Quantising a perfect fluid

Based on 1406.4422, supervised by Ben Gripaios

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The perfect fluid Lagrangian

Each fluid element is labelled by its position at a known time in the past. The map $\phi^i(x,t)$ returns the label of the fluid element at x at time t.

$$B^{ij} = \partial_\mu \phi^i \partial^\mu \phi^j \qquad B = \det(B^{ij}) \tag{1}$$

B is invariant under transformations $\phi \to \phi'$ where $\det\left(\frac{\partial \phi'^i}{\partial \phi^j}\right) = 1$ everywhere.

The action is an arbitrary function f of B:

$$S = \int \mathrm{d}^d x \mathrm{d}t\, f(B) \tag{2}$$

Calculating correlators

Expand about a still vacuum:

$$
\begin{aligned}
\phi^i &= x^i + \pi^i \\
S &= \int \mathrm{d}^d x \mathrm{d}t \Big[\quad \frac{1}{2}(\dot{\pi}^2 - c^2(\partial_i \pi^i)^2) - \frac{(3c^2 + f_3)}{6}(\partial_i \pi^i)^3 \\
&\qquad + \frac{c^2}{2}(\partial_i \pi^i)(\partial_i \pi^j \partial^j \pi^i) + \dots \Big]
\end{aligned}
$$

$$
\text{propagator} = \frac{i}{\omega^2 - c^2 k^2}\left(\frac{k^i k^j}{k^2}\right) + \frac{i}{\omega^2}\left(\delta^{ij} - \frac{k^i k^j}{k^2}\right) \tag{3}
$$

Correlators diverge at late times T:

$$
\langle \pi^i \pi^j \rangle \sim \int \mathrm{d}\omega \frac{1}{\omega^2} e^{i\omega T} \tag{4}
$$

An analogy with $1+1d$ σ models

There are no Goldstone bosons in $1+1d$.

$$\langle \pi\pi \rangle \sim \int \mathrm{d}^2 k \frac{1}{k^2} e^{ikx} \tag{5}$$

"The vacuum is destroyed by quantum fluctuations"

However, you can still do useful calculations by computing the correlators of *symmetry invariants*.

$$\langle \phi\phi^\dagger \rangle = \langle e^{i\pi} e^{-i\pi} \rangle \sim 1 + \text{loop corrections} \tag{6}$$

Invariants in a fluid

Observables like p,ρ, u^μ are SDiff invariants.

We computed many 3- and 4-point SDiff invariant correlators at tree level. The $\frac{1}{\omega}$ terms cancel, and the correlators are finite.

We computed the tree and 1-loop parts of a 2-point density-density correlator in dim. reg.. Both the tree and loop parts are finite.

Regime of validity

This is strong evidence that the perfect fluid is a consistent (quantum) EFT! We can guess at the regime of validity by requiring the tree level part to be bigger than the loop level part.