# A simple, yet subtle, invariance of the two-body decay kinematics

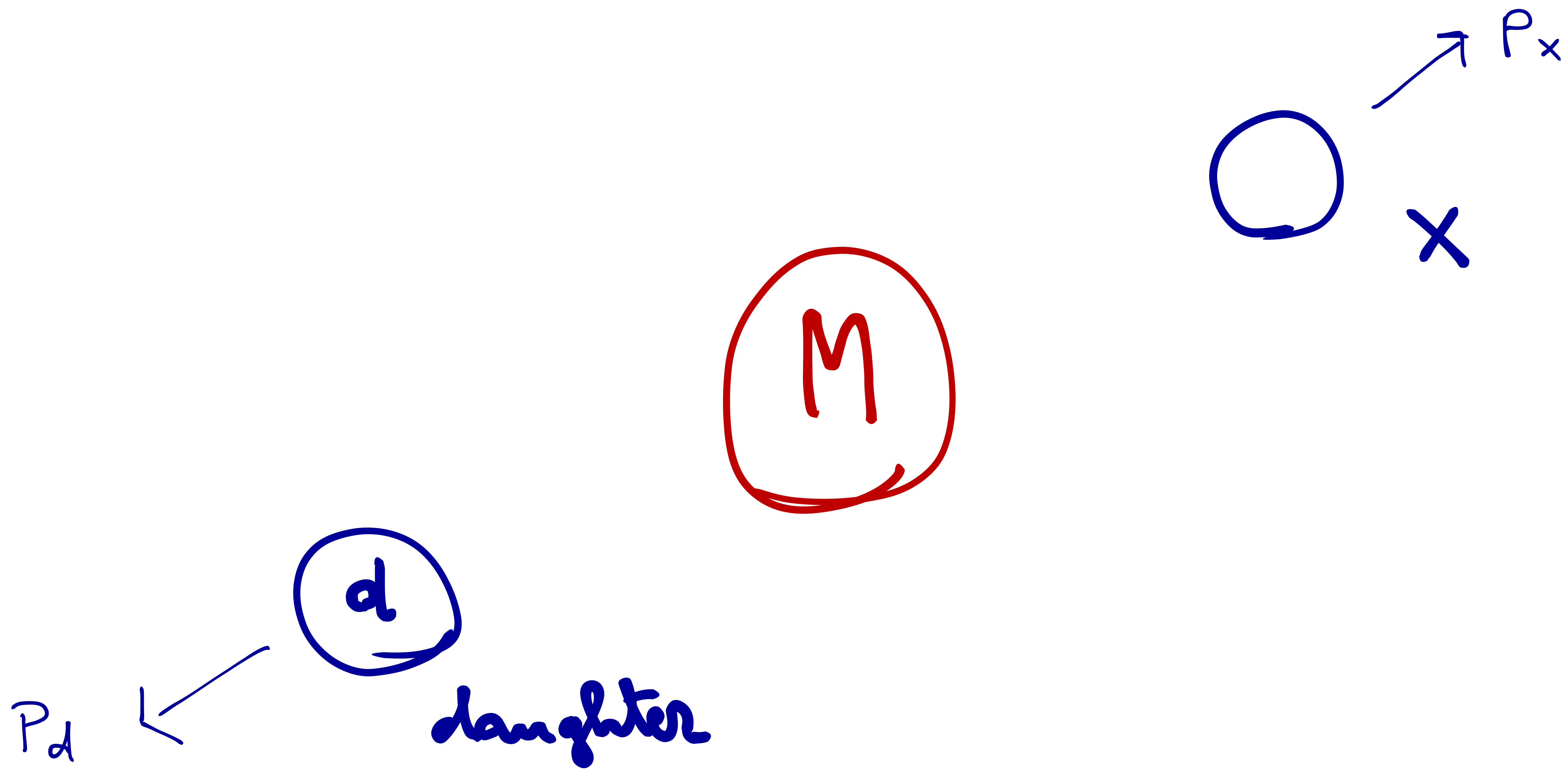
Roberto Franceschini (University of Maryland)

arXiv:1209.0772
with K.Agashe and D.Kim
arXiv:1212.5230
with K.Agashe, D.Kim, K. Wardlow
arXiv:1309.4776
with K.Agashe and D.Kim

TWO-BODY DECAY : $M \to d\, X$


$$p_M^{\mu} = p_d^{\mu} + p_x^{\mu}$$

KINEMATICS FULLY FIXED BY THE MASSES




$$E_d =: E_d^* = \frac{m_M^2 + m_d^2 - m_X^2}{2 m_M} \qquad \bar{p}_d + \bar{p}_X = 0$$

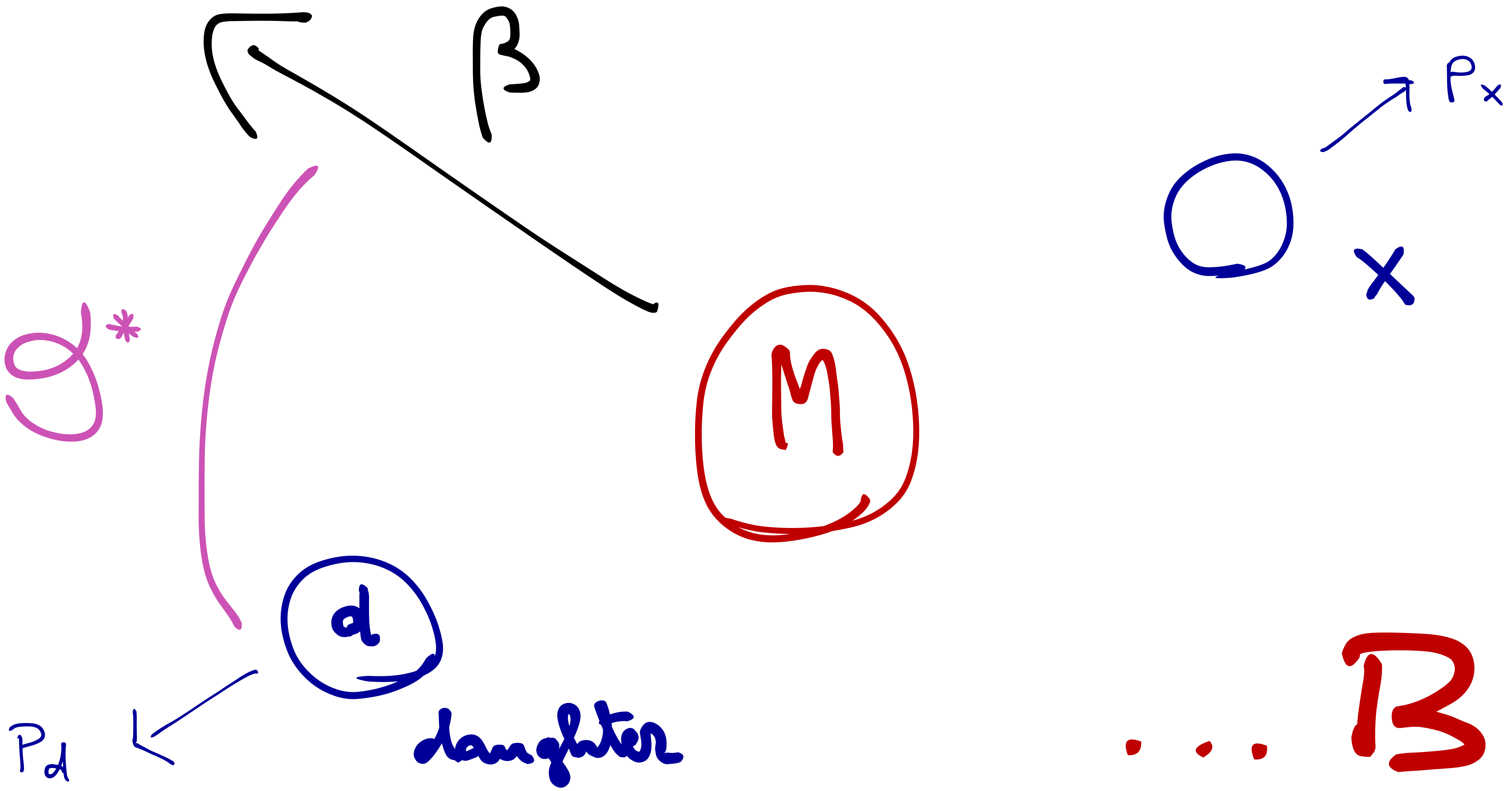
IN THE REST FRAME OF M

WHAT DOES IT LOOK LIKE IN ANOTHER FRAME?

IN GENERAL WE KNOW THE ANSWER

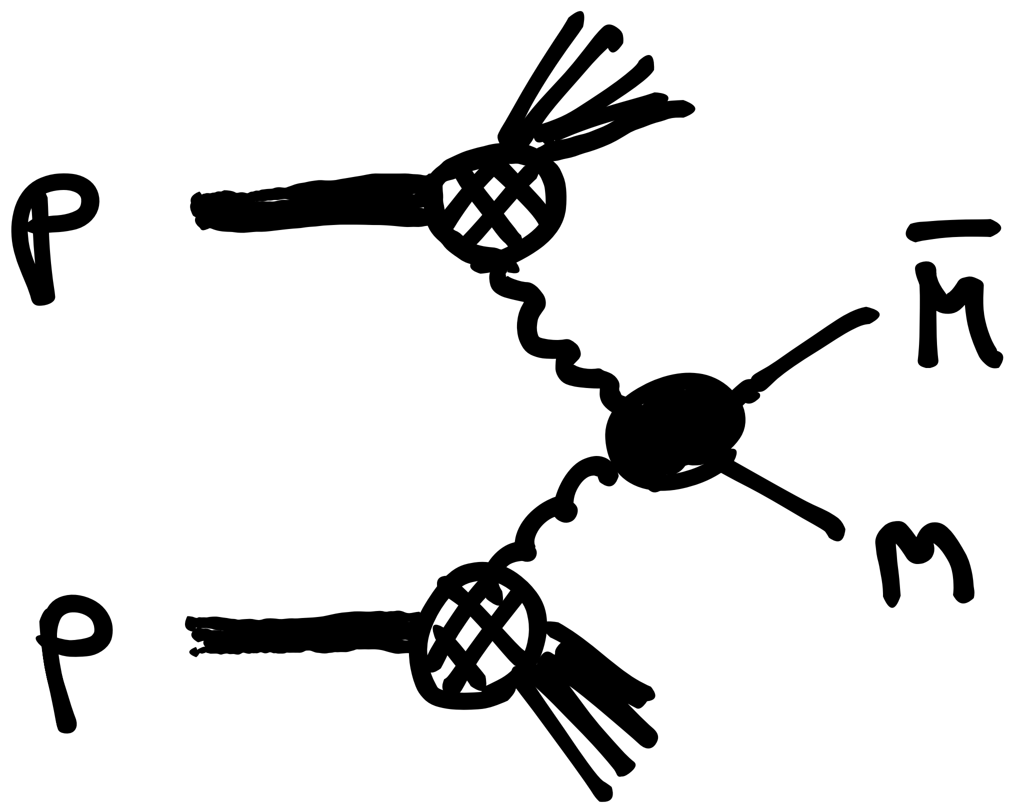
IF THE FRAME OF THE OBSERVER AND THAT OF REST OF THE MOTHER ARE CONNECTED BY A BOOST $\beta$

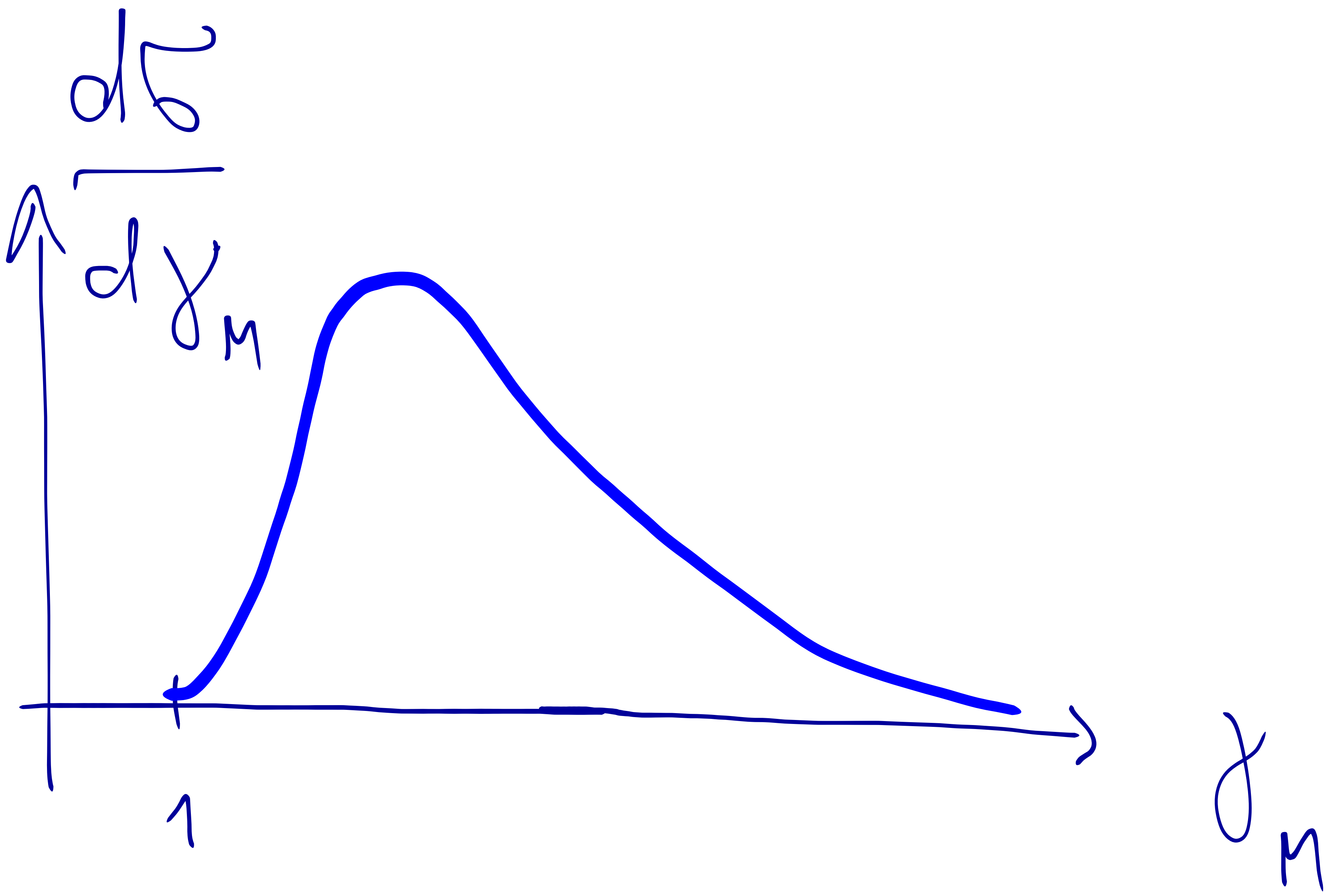
$$E'_d = E^*_d \gamma + p^*_d \gamma \beta \cos\vartheta^*$$

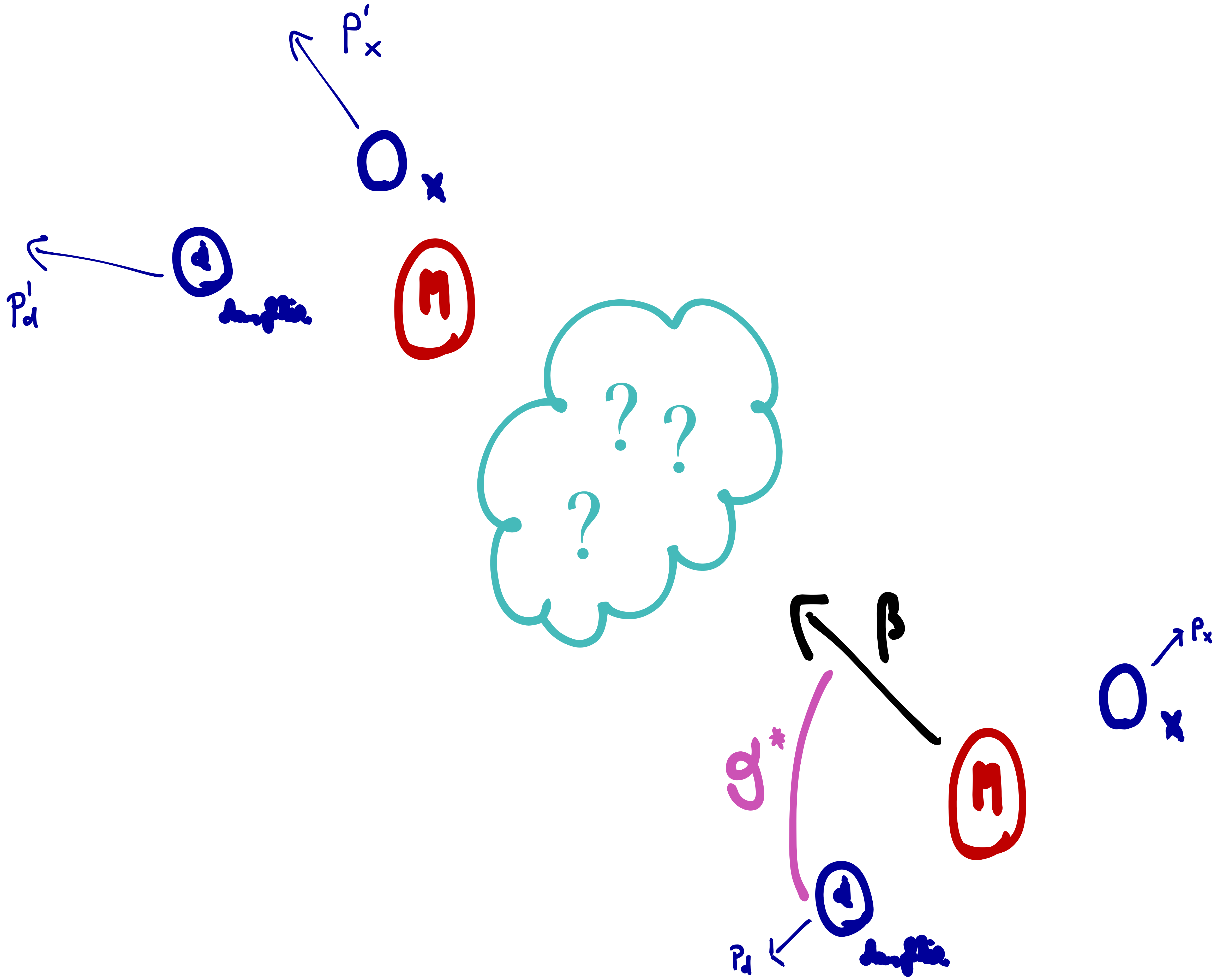



... BUT

# BUT IN MOST CASES WE DO NOT KNOW THE BOOST OF THE MOTHER



SOLUTION TO OVERCOME THE UNKNOWN BOOST

USE BOOST INVARIANT QUANTITIES

- CONSERVED EVENT BY EVENT
- SIMPLE TO UNDERSTAND
- UNIVERSAL (SPECIAL RELATIVITY IS THE SAME FOR ALL PARTICLES)

- DEMANDING
  - GENERICALLY THEY ARE FUNCTION OF SEVERAL QUANTITIES
    TO MAKE AN INVARIANT MASS YOU NEED TWO FOUR-VECTORS
    WITH BOTH ENERGY AND ANGLES

# IN THIS TALK:

LORENTZ VARIANT QUANTITIES
WITH SOME KIND OF "PHENOMENOLOGICAL INVARIANCE"
TO ACCESS INVARIANTS OF THE DECAY

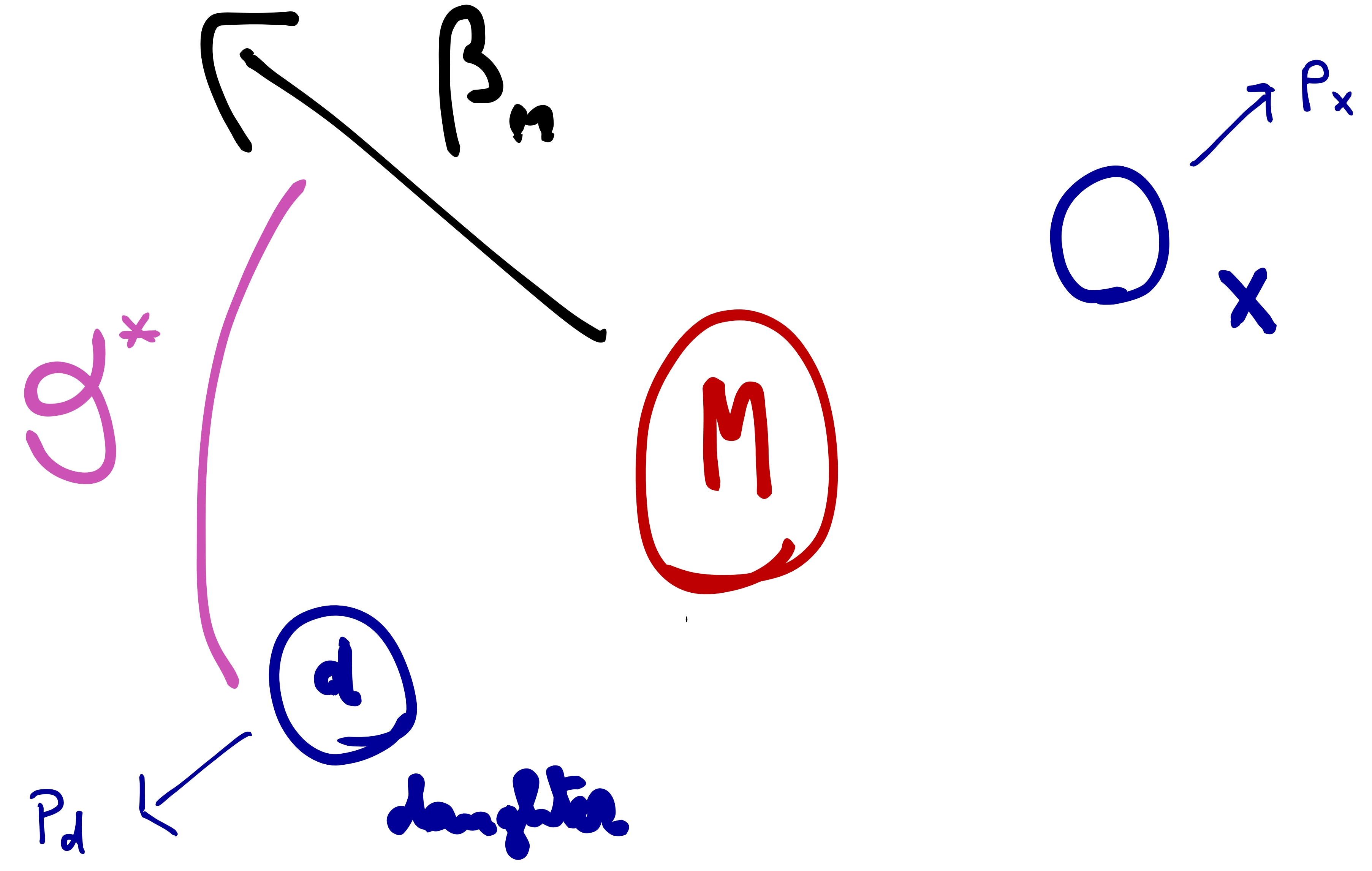
FOR INSTANCE:

THE OBSERVED ENERGY DEPENDS ON THE FRAME

THE <u>ENERGY</u> <u>DISTRIBUTION</u> IN PHENOMENOLOGICALLY RELEVANT
SITUATIONS HAS SOME INVARIANCE

- DAUGHTER $d$ IS MASSLESS (for now)

- IMAGINE THE MOTHER HAS A BOOST $\beta_M$ IN THE LAB FRAME

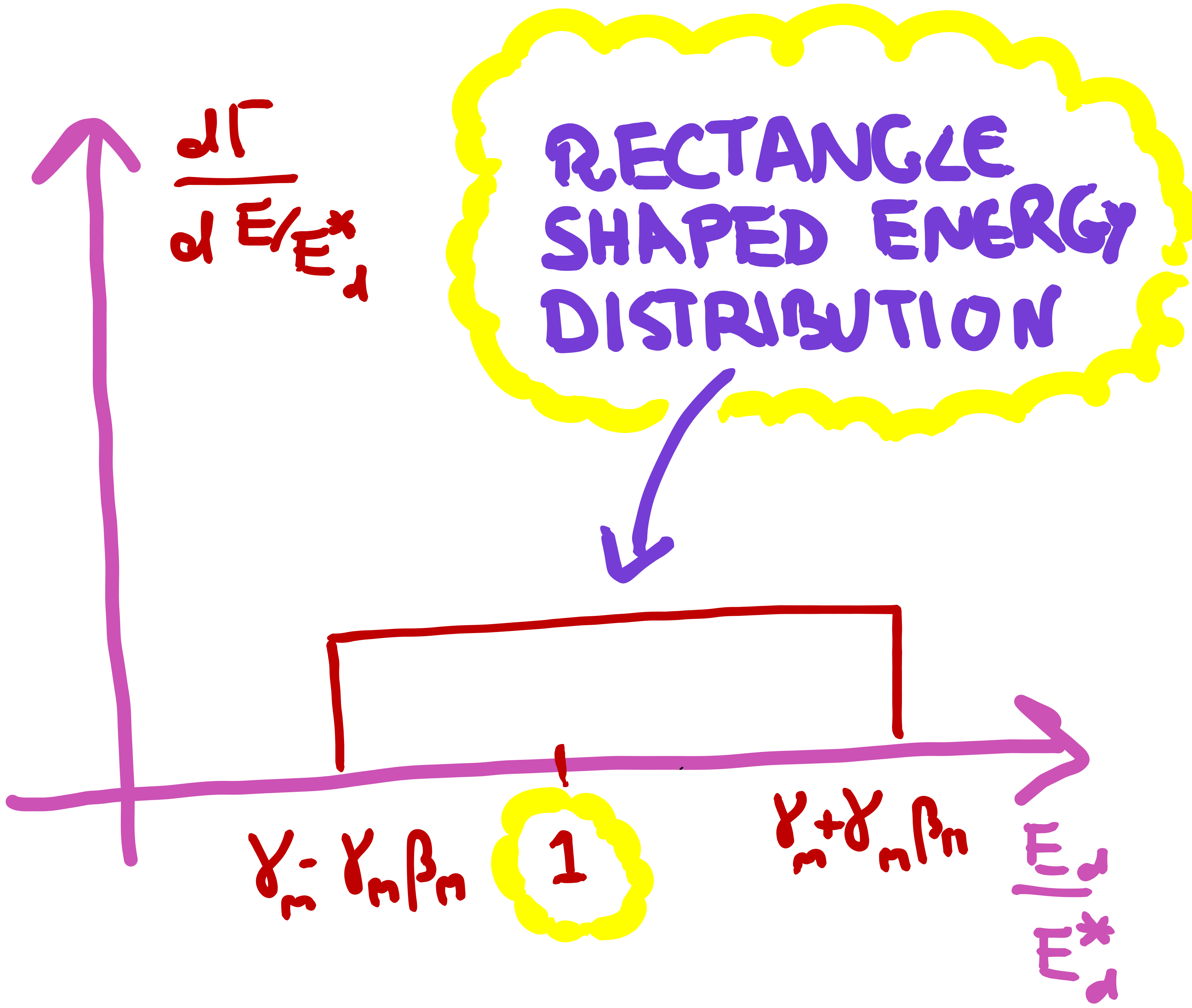


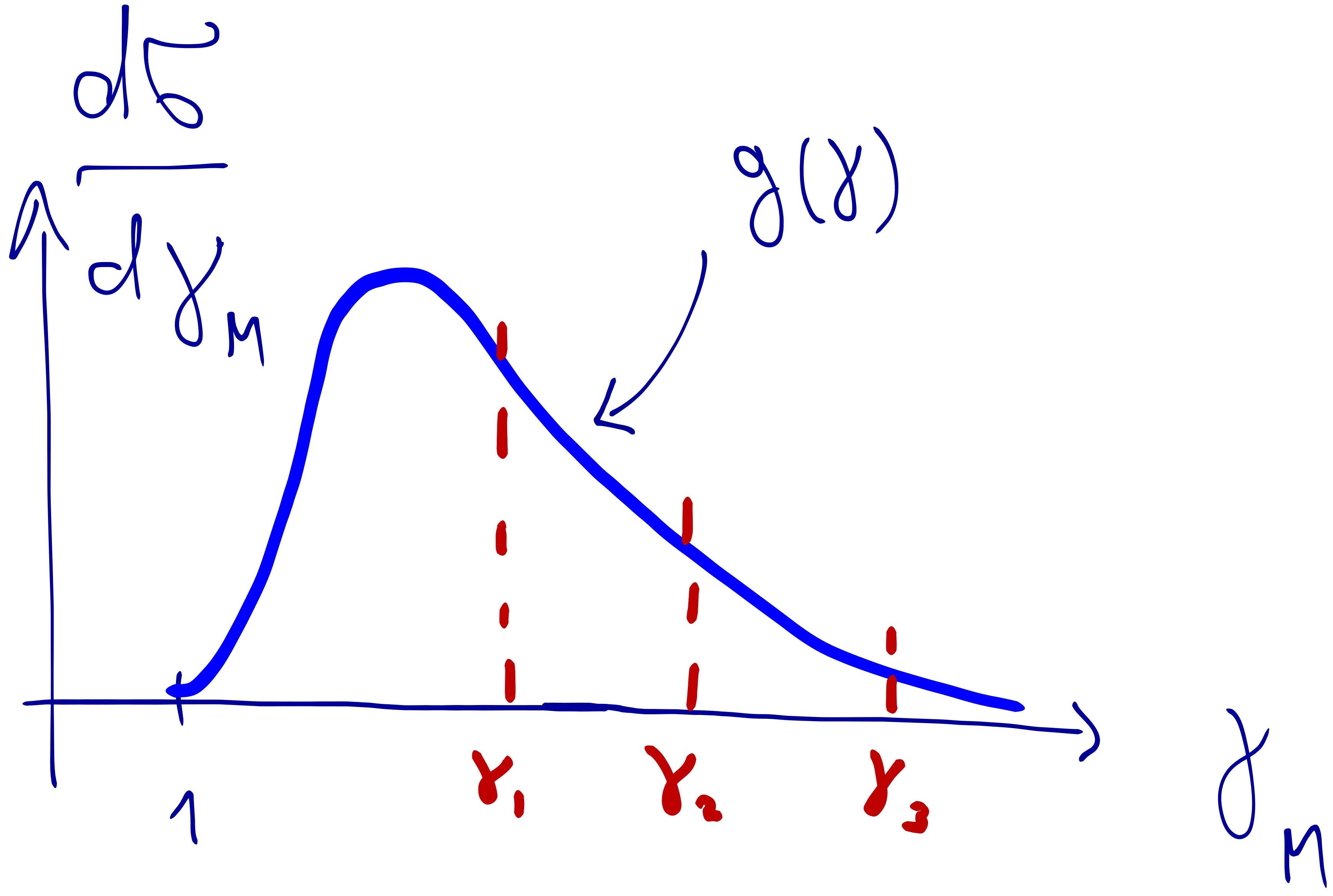

- THE DAUGHTER MOMENTUM IS AT AN ANGLE $\theta$ W.R.T. $\beta_M$

IN THE LAB

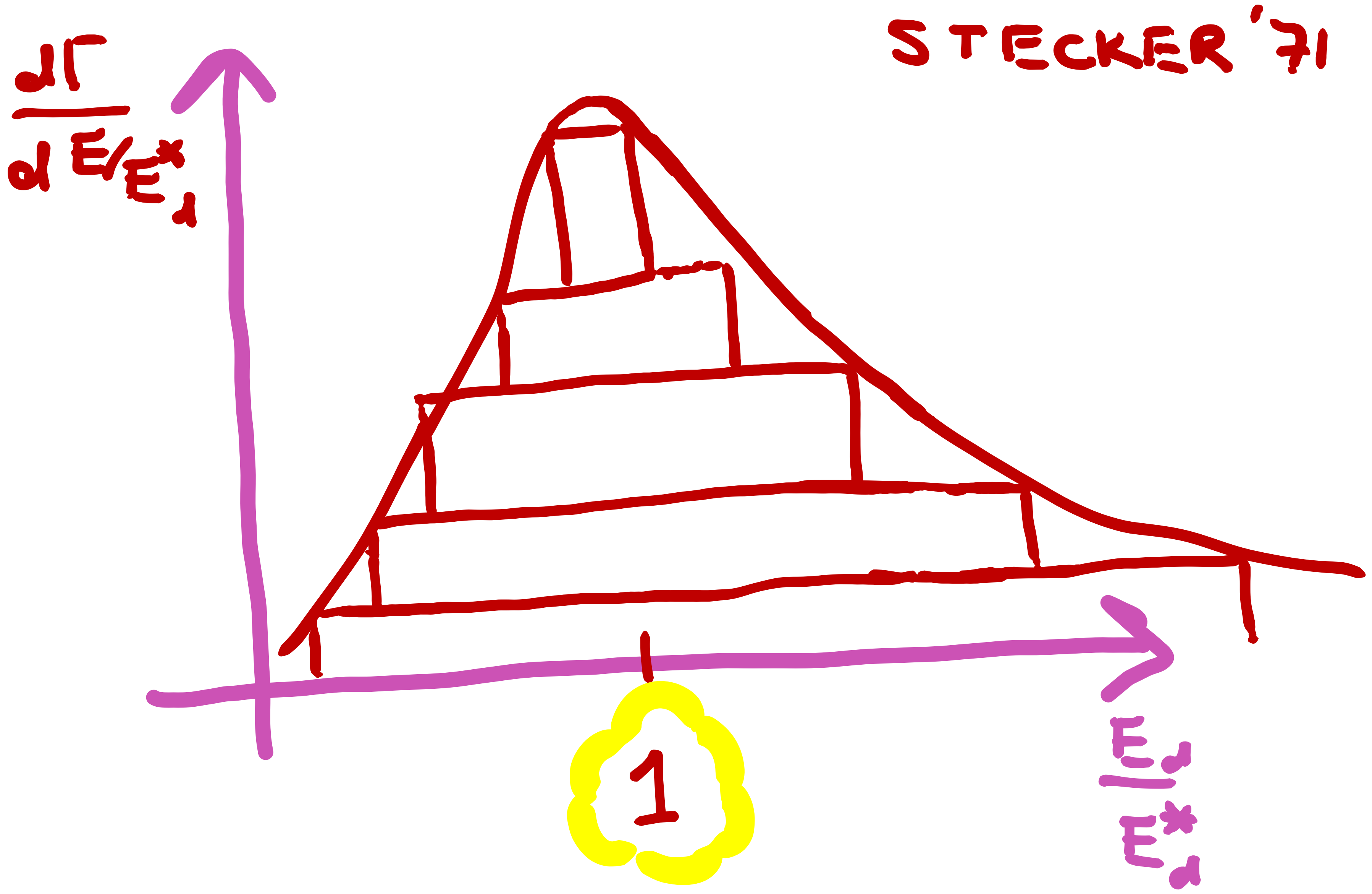
$$E_d = E_d^* \left( \gamma_M + \cos\theta^* \beta_M \gamma_M \right)$$

IF THE MOTHER IS A SCALAR $\cos\theta$ IS FLAT FROM $-1$ TO $1$

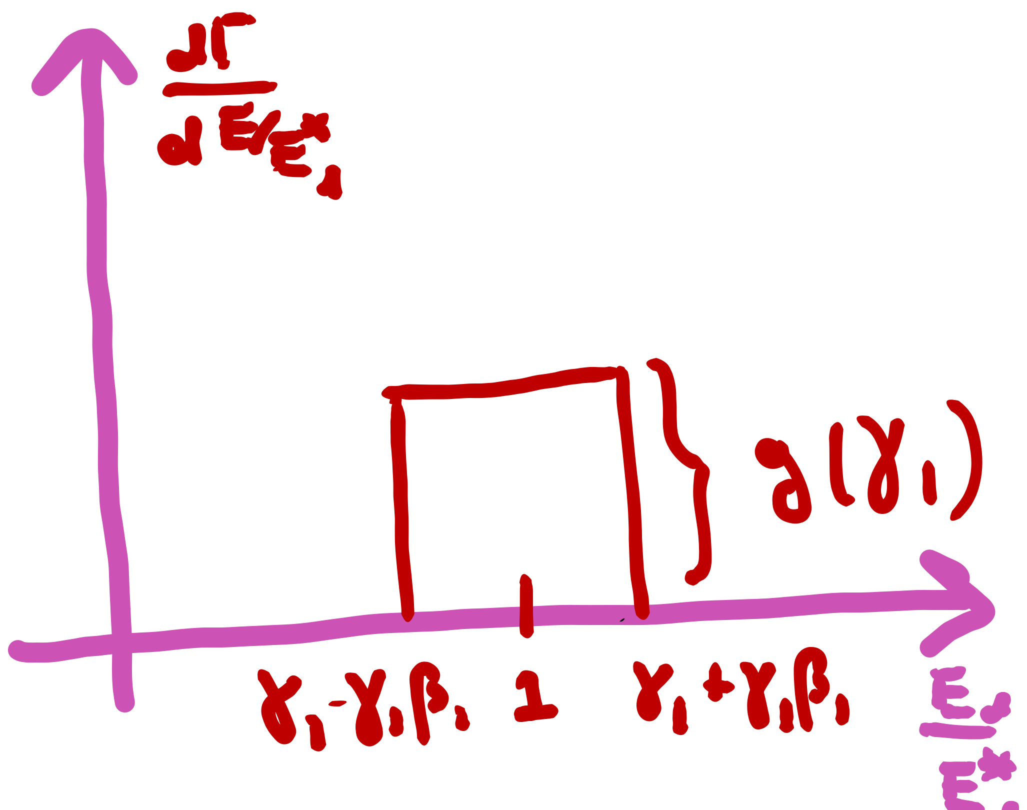


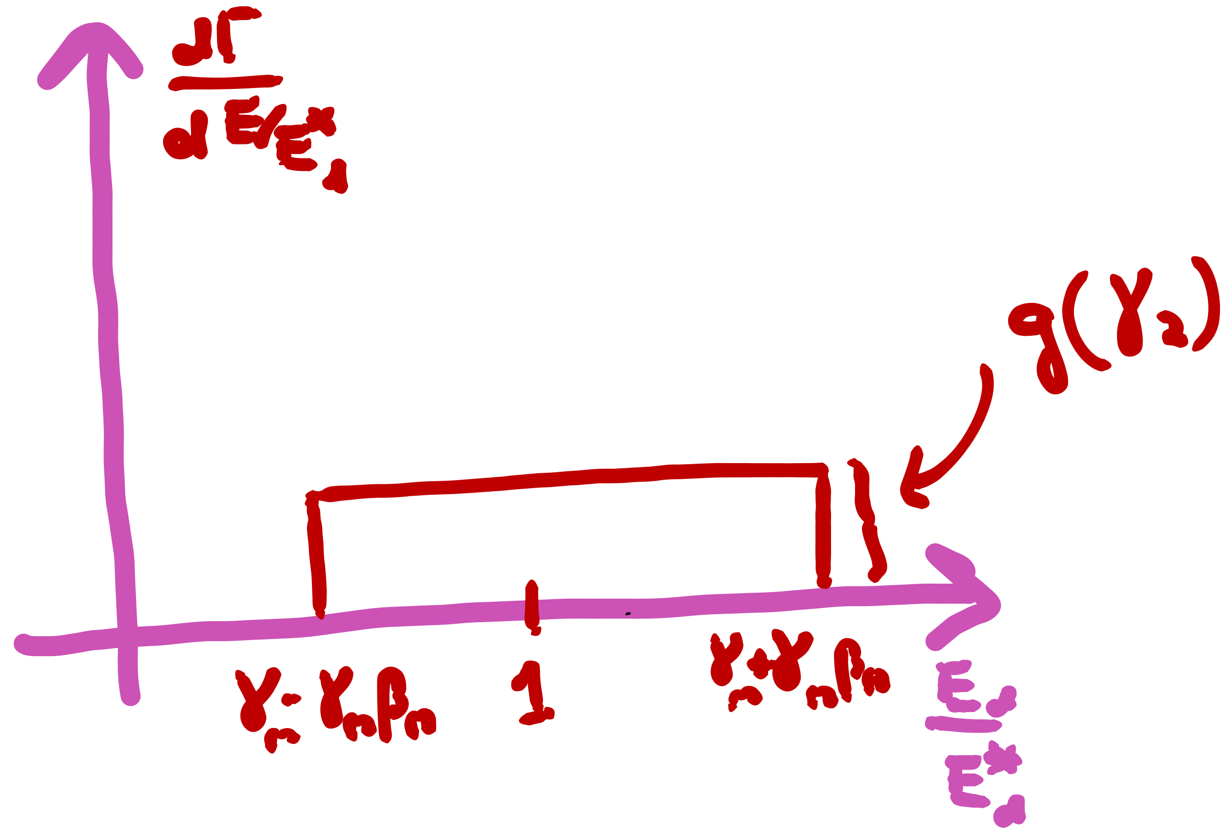


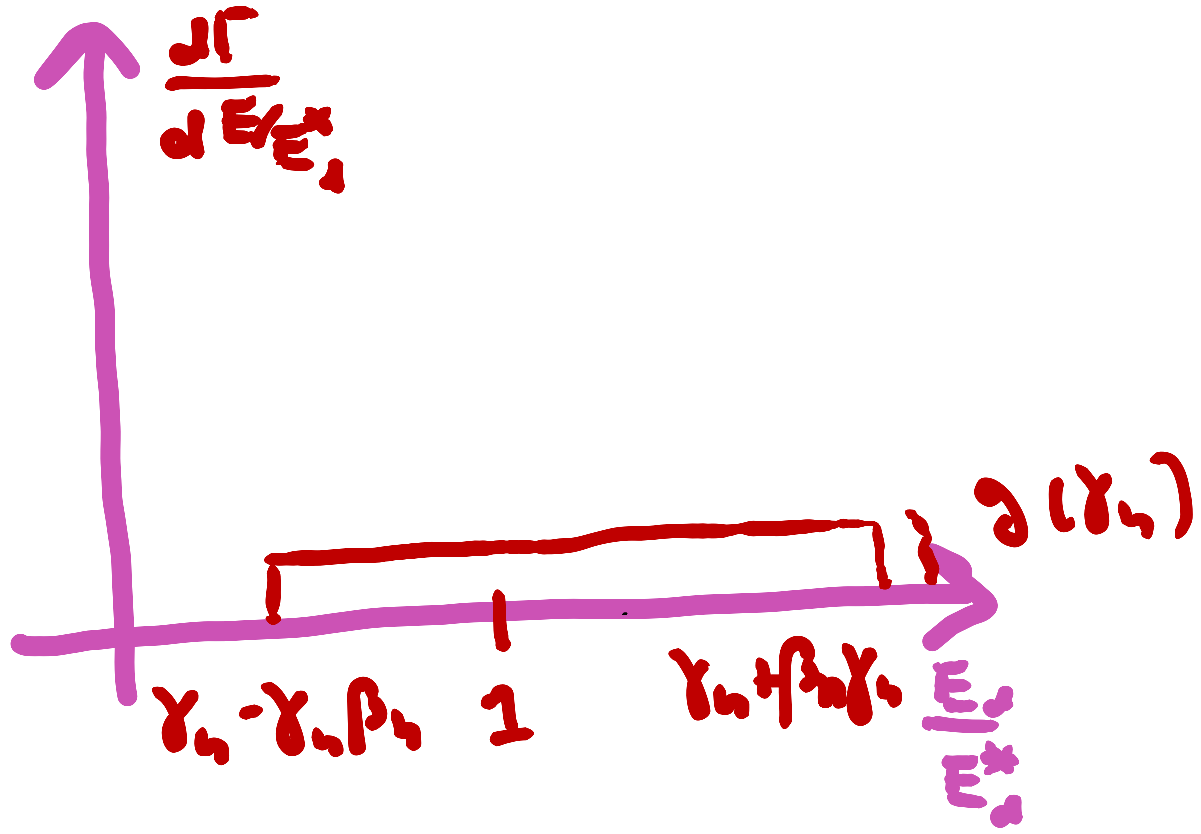

STECKER '71




THE ENERGY DISTRIBUTION IN THE LAB IS THE SUM OF ALL THE RECTANGLES



THOUGH NOT THAT USED

THIS PEAK WAS KNOWN IN COSMIC RAYS PHYSICS

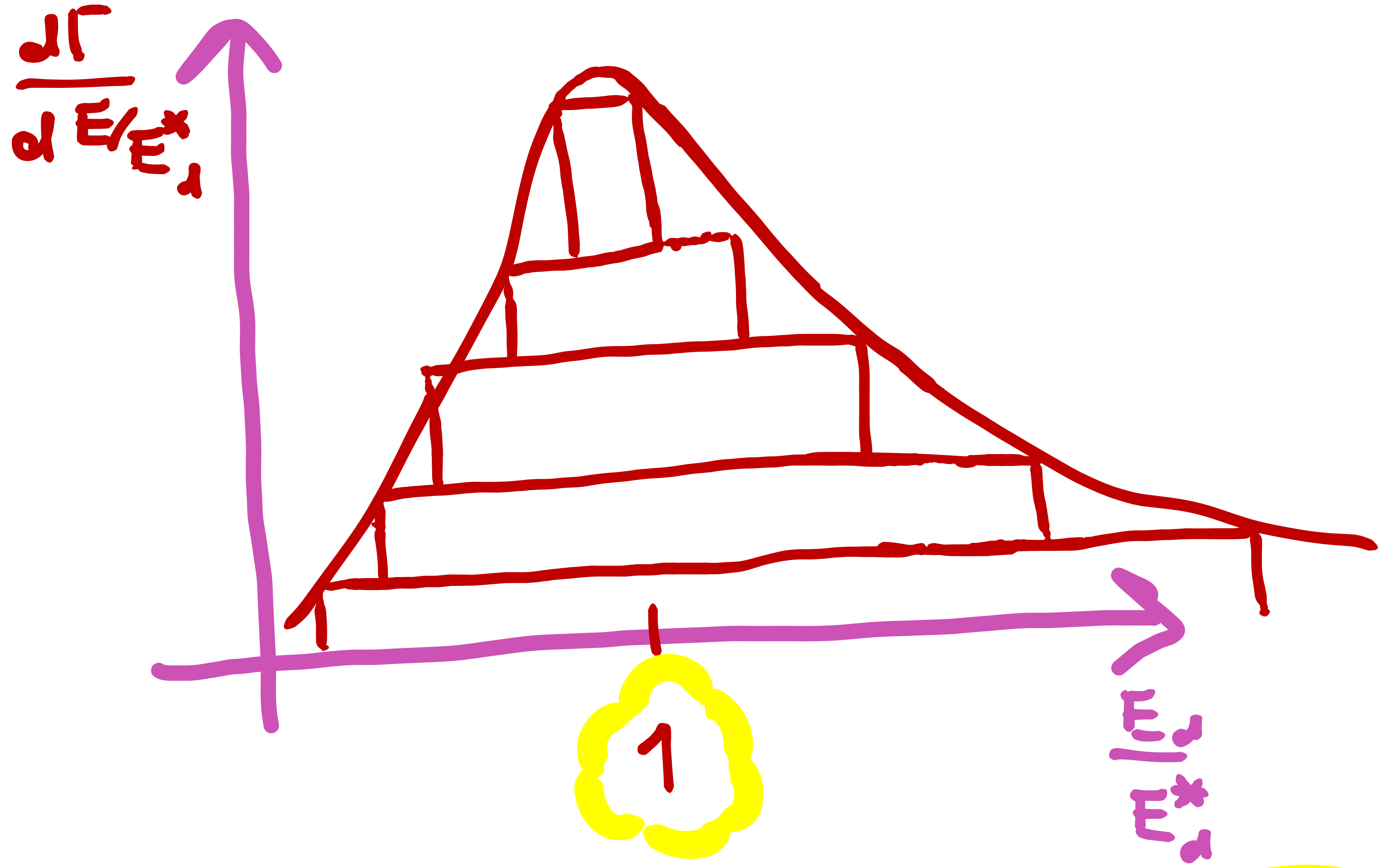
IS IT USEFUL FOR HIGH ENERGY PARTICLE PHYSICS?

# GENERALIZATIONS:

- INSTEAD OF A SCALAR MOTHER ONE CAN TAKE AN UNPOLARIZED ENSEMBLE OF PARTICLES WITH SPIN

- THE DAUGHTER CAN BE MASSIVE IF $g(\gamma) = 0$ for $\gamma \geq 2\gamma^{*2} - 1$ WHERE $\gamma^* = \frac{E_d^*}{m_d}$

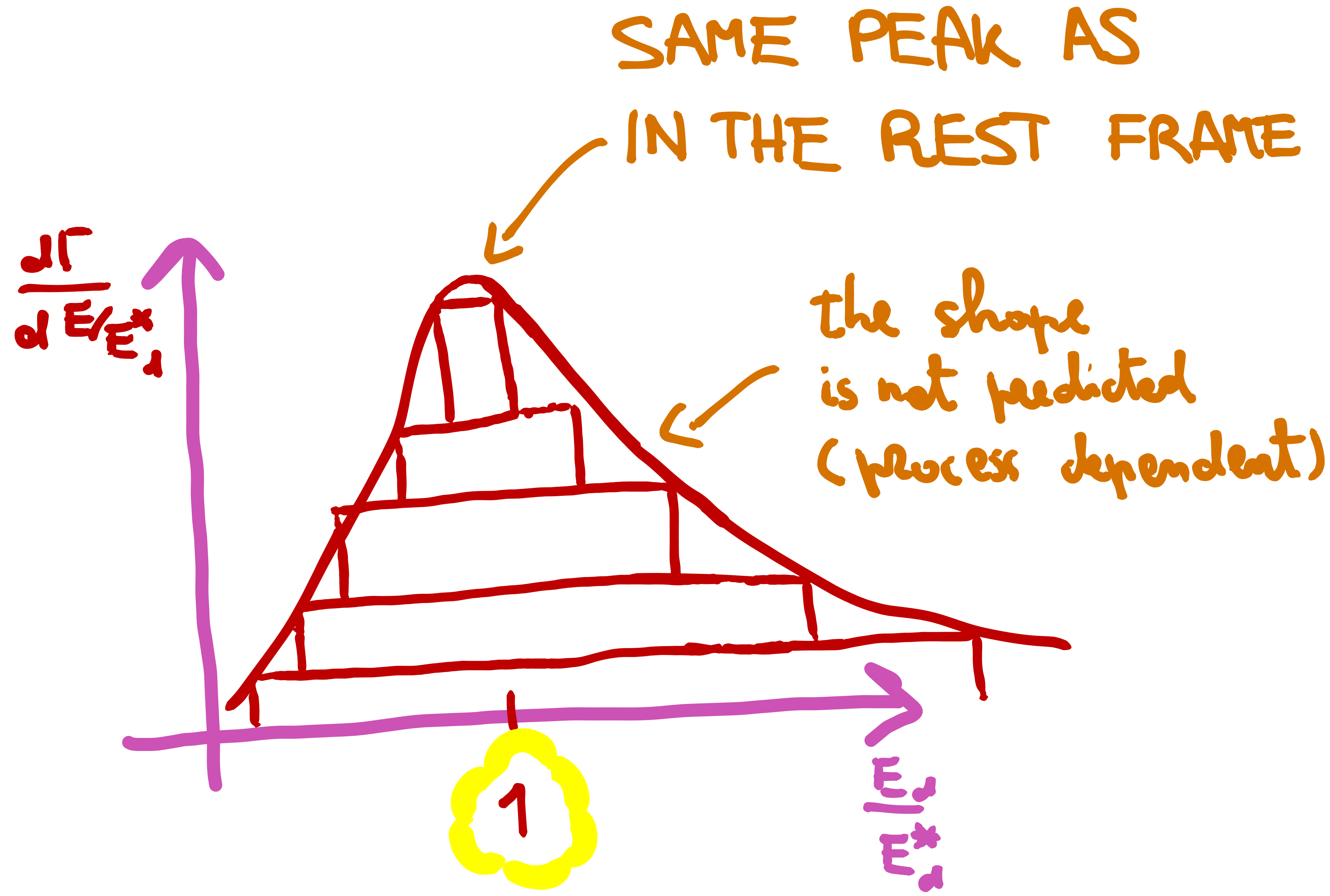
$$E_d^* = \frac{m_M^2 + m_d^2 - m_x^2}{2 m_M}$$




$E = E_d^*$ IS THE PEAK

THE FRAME-DEPENDENT ENERGY DISTRIBUTION ENCODES THE INVARIANT $E_d^*$ IN A VERY SIMPLE WAY

# ADVANTAGES (GENERAL: ALMOST ONLY KINEMATICS)

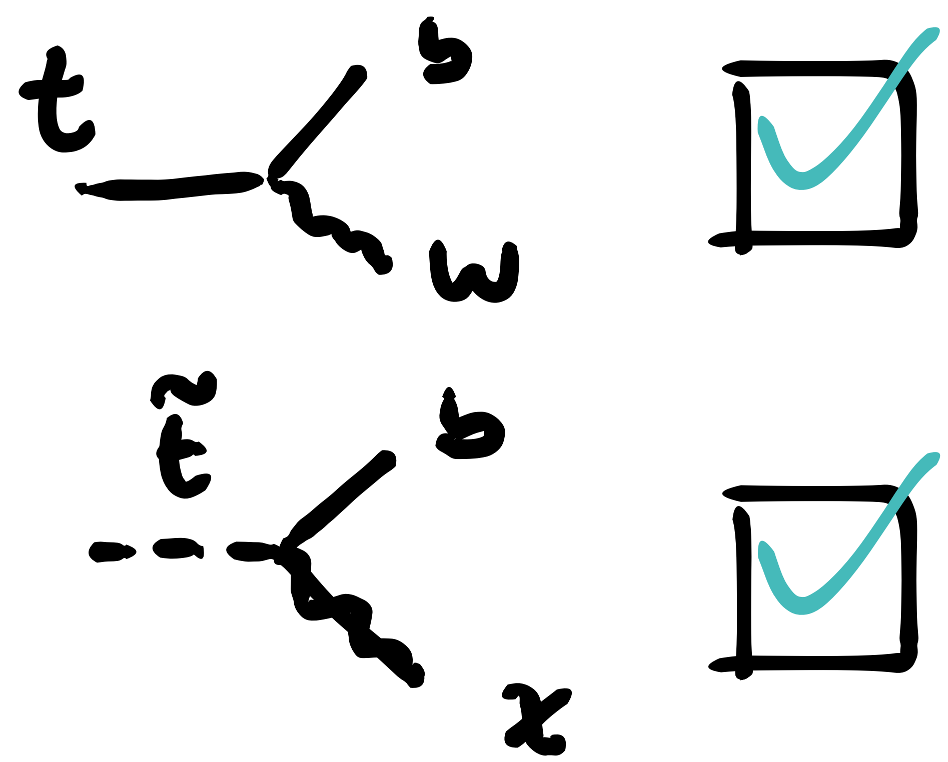



THE FRAME-DEPENDENT ENERGY DISTRIBUTION ENCODES THE INVARIANT $E_d^*$ IN A VERY SIMPLE WAY

THE ONLY DYNAMICAL ASSUMPTION WAS THE MOTHER TO BE NOT POLARIZED

⇓

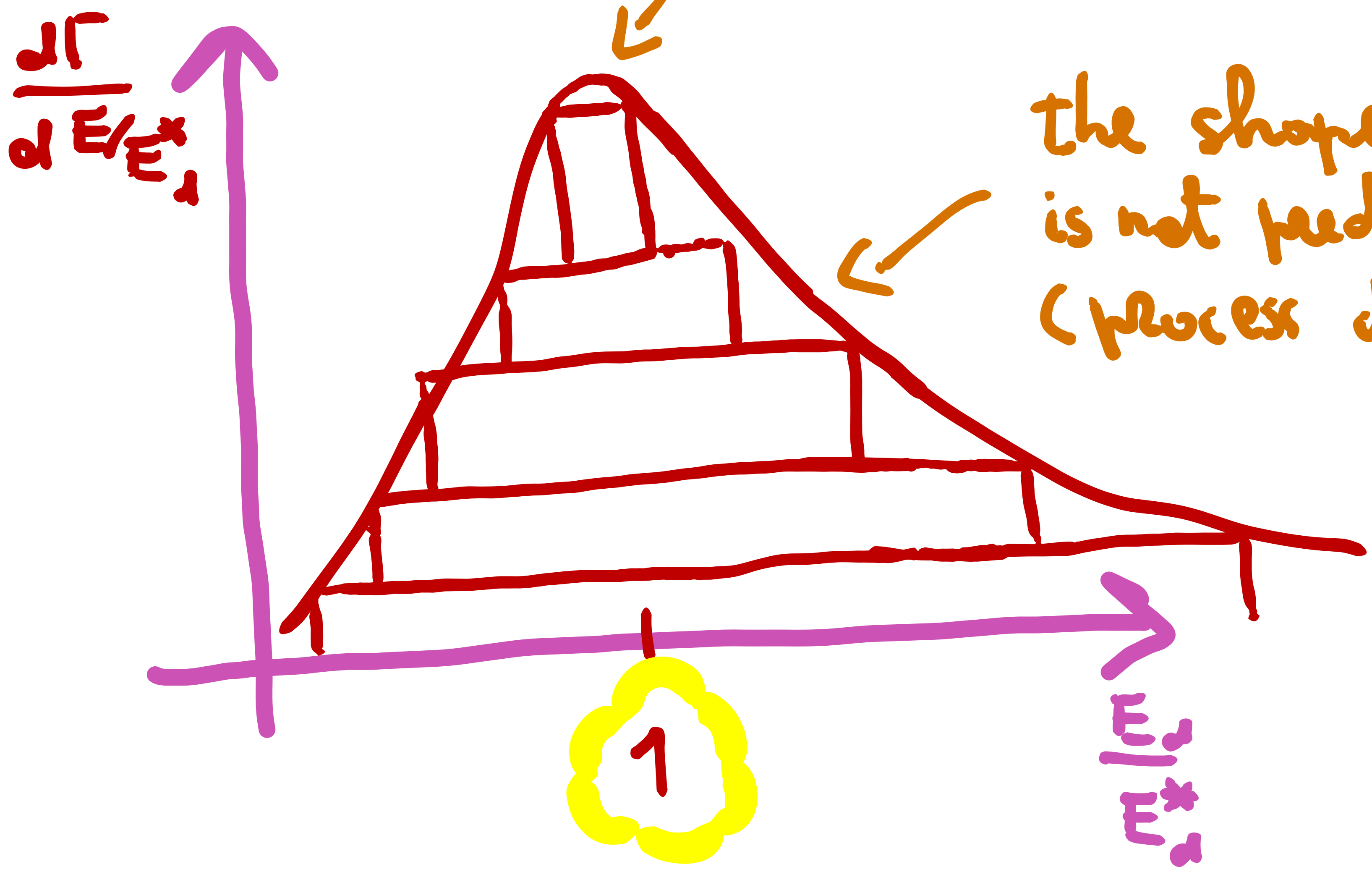
THE RESULT APPLIES FOR BOTH KNOW PARTICLES OF THE SM AND FOR NEW PHYSICS



# ADVANTAGES (GENERAL: ALMOST ONLY KINEMATICS)

SAME PEAK AS IN THE REST FRAME

the shape is not predicted (process dependent)

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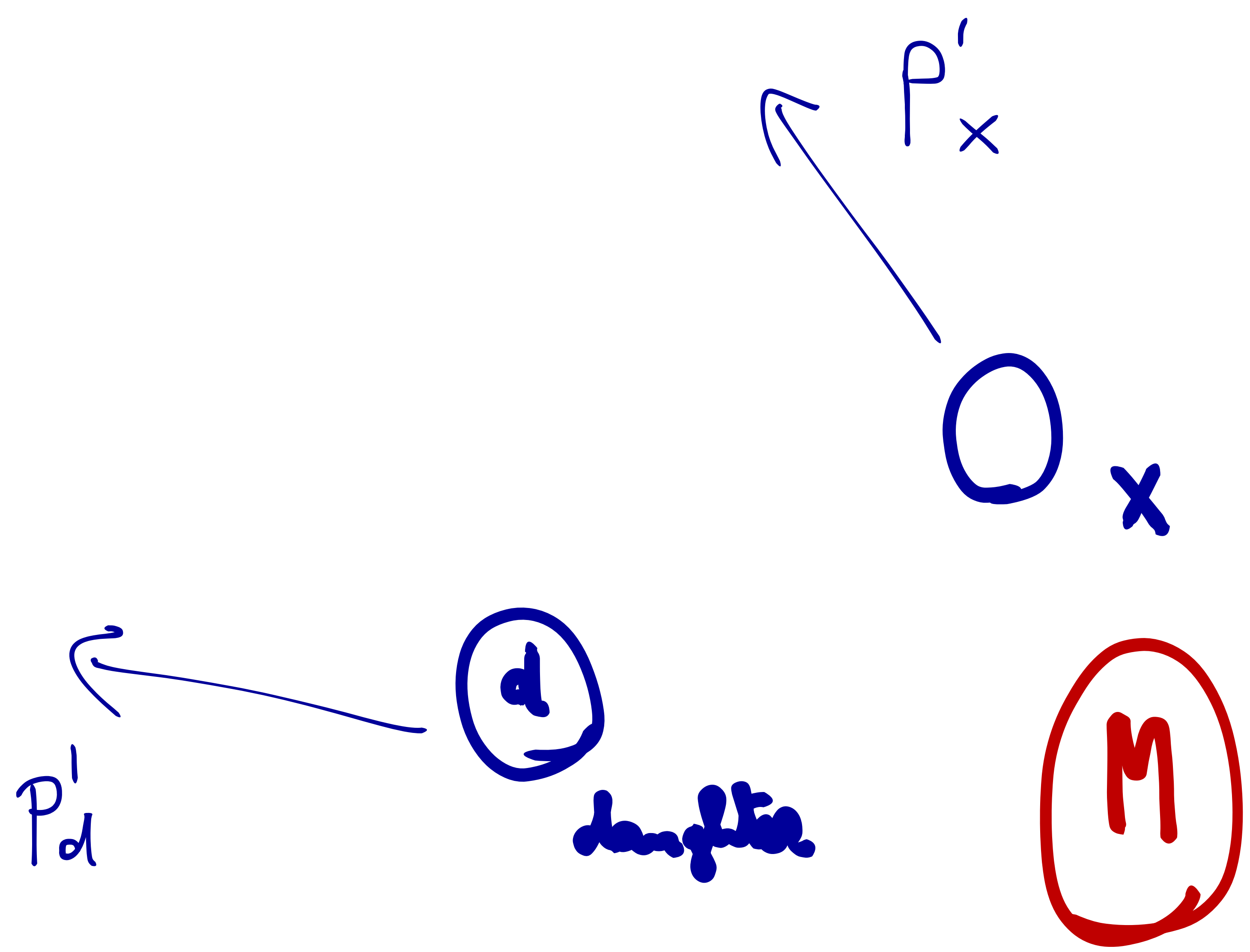
THE RESULT APPLIES FOR BOTH KNOW PARTICLES OF THE SM AND FOR NEW PHYSICS

THE FRAME-DEPENDENT ENERGY DISTRIBUTION ENCODES THE INVARIANT $E_d^*$ IN A VERY SIMPLE WAY

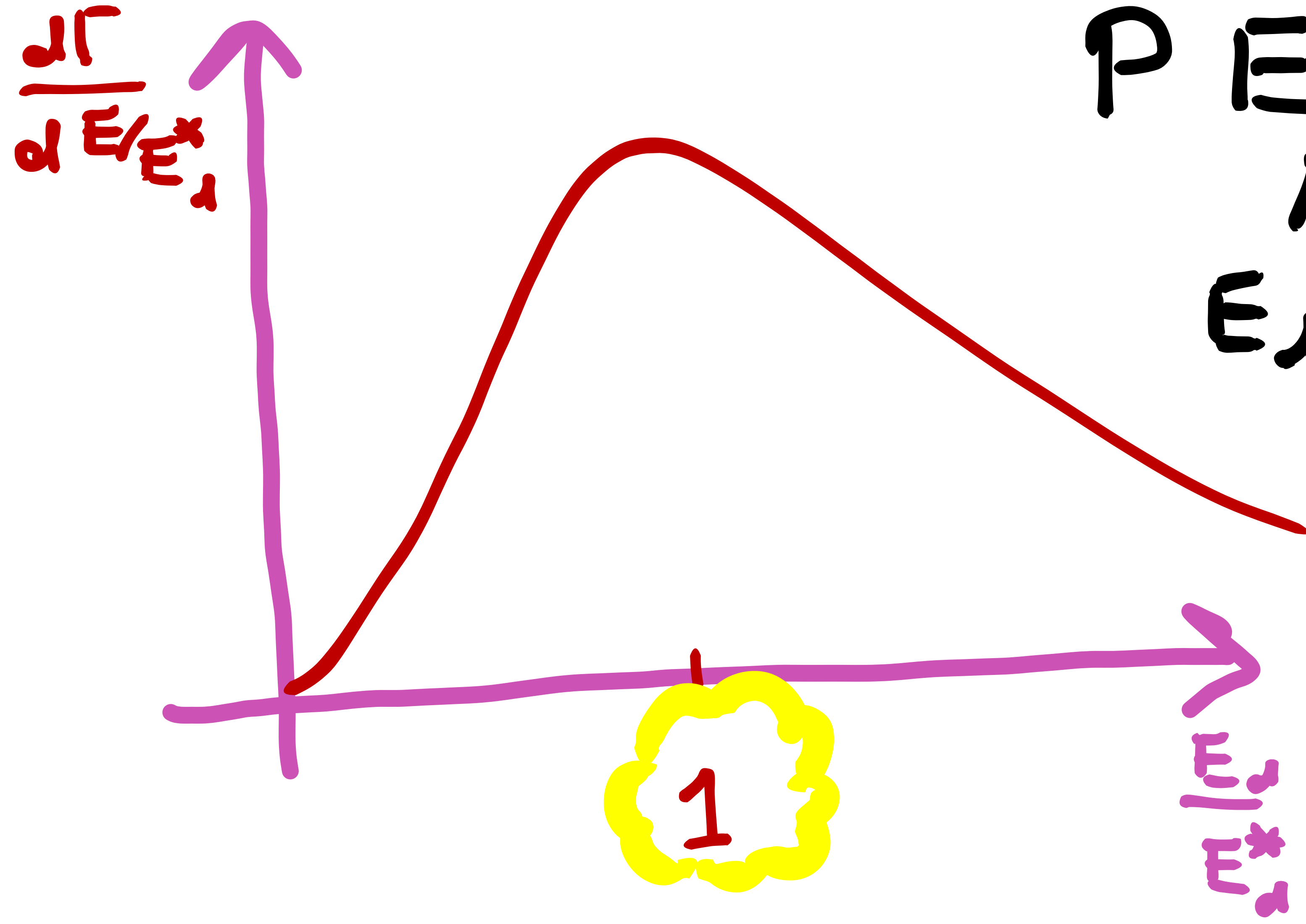
NO NEED TO KNOW THE EXACT PRODUCTION MECHANISM

# ADVANTAGES

(OVER INVARIANT MASS FOR INSTANCE)




$$E_d^* = \frac{m_M^2 + m_d^2 - m_x^2}{2 m_M}$$




PEAK AT $E_d = E_d^*$

- NO NEED TO MEASURE THE OTHER DECAY PRODUCT

⇓

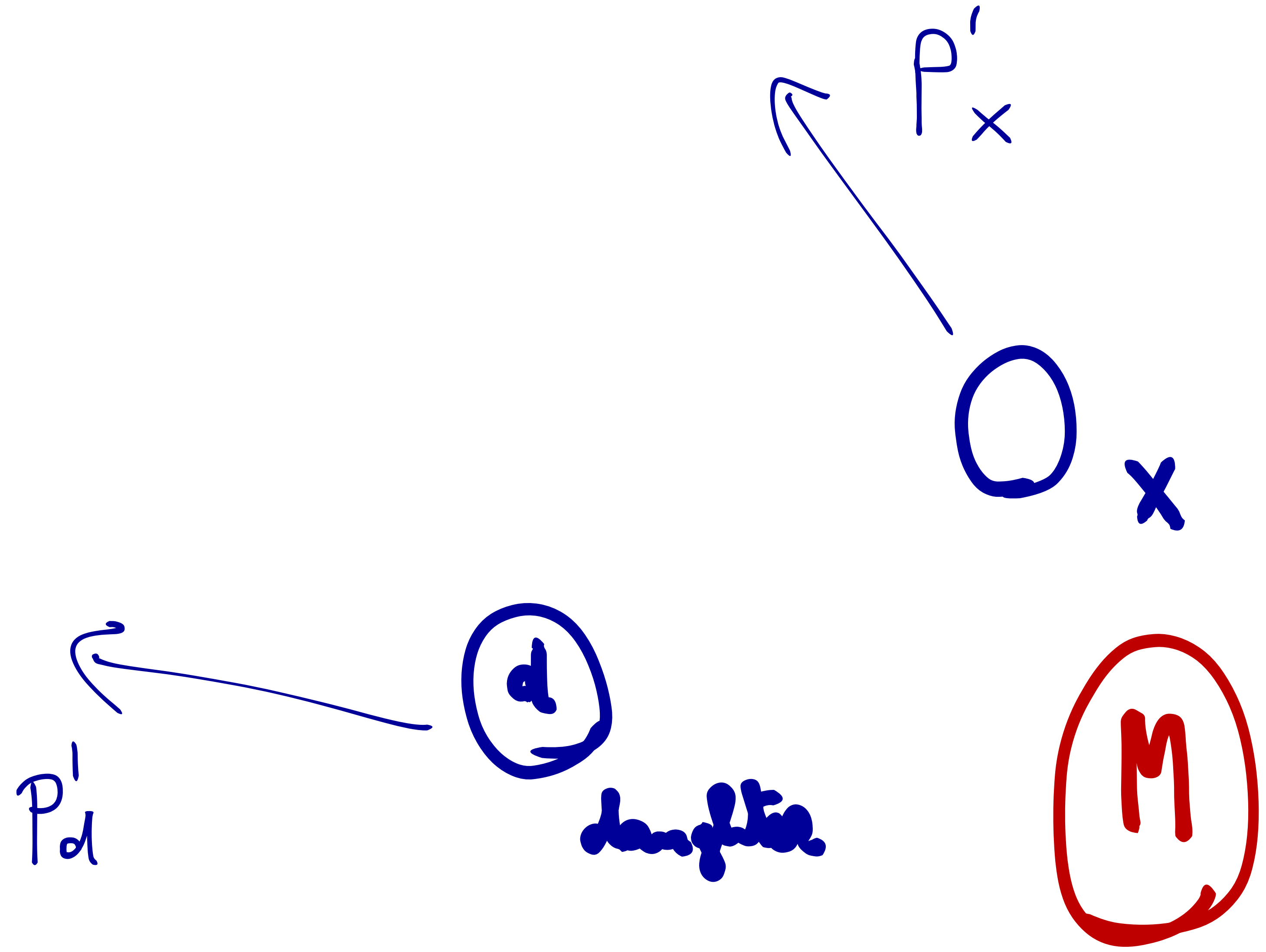
$\tilde{b} \to b \chi^0$ ← DARK MATTER

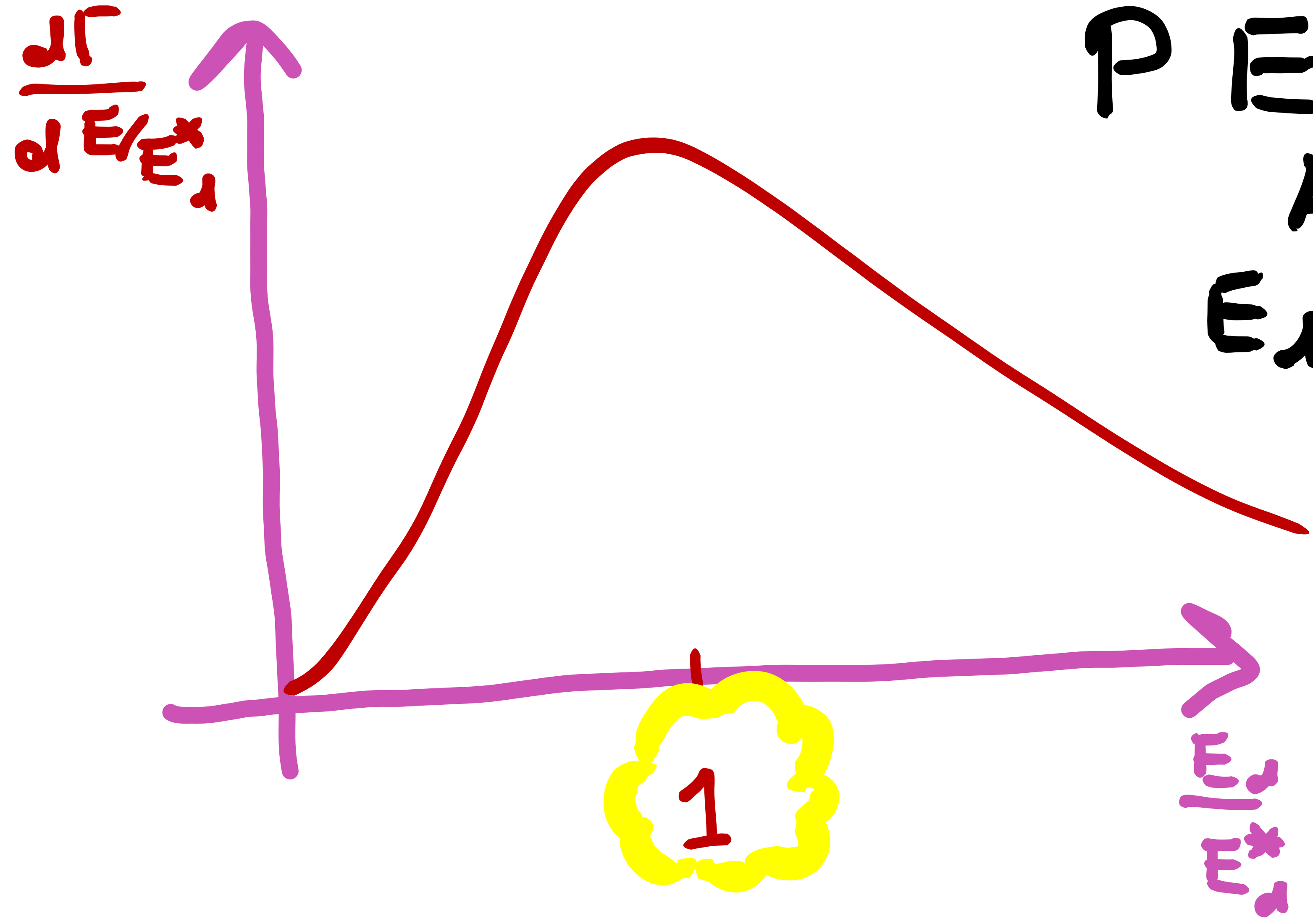
$W \to \ell \nu$ ← INVISIBLE

$t \to b W \to b \ell \nu$

# ADVANTAGES

OVER MANY TRANSVERSE KINEMATICAL VARIABLES IN USE IN COLLIDER PHYSICS ( $m_{T2}$ , ... )

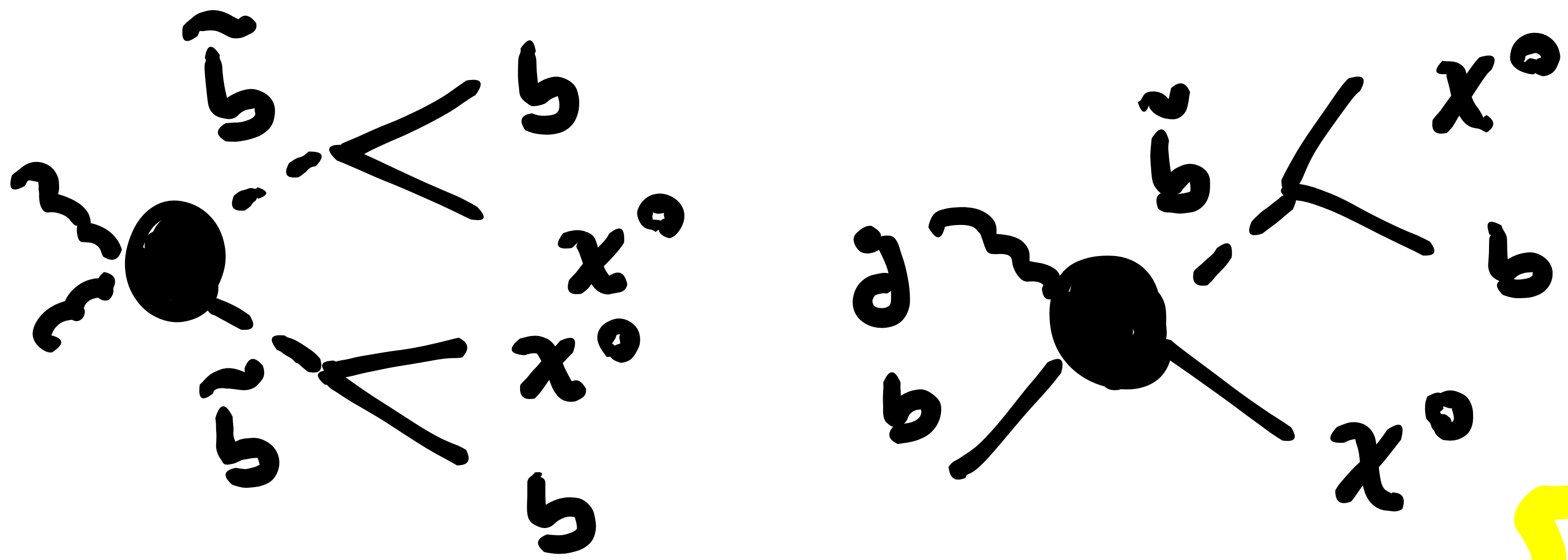



PEAK AT $E_d = E^*_d$

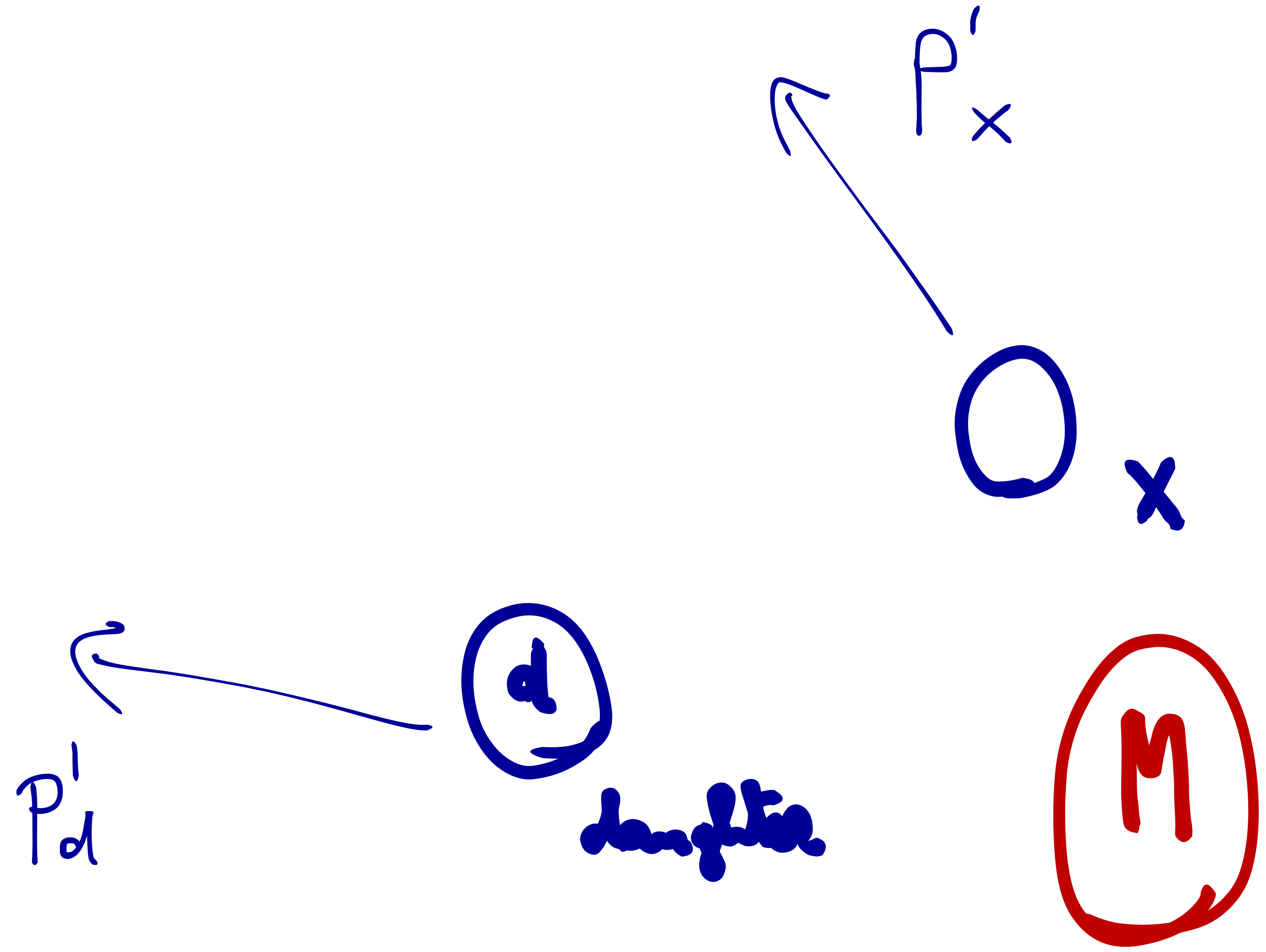
$$E^*_d = \frac{m_M^2 + m_d^2 - m_x^2}{2 m_M}$$

- NO NEED TO KNOW ANYTHING ABOUT THE REST OF THE EVENT



# ADVANTAGES

OVER MANY TRANSVERSE KINEMATICAL VARIABLES IN USE IN COLLIDER PHYSICS ($m_{T2}$, ...)

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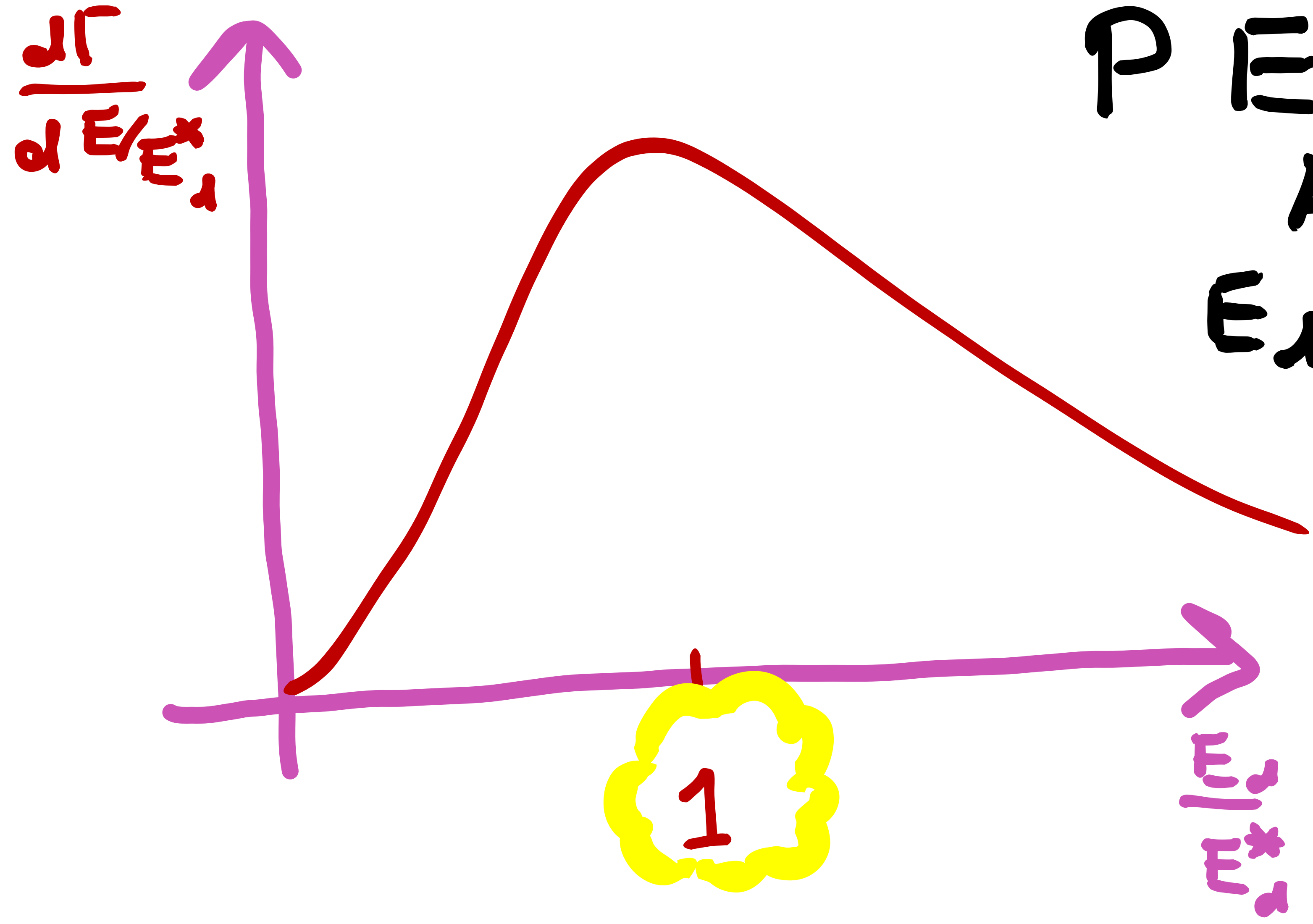


PEAK AT $E_d = E_d^*$

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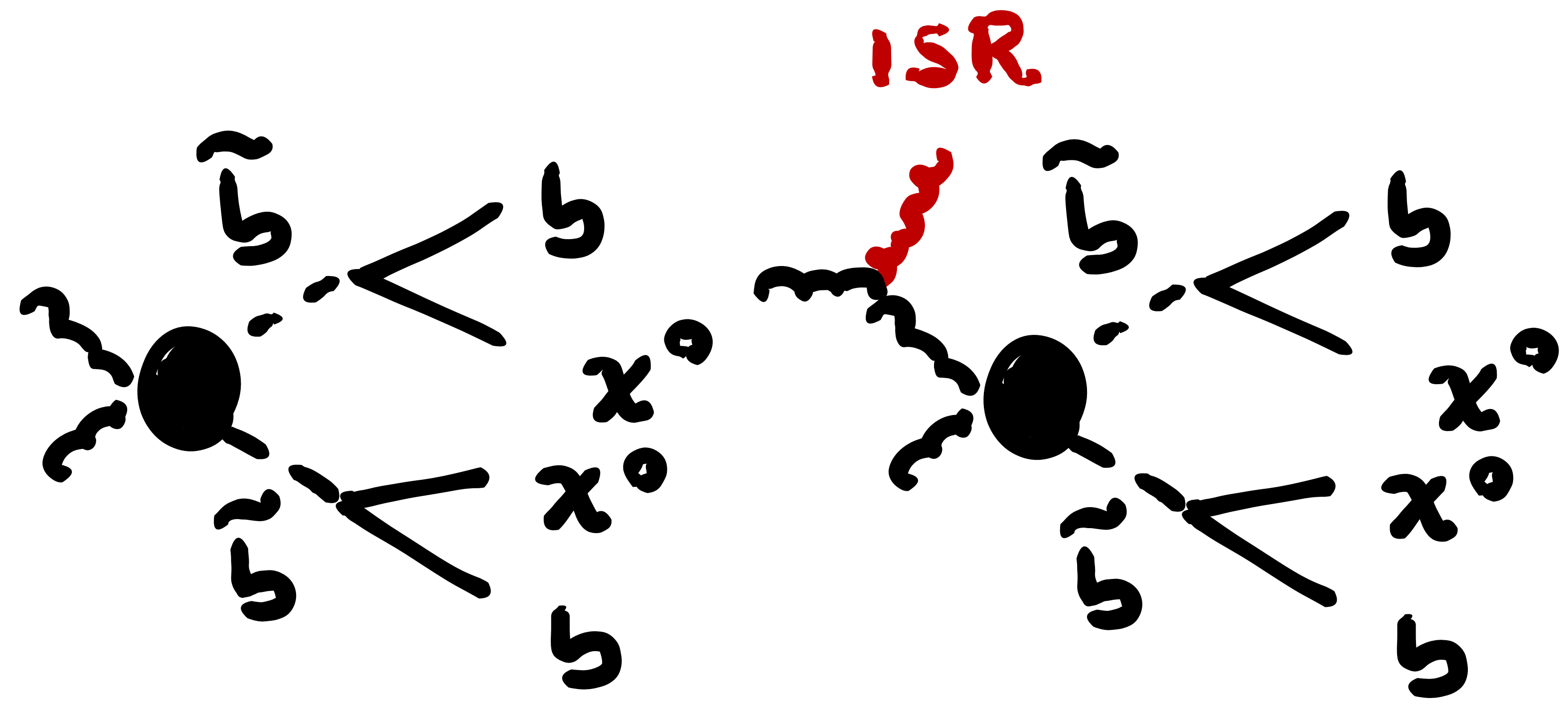


# SOME MORE INSIGHTS BY GOING THROUGH AN ANALYTIC PROOF:

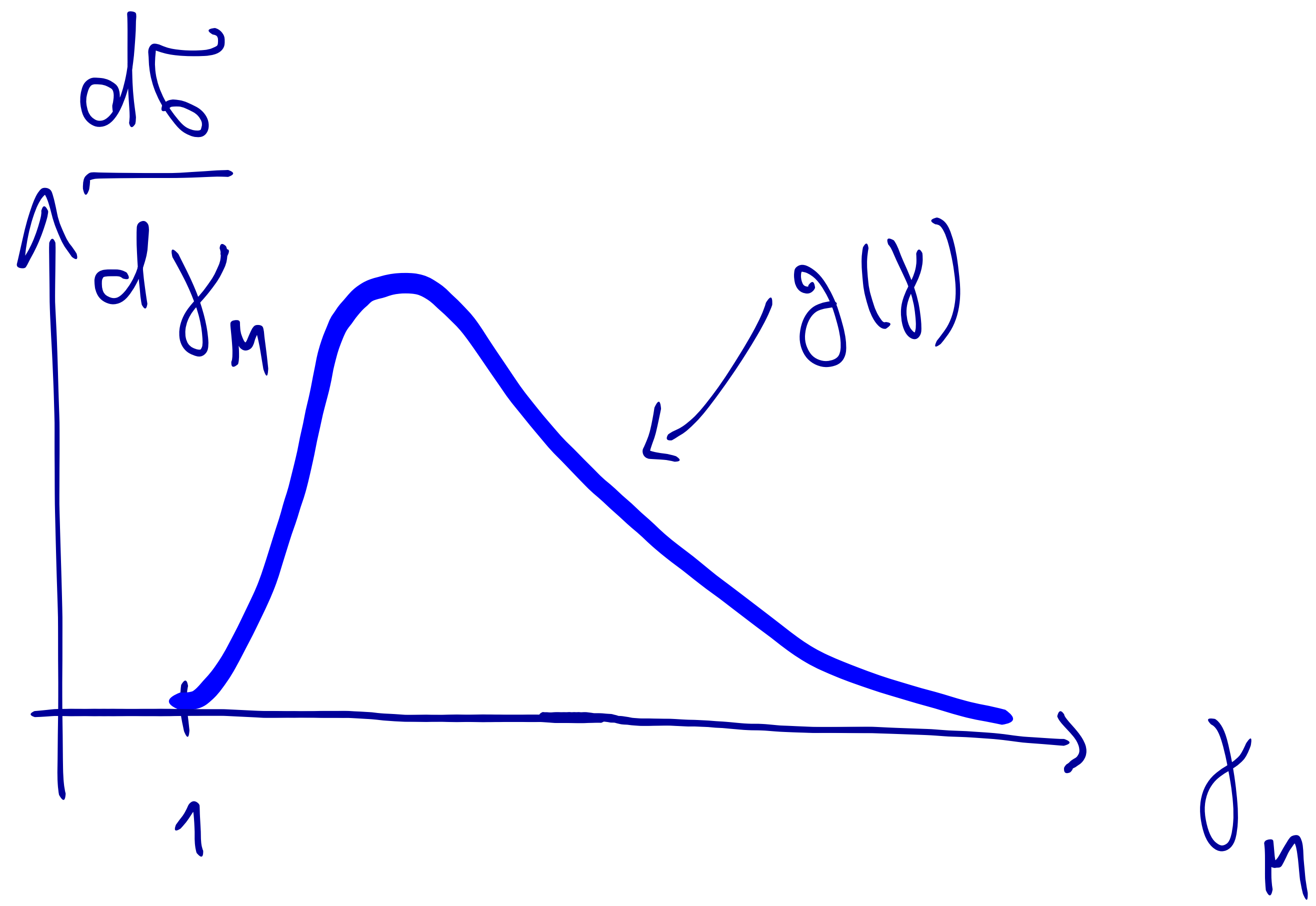
$$x := \frac{E_d}{E_d^*}$$

(MASSLESS DAUGHTER)

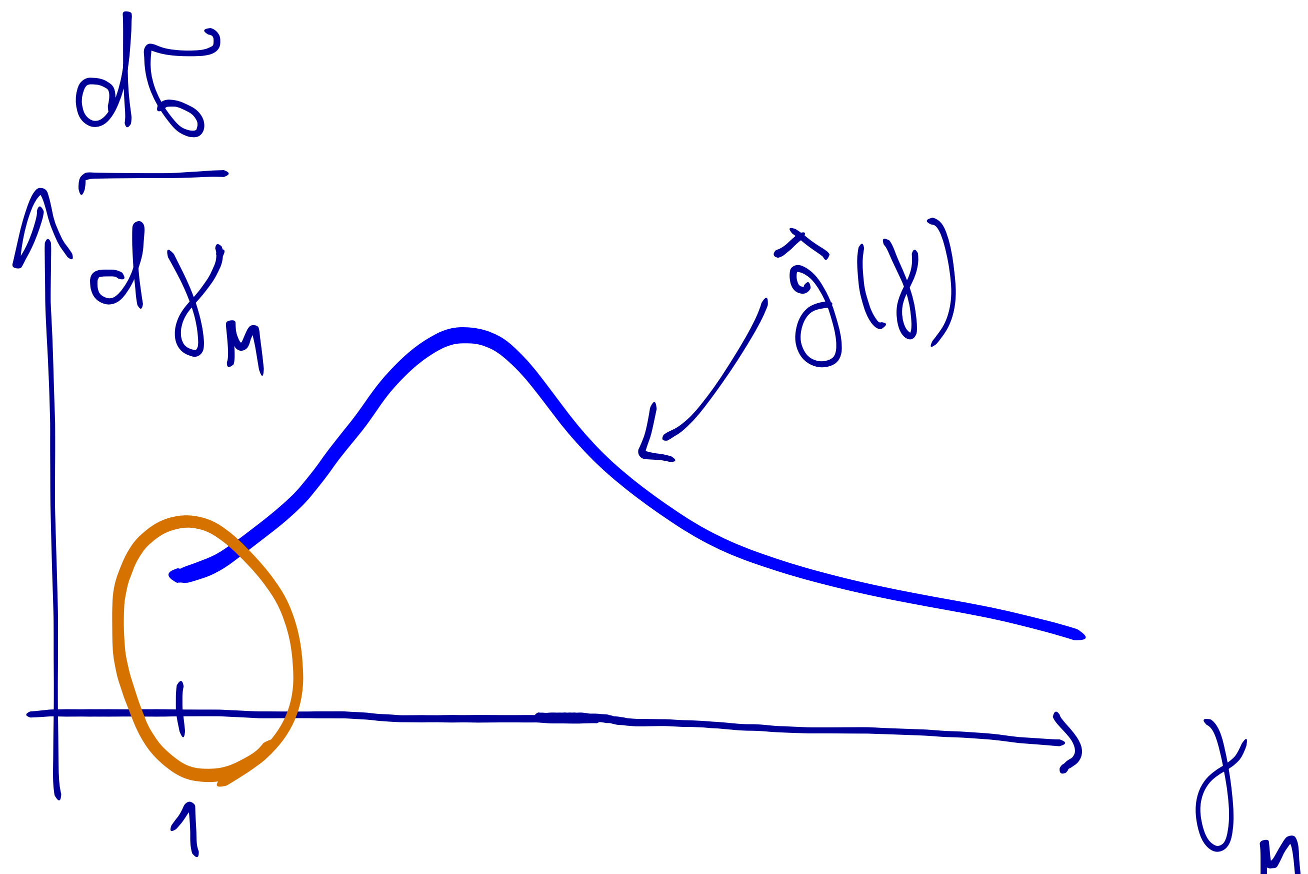
$$f(x) = \frac{1}{\Gamma}\frac{d\Gamma}{dx} = \int_{\frac{1}{2}\left(x+\frac{1}{x}\right)}^{\infty} d\gamma \, \frac{g(\gamma)}{2\sqrt{\gamma^2-1}}$$

$$f'(x) = \frac{\mathrm{sign}(1-x)}{2x}\, g\!\left(\frac{1}{2}\left(x+\frac{1}{x}\right)\right)$$

$g(1) = 0$




$g(1) \neq 0$ the derivative changes sign ⇒ **KINK IN THE OBSERVED ENERGY DISTRIBUTION**



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$$x := \frac{E_d}{E_d^*}$$

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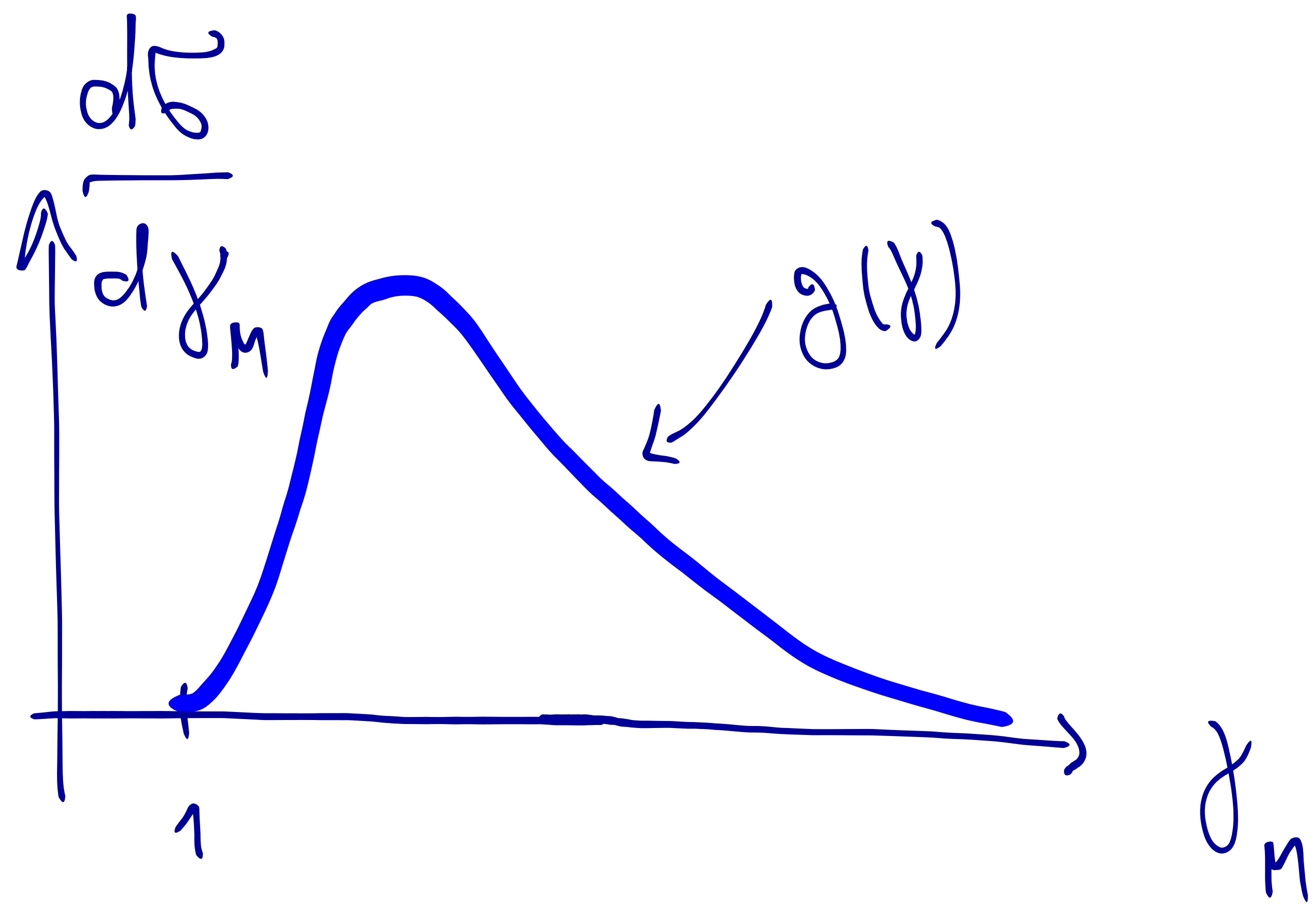
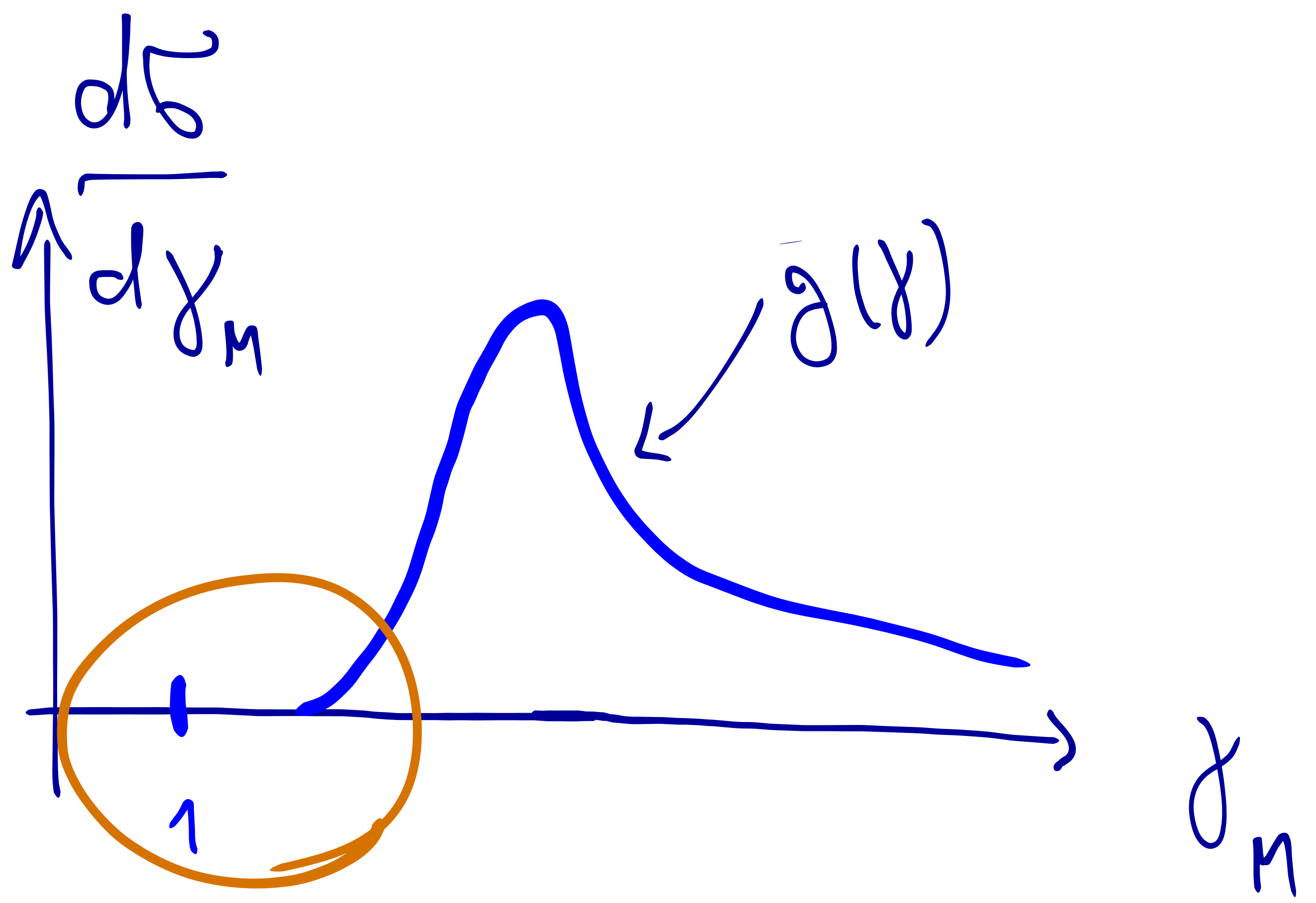
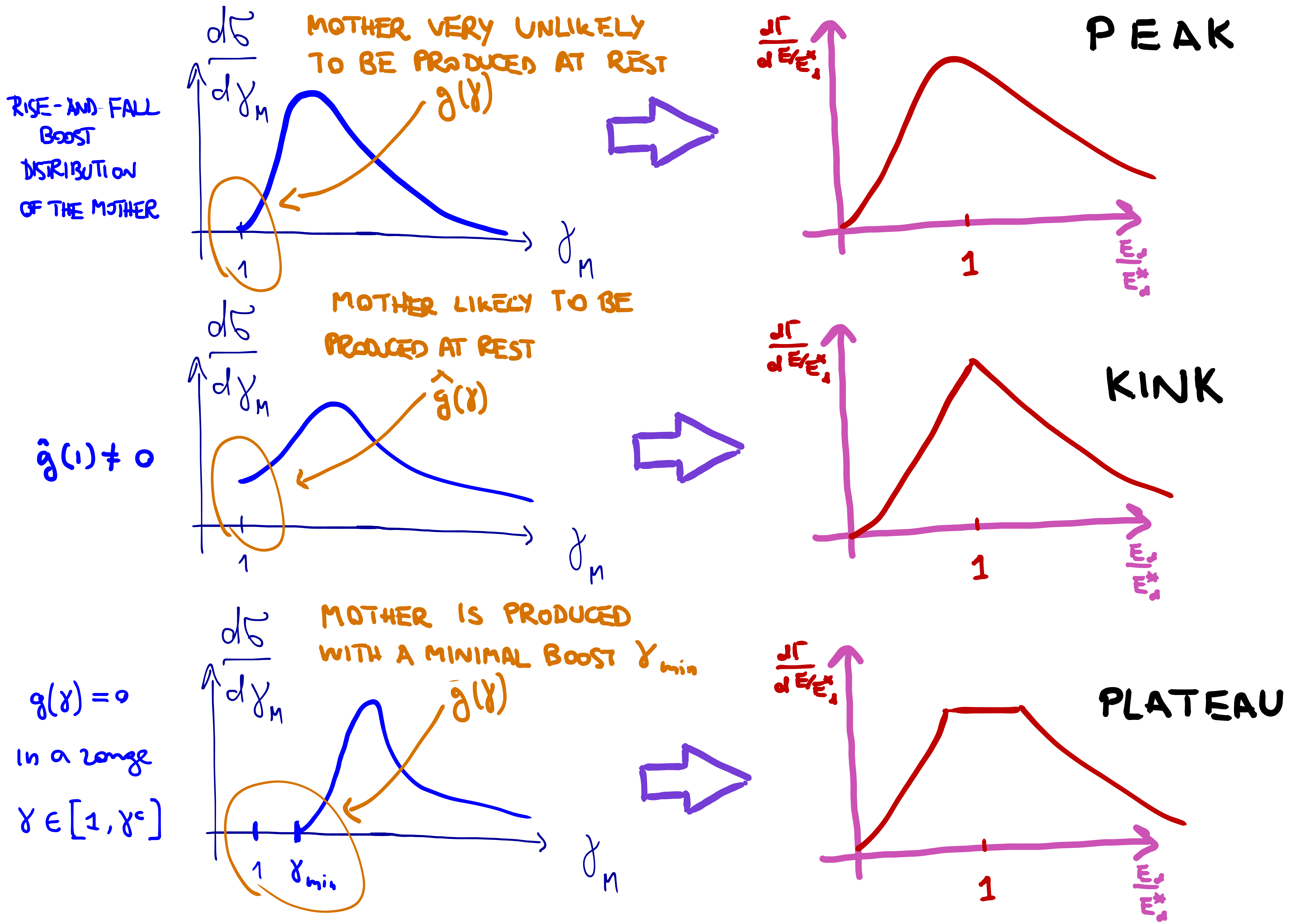
$$f'(x) = \frac{\text{sign}(1-x)}{2x} \, g\left(\frac{1}{2}\left(x+\frac{1}{x}\right)\right)$$

$g(\gamma) = 0$

IN A RANGE $[1, \gamma^c]$ ⇒ PLATEAU IN THE OBSERVED ENERGY DISTRIBUTION




RISE-AND-FALL BOOST DISTRIBUTION OF THE MOTHER
MOTHER VERY UNLIKELY TO BE PRODUCED AT REST
g(γ)
PEAK
ĝ(1) ≠ 0
MOTHER LIKELY TO BE PRODUCED AT REST
ĝ(γ)
KINK
g(γ) = 0
in a range
γ ∈ [1, γ^c]
MOTHER IS PRODUCED WITH A MINIMAL BOOST γ_min
ḡ(γ)
PLATEAU
dσ/dγ_M
γ_M
1
γ_min
dΓ/d E/E*_d
E_d/E*_d

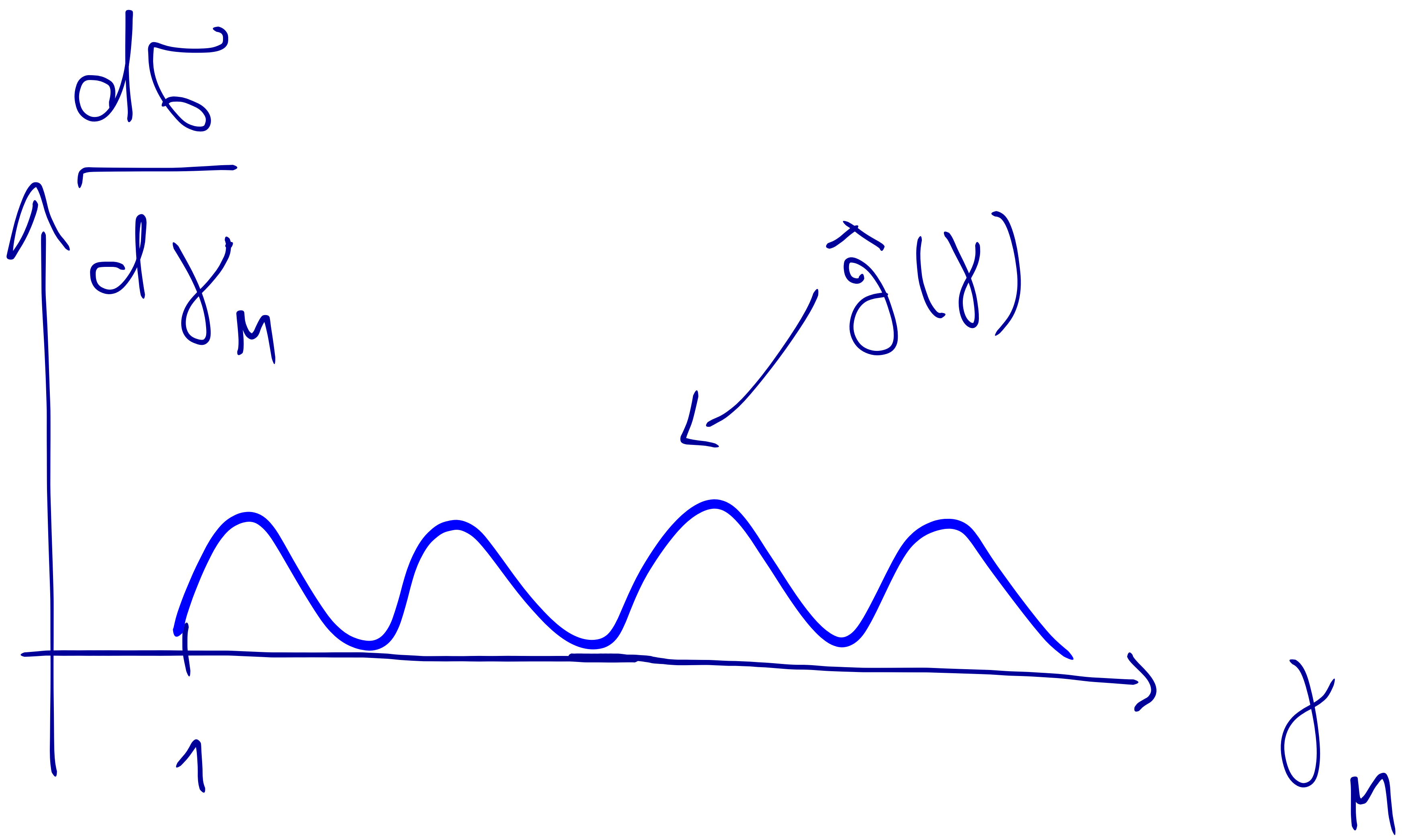
# WHEN AND WHY THIS BREAKS DOWN ?

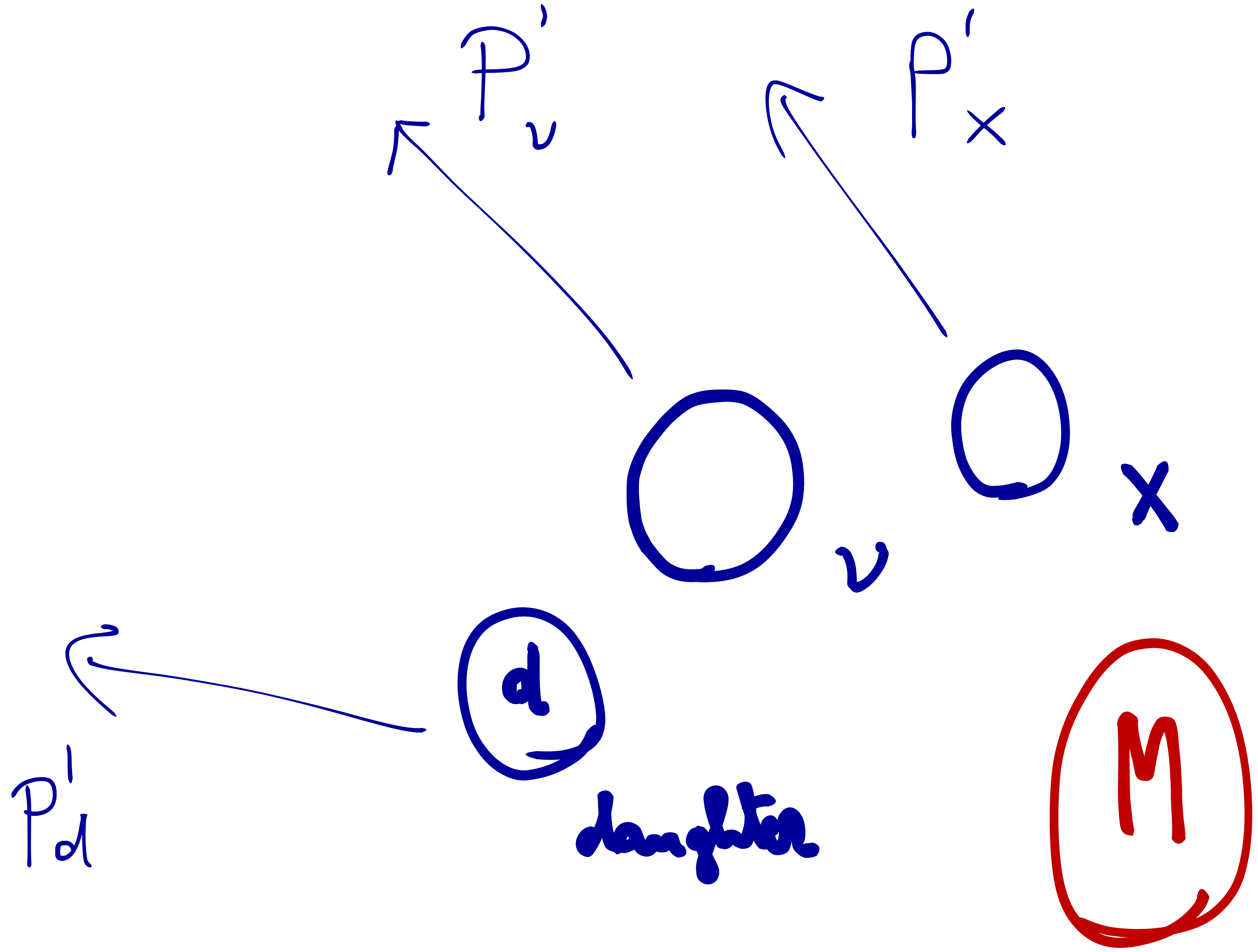
① BOOST DISTRIBUTION OF THE MOTHER WITH SPECIAL FEATURES

(MANY MINIMA, LARGE FLAT PORTIONS, ...)

② THE DECAY WAS NOT TWO-BODY

EXTRA INVISIBLE/UNDETECTED PARTICLES IN THE DECAY



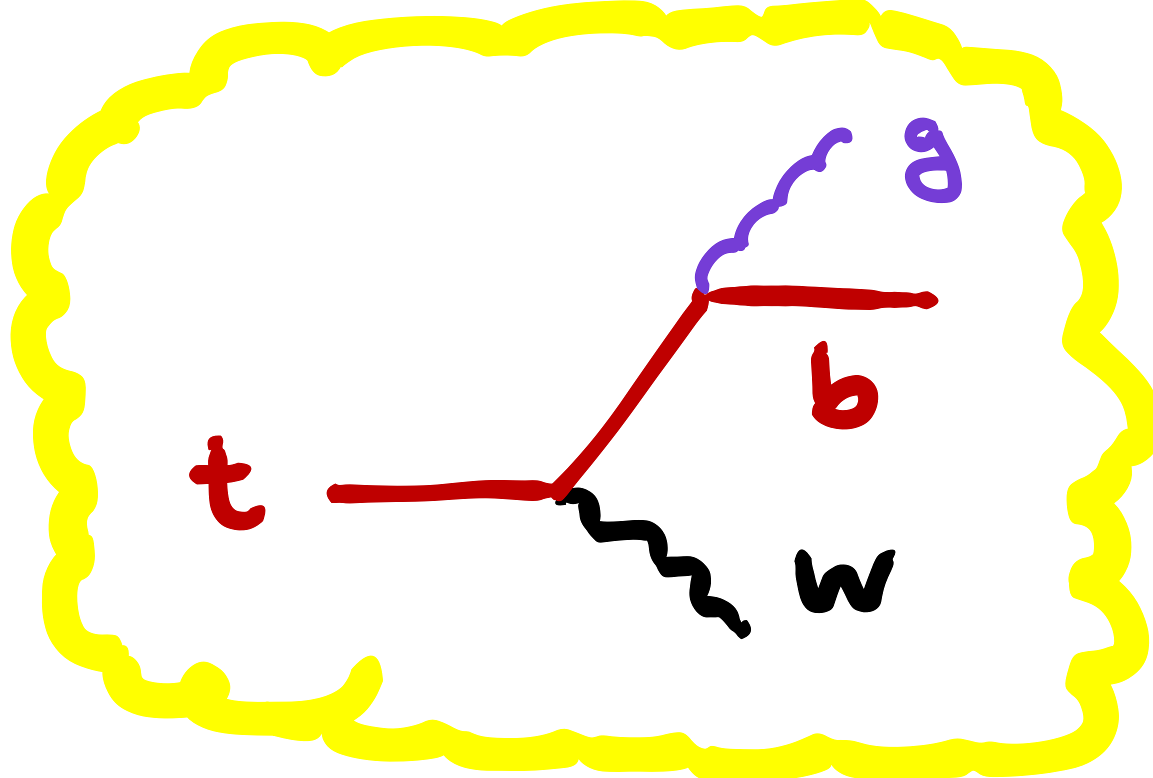



# WHEN AND WHY THIS BREAKS DOWN?

## RADIATION

$M \to d X$

IS TWO BODY ONLY UP TO EXTRA RADIATION




IF THE FINAL STATES ARE COLORED $M \to d X + \text{gluons}$

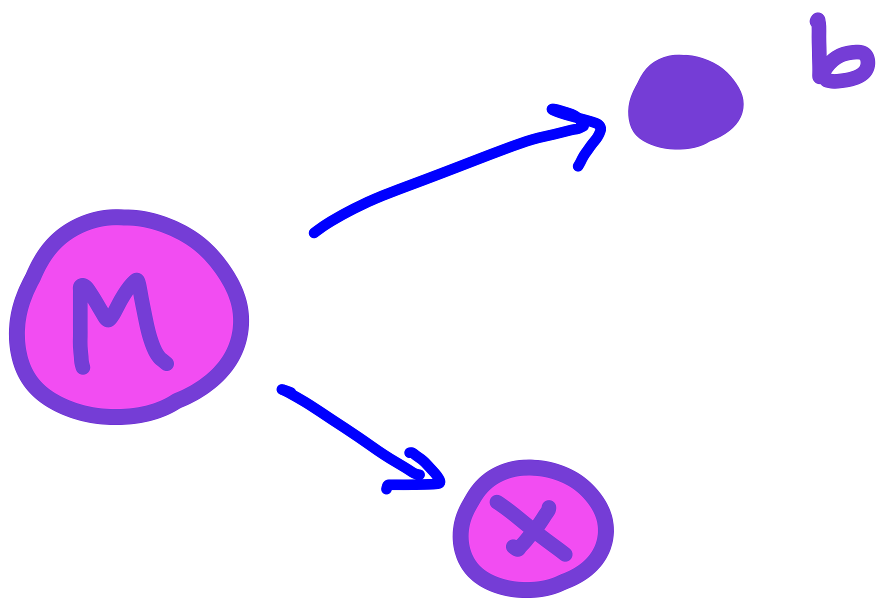
JET CLUSTERING SOLVES THIS ISSUE TO SOME EXTENT

HARD RADIATION MAY BE RESOLVABLE AND EFFECTIVELY GIVE RISE TO A THREE-BODY DECAY

RESOLVABLE RADIATION CAN BE VETOED

# WHEN AND WHY THIS BREAKS DOWN ?

## A MASSIVE DAUGTHER CAN BE AN OBVIOUS FAILURE




DECAY AT TRESHOLD

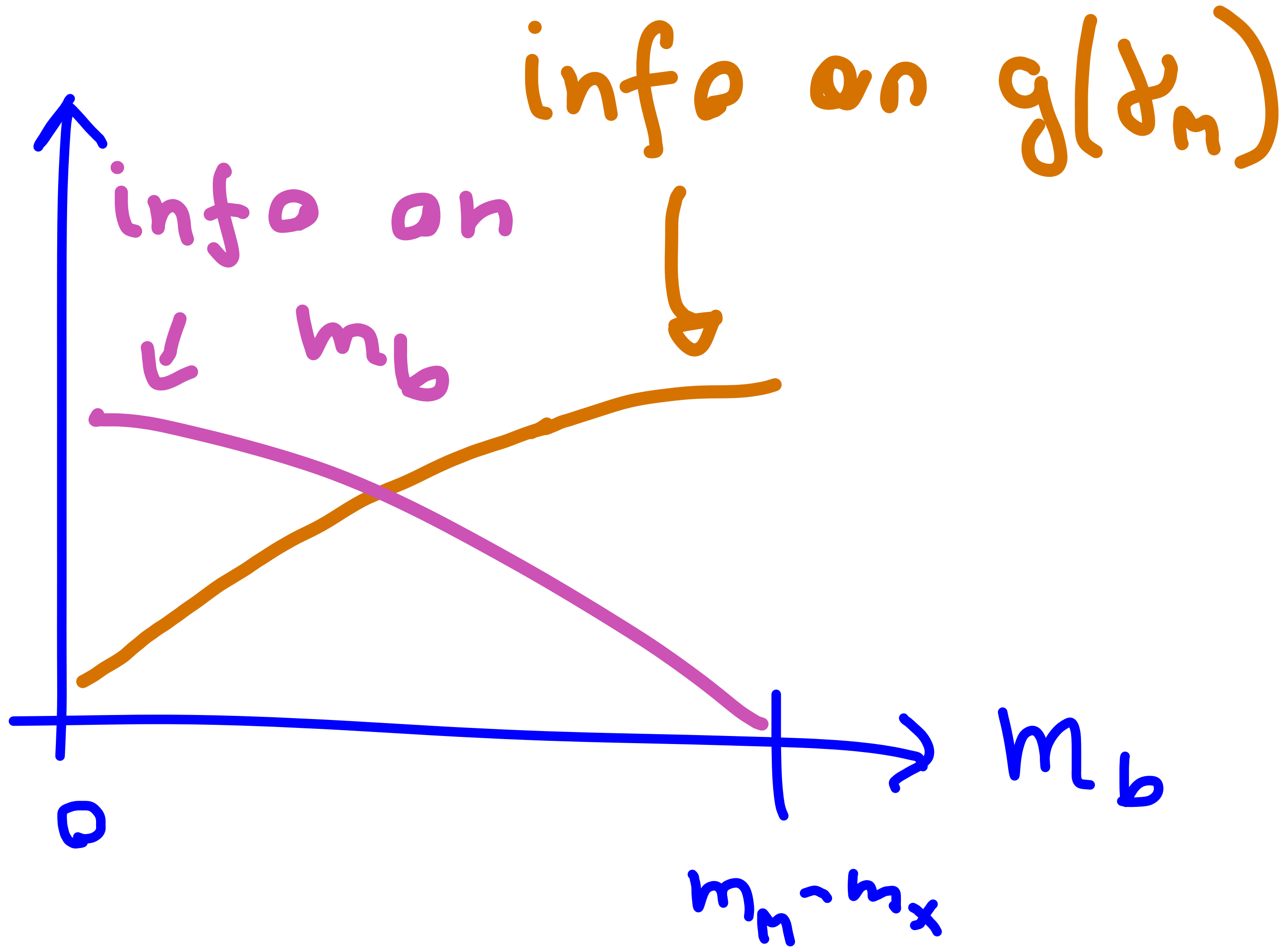
$$m_M = m_b + m_x$$

IN THE COM OF THE MOTHER
THE TWO DAUGHTER PARTICLES ARE AT REST

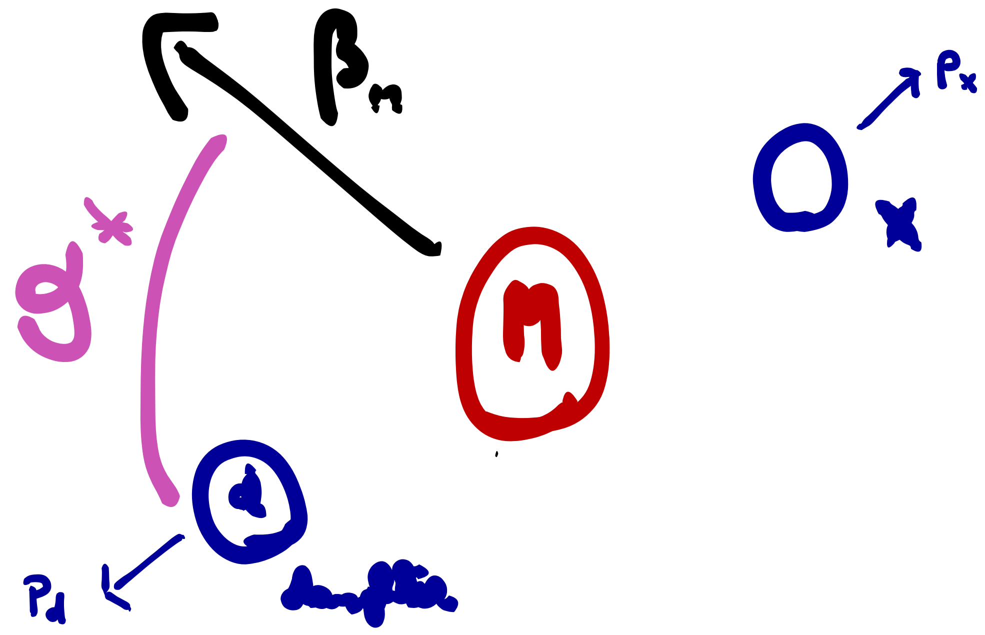
IN THE LAB FRAME $p_b^\mu = \Lambda(\gamma_M)\cdot(m_b,\vec{0})$

$\frac{d\Gamma}{dE_b}$ CARRIES MAXIMAL INFO ON THE BOOST DISTRIBUTION OF THE MOTHER

PEAK OF $\frac{d\Gamma}{dE_b}$ IS AN IMAGE OF THE PEAK OF $g(\gamma_M)$



# WHEN AND WHY THIS BREAKS DOWN ?




## THE DAUGHTER'S MASS

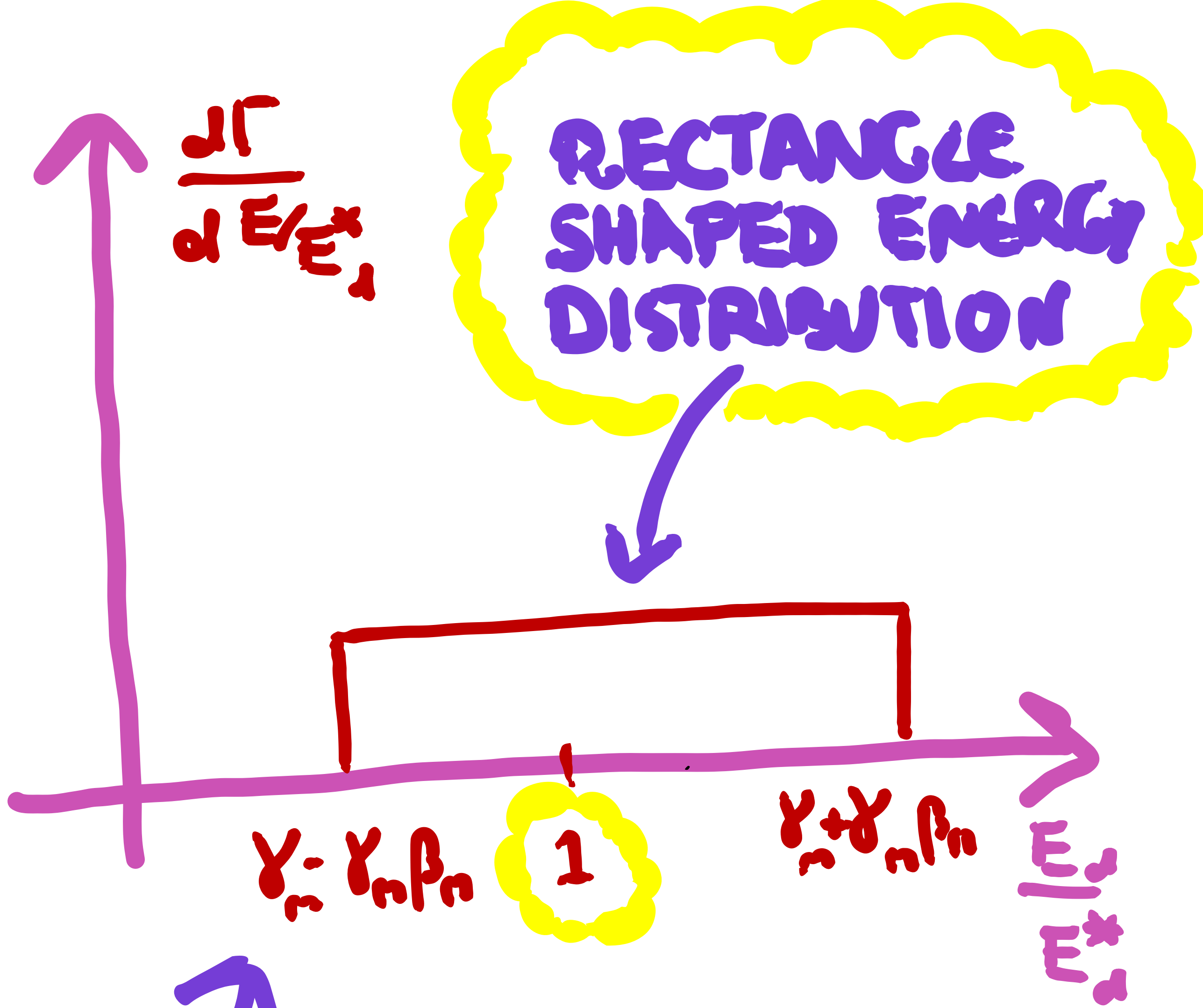
$$E'_d = E^*_d \gamma_M + \cos\theta^* \gamma_M \beta_M p^*_d$$

THE MINIMUM OF THIS QUANTITY AT $\theta^* = \pi$

(BOOST ANTI-PARALLEL TO THE MOMENTUM OF d)

IF $p^*_d = E^*_d$ (MASSLESS DAUGHTER)

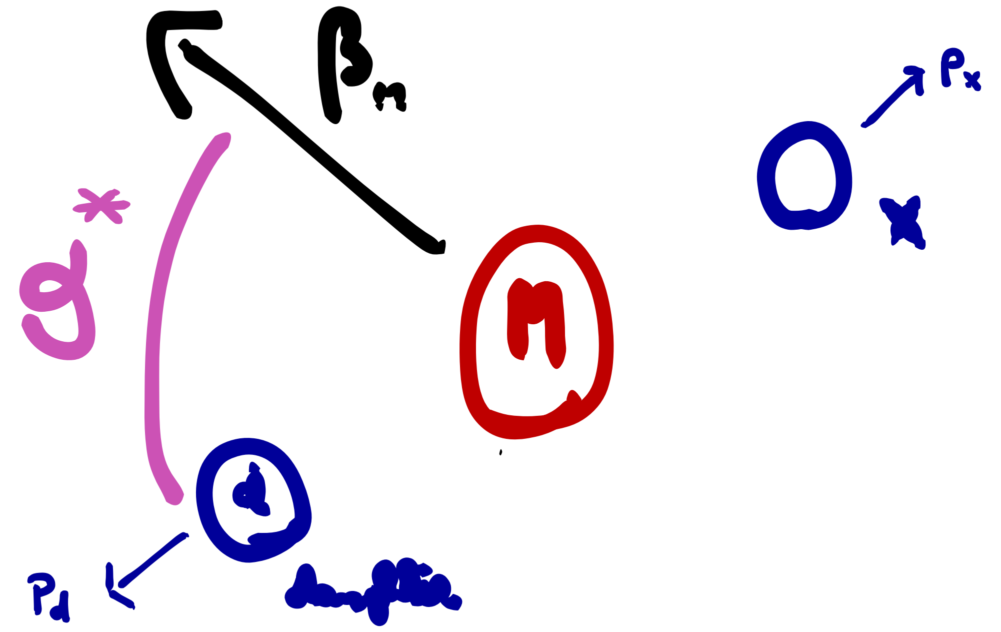
$$E'_{d,min} = E^*_d \left(\gamma_M - \sqrt{\gamma_M^2 - 1}\right) < E^*_d$$




LOWER EDGE OF EACH RECTANGLE **BELOW** $E^*$

# WHEN AND WHY THIS BREAKS DOWN ?

- THE DAUGHTER'S MASS




$$E'_d = E^*_d \gamma_M + \cos\vartheta^* \gamma_M \beta_M p^*_d$$

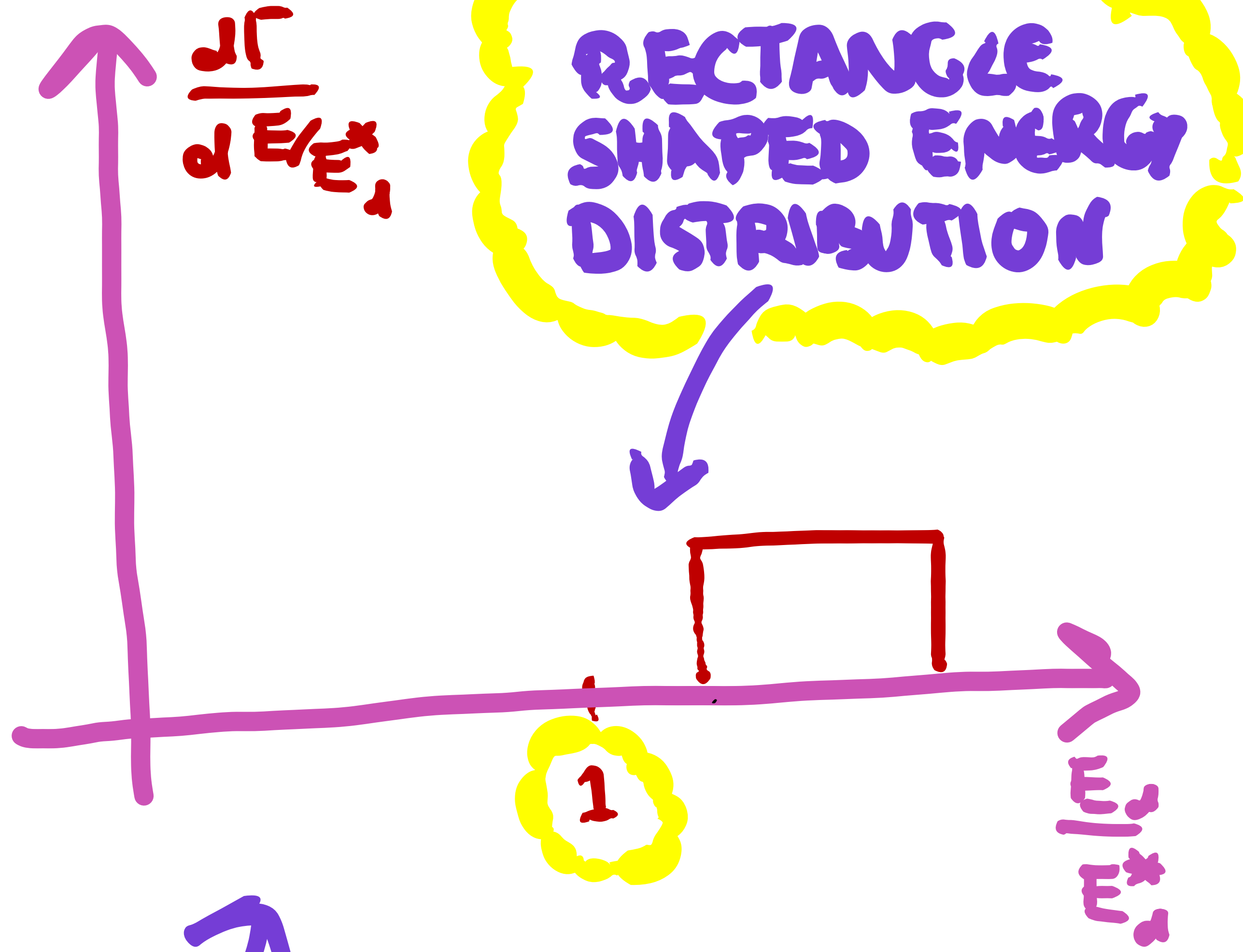
THE MINIMUM OF THIS QUANTITY AT $\vartheta^* = \pi$
(BOOST ANTI-PARALLEL TO THE MOMENTUM OF d)

IF $p^*_d \leq E^*_d$ (MASSIVE DAUGHTER)

$$p^*_d \to 0 \quad E^*_d \to m_d$$

$$E'_d = m_d \gamma_M + \ldots$$

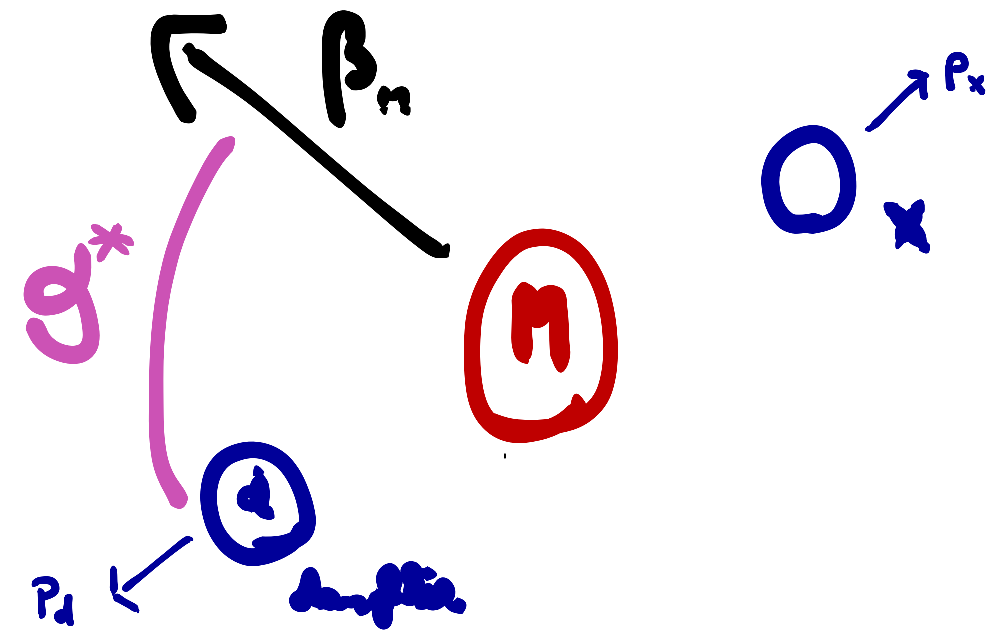
FOR $\gamma_M$ LARGE GIVES
RECTANGLES $E_{d,min} > E^*_d$




LOWER EDGE OF EACH RECTANGLE ABOVE $E^*$

# WHEN AND WHY THIS BREAKS DOWN ?

- THE DAUGHTER'S MASS




$$E'_d = E^*_d \gamma_M + \cos\theta^* \gamma_M \beta_M p^*_d$$

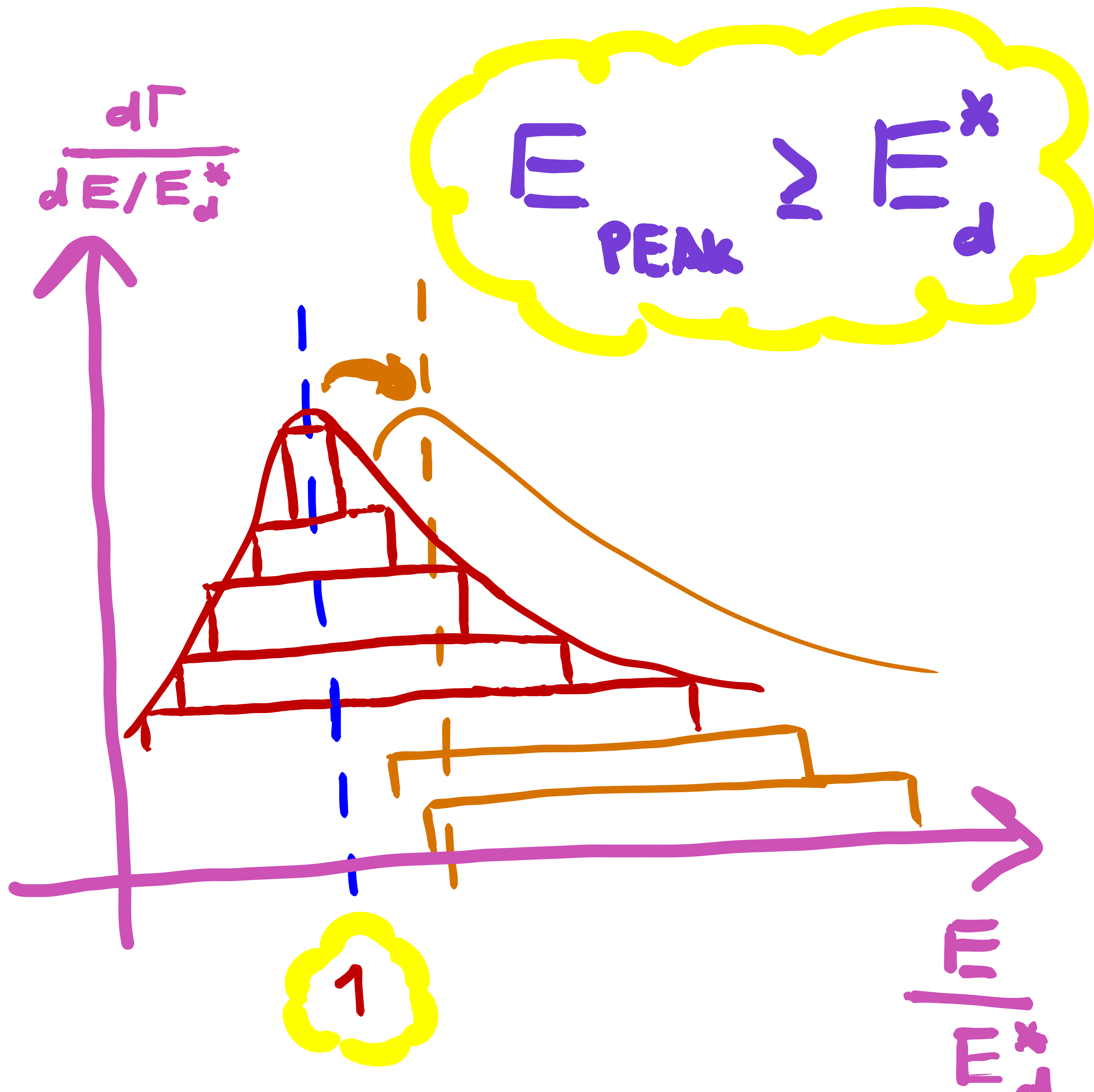
THE MINIMUM OF THIS QUANTITY AT $\theta^* = \pi$ (BOOST ANTI-PARALLEL TO THE MOMENTUM OF d)

IF $p^*_d \leq E^*_d$ (MASSIVE DAUGHTER)

$$p^*_d \to 0 \quad E^*_d \to m_d$$

$$E'_d = m_d \gamma_M + \dots$$

FOR $\gamma_M$ LARGE GIVES RECTANGLES $E_{d,min} \geq E^*_d$

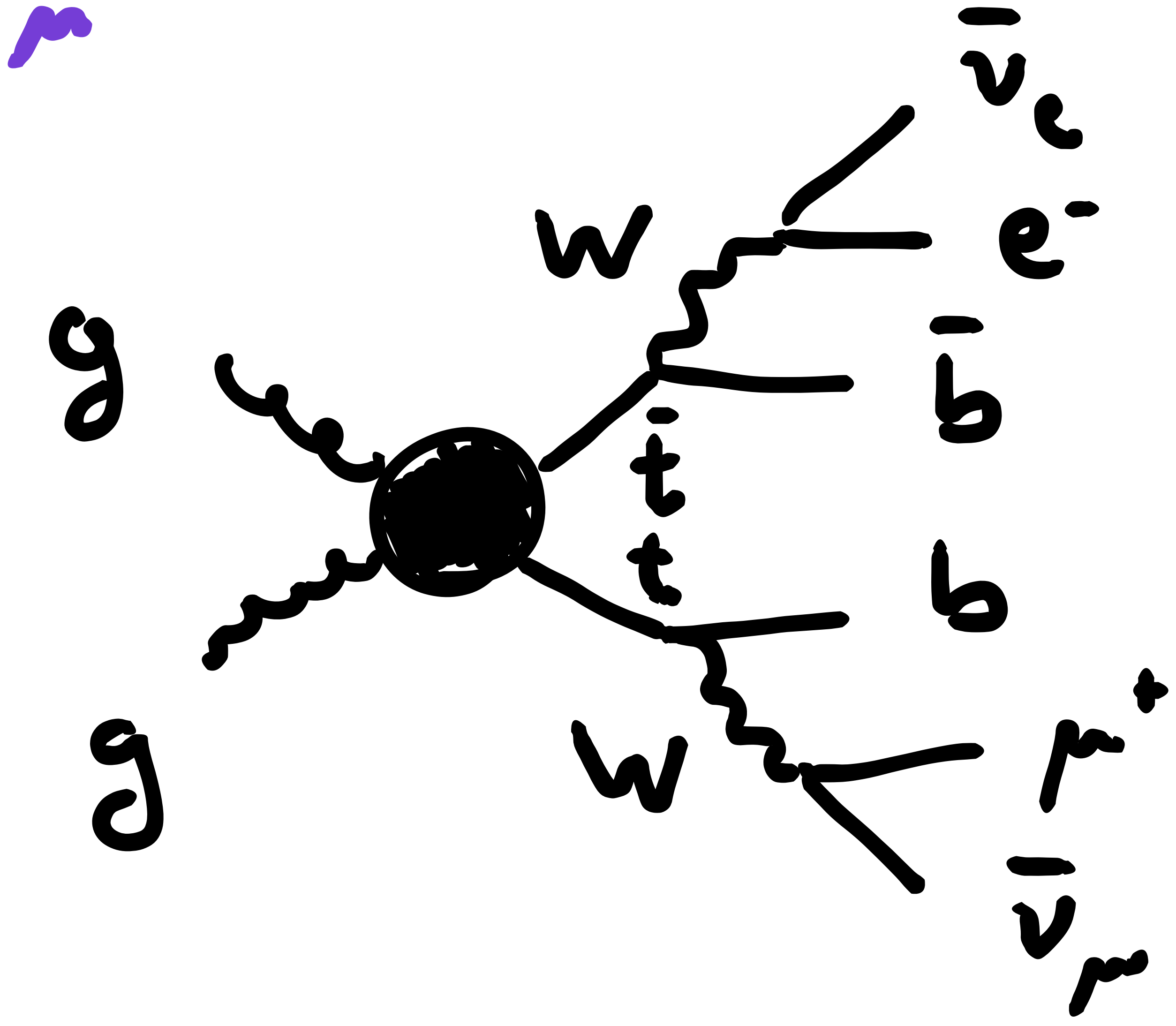


# APPLICATIONS (for mass measurements)

1) $pp \to t\bar{t} \to b\bar{b}\,\mu^+e^-\bar{\nu}_e\nu_\mu$

1209.0772

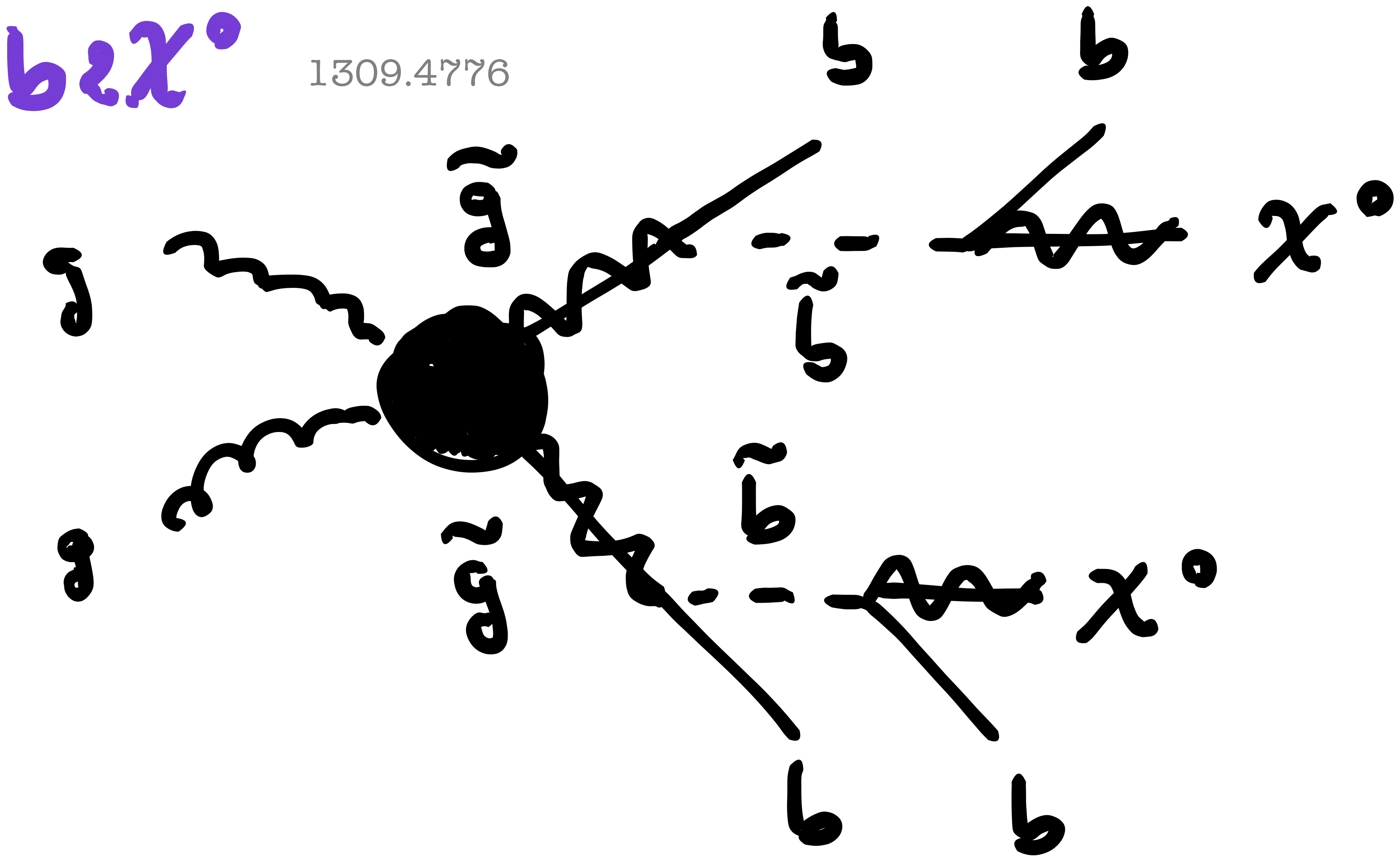
$m_{top}$ AS PROOF OF PRINCIPLE




2) $pp \to \tilde{g}\tilde{g} \to b\tilde{b}\,b\tilde{b} \to 4b\,2\chi^0$

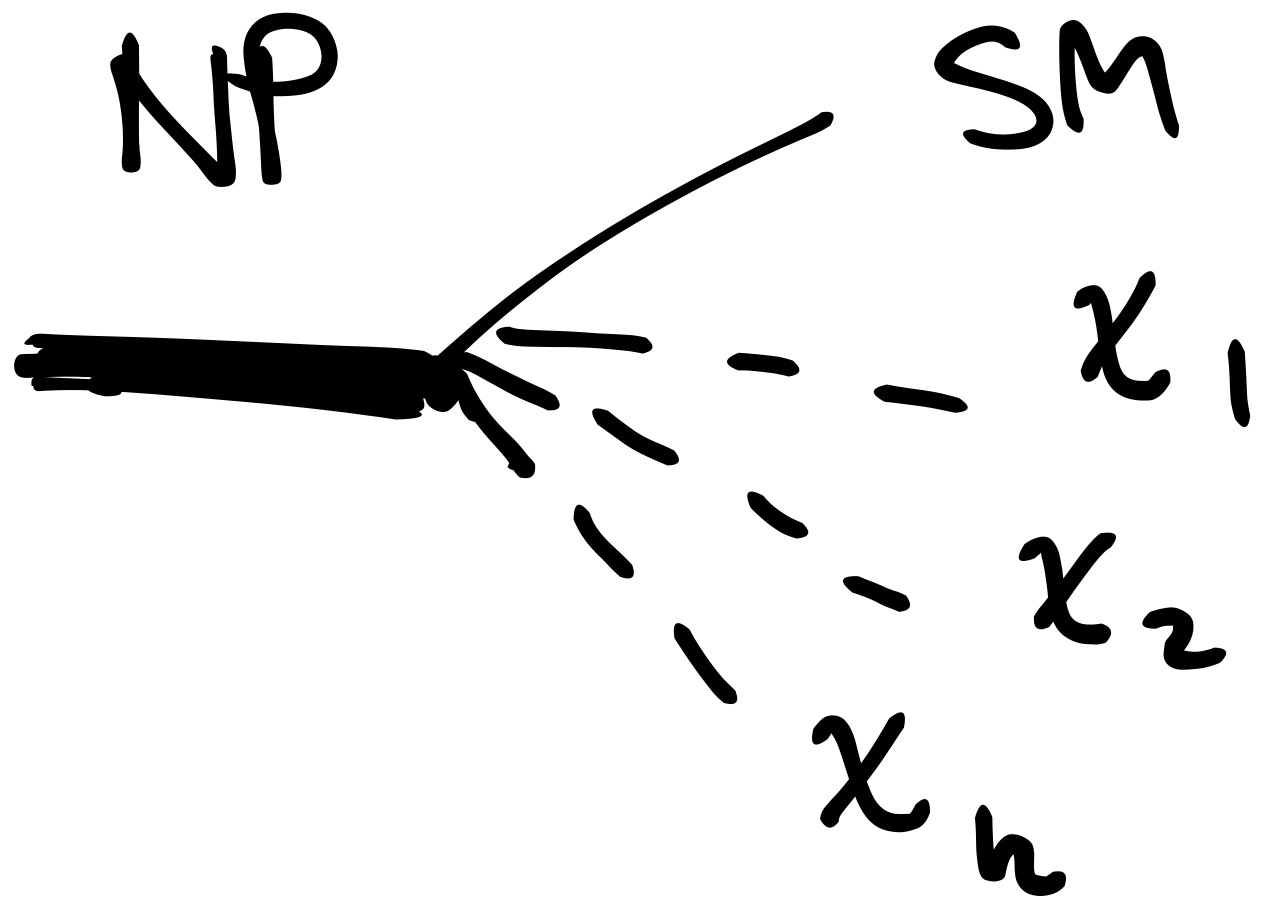
1309.4776

$m_{\tilde{g}}$, $m_{\tilde{b}}$, $m_{\chi}$



# APPLICATIONS

DISTINGUISHING BETWEEN 2-bodies AND 3-bodies 1212.5230



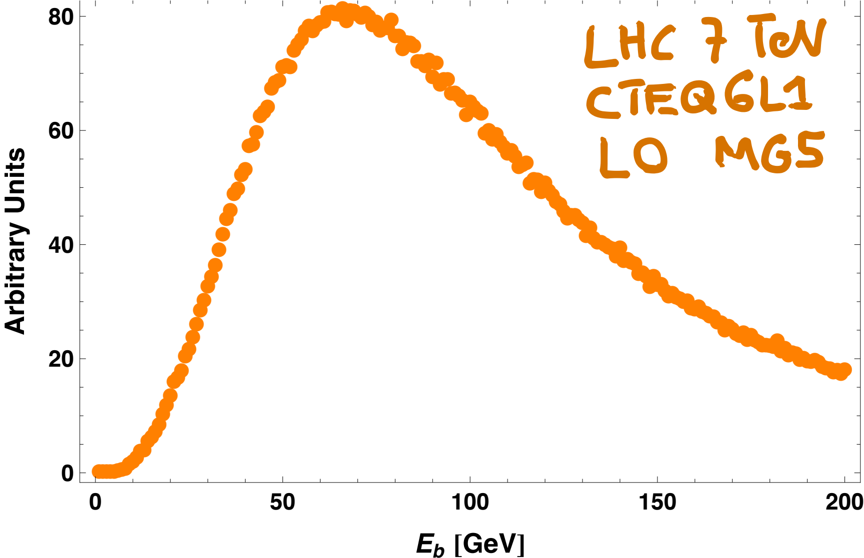

COUNT INVISIBLE PARTICLES IN SEMI-INVISIBLE DECAYS

$pp \to t\bar{t} \to b\bar{b}\, e\, \mu\, \not{E}_T$

- QCD PAIR PRODUCTION OF $t\bar{t}$ ENSURES THAT THE OVERALL SAMPLE OF TOP DECAYS IS UNPOLARIZED

- $$E_b^* = \frac{m_t^2 - m_W^2 + m_b^2}{2m_t} \simeq 67\ \text{GeV} \qquad E_b^* \gg m_b$$

THE b QUARK CAN BE TAKEN AS MASSLESS


LHC 7 TeV
CTEQ6L1
LO MG5
Arbitrary Units
0
20
40
60
80
0
50
100
150
200
$E_b$ [GeV]

# FINDING THE PEAK :

- LOOK BY EYE

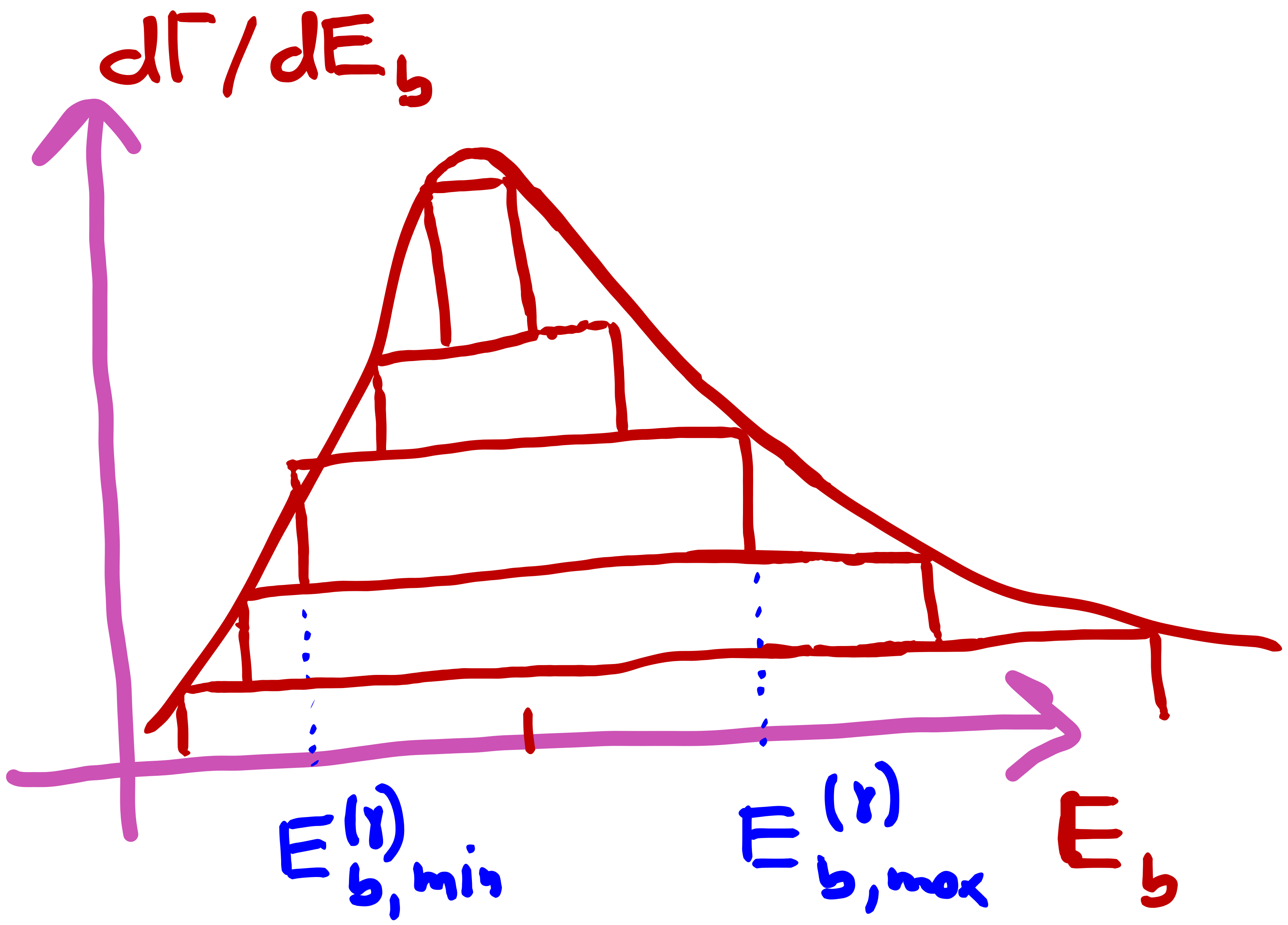
- **FIND A TEMPLATE AND USE IT TO FIT DATA**

  - A TEMPLATE MOTIVATED FROM PRIME PRINCIPLES SEEMS UNATTAINABLE BECAUSE IT DEPENDS ON PARTON DISTRIBUTION FUNCTIONS AND ON THE MATRIX ELEMENT FOR THE PRODUCTION PROCESS

# FIND A TEMPLATE AND USE IT TO FIT DATA

$$E_b = E_b^* \left( \gamma_n + \cos\vartheta^* \sqrt{\gamma_m^2 - 1} \right)$$

$$E_{b,\min/\max}^{(\gamma_n)} = E_b^* \left( \gamma_m \mp \sqrt{\gamma^2 - 1} \right)$$




$$\sqrt{E_{b,\min} E_{b,\max}} = E_b^*$$

$d\Gamma/dE_b$

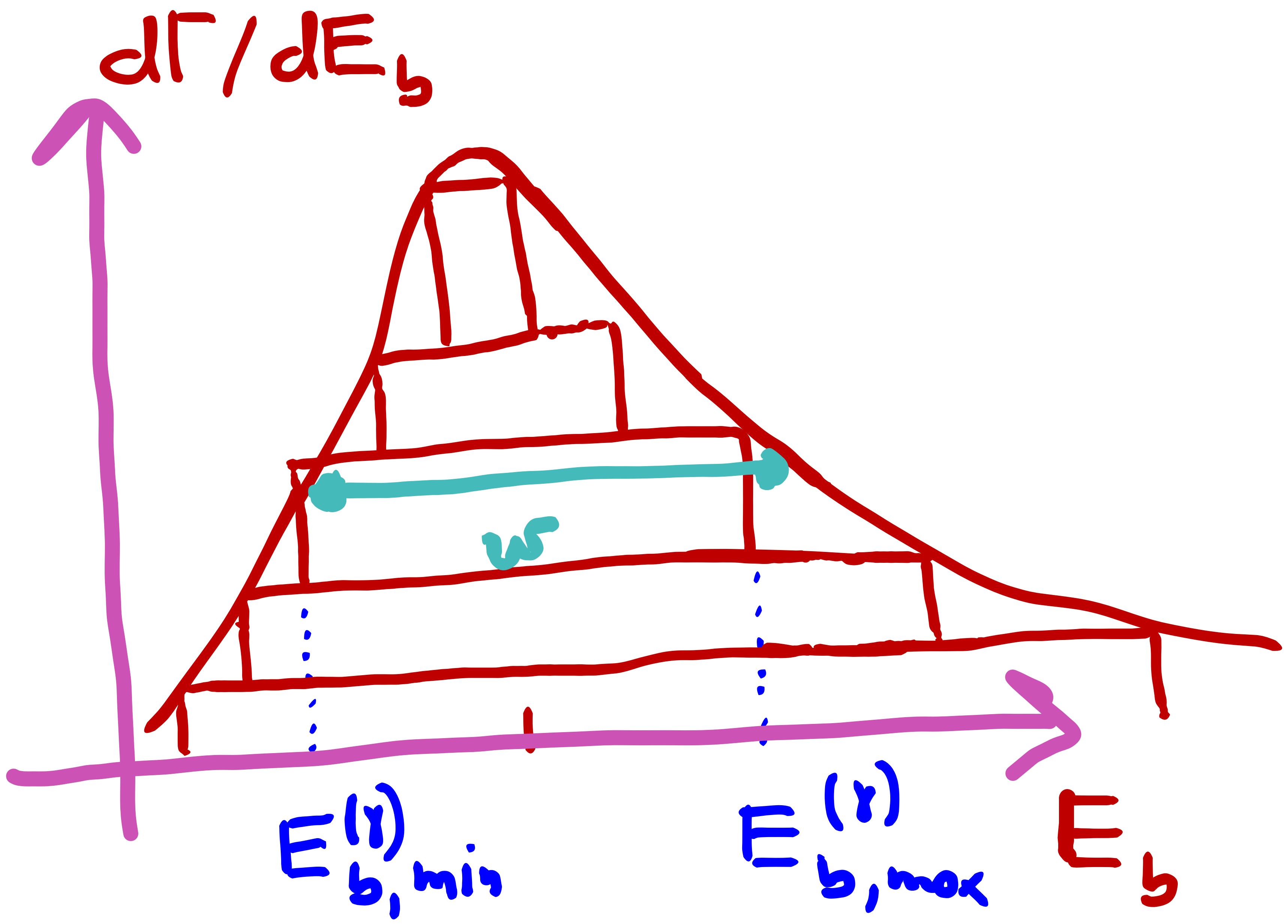
MUST BE A FUNCTION OF

$$\frac{E_b}{E_b^*} + \frac{E_b^*}{E_b}$$

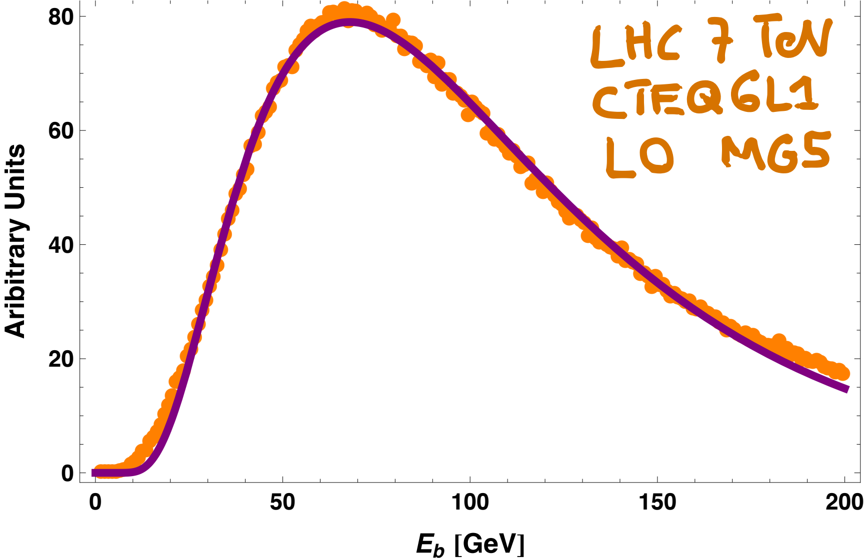
# FIND A TEMPLATE AND USE IT TO FIT DATA

- $d\Gamma/dE_b \xrightarrow{E_b \to 0,\infty} 0$ (at least)
- $d\Gamma/dE_b$ max at $E_b = E_b^*$
- IN SOME LIMIT SHOULD BE A $\delta$-FUNCTION (MOTHER AT REST)
- $d\Gamma/dE_b$ MUST BE A FUNCTION OF

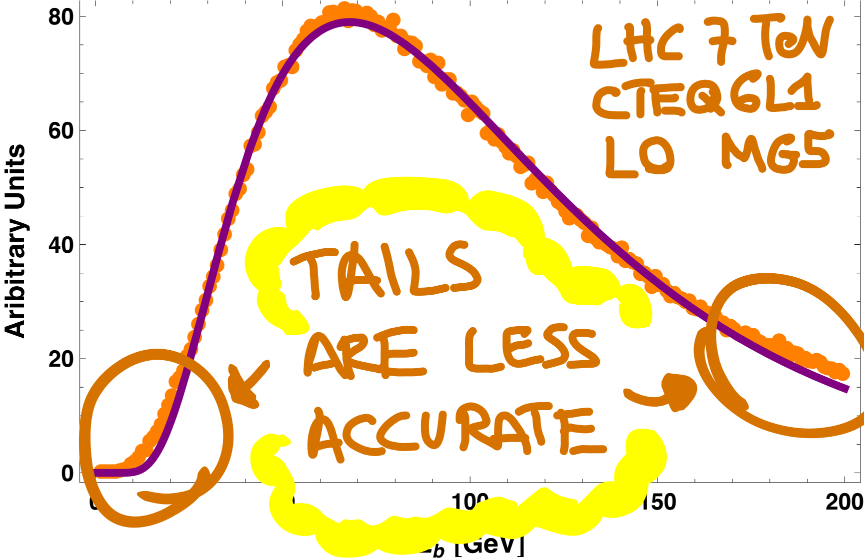
$$\frac{E_b}{E_b^*} + \frac{E_b^*}{E_b}$$




$$d\Gamma/dE \sim \exp\left(-w\left(\frac{E}{E^*} + \frac{E^*}{E}\right)\right)$$



LHC 7 TeV
CTEQ6L1
LO MG5
Aribitrary Units
$E_b$ [GeV]
0
20
40
60
80
0
50
100
150
200



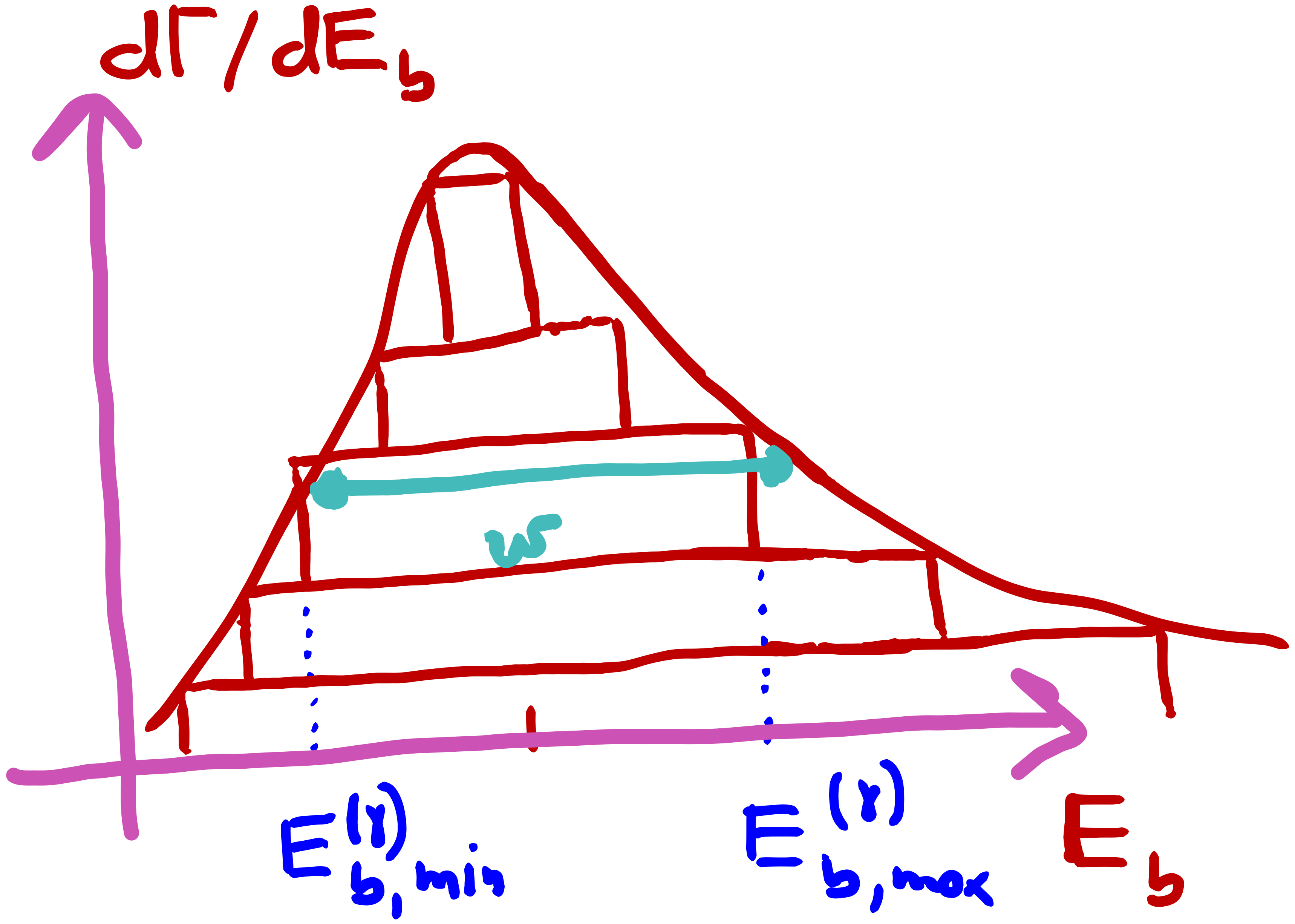
80
60
40
20
0
Aribitrary Units
100
150
200
[GeV]
LHC 7 TeV
CTEQ6L1
LO MG5
TAILS ARE LESS ACCURATE

# FIND A TEMPLATE AND USE IT TO FIT DATA

- $d\Gamma/dE_b \xrightarrow{E_b \to 0,\infty} 0$ ☑ ☒
- $d\Gamma/dE_b$ max at $E_b = E_b^*$ ☑
- IN SOME LIMIT SHOULD BE A δ-FUNCTION (MOTHER AT REST) ☑
- $d\Gamma/dE_b$ ☑

  MUST BE A FUNCTION OF

  $$\frac{E_b}{E_b^*} + \frac{E_b^*}{E_b}$$




$$d\Gamma/dE \sim \exp\left(-w\left(\frac{E}{E^*} + \frac{E^*}{E}\right)\right)$$

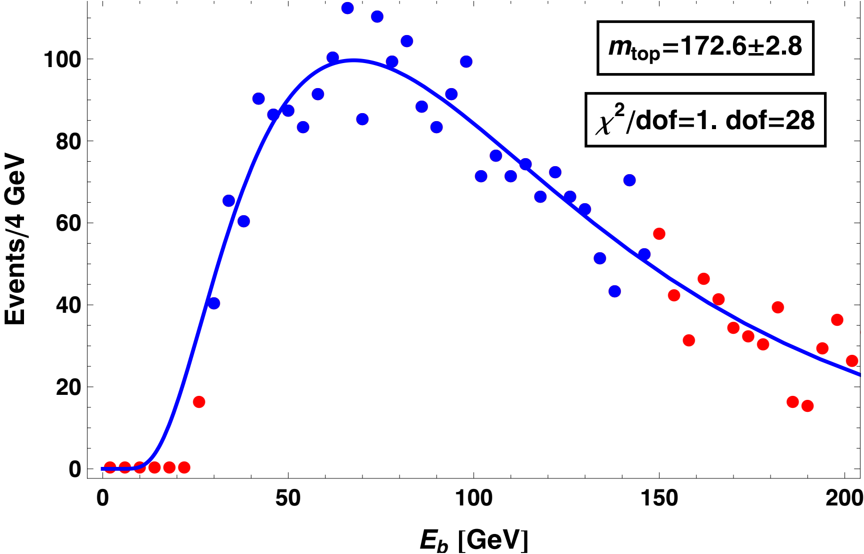
# CAN WE MEASURE PARTICLE MASSES ?

- $$E_b^* = \frac{m_t^2 - m_W^2 + m_b^2}{2m_t} \simeq 67\ \text{GeV}$$

FROM THE RESULT OF THE FIT TO THE LEADING ORDER MATRIX ELEMENT WE HAVE AT LEAST A CHANCE

NEED TO EVALUATE :

- DETECTOR EFFECTS ⟶ DELPHES 1.9
- EXTRA QCD RADIATION ⟶ SOFT QCD PYTHIA 6.4
- BIAS FROM EVENT SELECTION ⟶ ATLAS-CONF-2012-097



m_top=172.6±2.8
χ²/dof=1. dof=28
Events/4 GeV
E_b [GeV]
0
20
40
60
80
100
0
50
100
150
200

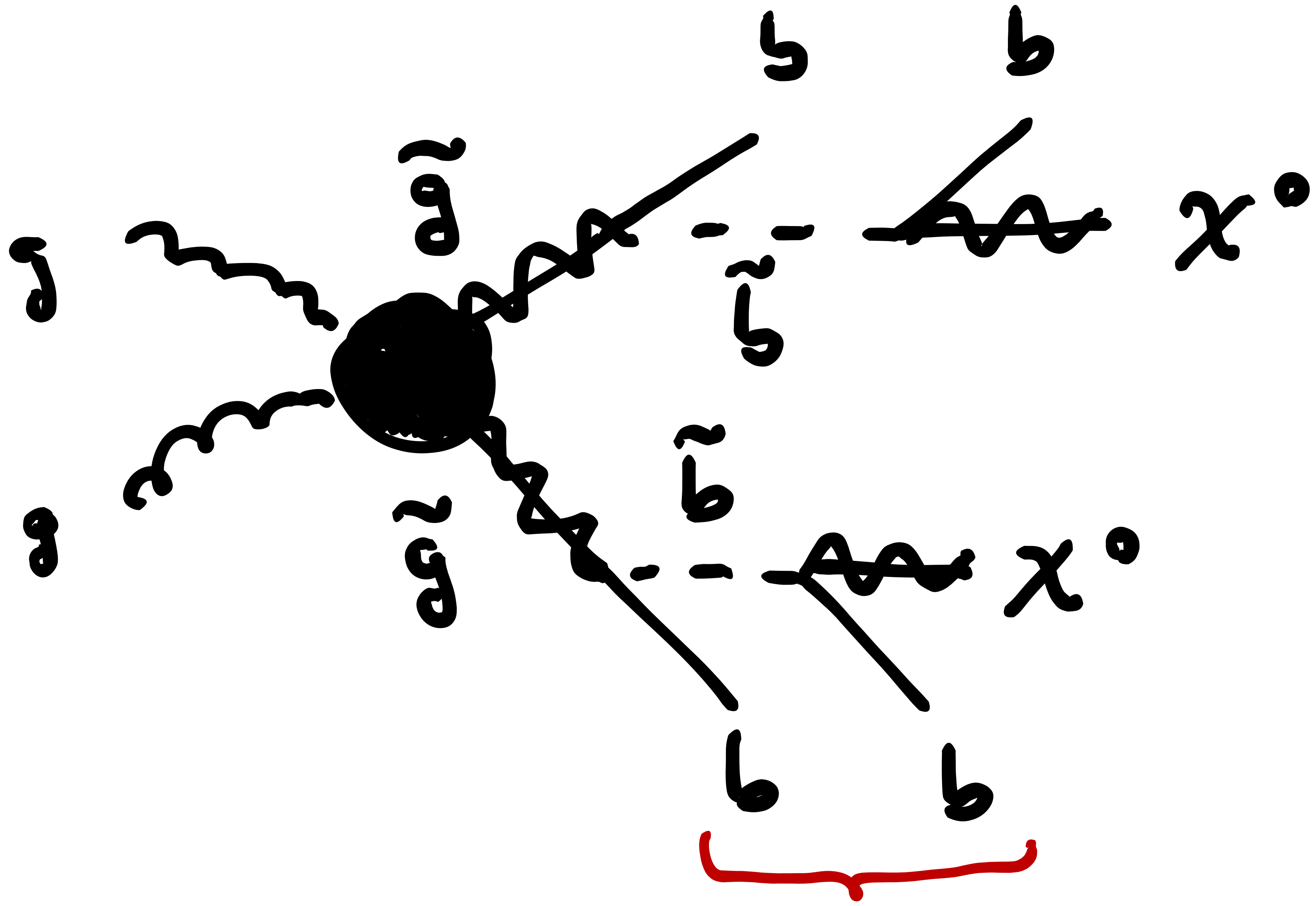
# CAN WE MEASURE PARTICLE MASSES ?

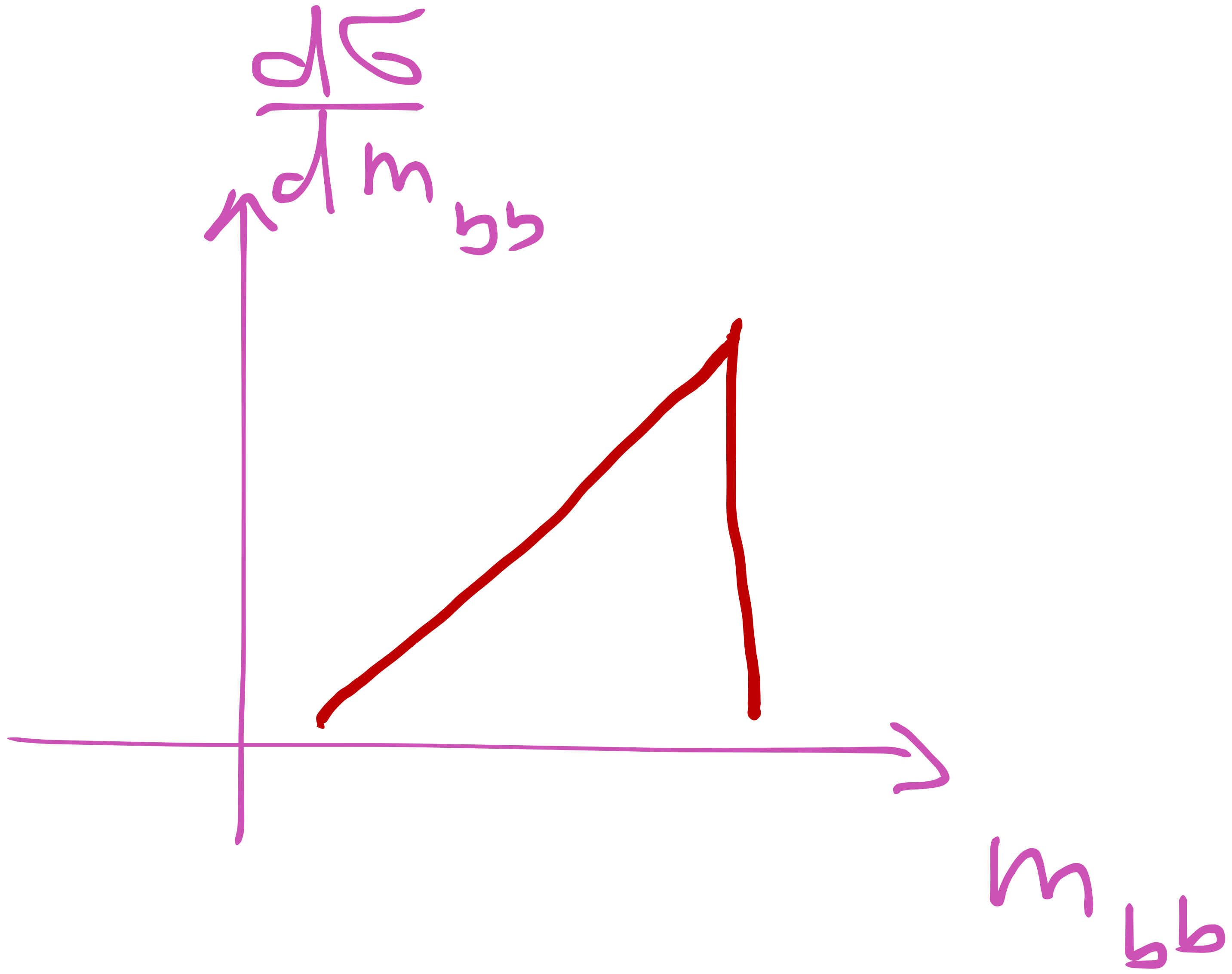
FROM 100 PSEUDO EXPERIMENTS
FOR LHC $\sqrt{s} = 7$ TeV AND $\mathcal{L} = 5/fb$
WE GET

$$m_{top} = 173.1 \pm 2.5 \text{ GeV}$$

- ALL THE EFFECTS AT LEADING ORDER ARE WELL UNDER CONTROL
- **HIGHER ORDER QCD WAS NOT INCLUDED ( $\lesssim 10\%$ )**
- WORK IN PROGRESS TO TEST THE METHOD ON CMS DATA

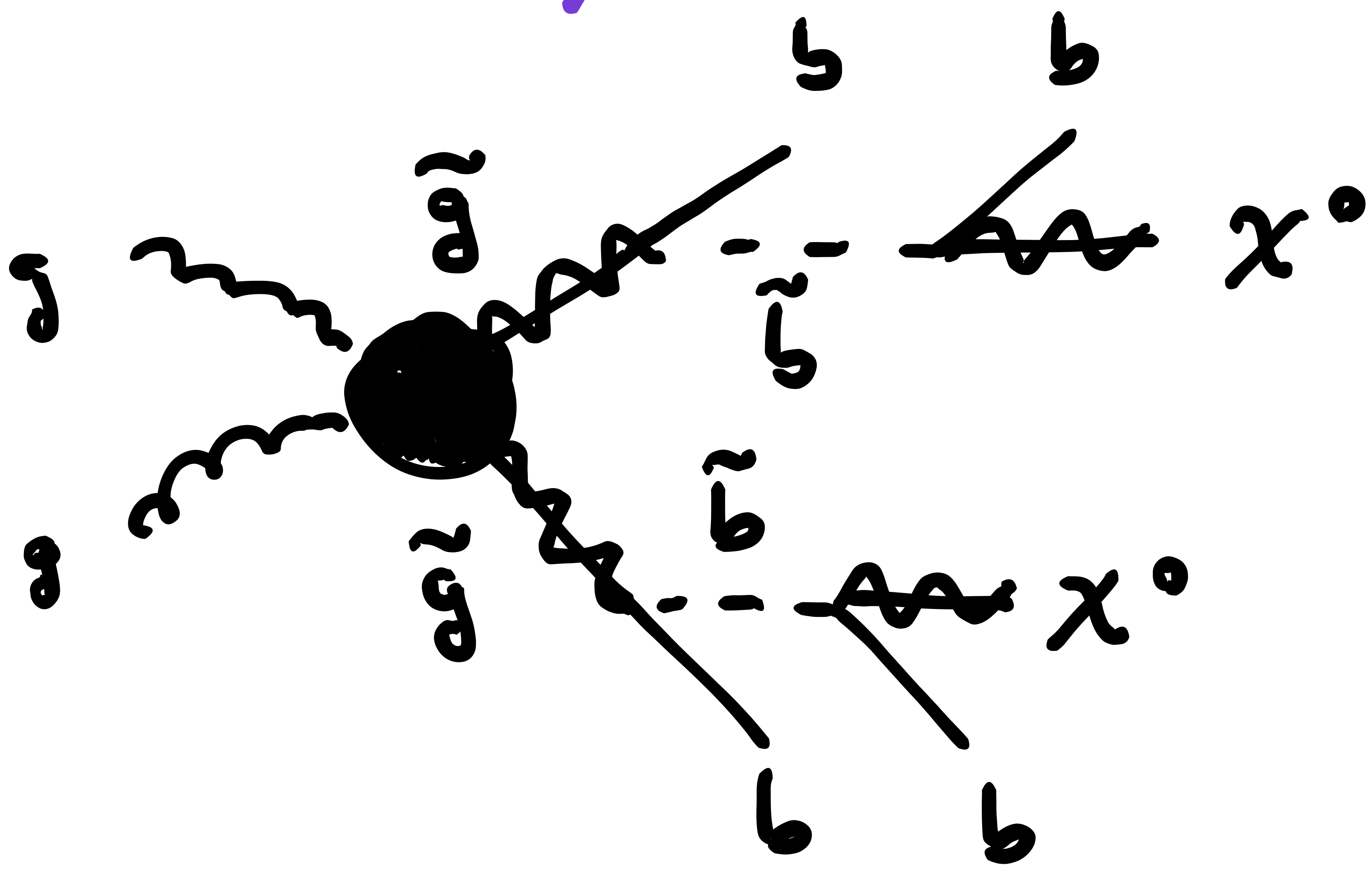
$$pp \to \tilde{g}\tilde{g} \to b\tilde{b}\, b\tilde{b} \to 4b\, \not{E}_T$$




$$m_{bb}^{max} = \sqrt{\frac{m_{\tilde{g}}^2 - m_{\tilde{b}}^2}{m_{\tilde{b}}} \cdot \frac{m_{\tilde{b}}^2 - m_{\chi}^2}{m_{\tilde{b}}}}$$

$$pp \to \tilde{g}\tilde{g} \to b\tilde{b}\,b\tilde{b} \to 4b\,\not{E}_T$$




TWO-STEPS DECAY $\Rightarrow$

$$E_b^{peak} = \frac{m_{\tilde{g}}^2 - m_{\tilde{b}}^2}{2m_{\tilde{g}}} \qquad E_b^{peak} = \frac{m_{\tilde{b}}^2 - m_{\chi}^2}{2m_{\tilde{b}}}$$

NO COMBINATORICAL ISSUES

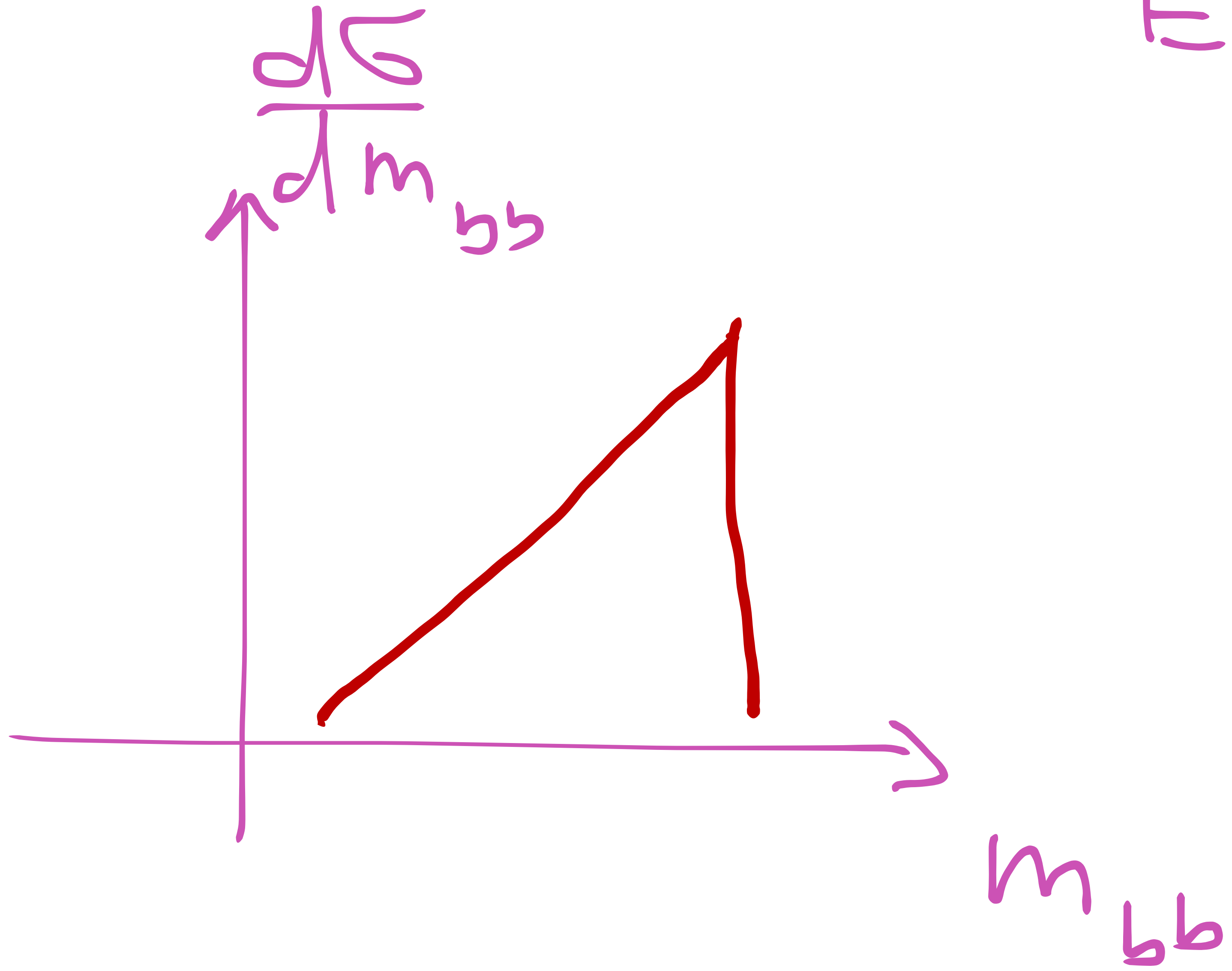
JUST LOOK AT $\frac{d\sigma}{dE_b}$

$pp \to \tilde{g}\tilde{g} \to b\tilde{b}\,b\tilde{b} \to 4b\,\not{E}_T$

$$E_{\tilde{b}_H} = \frac{m_{\tilde{b}}^2 - m_\chi^2}{2m_{\tilde{b}}}$$

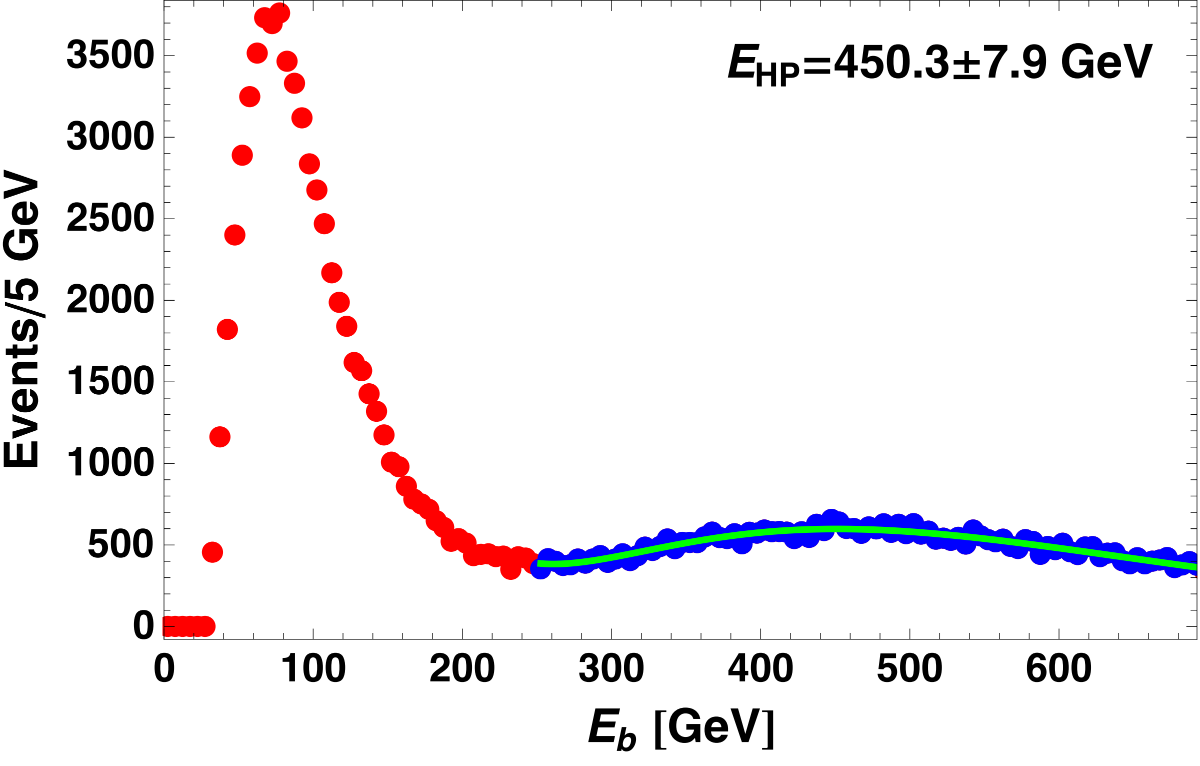
$$E_{\tilde{b}_L} = \frac{m_{\tilde{g}}^2 - m_{\tilde{b}}^2}{2m_{\tilde{g}}}$$

$$m_{bb}^{max} = \sqrt{4\frac{m_\chi}{m_{\tilde{b}}} E_{\tilde{b}_H} E_{\tilde{b}_L}}$$

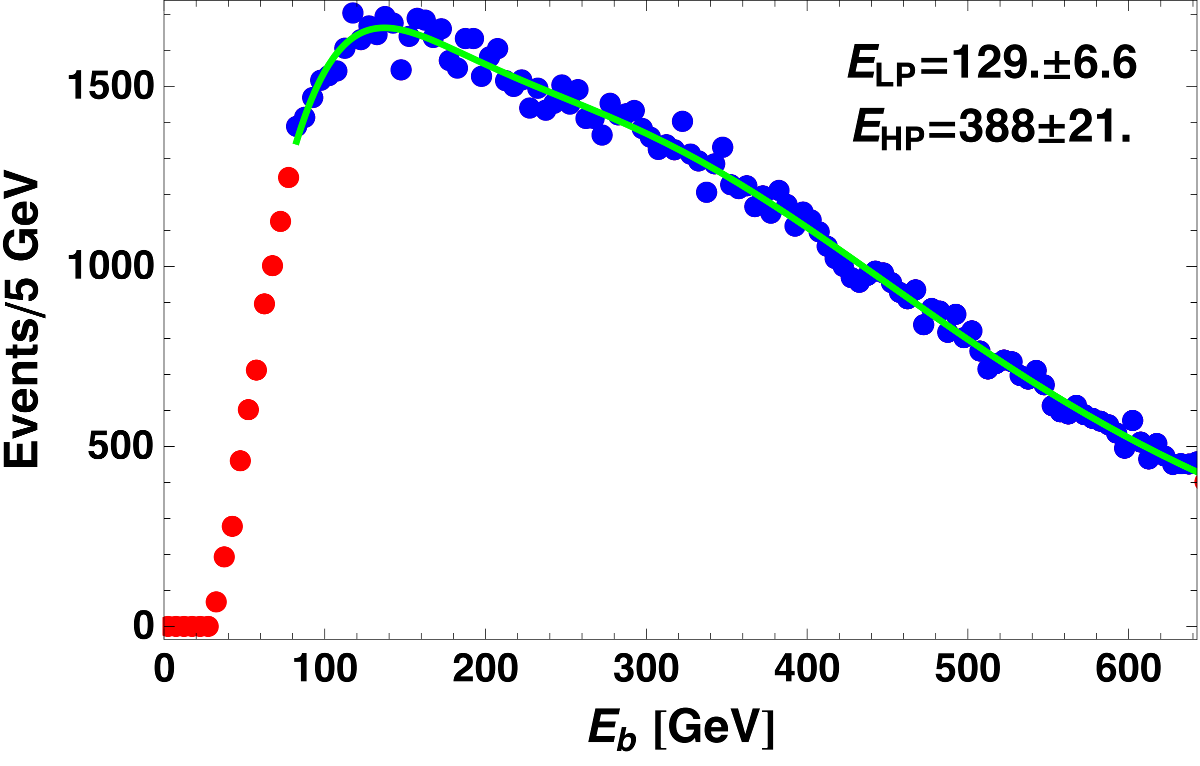




- BACKGROUNDS Z+jets (mostly Z bbbb) & $t\bar{t}b\bar{b}$ (subdom)
- CUTS MAY AFFECT THE ENERGY DISTRIBUTION

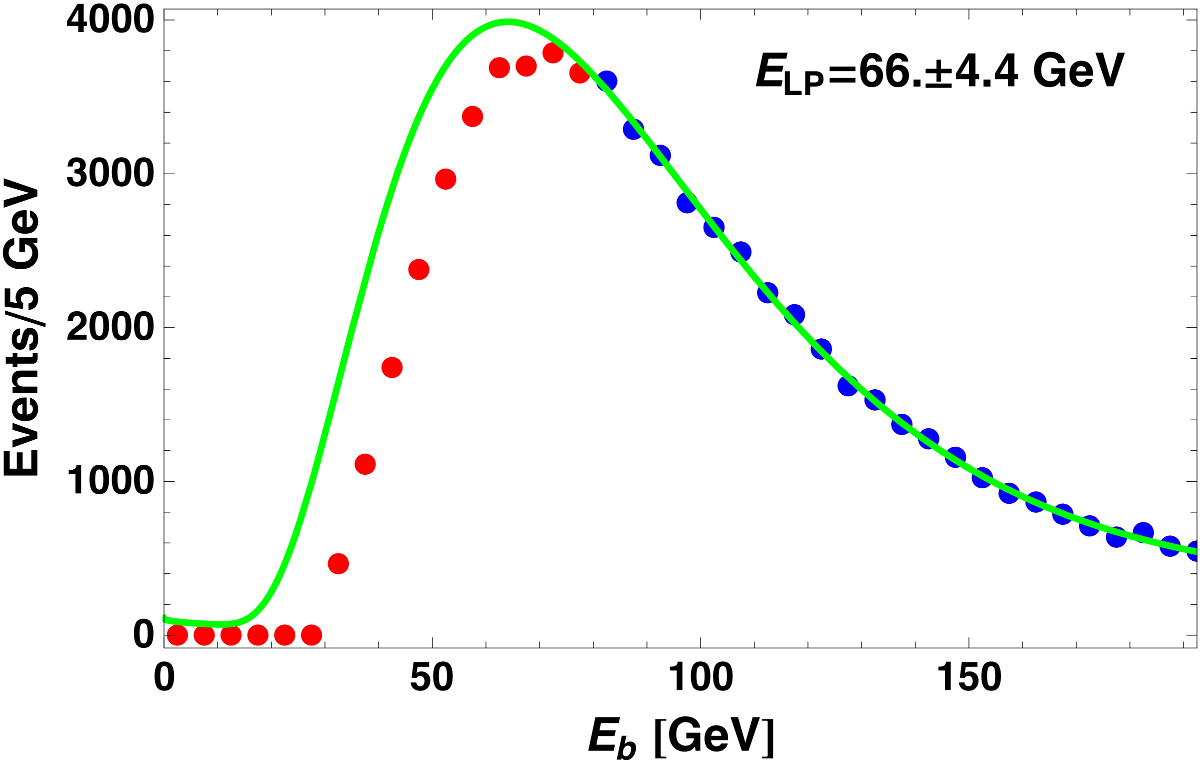
$P_{T_{jet}} > X \Rightarrow E_{jet} > X$



E_HP=450.3±7.9 GeV
Events/5 GeV
3500
3000
2500
2000
1500
1000
500
0
0
100
200
300
400
500
600
E_b [GeV]



$E_{LP}$=129.±6.6
$E_{HP}$=388±21.
Events/5 GeV
$E_b$ [GeV]
0
500
1000
1500
0
100
200
300
400
500
600



$E_{LP}$=66.±4.4 GeV
Events/5 GeV
4000
3000
2000
1000
0
0
50
100
150
$E_b$ [GeV]

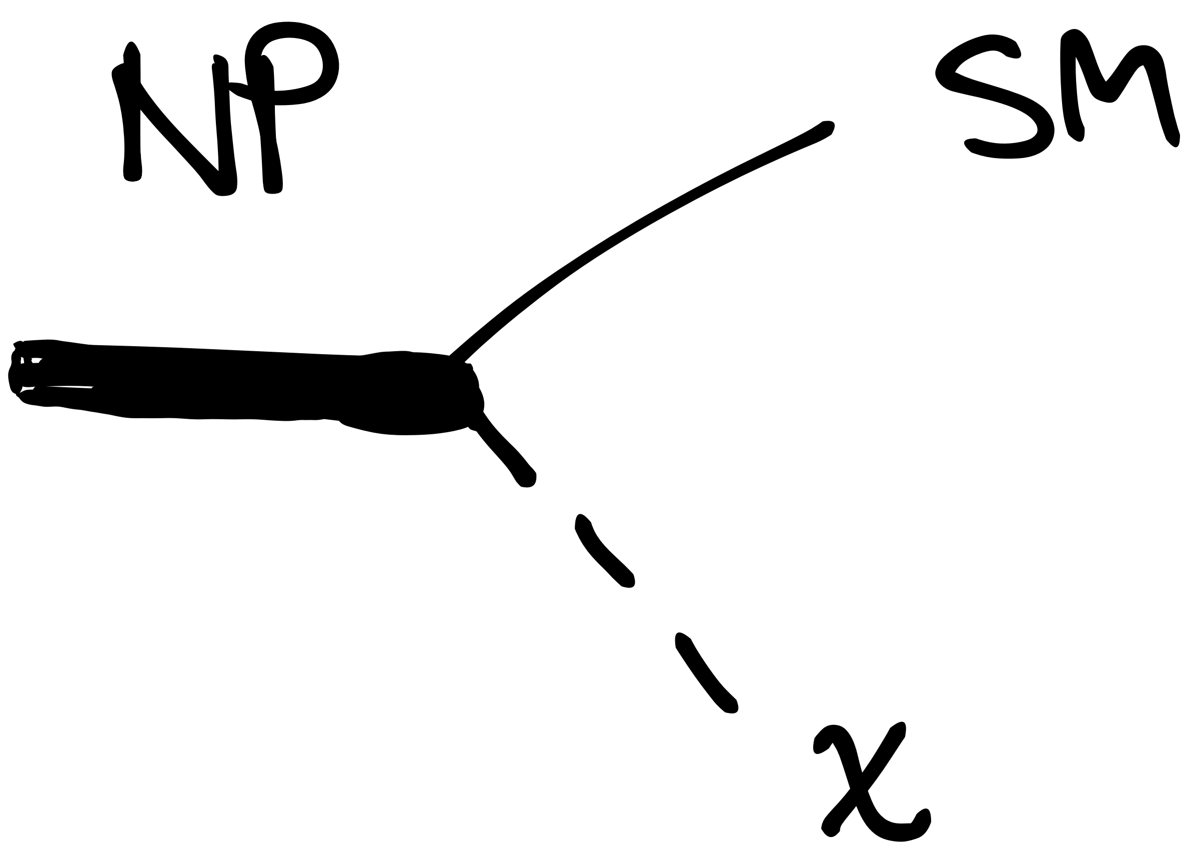
# COUNTING INVISIBLE PARTICLES

1212.5230

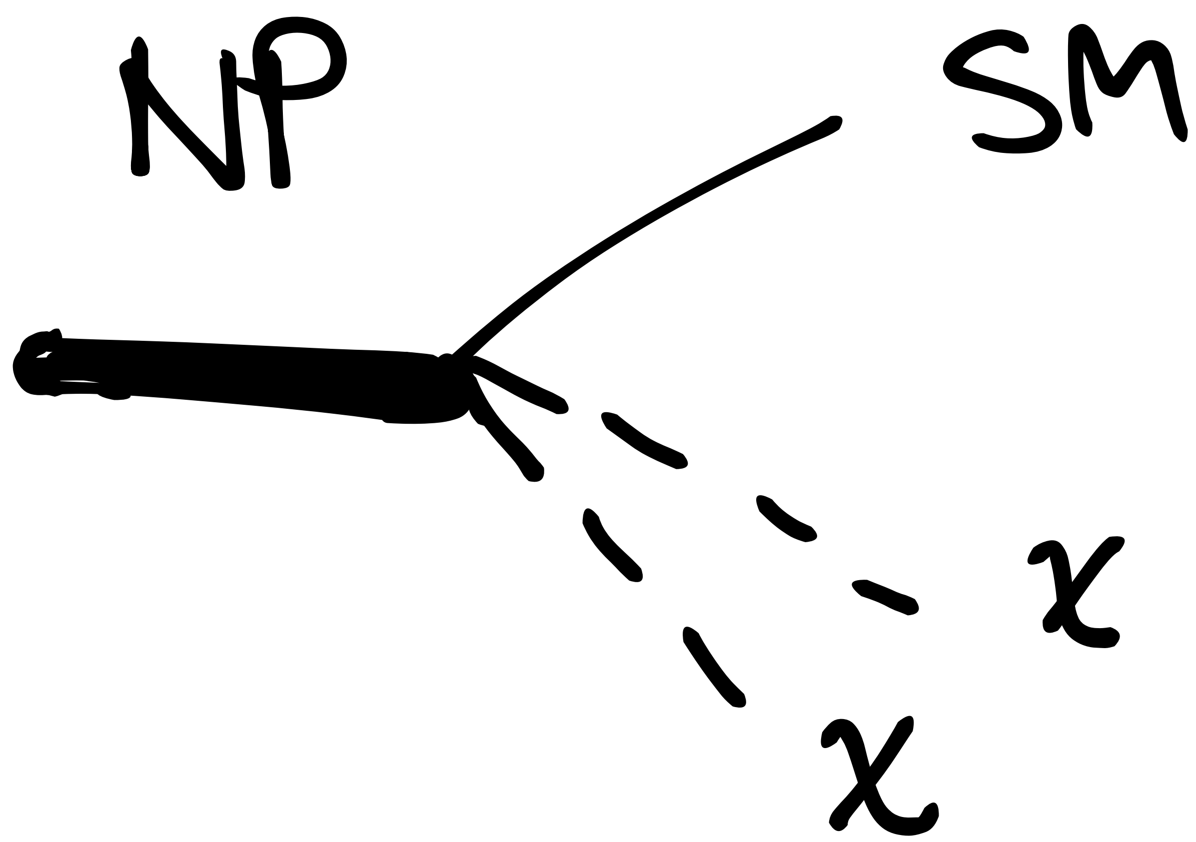
- A LOT OF LHC PHENOMENOLOGY INVOLVES INVISIBLE PARTICLES
- DARK MATTER IS AN INVISIBLE PARTICLE

WHAT SYMMETRY PREVENTS THE DM FROM DECAYING

2-BODIES

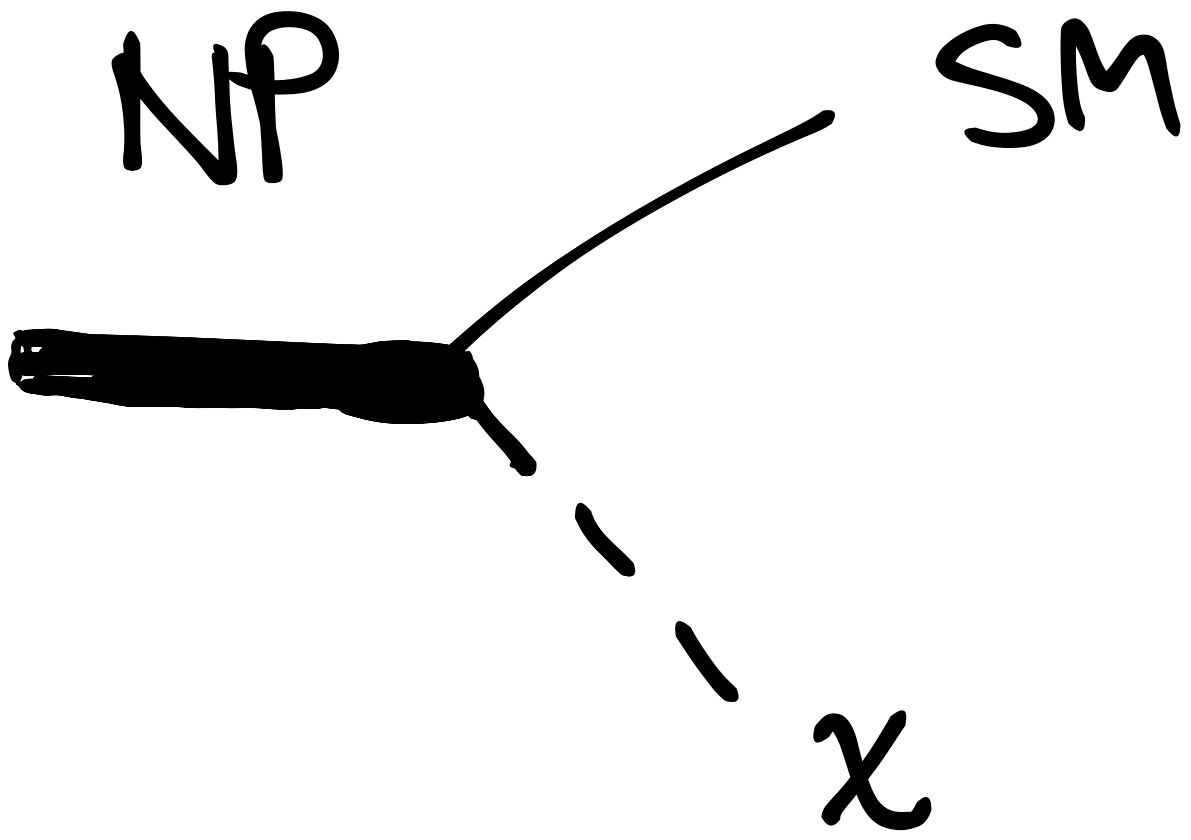



3-BODIES




ONE VISIBLE PARTICLE IN EACH DECAY : VERY LITTLE INFORMATION!

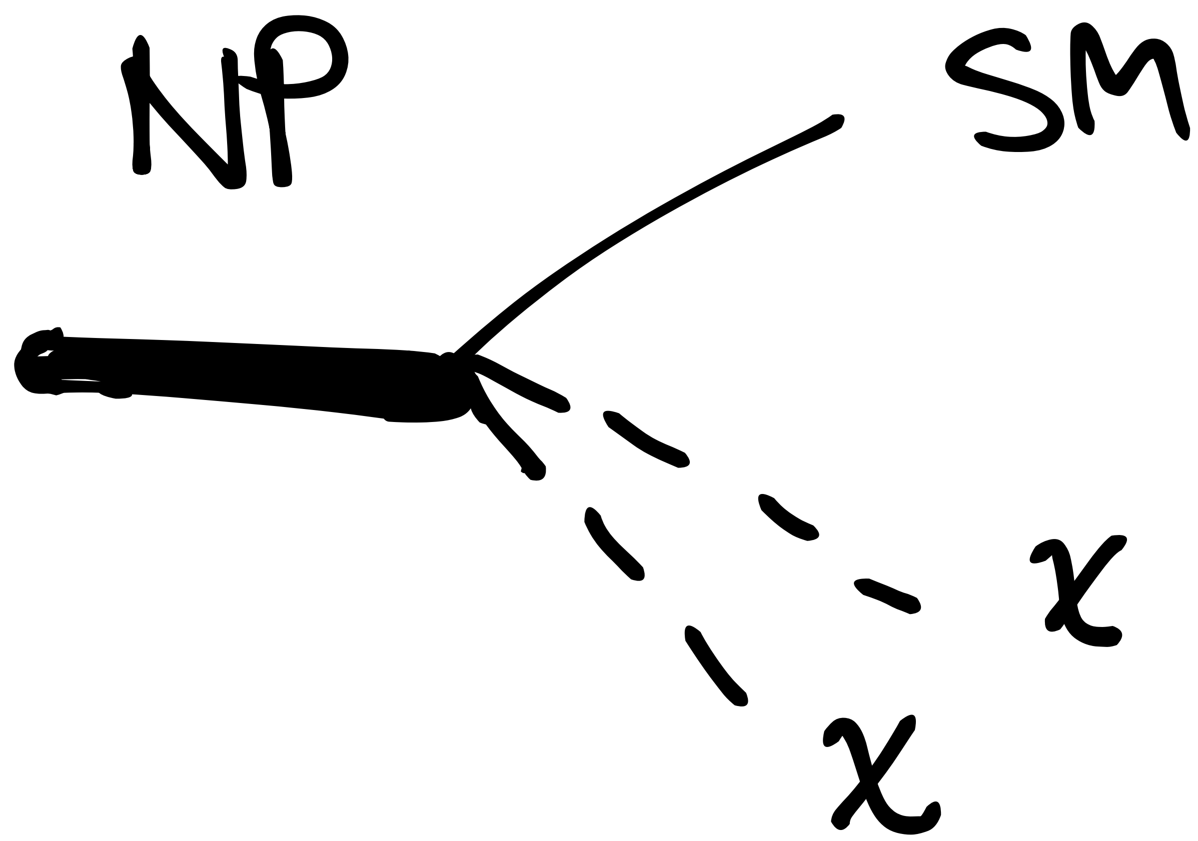
## 2-BODIES




$E_{SM}$ HAS A PEAK AT

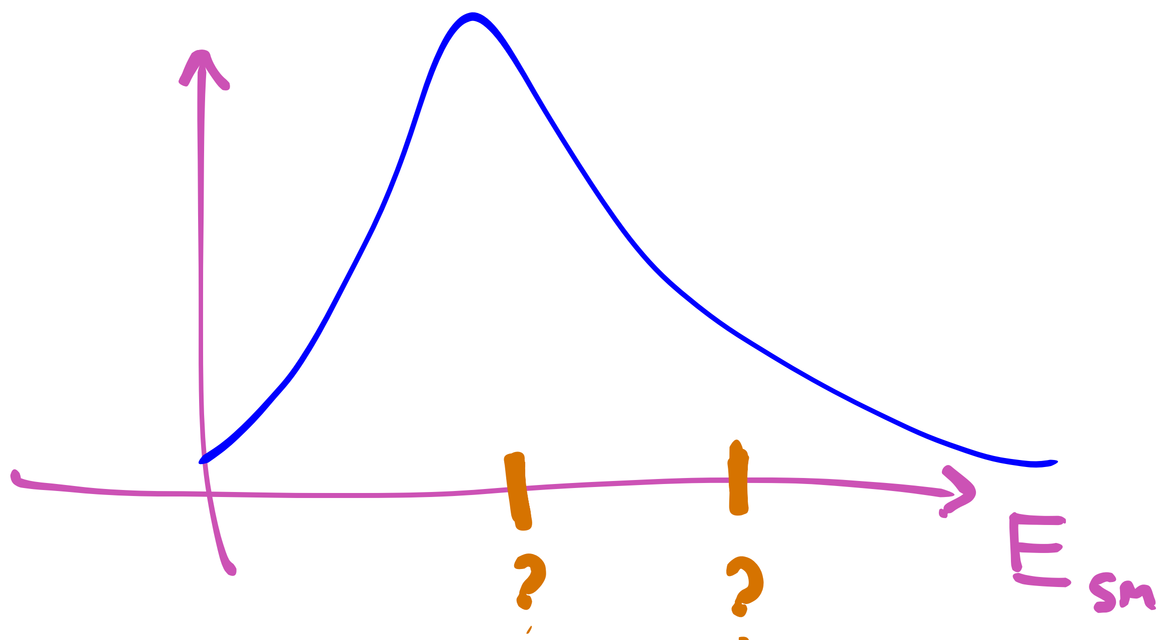
$$E_{peak} = \bar{E} = \frac{m_{NP}^2 - m_\chi^2 + m_{SM}^2}{2 m_{NP}}$$

## 3-BODIES



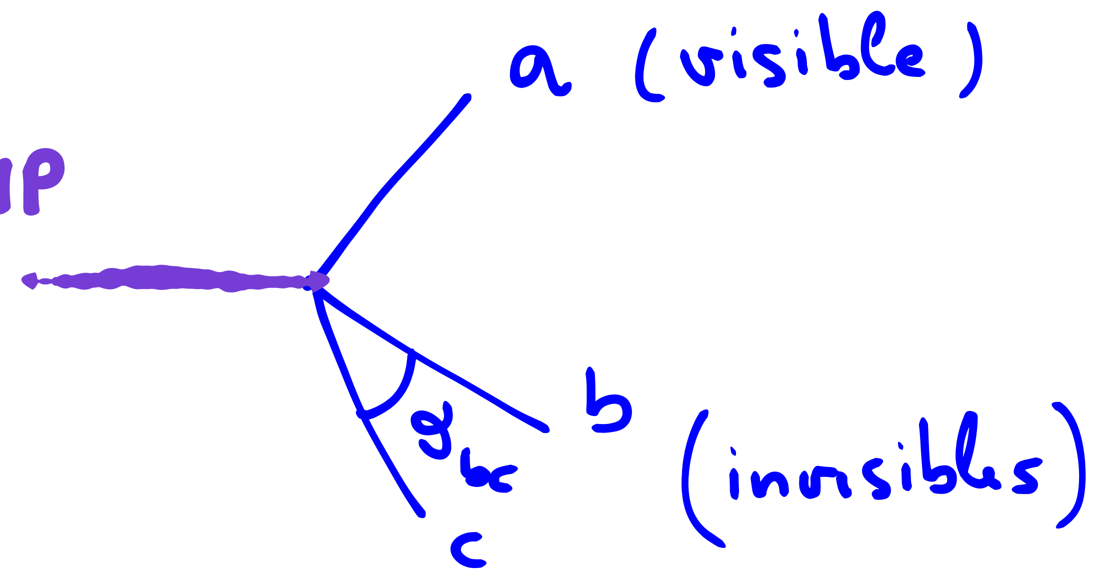

$E_{SM}$ HAS A PEAK BELOW

$$E_{peak} \ll \bar{E}' = \frac{m_{NP}^2 + m_{SM}^2 - 2 m_\chi^2}{2 m_{NP}}$$




**OBSERVING $E_{SM}$ ONLY WE CANNOT TELL**

# n-BODIES REDUCE TO FEWER WHEN BODIES ARE ALIGNED

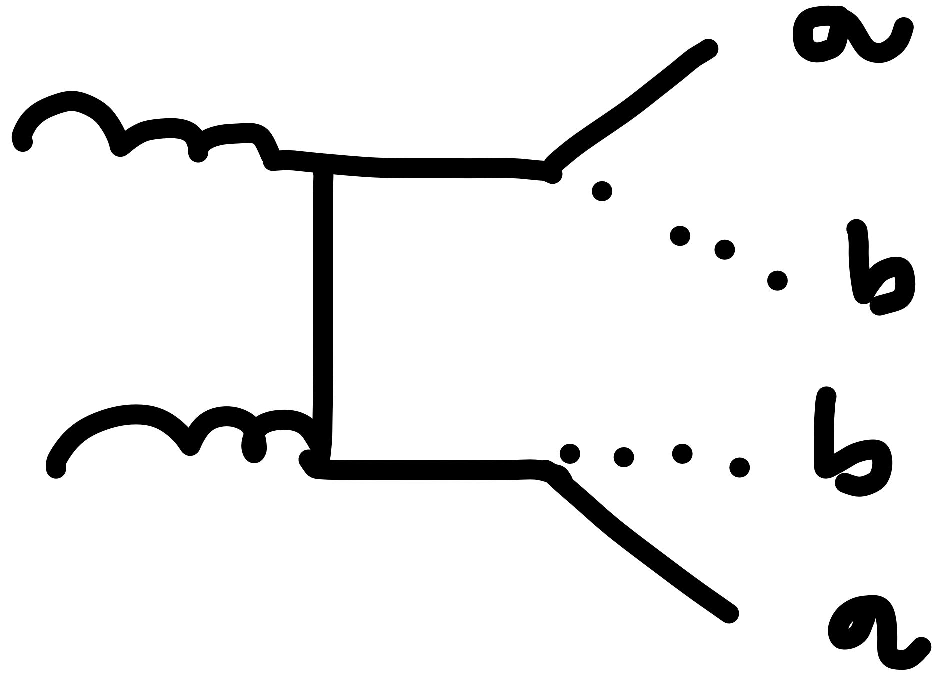



WHEN $\vartheta_{bc} \to 0$

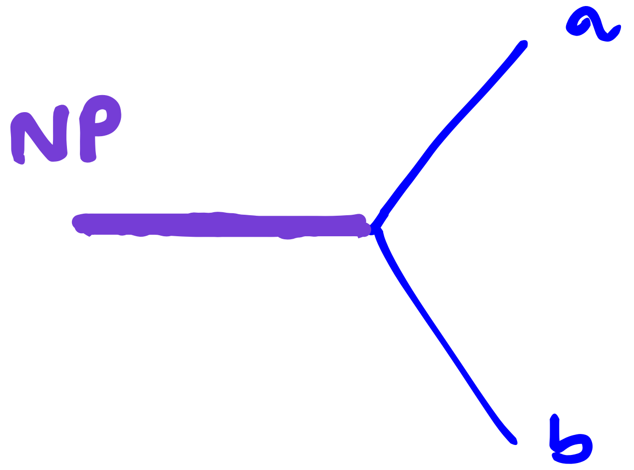
MEASURE SOME QUANTITY THAT SINGLES OUT THIS COLLINEAR CONFIGURATION

- b AND c ARE INVISIBLE
- MEASURE ONLY THE SUM OF THE INVISIBLES

# $m_T$ AND $m_{T_2}$



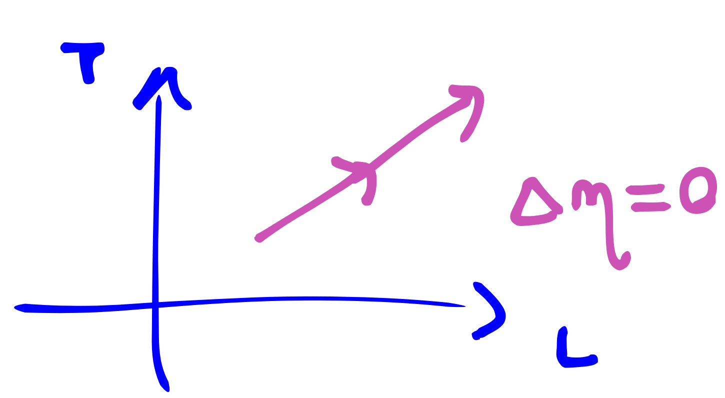

$m_T$ IS A PROJECTION OF THE INVARIANT MASS

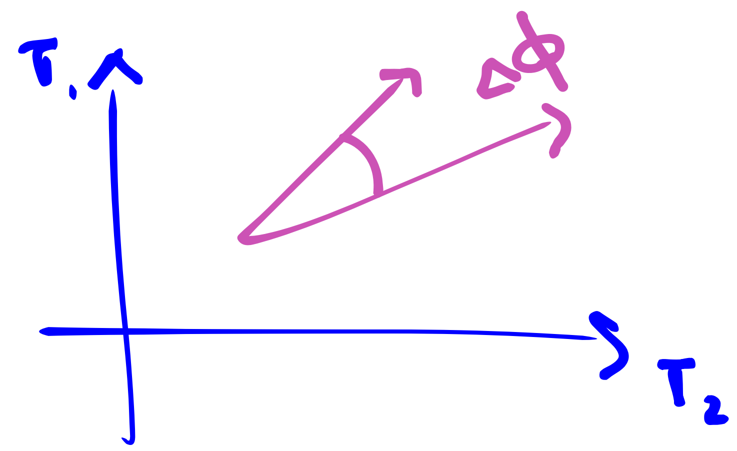



$$m^2_{NP} = m_a^2 + m_b^2 + 2\left( E_{T_a} E_{T_b} \cosh \Delta\eta_{ab} - p_{T_a} p_{T_b} \cos \Delta\phi_{ab} \right)$$

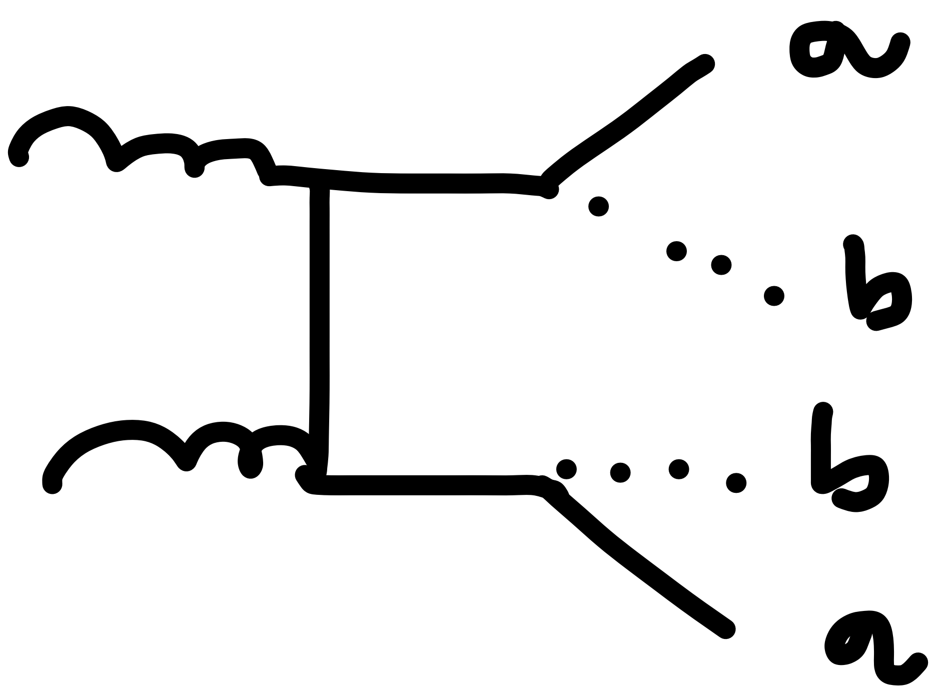
$$m^2_T = m_a^2 + m_b^2 + 2\left( E_{T_a} E_{T_b} - p_{T_a} p_{T_b} \cos \Delta\phi_{ab} \right)$$

$m_T \leq m_{NP}$ AND $m_T = m_{NP}$ $\Delta\eta_{ab} = 0$




THE MAX OF $m_T$ SINGLES OUT A KIND OF COLLINEAR CONFIGURATION




- BOTH a AND a' ARE INVISIBLE
- OBSERVE ONLY $\not{E}_T$ (THE SUM OF THE INVISIBLES)
- MAKE AN ANSATZ $\bar{P}_{T_a} + \bar{P}_{T_{a'}} = \not{E}_T$

$$m_{T2} = \min_{\text{ansatz}} \{ \max ( m_T , m_T ) \}$$

- min OVER ALL POSSIBLE WAYS TO SPLIT THE MISSING MOMENTUM

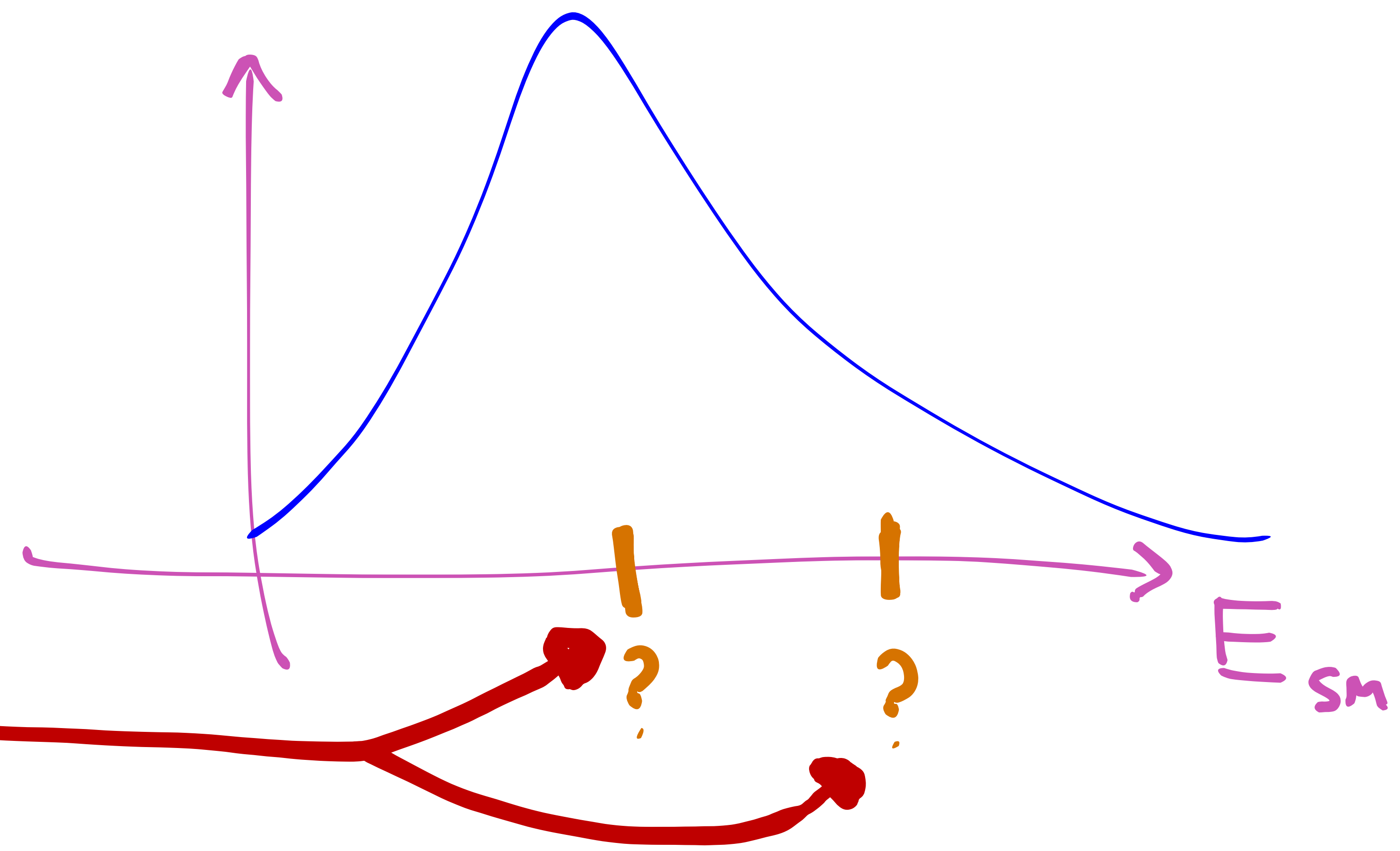
$m_{T_2}$ IS MAXIMIZED WHEN PARTICLES ARE ALIGNED

CLOSED FORM: "BALANCED CASE"

$$m_{T_2}^{max} = C + \sqrt{C^2 + \tilde{m}^2}$$

$$C_2 = \frac{m_{NP}^2 - m_a^2}{2 m_{NP}} = E_{PEAK}$$

$$C_n = \frac{m_{NP}^2 - n^2 m_a^2}{2 m_{NP}} \qquad m_a \to n\, m_a$$



# APPLICATION TO BOTTOM QUARK PARTNERS

$pp \to B' B'$ FOLLOWED BY $B' \to b\chi$ ($Z_2$-model), $B' \to b\chi\chi$ (non-$Z_2$ model)

POST-DISCOVERY ⇓ LARGE S/B

0 leptons with $|\eta_l| < 2.5$ and $p_{T\,l} > 20$ GeV for $l = e, \mu, \tau$,

2 $b$-tagged jets with $|\eta_b| < 2.5$ and $p_{T\,b_1} > 100$ GeV, $p_{T\,b_2} > 40$ GeV,

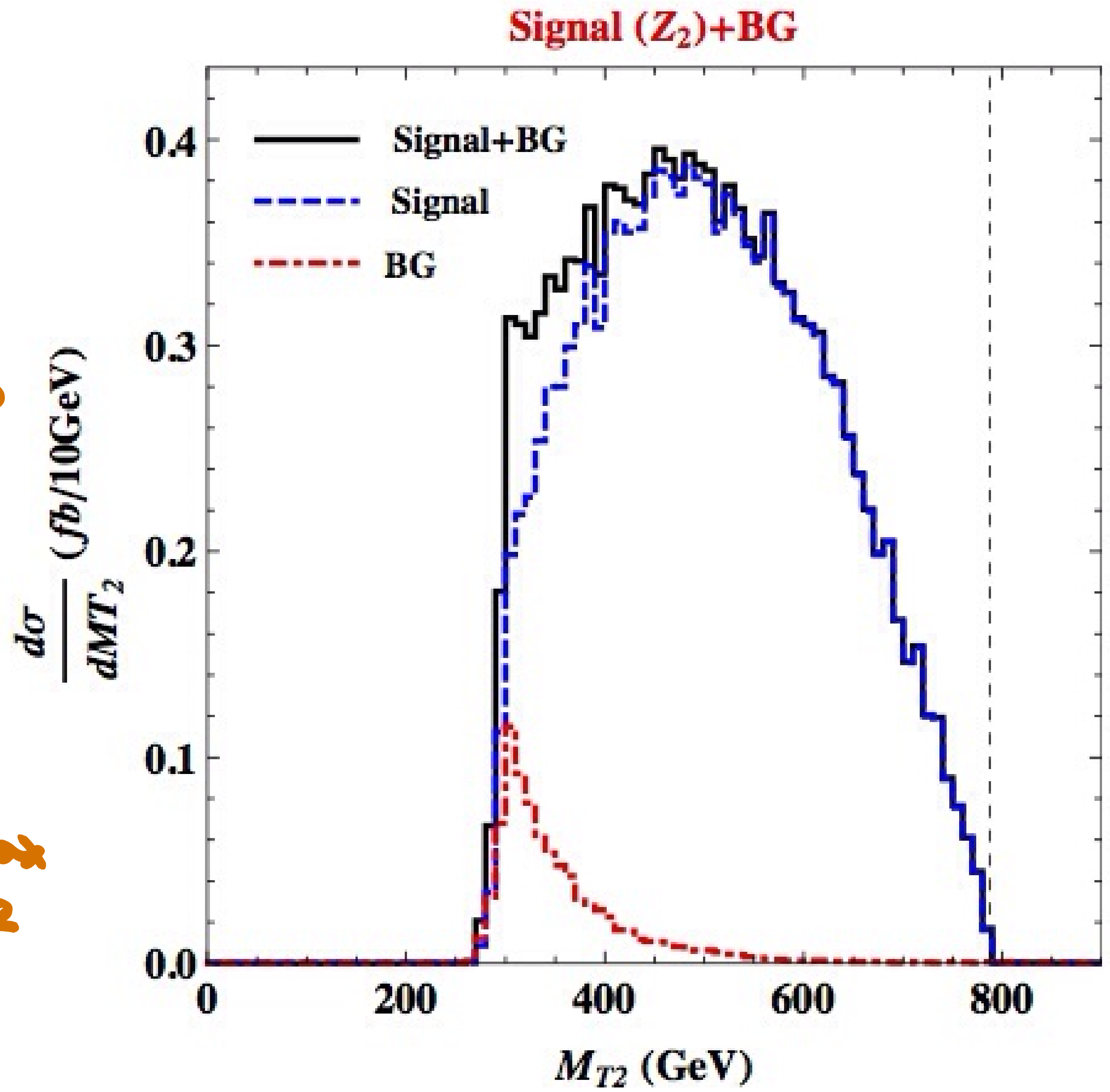
$\not{E}_T > 300$ GeV,

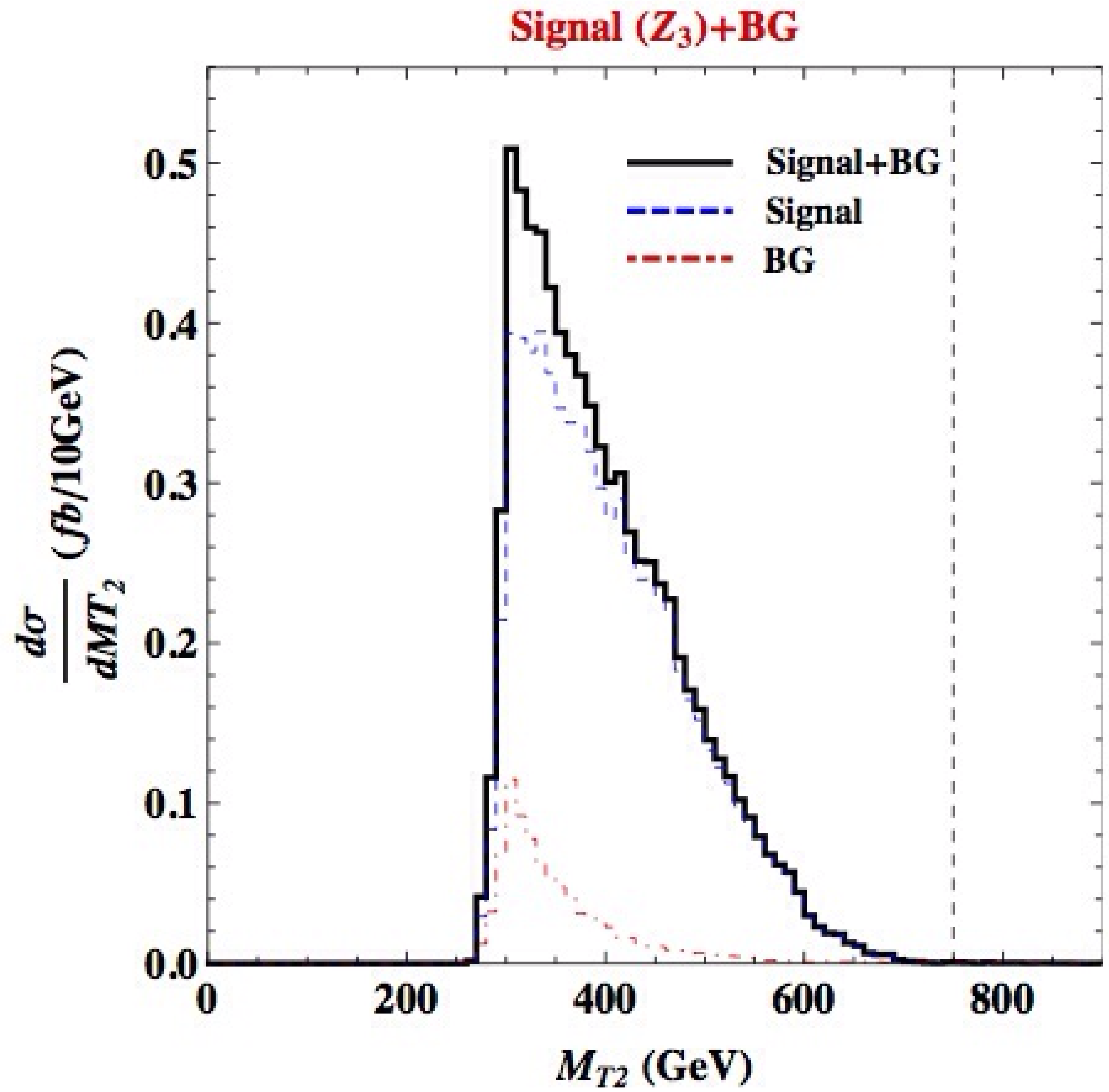
$S_T > 0.4$,

$f > 0.3$,

**STEP 1:** COMPUTE $M_{T2}$ FOR ALL EVENTS FOR A TRIAL MASS OF THE INVISIBLE

**STEP 2:** $c + \sqrt{c^2 + m^2_{TRIAL}} = m^{max}_{T2}$ TO GET $c$



## STEP 1:

COMPUTE $M_{T2}$ FOR ALL EVENTS FOR A TRIAL MASS OF THE INVISIBLE

## STEP 2:

$C+\sqrt{C^2+m^2_{TRIAL}} = m^{max}_{T2}$

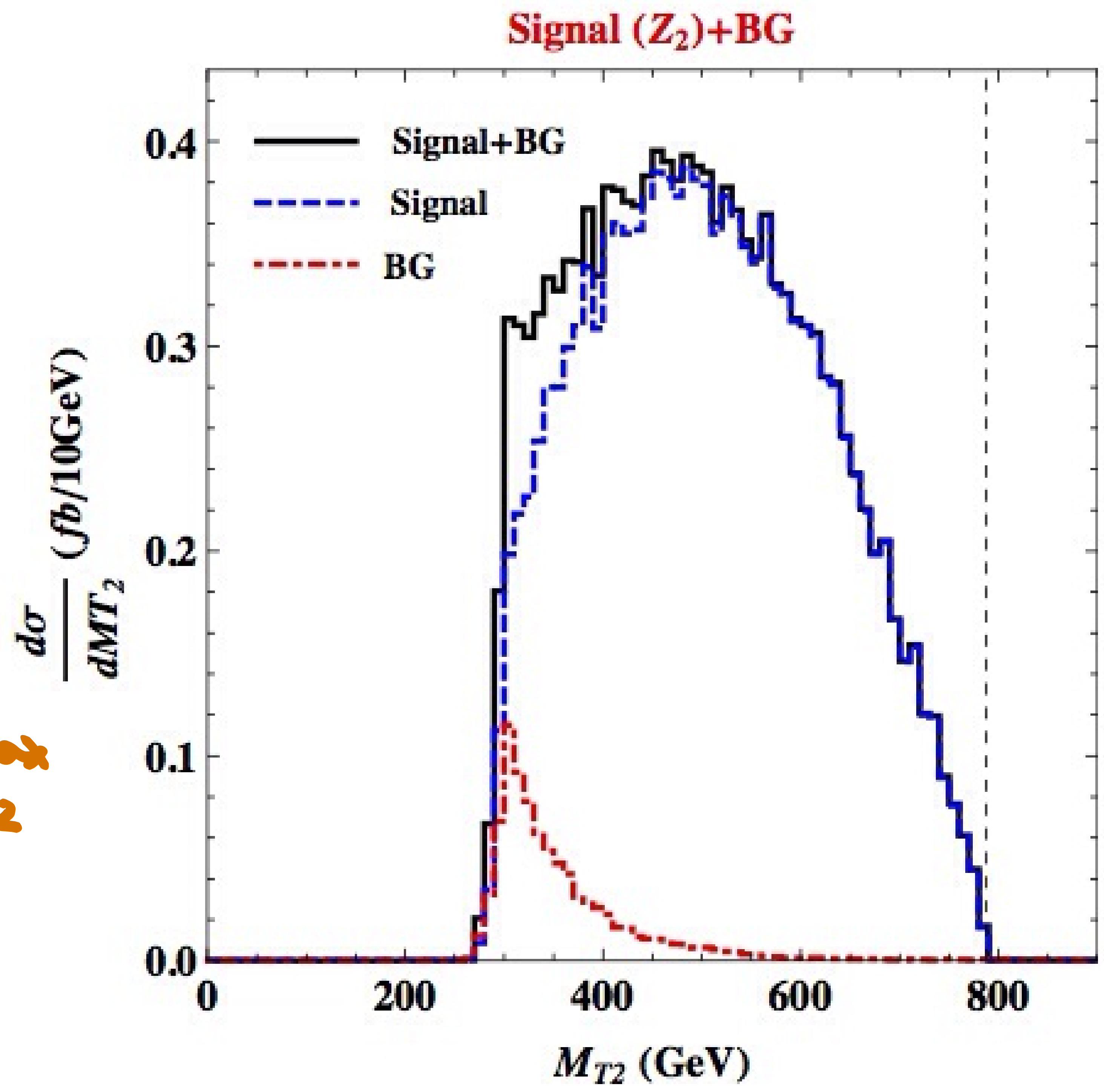
TO GET C

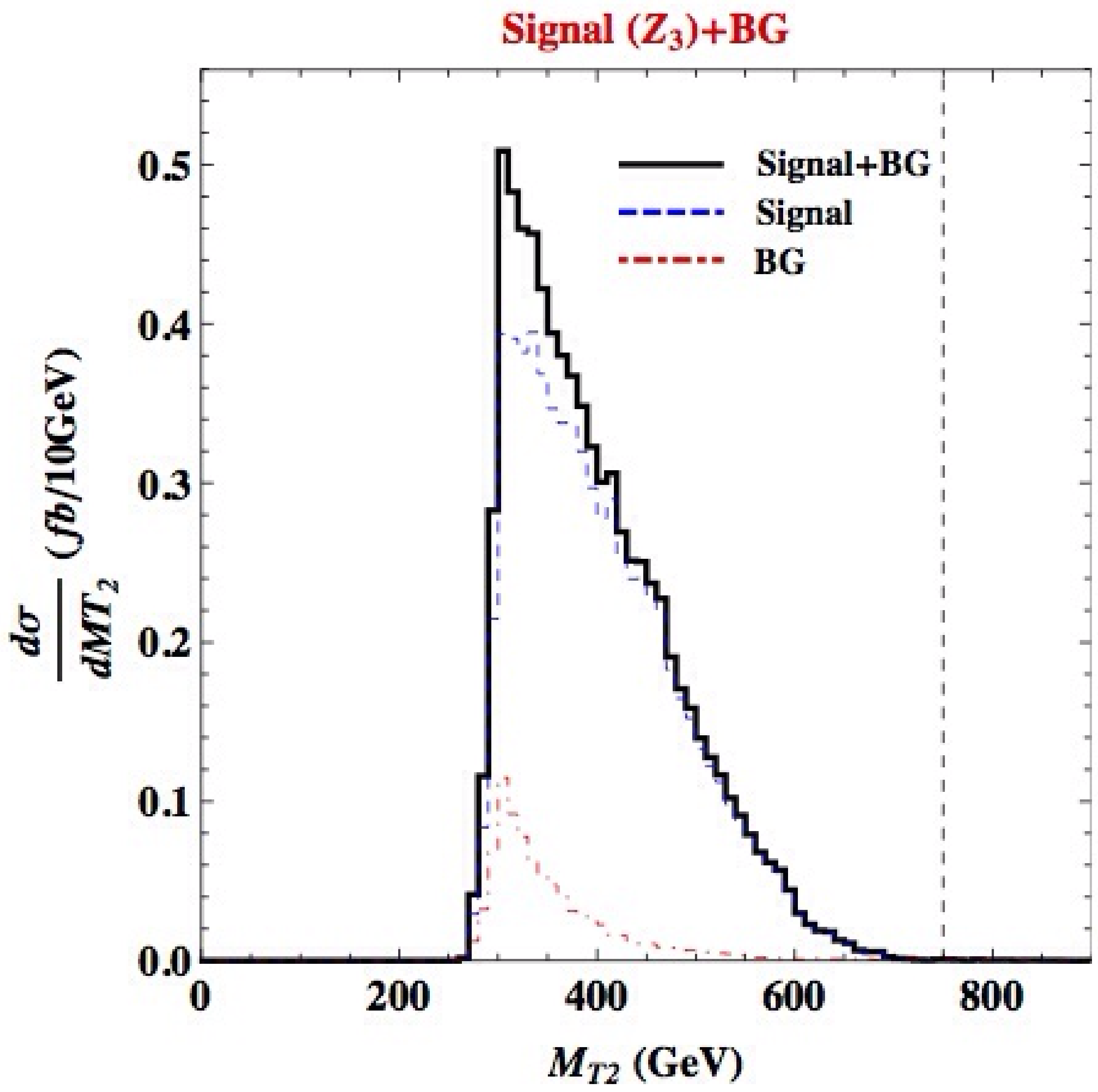
## STEP 3:

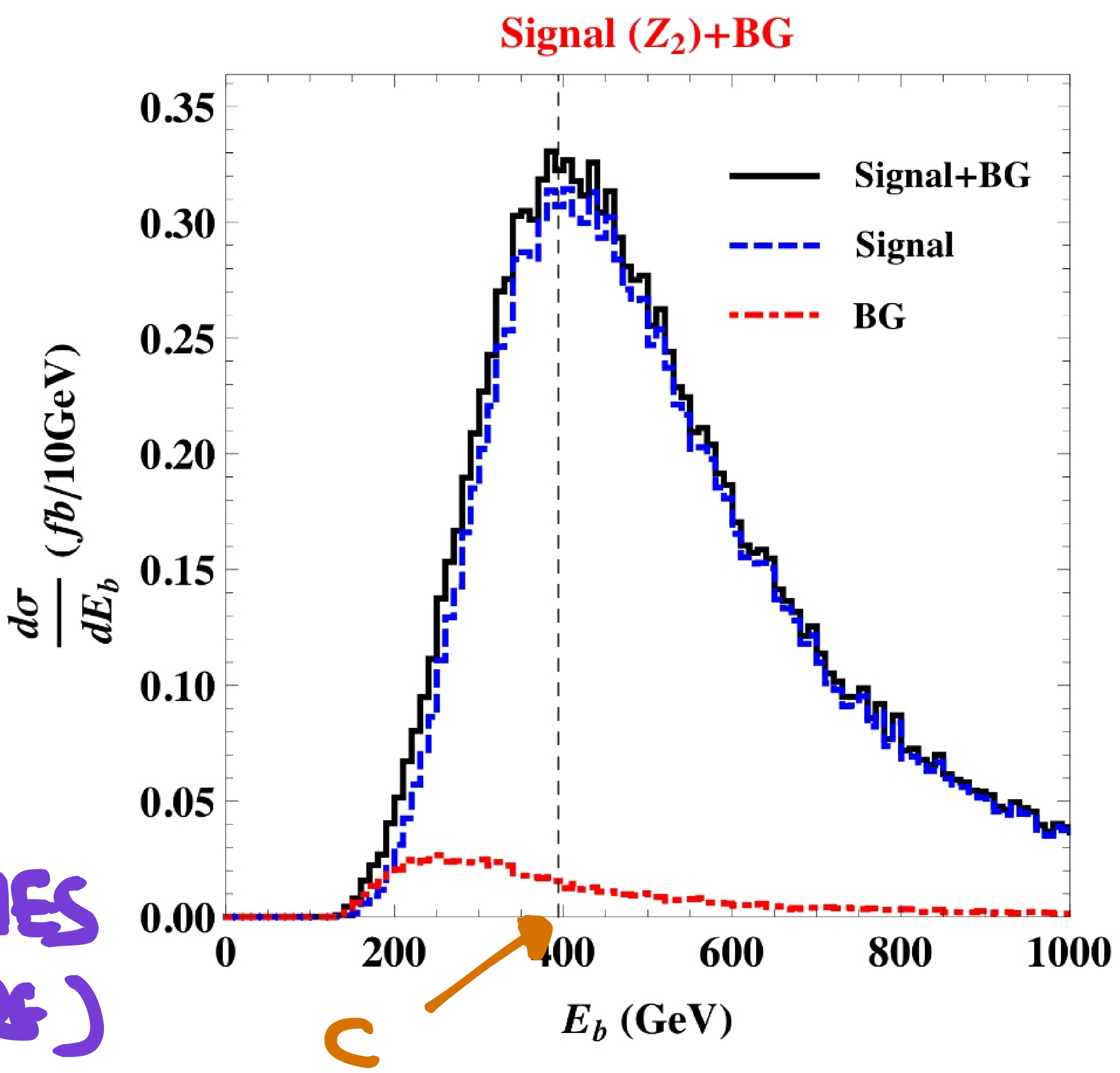
FIND THE PEAK OF THE ENERGY DISTRIBUTION (POSSIBLY REMOVING BG)

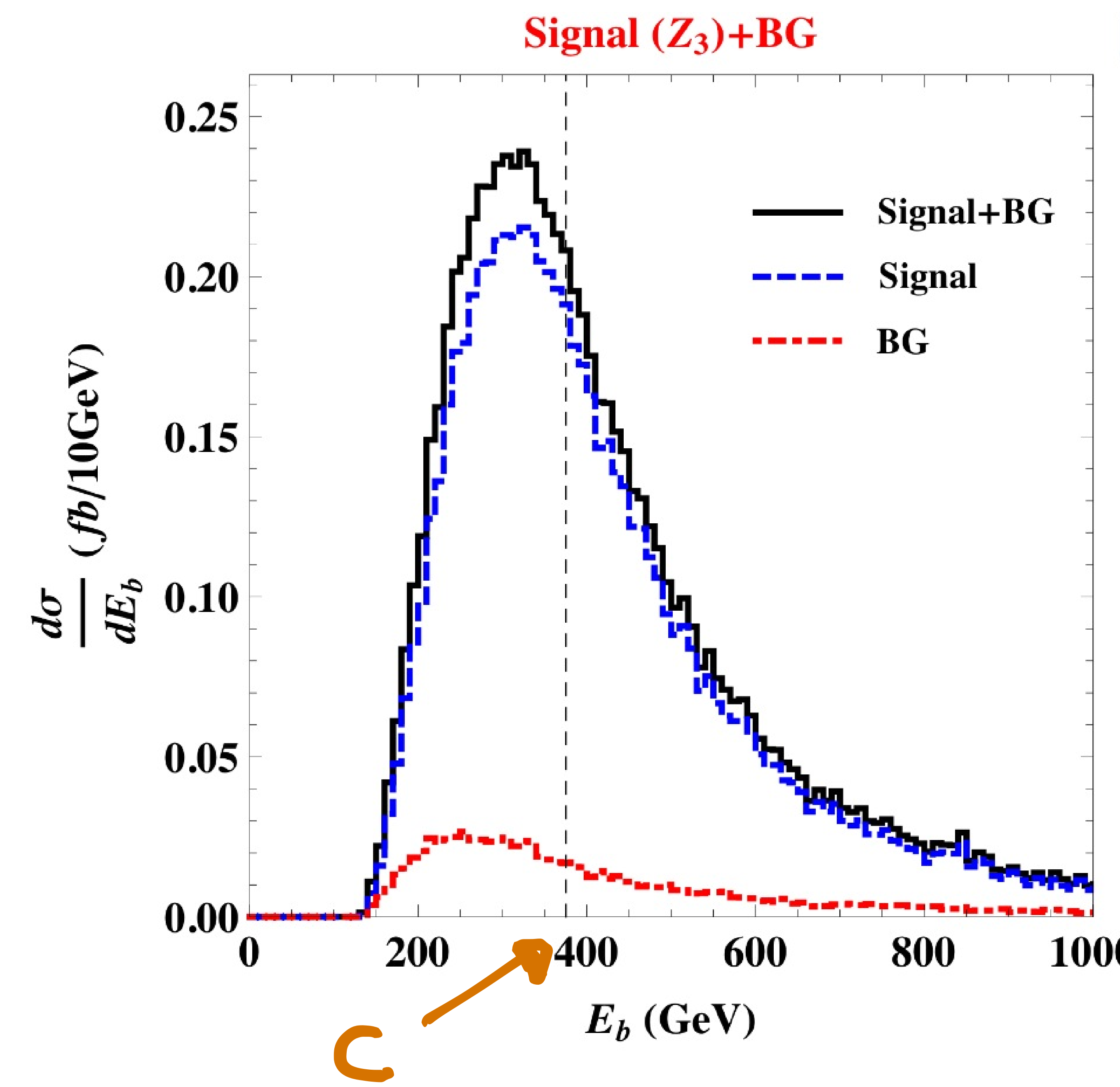
$E_{PEAK} = C \rightarrow$ 2 BODIES

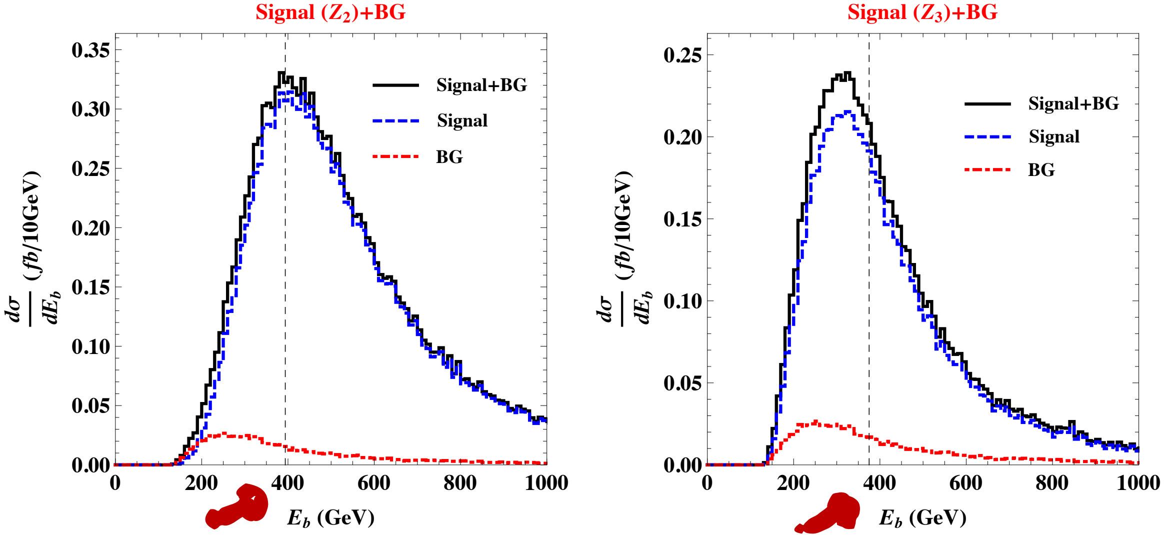
$E_{PEAK} < C \rightarrow$ 3 BODIES (OR MORE)



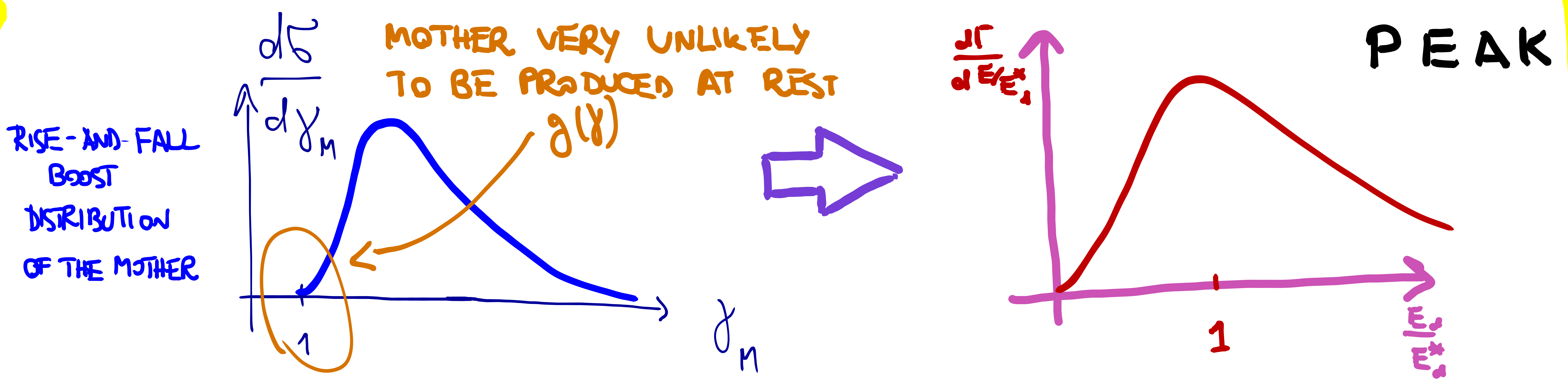


Signal ($Z_2$)+BG
Signal ($Z_3$)+BG
$\frac{d\sigma}{dE_b}$ ($fb$/10GeV)
$E_b$ (GeV)
Signal+BG
Signal
BG
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0
200
400
600
800
1000

# Conclusions

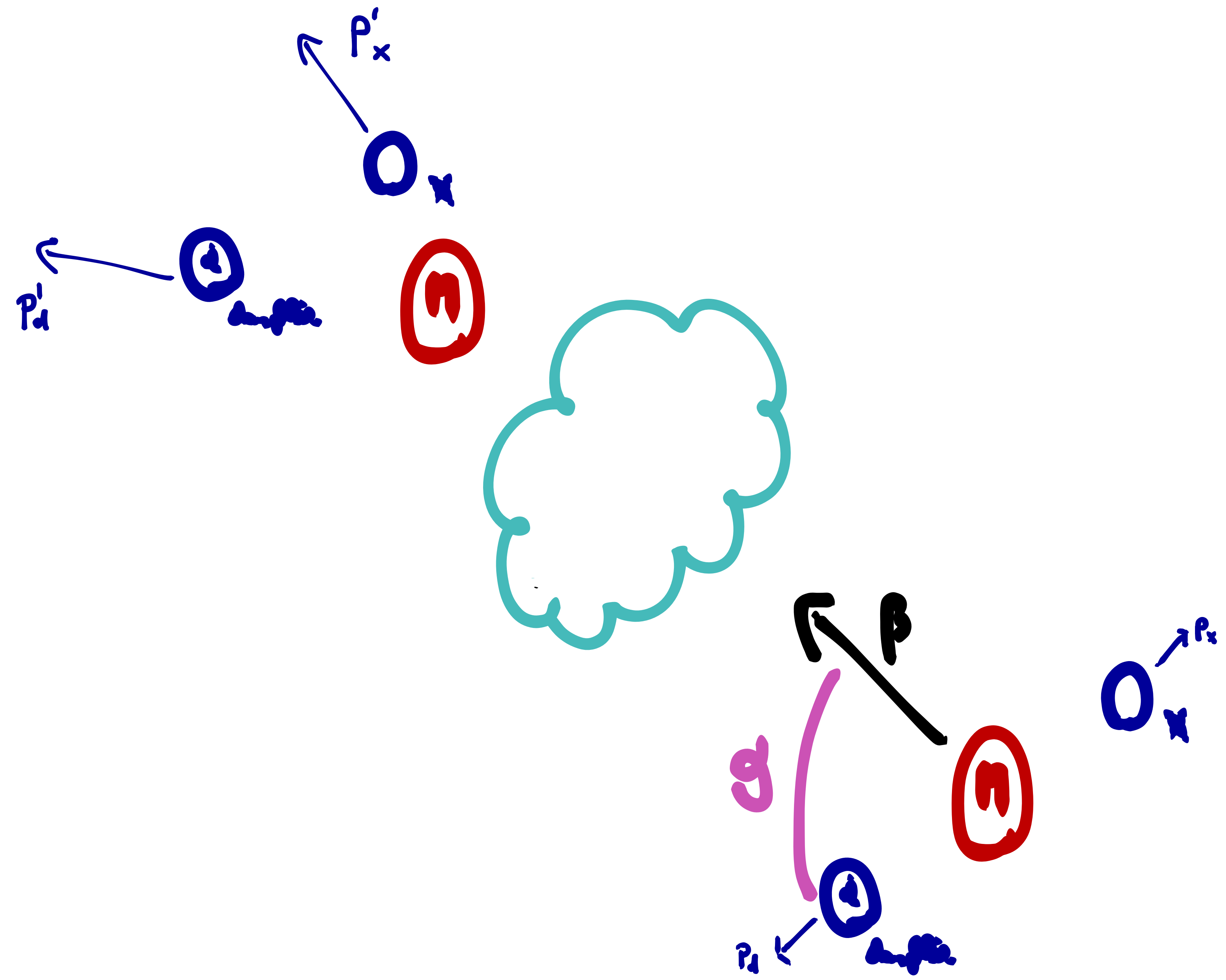
IN PHENOMENOLOGICALLY RELEVANT CASES (HIGH ENERGY COLLIDERS)

THE SPECTRUM OF ENERGY IN TWO BODY DECAYS ENCODES IN A SIMPLE WAY AN INVARIANT OF THE TWO BODY DECAY KINEMATICS




KINKS OR PLATEAUS ARE POSSIBLE AS WELL

# Conclusions




THE PEAK OF THE ENERGY DISTRIBUTION IS ROBUST FOR MASSLESS AND MASSIVE DAUGHTERS

$$E_{peak} = \frac{m_M^2 - m_x^2}{2 m_M}$$

$$E_{peak} \geq \frac{m_M^2 - m_x^2 + m_d^2}{2 m_M}$$

# LIMITING FACTORS:

- RADIATIVE CORRECTIONS

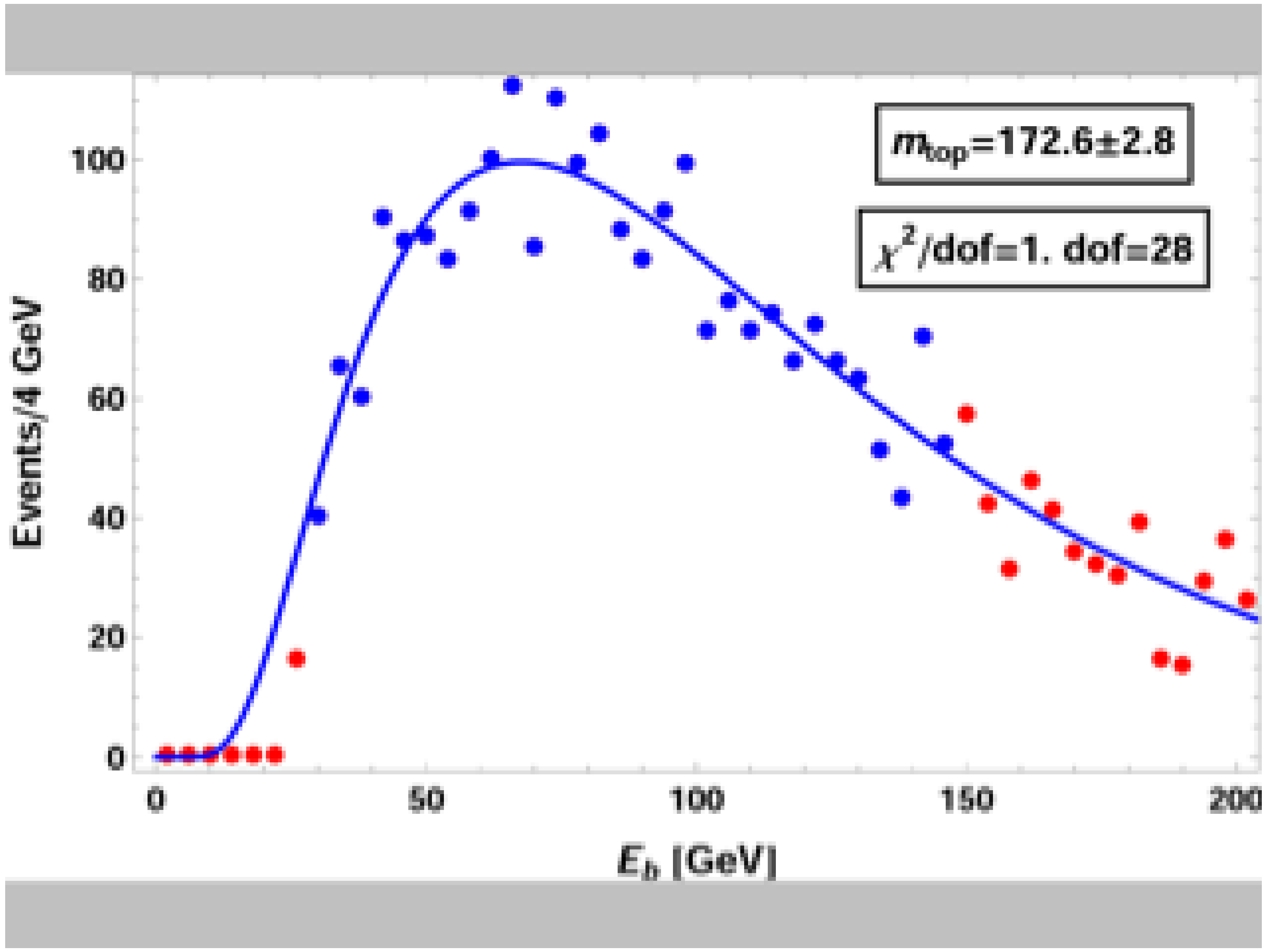
  EXTRA RADIATION MAKES THE DECAY 3-BODY

- TOO LARGE MASS OF THE OBSERVED DAUGHTER

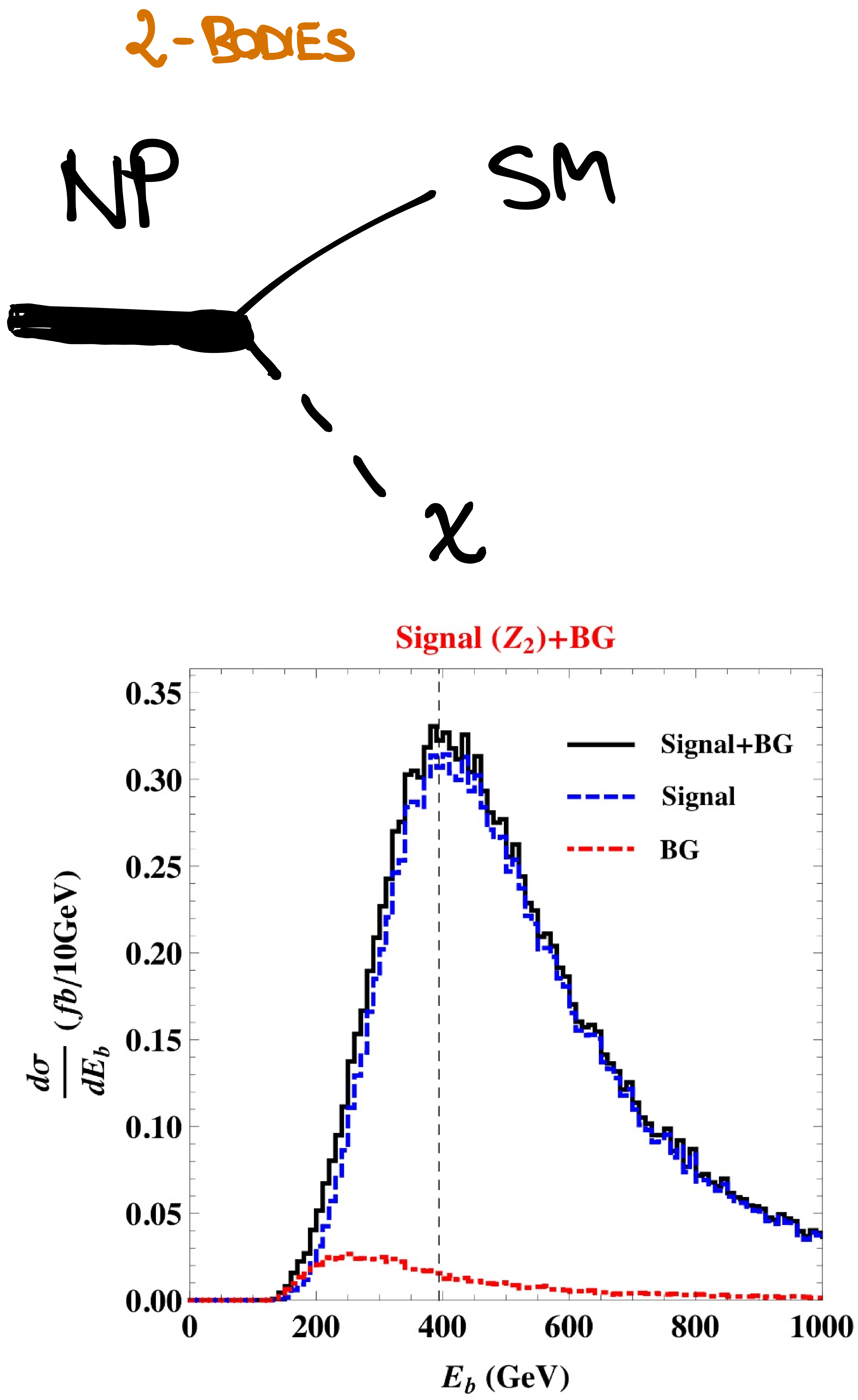
- MAY BE SENSITIVE TO SELECTION CUTS

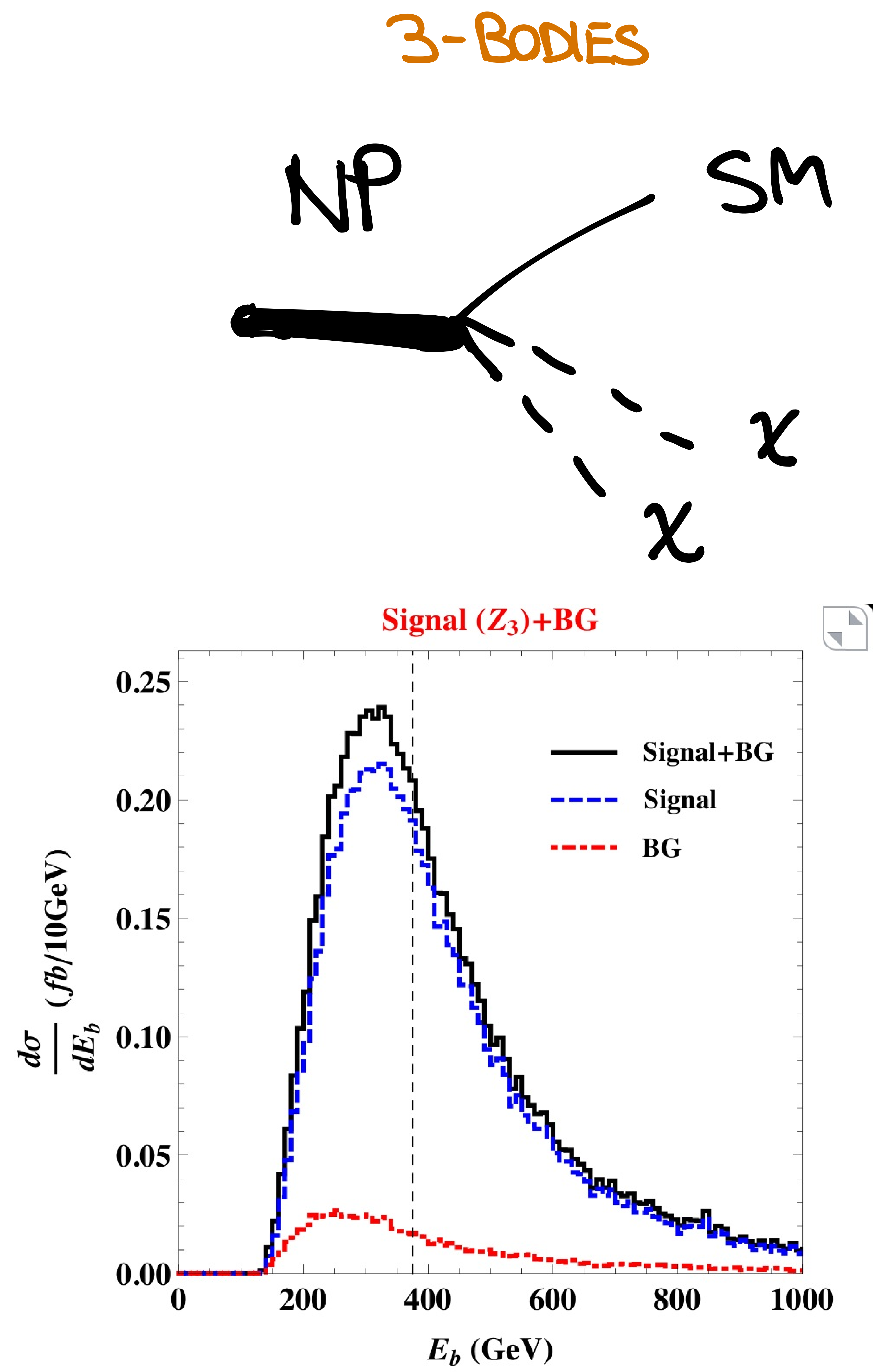
DESPITE THESE LIMITATIONS THE OBSERVATION CAN BE USED TO MEASURE PARTICLE MASSES WITH 10% ACCURACY OR BETTER

$t \to b \ell \nu$ in $pp \to t\bar{t}$ ⇒ $m_{top}$ FROM $d\sigma/dE_b$



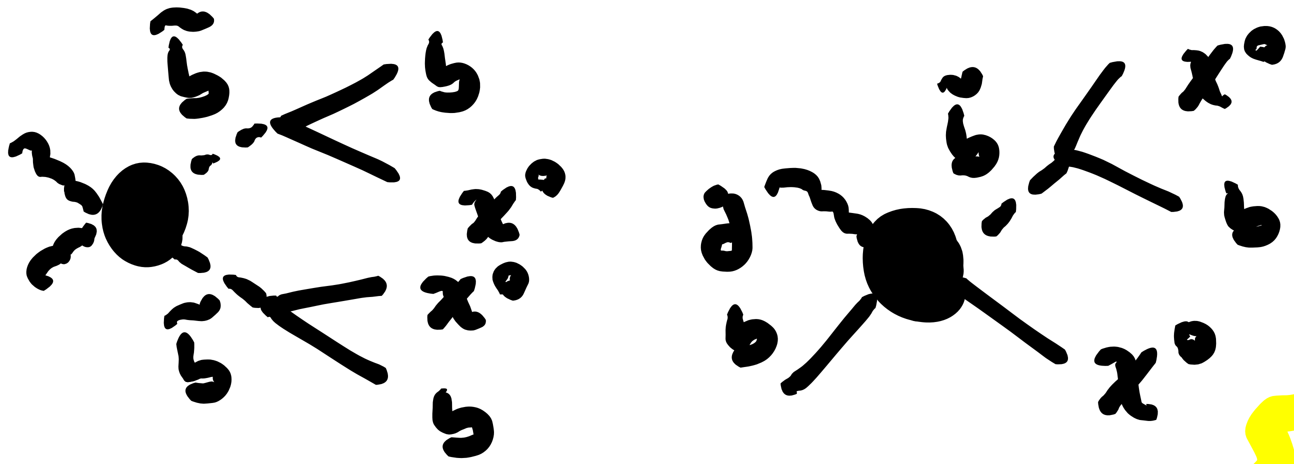

$g \to b\tilde{b} \to bbX$ AS WELL


2-BODIES
NP
SM
χ
Signal (Z2)+BG
Signal+BG
Signal
BG
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0
200
400
600
800
1000
Eb (GeV)
dσ/dEb (fb/10GeV)

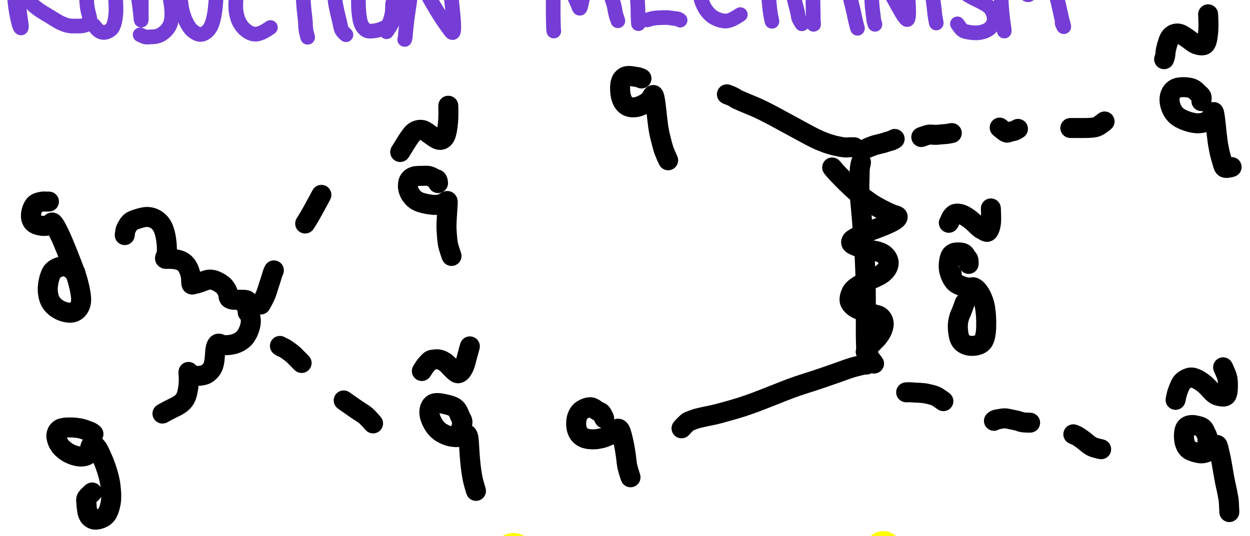

3-BODIES
NP
SM
χ
χ
Signal (Z3)+BG
Signal+BG
Signal
BG
0.00
0.05
0.10
0.15
0.20
0.25
0
200
400
600
800
1000
Eb (GeV)
dσ/dEb (fb/10GeV)

"LOCAL"

- NO NEED TO KNOW ANYTHING ABOUT THE REST OF THE EVENT




NO NEED TO KNOW THE EXACT PRODUCTION MECHANISM



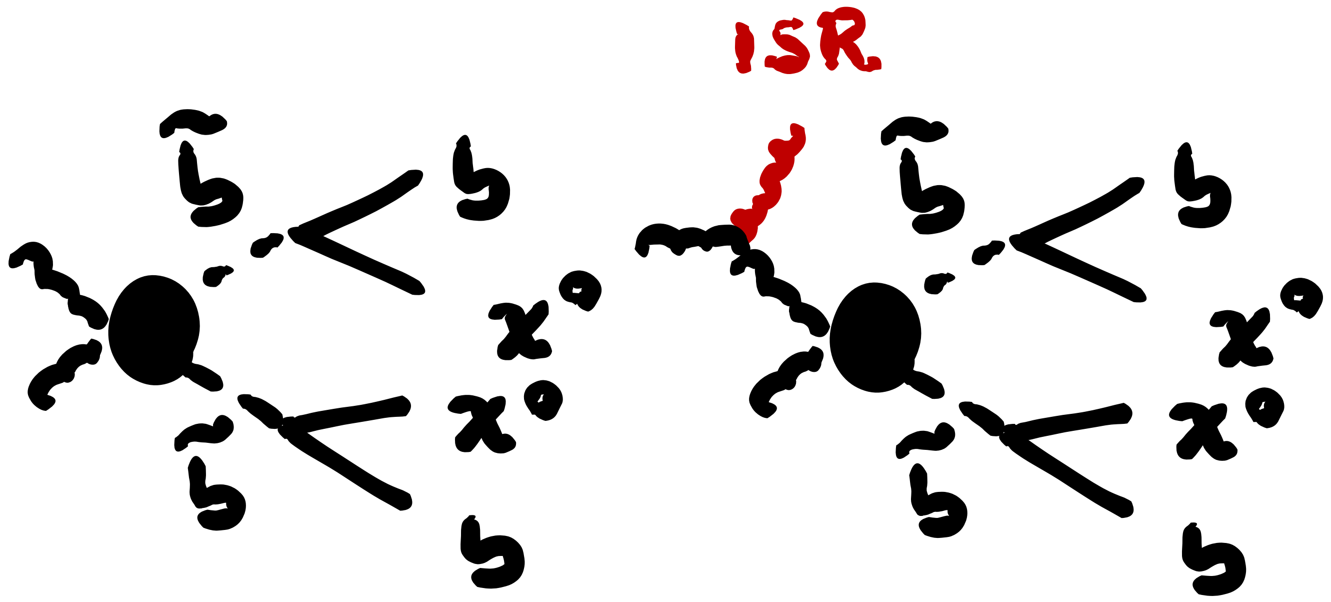

- NO NEED TO MEASURE THE OTHER DECAY PRODUCT

$\tilde{b} \to b \chi^0$

$W \to \ell \nu$

$t \to b W \to b \ell \nu$

- NO NEED TO KNOW ANYTHING ABOUT THE REST OF THE EVENT




SIMPLE

ROBUST

Also, since

$$\log E_\gamma = \tfrac{1}{2}(\log E_{\gamma,\,\min} + \log E_{\gamma,\,\max}) = \log \mu \qquad (1\text{–}225)$$

it follows that, on logarithmic plots of the energy spectra of these $\gamma$-rays, the rest-system energy $\mu$ will lie halfway between the extremum energies.

We are particularly concerned with decays that are isotropic in the rest system of the decaying particle, such as the $\pi^0$ and $\Sigma^0$ decays, which we have previously considered. For these decays, we have already shown that the resultant $\gamma$-ray energy distribution function is only a function of the momentum of the primary; indeed this function is a constant which is inversely proportional to this momentum for a given primary, within a range proportional to the momentum of the primary, and vanishes outside this range. Thus, for decays of parent particles with a wide range of primary energies, $\gamma$-ray spectra are generated which are made up of a superposition of rectangular spectra, as shown in figure 1–11. Higher energy primaries produce the $\gamma$-rays at the extremes of the spectrum. We therefore deduce a second important kinematic property, which holds for two-body decays that produce $\gamma$-rays isotropically in the rest system of the decaying primary; viz,

> The energy spectra of $\gamma$-rays produced isotropically in the rest system of the decaying primary will be symmetric on a logarithmic plot with respect to $E_\gamma = \mu$ and will peak at $E_\gamma = \mu$.

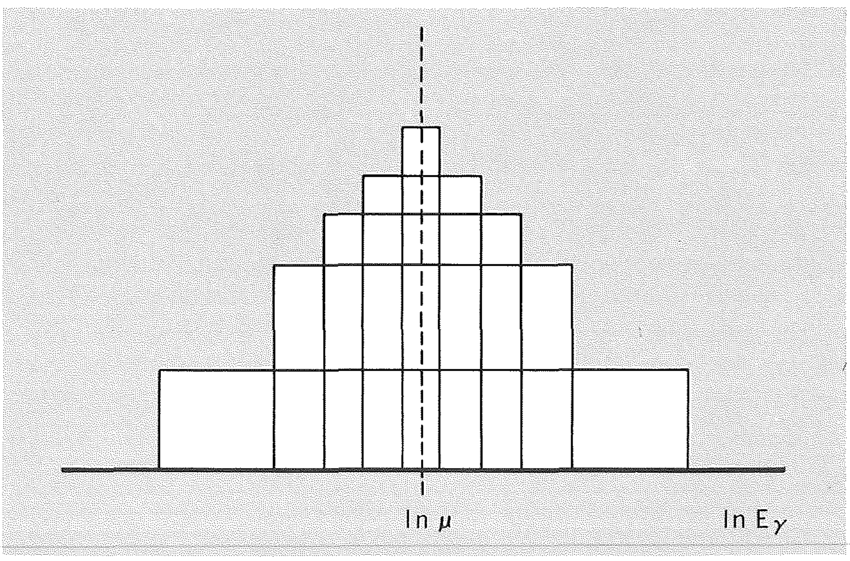
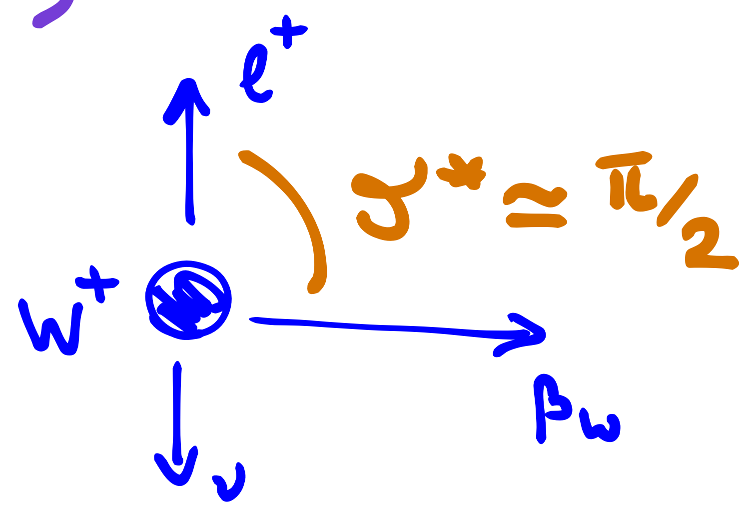



FIGURE 1–11.—Ideal superposition of $\gamma$-ray energy spectra from $\pi^0$ of $\Sigma^0$ particles having discrete values of energy.

## $\frac{d\sigma}{dp_T}$ JACOBIAN PEAK

- $p_T$ ( $E > p_T \Rightarrow$ NO PEAK IN $E$ FROM $p_T$ )
- USUALLY APPLIED FOR PARENT PARTICLES MOVING ALONG $\hat{z}$
- POLARIZATION DOES NOT MATTER
- END-POINT (UP TO RADIATION EFF)
-




$$\cos\vartheta^* \simeq 0$$

## ENERGY PEAK

- ENERGY
- VALID FOR PARENT PARTICLES MOVING ALONG ANY DIRECTION
- UNPOLARIZED PARENT PARTICLE
- MAXIMUM
-

$$E = E^* \gamma (1 + \cos\vartheta^* \beta)$$

$E = E^*$ at the peak implies

$$\cos\vartheta^* = -\sqrt{\frac{\gamma - 1}{\gamma + 1}}$$

FOR SOME BOOSTS THE EVENTS AT THE PEAK HAVE COM ANGLE $\neq \pi/2$

# RAZOR $M_R$ 1006.2727

- PAIR PRODUCTION
- BROAD PEAK
- $M_R = M_\Delta \not\Rightarrow E_1 = E^* \;\&\; E_2 = E^*$
- PEAK AROUND $2E^*$
- USUALLY APPLIED FOR CENTER OF MASS MOVING ALONG $\hat{z}$
- TRANSVERSE MOTION OF THE MOTHERS EDUCATEDLY GUESSED
- ASSUMPTIONS ON THE PRODUCTION
  $\gamma_{cm} \simeq 1 \quad S \ll \hat{s} \sim P_T$

# ENERGY PEAK

- SINGLE PRODUCTION AS WELL
- BROAD PEAK ( FWHM $\sim \langle\beta\rangle$ )
- $E_1 = E^* \;\&\; E_2 = E^* \Rightarrow M_R = M_\Delta$
- PEAK POSITION PREDICTABLE @ LO
- VALID FOR PARENT PARTICLES MOVING ALONG ANY DIRECTION
- SYMMETRIES OF THE INTERACTIONS GIVE UNPOLARIZED MOTHERS

## ENERGY AVERAGES

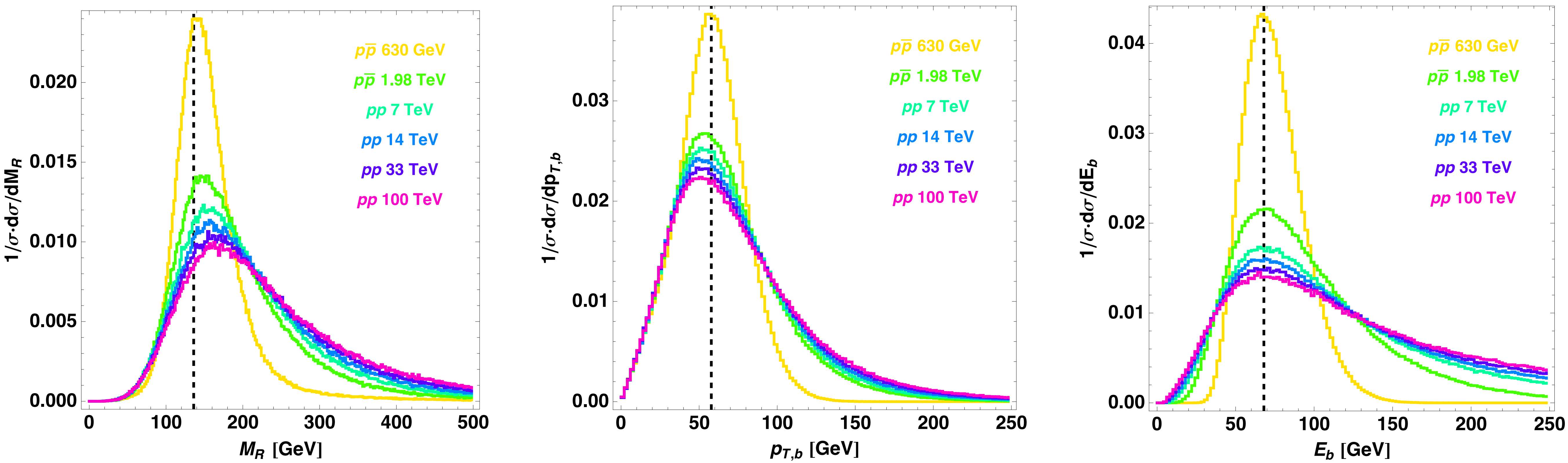
1107.4460
1305.6150

- GLOBAL PROPERTY → SENSITIVITY TO TAIL
- BACKGROUND EFFECTS
- GENERALIZATION TO LONGER CHAIN SEEMS HARD
- BOOST DISTRIBUTION IS COMPLETELY IRRELEVANT

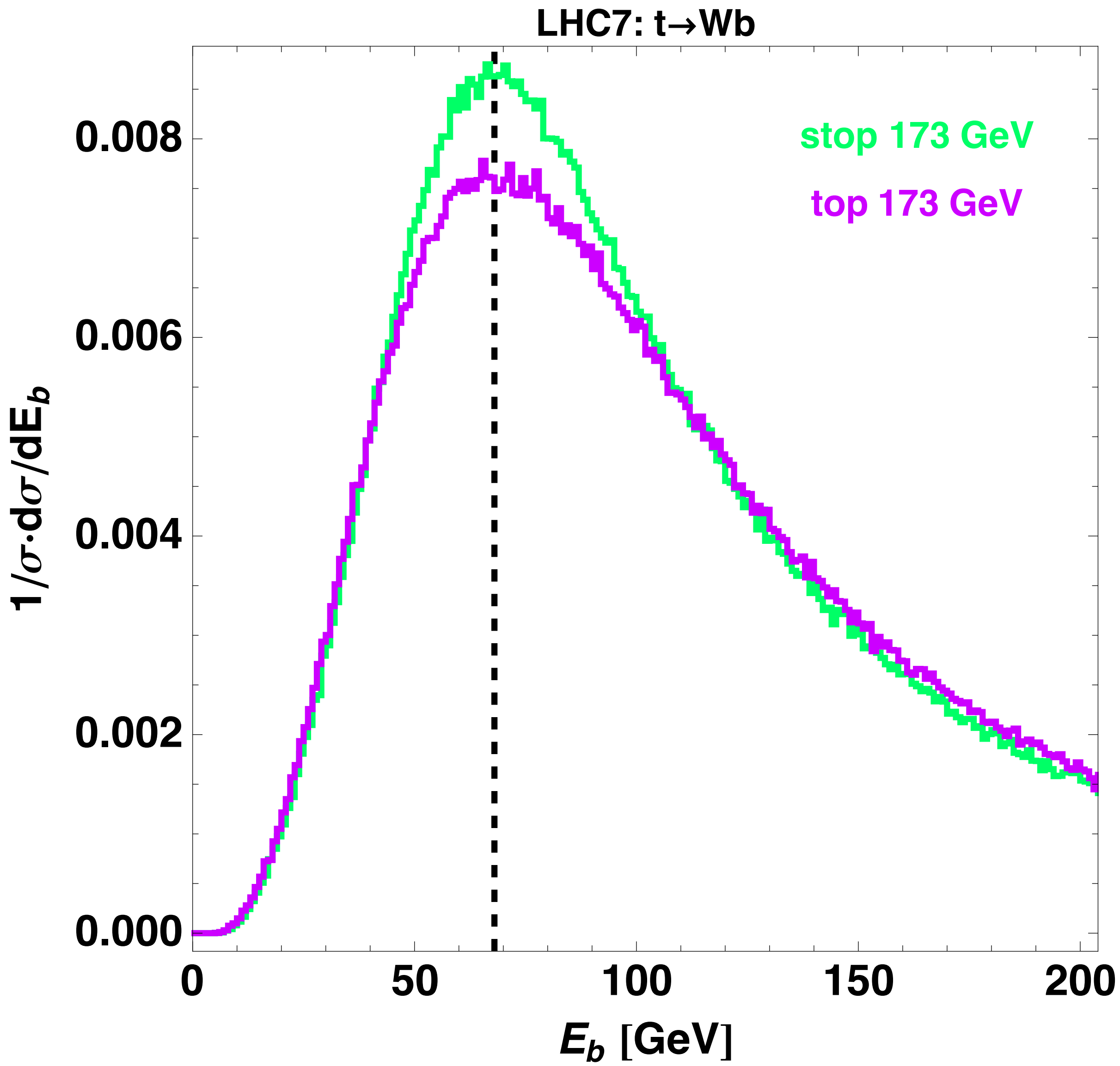
## ENERGY PEAK

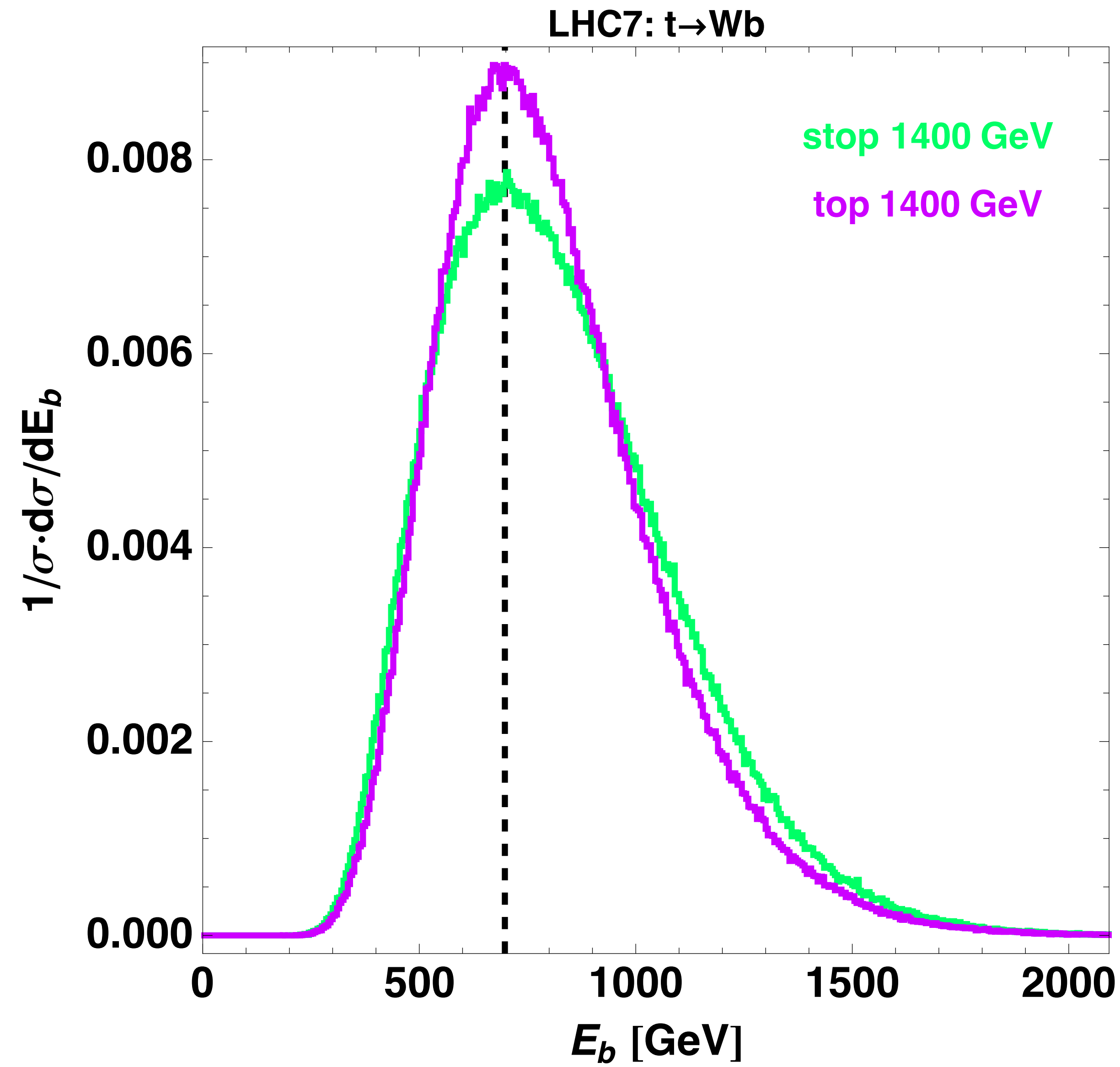
- LOCAL FEATURE
- MULTIPLE PEAKS SEARCH
- SOME DEPENDENCE ON BOOST DISTRIBUTION (PEAK, KINK, PLATEAU)

# How special is this invariance?



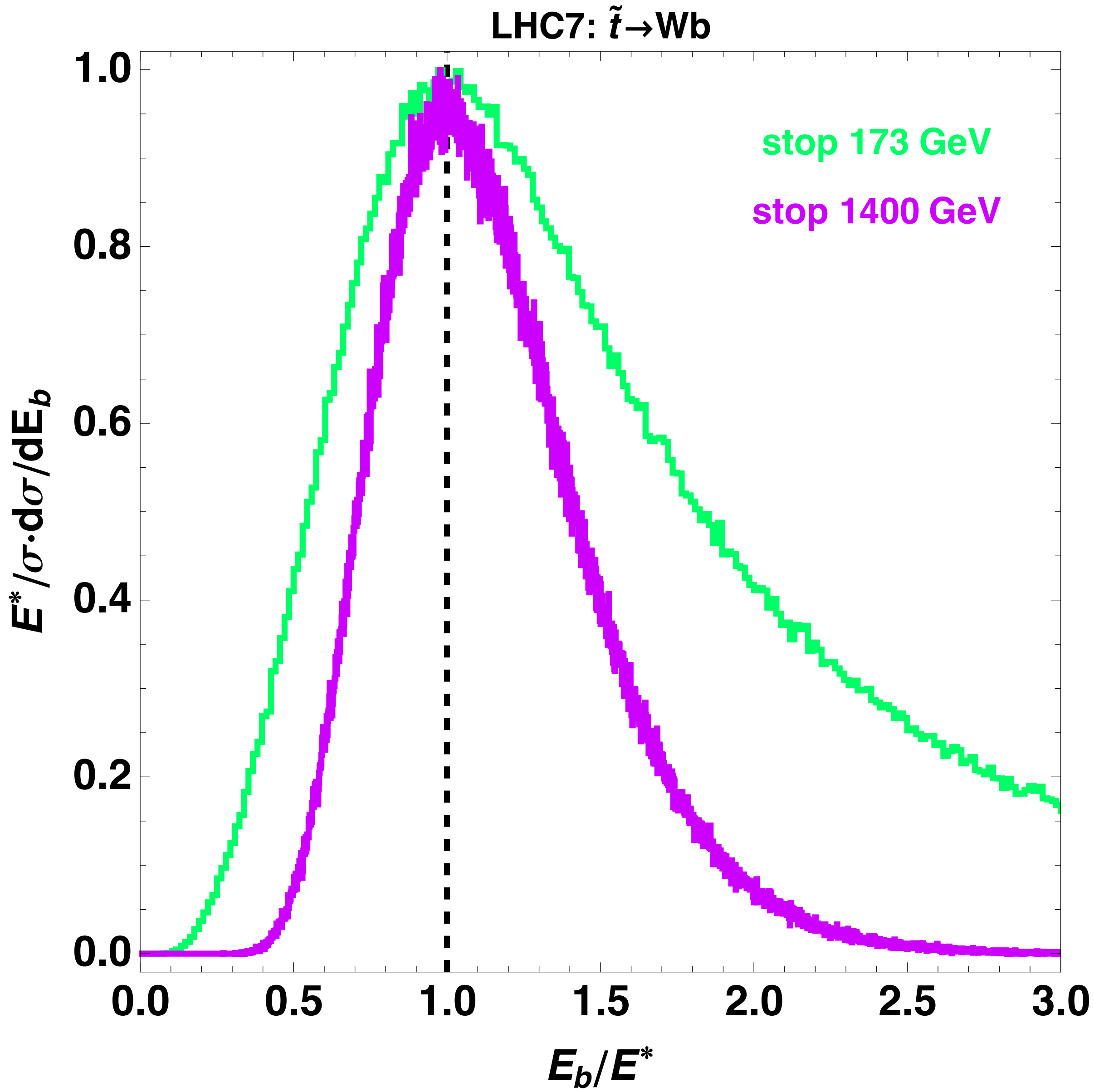
# Peak is invariant, what about the shape?



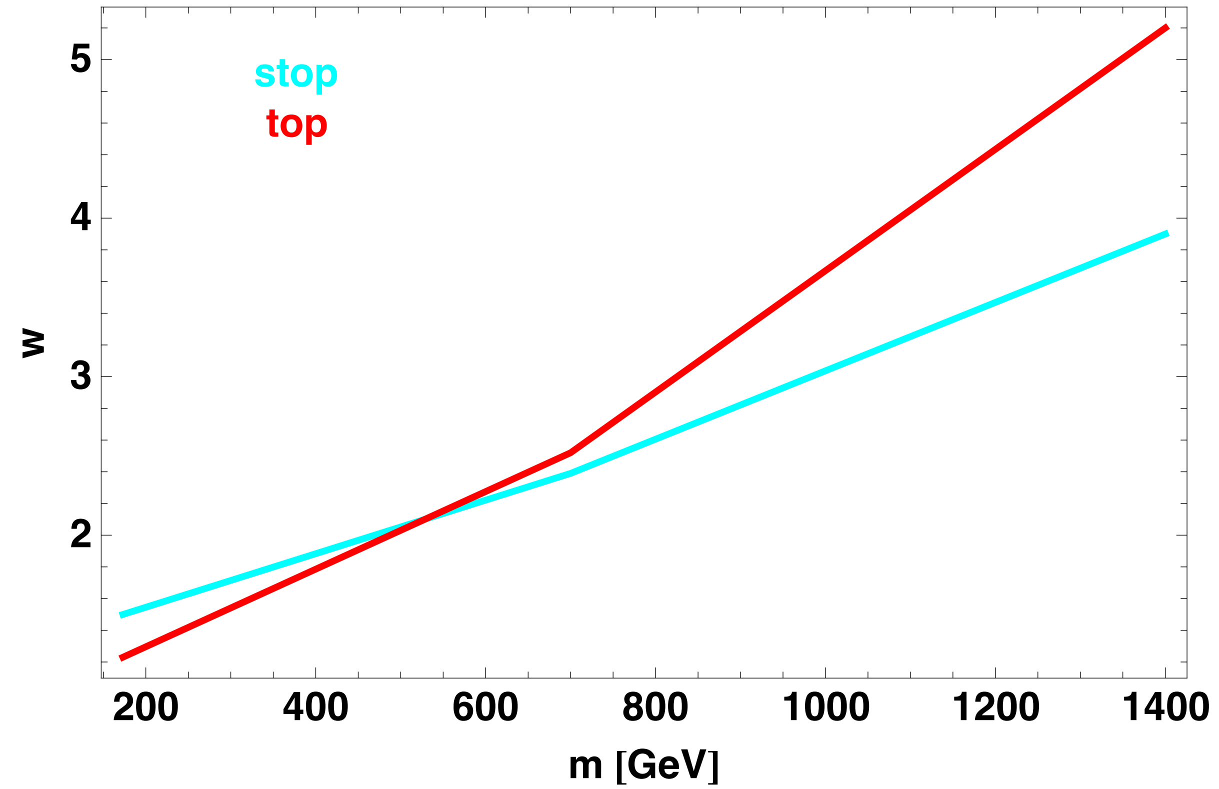

captures the peak for both stop and top: pure kinematics

# Shape of the energy spectrum




Exp(w(x+1/x))

w~average boost of the top




Can contribute to measure the spin? (is NLO under control?)

Can contribute to measure the PDFs?

# Side-by-Side

top pair production at hadron colliders

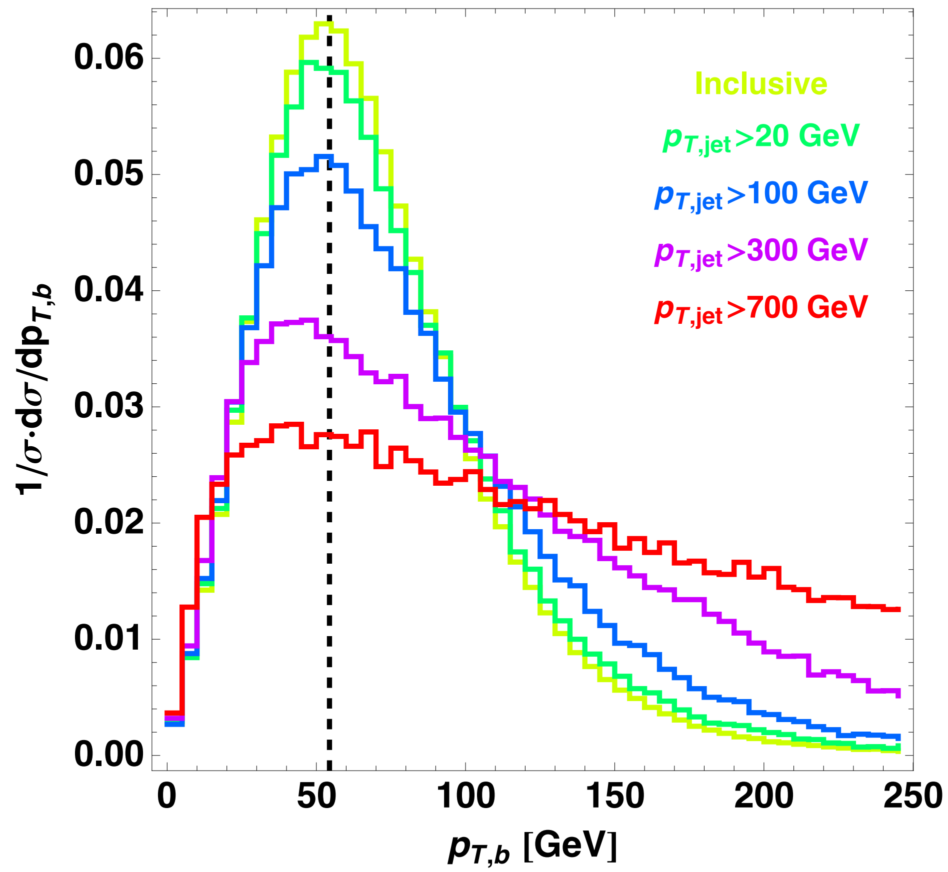
disclaimer:
pictures may be quite different for heavy new physics

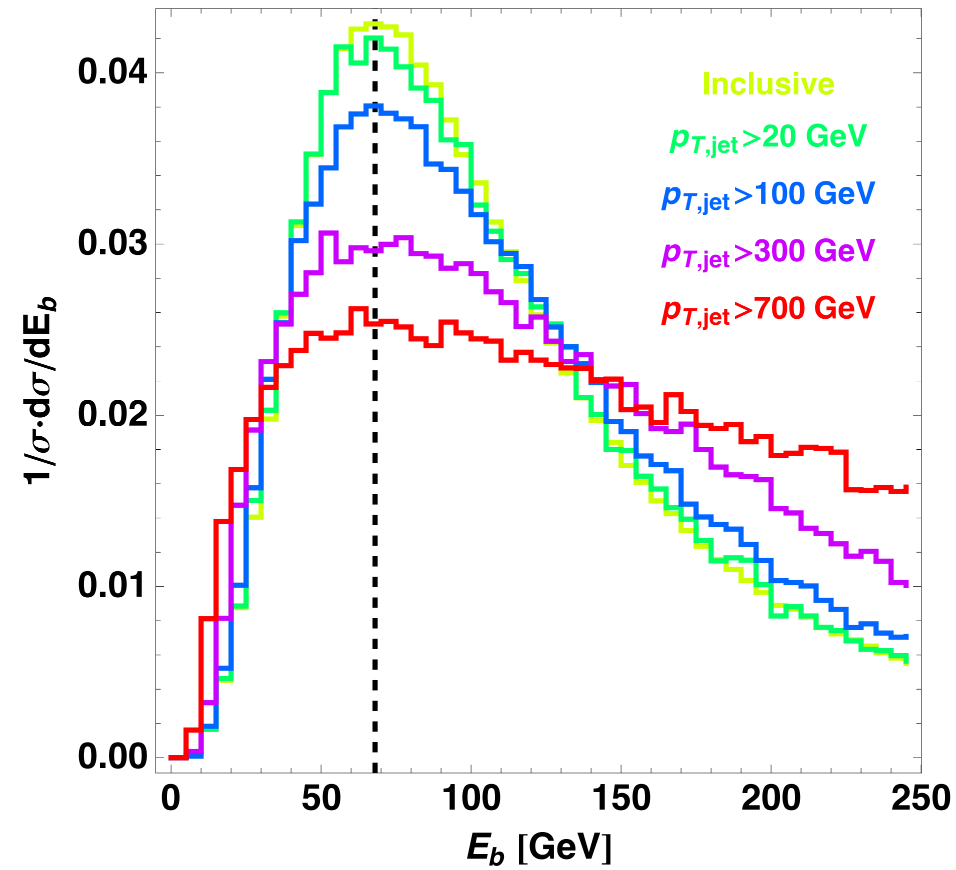
# Effect of ISR (LHC 7 TeV)

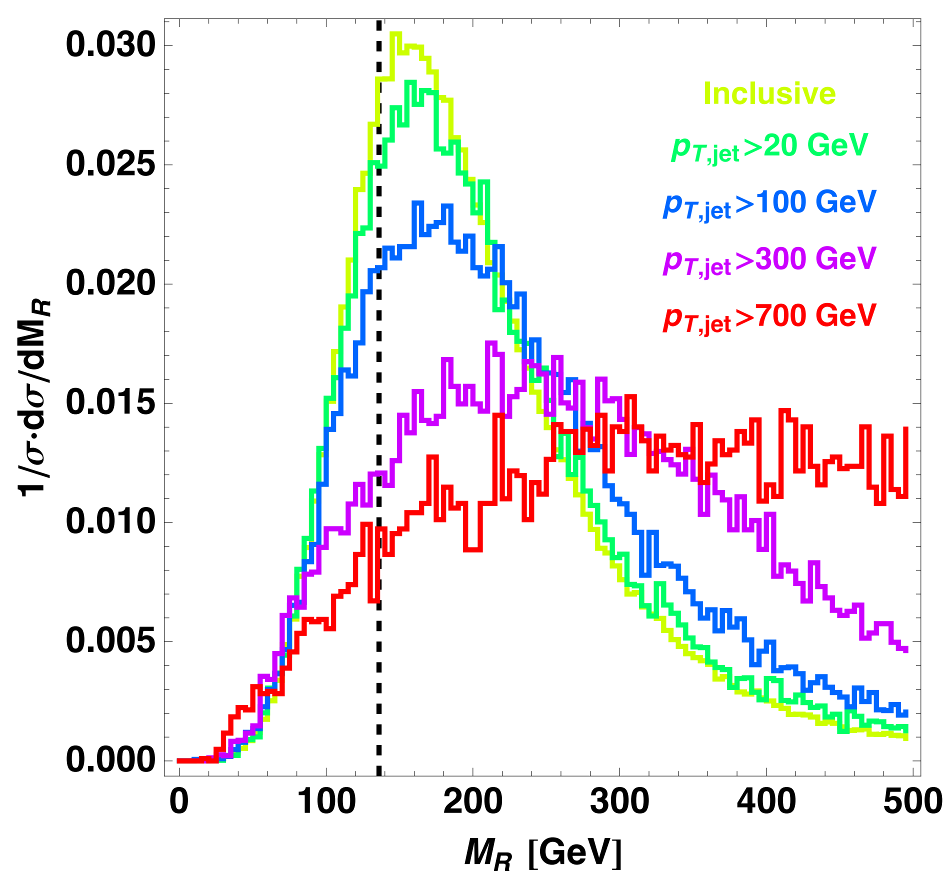
ISR messes up assumptions on the kinematics of the productions mechanism

Energy peak is invariant as long as:
1) there is an on-shell top
2) decay into 2 bodies

# Effect of ISR (LHC 7 TeV)




peak stability

# Sharpness of the peak

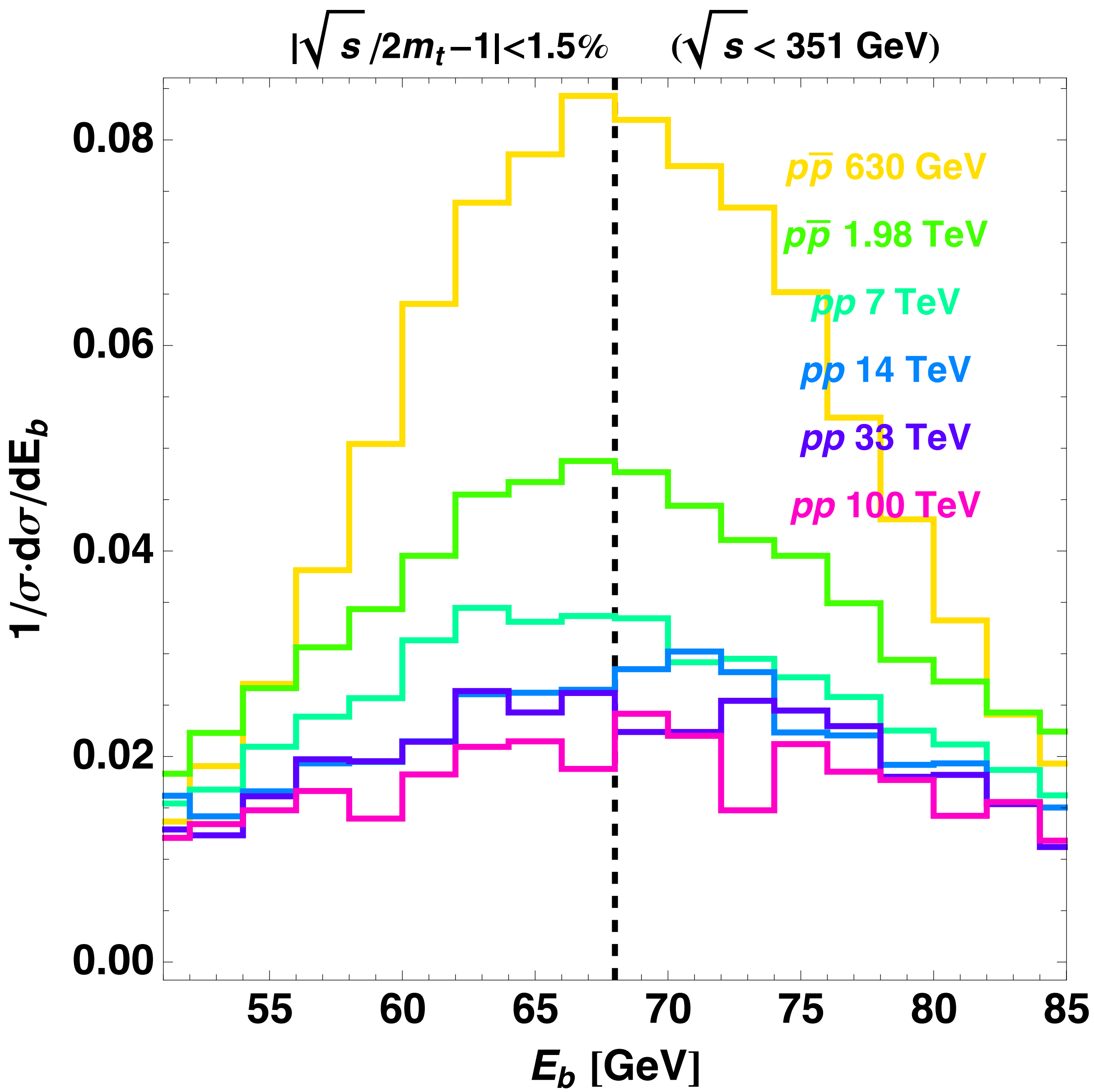
| | | |
|---|---|---|
| top and antitop at rest in the lab | → | top and antitop at rest in the tt-CoM |
| E~peak | → | MR~peak |

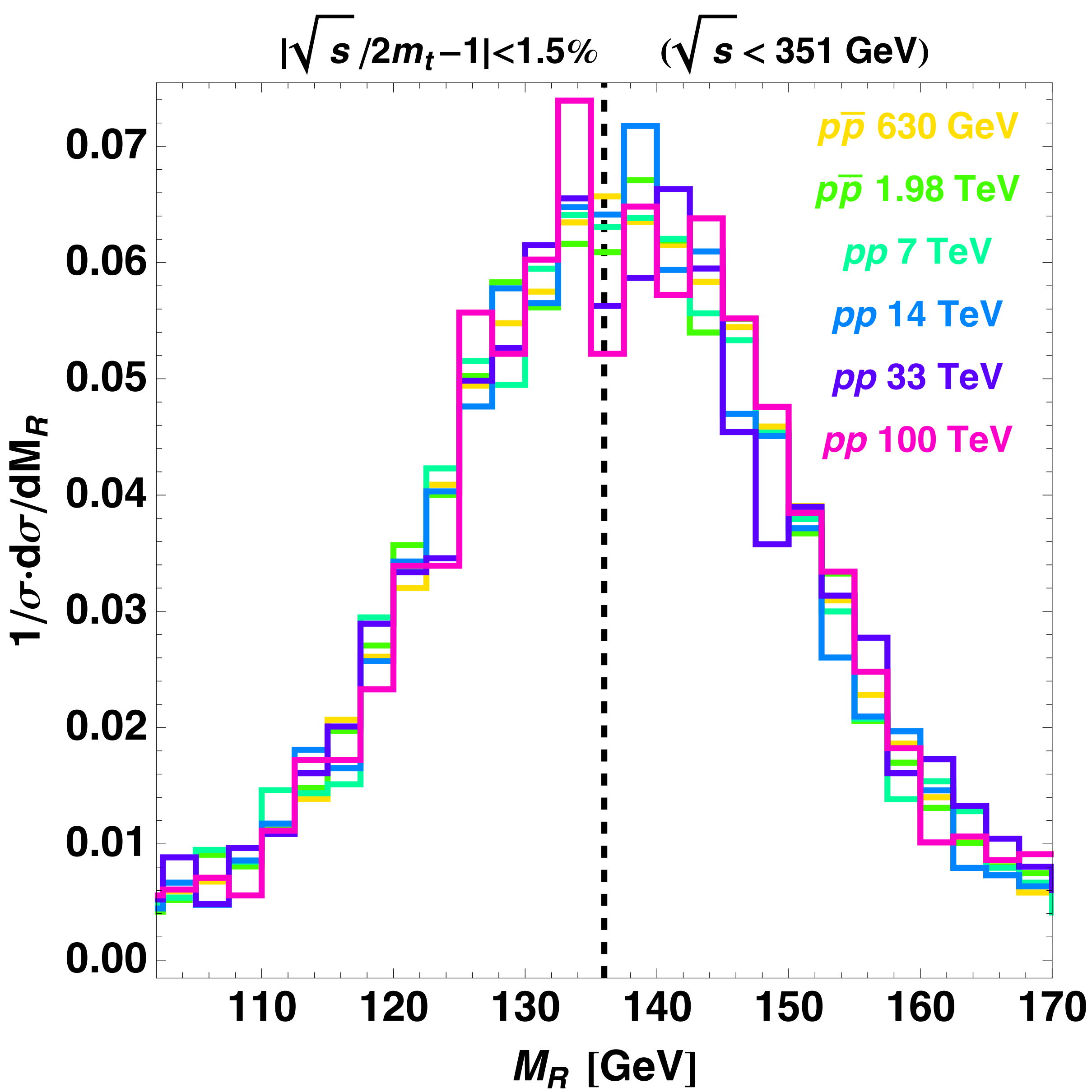
not true the converse!
the tt-CoM can still move in the Lab

↓

1) MR~peak has more phase space than E~peak
2) s~2m selects events for which MR is sharper than E

# Sharpness of the peak



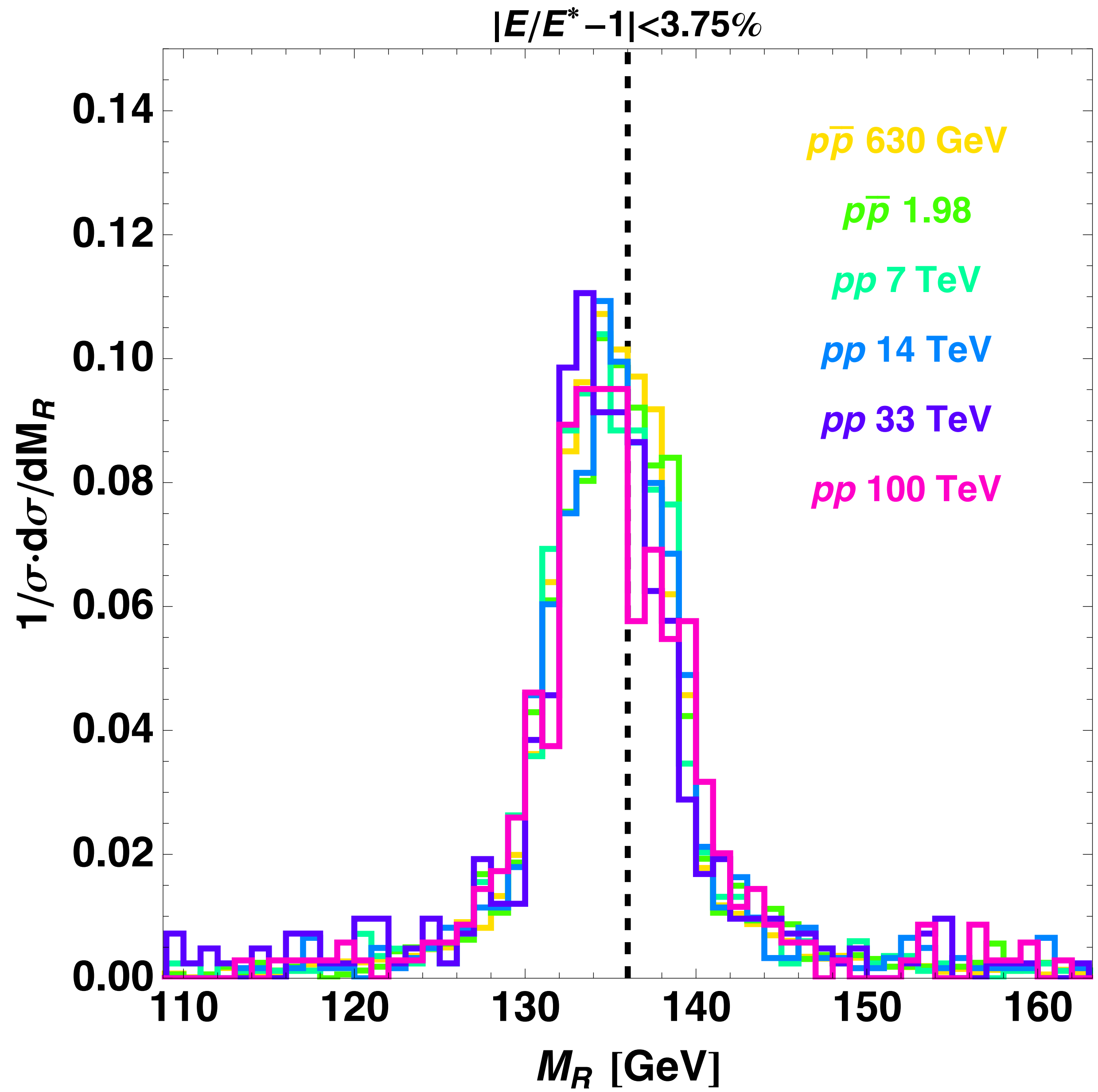

peak sharpness

# What is in the peak of the other?

Energy close to the peak

↓

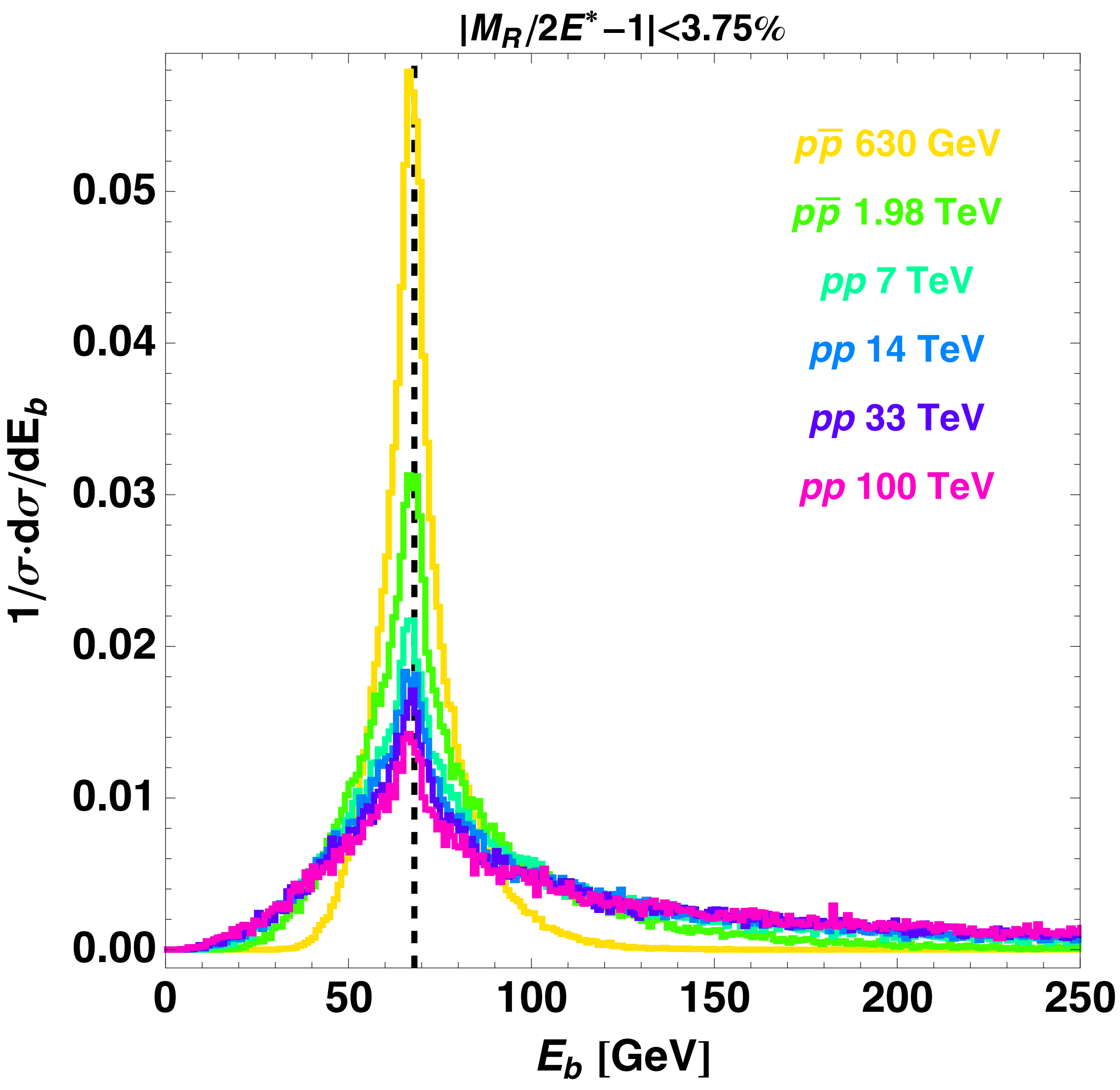
FWHM~dE

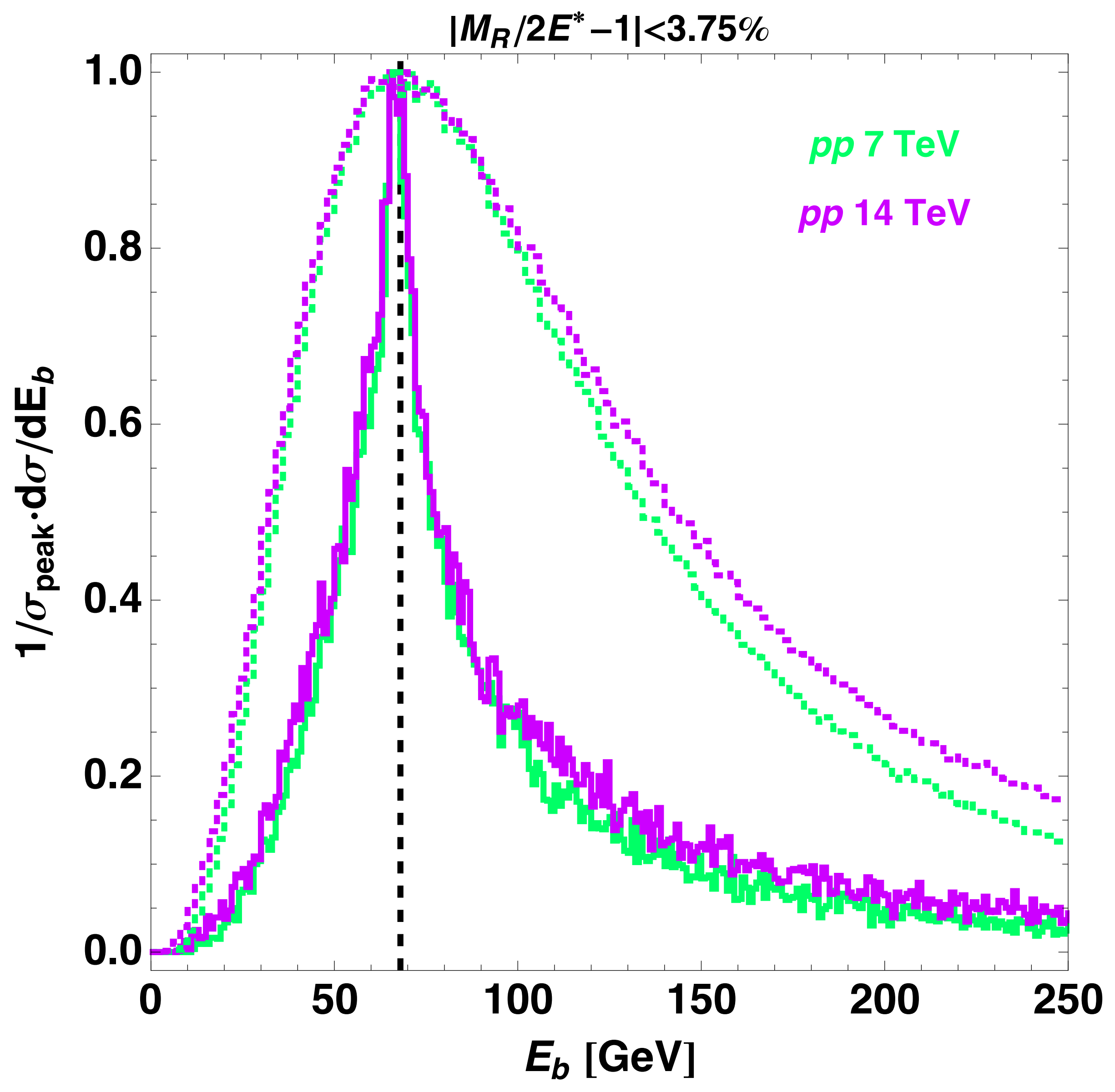


# What is in the peak of the other?

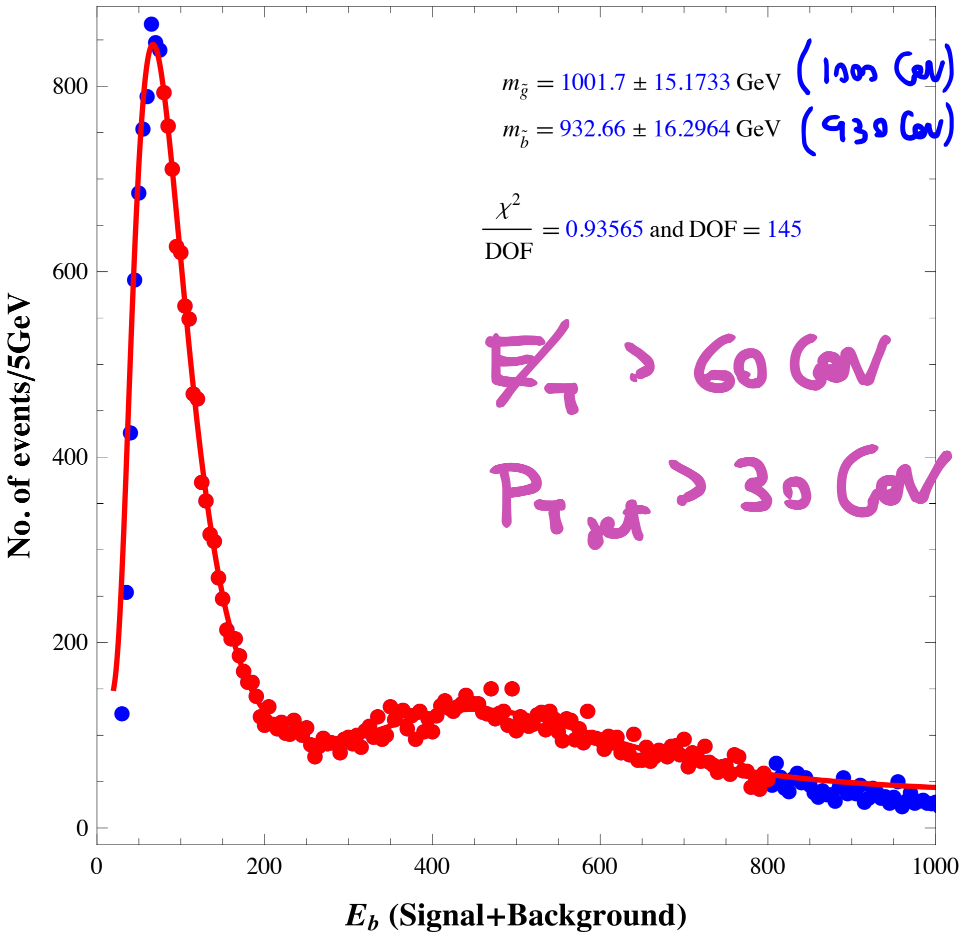
MR close to the peak → Energy peak more sharp

Why?

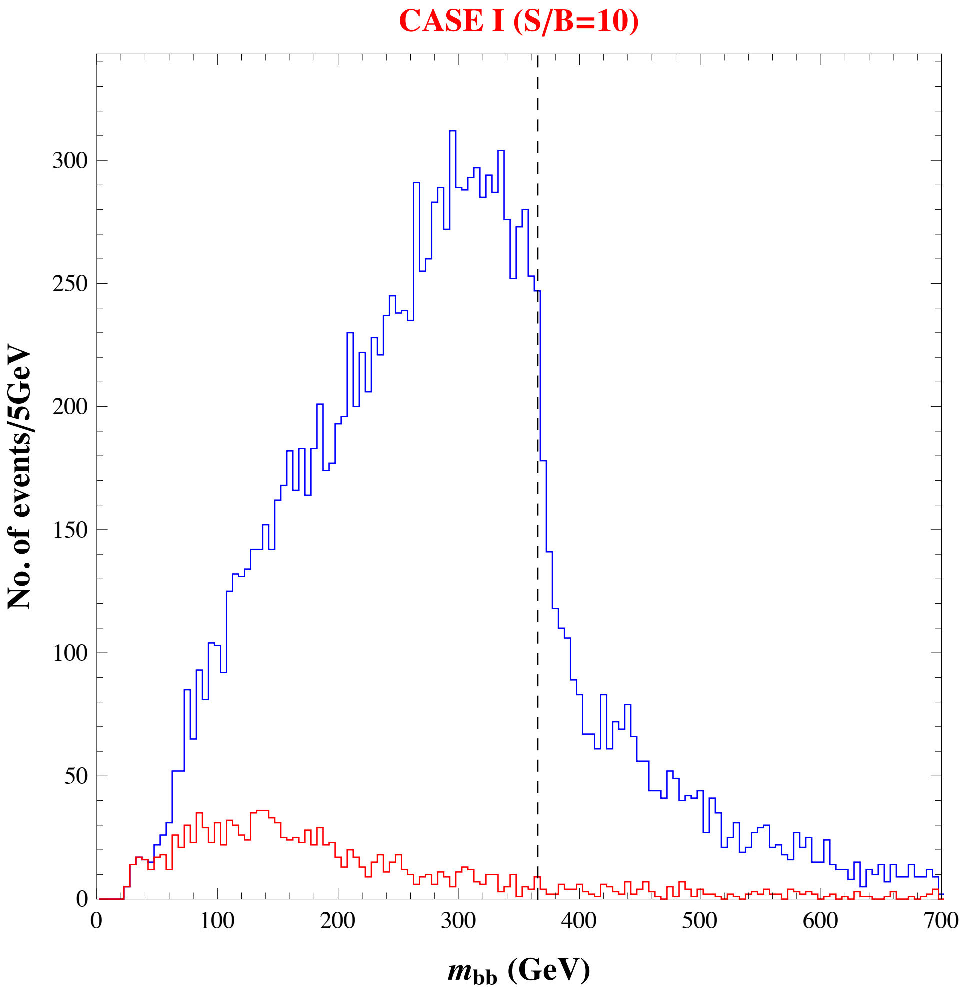


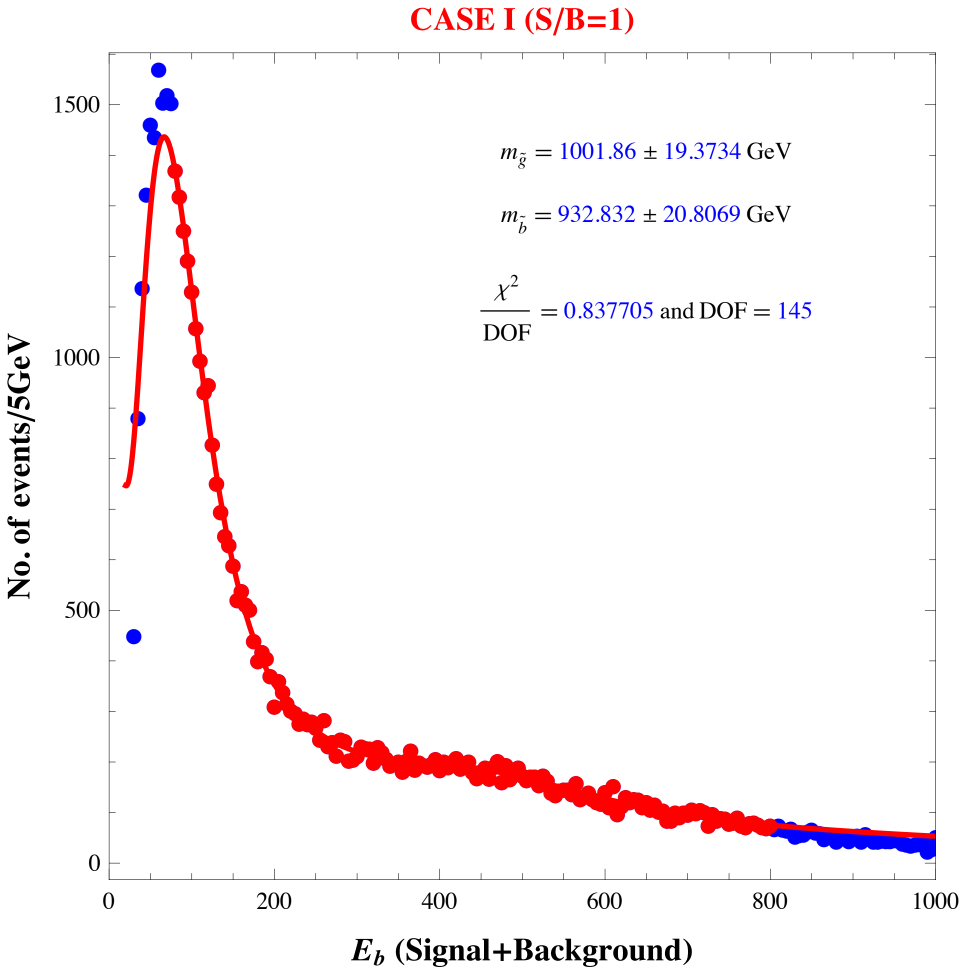
CASE I (S/B=10)

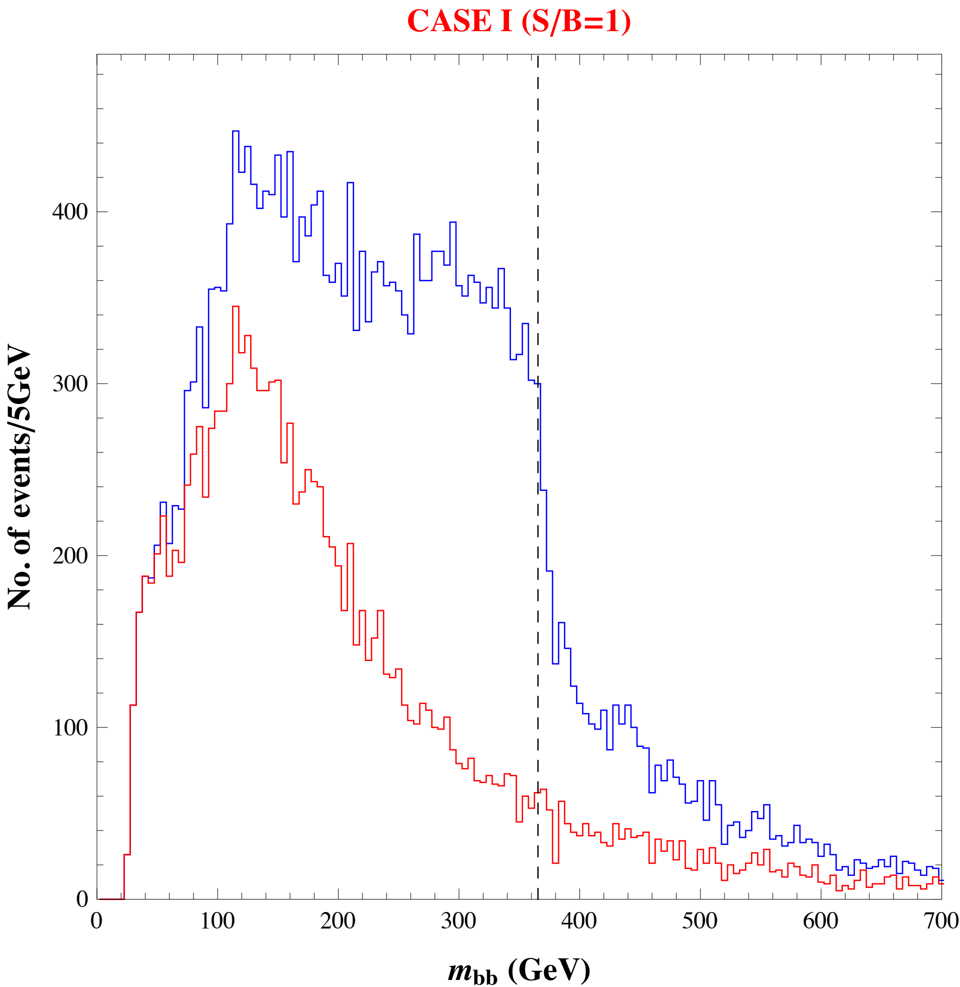


$m_{\tilde{g}} = 1001.7 \pm 15.1733$ GeV
$m_{\tilde{b}} = 932.66 \pm 16.2964$ GeV
$\frac{\chi^2}{\text{DOF}} = 0.93565$ and DOF = 145
No. of events/5GeV
$E_b$ (Signal+Background)
0
200
400
600
800
1000



CASE I (S/B=10)
No. of events/5GeV
$m_{bb}$ (GeV)
0
50
100
150
200
250
300
0
100
200
300
400
500
600
700


CASE I (S/B=1)
$m_{\tilde{g}} = 1001.86 \pm 19.3734$ GeV
$m_{\tilde{b}} = 932.832 \pm 20.8069$ GeV
$\frac{\chi^2}{DOF} = 0.837705$ and DOF = 145
No. of events/5GeV
$E_b$ (Signal+Background)
0
200
400
600
800
1000
500
1500


CASE I (S/B=1)
No. of events/5GeV
0
100
200
300
400
0
100
200
300
400
500
600
700
$m_{bb}$ (GeV)