Multi-Jet production at next-to-leading order with NJet

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Physics at the LHC

High multiplicity processes

Large rate of QCD processes

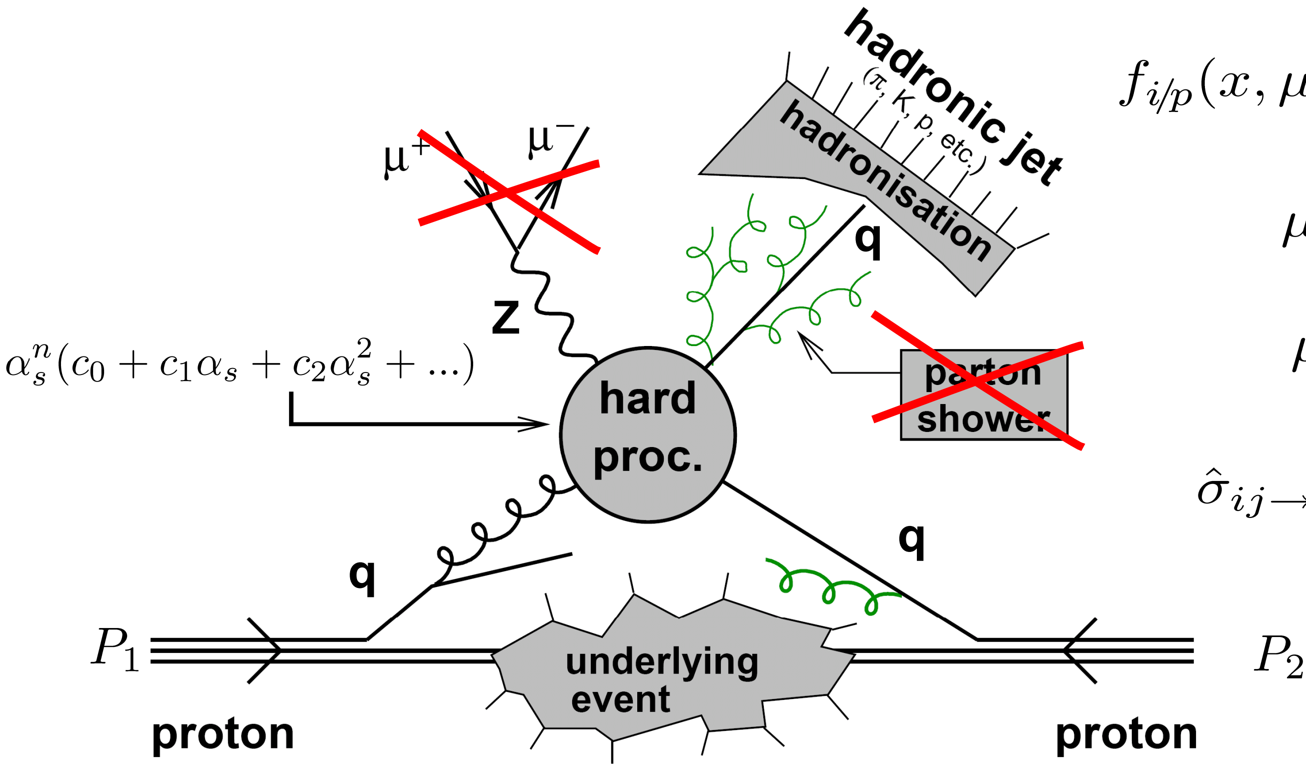
⟶ Interesting proceses often in association with Jets

Precise predictions of high multiplicity processes within the Standard Model crucial for a successful data analysis:

⟶ Simulation of the background

⟶ Constraining SM parameters (e.g. α_s from jet ratios)

Physics at the LHC



$f_{i/p}(x, \mu_f)$ parton distribution function (PDF)

μ_f factorisation scale

μ_r renormalisation scale

$\hat{\sigma}_{ij \to X}$ partonic cross section

$$\sigma(pp \to X) = \sum_{\substack{i,j \in \\ \{g,q,\bar{q}\}}} \int dx_1 dx_2 f_{i/p}(x_1, \mu_f) f_{j/p}(x_2, \mu_f) \hat{\sigma}_{ij \to X}(x_1 P_1, x_2 P_2, \mu_f, \mu_r)$$

Figure: Butterworth&Dissertori&Salam

In this talk: pure QCD-processes in fixed order α_s

Need for NLO accuracy

Leading order…

- well automated, tools like Madgraph, Sherpa, Alpgen for the **hard processes at tree-level**
- multiplicities of up to 12 jets possible

However…

- large uncertainty due to residual scale dependence
- QCD corrections in general large O(20 – x %)

For precision (jet-) physcis, next-to-leading order (NLO) accuracy required

Multi-Jet Production @ NLO

2-jet production:	[Ellis, Kunszt, Soper 1992] [Giele, Glover, Kosower 1993]	
3-jet production:	[Nagy 2002, 2003]	(all channels)
	[Trocsanyi 1996]	(gluon channel)
	[Kilgore, Giele 1997]	(gluon channel)
4-jet production:	[Bern, Diana, Dixon, Febres Cordero, Höche, Kosower, Ita, Maitre, Ozeren 2011] [Badger, BB, Uwer, Yundin 2012]	
5-jet production:	[Badger, BB, Uwer, Yundin 2013]	

pp $\rightarrow n$ jets @ NLO

$$d\sigma_n = \underbrace{d\sigma_n^{\mathrm{LO}}}_{\sim \alpha_s^n} + \underbrace{\delta d\sigma_n^{\mathrm{NLO}}}_{\sim \alpha_s^{n+1}} + O(\alpha_s^{n+2})$$

$$\delta\sigma_n^{\mathrm{NLO}} = \int_n d\sigma_n^{\mathrm{virt.}} \oplus \int_{n+1} d\sigma_{n+1}^{\mathrm{reell}}$$

- Factorisation of the initial state singularities into renormalised PDFs
- Choosing a suitable subtraction scheme ([Catani, Seymour 1996])

$$\delta\sigma_n^{\mathrm{NLO}} = \int_n d\bar{\sigma}_n^V + \int_n d\bar{\sigma}^I + \int_{n+1} d\bar{\sigma}_{n+1}^{RS}$$

Real Corrections with subtraction terms

Finite part of the virtual corrections	Finite contributions from subtraction scheme and Factorisation
NJet	**Sherpa with Comix**

Virtual Corrections

Rapid growth in complexity with increasing number of external legs

challenging for traditional Feynman diagram based methods

e.g. 6-gluon one-loop amplitude around 15’000 diagrams

7-gluon one-loop amplitude around 144’000 diagrams

→ use method of **generalised unitarity**

[Bern, Dixon, Dunbar, Kosower, Ossola, Papdopoulos, Pittau, Kunszt, Giele, Melnikov, Anastasiou, Britto, Feng, Mastroglia, Badger,…]

Ansätze for automation of virtual corrections:

`NGluon, NJet` [Badger, BB, Uwer, Yundin] ***public code***

And various others…

GoSam, Golem95, Helac-NLO, Blackhat, MadGraph5_aMC@NLO, OpenLoops, RECOLA, “numerical loop integration”…

Most of the codes meanwhile publicly available

Amplitude Structure

Colour summed squared matrix element

$$\sim 2\mathrm{Re}\left[(\mathcal{M}_{\mathrm{Born}})^{*}\cdot\mathcal{M}_{\text{1-loop}}\right]$$

$$\sim \left(\vec{A}^{\mathrm{tree}}_{\mathrm{prim}}\right)^{\dagger}\hat{\mathcal{C}}\,\vec{A}^{\text{1-loop}}_{\mathrm{prim}}$$

Full colour amplitude

$$\sim \mathcal{M}(\{p_i, a_i, h_i\}) \quad i \in \{1, ..., n\}$$

Partial amplitude

$$\sim C(a_1, \ldots, a_n)\,\mathcal{A}_n(\{p_1, h_1\}, \ldots, \{p_n, h_n\})$$

Primitive

$$\mathcal{A}_n(\{p_1, h_1\}, \ldots, \{p_n, h_n\}) = \sum_{P(\sigma)} A_{\mathrm{prim}}(\sigma(1, ..., n))$$

[Bern, Dixon, Dunbar, Kosower 1994]
[Bern, Dixon, Kosower 1995]
[Ellis, Kunszt, Melnikov, Zanderighi 2011]
[Schuster 2013, Weinzierl 2013]

NJet in a nutshell

Based on **NGluon** [Badger, BB, Uwer 2011]

generalised Unitarity with tree-level amplitudes as input
to compute 1-loop primitive amplitudes with **arbitrarily many legs**

Colour dressed Berends-Giele recursion for tree-level input [Berends,Giele 1986]

provides **full colour** summed 1-loop amplitudes interfered with the
Born for all channels of **2-jet, 3-jet, 4-jet** and **5-jet** production
in **massless QCD**. (Employs colour algorithm from [Ellis, Kunszt, Melnikov, Zanderighi])

Extended version **NJet 2.0** now available with primitive amplitudes including
vector bosons (W, Z, γ).
Full colour sums for **vector bosons with up to five jets, di-photon production with up to four jets**.
For first phaenomenological study of di-photon + 3 jets with NJet
see **arXiv:1312.5927**. [Badger, Guffanti, Yundin 2013]

NJet in a nutshell

Equipped with **Binoth Les Houches accord interface(s)** [Binoth et al. 2009, Alioli et al. 2013] to be linked trivially with standard Monte Carlo Programs

Download at: www.bitbucket.org/njet/njet

Includes a wiki that explains in detail event generation with NJet and Sherpa

Some technical details

Use scaling test to estimate accuracy per phase space point

$$A(p) = x^{n-4} A(xp) \qquad x \in R$$

Caching of tree subamplitudes for the unitarity cuts and scalar integrals via binary trees (efficient for permutation and helicity sums)

Basic channels for 5-jet production:

$$\begin{array}{ll} 0 \to 7g & 0 \to q\bar{q}5g \\ 0 \to q\bar{q}q'\bar{q}'3g & 0 \to q\bar{q}q'\bar{q}'q''\bar{q}''g \end{array}$$

Runtime and accuracy of the virtuals for 5-jet production

New in NJet 2.0: Split contribution in leading and subleading colour

virtual part	⟨time per event⟩	qp	qp w/ test	op
leading	17s	2%	0.5%	0.01%
sub-leading	112s	2.5%	1%	0.05%

1) Reprocess in quad precision if scaling test in double precision returns less than 5 valid digits.
   Use in addition **scaling test in quadruple precision**, if scaling test in double precision returns no or only one valid digit.
2) Switch to octuple precision, so far sufficient for all applications.

NB: 5-Jet production computed with **full colour + full helicity**

Numerical set up

- anti-kt jet algorithm as implemented in FastJet with jet radius like at ATLAS
  [Cacciari, Salam, Soyez 2012]
- NNPDF2.3 as standard NLO PDF set if not specified otherwise
- Massless QCD, 5-flavour scheme
- set $\mu_f = \mu_r \equiv \mu$ and use dynamical scale based on sum of the transverse momentum of the final state partons

$$\hat{H}_T = \sum_{i=1}^{N_{\text{parton}}} p_{T,i}^{\text{parton}} \qquad \mu = \hat{H}_T/2$$

- Scale variation: $\hat{H}_T/4 \leq \mu \leq \hat{H}_T$
- Kinematical cuts: Transverse momentum of the first jet pt > 80 GeV, subsequent jets at least pt > 60 GeV, Rapidity: eta < 2.8
  [ATLAS 2011]
- Use root NTuples to store generated events, convenient for PDF and scale variations

Five-jet inclusive cross section

Statistical uncertainty

Scalevariation

@ 7 TeV:

$$\sigma_{\text{5-jet}}^{\text{7TeV-LO}} = 0.699(0.004)^{0.419(+)}_{1.228(-)}\text{nb}$$

$$\sigma_{\text{5-jet}}^{\text{7TeV-NLO}} = 0.544(0.016)^{0.479(+)}_{0.367(-)}\text{nb}$$

@ 8 TeV:

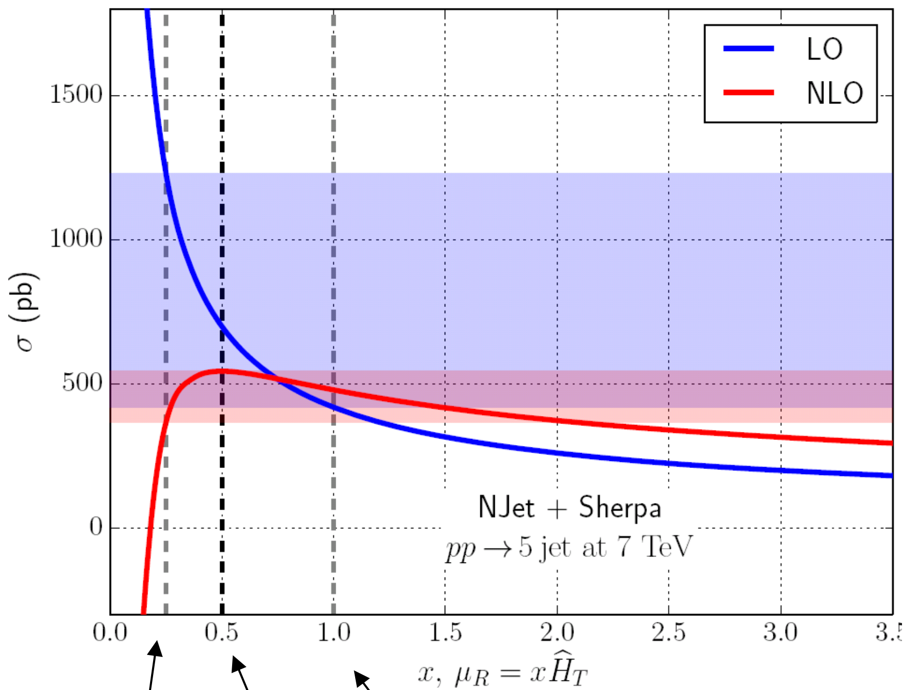
$$\sigma_{\text{5-jet}}^{\text{8TeV-LO}} = 1.044(0.006)^{0.631(+)}_{1.814(-)}\text{nb}$$

$$\sigma_{\text{5-jet}}^{\text{8TeV-NLO}} = 0.790(0.011)^{0.723(+)}_{0.477(-)}\text{nb}$$

NB: LO cross section calculated with LO PDF sets

Remarks on scale dependence

Use LO PDFs for LO X-section:

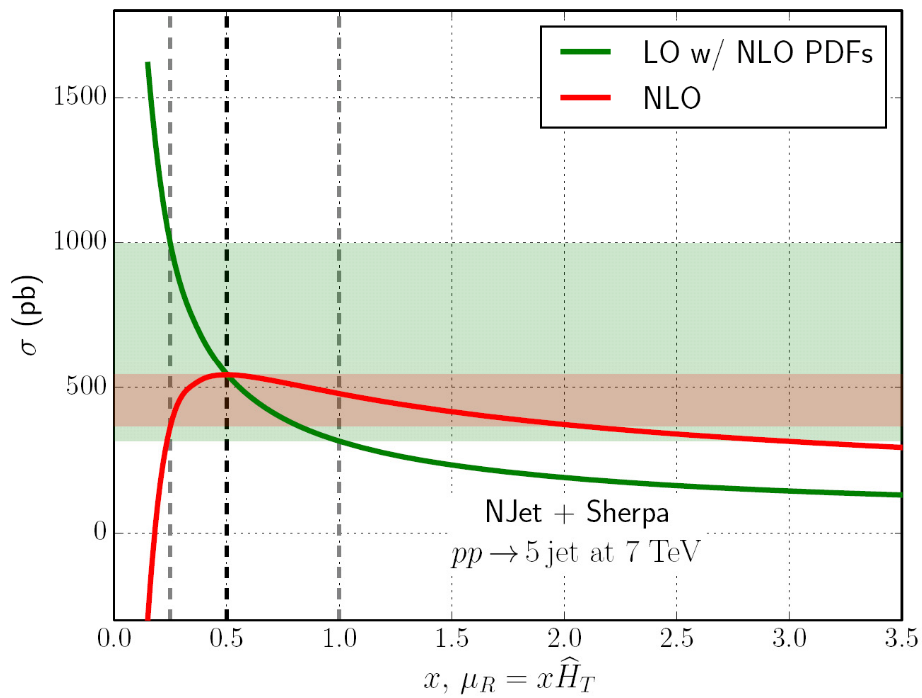


upper scale

lower scale

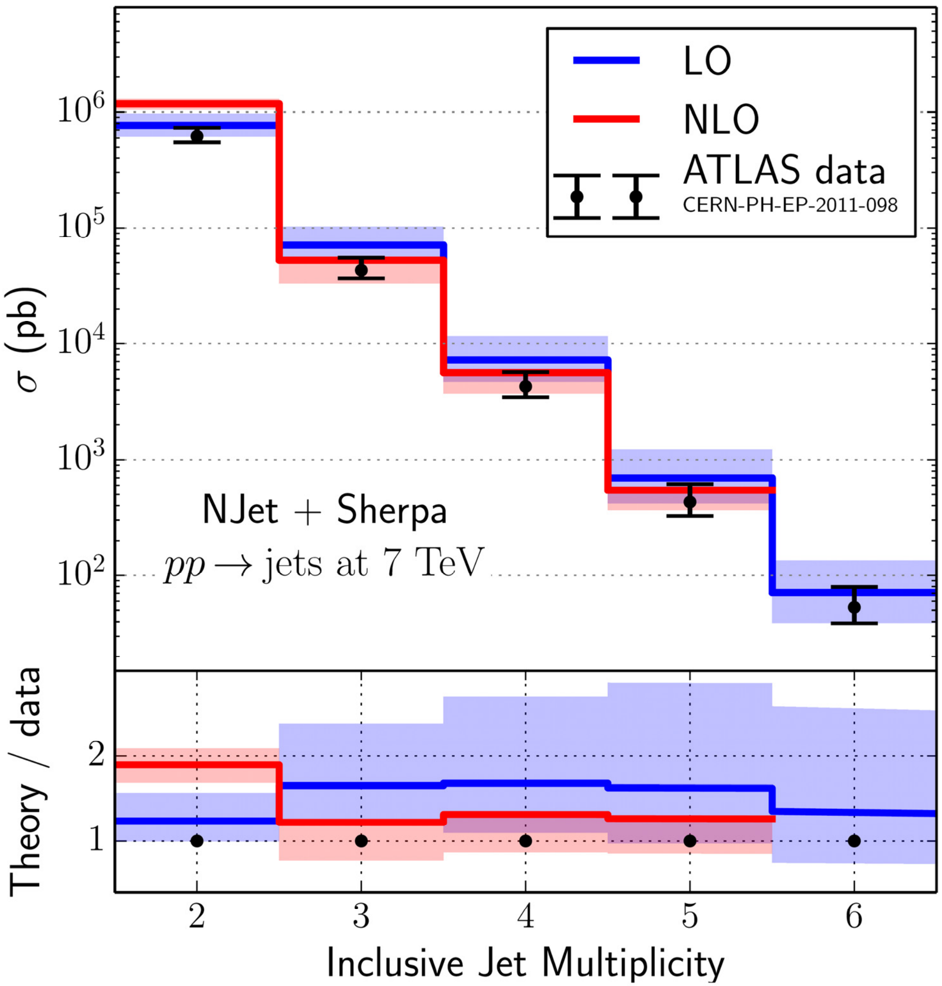
central scale

Use NLO PDFs for LO X-section



NLO PDFs for the LO X-section give better approximation to the NLO result.

Multi-Jet cross sections



$\sigma_2^{7\text{TeV-NLO}}$	$1175(3)^{1046(+)}_{1295(-)}$ nb
$\sigma_3^{7\text{TeV-NLO}}$	$52.5(0.3)^{54.4(+)}_{33.2(-)}$ nb
$\sigma_4^{7\text{TeV-NLO}}$	$5.65(0.07)^{5.36(+)}_{3.72(-)}$ nb
$\sigma_5^{7\text{TeV-NLO}}$	$0.544(0.016)^{0.479(+)}_{0.367(-)}$ nb
$\sigma_6^{7\text{TeV-LO}}$	$0.0496(0.0005)^{0.0263(+)}_{0.0992(-)}$ nb

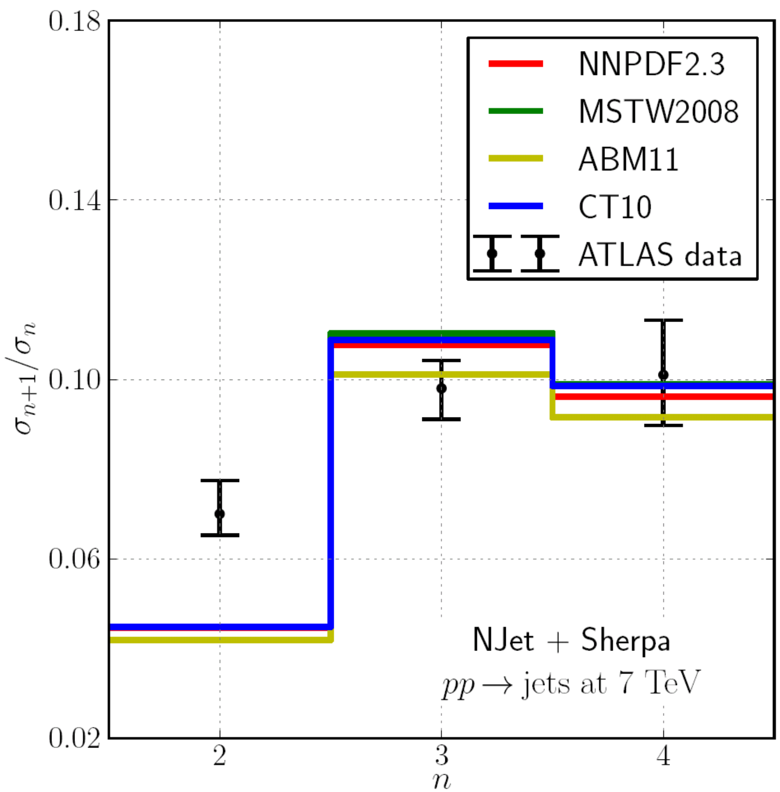
- ratio between theory and data about 1.2 – 1.3
- reduction of the cross section from LO -> NLO (except for 2-jets)

Inclusive jet ratios

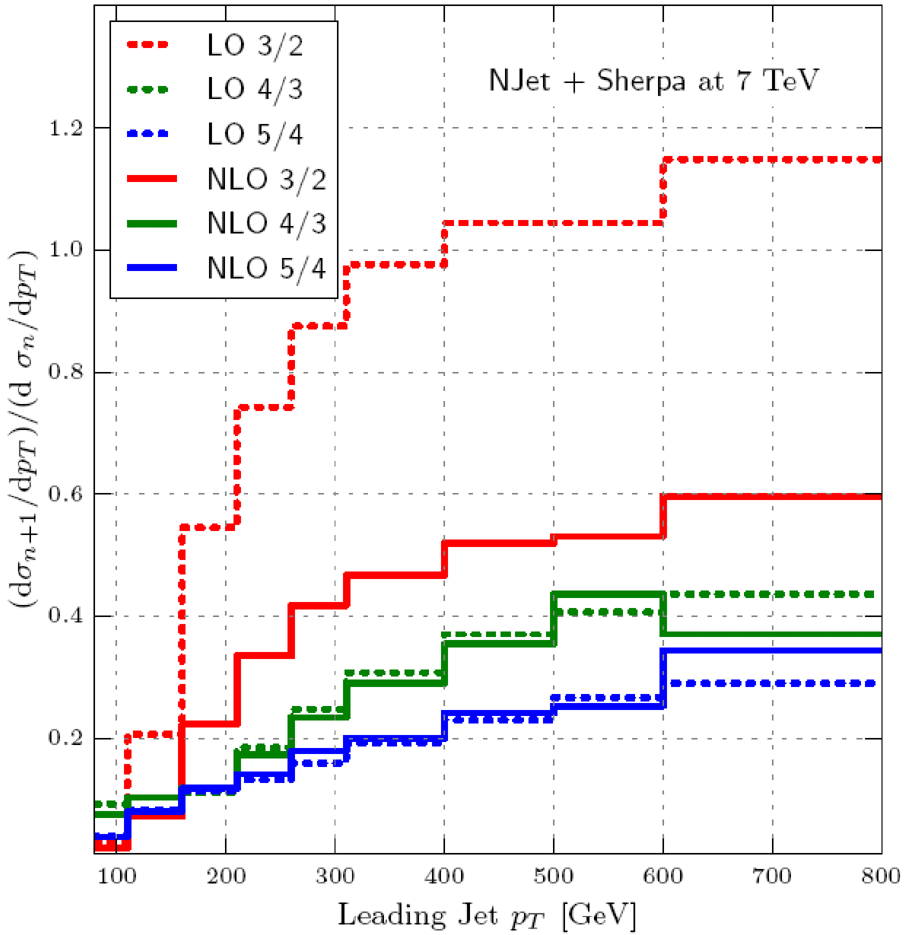
Jet ratio:

$$\mathcal{R}_n = \frac{\sigma_{(n+1)\text{-jet}}}{\sigma_{n\text{-jet}}}$$

- 4/3 and 5/4 jet ratio from ATLAS well described by fixed order QCD @ NLO
- NNPDF2.3, MSTW2008, CT10 compatible, ABM11 set slightly smaller
- obvious mismatch for 3/2 ratio



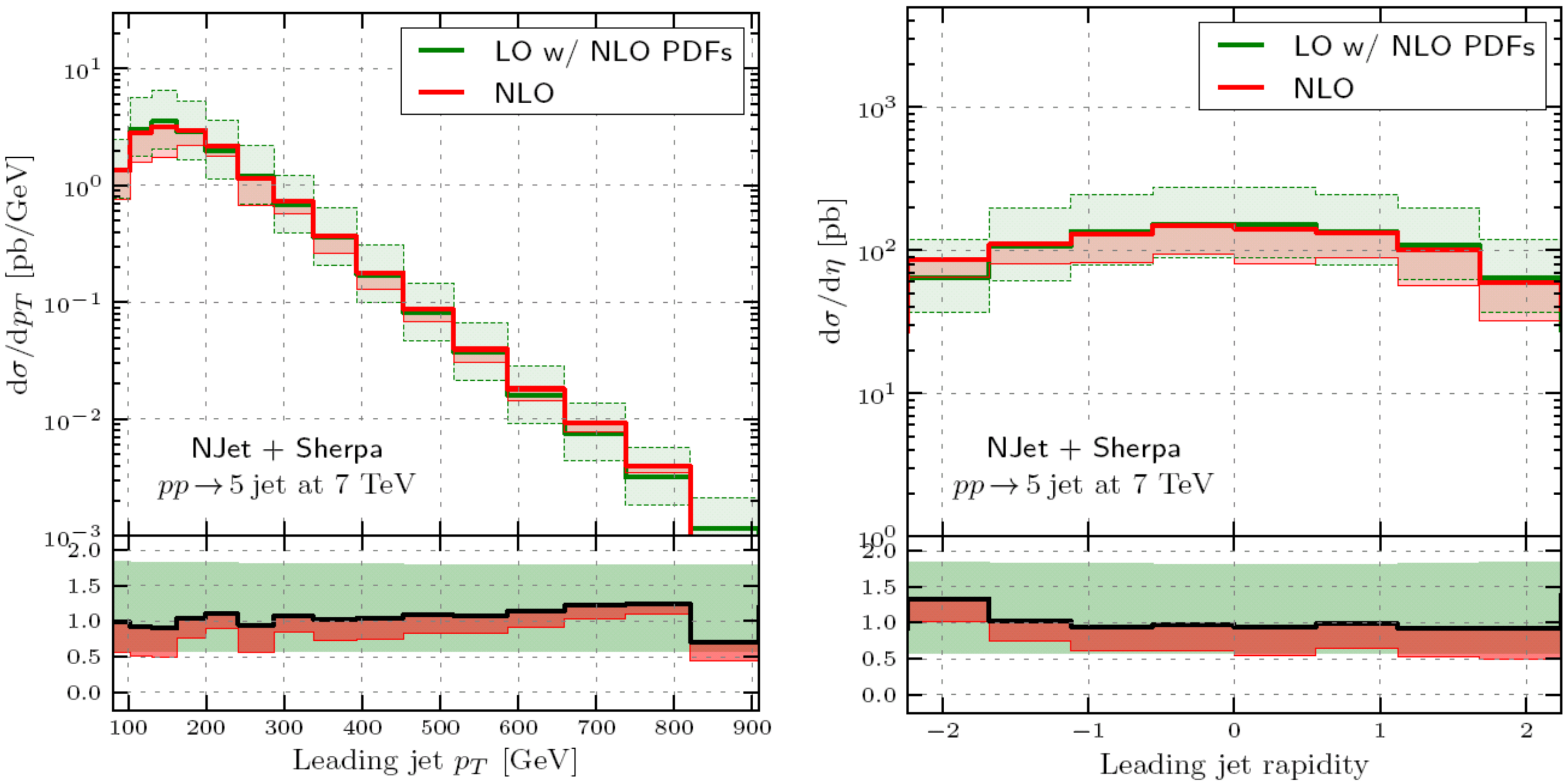
Comparing differential jet ratios



Higher multiplicities more stable against perturbative corrections!

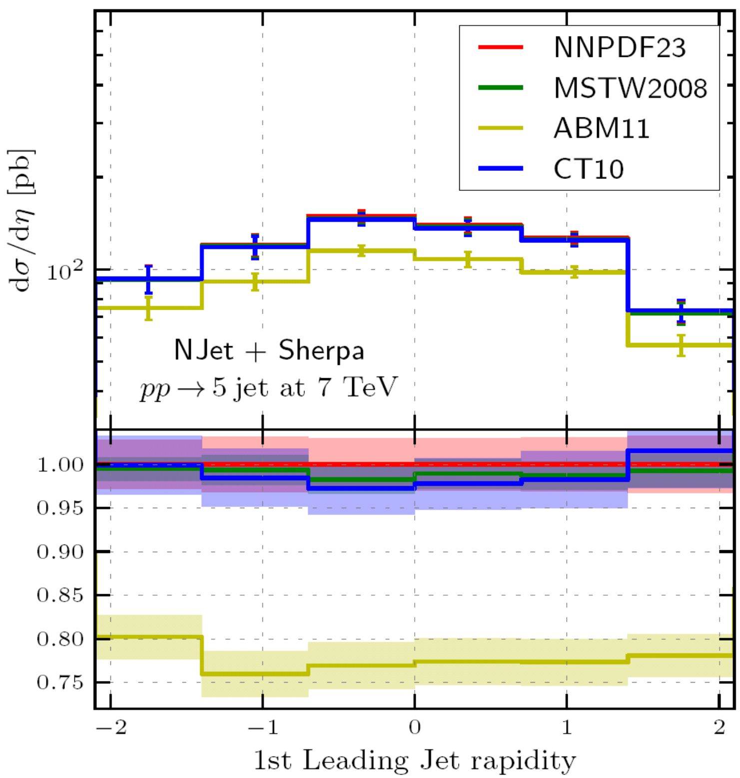
Use 4/3 and 5/4 jet ratio for future α_s measurement and use fixed order perturbation theory as theory input!

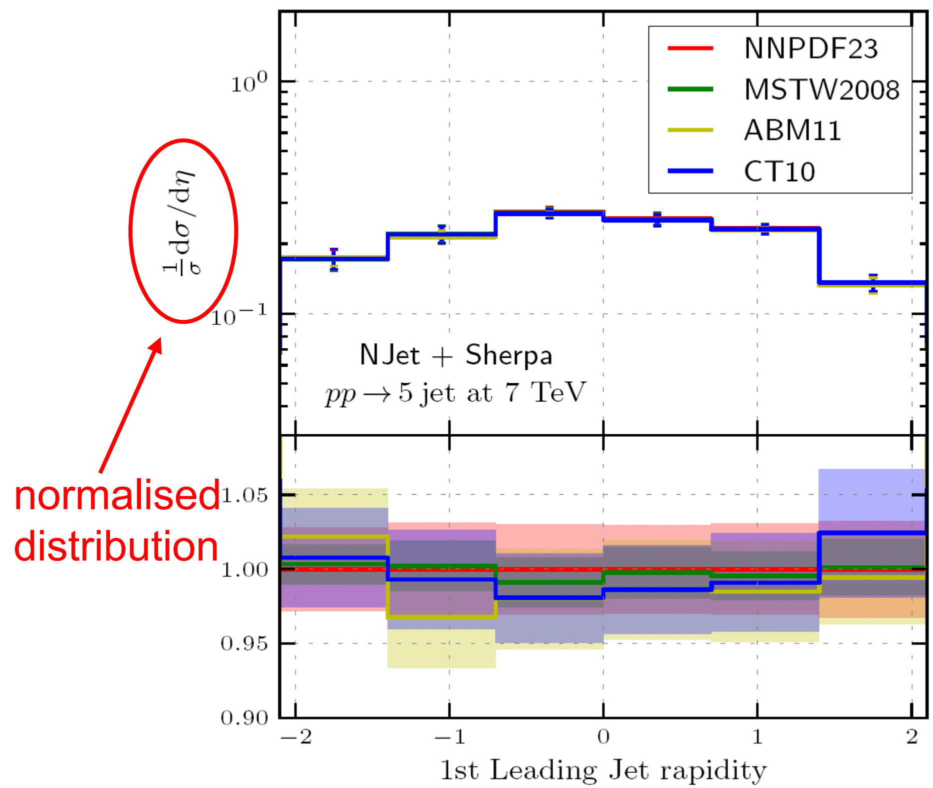
Differential 5-jet distributions



- reduced scale dependence at NLO
- remarkably constant K-Factor
- similar findings also for subleading jets

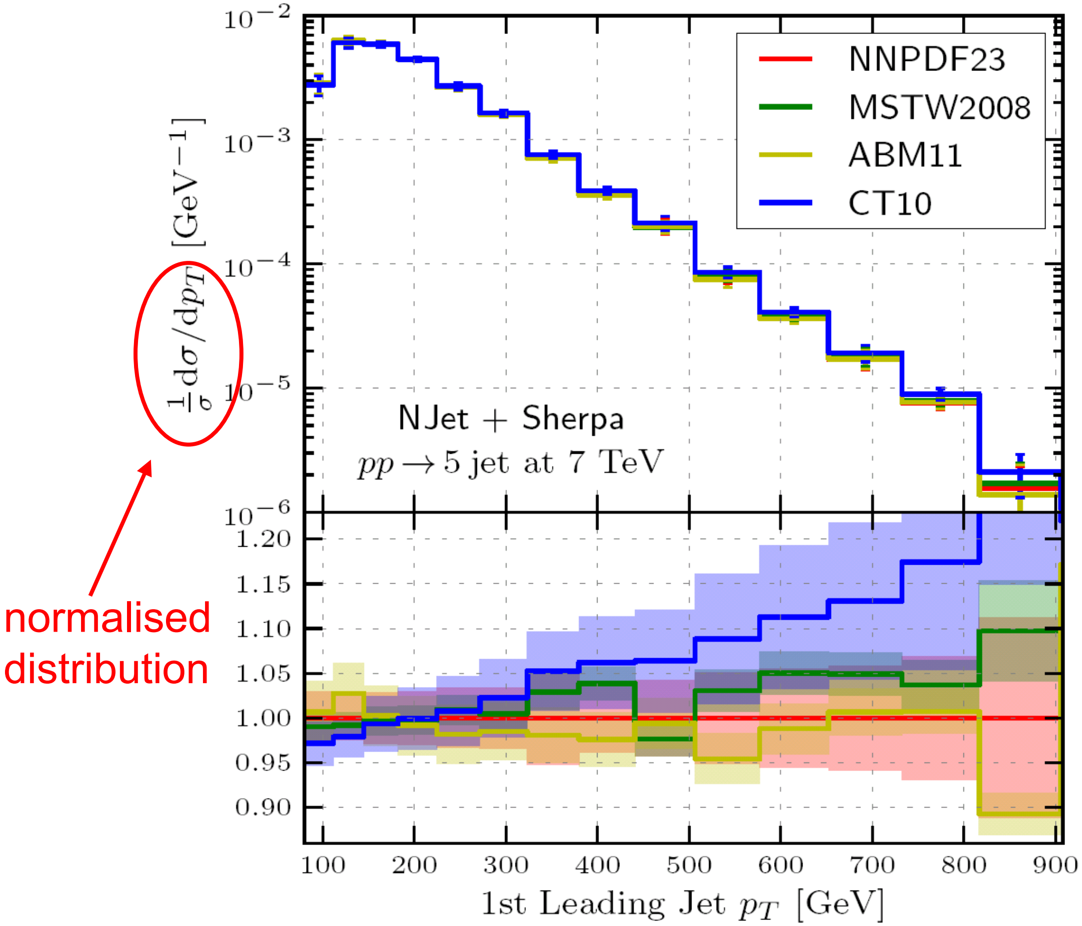
PDF dependence of the jet distributions





- ratio with respect to NNPDF2.3
- NNPDF2.3, MSTW2008 and CT10 compatible, ABM11 20% lower
- lower plots include also PDF uncertainty (around 3%)

PDF dependence of the transverse momentum distribution



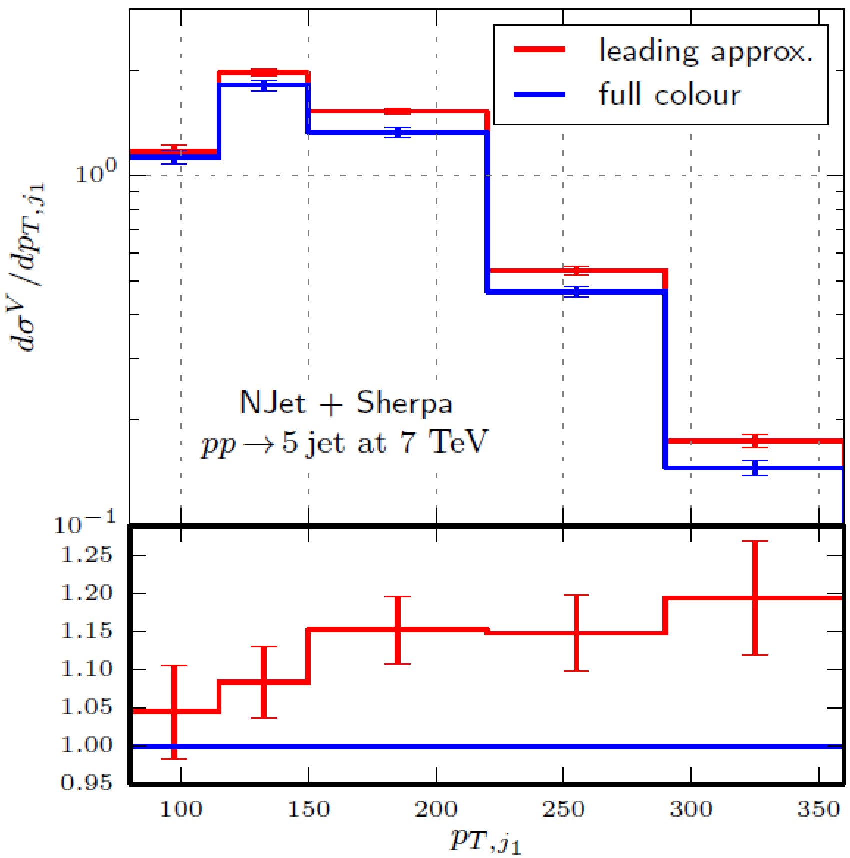
- normalised distributions compatible at low p_T
- for high p_T: NNPDF2.3, MSTW2008 and ABM11 compatible within 10-15%, CT10 around 20-30% larger than NNPDF2.3, MSTW2008 and ABM11

Conclusion

- NJet is publicly available, includes all necessary matrix elements for 2-jet, 3-jet, 4-jet and 5-jet production and primitive amplitudes of arbitrary multiplicity
- NJet 2.0 available, includes vector boson production in association with up to 5 jets, and di-photon production with up to four jets
- 5-jet production at 7 and 8 TeV for the LHC with NJet and Sherpa presented
- Moderate corrections at NLO of the order of 10% or less with respect to LO result using NLO PDFs at LO.
- Significant reduction of the scale dependence at NLO.
- Good agreement between theory and data from the ATLAS collaboration apart from 2-jet cross section.
- (n+1)/n jet ratio shows that 4/3 and 5/4 predictions are perturbatively more stable than 3/2 ratio with moderate NLO corrections
  $\longrightarrow$ suggestion for future α_s extractions

Extra slides

Leading colour approximation



Our leading colour definition: Multiquark processes in the Large N_c limit.
Include desymmetrised colour sums with gluons in the final state (exploit bosonic nature of the phase space).

$pp \rightarrow \gamma\gamma + \text{jets}$ @ NLO

- virtuals for up to 3 jets in NJET 2.0
- backgrounds to $pp \rightarrow H \rightarrow \gamma\gamma$

Badger, Guffanti, Yundin arXiv:1312.5927

$p_{T,j} > 30\,\text{GeV}$ $\quad |\eta_j| \leq 4.7$

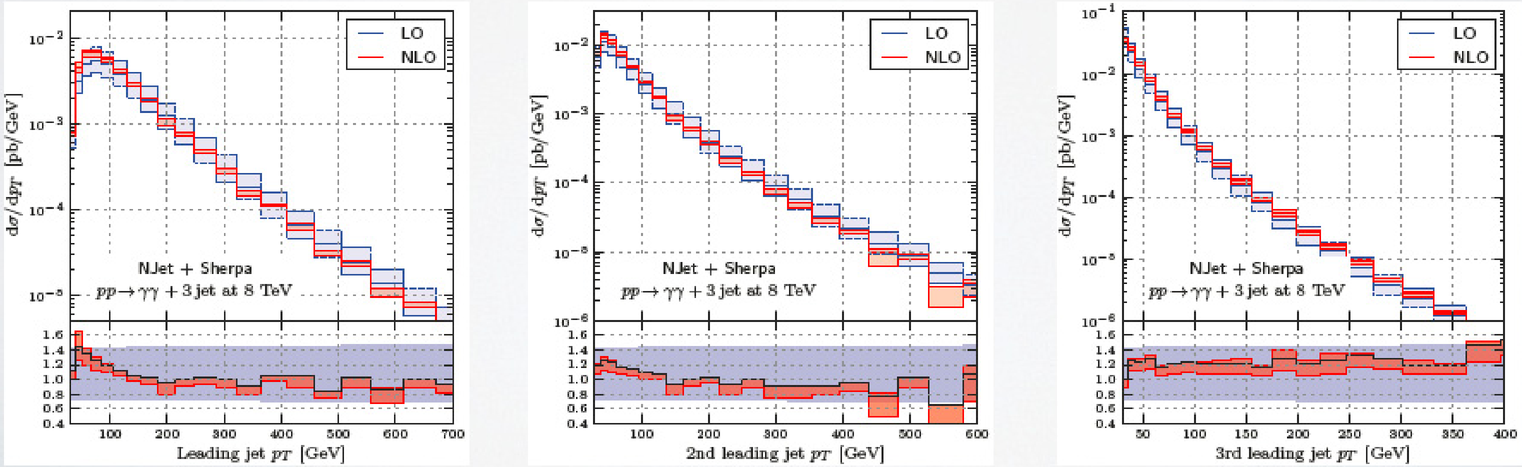
$p_{T,\gamma_1} > 40\,\text{GeV}$ $\quad p_{T,\gamma_2} > 25\,\text{GeV}$ $\quad |\eta_\gamma| \leq 2.5$

$R_{\gamma,j} = 0.5$ $\quad R_{\gamma,\gamma} = 0.45$

Frixione smooth cone photon isolation

CT10 PDF set

$$\sigma^{LO}_{\gamma\gamma+3j}(\widehat{H}'_T/2) = 0.643(0.003)^{+0.278}_{-0.180}\,\text{pb} \qquad \sigma^{NLO}_{\gamma\gamma+3j}(\widehat{H}'_T/2) = 0.785(0.010)^{+0.027}_{-0.085}\,\text{pb}$$

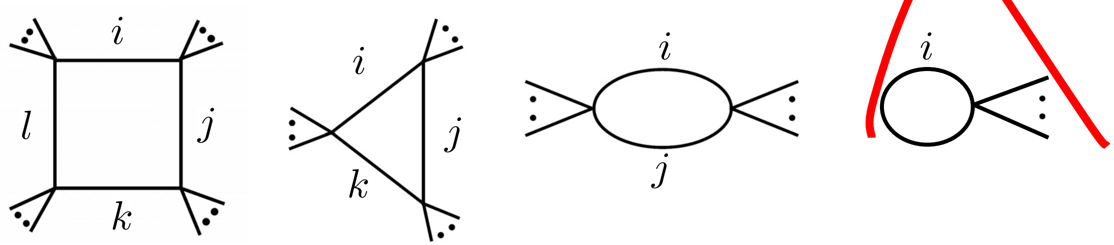


Scalar Integral Basis

Decomposition of an arbitrary one-loop amplitude: [Passarino,Veltman1979]

$$\mathcal{A}_n^{\text{loop}} = \sum_{\{ijkl\}} d_{ijkl}^{[\mathrm{d}]}\,\mathcal{I}_{ijkl}^{(4)} + \sum_{\{ijk\}} c_{ijk}^{[\mathrm{d}]}\,\mathcal{I}_{ijk}^{(3)} + \sum_{\{ij\}} b_{ij}^{[\mathrm{d}]}\,\mathcal{I}_{ij}^{(2)} + \cancel{\sum_{\{i\}} a_{i}^{[\mathrm{d}]}\,\mathcal{I}_{i}^{(1)}}$$

boxes triangles bubbles ~~tadpoles~~



Dimension:

$$[\mathrm{d}] = 4 - 2\varepsilon$$

Topology:

$$\{ijkl\}$$

Scalar integrals:

$$\mathcal{I}_{ijkl}^{(4)} = \int \mathrm{d}^{[\mathrm{d}]} l \frac{1}{D_i D_j D_k D_l}$$

$$D_i = (p_i + l)^2 - m_i^2$$

computation of one-loop amplitudes = determination of integral coefficients

No tadpoles in massless theories

Integrand Properties

Focus on the **integrand** $\mathcal{F}_n(l)$ of the amplitudes

$$\mathcal{A}_n^{\text{loop}} = \int \mathrm{d}^4 l\, \mathcal{F}_n(l) + \mathcal{A}_n^{\text{rat}}$$

[Ossola,Papadopoulos,Pittau]
[Ellis, Giele, Kunszt]
[Britto, Cachazo, Feng, Mastrolia]

$$\mathcal{F}_n(l) = \sum_{\{ijkl\}} \frac{\bar{d}_{ijkl}(l)}{D_i D_j D_k D_l} + \sum_{\{ijk\}} \frac{\bar{c}_{ijk}(l)}{D_i D_j D_k} + \sum_{\{ij\}} \frac{\bar{b}_{ij}(l)}{D_i D_j}$$

Numerators: loop-momentum independent part **+** spurious terms

Spurious terms: loop-momentum tensors which vanish after integration

$\longrightarrow$ Loop-momentum independent part is the desired integral coefficient

Box Example

Integrand $\mathcal{F}_n(l)$ **:**

$$\mathcal{F}_n(l) = \sum_{\{ijkl\}} \frac{\bar{d}_{ijkl}(l)}{D_i D_j D_k D_l} + \sum_{\{ijk\}} \frac{\bar{c}_{ijk}(l)}{D_i D_j D_k} + \sum_{\{ij\}} \frac{\bar{b}_{ij}(l)}{D_i D_j}$$

Tensor structure of a general box part $\bar{d}(l)$ well known:

$$\begin{aligned}\bar{d}(l) &= d_0 + \tilde{d}(l) \\ &= d_0 + d_1 \varepsilon_{\mu\nu\rho\sigma} p_1^\mu p_2^\nu p_3^\rho \, l^\sigma\end{aligned}$$

$$\int \mathrm{d}^4 l \frac{d_0 + \tilde{d}(l)}{D_i D_j D_k D_l} = d_0 \int \mathrm{d}^4 l \frac{1}{D_i D_j D_k D_l} = d_0 \, \mathcal{I}^{(4)}$$

Compute $\bar{d}(l)$ for two different l
$\hookrightarrow$ system of equations $\longrightarrow$ determine d_0

How do we get $\bar{d}(l)$?

Rational Part

1. Absorb epsilon dependence in effective mass:

[Bern, Dixon, Dunbar, Kosower 1997]
[Bern, Morgan 1996]

$$l_{[4-2\varepsilon]} = l_{[4]} + l_{[-2\varepsilon]}$$

$$l^2_{[4-2\varepsilon]} = l^2_{[4]} - l^2_{[-2\varepsilon]} \stackrel{!}{=} 0 \quad \longrightarrow \quad l^2_{[-2\varepsilon]} = -\mu^2$$

Integrand for the rational part is a polynomial in μ^2.

2. Expand integral basis in higher integer dimension and take ε → 0 limit

$$\mathcal{A}_n^{rat} = -\frac{1}{6}\sum_{i,j,k,l} C^{[4]}_{4;ijkl} - \frac{1}{2}\sum_{i,j,k} C^{[2]}_{3;ijk} - \sum_{i,j} \frac{s_{i,j-1}}{6} C^{[2]}_{2;ij}$$

"constant" integral

Integral coefficient

[Giele, Kunszt, Melnikov 2008]
[Badger 2009]

- Additional hidden pentagon contributions
- compute in background field gauge, make use of power counting criterion in order to interpret "rational gluons" as scalar contributions
  $\longrightarrow$ use the same 4D-techniques as in the cut-constructible case