Loop amplitudes and NLO simulations with OpenLoops

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based on

F. Cascioli, P. Maierhöfer and S.P., PRL **108** (2012) 111601 [arXiv:1111.5206]

F. Cascioli, S. Höche, F. Krauss, P. Maierhöfer, S. P. and F. Siegert, arXiv:1309.0500

F. Cascioli, P. Maierhöfer, N. Moretti, S. P. and F. Siegert, arXiv:1309.5912

F. Cascioli, S. Kallweit, P. Maierhöfer and S. P., arXiv:1312.0546

Zurich Phenomenology Workshop, 9 January 2014

Outline of the talk

(A) Introduction

(B) OpenLoops algorithm

(C) A unified NLO description of $\mathrm{t\bar{t}}$ and Wt production

(D) MC@NLO matching for $\mathrm{t\bar{t}b\bar{b}}$ production with $m_\mathrm{b} > 0$

(A) Introduction

NLO Revolution and Automation

NLO QCD calculations for $2 \to 4(5,6)$ processes at the LHC

- many recent results (2009-2013): $5j$, $\mathrm{W}+5j$, $\mathrm{Z}+4j$, $\mathrm{H}+3j$, $\mathrm{WW}jj$, $\mathrm{WZ}jj$, $\gamma\gamma+3j$, $\mathrm{W}\gamma\gamma j$, $\mathrm{WWb\bar{b}}$, $\mathrm{b\bar{b}b\bar{b}}$, $\mathrm{t\bar{t}b\bar{b}}$, $\mathrm{t\bar{t}}jj$, $\mathrm{t\bar{t}t\bar{t}}$, . . .
- NLO wish list closed since 2→4 NLO feasibility well established
- serious multi-particle simulations important for Run 2 ⇒ emphasis should move from proof-of-concept papers to complete simulations and nontrivial pheno studies
- technical frontier just shifted and still exciting to explore

NLO automation including matching and merging

- many tools: `CutTools`, `Samurai`, `HELAC-NLO`, `MadLoop`, `GoSam`, `BlackHat`, `NGluon`, `OpenLoops`, `Collier`, `Recola`, `MADGRAPH/aMC@NLO`, `POWHEG`, `Sherpa`, `Herwig`, `Pythia`
- new attitude towards R&D at NLO: think more in terms of general methodological features (e.g. EW corrections) and less in terms of single processes
- ... keeping in mind that simulation of every single process needs to be well understood and some processes will require more than “vanilla NLO”
- methodology and phenomenology at NLO much more involved wrt LO: usage, maintanance and development of tools requires much higher level of expertise and TH/EXP cross-talk
- algorithmic efficiency crucial in order to promote NLO to the default accuracy in LHC studies ⇒ don’t stop R&D

(B) The OpenLoops Algorithm [Cascioli, Maierhöfer, S.P '11]

OpenLoops Generator [Cascioli, Maierhöfer, S.P., PRL **108** (2012) 111601]

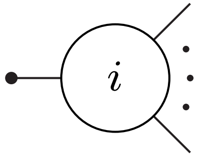
- fully automated generation of tree and loop amplitudes for NLO
- conceived to break **multi-particle** bottlenecks (fast, stable, flexible)
- NLO QCD for $2 \to 2, 3, 4$ SM processes ($2 \to 5$ and NLO EW possible)

Hybrid “tree–loop” algorithmic approach

- constructs process-dependent 1-loop ingredients with hybrid “tree–loop” approach based on diagrammatic building blocks (openloops)
- exploits pinch relations to obtain n-point diagrams from pre-computed $(n-1)$-point diagrams
- works in combination with both tensor-integral and OPP reduction
- numerical recursion inspired by 1-loop Dyson-Schwinger recursion [van Hameren ’09]

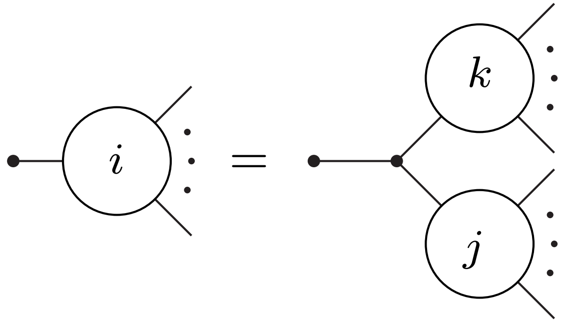
Tree generator

Colour-stripped tree diagrams are built numerically in terms of sub-trees

$w^{\beta}(i) =$ 

$\beta \leftrightarrow$ off-shell line spin

and recursively merged by attaching vertices and propagators

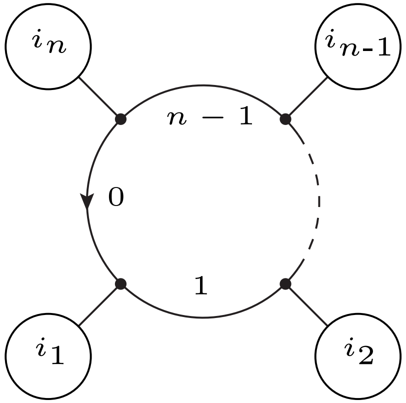


$$w^{\beta}(i) = \frac{X^{\beta}_{\gamma\delta}(i,j,k)}{p_i^2 - m_i^2}\, w^{\gamma}(j)\, w^{\delta}(k)$$

Flexible (only $\mathcal{L}_{\text{int}}$ dependent), **fast** (many diagrams share *common sub-trees*)

Loop diagrams and reduction to basis integrals

Colour-stripped loop diagrams are ordered sets of sub-trees connected by loop propagators $D_i = (q + p_i)^2 - m_i^2 + \mathrm{i}\varepsilon$,



$$= \int \frac{\mathrm{d}^D q \; \mathcal{N}(\mathcal{I}_n; q)}{D_0 D_1 \dots D_{n-1}} = \sum_{r=0}^{R} \mathcal{N}_{\mu_1 \dots \mu_r}(\mathcal{I}_n) \underbrace{\int \frac{\mathrm{d}^D q \; q^{\mu_1} \dots q^{\mu_r}}{D_0 D_1 \dots D_{n-1}}}_{\text{tensor integral}}$$

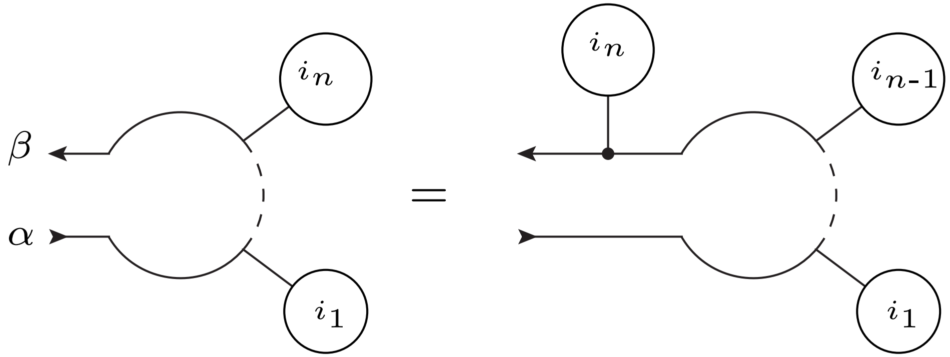
`OpenLoops` computes *symmetrised* $\mathcal{N}_{\mu_1 \dots \mu_r}(\mathcal{I}_n)$ *coefficients*

$\Rightarrow$ $\mathcal{O}(100)$ coefficients for nontrivial hexagon diagram

and applies **two alternative reductions:**

(A) **Tensor-integral reduction** [Denner/Dittmaier '05] **avoids instabilities** (Gram-determinant expansions)

(B) **OPP reduction** [Ossola, Papadopolous, Pittau '07] based on numerical evaluation of $\mathcal{N}(\mathcal{I}_n; q) = \sum \mathcal{N}_{\mu_1 \dots \mu_r}(\mathcal{I}_n) \; q^{\mu_1} \dots q^{\mu_r}$ at multiple q-values (**strong speed-up!**)



Tree generators for "usual" OPP-input $\mathcal{N}(\mathcal{I}_n; q)$

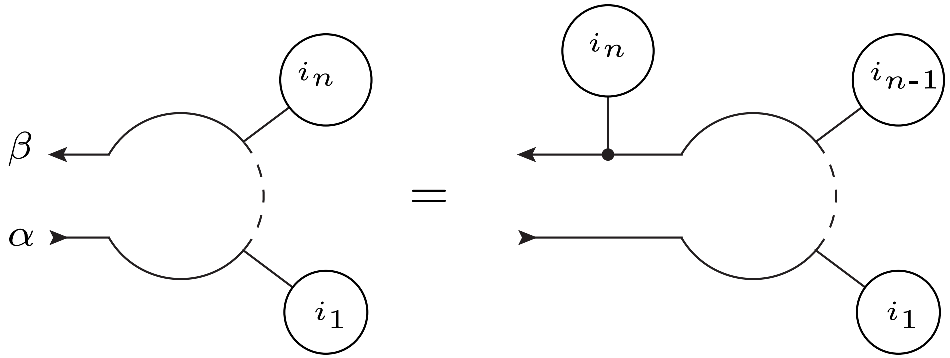
Cut-open loops can be built by recursively attaching external sub-trees

$$\mathcal{N}_\alpha^\beta(\mathcal{I}_n; q) = X^\beta_{\gamma\delta}(\mathcal{I}_n, i_n, \mathcal{I}_{n-1})\, \mathcal{N}_\alpha^\gamma(\mathcal{I}_{n-1}; q)\, w^\delta(i_n)$$

like in **conventional tree generators**

- one-loop automation in `Helac-NLO` (off-shell recursion) and `MadLoop` (diagrams)
- CPU expensive OPP reduction (multiple-q evaluations) since *tree algorithms conceived for fixed momenta*

Nature of loop amplitudes requires loop-momentum *functional* dependence!



OpenLoops recursion for $\mathcal{N}^{\beta}_{\mu_1\ldots\mu_r;\alpha}(\mathcal{I}_n)$

Handle building blocks of recursion as *polynomials* in the loop momentum q

$$\underbrace{\mathcal{N}^{\beta}_{\alpha}(\mathcal{I}_n;q)}_{\sum_{r=0}^{n}\mathcal{N}^{\beta}_{\mu_1\ldots\mu_r;\alpha}(\mathcal{I}_n)\;q^{\mu_1}\ldots q^{\mu_r}} \quad=\quad \underbrace{X^{\beta}_{\gamma\delta}(\mathcal{I}_n,i_n,\mathcal{I}_{n-1})}_{Y^{\beta}_{\gamma\delta}+q^{\nu}\;Z^{\beta}_{\nu;\gamma\delta}}\;\underbrace{\mathcal{N}^{\gamma}_{\alpha}(\mathcal{I}_{n-1};q)}_{\sum_{r=0}^{n-1}\mathcal{N}^{\beta}_{\mu_1\ldots\mu_r;\alpha}(\mathcal{I}_{n-1})\;q^{\mu_1}\ldots q^{\mu_r}}\;w^{\delta}(i_n)$$

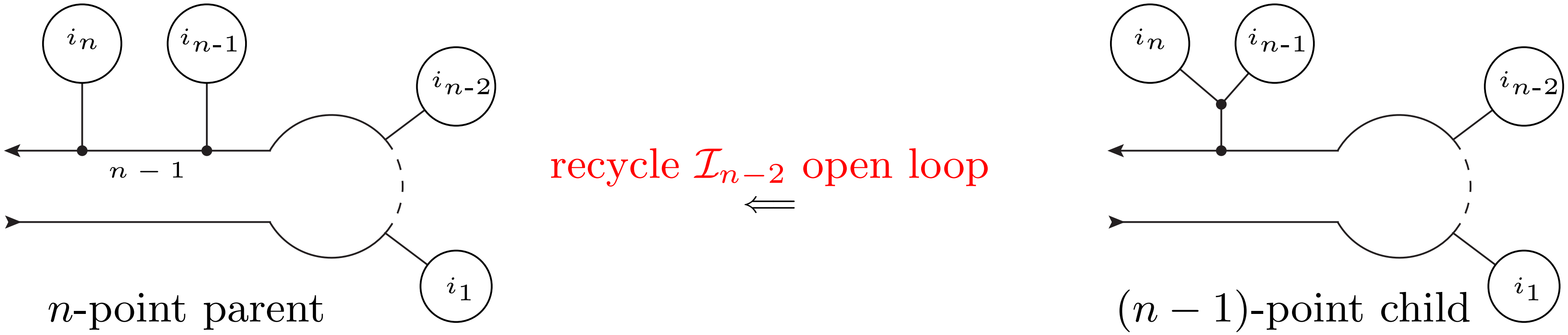
and construct polynomial coefficients with “open loops recursion”

$$\mathcal{N}^{\beta}_{\mu_1\ldots\mu_r;\alpha}(\mathcal{I}_n)=\left[Y^{\beta}_{\gamma\delta}\,\mathcal{N}^{\gamma}_{\mu_1\ldots\mu_r;\alpha}(\mathcal{I}_{n-1})+Z^{\beta}_{\mu_1;\gamma\delta}\,\mathcal{N}^{\gamma}_{\mu_2\ldots\mu_r;\alpha}(\mathcal{I}_{n-1})\right]\;w^{\delta}(i_n)$$

Key features

- **tree-like recursion** supplemented with **complete loop-momentum information**

- **fully flexible and automated** (universal kernels dictated by Feynman rules)
- **very fast thanks to**:
  - optimal implementation
  - helicity/colour/loop decoupling
  - pinch relations: in QCD any n-point loop diagram can be obtained starting from a pre-computed $(n-1)$-point child diagram



n-point parent

recycle $\mathcal{I}_{n-2}$ open loop $\Longleftarrow$

$(n-1)$-point child

`OpenLoops` Implementation and Technical Features

One-loop QCD corrections to SM processes fully automated

- process-definition file ⇒ `Fortran` 90 libraries for matrix elements

Other technical features

- interfaced to `Collier` library [Denner, Dittmaier, Hofer] for tensor integrals
- on-the-fly quadruple precision (very useful for benchmarks and NNLO)
- loop-induced processes
- speed of tree amplitudes optimised
- precision checks against independent in-house generator for > 100 processes
- ...

Flexibility and Automation

Process	size [MB]	t_{code} [s]
$u\bar{u} \to t\bar{t}$	0.1	2.2
$u\bar{u} \to W^+W^-$	0.1	7.2
$u\bar{d} \to W^+g$	0.1	4.2
$gg \to t\bar{t}$	0.2	5.4
$u\bar{u} \to t\bar{t}g$	0.4	12.8
$u\bar{u} \to W^+W^-g$	0.4	39.8
$u\bar{d} \to W^+gg$	0.5	22.9
$gg \to t\bar{t}g$	1.2	52.9
$u\bar{u} \to t\bar{t}gg$	3.6 (200)*	236 ($\sim 10^6$)*
$u\bar{u} \to W^+W^-gg$	2.5 (1000)*	381.7 ($\sim 10^6$)*
$u\bar{d} \to W^+ggg$	4.2	366.2
$gg \to t\bar{t}gg$	16.0	3005

Compact code

- 100 kB to few MB object files
- $\mathcal{O}(10^2$–$10^3)$ compression in $2 \to 4$

Fast code generation/compilation

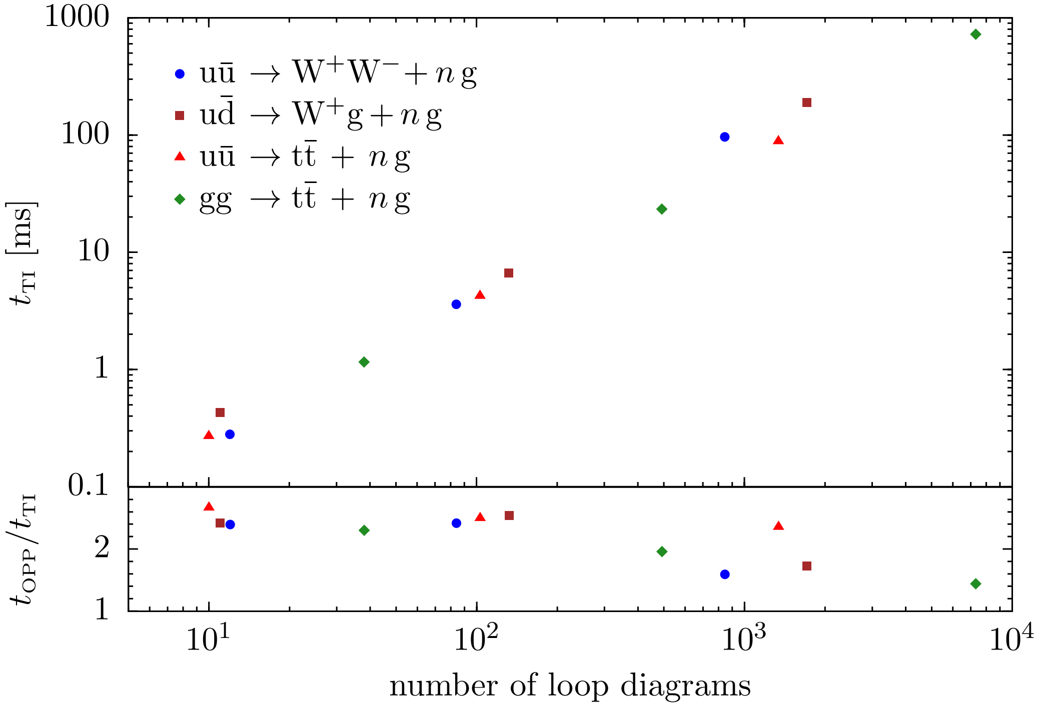
- few seconds to minutes
- $\mathcal{O}(10^3)$ speed-up in $2 \to 4$

Large-scale applicability!

*pp $\to \mathrm{t\bar{t}b\bar{b}}$ & $\mathrm{WWb\bar{b}}$ (Bredenstein, Denner, Dittmaier, Kallweit and S.P. ‘09–‘11)

High CPU efficiency for multi-particle processes

Timings including col/hel sums (`Intel i5-750` core)



$2 \to 4$ **amplitudes**

- $\mathcal{O}(10^3)$ diagrams in $\mathcal{O}(10^2)$ ms/point
- competitive with fastest codes

Scaling

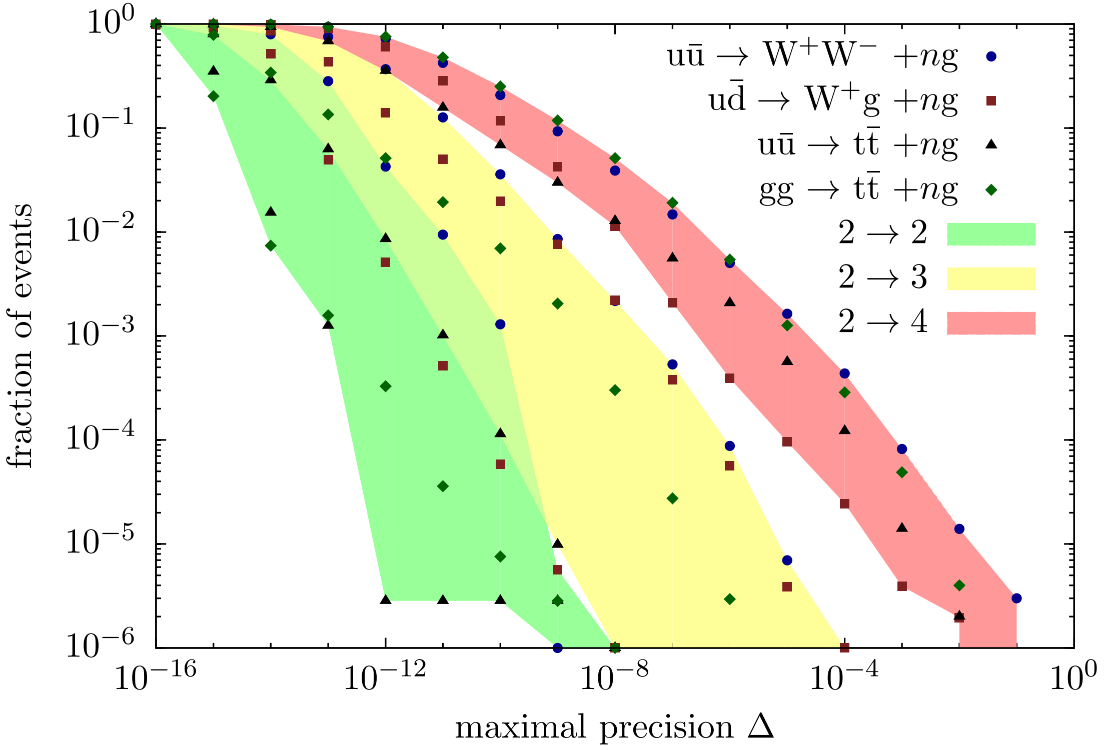
- linear n_{diag}-scaling $\Rightarrow$ $\mathcal{O}(10^5)$ diagrams feasible
- factor 20 per extra leg $\Rightarrow$ $2 \to 5$ feasible

Tensor-reduction vs OPP

- similar timings with `OpenLoops`!

Numerical stability with tensor reduction in double precision

Stability Δ in samples of 10^6 points ($\sqrt{\hat{s}} = 1\,\mathrm{TeV}$, $p_\mathrm{T} > 50\,\mathrm{GeV}$, $\Delta R_{ij} > 0.5$)



Average number of correct digits

- 11-15

Cross section accuracy

- depends on tails
- stability issues grow with n_part

$2 \to 4$ processes very stable

- $\lesssim 0.01\%$ prob. that $\Delta_\mathrm{S} < 10^{-3}$
- thanks to Gram-determinant expansions in `Collier`!

Real-life NLO applications

- $\mathcal{O}(10^{-4})$ unstable points in most challenging $2 \to 4$ calculations considered so far
- can be monitored and safely suppressed thanks to **online instability-trigger**

Interfacing OpenLoops with NLO Monte-Carlo Tools

Interface with various MC tools (IR subtraction, integration) provide complete automation from process definition to hadron-collider observables

- **dedicated interface to Sherpa2.0** (see talk by M. Schönherr)
  - automated matching (MC@NLO) to `Sherpa` shower and multi-jet merging (MEPS@NLO)
- **parton-level Monte-Carlo by S. Kallweit**
  - fully automated and very fast MC integrator
- **standard BLHA interface**
  - applicable to any other Monte-Carlo tool
  - completed very recently in combination with Herwig++ (see talk by S. Plätzer) and now under validation

First OpenLoops Applications

Recent papers

- MEPS@NLO for $\ell\ell\nu\nu$+0,1 jets, Cascioli, Höche, Krauss, Maierhöfer, S. P. and Siegert, arXiv:1309.0500
- MC@NLO for $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$ with $m_\mathrm{b} > 0$, Cascioli, Maierhöfer, Moretti, S. P. and Siegert, arXiv:1309.5912
- NLO for $\mathrm{pp} \to \mathrm{W^+W^-b\bar{b}}$ with $m_\mathrm{b} > 0$, Cascioli, Kallweit, Maierhöfer and S. P., arXiv:1312.0546
- NNLO for $\mathrm{pp} \to \gamma Z$ production, Grazzini, Kallweit, Rathlev and Torre, arXiv:1309.7000
- NLO merging for $\mathrm{pp} \to$ HH+0,1 jets, Maierhöfer and Papaefstathiou, arXiv:1401.0007

General motivation

- Higgs phenomenology
- technical stress test for OpenLoops: multi-particle and multi-scale processes, loop-induced processes, multiple resonances, . . .
- beyond parton-level NLO: MC@NLO, MEPS@NLO and NNLO applications

Publication Plans and Process Library

Towards OpenLoops publication

- all technical prerequisites essentially fulfilled: many processes validated, good experience in challenging real-life applications, BLHA interface almost ready
- we aim at code release in early 2014

The release is planned as NLO QCD library for $2 \to 2, 3, 4$ processes

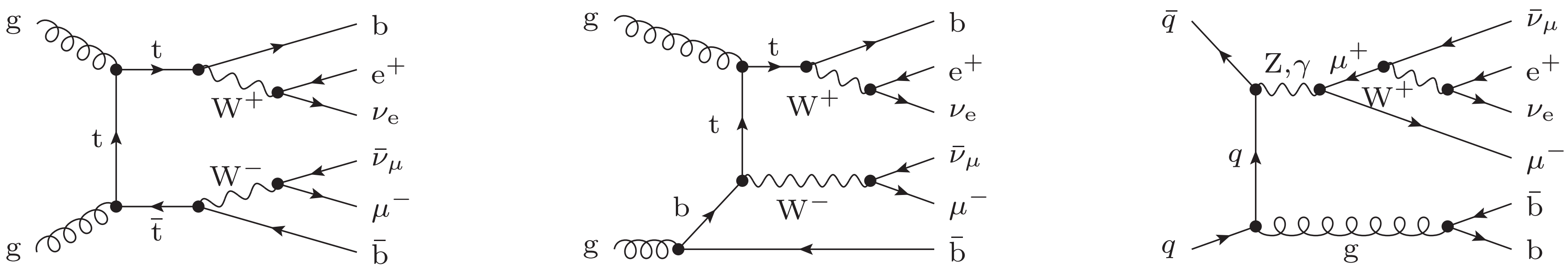
- first version already available to MCWGs of ATLAS/CMS
- new processes can/will be easily added (also upon user request)

W/Z	γ	jets	HQ pairs	single-top	Higgs
V+3j	γ+3j	3(4)j	$\mathrm{t\bar{t}}$+2j	tb+1j	(H+2j)
VV+2j	$\gamma\gamma$+2j		$\mathrm{t\bar{t}}V$+1j	t+1(2)j	VH+1j
gg → VV+1j	$V\gamma$+2j		$\mathrm{b\bar{b}}V$+1j	tW+0(1)j	$\mathrm{t\bar{t}H} + 1j$
VVV+1j					$qq \to \mathrm{H}qq$+0(1)j
gg → VVV					

lower jet multiplicities implicitly understood

(C) Unified $\mathrm{t}\bar{\mathrm{t}}$ and Wt description at NLO [Cascioli,Kallweit,Maieröfer,S.P. ’13]

Top-pair production plus (di-leptonic) decay at NLO



NWA [Bernreuther et al. ’04; Melnikov, Schulze ’09]

- Only $\mathrm{t\bar t}$ channels in $\Gamma_\mathrm{t} \to 0$ limit

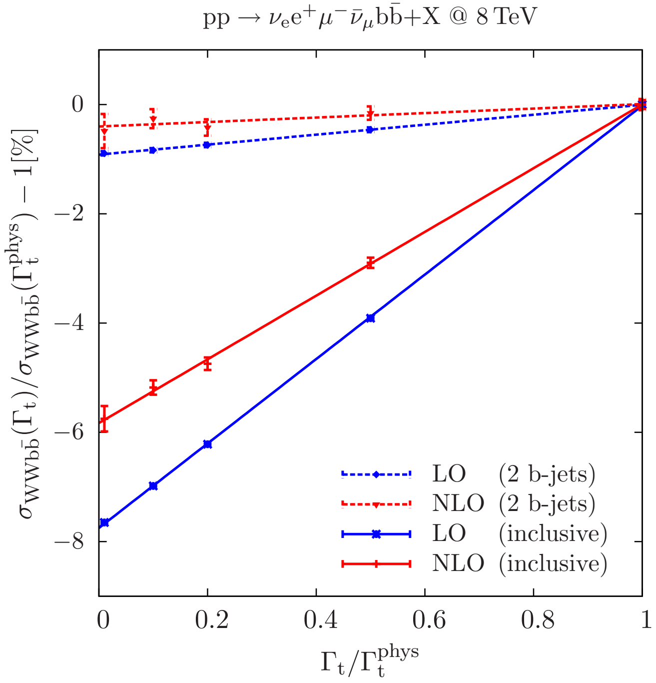
$\mathrm{pp} \to \mathrm{W^+W^-b\bar b}$ **in 5F scheme** [Denner, Dittmaier, Kallweit, S.P. ‘10; Bevilacqua et al. ’10; Heinrich et al. ‘13]

- off-shell, single- and non-resonant contributions
- small $\mathcal{O}(\Gamma_\mathrm{t}/m_\mathrm{t})$ effects for “inclusive” $\mathrm{t\bar t}$ cuts
- $m_\mathrm{b} = 0$ approx. requires two hard b-jets ($\mathrm{g} \to \mathrm{b\bar b}$ collinear singularities)

$\mathrm{pp} \to \mathrm{W^+W^-b\bar b}$ **in 4F scheme** [Frederix’13; Cascioli,Kallweit,Maieröfer,S.P. ’13]

- full b-quark phase space thanks to $m_\mathrm{b} > 0$
- first consistent $\mathrm{t\bar t}$ and Wt combination with interference at NLO
- important for top-backgrounds in 0- and 1-jet bins (e.g. in $\mathrm{H} \to \mathrm{WW}$)
- challenging multi-particle, multi-resonance, multi-scale $(m_\mathrm{b}, \ldots, m_{\mathrm{t\bar t}})$ process

Gauge-invariant separation of $\mathrm{t\bar{t}}$ and complementary contributions



Numerical NWA

$$d\sigma_{\mathrm{t\bar{t}}} = \lim_{\Gamma_{\mathrm{t}}\to 0}\left(\frac{\Gamma_{\mathrm{t}}}{\Gamma_{\mathrm{t}}^{\mathrm{phys}}}\right)^2 d\sigma_{\mathrm{W^+W^-b\bar{b}}}(\Gamma_{\mathrm{t}})$$

- permille-level convergence shows cancellation of soft-gluon $\ln(\Gamma_{\mathrm{t}}/m_{\mathrm{t}})$ singularities
- yields on-shell $\mathrm{t\bar{t}}$ production and decay

Finite-top-width remainder (FtW)

- contains all $\mathcal{O}(\Gamma_{\mathrm{t}}/m_{\mathrm{t}})$ effects: off-shell $\mathrm{t\bar{t}}$ production, single-top and non-resonant contributions with interferences
- from sub-percent for 2 b-jet final states to 6–8% effect in inclusive case

Ad-hoc dynamic scale choice

Idea: avoid large logs due to multi-channel/multi-scale nature of $\mathrm{W^+W^-b\bar{b}}$

- using $\mu_\mathrm{R} \sim m_\mathrm{t}$ also for $\mathrm{g \to b\bar{b}}$ splittings might generate corrections up to $\alpha_S(m_\mathrm{b})/\alpha_S(m_\mathrm{t}) \sim 2$ in Wt contribution

Appropriate scales for $\mathrm{t\bar{t}}$ **and** Wt **production**

$$\mu^2_\mathrm{t\bar{t}} = E_\mathrm{T,t} E_\mathrm{T,\bar{t}} \qquad \mu^2_\mathrm{tW^-} = E_\mathrm{T,t} E_\mathrm{T,\bar{b}} \qquad \Rightarrow \qquad \alpha^2_\mathrm{S}(\mu^2_\mathrm{tW^-}) \simeq \alpha_\mathrm{S}(E^2_\mathrm{T,t})\alpha_\mathrm{S}(E^2_\mathrm{T,\bar{b}})$$

Global "interpolating scale"

$$\mu^2_\mathrm{WWbb} = \mu_\mathrm{W^+b}\,\mu_\mathrm{W^-\bar{b}} \quad \text{with} \quad \mu_\mathrm{Wb} = P_\mathrm{b}(p_\mathrm{W,b})\,E_\mathrm{T,b} + P_\mathrm{t}(p_\mathrm{W,b})\,E_\mathrm{T,t}$$

$\mathrm{g \to b\bar{b}}$ and $\mathrm{t \to Wb}$ **probabilities dictated by** respective **singularity structures**

$$\frac{P_\mathrm{b}}{P_\mathrm{t}} \propto \frac{\chi_\mathrm{b}}{\chi_\mathrm{t}} \qquad \text{with} \qquad \chi_\mathrm{b} = \frac{m_\mathrm{t}^2}{E^2_\mathrm{T,b}}, \quad \chi_\mathrm{t} = \frac{m_\mathrm{t}^4}{[(p_\mathrm{W}+p_\mathrm{b})^2 - m_\mathrm{t}^2]^2 + \Gamma_\mathrm{t}^2 m_\mathrm{t}^2},$$

and free constants fixed by **natural normalisation conditions**

$$P_\mathrm{b} + P_\mathrm{t} = 1, \qquad \text{and} \qquad \int \mathrm{d}\sigma^\mathrm{FtW}_\mathrm{W^+W^-b\bar{b}} = \int \mathrm{d}\Phi\,[1 - P_\mathrm{t}(\Phi)P_\mathrm{\bar{t}}(\Phi)]\,\frac{\mathrm{d}\sigma_\mathrm{W^+W^-b\bar{b}}}{\mathrm{d}\Phi}$$

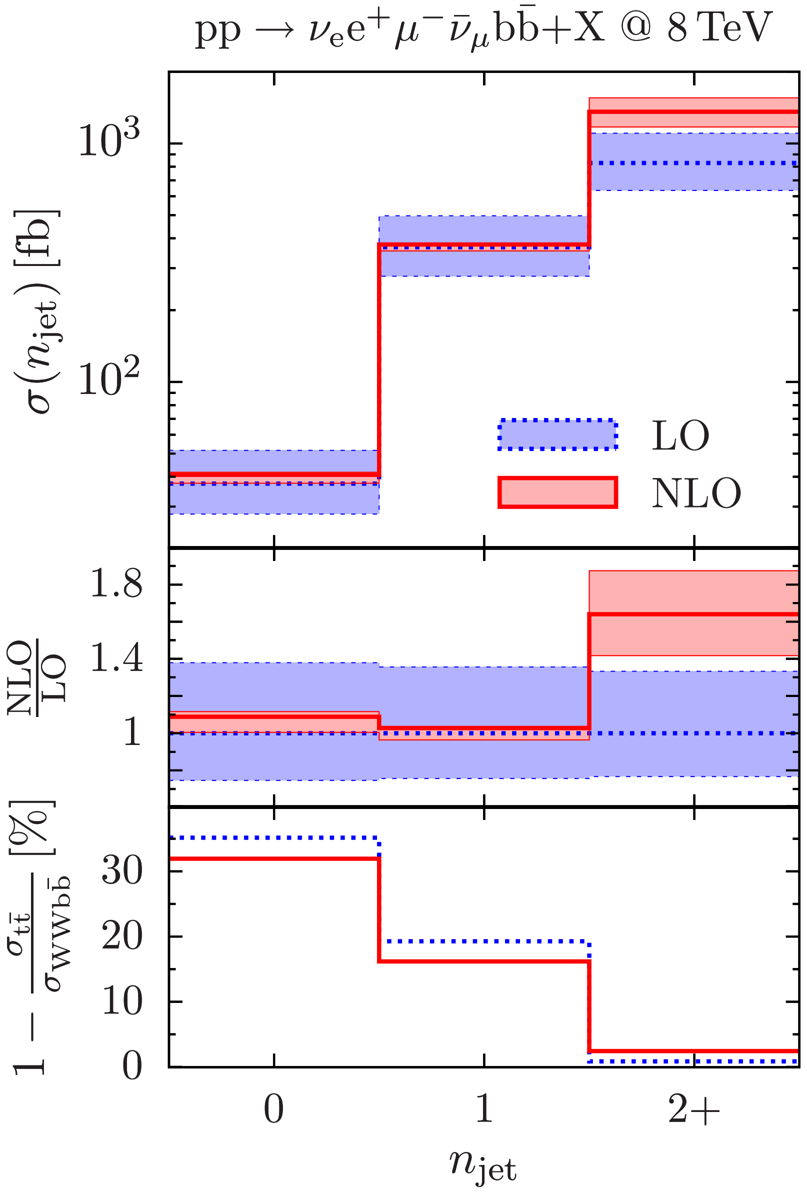
NLO and FtW effects in jet bins ($p_\mathrm{T} > 30\,\mathrm{GeV}$)

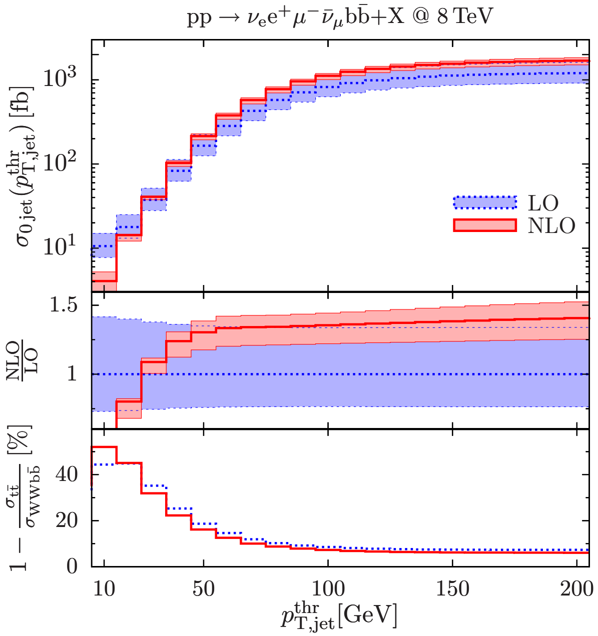
Jet bins relevant for $\mathrm{t\bar{t}}$-suppression and most interesting application of $m_\mathrm{b} > 0$

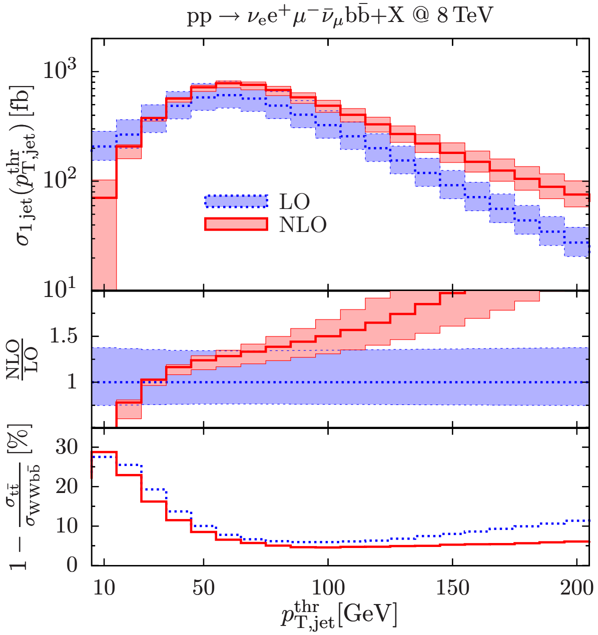
- 40% inclusive NLO correction driven by 2-jet bin, with very stable 0/1-jet bins
- only ∼10% NLO uncertainty in all bins!
- FtW contribution bin-dependent (2% to 30%) and strongly enhanced in 0/1-jet bins!
- also FtW part perturbatively stable (not shown here)

Success of "ad-hoc" scale choice

- but naive $\mu = m_\mathrm{t}$ choice yields surprisingly similar stability in jet bins!
- "ad-hoc scale" might be superior for more exclusive observables…







Jet-Veto and Binning Effects

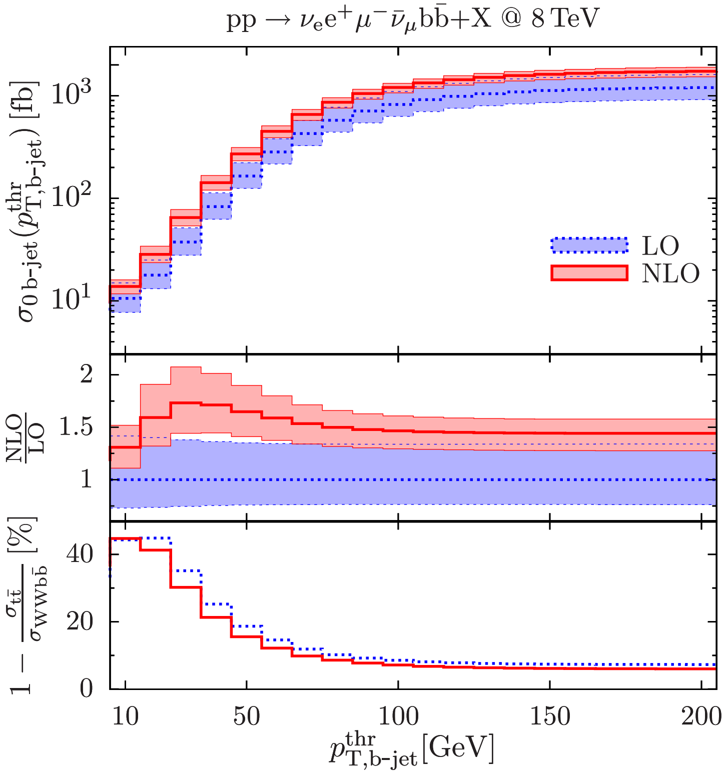
0-jet bin vs p_{T}-veto

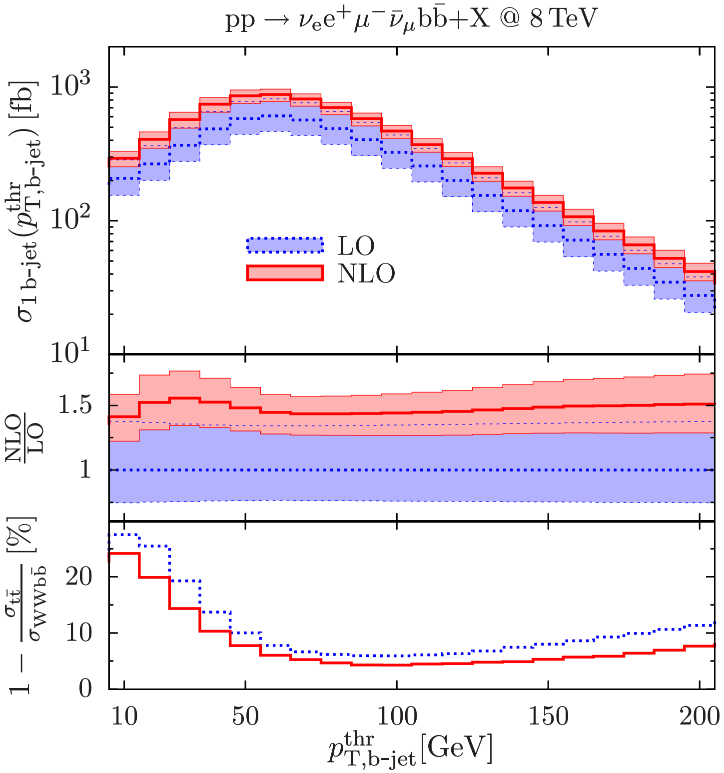
- smooth inclusive limit at large p_{T} and very strong p_{T} sensitivity below 50 GeV:
  - FtW effects increase up to 50%
  - K-factor falls very fast
- at low p_{T} IR singularity calls for NLO+PS matching
- typical veto $p_{\mathrm{T}} \sim 30$ GeV yields 98% suppression and still decent NLO stability ($K \sim 1$)

1-jet bin vs p_{T} threshold

- low p_{T} behaviour driven by veto on 2nd jet and analogous to 0-jet case
- high p_{T} region driven by 1st jet and NLO radiation dominates over b-jets from $W^+W^-b\bar{b}$

B-Jet-Veto and Binning Effects

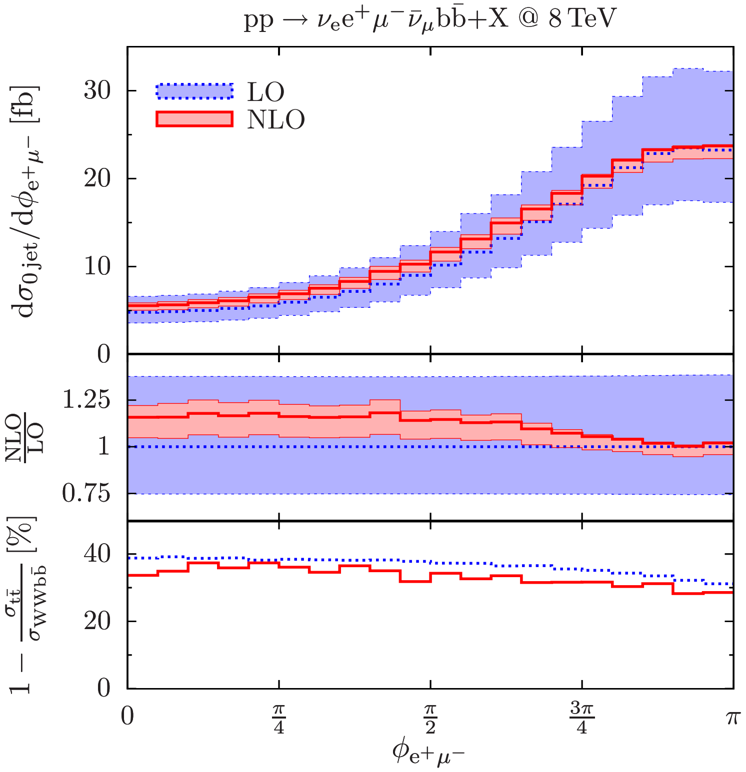


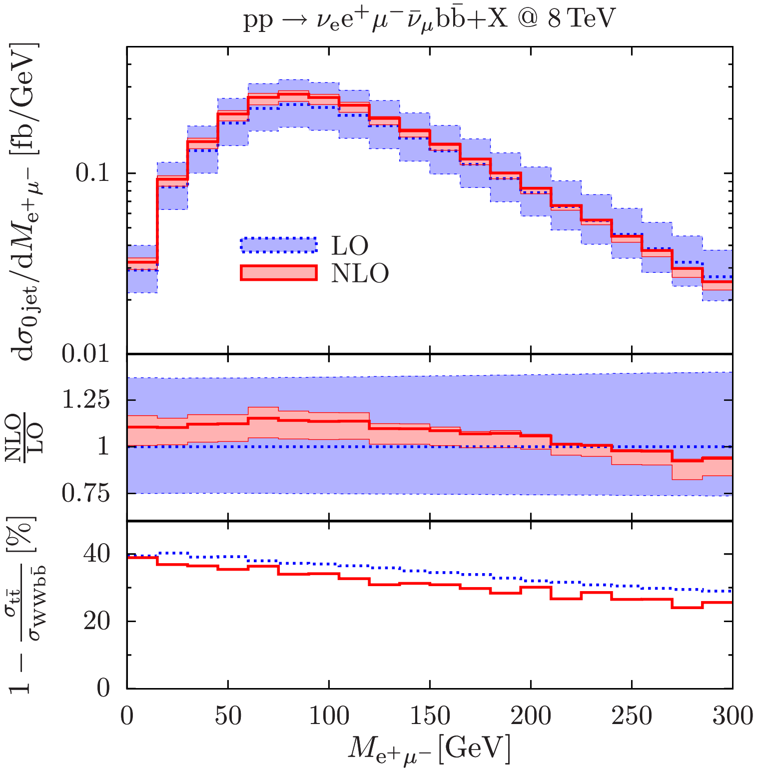


- NLO radiation doesn't change b-jet multiplicity ⇒ rather stable K-factor and uncertainties
- single-top and off-shell effects still enhanced at small b-jet p_T

In general: nontrivial interplay of NLO and off-shell/single-top effects

$t\bar{t}$ and Wt background to $H \to W^+W^-$ in 0-jet bin

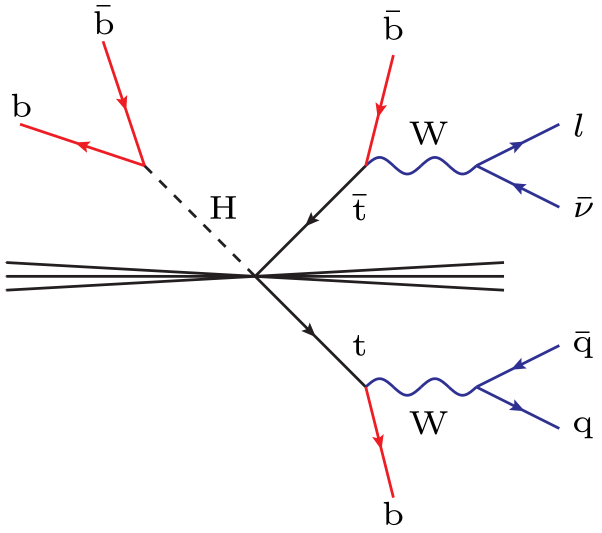




- $\Delta\phi_{e^+\mu^-}$ and $M_{e^+\mu^-}$ distributions feature 10% NLO uncertainty
- significant (although moderate) NLO shape distortions
- 30–40% FtW contributions (nontrivial $t\bar{t}$/Wt mix)

(D) MC@NLO for 4F $t\bar{t}b\bar{b}$ production [Cascioli,Maieröfer,Moretti,S.P. ,Siegert '13]

$\mathrm{t\bar{t}H(b\bar{b})}$ **Analyses at the LHC and Irreducible** $\mathrm{t\bar{t}b\bar{b}}$ **Background**



- complicated $\mathrm{b\bar{b}b\bar{b}}\ell\nu jj$ final state hampers $\mathrm{H \to b\bar{b}}$ peak reconstruction

- signal still hidden in huge QCD background and search dominated by systematics

- theory uncertainty of irreducible $\mathrm{t\bar{t}b\bar{b}}$ background crucial (normalisation in control region quite difficult)

Theory predictions for $\mathrm{t\bar{t}b\bar{b}}$ **background**

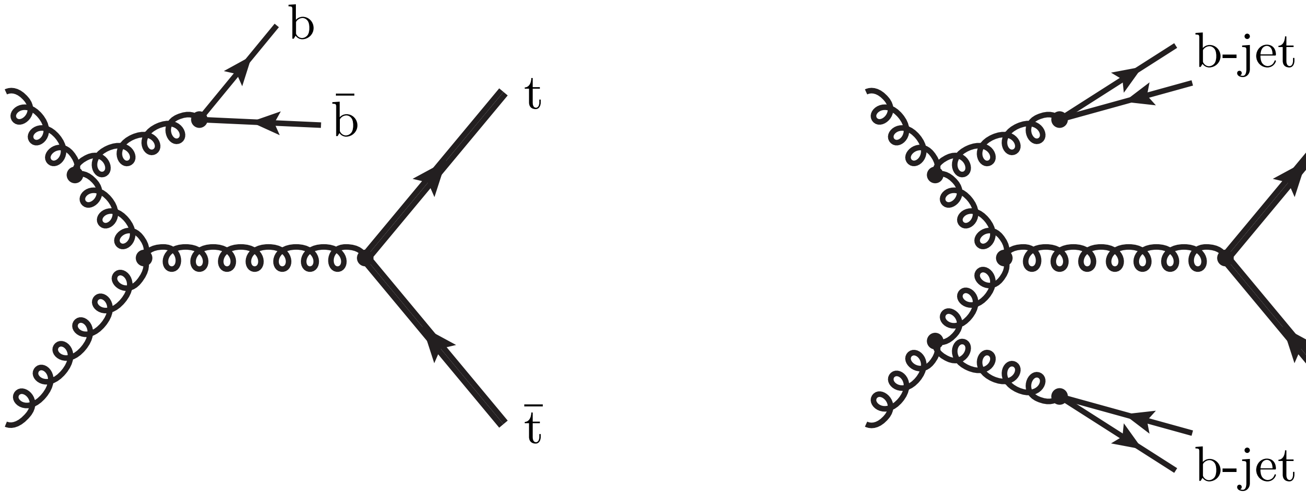
- NLO reduces scale uncertainty from 80% to 20–30% [Bredenstein, Denner, Dittmaier, S. P. '09/'10; Bevilacqua, Czakon, Papadopoulos, Pittau, Worek '09]

- application to ATLAS/CMS analyses requires matching to parton showers

→ POWHEG matching in **5F scheme** [Kardos, Trocsanyi '13]

→ Sherpa-MC@NLO matching in **4F scheme** [Cascioli, Maierhoefer, Moretti, S. P., Siegert '13]

NLO matching for $\mathrm{t\bar{t}b\bar{b}}$ production in 5F vs 4F schemes



5F scheme ($m_\mathrm{b} = 0$): $\mathrm{t\bar{t}b\bar{b}}$ MEs cannot describe collinear $\mathrm{g} \to \mathrm{b\bar{b}}$ splittings

⇒ inclusive $\mathrm{t\bar{t}}$+b-jets simulation requires $\mathrm{t\bar{t}g}$+PS, i.e. $\mathrm{t\bar{t}}+ \leq 2$ jets NLO merging

4F scheme ($m_\mathrm{b} > 0$): $\mathrm{t\bar{t}b\bar{b}}$ MEs cover full b-quark phase space

⇒ MC@NLO $\mathrm{t\bar{t}b\bar{b}}$ sufficient for inclusive $\mathrm{t\bar{t}}$+b-jets simulation

- access to **new** $\mathrm{t\bar{t}} + 2$**b-jets production mechanism** wrt 5F scheme: double collinear $\mathrm{g} \to \mathrm{b\bar{b}}$ splittings (surprisingly important impact on $\mathrm{t\bar{t}H(b\bar{b})}$ analysis!)

Sherpa Formulation [Höche, Krauss, Schönherr, Siegert '11] of MC@NLO Matching [Frixione, Webber '02]

Sherpa parton shower based on CS dipoles (no Pythia/Herwig)

$$U(t_0,\mu_Q^2) = \Delta(t_0,\mu_Q^2)\mathcal{O}(\Phi_B) + \sum_{ijk}\int_{t_0}^{\mu_Q^2} \mathrm{d}\Phi_{R|B}\,\frac{D_{ijk}(\Phi_R)}{B(\Phi_B)}\Delta(t,\mu_Q^2)\mathcal{O}(\Phi_R),$$

MC@NLO matching (in-house PS ⇒ exact and automated colour treatment)

$$\langle\mathcal{O}\rangle = \int \mathrm{d}\Phi_B \Big[B(\Phi_B) + V(\Phi_B) + I(\Phi_B)\Big] U(t_0,\mu_Q^2)$$
$$+ \int \mathrm{d}\Phi_R \Big[R(\Phi_R) - \sum_{ijk} D_{ijk}(\Phi_R)\theta(\mu_Q^2 - t)\Big]\mathcal{O}(\Phi_R).$$

Integrated subtraction terms (note resummation scale μ_Q!)

$$I(\Phi_B) = \sum_{ijk}\int \mathrm{d}\Phi_{R|B} D_{ijk}(\Phi_R)\theta(\mu_Q^2 - t),$$

Resummation scale μ_Q (parton-shower starting scale) restricts shower to meaningful region and its variations provide systematic shower-uncertainty estimates

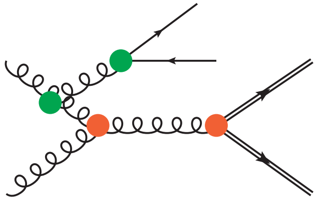
Scale choice and b-jet selections

Factorisation and Resummation scales (available phase space for QCD emission)

$$\mu_{\rm F} = \mu_Q = \frac{1}{2}(E_{\rm T,t} + E_{\rm T,\bar{t}})$$

Scale choice crucial due to $\alpha_S^4(\mu^2)$ dependence (80% LO variation)

- widely separated scales $m_{\rm b} \le Q_{ij} \lesssim m_{\rm t\bar{t}b\bar{b}}$ can generate huge logs
- CKKW inspired scale adapts to b-jet $p_{\rm T}$ and guarantees good pert. convergence



$$\mu_{\rm R}^4 = E_{\rm T,t}E_{\rm T,\bar{t}}E_{\rm T,b}E_{\rm T,\bar{b}} \quad \Rightarrow \quad \alpha_S^4(\mu_{\rm R}^2) = \alpha_S(E^2_{\rm T,t})\alpha_S(E^2_{\rm T,\bar{t}})\alpha_S(E^2_{\rm T,b})\alpha_S(E^2_{\rm T,\bar{b}})$$

***ttb*, *ttbb* and *ttbb*$_{100}$ analyses with stable tops**

- *ttb* analysis ($N_{\rm b} \ge 1$)
- *ttbb* analysis ($N_{\rm b} \ge 2$)
- *ttbb*$_{100}$ ($N_{\rm b} \ge 2$) analysis in the signal region $m_{\rm bb} > 100\,{\rm GeV}$

($N_{\rm b}$= number of QCD b-jets with $p_{\rm T} > 25\,{\rm GeV}$, $|\eta| < 2.5$ and at least one b-quark)

NLO corrections and uncertainties for *ttb* and *ttbb* cross sections

	ttb	ttbb	ttbb($m_{\mathrm{bb}} > 100$)
σ_{LO}[fb]	$2547^{+71\%+14\%}_{-37\%-11\%}$	$463.9^{+66\%+15\%}_{-36\%-12\%}$	$123.7^{+62\%+17\%}_{-35\%-13\%}$
σ_{NLO}[fb]	$3192^{+33\%+4.6\%}_{-25\%-4.9\%}$	$557^{+28\%+5.6\%}_{-24\%-4.0\%}$	$141^{+25\%+8.6\%}_{-22\%-3.8\%}$
$\sigma_{\mathrm{NLO}}/\sigma_{\mathrm{LO}}$	1.25	1.20	1.14

Good perturbative convergence (also for *ttb*)

- K-factors and uncertainties rather independent of selection
- +20% correction mainly from b-quark contribution to α_{S} running in 4F scheme ($K \simeq 1$ with 5F running)
- 20–30% residual uncertainty dominated by μ_{R} variations (1st uncertainty)
- only 5-10% uncertainty from combined μ_{F} and μ_Q variations (2nd uncertainty)

M@NLO corrections wrt NLO in *ttb* and *ttbb* cross sections

	ttb	ttbb	ttbb($m_{\mathrm{bb}} > 100$)
$\sigma_{\mathrm{MC@NLO}}$[fb]	$3223^{+33\%+4.3\%}_{-25\%-2.5\%}$	$607^{+25\%+2.2\%}_{-22\%-2.8\%}$	$186^{+21\%+5.4\%}_{-20\%-4.7\%}$
$\sigma_{\mathrm{MC@NLO}}/\sigma_{\mathrm{NLO}}$	1.01	1.09	1.32
$\sigma^{2\mathrm{b}}_{\mathrm{MC@NLO}}$[fb]	3176	539	145
$\sigma^{2\mathrm{b}}_{\mathrm{MC@NLO}}/\sigma_{\mathrm{NLO}}$	0.99	0.97	1.03

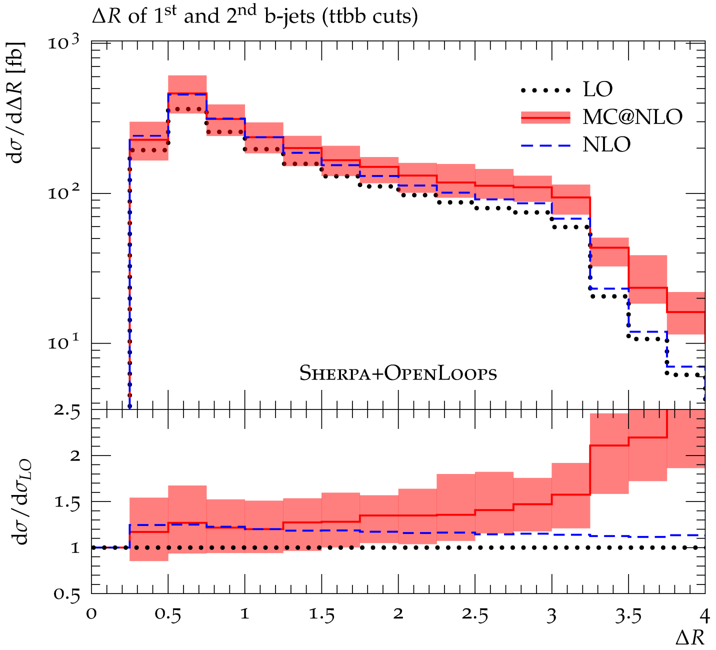
Nontrivial MC@NLO effects

- μ_{R}, μ_{F} and μ_Q uncertainties similar as for NLO
- negligible(moderate) MC@NLO/NLO differences with standard *ttb*(*ttbb*) selections
- large MC@NLO effect (∼30%) in Higgs-signal region of *ttbb*
- disappears in MC@NLO$_{2\mathrm{b}}$, where $\mathrm{g} \to \mathrm{b}\bar{\mathrm{b}}$ shower splittings are switched off (see more details in distributions)

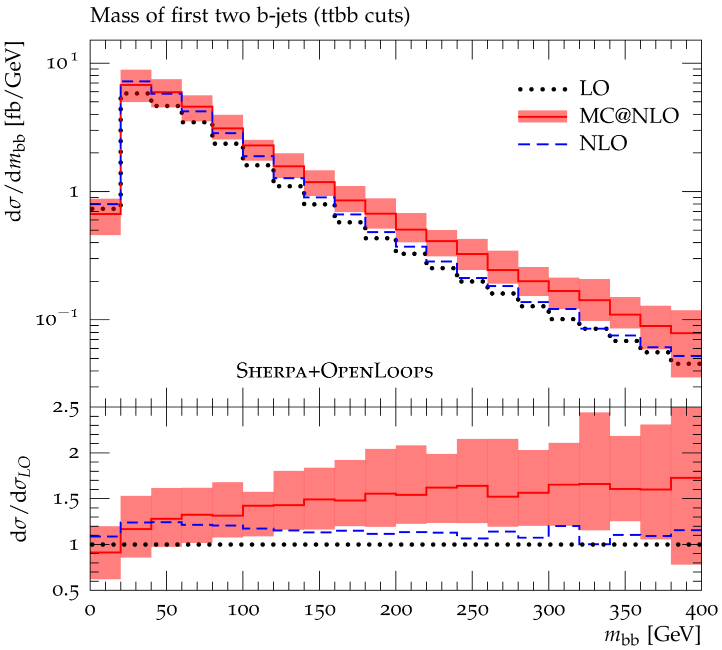
NLO and MC@NLO effects in distributions

***ttbb* analysis ($N_b \geq 2$):** b-jet correlations

$\Delta R_{\mathrm{b}_1\mathrm{b}_2}$



$m_{\mathrm{b}_1\mathrm{b}_2}$

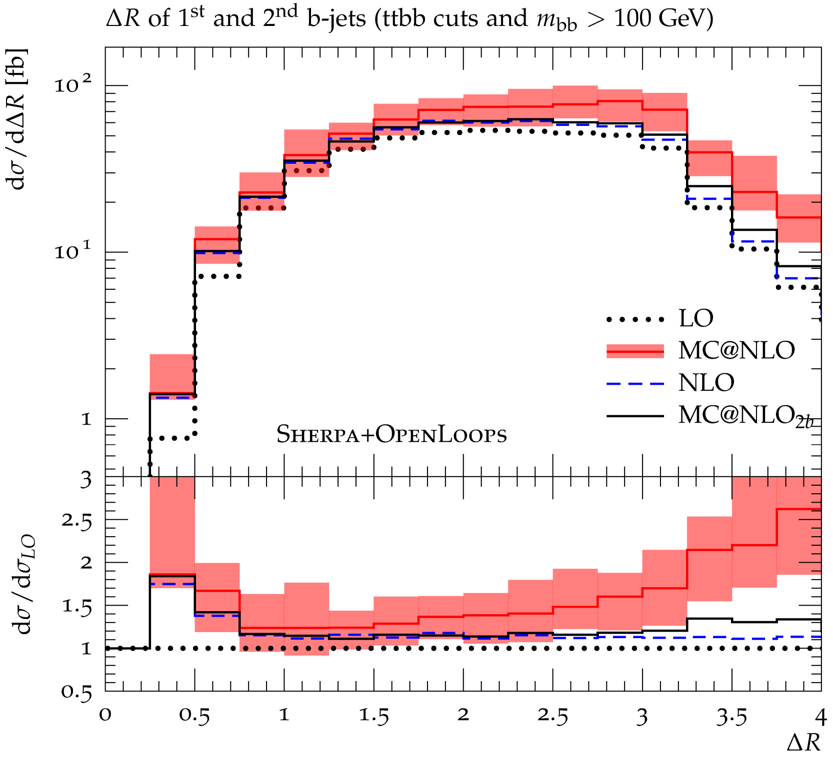


Unexpected behaviour

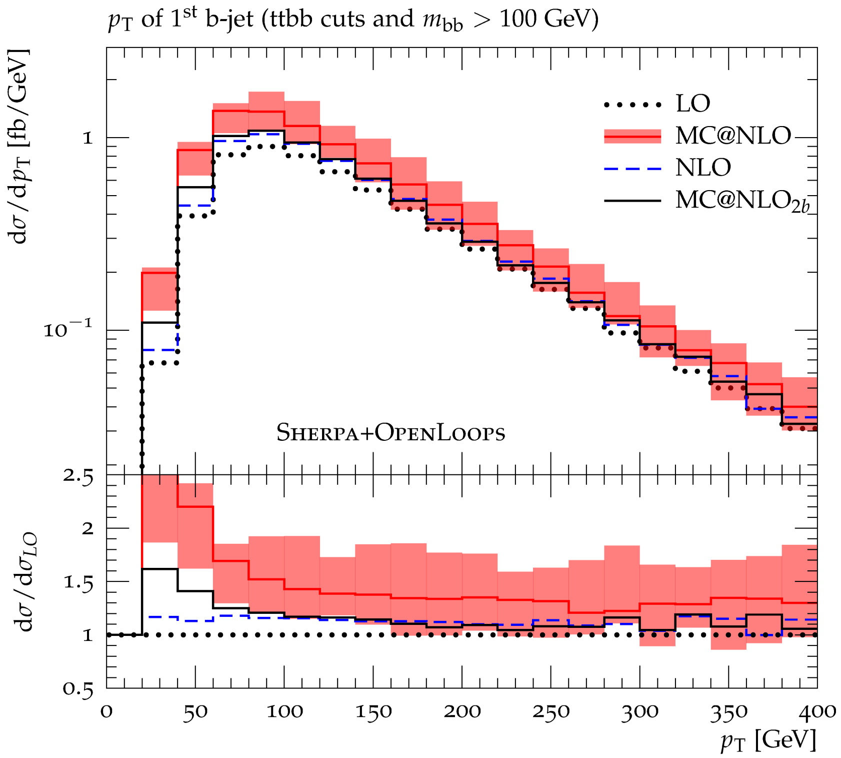
- NLO corrections quite flat
- **pronounced MC@NLO enhancement at large $\Delta R_{\mathrm{b}_1\mathrm{b}_2}$ and large $m_{\mathrm{b}_1\mathrm{b}_2}$**
- reaches 30–40% at $m_{\mathrm{b}_1\mathrm{b}_2} \sim 125\,\mathrm{GeV}$ (largely exceeds $\mathrm{t\bar{t}H(b\bar{b})}$ signal!)

$ttbb$ **analysis** $(N_b \geq 2)$ **with** $m_{\mathrm{b_1 b_2}} > 100\,\mathrm{GeV}$: b-jet observables

$\Delta R_{\mathrm{b_1 b_2}}$

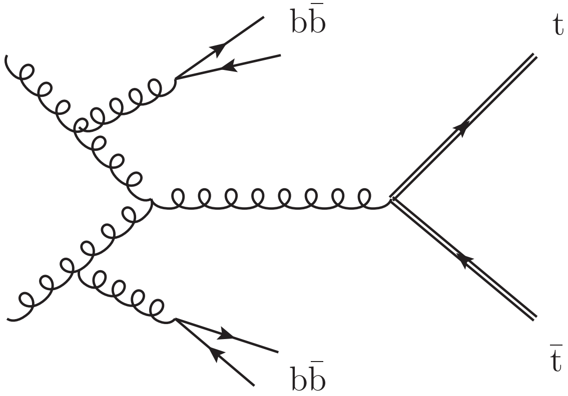


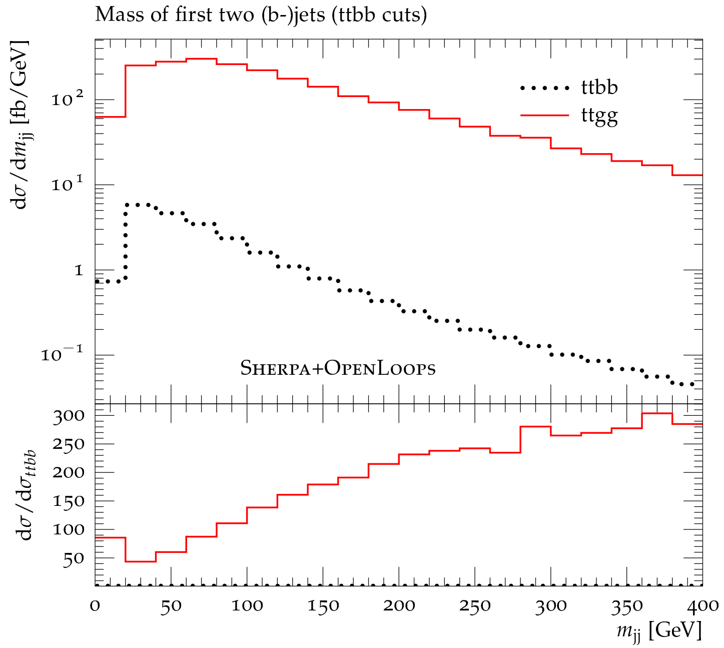
$p_{\mathrm{T,b_1}}$



MC@NLO excess at large m_{bb} **from back-to-back soft jets**

- factor-2 enhancement at $\Delta R \sim \pi$ and at small p_{T}
- disappears almost completely in MC@NLO$_{\mathrm{2b}}$ where $\mathrm{g} \to \mathrm{b\bar{b}}$ splittings are switched off in the parton shower (double $\mathrm{g} \to \mathrm{b\bar{b}}$ splittings "smoking gun")

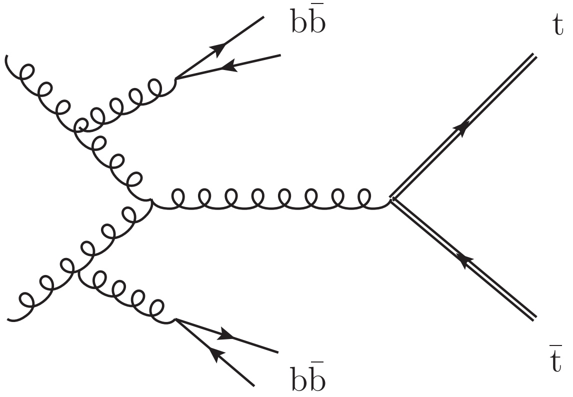




MC@NLO enhancement consistent with double $g \to b\bar{b}$ splittings mechanism

- "double splittings" kinematically favoured at large m_{bb} since $t\bar{t}gg/t\bar{t}b\bar{b}$ ratio grows and $g \to b\bar{b}$ splitting probability does not decrease at large m_{gg}
- emission of parent gluons is strongly enhanced at small p_T due to double (soft-collinear) singularity associated to IS gluon emission $\Rightarrow$ at large invariant mass the di-jet system tends to have the smallest possible p_T and $\Delta R \sim \pi$
- kinematic reconstruction of double $g \to b\bar{b}$ splitting nontrivial since typically $\Delta R_{b\bar{b}} > 0.4$ and one of the b-quarks can be outside acceptance

Implications of (double) $\mathrm{g} \to \mathrm{b\bar{b}}$ splitting contributions



Double splittings change conventional hard-scattering picture

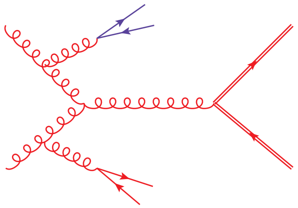
- this kind of contributions have always been present in $\mathrm{t\bar{t}}$+jets LO merged samples
- however, their large impact on the $\mathrm{t\bar{t}H(b\bar{b})}$ signal region is surprising and does not fit into the conventional hard-scattering picture of $\mathrm{t\bar{t}b\bar{b}}$ production based on a *single* and *non-collinear* $\mathrm{b\bar{b}}$ pair

Implications for theory systematics in $\mathrm{t\bar{t}}$+HF

- matching to shower essential (4F $\mathrm{t\bar{t}b\bar{b}}$ NLO matching or 5F $\mathrm{t\bar{t}}$+jets NLO merging)
- MC@NLO $\mathrm{t\bar{t}b\bar{b}}$ simulation provides NLO accuracy for $\mathrm{t\bar{t}}$+2 b-jets with hard b-quark jets: NLO or LO+PS accuracy for “double-splittings”?

Accuracy of “double splittings” in MC@NLO $ttbb$ simulation

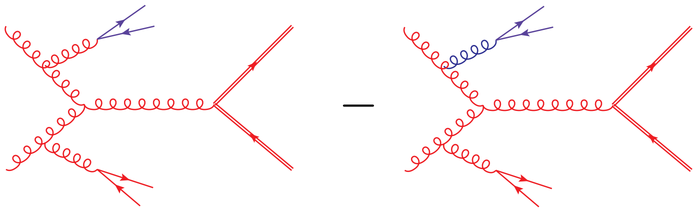
Naive picture



real-emission $\mathrm{t\bar{t}b\bar{b}g}$ MEs plus $\mathrm{g} \to \mathrm{b\bar{b}}$ shower splitting

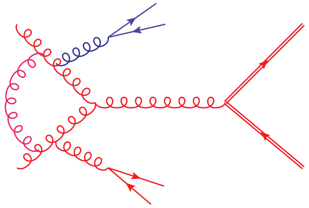
⇒ only LO+PS accuracy as in usual LO merging

Correct MC@NLO picture: interplay of three different contributions



$\mathrm{t\bar{t}b\bar{b}g}$ MEs plus PS $\mathrm{g} \to \mathrm{b\bar{b}}$ emission

- LO $\mathrm{t\bar{t}b\bar{b}g}$ uncertainty ∼100% at large p_{T}
- largely cancelled by PS-matching at small p_{T}

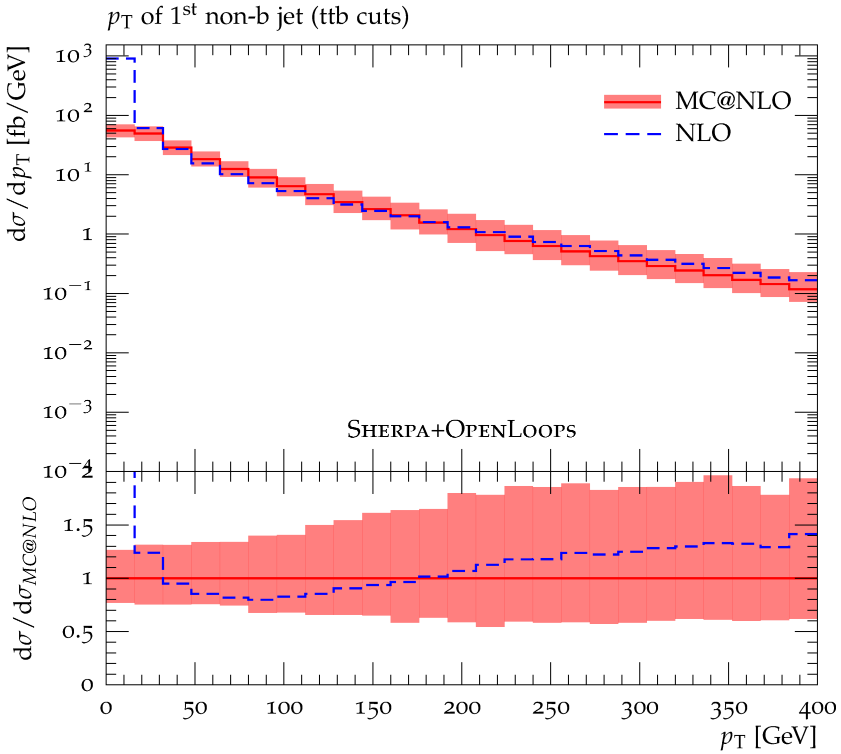


$\mathrm{t\bar{t}b\bar{b}}$ MEs plus PS gluon and $\mathrm{g} \to \mathrm{b\bar{b}}$ emissions

- dominates at small p_{T}
- NLO $\mathrm{t\bar{t}b\bar{b}}$ accuracy ∼25%

Well reflected in scale uncertainty of 1^{st} light-jet emission on top of $\mathrm{t\bar{t}b\bar{b}}$…

ttb **analysis** ($N_b \geq 1$)**:** 1^{st} light-jet p_{T} distribution (responsible for double splittings)



MC@NLO vs NLO

- Sudakov damping of NLO IR singularity at $p_{\text{T}} \to 0$
- 30% NLO excess in the hard tail (probably due to dynamic μ_Q, multi-jet final state, unresolved b-quark)

MC@NLO scale uncertainty

- LO-like uncertainty (∼100%) in the tail irrelevant for $\mathrm{t\bar{t}H(b\bar{b})}$
- NLO-like accuracy (∼30%) up to 70 GeV

⇒ **NLO-like accuracy in the region relevant for** $\mathrm{t\bar{t}H(b\bar{b})}$

Conclusions

OpenLoops

- handles $2 \to 2, 3, 4$ SM process at NLO QCD very efficiently
- well tested, working for nontrivial LHC studies, ready for publication

Examples of first applications ($\mathrm{W^+W^-b\bar{b}}$ and $\mathrm{t\bar{t}b\bar{b}}$)

- $m_\mathrm{b} > 0$ and NLO matching give access to new important physics ingredients (single-top, double splittings) and crucial for applicability to exp analysis
- ~ 4 years after first NLO papers (2009, 2011) and not yet the end of the story (top decays in $\mathrm{t\bar{t}b\bar{b}}$, NLO matching for $\mathrm{W^+W^-b\bar{b}}$, nontrivial pheno applications like m_t measurements,. . .)

Lesson

- NLO $\mathrm{t\bar{t}}$ still very active business 25 years after first pioneering result
- NLO automation is just moving the first (very promising) steps
- the very wide applicability range and high relevance for the LHC will stimulate further exciting progress