Standard Model and New Physics contributions to K_L and K_S into four leptons

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Workshop on Rare Strange Decays at LHCb – CERN

The recent LHCb measurement on $K_S \to \mu\overline{\mu}$ is getting closer to the Standard Model (SM) prediction

Br $(K_S \to \mu\overline{\mu})\big|_{\text{LHCb}} < 9 \times 10^{-9}$ at 90% CL   Br $(K_S \to \mu\overline{\mu})\big|_{\text{SM}} = (5.0 \pm 1.5) \times 10^{-12}$

This has motivated our interest in studying other feasible decays at LHC or other facilities for

$$K_{L,S} \to \mu\overline{\mu}\mu\overline{\mu}\ ,\ K_{L,S} \to e\overline{e}\mu\overline{\mu}\ ,\ K_{L,S} \to \ e\overline{e}e\overline{e}$$

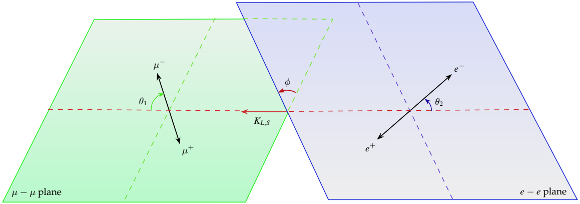
- We have introduced a form factor, motivated by **V**ector **M**eson **D**ominance.
- The measurement of the time interference of $\mathcal{A}(K_L \to \ell\bar{\ell}\ell\bar{\ell})$ with $\mathcal{A}(K_S \to \ell\bar{\ell}\ell\bar{\ell})$ would allow the determination of the sign of $\mathcal{A}(K_L \to \gamma\gamma)$. This experimental determination is very welcome for CKM.
- Two possible New Physics (NP) models that can be studied by measuring measurements of $\mathcal{A}(K_{L,S} \to \ell\bar{\ell}\ell\bar{\ell})$:
  1. a direct NP coupling for $K_L\gamma\gamma$.
  2. a Bremsstrahlung part from $K_{L,S} \to \mu\overline{\mu}$.

The Cabibbo-Maksymovych approach

N. Cabibbo and A. Maksymovicz, Phys. Rev. B 137, 438-443 (1965)

Decay width

$$\mathrm{d}\Gamma(K_{L,S} \to \ell_1\bar{\ell}_1\ell_2\bar{\ell}_2) = \frac{(2\pi)^{-8}}{2M_K}|\mathcal{M}|^2\mathrm{d}\Phi_4$$



$$\mathrm{d}\Phi_4 = \frac{\pi^2}{2^6 M_K^2}\,\sigma_1\,\sigma_2\,\lambda^{1/2}(M_K^2, q_1^2, q_2^2)\,\mathrm{d}q_1^2\,\mathrm{d}q_2^2\,\mathrm{d}(\cos\theta_1)\,\mathrm{d}\phi\,\mathrm{d}(\cos\theta_2),$$

with

$$\sigma_i = \left(1 - 4\frac{m_i^2}{q_i^2}\right)^{1/2}, \quad i = 1, 2$$

$$\lambda(x, y, z) = (x - y - z)^2 - 4yz$$

$$\begin{cases} 4m_\mu^2 \leq q_1^2 \leq (M_K - 2m)^2 \\ 4m^2 \leq q_2^2 \leq (M_K - \sqrt{q_1^2})^2 \\ 0 \leq \theta_1, \theta_2 \leq \pi \\ 0 \leq \phi \leq 2\pi \end{cases}$$

$K_L \to \ell\bar{\ell}\ell\bar{\ell}$

G. D' Ambrosio, G. Isidori and J. Portolés, Phys. Rev. B 423, 385–394 (1998)

Analysis from $K_L \to \mu^+\mu^-$

The **S**hort **D**istance and **L**ong **D**istance constraints drive to a form factor inspired by **V**ector **M**eson **D**ominance

$$\mathrm{F}_L(q_1^2, q_2^2) \doteq \mathrm{F}_L(0,0)\left[1 + \boldsymbol{\alpha_L}\left(\frac{q_1^2}{q_1^2 - M_V^2} + \frac{q_2^2}{q_2^2 - M_V^2}\right) + \boldsymbol{\beta_L}\frac{q_1^2 q_2^2}{(q_1^2 - M_V^2)(q_2^2 - M_V^2)}\right]$$

with the condition

$$1 + 2\boldsymbol{\alpha_L} + \boldsymbol{\beta_L} = 0.3$$

$K_L \to \ell\bar{\ell}\ell\bar{\ell}$

$$\mathcal{A}\left(K_L \to \gamma^*\gamma^*\right) = i\varepsilon_{\mu\nu\rho\sigma} q_1^\rho q_2^\sigma\ \epsilon_1^\mu\ \epsilon_2^\nu\ \mathrm{F}_L(q_1^2, q_2^2)$$

with the effective lagrangian $\mathcal{L}_{\rm eff} = -\frac{\mathrm{F}_L(0,0)}{8}\ \varepsilon_{\mu\nu\rho\sigma}\ K_2 F^{\mu\nu} F^{\rho\sigma}$, one fixes

$$|\mathrm{F}_L(0,0)| = \left[\frac{64\pi\,\Gamma(K_L \to \gamma\gamma)}{M_K^3}\right]^{1/2} = (5.61 \pm 0.06) \times 10^{-9}\ \mathrm{GeV}^{-1}$$

From $K_L \to \gamma\gamma^*$ experimental decay, one has $\boldsymbol{\alpha_L} = -1.69 \pm 0.08$.

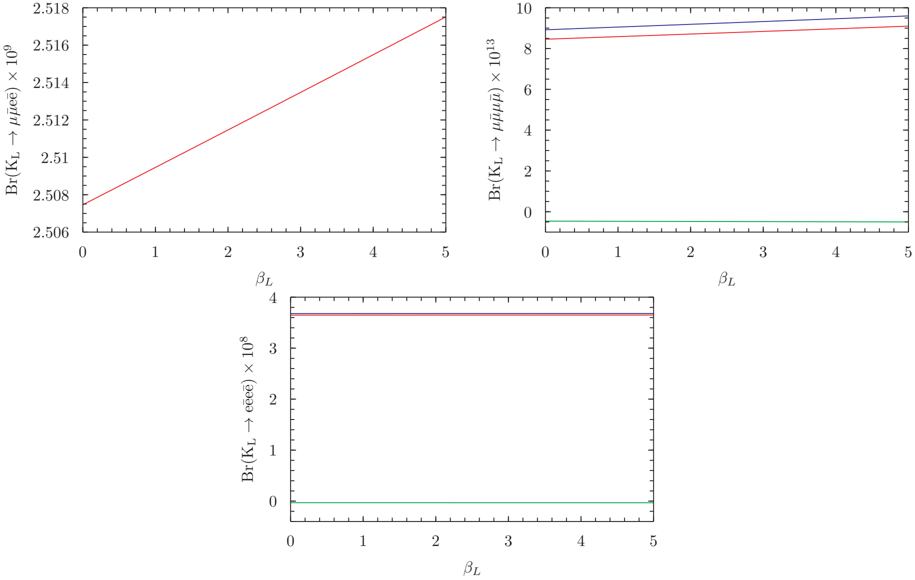
$K_L \to \ell\bar{\ell}\ell\bar{\ell}$

T. Miyazaki and E. Takasugi, Phys. Rev. D 8, 2051–2062 (1973)

J. Beringer *et al.* (Particle Data Group), Phys. Rev. D 86, 010001 (2012)

	$\alpha_L = \beta_L = 0$		$\alpha_L = -1.63$ $\beta_L = 0.3 - 1 - 2\alpha_L$	Experiment
	This work	Miyazaki *et al.*	This Work	PDG
$K_L \to \mu\bar{\mu}\mu\bar{\mu}$	4.82×10^{-13}	5.17×10^{-13}	8.78×10^{-13}	
$K_L \to e\bar{e}e\bar{e}$	3.40×10^{-8}	3.22×10^{-8}	3.65×10^{-8}	$(3.56 \pm 0.21) \times 10^{-8}$
$K_L \to \mu\bar{\mu}e\bar{e}$	1.55×10^{-9}	7.77×10^{-10}	2.51×10^{-9}	$(2.69 \pm 0.27) \times 10^{-9}$

$K_L \to \ell\bar{\ell}\ell\bar{\ell}$



$K_S \to \ell\bar{\ell}\ell\bar{\ell}$

G. Ecker and A. Pich, Nucl. Phys. B **366**, 189 (1991)

Analysis from $K_S \to \gamma^*\gamma^*$

The first non-trivial ChPT contribution to $K_S \to \gamma^*\gamma^*$ appears at $\mathcal{O}(p^4)$ (the chiral loops are finite then). We consider two important $\mathcal{O}(p^6)$ effects:

1. We need to add local $\mathcal{O}(p^6)$ contributions to the $\mathcal{O}(p^4)$ $K_S \to \gamma\gamma$ to match exactly the experimental value.
2. Potentially important VMD contribution $\mathcal{O}(p^6)$ to $K_S \to \gamma^*\gamma^*$ generated by the $\mathcal{O}(p^4)$ electromagnetic form factor of the pion.

$K_L \to \ell\bar{\ell}\ell\bar{\ell}$

One has

$$\mathcal{A}(K_S \to \gamma^*\gamma^*) = i\left[(q_1 \cdot q_2)g^{\mu\nu} - q_2^\mu q_1^\nu\right] \epsilon_{1\mu}\, \epsilon_{2\nu}\, \mathrm{F}_S(q_1^2, q_2^2)$$

where experimentally,

$$|\mathrm{F}_S(0,0)| = \left[\frac{64\pi\,\Gamma(K_S \to \gamma\gamma)}{M_K^3}\right]^{1/2} = (3.38 \pm 0.03) \times 10^{-9}\ \text{GeV}^{-1}.$$

$K_S \to \ell\bar{\ell}\ell\bar{\ell}$

G. Buchalla, G. D'Ambrosio and G. Isidori, Nucl. Phys. B **672**, 387 (2003)
G. D'Ambrosio and D. Espriu, *Phys. Lett.* **B 175** 237 (1986)
J.L. Goity, *Z. Phys.* **C 34,** (1987) 341
G. D'Ambrosio, G. Isidori and J. Portolés, Phys. Rev. B 423, 385-394 (1998)

Form Factor properties

1. Factorising the pion loop effects in $K_S \to \gamma^*\gamma^*$.
2. Including a large contribution from VMD and unitarity at order $\mathcal{O}(p^6)$.

$$\mathrm{F}_S(q_1^2, q_2^2) \doteq \mathrm{F}_S(0,0)\left[1 + \alpha_S\left(\frac{q_1^2}{q_1^2 - M_V^2} + \frac{q_2^2}{q_2^2 - M_V^2}\right) + \beta_S \frac{q_1^2 q_2^2}{(q_1^2 - M_V^2)(q_2^2 - M_V^2)}\right]$$

with the exact SD constraint

$$1 + 2\alpha_S + \beta_S = 0$$

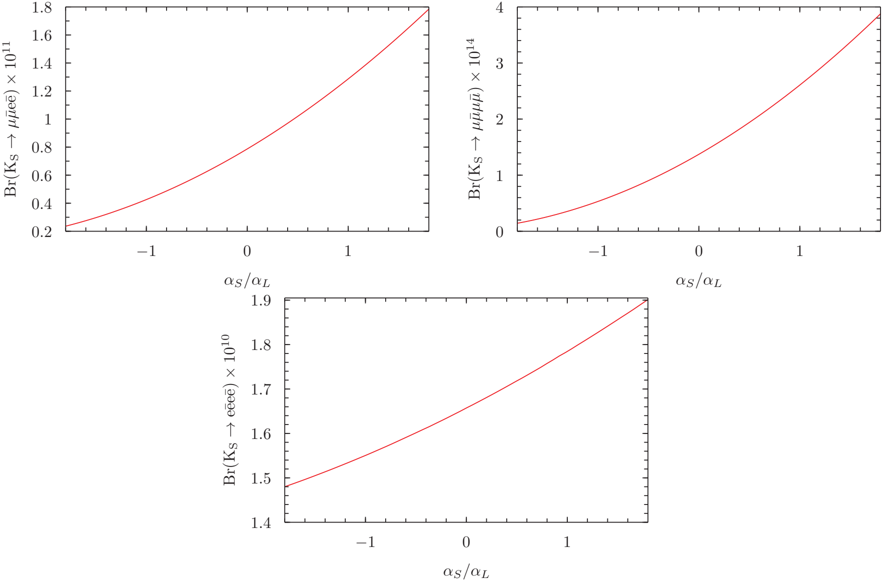
$K_S \to \ell\bar{\ell}\ell\bar{\ell}$

W. Birkfellner, Diploma Thesis, Univ. of Vienna (1996)

	This work			Birkfellner
	$\alpha_S = 0$, $\beta_S = 0$	$\alpha_S = 0$, $\beta_S = -1 - 2\alpha_S$	$\alpha_S = \alpha_L$, $\beta_S = -1 - 2\alpha_S$	
$K_S \to \mu\bar{\mu}\mu\bar{\mu}$	1.40×10^{-14}	1.37×10^{-14}	2.61×10^{-14}	1×10^{-14}
$K_S \to e\bar{e}e\bar{e}$	1.66×10^{-10}	1.66×10^{-10}	1.78×10^{-10}	7×10^{-11}
$K_S \to \mu\bar{\mu}e\bar{e}$	7.88×10^{-12}	7.87×10^{-12}	1.29×10^{-11}	8×10^{-12}

$K_S \to \ell\bar{\ell}\ell\bar{\ell}$

$(\alpha_S = -1.63)$



CP conserving interferences

G. D'Ambrosio *et al.* Phys. Rev. D 50, 5767–5774 (1994)

P. Heiliger and L. M. Sehgal, Phys. Rev. D **48** (1993) 4146

Since the K_L and K_S are composite systems in the point of view of CP violation,

$$\begin{pmatrix} K_S \\ K_L \end{pmatrix} = \frac{1}{\sqrt{1+|\varepsilon|^2}} \begin{pmatrix} 1 & \varepsilon \\ \varepsilon & 1 \end{pmatrix} \begin{pmatrix} K_1 \\ K_2 \end{pmatrix} = \frac{1}{\sqrt{2(1+|\varepsilon|^2)}} \begin{pmatrix} 1 & \varepsilon \\ \varepsilon & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} K_0 \\ \bar{K}^0 \end{pmatrix},$$

with Re $\varepsilon = 1.66 \times 10^{-3}$ and Im $\varepsilon = 1.57 \times 10^{-3}$.

Asymmetry measurement in CP conserving part

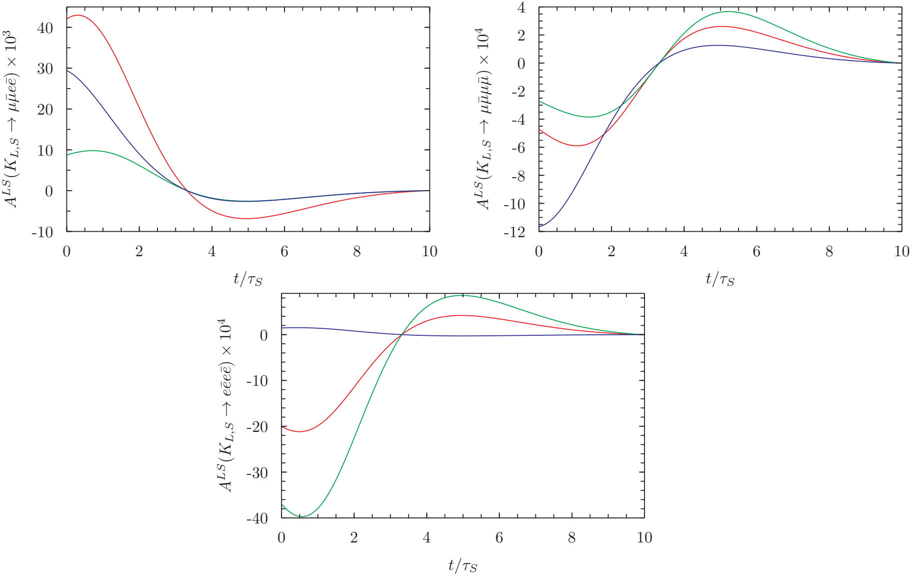
$$A^{LS}(t) = \frac{2\mathrm{e}^{-\Gamma t} \displaystyle\int \mathrm{d}\Phi_4 f(X,Y)\, \mathrm{Re} A_L\, \mathrm{Re} A_S}{\displaystyle\int \mathrm{d}\Phi_4 \left[\mathrm{e}^{-\Gamma_S t}|A_S|^2 + \mathrm{e}^{-\Gamma_L t}|A_L|^2\right]} \cos(\Delta M\, t),$$

for some weight function $f(X,Y)$ here $f(X,Y) \doteq \mathrm{sgn}(\cos\phi \sin\phi)$, and

$$\Delta M = M_L - M_S \text{ and } \Gamma = \frac{1}{2}(\Gamma_L + \Gamma_S)\,.$$

CP conserving interferences

$\alpha_S = -3$, $\alpha_S = 0$, $\alpha_S = 3$



New Physics contributions

$$A_S = |A_1|e^{i\delta} + \varepsilon|A_2|e^{i\delta'} + i|A_1^B| + \boxed{i|A_1'|e^{i\delta'}}$$

- $|A_1|e^{i\delta}$ is the K_S CP conserving part. A_1 is just the SM amplitude and δ is related to the absorptive part of $\mathcal{A}(K_S \to \pi\pi \to \gamma^*\gamma^*)$:

$$\arctan\delta = \frac{\mathrm{Im}\,\mathcal{A}(K_S \to \pi\pi \to \gamma^*\gamma^*)}{\mathrm{Re}\,\mathcal{A}(K_S \to \pi\pi \to \gamma^*\gamma^*)},$$

  with $\delta \approx -27.54^\circ \approx -0.48065$.
- $\varepsilon|A_2|e^{i\delta'}$ is the K_L CP-violating part. A_2 is the SM amplitude and ε parametrizes the indirect CP violation and δ':

$$\arctan\delta' = \frac{\mathrm{Im}\,\mathcal{A}(K_L \to \mu\bar{\mu})}{\mathrm{Re}\,\mathcal{A}(K_L \to \mu\bar{\mu})},$$

  We get $\delta' \approx -82.48^\circ \approx -1.43952$.
- $i|A_1^B|$ is the K_S Bremsstrahlung CP-violating part.
- $|A_1'|e^{i\delta'}$ is the K_S CP-violating part. A_1' is a CP-violation part of the K_S coming from $K_S\,\varepsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$. Therefore

$$A_1' \doteq \xi\, A_2$$

  with $\xi \sim 10^{-1}$. δ' is the same as for the second term due to universality. This constitutes our NP implementation.

NP contributions to CP violation interferences

Interferences measurement

$$A^{LS}(t) = \frac{e^{-\Gamma t} \int_0^{\phi_0} d\phi \int \frac{d\Phi_4}{d\phi} \, \mathrm{sgn}(\cos\phi \sin\phi) \left[\mathrm{Re}(A_L A_S^*) \cos(\Delta M \, t) + \mathrm{Im}(A_L A_S^*) \sin(\Delta M \, t) \right]}{\int_0^{\phi_0} d\phi \int \frac{d\Phi_4}{d\phi} \left[e^{-\Gamma_S t} |A_S|^2 + e^{-\Gamma_L t} |A_L|^2 \right]}$$

where it is maximized for $\phi_0 = \pi$ for $K_S \to \bar{e}e\bar{e}e$, $K_S \to \bar{\mu}\mu\bar{\mu}\mu$ and $\phi_0 = \pi/2$ for $K_S \to \bar{\mu}\mu\bar{e}e$.

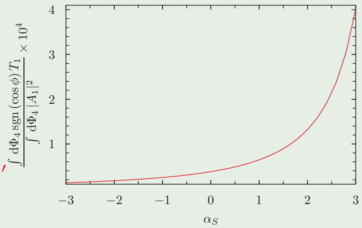
Dominant contribution

Dominant NP contribution to $K_S \to 4\mu$:

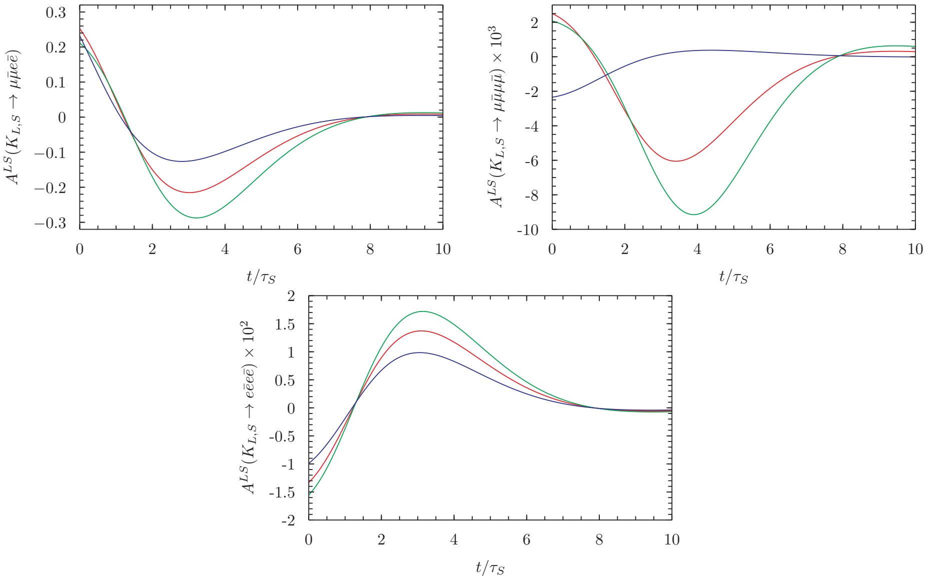
$$T_1 = \left(|\varepsilon|^2 + \xi^2 + 2\xi \, \mathrm{Im} \, \varepsilon \right) |A_2|^2$$

We will consider only:

$$A_L = |A_2|,$$
$$A_S = |A_1| e^{i\delta} + i|A_1'| e^{i\delta'} = |A_1| e^{i\delta} + i\xi |A_2| e^{i\delta'}$$



NP contributions to CP violation interferences



Conclusions

- We have shown that it is possible to obtain good predictions for the branching ratios for the decays of the K_L into four leptons comparing to the experimental data through a VMD inspired form factor. It is natural then to consider that the same approach is pertinent for the case of the K_S into four leptons, since the model is more constrained from short distance behaviour.

- A direct consequence of the experimental data in our approach would be to fix the α and β parameters for K_L and K_S and then give the sign of $\mathcal{A}(K_L \to \gamma\gamma)$. Moreover, we have shown that a simple assumption on the existence of a NP operator in the lagrangian could be verified with interferences in those decays.

- It appears that now it is important to obtain some experimental data in these channels involving the K_S decays (particularly the muons ones) and considering our predictions, we hope that the LHCb processes for tagging the muons allow us to reach a sufficient level of accuracy.