Some summits reached and the climb ahead

ZPW2014
9 January, 2014

James "Jamie" Gainer
University of Florida

Outline

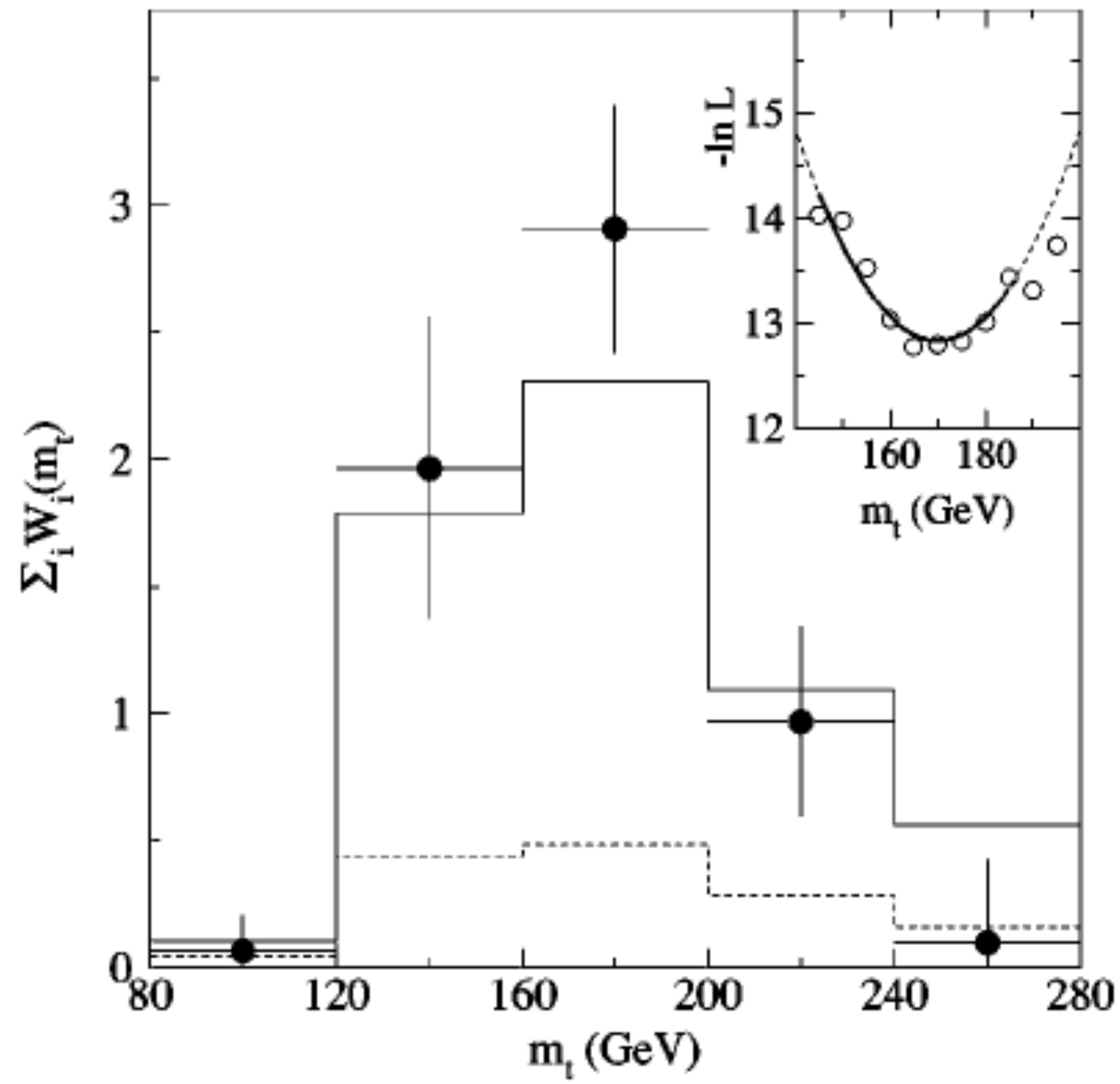
- Some Summits Reached
  - Focus on Higgs discovery in 4ℓ
- Climb ahead
  - Higgs Properties Measurement
  - The Future of the MEM

Matrix Element Method

- MEM = The use of the likelihood for all kinematic variables as a test statistic.
- This likelihood is, in some well-defined sense, the optimal test statistic (Neyman-Pearson lemma).
- This likelihood is taken to be the normalized differential cross section
  (~ squared matrix element)
  for the appropriate parton level process
- If there are invisible particles in the final state, need to integrate over their potential momenta.
- One takes finite detector resolution into account by integrating over transfer functions that give the probability of true momenta for given observed momenta.

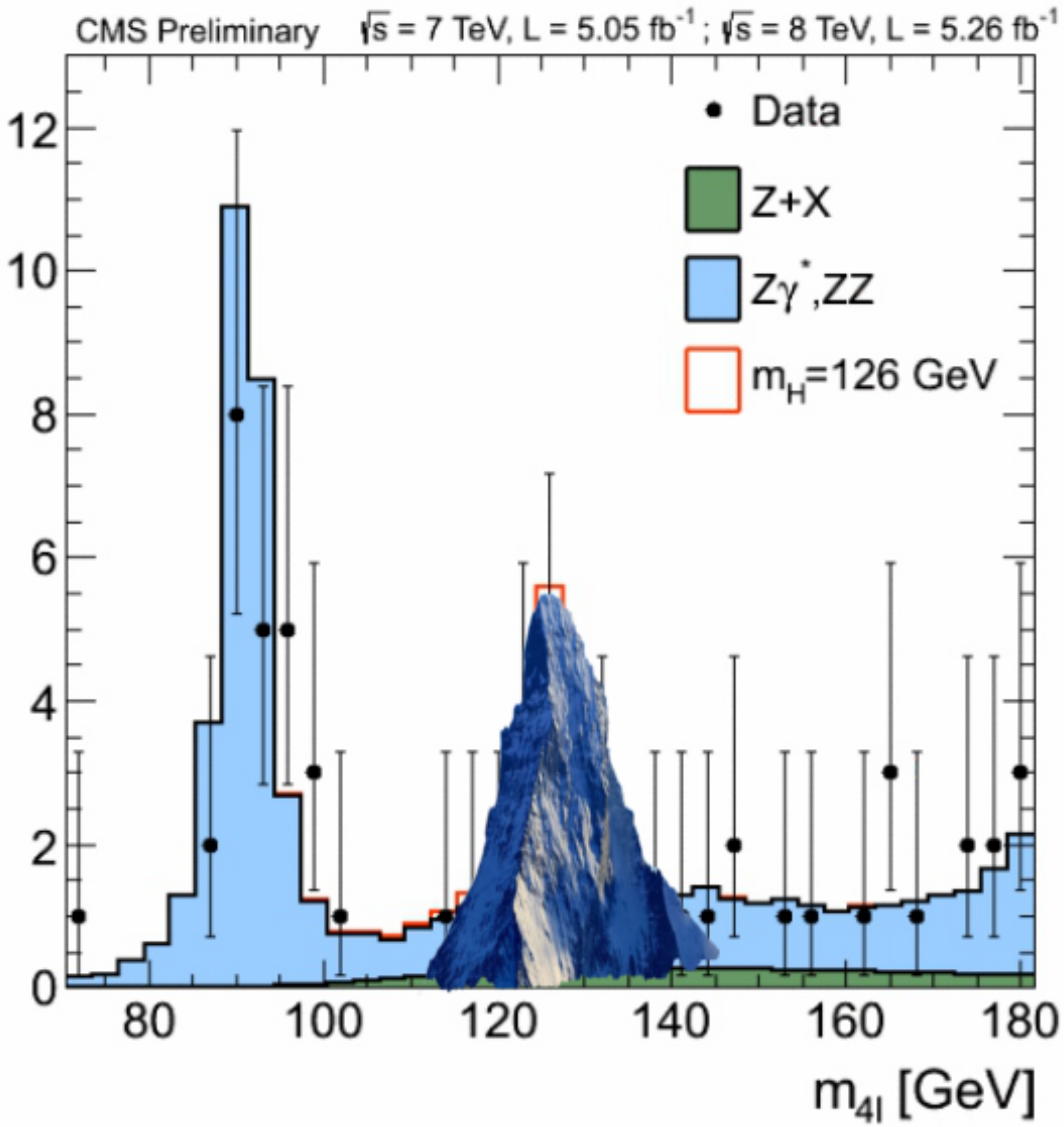
Some Summits Reached

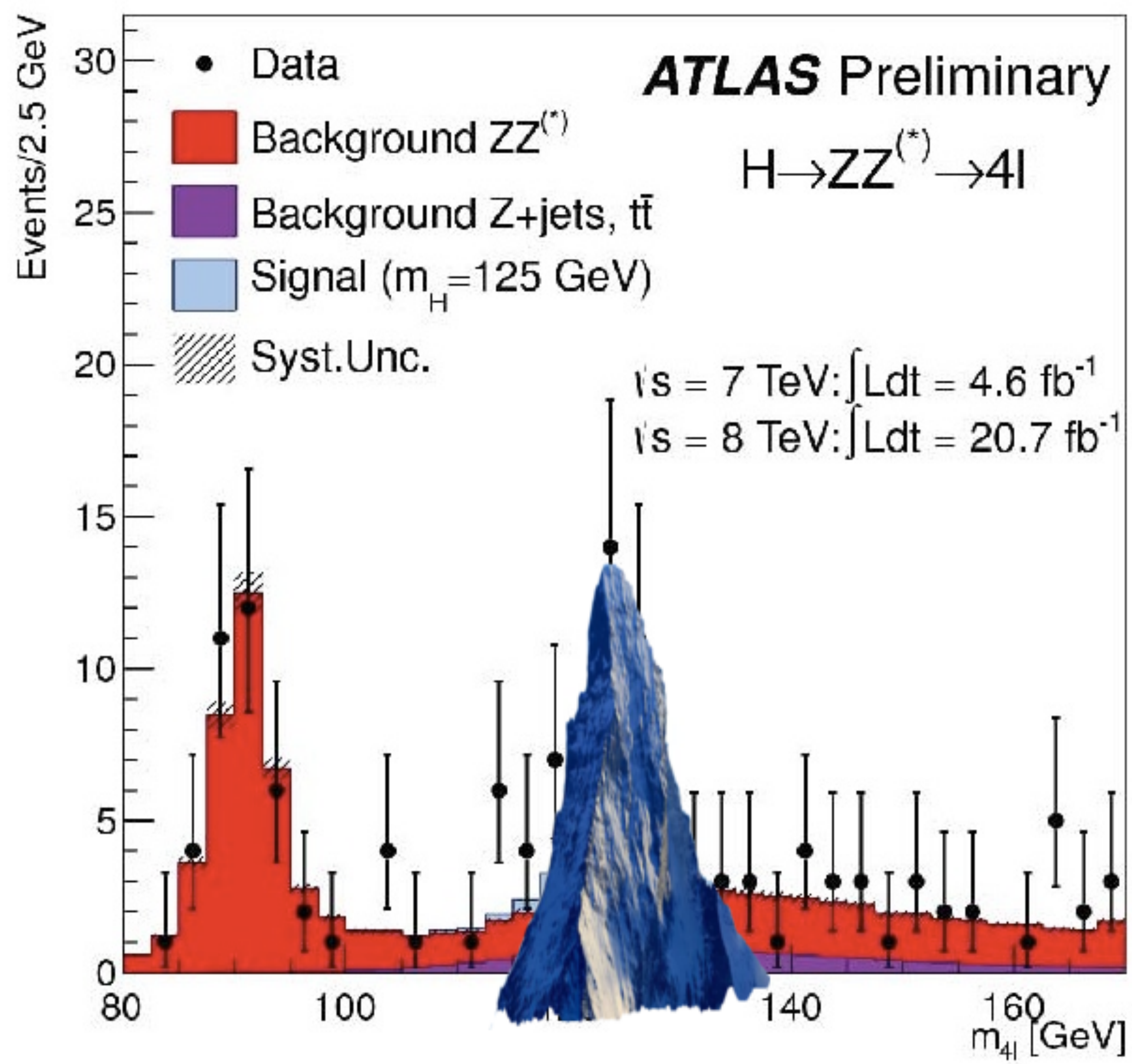
- Top Mass Measurement
- B physics
- Single top
- …
- Other talks at this workshop



From D0 top mass measurement: PRD 60, 052001.

MEM for Higgs Discovery in H→ZZ →4ℓ





- Can the MEM help distinguish Higgs signal from background for 4ℓ final states?
- If so, how? What is the physical reason for the sensitivity?

Physics of Signal-Background Separation

- **Initial state radiation.**
  SM signal is mostly gg, irreducible background is from $q\bar{q}$.
  (See Ciaran Williams' talk on how to include NLO effects)

- **Rapidity distributions.**
  Also sensitive to different initial states, though demanding four central leptons forces $|\eta|$ for the event to be relatively small, reducing the ability to separate signal and background.

- **Z polarizations.**
  Z polarizations different for different signal, background processes. See next slide.

- **Propagators.**
  Invariant mass distribution of off-shell "Z" different depending on likelihood for that resonance to be γ^* versus Z^*.

ZZ Polarizations

$$\Delta\lambda = \pm 2 : \quad \mathcal{A}^{\Delta\sigma}_{\pm\mp} = -\sqrt{2}(1+\beta_1\beta_2)\ ,$$

$$\begin{aligned}\Delta\lambda = \pm 1 : \quad \mathcal{A}^{\Delta\sigma}_{\pm 0} = \frac{1}{\gamma_2(1+x)}\Bigg[&(\Delta\sigma\Delta\lambda)\left(1+\frac{\beta_1^2+\beta_2^2}{2}\right) - 2\cos\Theta \\ &-(\Delta\sigma\Delta\lambda)(\beta_2^2-\beta_1^2)x - 2x\cos\Theta - (\Delta\sigma\Delta\lambda)\left(1-\frac{\beta_1^2+\beta_2^2}{2}\right)x^2\Bigg]\end{aligned}$$

$$\begin{aligned}: \quad \mathcal{A}^{\Delta\sigma}_{0\pm} = \frac{1}{\gamma_1(1-x)}\Bigg[&(\Delta\sigma\Delta\lambda)\left(1+\frac{\beta_1^2+\beta_2^2}{2}\right) - 2\cos\Theta \\ &-(\Delta\sigma\Delta\lambda)(\beta_2^2-\beta_1^2)x + 2x\cos\Theta - (\Delta\sigma\Delta\lambda)\left(1-\frac{\beta_1^2+\beta_2^2}{2}\right)x^2\Bigg]\end{aligned}$$

$$\Delta\lambda = 0 : \quad \mathcal{A}^{\Delta\sigma}_{\pm\pm} = -(1-\beta_1\beta_2)\cos\Theta - \lambda_1\Delta\sigma(1+\beta_1\beta_2)x\ ,$$

$$\Delta\lambda = 0 : \quad \mathcal{A}^{\Delta\sigma}_{00} = 2\gamma_1\gamma_2\cos\Theta\left[((1-x)\beta_1+(1+x)\beta_2)\sqrt{\frac{\beta_1\beta_2}{1-x^2}} - (1+\beta_1^2\beta_2^2)\right]$$

JG, Kumar, Low, Vega-Morales, 2011

TABLE 8
Coefficients for the helicity amplitudes for the processes
$e^+e^- \to ZZ$ and $e^+e^- \to Z\gamma$

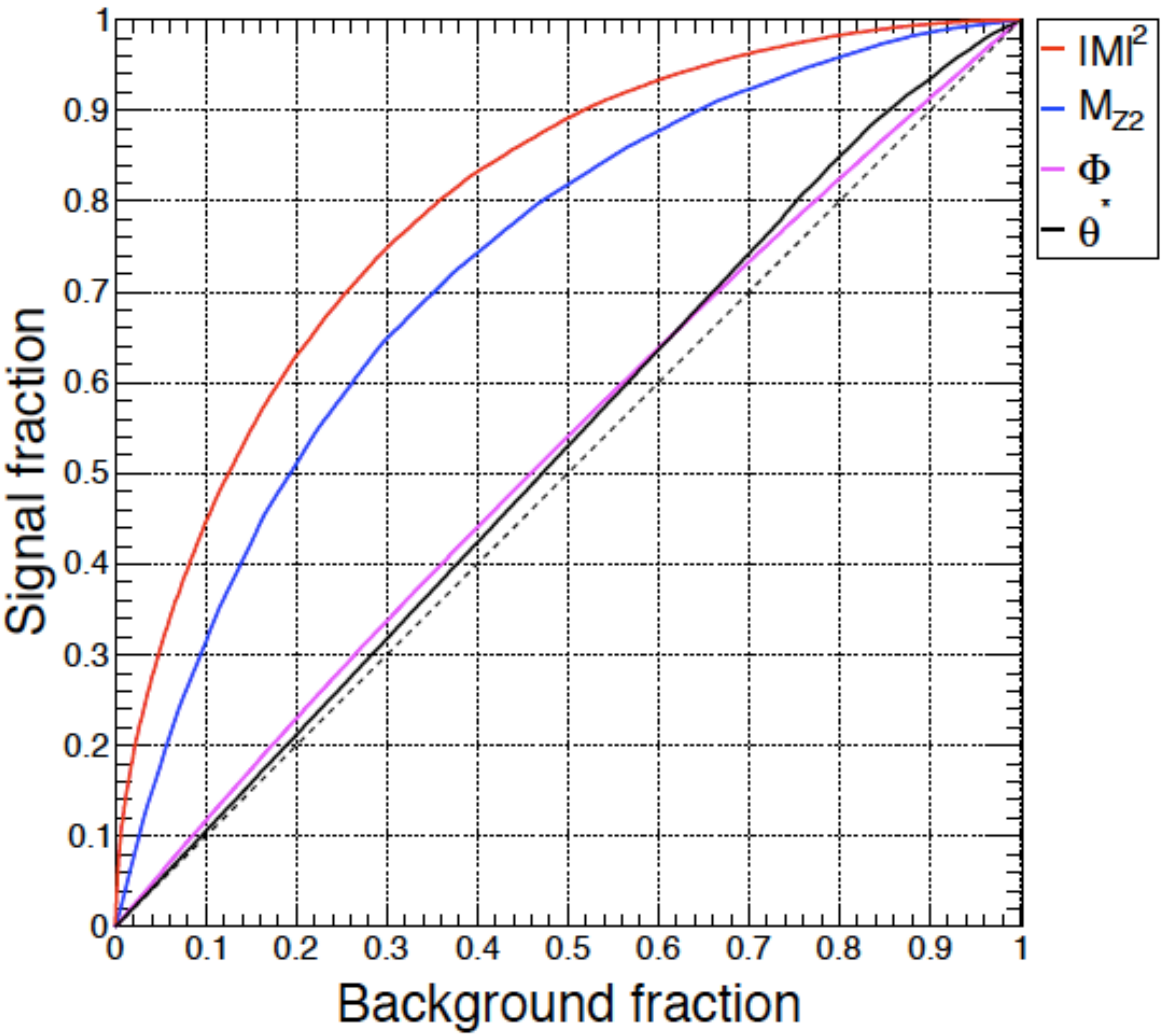
$\Delta\lambda$	$(\lambda_1\lambda_2)$	$\mathscr{A}_{\lambda_1\lambda_2}$	$\mathscr{B}_{\lambda_1\lambda_2}$
± 2	$(\pm\mp)$	$-\sqrt{2}(1+\beta^2)$	$\sqrt{2}$
± 1	(± 0)	$\gamma^{-1}[\Delta\sigma\cdot\Delta\lambda(1+\beta^2) - 2\cos\Theta]$	
± 1	$(0\pm)$	$\gamma^{-1}[\Delta\sigma\cdot\Delta\lambda(1+\beta^2) - 2\cos\Theta]$	$2r(\cos\Theta + \Delta\sigma\cdot\lambda_2)$
0	$(\pm\pm)$	$-\gamma^{-2}\cos\Theta$	$r^2(\cos\Theta + \Delta\sigma\cdot\lambda_2)$
0	(00)	$-2\gamma^{-2}\cos\Theta$	

Hagiwara, Peccei, Zeppenfeld, Hikasa, 1987

- At high energies the dominant polarizations of the Zs are +- for background
- Amplitudes non-vanishing for all choices for Z_1 and Z_2 polarizations
- For spin-zero resonances, only ZZ polarization possibilities are ++, - -, and 00
- So Z polarizations → lepton angular distributions, Z invariant mass distributions allows for signal and background separation for heavy Higgses ($M_H \gtrsim 2\ M_Z$), cf., e.g., JG, Kumar, Low, Vega-Morales, 2011

Propagator Drives Sensitivity for 125 GeV Higgs

- Full Matrix Element best variable for signal-background discrimination
- Invariant mass of lighter "Z", M_{Z2}, also very sensitive
- Φ (angle between decay planes) and θ^* (angle between Z_1 direction and collision axis in X rest frame) are less sensitive.
- Sensitivity to M_{Z2} due to very different signal and background distributions:
    - Background distribution peaks at M_{Z2} due to γ^* propagator
    - Signal distribution does not.

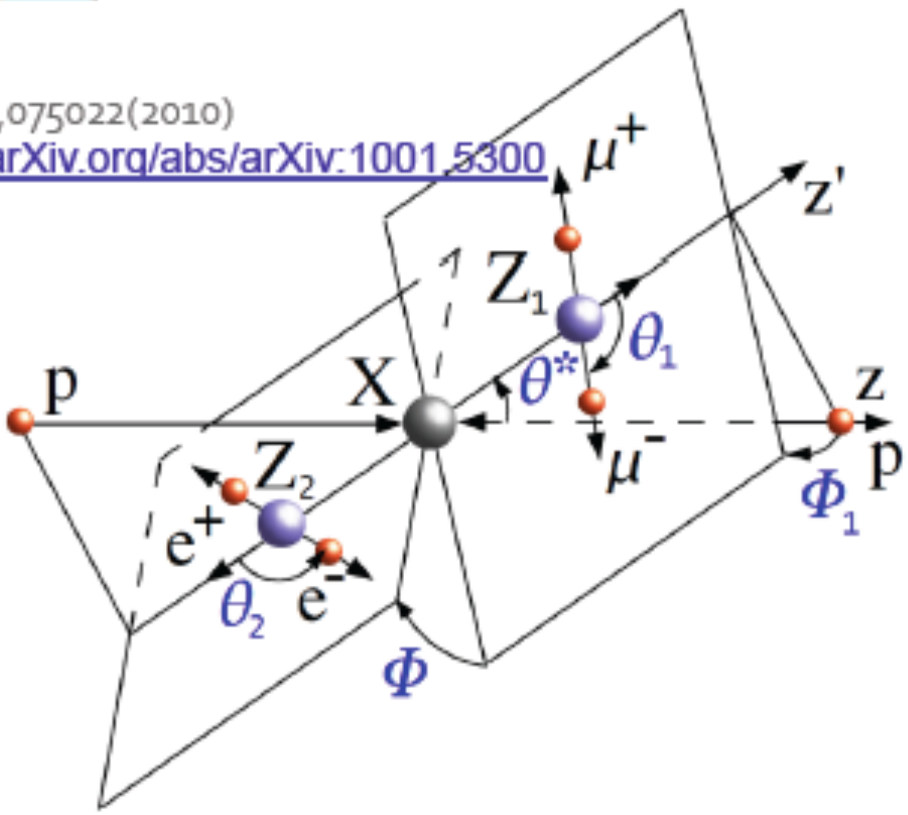


from Phys.Rev. D87 (2013) 055006 (Avery, Bourilkov, Chen, Cheng, Drozdetskiy, JG, Korytov, Matchev, Milenovic, Mitselmakher, Park, Rinkevicius, and Snowball.
In this paper we presented **MEKD**, a publicly available tool for MEM analyses in this channel

1210.0896

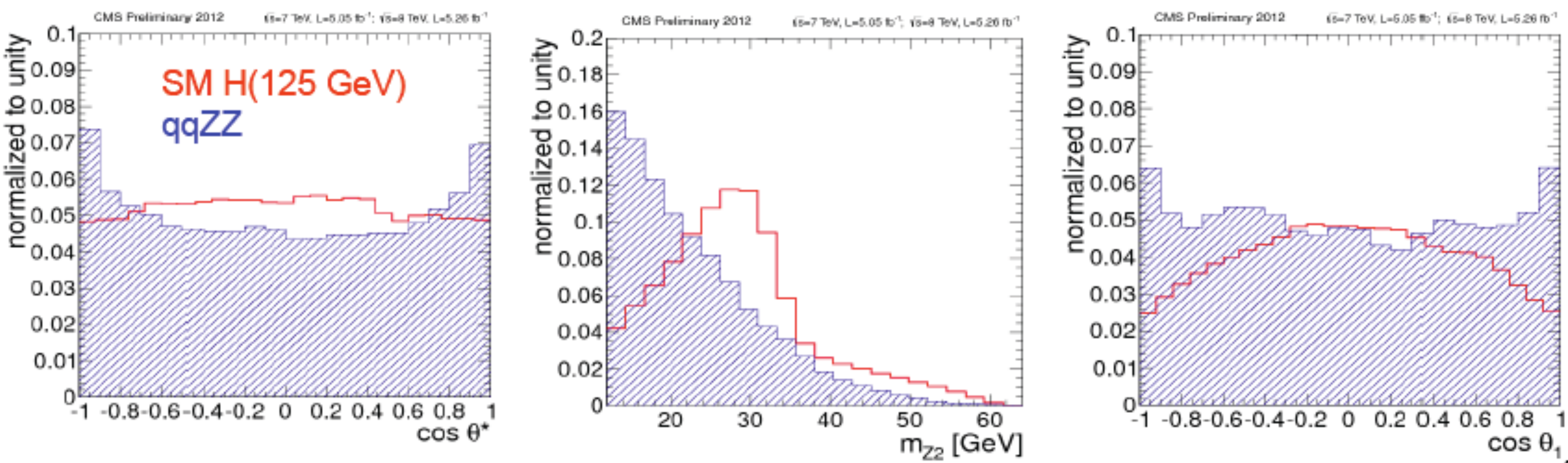
MELA

PRD81,075022(2010)
http://arXiv.org/abs/arXiv:1001.5300



Matrix Element Likelihood Analysis:
uses kinematic inputs for
signal to background discrimination
$\{m_1, m_2, \theta_1, \theta_2, \theta^*, \Phi, \Phi_1\}$

$$\mathrm{MELA} = \left[1 + \frac{\mathcal{P}_{\mathrm{bkg}}(m_1, m_2, \theta_1, \theta_2, \Phi, \theta^*, \Phi_1 | m_{4\ell})}{\mathcal{P}_{\mathrm{sig}}(m_1, m_2, \theta_1, \theta_2, \Phi, \theta^*, \Phi_1 | m_{4\ell})}\right]^{-1}$$



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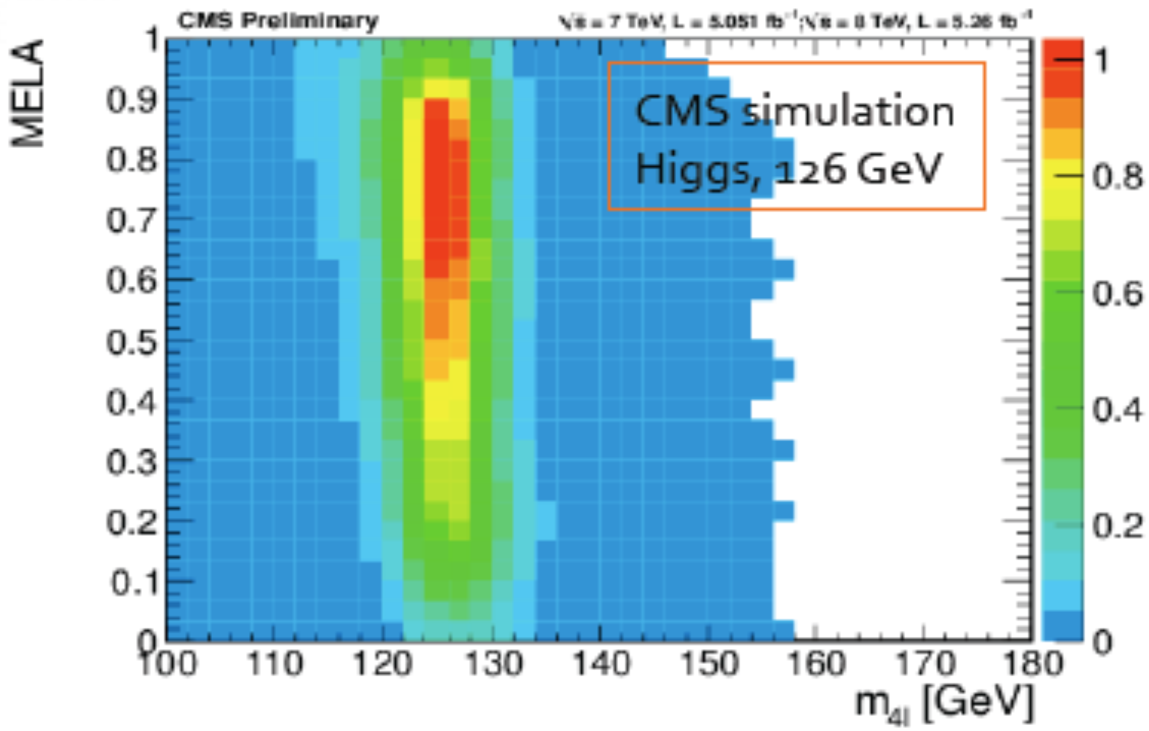
(Slide from CMS Higgs Discovery Talk)

MELA

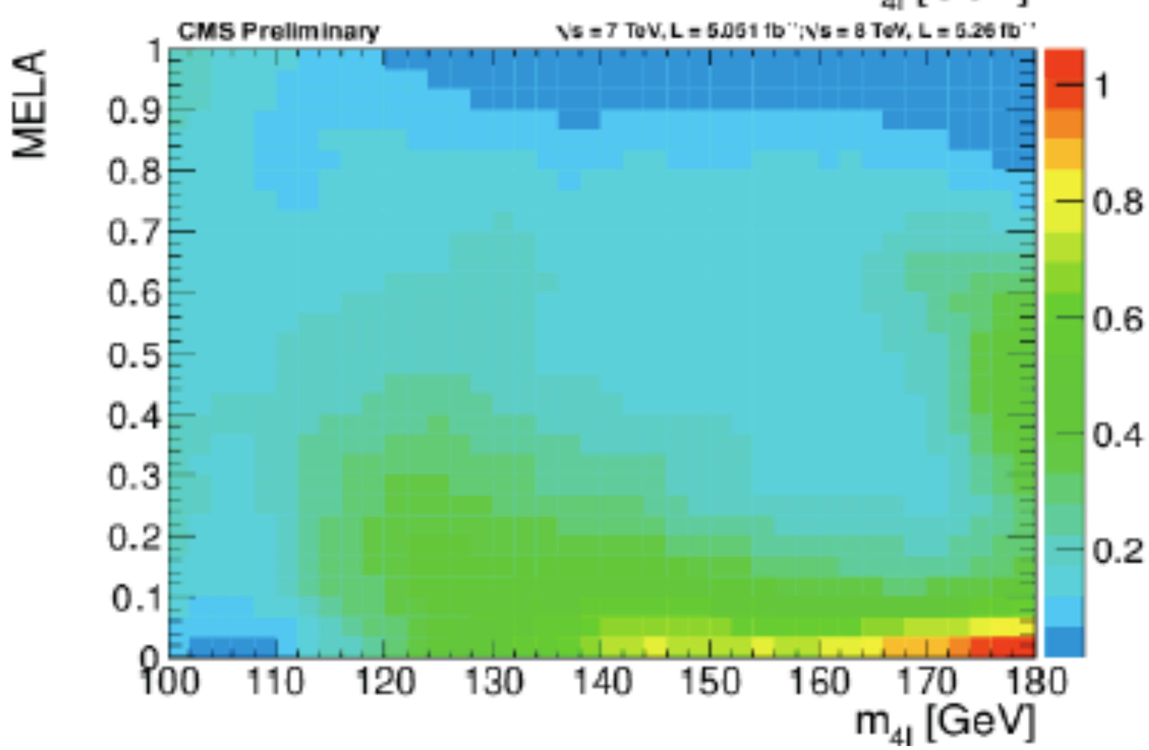
July 4th 2012 The Status of the Higgs Search J. Incandela for the CMS COLLABORATION

2D analysis using {m_{4l}, MELA}

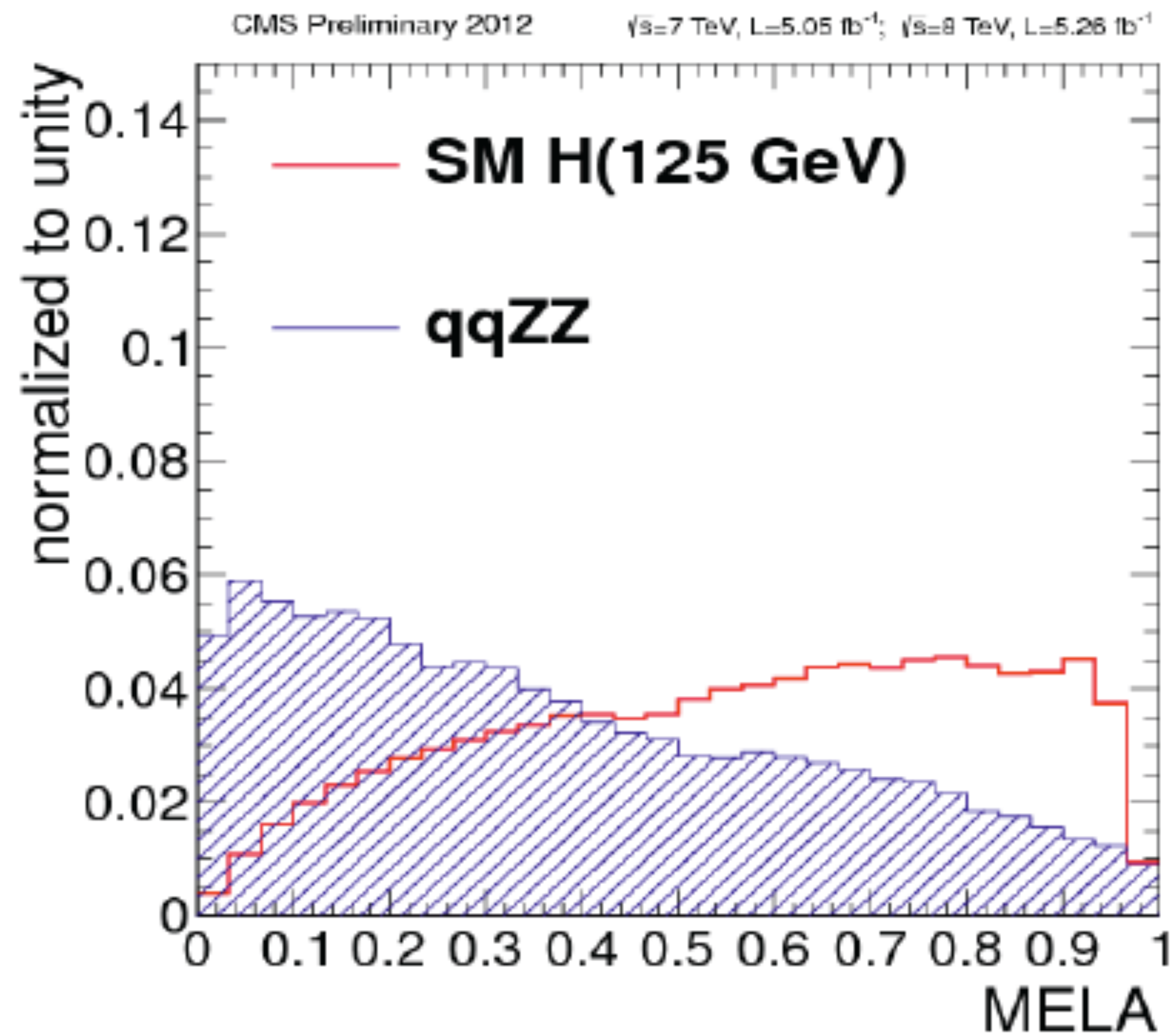
signal



background



MELA offers powerful discrimination of background



technique applicable for signal hypothesis testing

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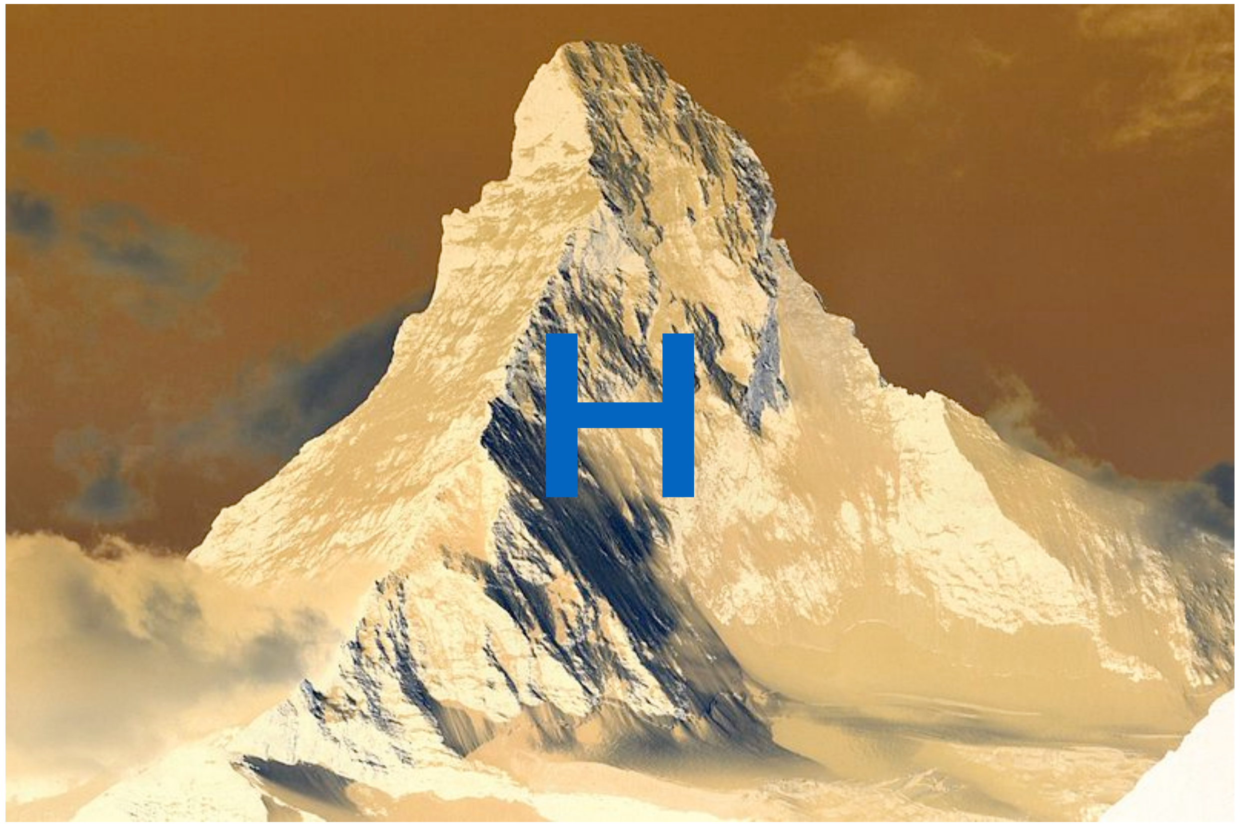
(Slide from CMS Higgs Discovery Talk)

Starting the Climb…

MEM for Higgs Properties Measurement

or



Goal: Measure XZZ Couplings

(I'm going to refer to the ≈125 GeV scalar as "X", since we are trying to determine whether it really is an "H")

Mainly Following

- Phys.Rev. D87 (2013) 055006
  Avery, Bourilkov, Chen, Cheng, Drozdetskiy, JG, Korytov, Matchev, Milenovic, Mitselmakher, Park, Rinkevicius, and Snowball.
  (1210.0896)
- PRL 111 (2013) 041801
  JG, Lykken, Matchev, Mrenna, and Park.
  (1304.4936)
- arXiv:**1310.1397** (accepted by PRD)
  Chen, Cheng, Gainer, Korytov, Matchev, Milenovic, Mitselmakher, Park, Rinkevicius, and Snowball
- In progress (arXiv: **1410.SOON)**
  JG, Lykken, Matchev, Mrenna, and Park

but also

- Gao, Gritsan, Guo, Melnikov, Schulze, Tran (2010)
- De Rujula, Lykken, Pierini, Rogan, Spiropulu (2010)
- Bolognesi, Gao, Gritsan, Melnikov, Schulze, Tran, Whitbeck (2012)
- Artoisenet, de Aquino, Demartin, Frixione, Maltoni, Mandal, Mathews, Mawatari, Ravindran, Seth, Torrielli, Zaro, 2013
- …
- Many, more, see e.g. the bibliography of the journal version of 1310.1397

Also, I'm not going to say anything about experimental status, since this is and has been covered in other talks.

- To use the MEM, to distinguish between signal hypotheses, we need to specify these hypotheses precisely

- To specify XZZ coupling use either

  - EFT with all terms up to a specified mass dimension

  - Amplitude with all structures up to a specified mass dimension

In both cases there is freedom in how couplings are parameterized.

1401.SOON

EFT

$$\mathcal{L} \supset \sum_{i=1}^{5} \kappa_i \mathcal{O}_i = -\kappa_1 \frac{m_Z^2}{v} X Z_\mu Z^\mu - \frac{\kappa_2}{2v} X F_{\mu\nu} F^{\mu\nu} - \frac{\kappa_3}{2v} X F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{\kappa_4 m_Z^2}{M_X^2 v} \Box X Z_\mu Z^\mu + \frac{2\kappa_5}{v} X Z_\mu \Box Z^\mu$$

- **Most general EFT with all terms with mass dimension ≤ 5 (+1 if we assume Higgs mechanism)**
- **I'll say more about all of these operators later.**

1401.SOON

Amplitude

$$\mathcal{A} = -\frac{2i}{v}\epsilon_1^{*\mu}\epsilon_2^{*\nu}\left(a_1 g_{\mu\nu} + a_2 p_{1\nu}p_{2\mu} + a_3\epsilon_{\mu\nu\rho\sigma}\, p_1^\rho p_2^\sigma\right)$$

$$a_1(p_1,p_2) = a_{10} + \frac{1}{\Lambda^2}\left(a_{11}(p_1\cdot p_2) + a_{13}(p_1^2+p_2^2)\right)$$

cf. analogous expressions in Bolognesi et al. (2012) and Gao et al. (2010)

- Most general Lorentz invariant, Bose symmetric coupling of scalar, X, to Z bosons with momenta p_1 and p_2, containing only terms with two or fewer powers of momenta

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Translation

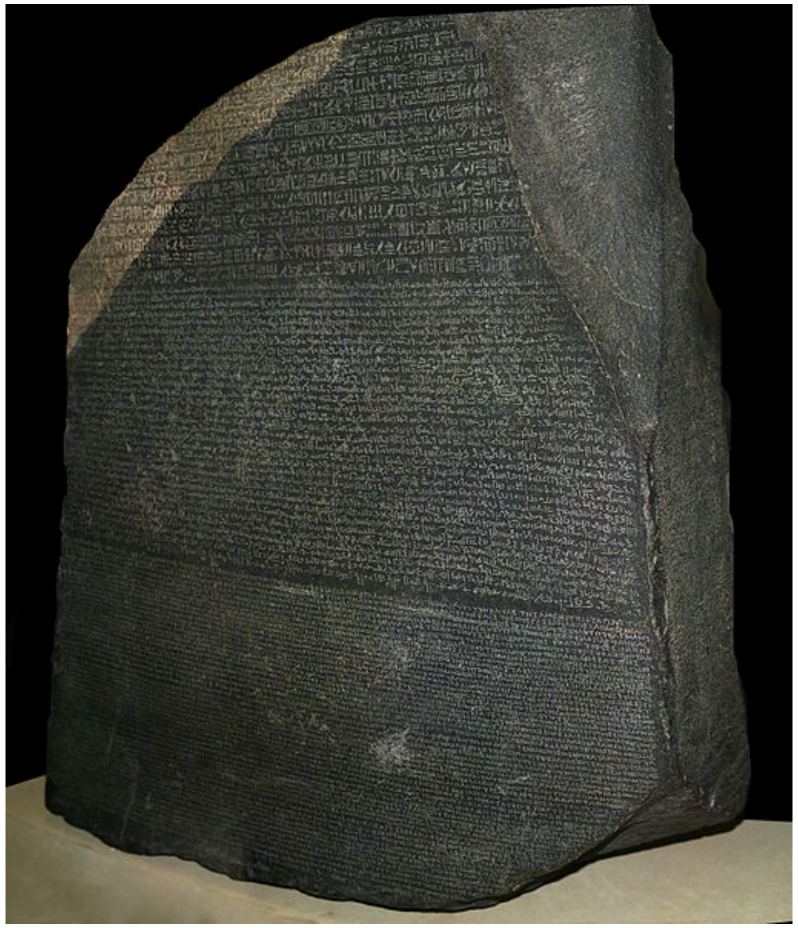
$$i\ \epsilon_1^* \cdot \epsilon_2^* \quad \Longleftrightarrow \quad -\frac{1}{2} X Z_\mu Z^\mu$$

$$i\ (p_1 \cdot p_2)(\epsilon_1^* \cdot \epsilon_2^*) \quad \Longleftrightarrow \quad \frac{1}{2} X \partial_\mu Z_\nu \partial^\mu Z^\nu$$

$$i\ (p_1 \cdot \epsilon_2^*)(p_2 \cdot \epsilon_1^*) \quad \Longleftrightarrow \quad \frac{1}{2} X \partial_\mu Z_\nu \partial^\nu Z^\mu$$

$$i\ \epsilon_{\mu\nu\rho\sigma} \epsilon_1^{*,\mu} \epsilon_2^{*,\nu} p_1^\rho p_2^\sigma \quad \Longleftrightarrow \quad -\frac{1}{2} \epsilon_{\mu\nu\rho\sigma} \partial^\mu Z^\nu \partial^\rho Z^\sigma$$

$$i\ (p_1^2 + p_2^2)(\epsilon_1^* \cdot \epsilon_2^*) \quad \Longleftrightarrow \quad X Z_\mu \Box Z^\mu$$



$$a_1 \equiv \kappa_1 m_z^2 + (2(m_Z^2/m_X^2)\kappa_4 - \kappa_2) p_1 \cdot p_2 + ((m_Z^2/m_X^2)\kappa_4 + \kappa_5)(p_1^2 + p_2^2),$$

$$a_2 \equiv \kappa_2,$$

$$a_3 \equiv \kappa_3.$$

1401.SOON

How General?

$$\mathcal{L} \supset \sum_{i=1}^{5} \kappa_i \mathcal{O}_i = -\kappa_1 \frac{m_Z^2}{v} X Z_\mu Z^\mu - \frac{\kappa_2}{2v} X F_{\mu\nu} F^{\mu\nu} - \frac{\kappa_3}{2v} X F_{\mu\nu} \tilde{F}^{\mu\nu}$$
$$+ \frac{\kappa_4 m_Z^2}{M_X^2 v} \Box X Z_\mu Z^\mu + \frac{2\kappa_5}{v} X Z_\mu \Box Z^\mu$$

- Can constrain all couplings
- Makes sense to narrow focus somewhat in early running based on sensitivity, likelihood of deviations, etc.

1401.SOON

Simplify Lagrangian

Keeping only the lowest dimensional operators with each of three symmetry properties:

1. CP-even, (naively) gauge invariance violating
2. CP-even, gauge invariant
3. CP-odd, gauge invariant

$$\mathcal{L} = -X \left[\kappa_1 \frac{m_Z^2}{v} Z_\mu Z^\mu + \frac{\kappa_2}{2v} F_{\mu\nu} F^{\mu\nu} + \frac{\kappa_3}{2v} F_{\mu\nu} \tilde{F}^{\mu\nu} \right]$$

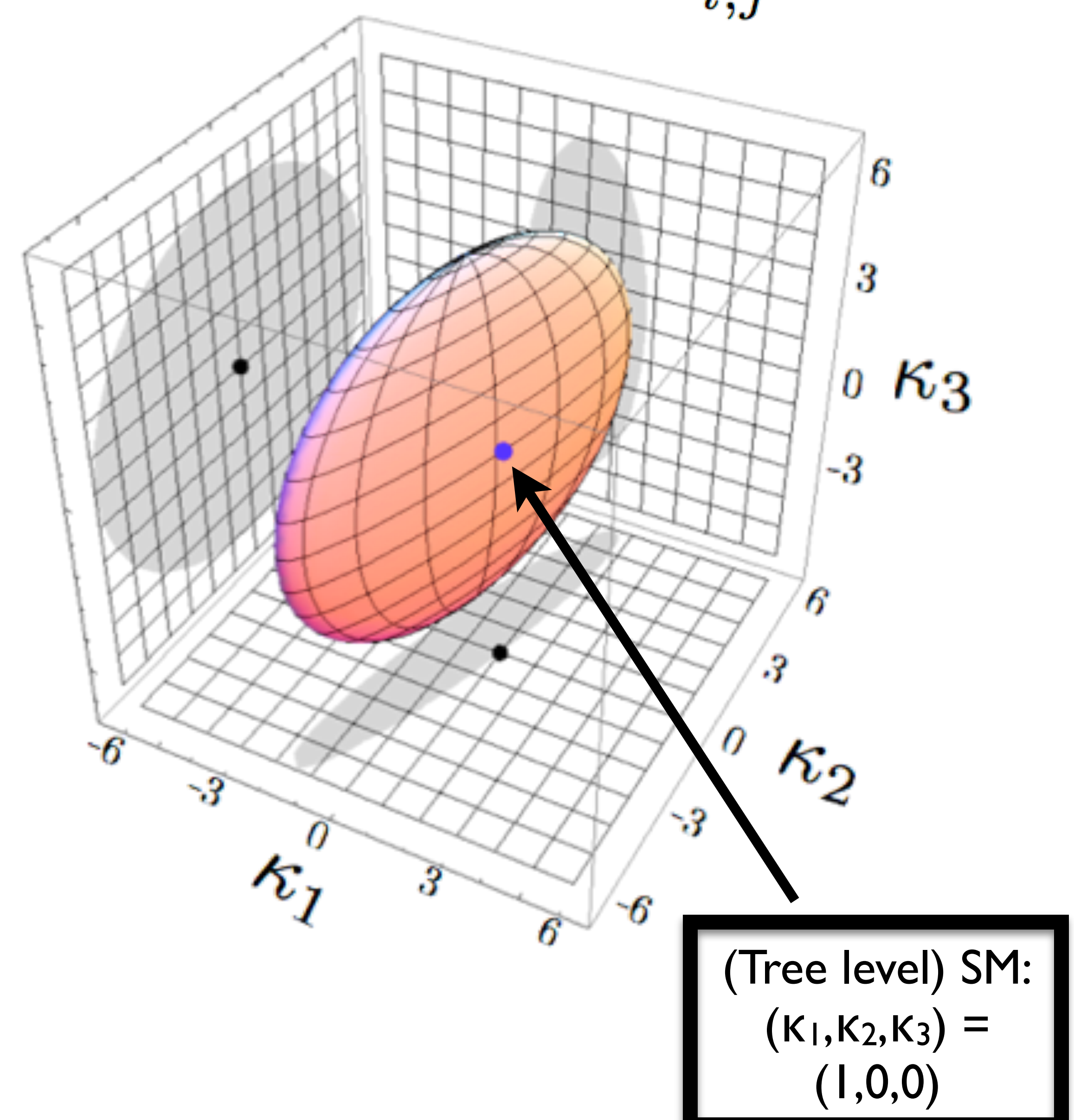
These operators contribute to the three Lorentz structures in the amplitude, a_1, a_2, and a_3.

1304.4936

Apply Rate Constraint

- In terms of these three real couplings, we can write the partial width for a point in parameter space as
- Measured rate implies correlations among couplings
- Defines an ellipsoidal "pancake" in κ space
- But shape stays the same
- Separates rate— leaves us with two parameters that are independent of rate
- Helpful when maximizing likelihoods when using the MEM, as likelihoods involve normalized probabilities

$$\Gamma(X \to ZZ) = \Gamma_{SM} \sum_{i,j} \gamma_{ij} \kappa_i \kappa_j$$



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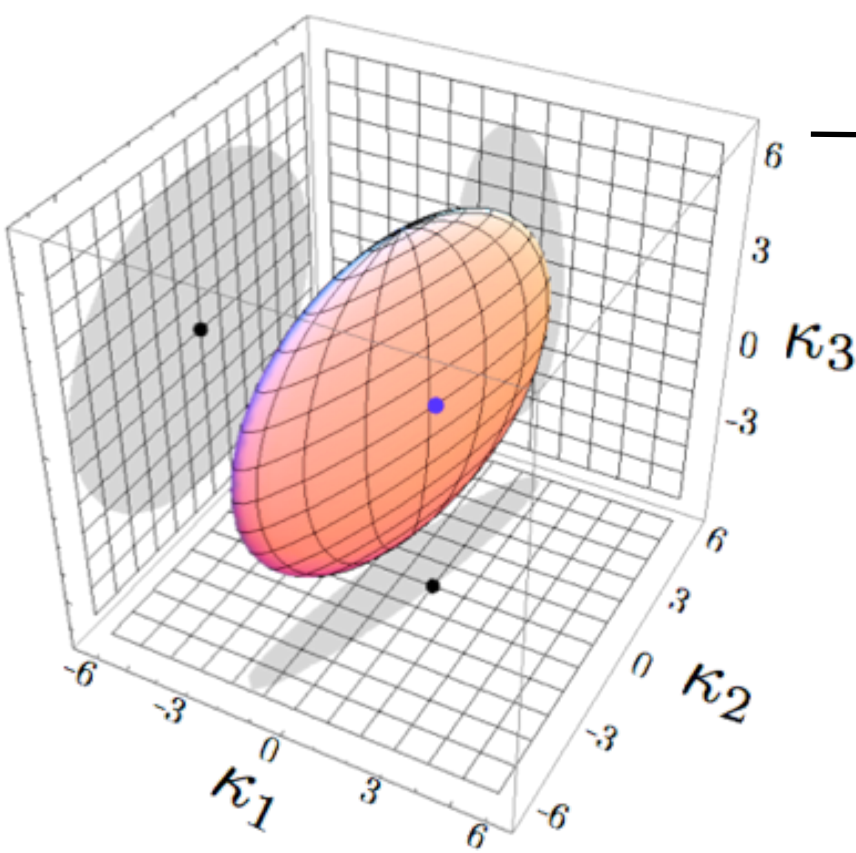
Parametrize the Pancake

- One can describe points on the equal rate “pancake” using, e.g., spherical coordinates
- Alternatively one can change variables to deform the pancake into an “equal rate sphere”
- This involves a linear transformation:

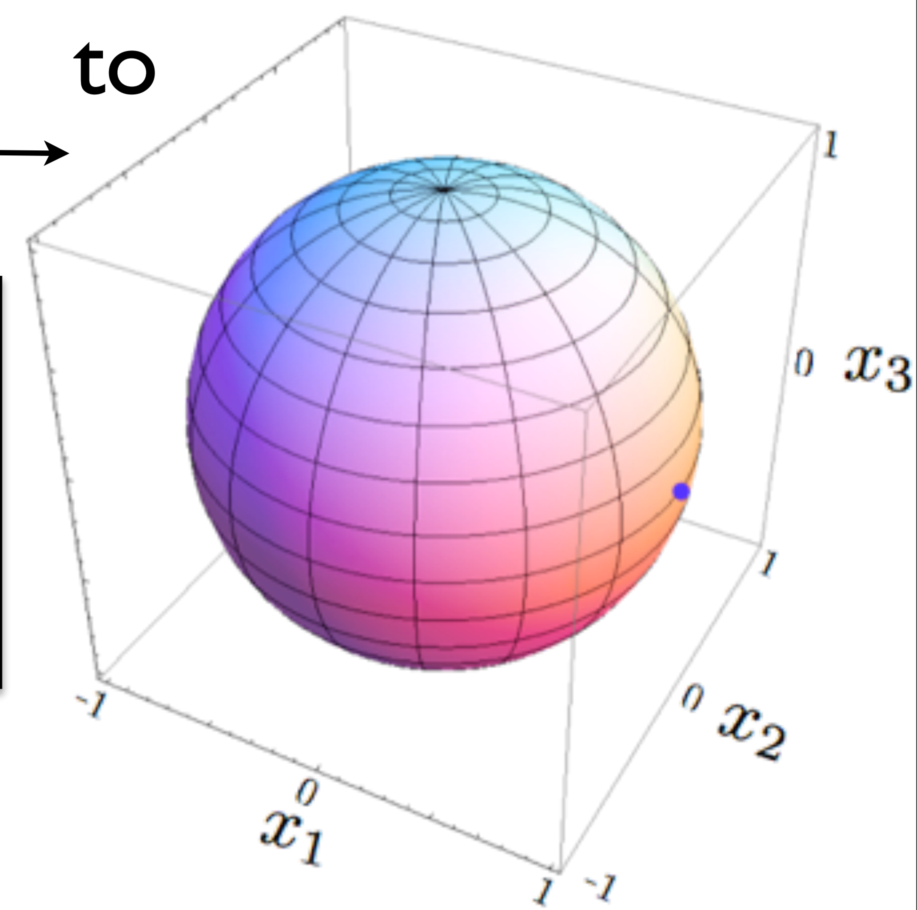


We go from → to

using

$$
\begin{aligned}
x_1 &= \kappa_1 - 0.25\ \kappa_2 \\
x_2 &= 0.17\ \kappa_2 \\
x_3 &= 0.19\ \kappa_3
\end{aligned}
$$

DF, before cuts



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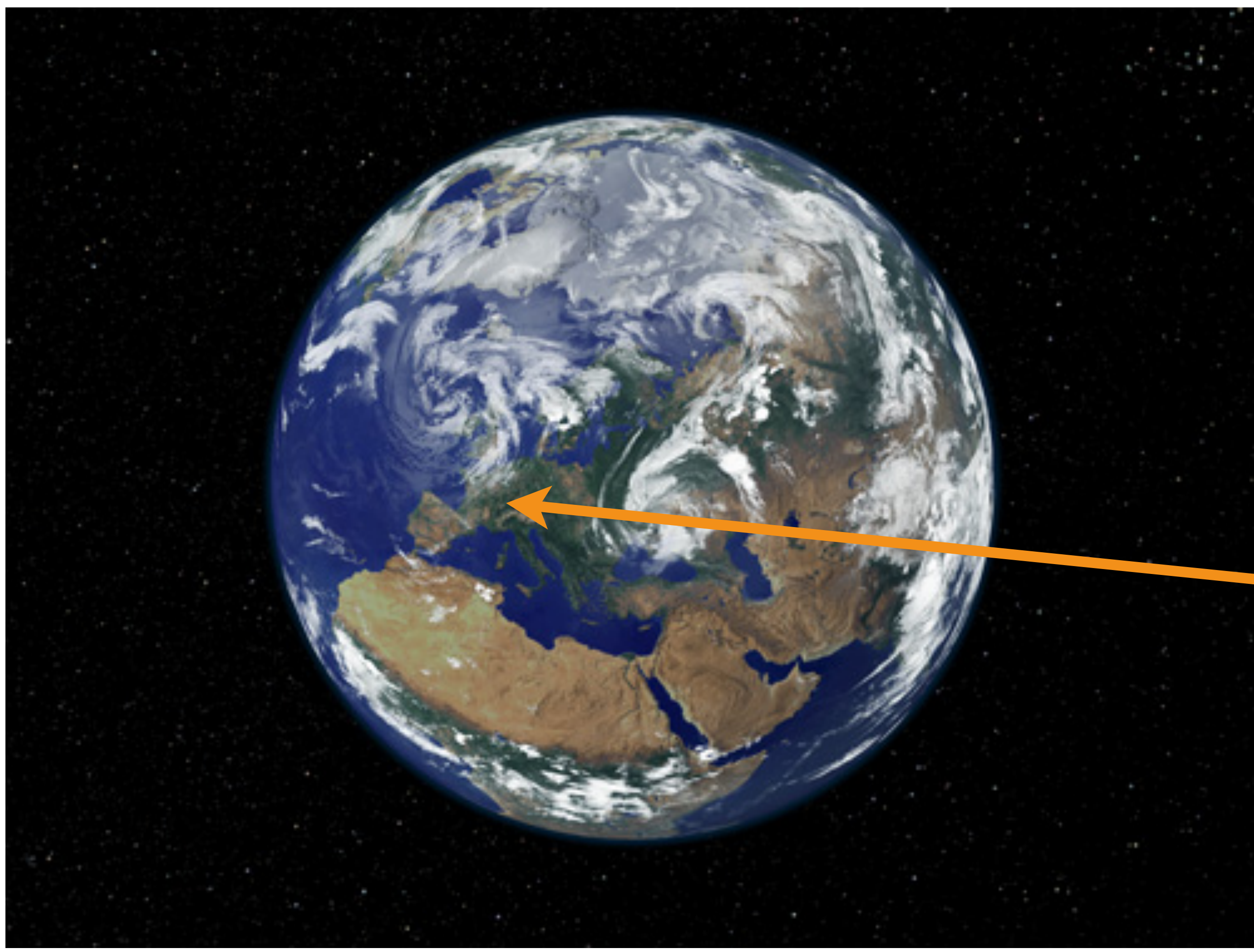
Geolocating the Higgs



Any given value of
$(\kappa_1, \kappa_2, \kappa_3)$,
corresponding to a given rate,
maps to a point on the sphere

“Measuring parameters of Higgs-ZZ couplings = “Geolocating the Higgs”

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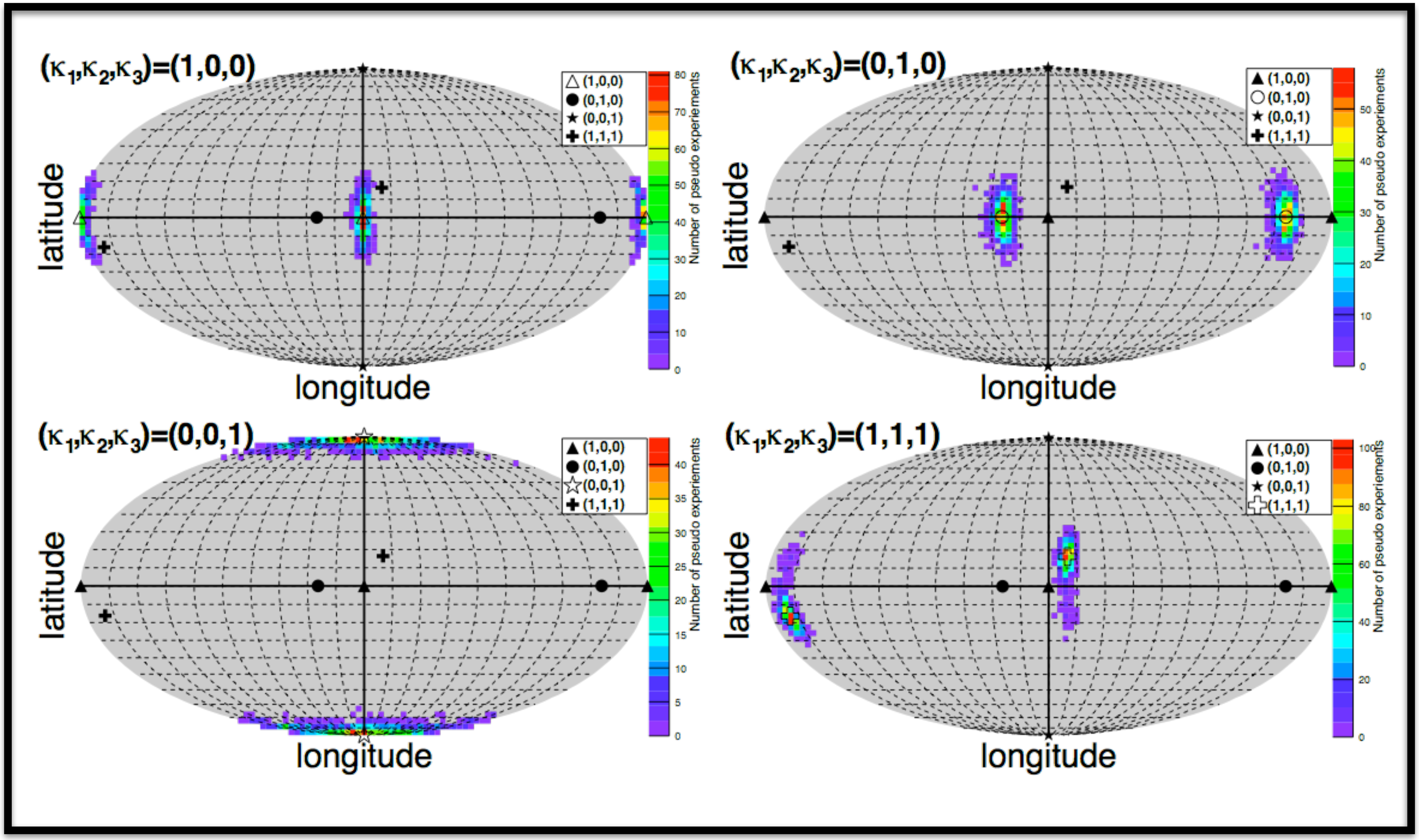
Geolocation Example

- We illustrate the use of the sphere for displaying results with a toy analysis
- We generate 1000 pseudoexperiments
    - 300 DF signal events for each of 4 benchmark points (~300 fb-1 at 14 TeV): three pure states and one completely mixed state
    - Impose cuts (pT, |η|, MZ1, MZ2)
    - Find the point on the sphere that maximizes the likelihood for each pseudoexperiment and plot



Note: a point and its antipode are effectively equivalent

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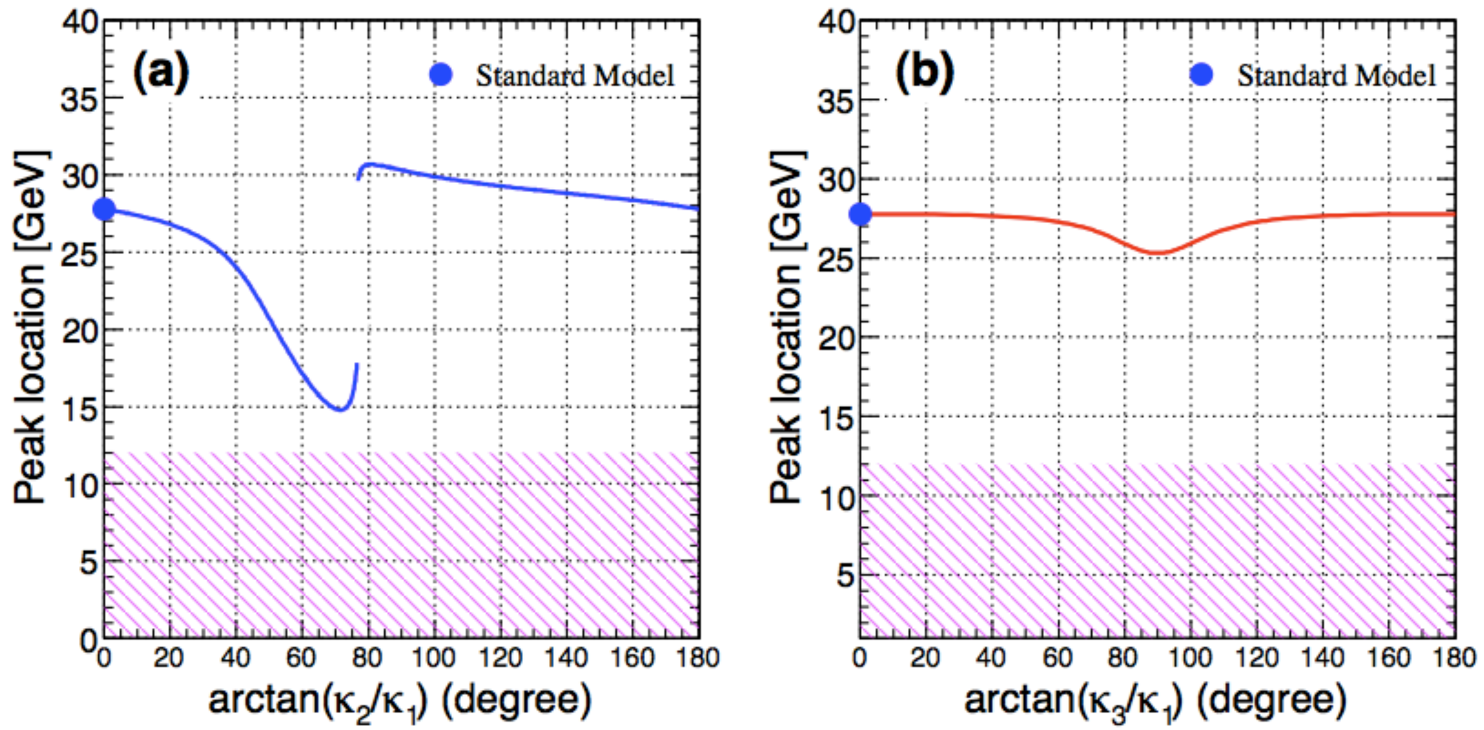
Importance of Interference

One goal of the geolocating framework is to have measurements with multiple non-zero couplings, due to the potential importance of interference between operators.



Peak of M_{Z2} distribution displays "first order phase transition" from κ_1-κ_2 interference , no such feature when considering κ_1 and κ_3

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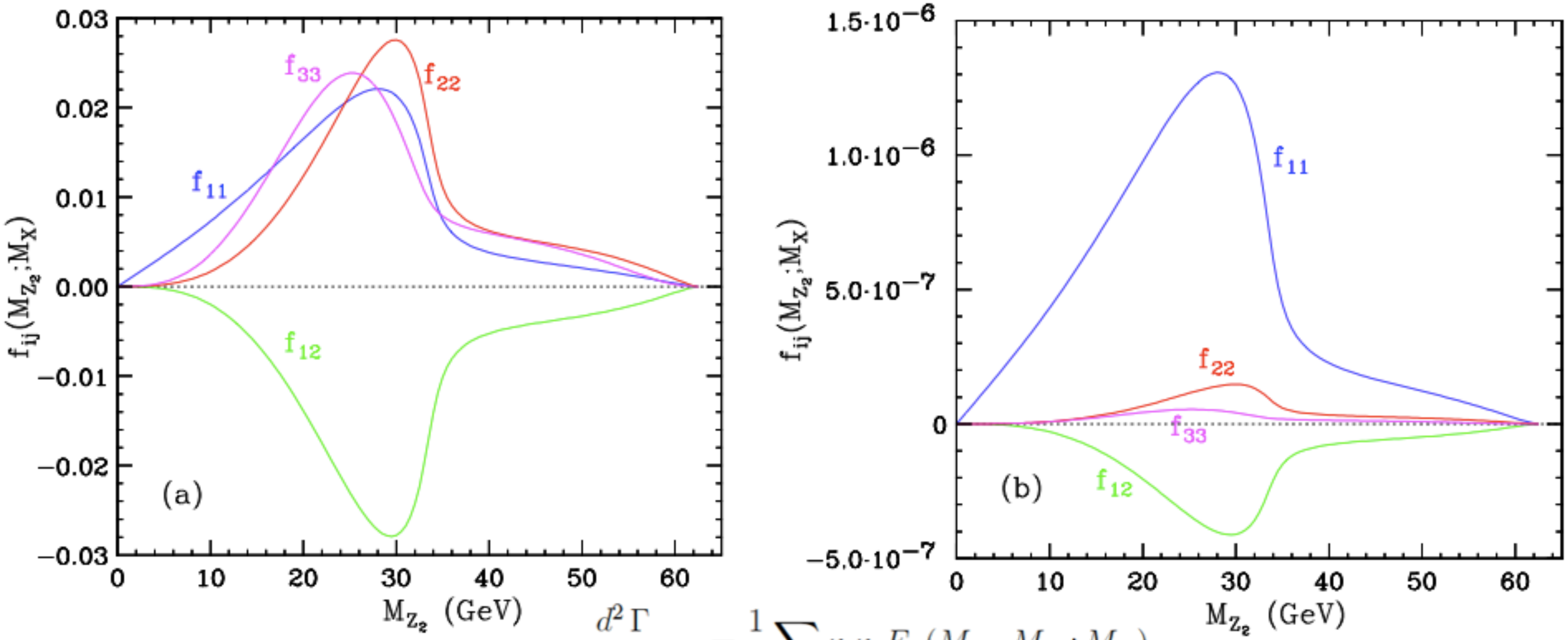
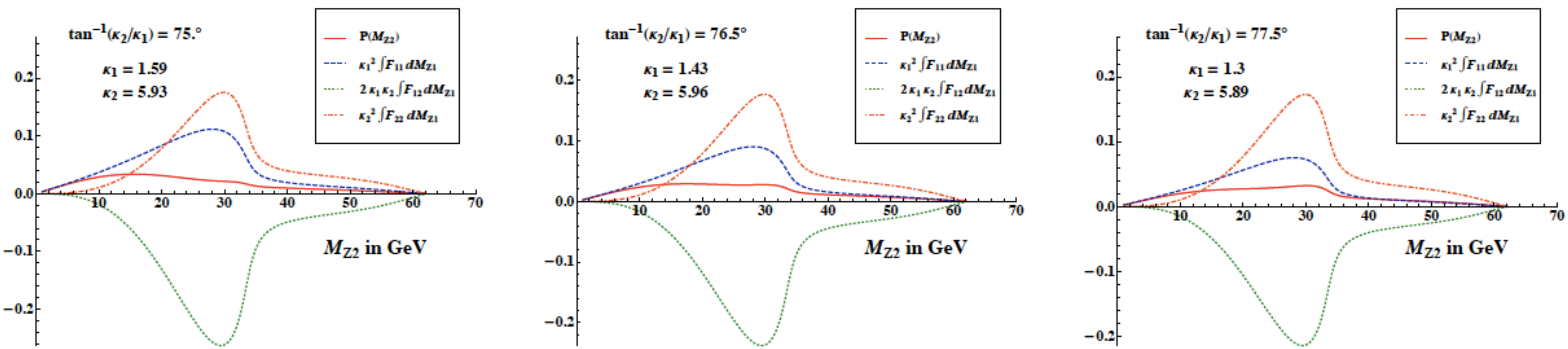


$$\frac{d^2\Gamma}{dM_{Z_1}dM_{Z_2}} = \frac{1}{v}\sum_{i,j}\kappa_i\kappa_j F_{ij}(M_{Z_1}, M_{Z_2}; M_X)$$

obtain f_{ij} from integrating F_{ij} over M_{Z1}



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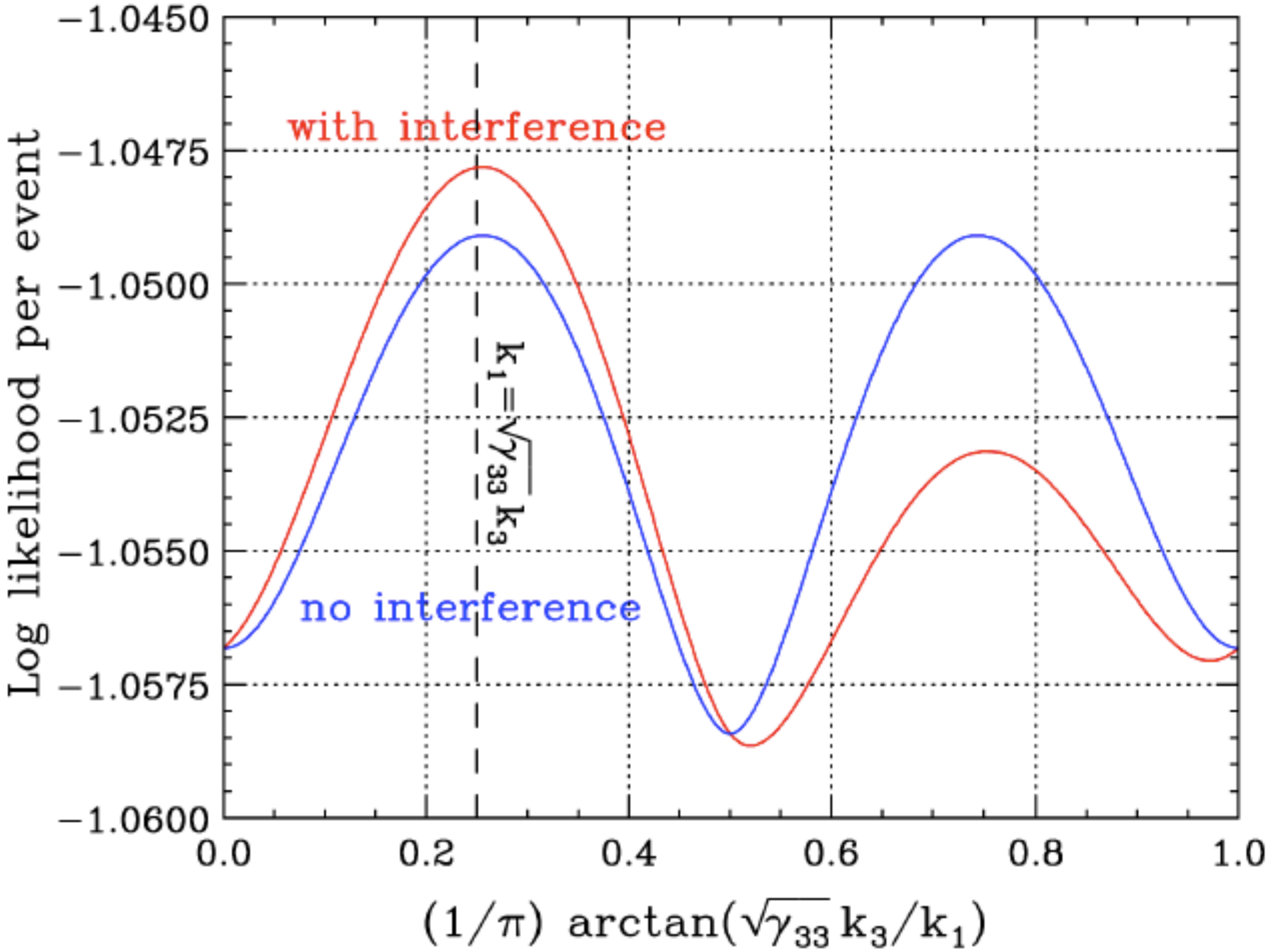
Importance of Interference



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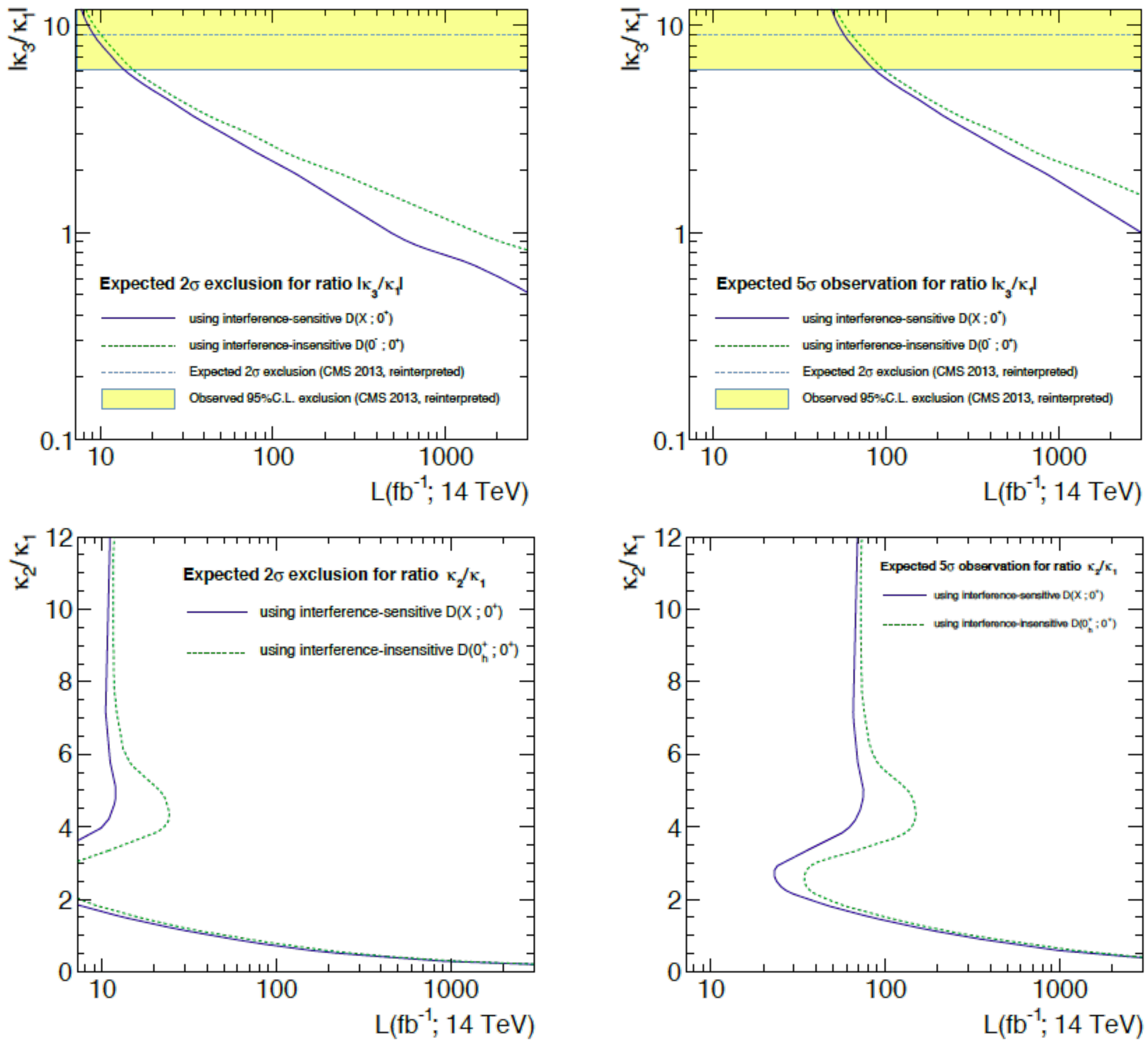
Projections



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The Other Operators

- Remember, we started with the EFT Lagrangian

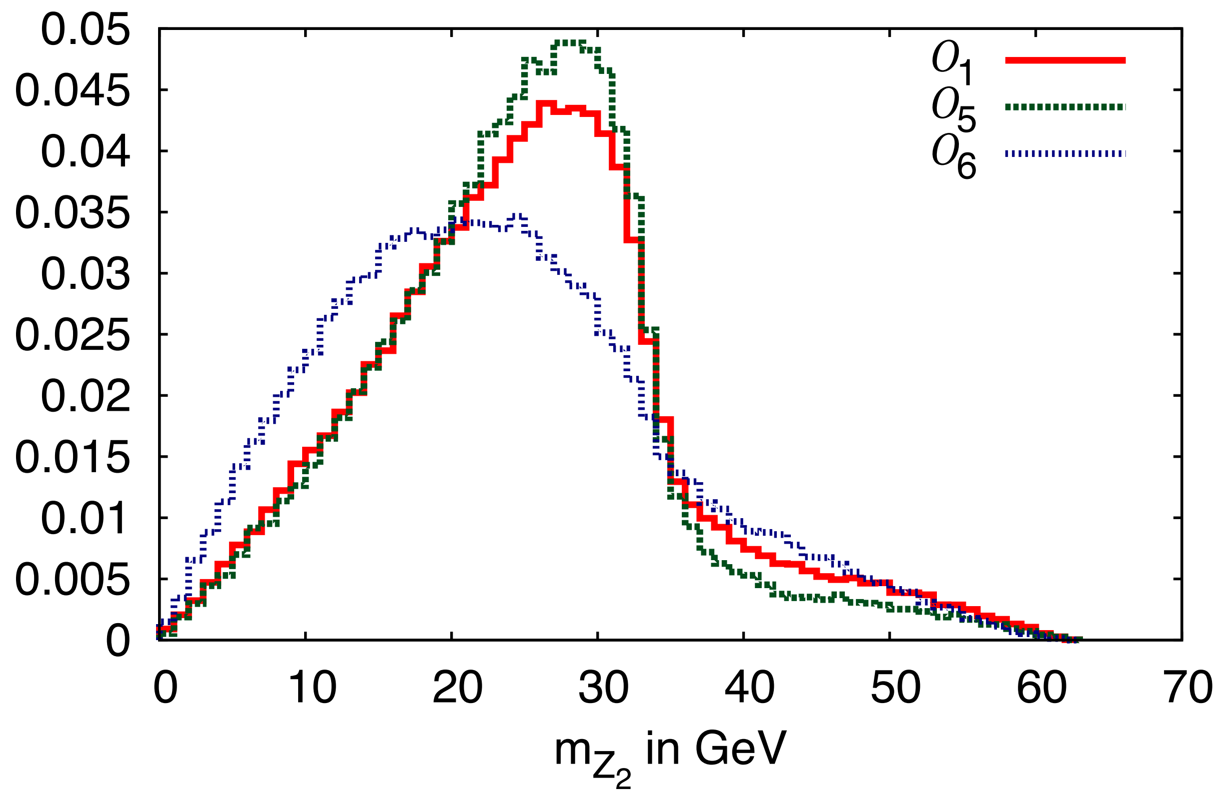
$$\mathcal{L} \supset \sum_{i=1}^{5} \kappa_i \mathcal{O}_i = -\kappa_1 \frac{m_Z^2}{v} X Z_\mu Z^\mu - \frac{\kappa_2}{2v} X F_{\mu\nu} F^{\mu\nu} - \frac{\kappa_3}{2v} X F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{\kappa_4 m_Z^2}{M_X^2 v} \Box X Z_\mu Z^\mu + \frac{2\kappa_5}{v} X Z_\mu \Box Z^\mu$$

but only considered the first three operators.

- κ_5 operator can also be written as

$$Z_\nu \partial_\mu Z^{\mu\nu}$$

- Its main effect is a subtle modification of the M_{Z2} distribution



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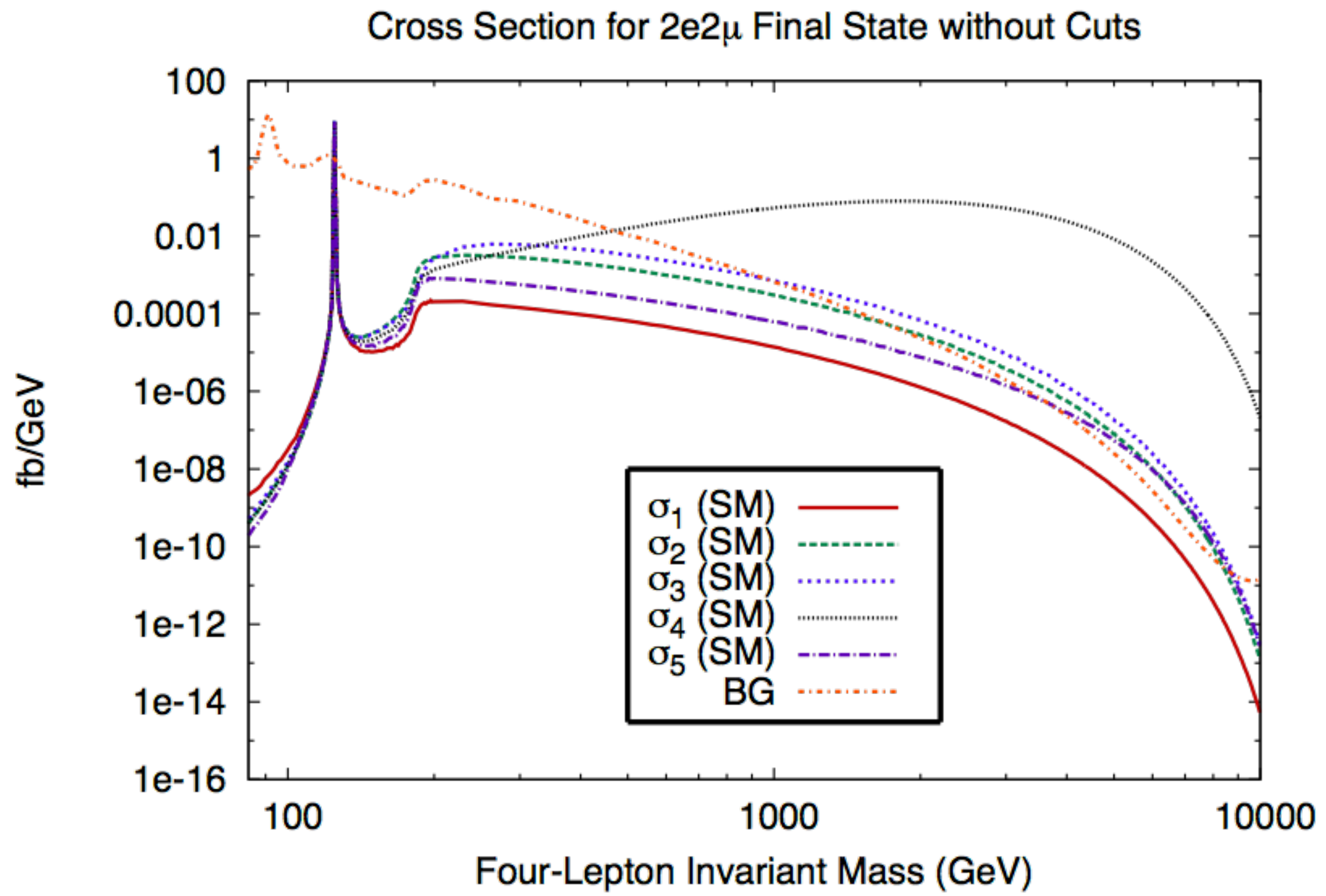
The Other Operators

- The κ_4 operator $\Box X Z_\mu Z^\mu$

  is indistinguishable from the κ_1 (SM) operator on the Higgs peak

- Much larger cross section for off-shell Higgs production



- **Strong unitarity bounds on this operator**

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The Other Operators

	$\mathcal{O}_1$	$\mathcal{O}_2$	$\mathcal{O}_3$	$\mathcal{O}_4$	$\mathcal{O}_5$	$\mathcal{O}_6$
$2\langle \Delta \log \mathcal{L} \rangle_{SM}$	0	−0.747	−1.017	0	−0.178	−0.503
Events for 3σ Limit	———	12.0	8.85	———	50.5	17.9

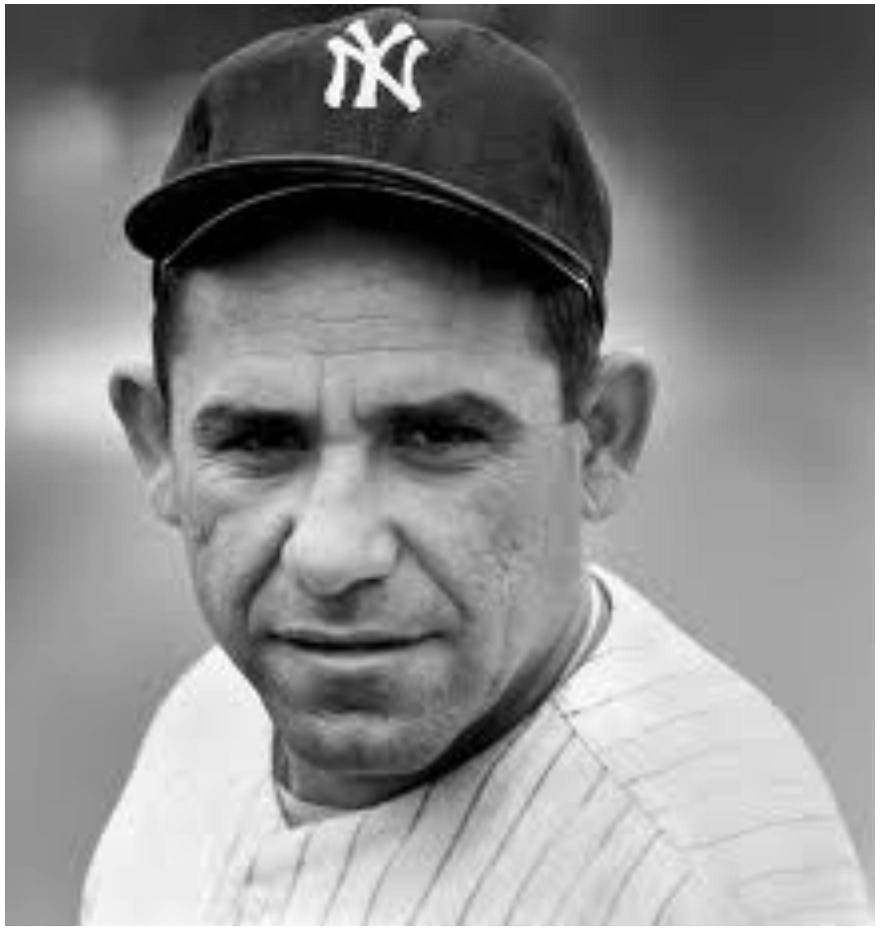
- To quantify the sensitivity of various operators, we can calculate the difference in log likelihood, with respect to the SM, per signal event.
- Since in the high statistics limit, $2\ \Delta \log \mathscr{L} \sim \Delta\ \chi^2$, we can estimate the number of events needed to get a 3σ limit, (assuming true hypothesis is SM) by dividing 3^2 by twice the average per-event difference in log likelihood
- Here we show values for various pure operator couplings, normalized to give the SM rate on peak.
- These values are on the small side because we are assuming a perfect detector and turning off backgrounds.
- Still, they suggest that the easiest pure state to exclude is the pure pseudo scalar, followed by the k_2 coupling, followed by the k_5 coupling. (k_4 can not be distinguished from the SM using only on-peak events.)

1401.SOON

Distant Summits

- I was asked to talk about a Snowmass whitepaper on the Matrix Element Method (Gainer, Lykken, Matchev, Mrenna, and Park), arXiv:1307.3546.
- I think the most interesting part of this paper for this audience involves speculations about the future of the MEM.
- So I'm going to switch gears and speculate about this future for the rest of my talk
- (Unless I'm out of time, in which case, thanks for your attention!)

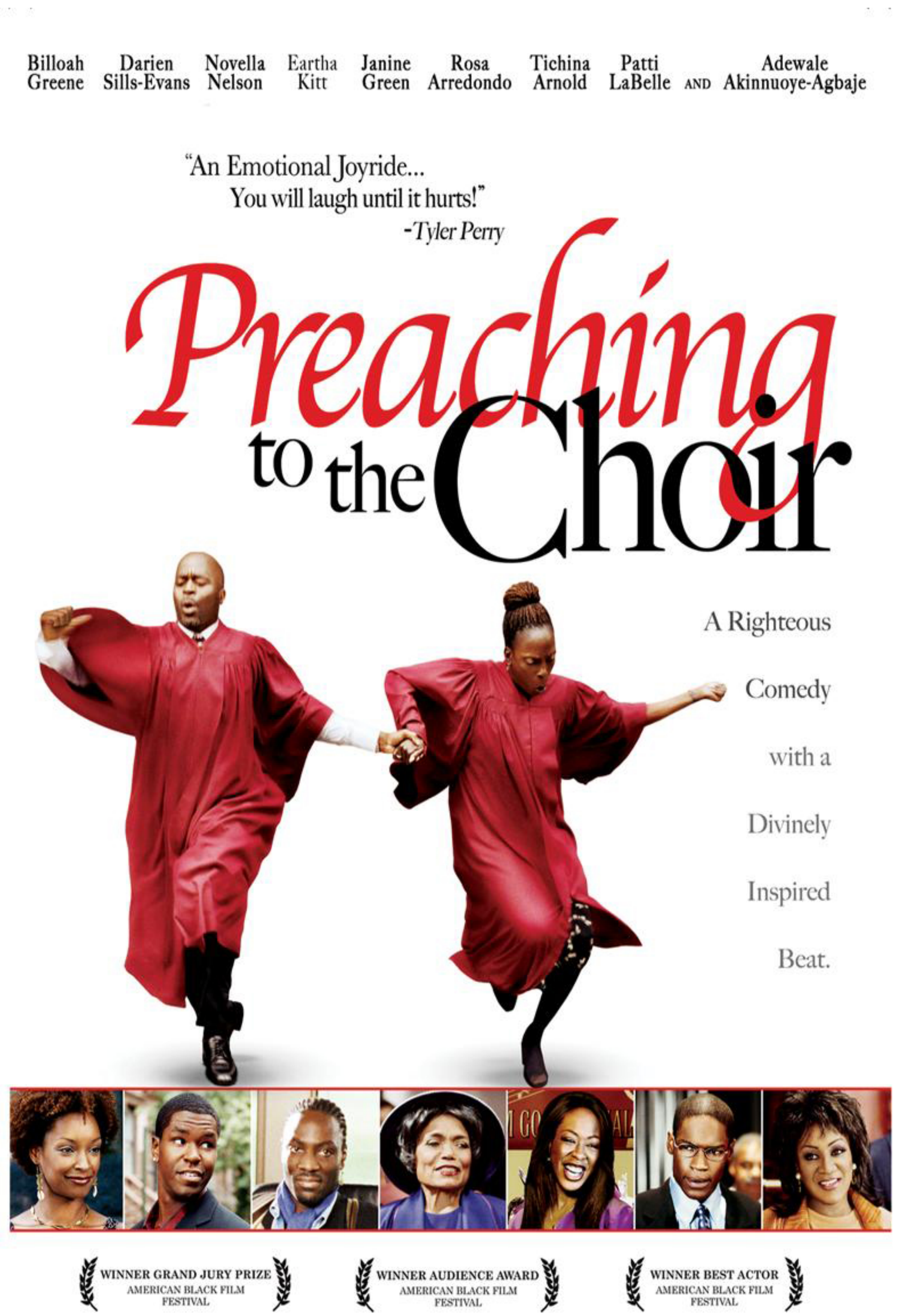
“Prediction is Hard, Especially About the Future”



Quote attributed to American baseball player Yogi Berra
and Danish (association) football player Niels Bohr among others

More MEM

- Multivariate analyses are essential
- MEM is more transparent (especially for theorists/ phenomenologists) than neural nets, other MVAs
- Sensitivity (or lack thereof) of an analysis can be related to properties of signal and background differential cross section.
- Previous summits motivate broader use of the MEM



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More Power: Moore's Law

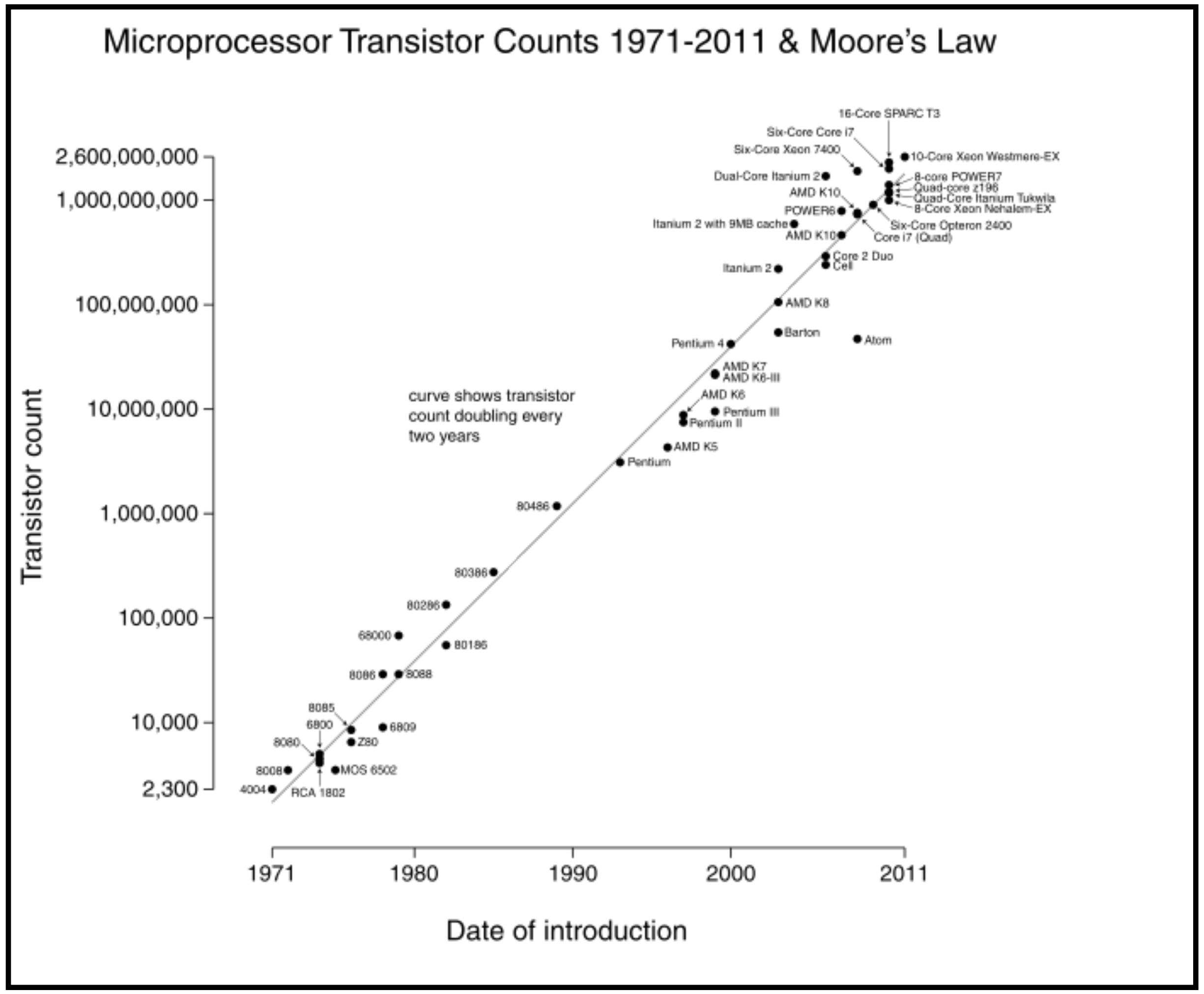
- Specifically, refers to exponential growth in the number of transistors on a chip

- Also used to refer to related exponential growth in processing speed per unit cost

- One of the main drawbacks of the MEM is that it is computationally intensive, especially due to transfer functions, integrating over invisible momenta, etc.

  though see (Artoisenet, Lemaitre, Maltoni, and Mattelaer, 2010) on optimizing the relevant phase space integrals

- Ultimately increased computer power will make this drawback of the MEM obsolete



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Futurism: Transfer Functions

- With increased computer power, more sophistication is possible
- One possibility: use "transfer functions" to relate partons to detector-level information (tracks, energy deposition, etc.) rather than reconstructed physics objects. Ideally these transfer functions would involve the specific properties of each cell in the detector, with time dependence as appropriate.
  Goal is the precise characterization of reducible backgrounds.
- In the shorter term, use of "transfer functions" for jets that also include substructure information, could significantly improve sensitivity in some analyses.

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Kinematic Variables

- Since the MEM uses the likelihood, and is hence optimal for deciding between two hypotheses, will it eventually replace all other analyses?
(Konstantin and I talk about this a lot.)

- Maybe. One question I have (to which I do not know the answer) is whether kinematic variable techniques may still do better for distinguishing one class of hypotheses (e.g. those with signal) from another (e.g. those without).

- What I mean is, in the limit that one can use the MEM (computational resources exist, etc.) then the MEM applied to a specific signal process S_1 and a specific background B_1 will be more sensitive, than, e.g., a search based on M_{T2}.

- But if we have to consider all signal topologies (i.e. S_1, S_2, …, S_N) and a possibly not-quite understood mixture of background processes, is this still true?

- My guess is that the answer is "no", because scanning over possible topologies is like adding nuisance parameters, which reduce the significance of any given difference in log likelihood.

- On the other hand, maybe there is a well-defined extension of the MEM ("Super-MEM"?) that can be used for a set of related signal hypotheses.

Conclusions

- The Matrix Element has been an important tool for well over a decade.
- It played an important role in Higgs discovery in the 4ℓ final state,
- And is playing an important role in Higgs properties measurement in the 4ℓ final state and beyond.
- The future looks bright, both because of the successes of MEM-based analyses,
- and because computing power will keep getting cheaper in the future.
- This is a good thing as the MEM is an (optimally, in some sense) sensitive MVA that preserves physical transparency.
- I am looking forward to discoveries at the LHC from MEM-based analyses.