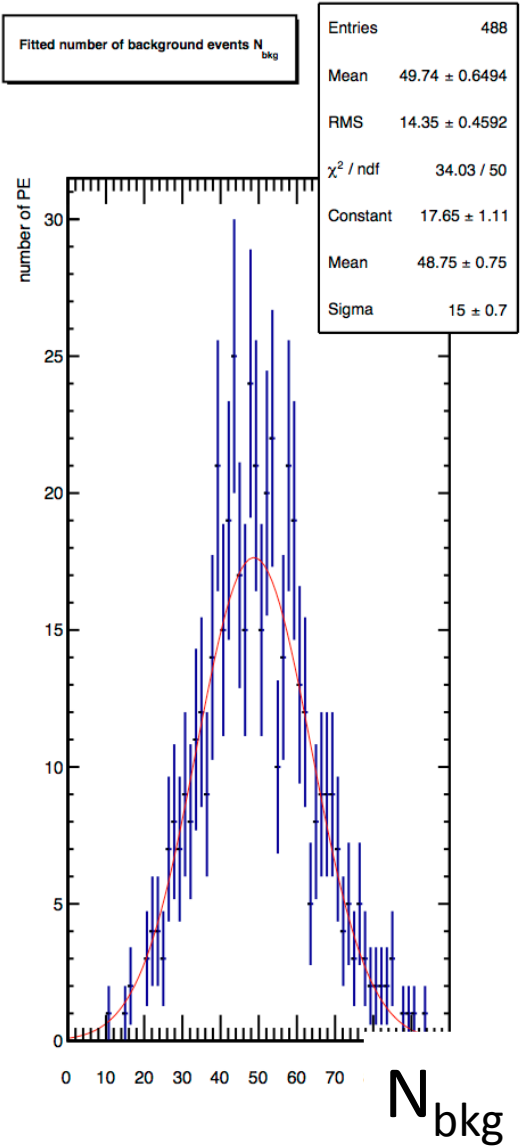
Closure Tests: Spin Correlation Fraction



Closure Tests: Spin Correlation Fraction



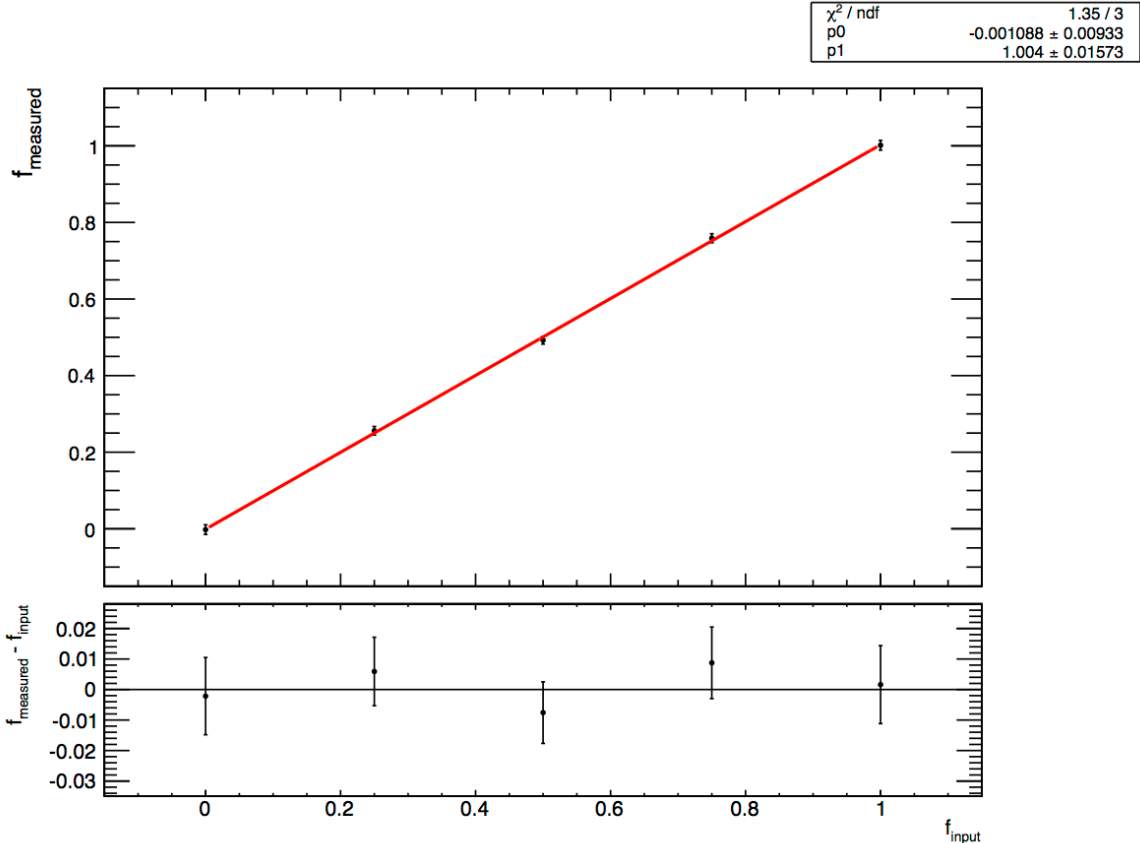
Closure Tests: Spin Correlation Fraction



Closure Tests: Spin Correlation Fraction

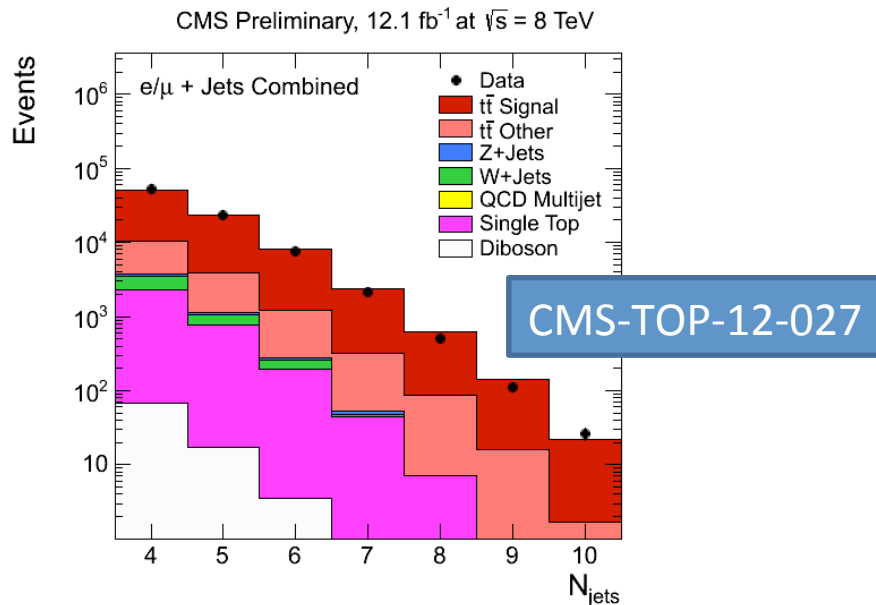
- We manually mix the two event pools (SM and spin-uncorrelated) to simulate samples with varying degrees of mixing.
- *We can extract any fraction f precisely without a bias.*

We correct the fit errors for correlations between the data points.



Closure Tests

- Closure tests are successful: we can extract the signal f , $t\bar{t}$ and background cross sections simultaneously.
- The closure tests assumed correct jet-parton assignments with no extra jets in the event.



+ MEs used in MEM are LO , i.e. can take only 4 jets as input.

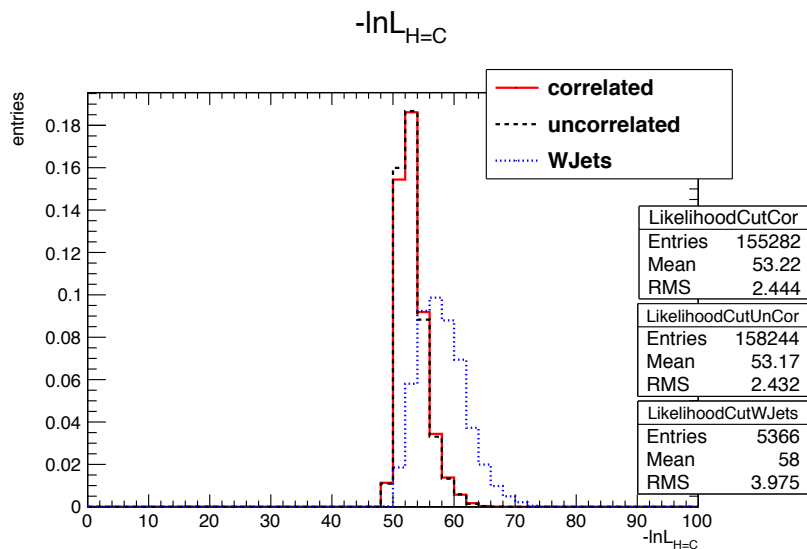
MEM+Kinematic Fitter

- Require ≥ 2 b-tags.
- Use a kinematic fitter
 - ◆ to choose the 4 jets coming from the $t\bar{t}$ decay
 - ◆ and calculate the likelihood only for the selected permutation
 - significant decrease in CPU requirements.

- **Kinematic fitter** modifies the kinematics within object resolutions to obtain kinematics most suitable with constraints (e.g. $W_{\text{had}}=80.4$ GeV, $m_{\text{top}}=m_{\text{anti-top}}$)
- The solution with the lowest χ^2 and consistent with b-tagging information is taken as the best estimate for the correct permutation.
 - ◆ *Can it simplify the transfer functions or make them obsolete?*

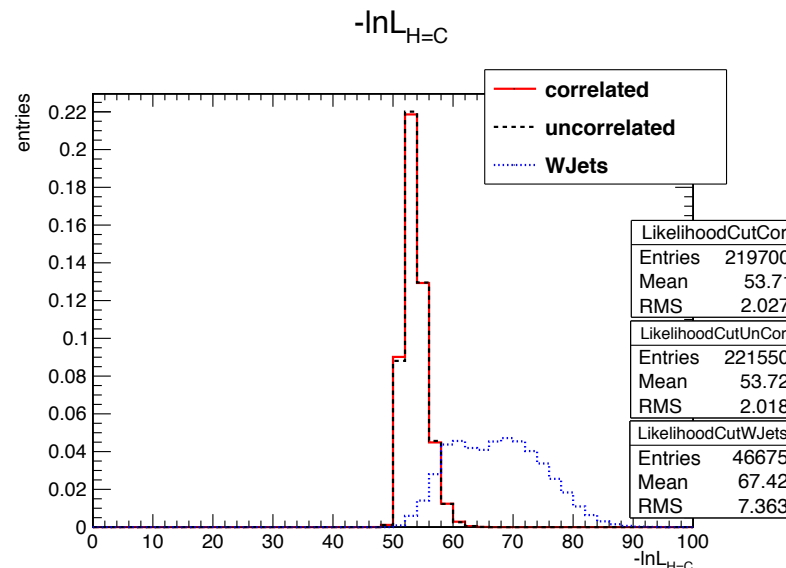
Likelihoods and Kinematic fitter

Kinematic fitter kinematics with quality cuts



No Kinematic fitter

All permutations in MadWeight



→ Likelihoods from SM & spin-uncor evts and bkg sample under the spin corr hypothesis ($H=C$)

Using the kinematics determined by kinematic fitter (or adding extra quality criteria) forces the background to be $t\bar{t}$ -like.

+ A certain contamination of wrong jet-parton permutations will always be present

→ Increases the uncertainty.

→ Running over all permutations in MadWeight ensures the correct one is always considered and yields smaller uncertainty.

I/FSR

- Our LO MEs do not treat radiated jets.
- Ignoring these means working with wrong kinematics.
 - ◆ Use only 4 and 5 jet events and treat them separately (or not).
 - ◆ Test the ISR treatment in MadWeight.
 - ◆ ..

Systematic Uncertainties

- Three types:
 - ◆ Type 0: Normalization uncertainties
 - absolute and background normalizations, integrated luminosity.
 - ◆ Type 1: Effects that change the template shapes but can be estimated w/o re-running MadWeight.
 - lepton, trigger, b-tag efficiencies vs pT and eta.
 - pile-up
 - Template stats., PDF uncertainties through weights
 - Kinematic Fit (effect of top mass window and chi2 cut if used).
 - ◆ Type 2: Effects that require re-running MadWeight →

Systematic Uncertainties

- Effects that require re-running MadWeight.
 - ◆ Jet energy scale and resolution
 - light jets and b jets
 - any uncertainty due to resolution or transfer functions are included in jet energy scale and resolution uncertainties.
 - Taylor expansion of the TFs suggested by Rieck et al. (yesterday) seems reasonable (to be tested. If works → type 1.).
 - ◆ Top quark mass
 - Samples with different assumed top mass values.
 - But re-weighting might work within δm_t (then → type 1)
 - ◆ ISR/FSR
 - ◆ Samples with different renorm. & fact. scales. → Reweighting if only SM samples exist.
 - ◆ Method calibration
 - ◆ Signal and background modeling
 - ◆ Hadronization
 - ◆ Underlying event, Color reconnection
 - ◆ ...

Summary

- Spin correlated and uncorrelated matrix elements implemented and tested.
- Validation and study of statistical properties at the generator level + generator level with smeared jets done.
- Studies with kinematic fit + MEM.
- *ttbar spin correlation can be measured with high statistical precision using template fits to likelihood ratios determined from matrix element calculations using MadWeight.*
- *A kinematic fitter helps the measurement in selecting the correct 4 jets to be used as input to MadWeight, however, using the updated kinematics from the kinematic fitter worsens the discriminating power and the precision of the measurement.*
- *Measurement to come in 2014 with 8 TeV LHC data!*

Thanks!

- **W. Bernreuther** for providing all necessary matrix elements.
- **O. Mattelaer, P. Artoisenet** for their great help in MadGraph and MadWeight.
- And **S. Frixione** for discussions and his help in theoretical issues.
- **V. Adler** for providing the transfer and resolution functions.
- Workshop organizers.