Matrix elements + PYTHIA 8

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(DESY)

(with Leif Lönnblad)

Zürich Phenomenology Workshop 2014,
January 08

Outline

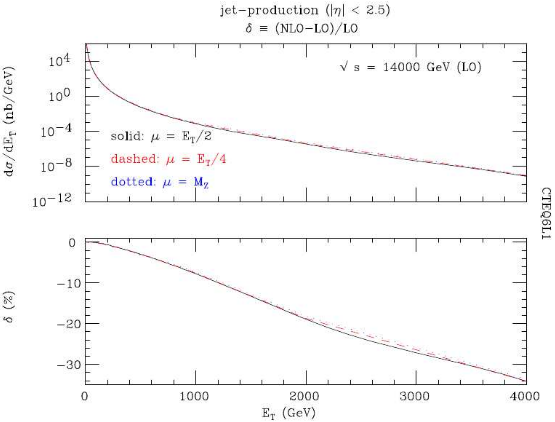
- Comments on leading-order ME+PS merging.
- NLO multi-jet merging in PYTHIA 8.
- NNLO outlook.
- Summary.

Why PYTHIA 8?

- \- No *ep*, γp or $\gamma\gamma$ incoming beams.
- \- Fewer Technicolour and SUSY in-built processes.
- \+ Evolved MPI model, sophisticated diffractive machinery.
- \+ τ polarisation in production and decay.
- \+ Hidden valley showers, R-hadrons.
- \+ Weak showers.
- \+ More perturbative physics: Matching and Merging!
- \+ Simple card files: Should be better match for software frameworks.

PYTHIA 6 development has stopped.
PYTHIA 6 support will stop after the shutdown.

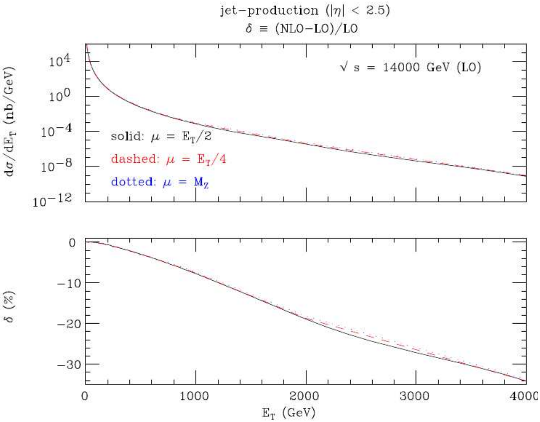
Weak corrections to showers?



Nucl.Phys.B759(2006)50

- ◇ Weak correction is $\sim \alpha_w \ln^2(\hat{s}/M_w)$.
- ◇ Is W/Z-boson radiation a necessary ingredient for TeV-jets?

Weak corrections to showers?



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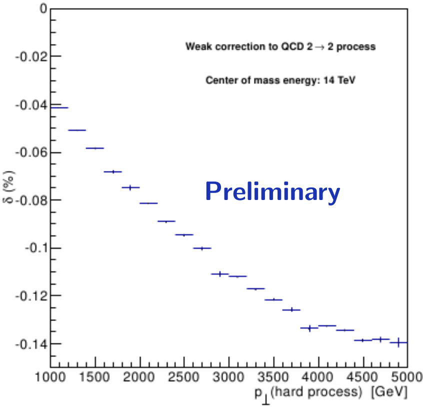
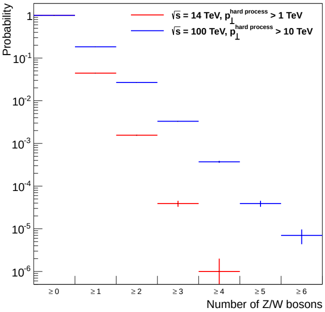
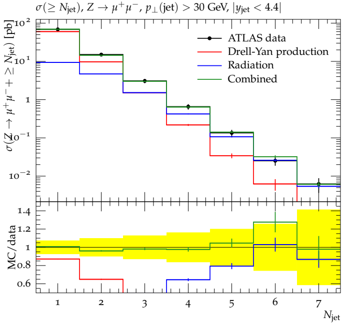


- ◇ Weak correction is $\sim \alpha_w \ln^2(\hat{s}/M_w)$.
- ◇ Is W/Z-boson radiation a necessary ingredient for TeV-jets?
- ◇ Idea: Implement W/Z-shower off QCD processes, and check!

Electroweak showers in PYTHIA 8: Preliminary results

- Effect of weak emissions in high $p_\perp$-jets.
- Possible to give a better description of the W/Z+jets production than the normal PS?
- Needed step to be able to recluster all PS histories in the merging/matching approach.

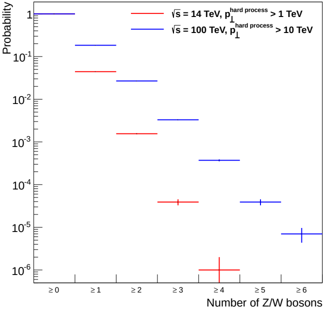


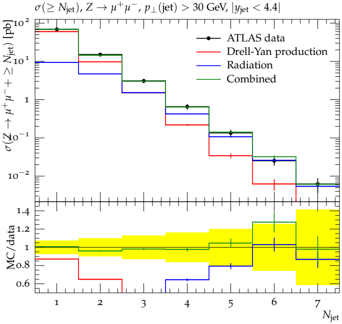


Electroweak showers in PYTHIA 8: Preliminary results

Uses s- or $u+t$-channel ME's as splitting probabilities. Multiple boson emissions very rare.

- Effect of weak emissions in high $p_{\perp}$-jets.
- Possible to give a better description of the W/Z+jets production than the normal PS?
- Needed step to be able to recluster all PS histories in the merging/matching approach.





Fixed-order improvements: The ME+PS merging problem

Problem: We want to describe soft/collinear and hard jets in one sample, but we do not know the boundary between "soft" and "hard". Parton showers are good for describing soft/collinear jets. Fixed-order matrix elements are good for well-separated jets.

$\Rightarrow$ Should we just "add" several PS and ME samples?

- Just adding ME and PS gives massive double counting.

$\longrightarrow$ Instead, use ME above a cut t_{MS}, and PS below t_{MS}.

- This still has problems: ME overlaps and cut dependence.

$\longrightarrow$ Apply the same weights above and below the cut.

$\longrightarrow$ MEPS merging

CKKW(-L)[1] merging

The CKKW-L prescription is:

- Calculate the tree-level MEs.

$$\langle \mathcal{O} \rangle = \mathrm{B}_0 \qquad \mathcal{O}(S_{+0j})$$

$$\int \mathrm{B}_1 \Theta\left(t\left(S_{+1}\right) - t_{\mathrm{MS}}\right) \qquad \mathcal{O}(S_{+1j})$$

[1] JHEP 0111 (2001) 063 (Catani, Krauss, Kuhn, Webber), JHEP 0205 (2002) 046 (Lönnblad) ...

CKKW(-L)[1] merging

The CKKW-L prescription is:

- Calculate the tree-level MEs.
- Reweight with Sudakovs, α_{s}- and PDF-ratios.

$$\langle\mathcal{O}\rangle = \mathrm{B}_0 \quad \times \ \Pi_{S_{+0}}(\rho_0, t_{\mathrm{MS}}) \quad \mathcal{O}(S_{+0j})$$

$$\int \mathrm{B}_1 \Theta\left(t\left(S_{+1}\right) - t_{\mathrm{MS}}\right) w_f^0 w_{\alpha_{\mathrm{s}}}^0 \Pi_{S_{+0}}(\rho_0, \rho_1) \mathcal{O}(S_{+1j})$$

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Illustration

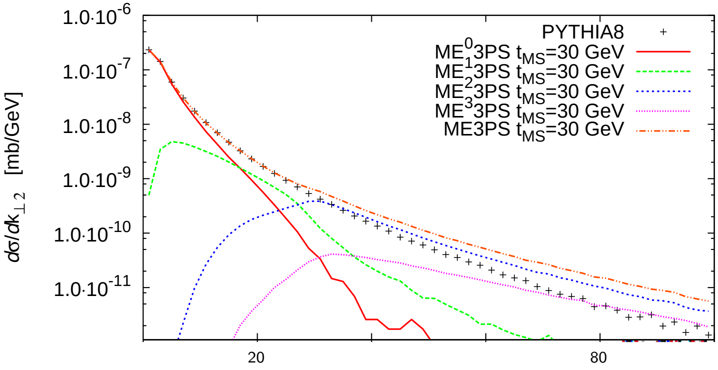

Figure: Separation between the first and second jet for W+jets, when clustering to exactly two jets. The coloured lines show the different reweighted multi-parton MEs.

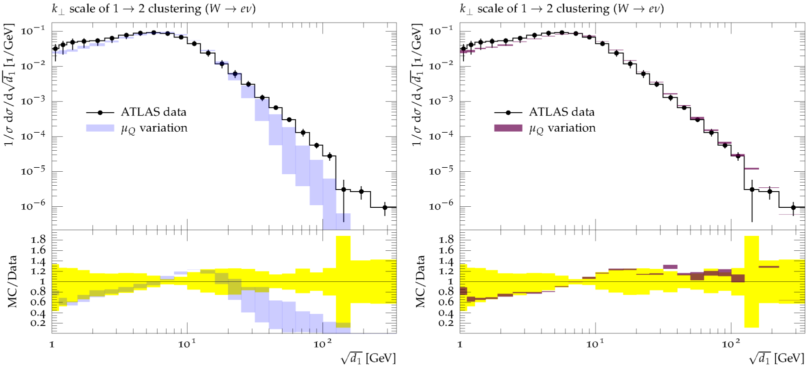

Figure: $k_\perp$-separation between the first and second jet for W+jets, when clustering to exactly two jets. The bands are obtained by varying the PS starting scale in $\mu_Q \in [\frac{1}{2}\mathrm{M_W}, 2\mathrm{M_W}]$

. . . however

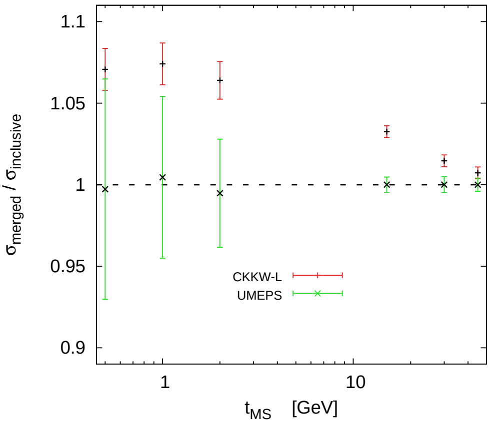

Figure: Ratio of the inclusive cross section after merging, compared to the tree-level inclusive cross section.

The problem with CKKW-L

The ME includes terms that are not compensated by the PS approximate virtual corrections (i.e. Sudakov factors).

These are the improvements that we need to describe multiple hard jets!
But if we simply add samples, the "improvements" will degrade the inclusive (0-jet) cross section!

Traditional approach: Don't use a too small merging scale.

$\rightarrow$ Uncancelled terms numerically not important.

Unitary approach[1]:

Use a (PS) unitarity inspired approach exactly cancel the dependence of the inclusive cross section on ρ_{MS}, thus preserving the inclusive cross section.

[1] JHEP1302(2013)094 (Leif Lönnblad, SP), JHEP1308(2013)114 (Simon Plätzer)

CKKW(-L)[1] merging

The CKKW-L prescription is:

- ◇ Calculate the tree-level MEs.
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- ◇ Start the PS on the reweighted ME configuration.
  Veto the event if the PS emission if it gives a state in the ME region.
- ◇ Combine by adding all accepted events.

$$\langle \mathcal{O} \rangle = \mathrm{B}_0 \quad \times \ \Pi_{S_{+0}}(\rho_0, t_{\mathrm{MS}}) \quad \mathcal{O}(S_{+0j})$$

$$+ \int \mathrm{B}_1 \Theta \left(t\left(S_{+1}\right) - t_{\mathrm{MS}} \right) w_f^0 w_{\alpha_s}^0 \Pi_{S_{+0}}(\rho_0, \rho_1) \mathcal{O}(S_{+1j})$$

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  Veto the event if the PS emission if it gives a state in the ME region.
- ◇ Combine by adding all accepted events.

$$\langle \mathcal{O} \rangle = \mathrm{B}_0 \left(1 - \int d\rho w_f^0 w_{\alpha_s}^0 P_0(\rho) \Pi_{S_{+0}}(\rho_0, \rho) \Theta\left(t\left(S_{+1}\right) - t_{\mathrm{MS}}\right) \right) \mathcal{O}(S_{+0j})$$

$$+ \int \mathrm{B}_1 \Theta\left(t\left(S_{+1}\right) - t_{\mathrm{MS}}\right) w_f^0 w_{\alpha_s}^0 \Pi_{S_{+0}}(\rho_0, \rho_1) \mathcal{O}(S_{+1j})$$

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The UMEPS prescription is:

- ◇ Calculate the tree-level MEs.
- ◇ Reweight with Sudakovs, α_s- and PDF-ratios.
- ◇ Process 1j-ME again: Reweight, then project onto 0-parton state.
- ◇ Start the PS on the reweighted ME configuration.
  Veto the PS emission if it gives a state in the ME region.
- ◇ Combine by adding reweighted events, and subtracting reweighted-projected events.

$$\langle\mathcal{O}\rangle = \mathrm{B}_0 \left(1 - \int d\rho w_f^0 w_{\alpha_s}^0 \frac{\mathrm{B}_1}{\mathrm{B}_0} \Pi_{S_{+0}}(\rho_0, \rho) \Theta\left(t\left(S_{+1}\right) - t_{\mathrm{MS}}\right)\right) \mathcal{O}(S_{+0j})$$

$$+ \int \mathrm{B}_1 \Theta\left(t\left(S_{+1}\right) - t_{\mathrm{MS}}\right) w_f^0 w_{\alpha_s}^0 \Pi_{S_{+0}}(\rho_0, \rho_1) \mathcal{O}(S_{+1j})$$

[1] JHEP1302(2013)094 (Leif Lönnblad, SP), JHEP1308(2013)114 (Simon Plätzer)

Comments on UMEPS

This sketch directly extends to the case of (α_s- or Sudakov-) weighted $+n$-jet states ($\widehat{\mathrm{B}}_n$), e.g. two-jet UMEPS merging:

$$
\begin{aligned}
\langle \mathcal{O} \rangle \quad = \quad & \int d\phi_0 \Big\{ \mathcal{O}(S_{+0j}) \left[\mathrm{B}_0 \; - \; \int_s \widehat{\mathrm{B}}_{1\to 0} \; - \; \int_s \widehat{\mathrm{B}}_{2\to 0} \right] \\
& + \int \mathcal{O}(S_{+1j}) \left[\widehat{\mathrm{B}}_1 \; - \; \int_s \widehat{\mathrm{B}}_{2\to 1} \right] \\
& + \int \cdots \int \mathcal{O}(S_{+2j}) \; \widehat{\mathrm{B}}_2 \Big\}
\end{aligned}
$$

The merging scale is finally a *technical* parameter, much in the same way that the shower cut-off (or `ptminsq` in the POWHEG-BOX) is.

Integrated version of the real-emission matrix elements are available anyway in MEPS since we need them to perform the Sudakov weighting.

The "subtract what you add" prescription means that this will produce *counter-events* with negative weight → Statistics might be problematic, if the integrations are done separately.

Unitarisation should be helpful for inclusive observables, heavy flavour (5F vs. 4F, 4F vs. 3F) treatments, MC tuning in the presence of merging,. . .

UMEPS results

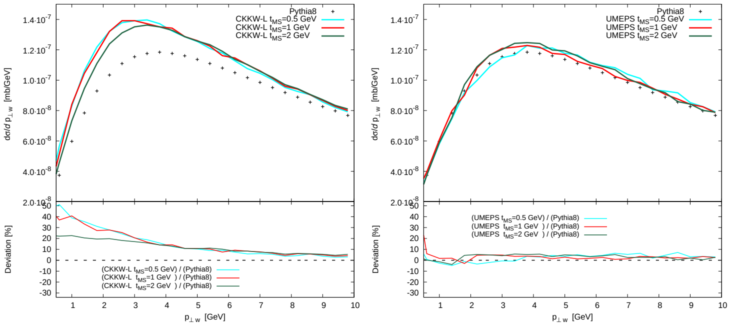


Figure: $p_\perp$ of the W-boson in the Sudakov region (for 2-jet merging, $E_{CM} = 7$ TeV)

⇒ CKKW-L overshoots for (very) low merging scales.

⇒ UMEPS describes the Sudakov peak nicely.

(For jet observables (high-$p_\perp$ tails etc.) UMEPS does as nicely as CKKW-L.)

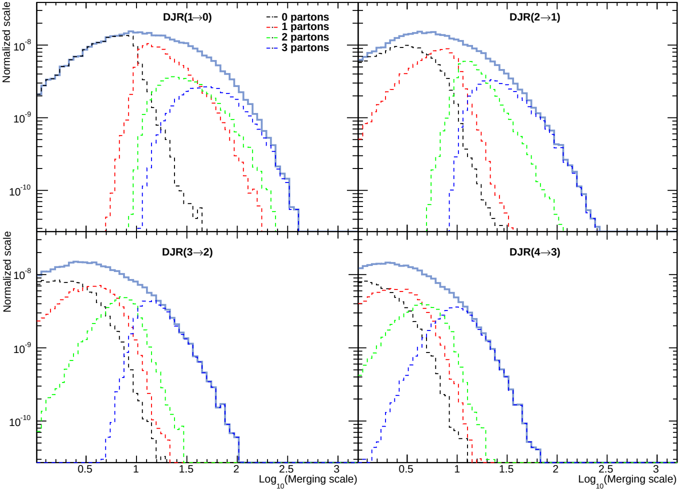
Jet matching in PYTHIA 8

PYTHIA 8 also includes MLM jet matching facilities. These include

- Alpgen-style MLM jet matching
- Madgraph-style kT-MLM jet matching
- New: Madgraph-style Shower-kT jet matching (to come soon)

Jet matching in PYTHIA 8

PYT clude



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-
- soon)

Is leading-order enough?

UMEPS is a leading-order method, i.e. it contains only approximate virtual corrections.

We want to use the full NLO results whenever possible.

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We want to use the full NLO results whenever possible.

Basic idea: Do NLO multi-jet merging for UMEPS:

- Subtract approximate UMEPS $\mathcal{O}(\alpha_s)$-terms, add back full NLO.
- To preserve the inclusive (NLO) cross section, add approximate NNLO.

⇒ UNLOPS[1].

For UNLOPS merging, we need exclusive NLO inputs:

$$\widetilde{\mathrm{B}}_n = \mathrm{B}_n + \mathrm{V}_n + \mathrm{I}_{n+1|n} + \int d\Phi_{\mathrm{rad}} \left(\mathrm{B}_{n+1|n} \Theta \left(\rho_{\mathrm{MS}} - t \left(S_{+n+1}, \rho \right) \right) - \mathrm{D}_{n+1|n} \right)$$

We can get these e.g. by massaging POWHEG-BOX output.

[1] JHEP1303(2013)166 (Leif Lönnblad, SP), Similar scheme in JHEP1308(2013)114 (Simon Plätzer)

The UNLOPS method

Start with UMEPS:

$$\langle \mathcal{O} \rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\mathrm{B}_0 + \quad - \int_s \widehat{\mathrm{B}}_{1\to 0} \quad - \int_s \widehat{\mathrm{B}}_{2\to 0} \Bigg)$$

$$+ \int \mathcal{O}(S_{+1j}) \Bigg(\quad \widehat{\mathrm{B}}_1 \quad - \int_s \widehat{\mathrm{B}}_{2\to 1} \quad \Bigg) + \int\!\!\int \mathcal{O}(S_{+2j}) \widehat{\mathrm{B}}_2 \Bigg\}$$

The UNLOPS method

Add full NLO results:

$$\langle \mathcal{O} \rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\mathrm{B}_0 + \widetilde{\mathrm{B}}_0 - \int_s \widetilde{\mathrm{B}}_{1\to 0} - \int_s \widehat{\mathrm{B}}_{1\to 0} - \int_s \widehat{\mathrm{B}}_{2\to 0} \Bigg)$$

$$+ \int \mathcal{O}(S_{+1j}) \Bigg(\widetilde{\mathrm{B}}_1 + \widehat{\mathrm{B}}_1 - \int_s \widehat{\mathrm{B}}_{2\to 1} \Bigg) + \int\!\!\int \mathcal{O}(S_{+2j}) \widehat{\mathrm{B}}_2 \Bigg\}$$

The UNLOPS method

Remove all unwanted $\mathcal{O}(\alpha_s^n)$- and $\mathcal{O}(\alpha_s^{n+1})$-terms:

$$\langle\mathcal{O}\rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\widetilde{\mathrm{B}}_0 - \int_s \widetilde{\mathrm{B}}_{1\to 0} + \int_s \mathrm{B}_{1\to 0} - \left[\int_s \widehat{\mathrm{B}}_{1\to 0} \right]_{-1,2} - \int_s \mathrm{B}^{\uparrow}_{2\to 0} - \int_s \widehat{\mathrm{B}}_{2\to 0} \Bigg)$$

$$+ \int \mathcal{O}(S_{+1j}) \Bigg(\widetilde{\mathrm{B}}_1 + \left[\widehat{\mathrm{B}}_1 \right]_{-1,2} - \left[\int_s \widehat{\mathrm{B}}_{2\to 1} \right]_{-2} \Bigg) + \int\!\!\int \mathcal{O}(S_{+2j}) \widehat{\mathrm{B}}_2 \Bigg\}$$

The UNLOPS method

UNLOPS merging of zero and one parton at NLO:

$$\langle \mathcal{O} \rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\widetilde{\mathrm{B}}_0 - \int_s \widetilde{\mathrm{B}}_{1\to 0} + \int_s \mathrm{B}_{1\to 0} - \left[\int_s \widehat{\mathrm{B}}_{1\to 0} \right]_{-1,2} - \int_s \mathrm{B}^{\uparrow}_{2\to 0} - \int_s \widehat{\mathrm{B}}_{2\to 0} \Bigg)$$

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Or for the case of M different NLO calculations, and N tree-level calculations:

$$\langle \mathcal{O} \rangle = \sum_{m=0}^{M-1} \int d\phi_0 \int \cdots \int \mathcal{O}(S_{+mj}) \Bigg\{ \widetilde{\mathrm{B}}_m + \left[\widehat{\mathrm{B}}_m \right]_{-m,m+1} + \int_s \mathrm{B}_{m+1\to m}$$

$$- \sum_{i=m+1}^{M} \int_s \widetilde{\mathrm{B}}_{i\to m} - \sum_{i=m+1}^{M} \left[\int_s \widehat{\mathrm{B}}_{i\to m} \right]_{-i,i+1} - \sum_{i=m+1}^{M} \int_s \mathrm{B}^{\uparrow}_{i+1\to m} - \sum_{i=M+1}^{N} \int_s \widehat{\mathrm{B}}_{i\to m} \Bigg\}$$

$$+ \int d\phi_0 \int \cdots \int \mathcal{O}(S_{+Mj}) \Bigg\{ \widetilde{\mathrm{B}}_M + \left[\widehat{\mathrm{B}}_M \right]_{-M,M+1} - \left[\int_s \widehat{\mathrm{B}}_{M+1\to M} \right]_{-M} - \sum_{i=M+1}^{N} \int_s \widehat{\mathrm{B}}_{i+1\to M} \Bigg\}$$

$$+ \sum_{n=M+1}^{N} \int d\phi_0 \int \cdots \int \mathcal{O}(S_{+nj}) \Bigg\{ \widehat{\mathrm{B}}_n - \sum_{i=n+1}^{N} \int_s \widehat{\mathrm{B}}_{i\to n} \Bigg\}$$

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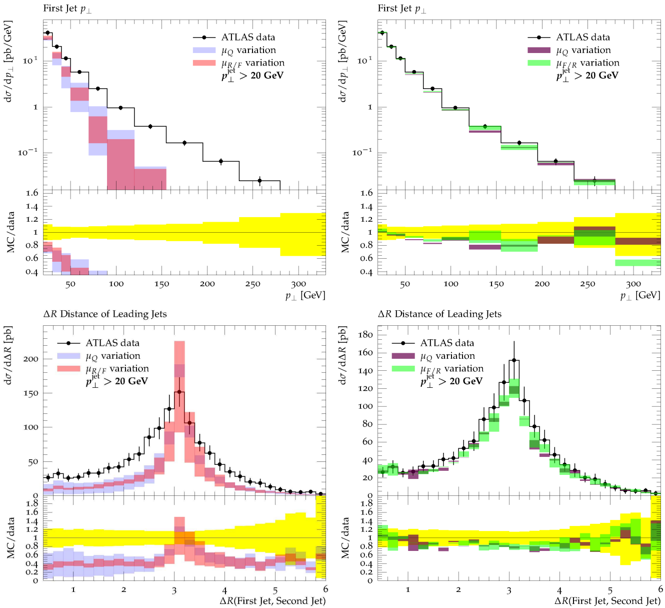
$$+ \sum_{n=M+1}^{N} \int d\phi_0 \int \cdots \int \mathcal{O}(S_{+nj}) \Bigg\{ \widehat{\mathrm{B}}_n - \sum_{i=n+1}^{N} \int_s \widehat{\mathrm{B}}_{i\to n} \Bigg\}$$

Inputs (B_n, $\overline{\mathrm{B}}_n$) taken from standard (external) tools.

Merging done internally in PYTHIA 8.

UNLOPS results (W+jets)



Inclusive sample containing (W + no resolved)@NLO, (W + one resolved)@NLO and (W + two resolved)@LO.

NLO merged results (H+jets, the real reason why we unitarise)

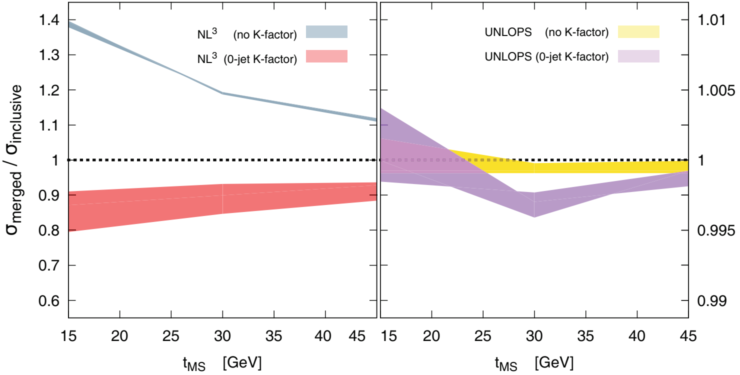


Figure: Ratio of the inclusive cross section for gg→H after merging (H+0)@NLO, (H+1)@NLO and (H+2)@LO, compared to the NLO inclusive cross section.

⇒ NL^3 (=CKKW-L@NLO) has problems for processes with large, loop-driven NLO corrections.

NLO merged results (squarks+jets)

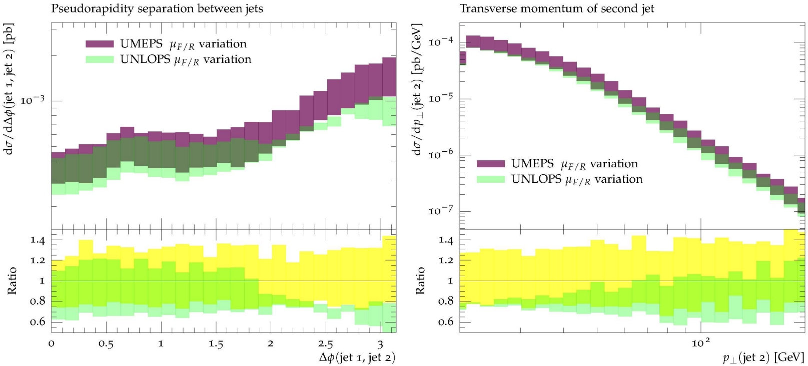


Figure: $\Delta\phi_{12}$ and $p_{\perp 2}$ for u-squark pair production ($m_{\widetilde{u}} = 500$ GeV, $m_{\chi_0} = 500$ GeV, BR($\widetilde{u} \rightarrow u\chi_0) \approx 1$) after merging (squarks+0)@NLO[1], (squarks+1)@LO and (squarks+2)@LO.

[1] arXiv:1305.4061 (Gavin, Hangst, Krämer, Mühlleitner, Pellen, Popenda, Spira)

Comparison to other schemes

Other public codes don't go through the trouble of unitarisation. Why?

FxFx: Restricts the range of merging scales. Violation numerically small.

Probably fewest counter events.

MEPS@NLO: Improved, colour-correct Sudakov of MC@NLO for the first emission. Small (non-logarithmic?) violation only comes from (R-D)-type events.

Improved resummation in process-independent way.

MiNLO: applies analytical NNLL Sudakov factors, which cancel the neccesary terms when merging two multiplicities.

Can be (and was) moulded into an NNLO matching.

So what's next? NNLO?

Note that in UNLOPS, the lowest-multiplicity input cross section is *not* reweighted. To go to NNLO, we have to identify terms of...

$$
\begin{aligned}
\langle \mathcal{O} \rangle \;=\; \int d\phi_0 \Bigg\{ & \mathcal{O}(S_{+0j}) \left(\widetilde{\mathrm{B}}_0 - \int_s \widetilde{\mathrm{B}}_{1\to 0} + \int_s \mathrm{B}_{1\to 0} - \left[\int_s \widehat{\mathrm{B}}_{1\to 0} \right]_{-1,2} \right. \\
& \left. - \int_s \mathrm{B}^{\uparrow}_{2\to 0} - \int_s \widehat{\mathrm{B}}_{2\to 0} \right) \\
& + \int \mathcal{O}(S_{+1j}) \left(\widetilde{\mathrm{B}}_1 + \left[\widehat{\mathrm{B}}_1 \right]_{-1,2} - \left[\int_s \widehat{\mathrm{B}}_{2\to 1} \right]_{-2} \right) \\
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\end{aligned}
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Note that in UNLOPS, the lowest-multiplicity input cross section is *not* reweighted. To go to NNLO, we have to identify terms of...

$$\langle\mathcal{O}\rangle = \int d\phi_0 \Big\{ \mathcal{O}(S_{+0j}) \Big(\widetilde{\mathrm{B}}_0 - \int_s \widetilde{\mathrm{B}}_{1\to 0} + \int_s \mathrm{B}_{1\to 0} - \Big[\int_s \widehat{\mathrm{B}}_{1\to 0}\Big]_{-1,2}$$

$$- \int_s \mathrm{B}^{\uparrow}_{2\to 0} - \int_s \widehat{\mathrm{B}}_{2\to 0} \Big)$$

$\mathcal{O}(\alpha_s)$

$$+ \int \mathcal{O}(S_{+1j}) \Big(\widetilde{\mathrm{B}}_1 + \Big[\widehat{\mathrm{B}}_1\Big]_{-1,2} - \Big[\int_s \widehat{\mathrm{B}}_{2\to 1}\Big]_{-2} \Big)$$

$$+ \int\int \mathcal{O}(S_{+2j})\, \widehat{\mathrm{B}}_2$$

So what's next? NNLO?

Note that in UNLOPS, the lowest-multiplicity input cross section is *not* reweighted. To go to NNLO, we have to identify terms of...

$$\langle\mathcal{O}\rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\widetilde{\mathrm{B}}_0 - \int_s \widetilde{\mathrm{B}}_{1\to 0} + \int_s \mathrm{B}_{1\to 0} - \left[\int_s \widehat{\mathrm{B}}_{1\to 0}\right]_{-1,2}$$

$$\mathcal{O}(\alpha_s) \qquad - \int_s \mathrm{B}^{\uparrow}_{2\to 0} - \int_s \widehat{\mathrm{B}}_{2\to 0} \Bigg)$$

$$\mathcal{O}(\alpha_s^2) \qquad + \int \mathcal{O}(S_{+1j}) \Bigg(\widetilde{\mathrm{B}}_1 + \left[\widehat{\mathrm{B}}_1\right]_{-1,2} - \left[\int_s \widehat{\mathrm{B}}_{2\to 1}\right]_{-2} \Bigg)$$

$$+ \int\!\!\int \mathcal{O}(S_{+2j})\, \widehat{\mathrm{B}}_2$$

Terms entering due to PS weights are $\mathcal{O}(\alpha_s^2)_{\mathrm{PS}} \times \mathcal{O}(\alpha_s^1)_{\mathrm{ME}}$ and $\mathcal{O}(\alpha_s^0)_{\mathrm{PS}} \times \mathcal{O}(\alpha_s^2)_{\mathrm{ME}}$

So what's next? NNLO?

Note that in UNLOPS, the lowest-multiplicity input cross section is *not* reweighted. To go to NNLO, we have to identify terms of...

$$
\begin{aligned}
\langle \mathcal{O} \rangle \quad = \quad \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(& \qquad\qquad\qquad\qquad \left[\int_s \widehat{\mathrm{B}}_{1\rightarrow 0} \right]_{-1,2} \\
& - \left[\int_s \widehat{\mathrm{B}}_{2\rightarrow 0} \right]_{-2} \Bigg) \\
& + \int \mathcal{O}(S_{+1j}) \left(\widetilde{\mathrm{B}}_1 + \left[\widehat{\mathrm{B}}_1 \right]_{-1,2} - \left[\int_s \widehat{\mathrm{B}}_{2\rightarrow 1} \right]_{-2} \right) \\
& + \int\!\!\int \mathcal{O}(S_{+2j})\, \widehat{\mathrm{B}}_2
\end{aligned}
$$

... now remove these terms

So what's next? NNLO?

Note that in UNLOPS, the lowest-multiplicity input cross section is *not* reweighted. To go to NNLO, we have to identify terms of...

$$
\begin{aligned}
\langle \mathcal{O} \rangle \quad = \quad \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\widetilde{\widetilde{\mathrm{B}}}_0 \qquad\qquad\qquad & \left[\int_s \widehat{\mathrm{B}}_{1\to 0} \right]_{-1,2} \\
- \left[\int_s \widehat{\mathrm{B}}_{2\to 0} \right]_{-2} \Bigg) & \\
+ \int \mathcal{O}(S_{+1j}) \left(\widetilde{\mathrm{B}}_1 + \left[\widehat{\mathrm{B}}_1 \right]_{-1,2} - \left[\int_s \widehat{\mathrm{B}}_{2\to 1} \right]_{-2} \right) & \\
+ \int\!\!\int \mathcal{O}(S_{+2j})\, \widehat{\mathrm{B}}_2 &
\end{aligned}
$$

...now remove these terms ...and instead use the exclusive NNLO result.

The matching formula at NNLO is:

$$
\begin{aligned}
\langle\mathcal{O}\rangle \;=\; \int d\phi_0 \Bigg\{ & \mathcal{O}(S_{+0j}) \left(\widetilde{\widetilde{\mathrm{B}}}_0 \;-\; \left[\int_s \widehat{\mathrm{B}}_{1\to 0} \right]_{-1,2} \;-\; \left[\int_s \widehat{\mathrm{B}}_{2\to 0} \right]_{-2} \right) \\
& + \int \mathcal{O}(S_{+1j}) \left(\widetilde{\mathrm{B}}_1 + \left[\widehat{\mathrm{B}}_1 \right]_{-1,2} \;-\; \left[\int_s \widehat{\mathrm{B}}_{2\to 1} \right]_{-2} \right) \\
& + \int\!\!\int \mathcal{O}(S_{+2j})\, \widehat{\mathrm{B}}_2 \;\Bigg\}
\end{aligned}
$$

The inclusive cross section is

$$
\int d\phi_0 \mathcal{O}(S_{+0j}) \left(\widetilde{\widetilde{\mathrm{B}}}_0 \;+\; \int_s \mathrm{B}_{2\to 0} \right) \;+\; \int d\phi_0 \int \mathcal{O}(S_{+1j}) \left(\widetilde{\mathrm{B}}_1 \;+\; \int_s \mathrm{B}_{2\to 1} \right)
$$

which is just the NNLO result, provided that $\widetilde{\widetilde{\mathrm{B}}}_0$ does not contain phase space regions with jet separations above ρ_{MS} (see back-up).

NNLO with UNLOPS

Alternatively, we can use the inclusive cross section for NNLO matching:

$$\langle\mathcal{O}\rangle = \int d\phi_0 \Bigg\{ \mathcal{O}(S_{+0j}) \Bigg(\overline{\overline{\mathrm{B}}}_0 - \int_s \overline{\mathrm{B}}_{1\to 0} - \Big[\int_s \widehat{\mathrm{B}}_{1\to 0}\Big]_{-1,2} - \Big[\int_s \widehat{\mathrm{B}}_{2\to 0}\Big]_{-2} \Bigg)$$
$$+ \int \mathcal{O}(S_{+1j}) \Bigg(\overline{\mathrm{B}}_1 + \Big[\widehat{\mathrm{B}}_1\Big]_{-1,2} - \int_s \widehat{\mathrm{B}}_{2\to 1} \Bigg)$$
$$+ \int\int \mathcal{O}(S_{+2j})\, \widehat{\mathrm{B}}_2 \Bigg\}$$

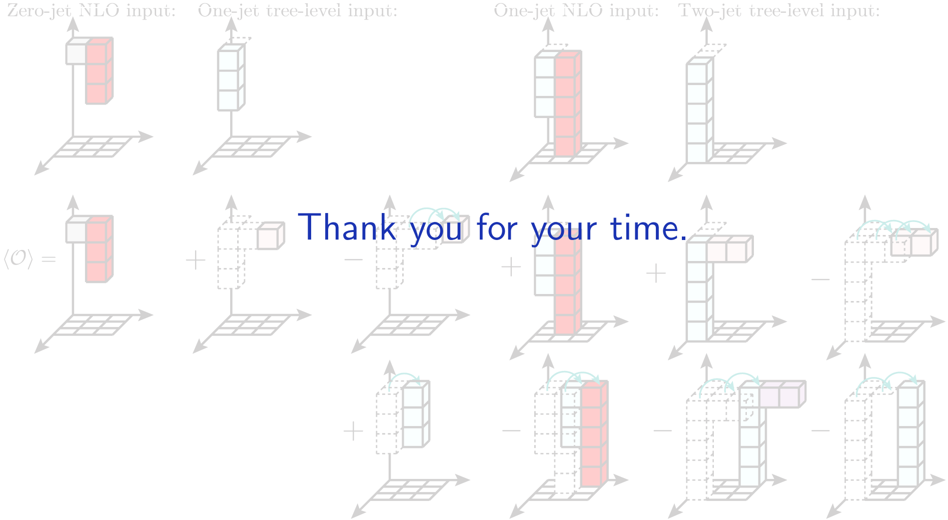
- Relatively clean only because the NLO cross section was not reweighted. Process-independent.
- ρ_{MS}-definition must be infrared safe at NNLO[1].
- Needs NNLO generator to produce $\widetilde{\widetilde{\mathrm{B}}}_0$ or $\overline{\overline{\mathrm{B}}}_0$. For $\widetilde{\widetilde{\mathrm{B}}}_0$, the ρ_{MS}-definition has to be known within the NNLO calculation. Otherwise, $\overline{\overline{\mathrm{B}}}_0$ should best be fully differential, so that the PS can project onto 0-parton states.

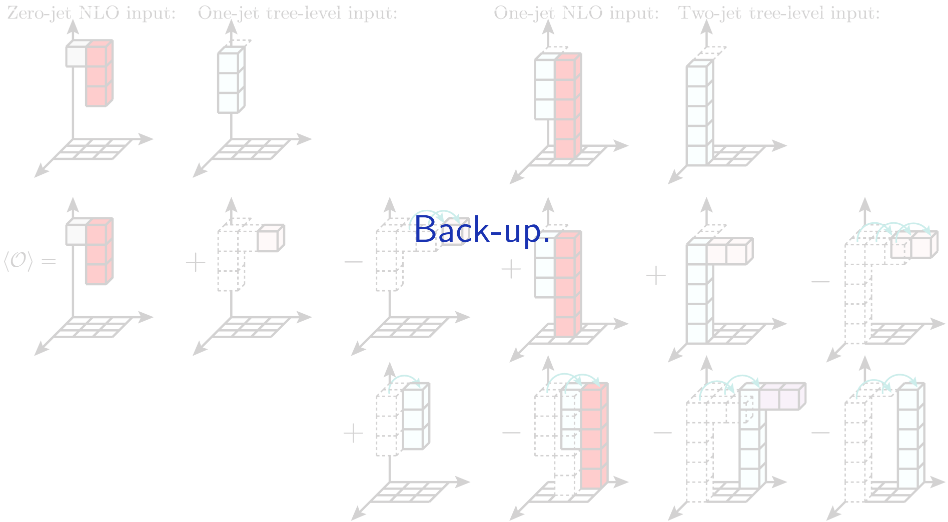
[1] see e.g. discussion in arXiv:1311.0286 (Alioli, Bauer, Berggren, Tackmann, Walsh, Zuberi)

Summary

- To describe data, we need to infuse parton showers with matrix elements.
- CKKW-L tree-level merging is included in PYTHIA 8.
  But it does not work for very small merging scales.
- UMEPS tree-level merging is included in PYTHIA 8.
  UMEPS almost cancels the merging scale dependence.
  But it's not NLO.
- Two NLO merging schemes are implemented in PYTHIA 8:
  NL^3 and UNLOPS. The latter is our preferred choice.
- All merging schemes in PYTHIA 8 run on LHEF input, e.g. POWHEG-BOX or MADEVENT input. aMC@NLO in testing.
- PYTHIA 8 doesn't have to be the "bad example" if you feed it LHEFs
  . . . if you use these features!

Thank you for your time.





Zero-jet NLO input:
One-jet tree-level input:
One-jet NLO input:
Two-jet tree-level input:
$\langle \mathcal{O} \rangle =$

Back-up.

$$
\begin{aligned}
w_n =& \frac{x_n^+ f_n^+(x_n^+, \rho_n)}{x_n^+ f_n^+(x_n^+, \mu_F)} \frac{x_n^- f_n^-(x_n^-, \rho_n)}{x_n^- f_n^-(x_n^-, \mu_F)} \\
& \times \prod_{i=1}^{n} \left[\frac{\alpha_s(\rho_i)}{\alpha_s(\mu_R)} \frac{x_{i-1}^+ f_{i-1}^+(x_{i-1}^+, \rho_{i-1})}{x_{i-1}^+ f_{i-1}^+(x_{i-1}^+, \rho_i)} \frac{x_{i-1}^- f_{i-1}^-(x_{i-1}^-, \rho_{i-1})}{x_{i-1}^- f_{i-1}^-(x_{i-1}^-, \rho_i)} \Pi_{S_{+i-1}}(x_{i-1}, \rho_{i-1}, \rho_i) \right] \\
\widehat{\mathrm{B}}_n =& \mathrm{B}_n w_n
\end{aligned}
$$

$$
\int_s \widehat{\mathrm{B}}_{n\to m} = \left[\prod_{a=m+1}^{n-1} \int d\rho_a dz_a d\varphi_a \Theta(\rho_{\mathrm{MS}} - \rho_a) \right] \int d\rho_n dz_n d\varphi_n \mathrm{B}_n w_n
$$

$$
\langle \mathcal{O} \rangle = \sum_{n=0}^{N} \int d\phi_0 \int \dots \int \mathcal{O}(S_{+nj}) \left\{ \widehat{\mathrm{B}}_n - \sum_{i=n+1}^{N} \int_s \widehat{\mathrm{B}}_{i\to n} \right\} .
$$

In CKKW-L, w_n contains an additonal factor $\Pi_{S_{+n}}(x_n, \rho_n, \rho_{\mathrm{MS}})$.
UMEPS induces this through $\int_s \widehat{\mathrm{B}}_{n+1\to n}$ instead.

Here, we illustrate how to include the full kinematical information also below the merging scale.

$$d\sigma^u_{+0} \equiv \widehat{\mathrm{B}}_{+0} - \int_s \widehat{\mathrm{B}}_{0+1\to 0} \qquad d\sigma^u_{+1} \qquad \equiv \widehat{\mathrm{B}}_{+1}$$

So far, we have

$$\begin{aligned}\langle\mathcal{O}\rangle = & d\sigma^u_{+0} \times [\Delta(\rho_0,\rho_c)\mathcal{O}_0(S_{+0}) \;+\; \Delta(\rho_{\mathrm{MS}},\rho_1)d\rho_1 dz_1 P(\rho_1,z_1) \times (\Delta(\rho_1,\rho_c)\mathcal{O}_1(S_{+1}) \;+\ldots)] \\ & + d\sigma^u_{+1} \times [\Delta(\rho_1,\rho_c)\mathcal{O}_1(S_{+1}) \;+\; \Delta(\rho_1,\rho_2)d\rho_2 dz_2 P(\rho_2,z_2) \times (\Delta(\rho_2,\rho_c)\mathcal{O}_2(S_{+2}) \;+\ldots)] \\ & \qquad \times \Theta(\rho_1 - \rho_{\mathrm{MS}})\end{aligned}$$

But we could write

$$\begin{aligned}\langle\mathcal{O}\rangle = & d\sigma^u_{+0} \times [\Delta(\rho_0,\rho_c)\mathcal{O}_0(S_{+0}) \;+\; \Delta(\rho_{\mathrm{MS}},\rho_1)d\rho_1 dz_1 P(\rho_1,z_1) \times (\Delta(\rho_1,\rho_c)\mathcal{O}_1(S_{+1}) \qquad)] \\ & + d\sigma^u_{+1} \times [\Delta(\rho_1,\rho_c)\mathcal{O}_1(S_{+1}) \;+\; \Delta(\rho_1,\rho_2)d\rho_2 dz_2 P(\rho_2,z_2) \times (\Delta(\rho_2,\rho_c)\mathcal{O}_2(S_{+2}) \;+\ldots)] \\ & \qquad \times \Theta(\rho_1 - \rho_{\mathrm{MS}}) \\ & + d\sigma^d_{+1} \times [\Delta(\rho_1,\rho_c)\mathcal{O}_1(S_{+1}) \;+\; \Delta(\rho_1,\rho_2)d\rho_2 dz_2 P(\rho_2,z_2) \times (\Delta(\rho_2,\rho_c)\mathcal{O}_2(S_{+2}) \;+\ldots)] \\ & \qquad \times \Theta(\rho_{\mathrm{MS}} - \rho_1) \\ & - d\sigma^u_{+0} \times \Delta(\rho_{\mathrm{MS}},\rho_1)d\rho_1 dz_1 P(\rho_1,z_1) \times \Delta(\rho_1,\rho_c)\mathcal{O}_1(S_{+1}) \\ & - \int \Big\{ d\sigma^d_{+1} \times [\Delta(\rho_1,\rho_c) \;+\; \Delta(\rho_1,\rho_2)d\rho_2 dz_2 P(\rho_2,z_2) \times (\Delta(\rho_2,\rho_c) \;+\ldots)] \times \Theta(\rho_{\mathrm{MS}} - \rho_1) \\ & - d\sigma^u_{+0} \times \Delta(\rho_{\mathrm{MS}},\rho_1)d\rho_1 dz_1 P(\rho_1,z_1) \times \Delta(\rho_1,\rho_c)\Big\}\mathcal{O}_0(S_{+0})\end{aligned}$$

UMEPS ' = UMEPS

This is just

$$\begin{aligned}\langle\mathcal{O}\rangle = &\, d\sigma^{u}_{+0}\mathcal{O}_0(S_{+0}) - \int d\sigma^{d}_{+1} \times \Theta(\rho_{\mathrm{MS}} - \rho_1)\mathcal{O}_0(S_{+0}) \\ &+ d\sigma^{u}_{+1} \times [\Delta(\rho_1,\rho_c)\mathcal{O}_1(S_{+1}) \; + \; \Delta(\rho_1,\rho_2)d\rho_2 dz_2 P(\rho_2,z_2) \times (\Delta(\rho_2,\rho_c)\mathcal{O}_2(S_{+2}) \; + \ldots)] \\ &\qquad \times \Theta(\rho_1 - \rho_{\mathrm{MS}}) \\ &+ d\sigma^{d}_{+1} \times [\Delta(\rho_1,\rho_c)\mathcal{O}_1(S_{+1}) \; + \; \Delta(\rho_1,\rho_2)d\rho_2 dz_2 P(\rho_2,z_2) \times (\Delta(\rho_2,\rho_c)\mathcal{O}_2(S_{+2}) \; + \ldots)] \\ &\qquad \times \Theta(\rho_{\mathrm{MS}} - \rho_1)\end{aligned}$$

which for $d\sigma^{d}_{+1} = d\sigma^{u}_{+1}$ becomes

$$\begin{aligned}\langle\mathcal{O}\rangle = &\, d\sigma^{u}_{+0}\mathcal{O}_0(S_{+0}) \\ &+ d\sigma^{u}_{+1} \times [\Delta(\rho_1,\rho_c)\mathcal{O}_1(S_{+1}) \; + \; \Delta(\rho_1,\rho_2)d\rho_2 dz_2 P(\rho_2,z_2) \times (\Delta(\rho_2,\rho_c)\mathcal{O}_2(S_{+2}) \; + \ldots)] \\ &\qquad \times \Theta(\rho_1 - \rho_c)\end{aligned}$$

i.e. simply the UMEPS result for $\rho_{\mathrm{MS}} = \rho_c$.
Note that this does not depend on the form of $d\sigma^{u}_{+0}$ or $d\sigma^{u}_{+1}$, and can thus also be applied for UNLOPS to add real-emission kinematics with $\rho(S_{+n+1}) < \rho_{\mathrm{MS}}$.

Extreme merging scale values

In unitarised merging, t_{MS} can be varied between the PS cut-off ρ_c and ∞. The t_{MS} dependence is almost exactly cancelled (caveat: jet definition)

For $t_{\mathrm{MS}} \to \infty$, the spectrum of the first add. emission is given by the PS. The below-t_{MS} behaviour can be fixed[1], leading e.g. to the following form:

$$\langle \mathcal{O} \rangle = \left[d\sigma^u_{+0} - \int d\sigma^d_{+1} \times \Theta(\rho_{\mathrm{MS}} - \rho_1) \right] \mathcal{O}_0(S_{+0}) + \mathbf{PS}\left[d\sigma^u_{+1} \Theta(\rho_1 - \rho_{\mathrm{MS}}) \right] + \mathbf{PS}\left[d\sigma^d_{+1} \Theta(\rho_{\mathrm{MS}} - \rho_1) \right]$$

which for $d\sigma^d_{+1} = d\sigma^u_{+1}$ becomes

$$\langle \mathcal{O} \rangle = d\sigma^u_{+0} \mathcal{O}_0(S_{+0}) + \mathbf{PS}\left[d\sigma^u_{+1} \Theta(\rho_1 - \rho_c) \right]$$

i.e. simply the UMEPS result for $\rho_{\mathrm{MS}} = \rho_c$.

The merging scale is a *technical* parameter, much in the same way that the shower cut-off (or `ptminsq`/`hfact` in the POWHEG-BOX) is.

[1] See also arXiv:1211.5467 (Simon Plätzer)

How do we get $\widetilde{\mathrm{B}}_n$?

Like everyone else, we need exclusive NLO cross sections as input:

$$\widetilde{\mathrm{B}}_n = \mathrm{B}_n + \mathrm{V}_n + \mathrm{I}_{n+1|n} + \int d\Phi_{\mathrm{rad}} \left(\mathrm{B}_{n+1|n}\Theta\left(\rho_{\mathrm{MS}} - t\left(S_{+n+1}, \rho\right)\right) - \mathrm{D}_{n+1|n}\right)$$

First, observe that

$$\widetilde{\mathrm{B}}_n = \mathrm{B}_n + \mathrm{V}_n + \mathrm{I}_{n+1|n} + \int d\Phi_{\mathrm{rad}} \left(\mathrm{B}_{n+1|n} - \mathrm{D}_{n+1|n}\right) - \int d\Phi_{\mathrm{rad}} \mathrm{B}_{n+1|n}\Theta\left(t\left(S_{+n+1}, \rho\right) - \rho_{\mathrm{MS}}\right)$$
$$= \overline{\mathrm{B}}_n - \int_{\rho_{\mathrm{MS}}} d\Phi_{\mathrm{rad}} \mathrm{B}_{n+1|n}$$

and remember that the POWHEG-BOX produces

$$\overline{B}_n \Delta(p_{\perp min}) + \overline{B}_n \frac{B_{n+1}}{B_n} \Delta(p_\perp) = \overline{B}_n \left[1 - \int_{p_\perp min} \frac{B_{n+1}}{B_n} \Delta(p_\perp)\right] + \overline{B}_n \frac{B_{n+1}}{B_n} \Delta(p_\perp)$$

Thus, if we project onto an underlying Born for radiative events, we get $\overline{B}_n$.
By having a subtraction sample $\int_{\rho_{\mathrm{MS}}} d\Phi_{\mathrm{rad}} \mathrm{B}_{n+1|n}$, we get $\widetilde{\mathrm{B}}_n$.

What is $\widetilde{\widetilde{\mathrm{B}}}_0$?

For UNLOPS@NNLO, $\widetilde{\widetilde{\mathrm{B}}}_0$ is given by

$$\widetilde{\widetilde{\mathrm{B}}}_0 \mathcal{O}_{+0} = \left[\overline{\overline{\mathrm{B}}}_0 - \overline{\mathrm{B}}_1 \Theta \left(\rho \left(S_{+1}, \rho \right) - \rho_{\mathrm{MS}} \right) \right] \mathcal{O}_{+0}$$

where we need to project

$$\begin{aligned}
\overline{\overline{\mathrm{B}}}_0 \quad = & \quad \left[\mathrm{B}_0 + \mathrm{V}_0 + \int \mathrm{D}_{1|0} \right] \mathcal{O}_{+0} + \int \left[\mathrm{B}_1 \mathcal{O}_{+1} - \mathrm{D}_{1|0} \mathcal{O}_{+0} \right] \\
+ & \quad \left[\mathrm{W}_0 + \iint \mathrm{G}_{2|0} + \iint (1-h) \mathrm{H}_{2|0} \right] \mathcal{O}_{+0} \\
+ & \quad \int \left[\mathrm{V}_1 \mathcal{O}_{+1} + \int \mathrm{D}_{2|1} \mathcal{O}_{+1} - \int \mathrm{E}_{2|0} \mathcal{O}_{+0} + \int \mathrm{F}_{2|0} \mathcal{O}_{+0} + \int h \mathrm{H}_{2|0} \mathcal{O}_{+0} \right] \\
+ & \quad \iint \left[\mathrm{B}_2 \mathcal{O}_{+2} - \mathrm{D}_{2|1} \mathcal{O}_{+1} + \mathrm{E}_{2|0} \mathcal{O}_{+0} - \mathrm{F}_{2|0} \mathcal{O}_{+0} - \mathrm{G}_{2|0} \mathcal{O}_{+0} - \mathrm{H}_{2|0} \mathcal{O}_{+0} \right]
\end{aligned}$$

onto 0-parton states.

$\mathrm{E}_{2|0}$ cancels spurious div's from integrating $\mathrm{D}_{2|1}$ over the full 2-parton PS;
$\mathrm{F}_{2|0}$ cancels double-unresolved div's in $\iint \mathrm{B}_2$ that would otherwise cancel against $\int \mathrm{V}_1$;
$\mathrm{G}_{2|0}$ cancels double-unresolved div's in $\iint \mathrm{B}_2$ that would cancel against W_0;
$\mathrm{H}_{2|0}$ cancels div's that are genuinely shared between $\iint \mathrm{B}_2$, $\int \mathrm{V}_1$ and W_0 (if present), or other spurious div's (if existing).

What is $\tilde{\tilde{\mathrm{B}}}_0$?

With $\Theta_n(>) = \Theta(t(S_{+n},\rho) - \rho_{\mathrm{MS}})$ and $\Theta_n(<) = \Theta(\rho_{\mathrm{MS}} - t(S_{+n},\rho))$, we find

$$
\begin{aligned}
\tilde{\tilde{\mathrm{B}}}_0 \quad = \quad & \left[\mathrm{B}_0 + \mathrm{V}_0 + \int \mathrm{D}_{1|0}\right]\mathcal{O}_{+0} \\
+ \quad & \int \left[\mathrm{B}_1\Theta_1(<)\,\mathcal{O}_{+1} - \mathrm{D}_{1|0}\mathcal{O}_{+0}\right] + \left[\mathrm{W}_0 + \iint \mathrm{G}_{2|0} + \iint (1-h)\mathrm{H}_{2|0}\right]\mathcal{O}_{+0} \\
+ \quad & \int \left[\mathrm{V}_1\Theta_1(<)\,\mathcal{O}_{+1} + \int \mathrm{D}_{2|1}\Theta_1(<)\,\mathcal{O}_{+1} - \int \mathrm{E}_{2|0}\mathcal{O}_{+0} + \int \mathrm{F}_{2|0}\mathcal{O}_{+0} + \int h\mathrm{H}_{2|0}\right. \\
+ \quad & \iint \left[\mathrm{B}_2\Theta_1(<)\,\Theta_2(>)\,\mathcal{O}_{+2} + \mathrm{B}_2\Theta_1(<)\,\Theta_2(<)\,\mathcal{O}_{+2} - \mathrm{D}_{2|1}\Theta_1(<)\,\mathcal{O}_{+1}\right. \\
& \left. + \mathrm{E}_{2|0}\mathcal{O}_{+0} - \mathrm{F}_{2|0}\mathcal{O}_{+0} - \mathrm{G}_{2|0}\mathcal{O}_{+0} - \mathrm{H}_{2|0}\mathcal{O}_{+0}\right]
\end{aligned}
$$

Note the term $\mathrm{B}_2\Theta_1(<)\,\Theta_2(>)\,\mathcal{O}_{+2}$, which contains a state S_{+2} with two resolved partons, but no state with only one resolved parton. Alternatively, we could have defined $\tilde{\tilde{\mathrm{B}}}_0$ without this term. Then, it would be, in an NNLO-matched calculation, be included e.g. through $\int_s \widehat{\mathrm{B}}_{2\to 0}$. This is how we defined $\tilde{\tilde{\mathrm{B}}}_0$ in the main text. Above, we also defined the sum $\overline{\overline{\mathrm{B}}}_0 - \overline{\mathrm{B}}_1$ to not contain such terms.