Dispersive Approach to Hadronic Light-by-Light Scattering and the Muon $g-2$

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`arXiv:1402.7081` in collaboration with

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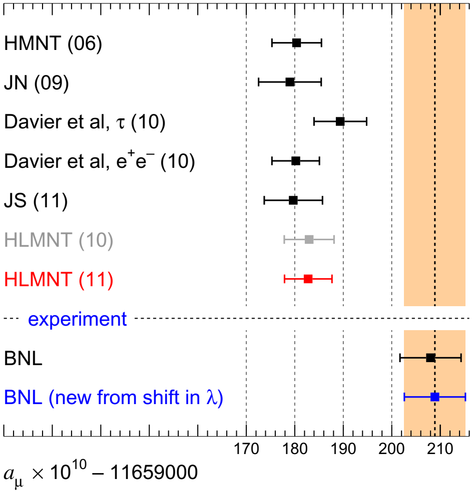
22nd July 2014

Cargèse 2014 International Summer School

$a_\mu = (g_\mu - 2)/2$: theory vs. experiment

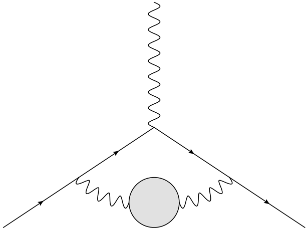


→ Hagiwara et al. 2012

a_μ: theory vs. experiment

- Theory error completely dominated by hadronic effects
- Discrepancy between Standard Model and experiment $\sim 3\sigma$
- Hint to new physics?
- New experiments (FNAL, J-PARC) aim at reducing the experimental error by a factor of 4

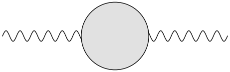
Leading hadronic contribution: $\mathcal{O}(\alpha^2)$



- Problem: QCD is non-perturbative at low energies
- First principle calculations (lattice QCD) may become available in the future
- Current evaluations based on dispersion relations and data

Leading hadronic contribution: $\mathcal{O}(\alpha^2)$

Photon vacuum polarisation function:



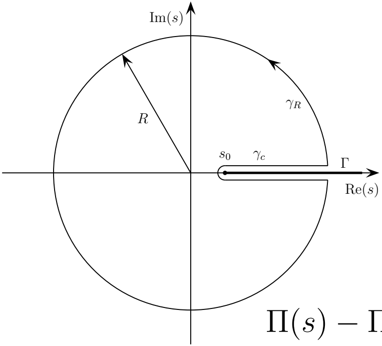
$$= -i(q^2 g_{\mu\nu} - q_\mu q_\nu)\Pi(q^2)$$

Unitarity of the S-matrix implies the optical theorem:

$$\mathrm{Im}\Pi(s) = \frac{s}{e(s)^2}\sigma_{\mathrm{tot}}(e^+e^- \to \gamma^* \to \mathrm{hadrons})$$

Dispersion relation

Causality implies analyticity:



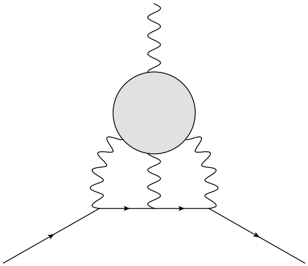
Cauchy integral formula:

$$\Pi(s) = \frac{1}{2\pi i} \oint_\gamma \frac{\Pi(s')}{s' - s} ds'$$

Deform integration path:

$$\Pi(s) - \Pi(0) = \frac{s}{\pi} \int_{4M_\pi^2}^{\infty} \frac{\mathrm{Im}\Pi(s')}{(s' - s - i\epsilon)s'} ds'$$

How to improve HLbL calculation?



- Relate HLbL to experimentally accessible quantities
- Make use of unitarity, analyticity, gauge invariance and crossing symmetry

Mandelstam representation

- We limit ourselves to intermediate states of at most two pions
- Writing down a double-spectral (Mandelstam) representation allows us to split up the HLbL tensor:

$$\Pi_{\mu\nu\lambda\sigma} = \Pi^{\pi^0\text{-pole}}_{\mu\nu\lambda\sigma} + \Pi^{\text{FsQED}}_{\mu\nu\lambda\sigma} + \bar{\Pi}_{\mu\nu\lambda\sigma} + \dots$$

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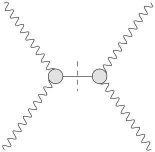
One-pion intermediate state:



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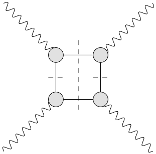
Two-pion intermediate state in both channels:



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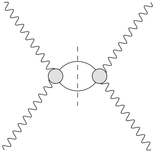
Two-pion intermediate state in first channel:



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Neglected: higher intermediate states

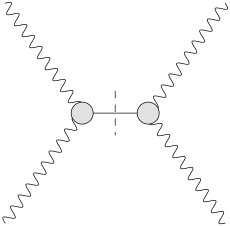
Summary

- Our dispersive approach to HLbL scattering is based on fundamental principles: unitarity, analyticity, crossing, gauge invariance
- We take into account the lowest intermediate states: π^0-pole and $\pi\pi$-cuts
- Relation to experimentally accessible (or again with data dispersively reconstructed) quantities
- A step towards a model-independent calculation of a_μ
- Numerical evaluation is work in progress

Backup

	$10^{11} \cdot a_\mu$	$10^{11} \cdot \Delta a_\mu$	
BNL E821	$116\,592\,091$	63	→ PDG 2013
QED total	$116\,584\,718.95$	0.08	→ Kinoshita et al. 2012
EW	153.6	1.0	
LO HVP	$6\,949$	43	→ Hagiwara et al. 2011
NLO HVP	-98	1	→ Hagiwara et al. 2011
NNLO HVP	12.4	0.1	→ Kurz et al. 2014
LO HLbL	116	40	→ Jegerlehner, Nyffeler 2009
NLO HLbL	3	2	→ Colangelo et al. 2014
Hadronic total	6982	59	
Theory total	$116\,591\,855$	59	

Pion pole

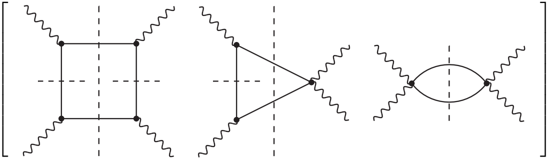


- Input the doubly-virtual and singly-virtual pion transition form factors $\mathcal{F}_{\gamma^*\gamma^*\pi^0}$ and $\mathcal{F}_{\gamma^*\gamma\pi^0}$
- Dispersive analysis of transition form factors in progress

$\rightarrow$ B. Kubis, Amherst workshop 2014

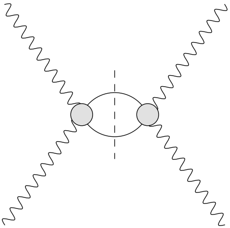
FsQED

$F_\pi^V(q_1^2)F_\pi^V(q_2^2)F_\pi^V(q_3^2) \times$



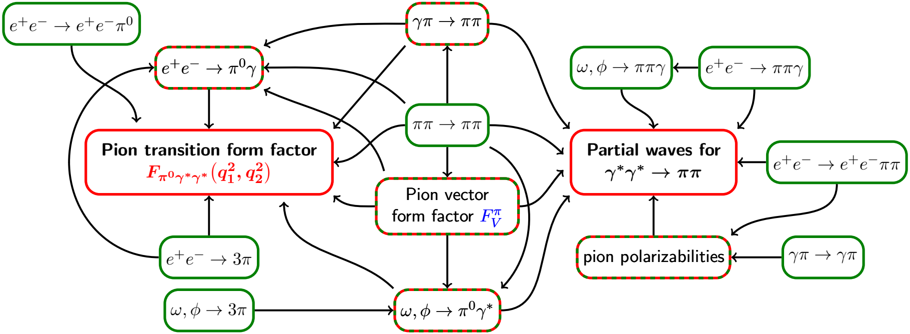
- Simultaneous two-pion cuts in two channels
- Analytic properties correspond to sQED box diagram
- Gauge invariance requires triangle and bulb diagrams
- q^2-dependence given by multiplication with pion vector form factor $F_\pi^V(q^2)$ for each off-shell photon

Remainder



- Two-pion cut in only one channel $\Rightarrow$ scalar functions have only a right-hand cut
- Expand into partial waves
- Unitarity relates it to the helicity amplitudes of the subprocess $\gamma^*\gamma^{(*)} \to \pi\pi$
- Dispersive integrals over the imaginary parts give $\bar{\Pi}_{\mu\nu\lambda\sigma}$

A roadmap for HLbL



→ Flowchart by M. Hoferichter