Two lectures on top quark physics

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Contents:

- ✓ The basics
- ✓ Top and EW theory; precision fits
- ✓ Top pair production at hadron colliders [FO, threshold resummation]
- ✓ Top decay
  - ✓ Factorizable and non-factorizable corrections in top production and decay.
- ✓ Does it make sense to even speak of tops?
- ✓ Single top production
- ✓ Top quark mass
- ✓ e^+e^- colliders: similarities and differences w/r to hadron colliders

Additional point to reflect on:

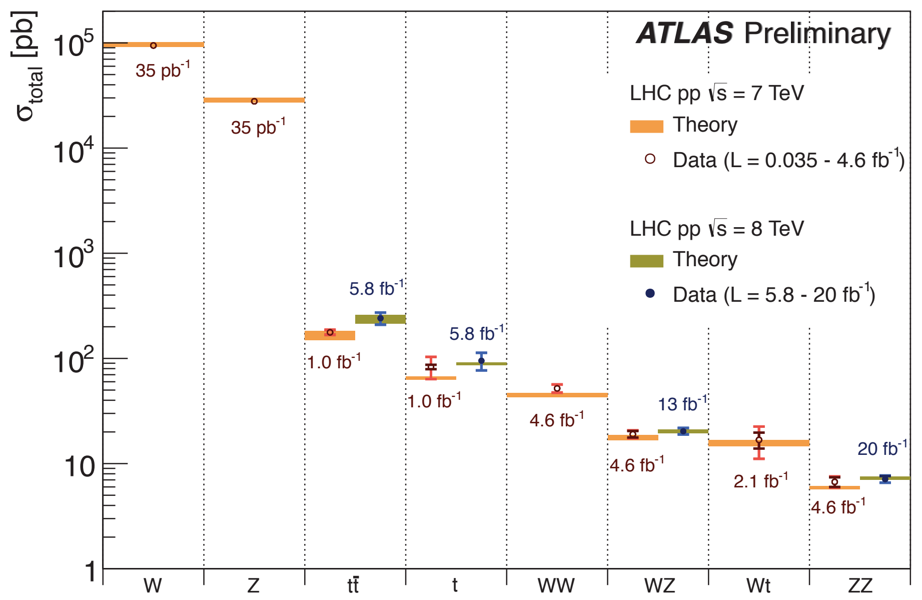
- ✓ Heavy flavor production; how top differs from the other massive quarks

Additional readings:

- Werner Bernreuther [arXiv:0805.1333]

Introduction: the spirit of these lectures

- Audience with diverse interests (QCD/SM, bSM): discussion should be useful for both
- Times are changing:
  - It was only few years ago when we were thinking of physics at the Born level focusing almost exclusively on the ideas …



Moriond 2013

- Such a way of thinking of physics is not adequate any more:
- LHC Run I showed an impressive level of agreement with SM (above)
- Will try to emphasize the important issues from modern prospective

Modern = today, not last year …

Introduction: why bother with top physics,
or
why the people who do not work on top physics should nevertheless follow these lectures
(comments welcome)

✓ The SM prospective

- Top production is the most complex SM process: tame tt, tame the SM (needed for bSM)
  - massive: addition of a mass in a problem adds a dimension to its complexity
  - colored
  - large QCD corrections
  - important EW interactions (strongly interacts with all SM particles)
  - results in very complex final states
- Top <u>can</u> be studied perturbatively: (record-) high accuracy expected (both TH and EXP)
- The only *bare* quark: gives direct access to the SM Lagrangian (with caveats, of course)

✓ The bSM prospective

- Top is a major background for many (most?) bSM processes: search for bSM "beneath" top
- The most prominent current discrepancy w/r to SM: Tevatron top A_{FB}
- Decays to tops; top loop effects
- Very large coupling to Higgs: if anything in the SM matters for Higgs, this is top.
- In summary, top matters in 2 ways:
  - through its *parametric* values (e.g. M_{top} and EW vacuum stability)
  - directly (through its production rates)

Top pair : the basics

Top quark: the basics

- Is the top special (as we hear all the time?): it depends!
  - From the viewpoint of QCD: NO
  - From the viewpoint EW : YES

- Top gets most of its corrections – and production rates – from QCD effects. But it gets its properties from EW interactions. ==> both are very important.

- Top's main attribute: its very large mass: $M_{top} \approx 173$ GeV . Compare:

  * $M_H \approx 125$ GeV
  * $M_W \approx 80$ GeV
  * $M_b \approx 5$ GeV
  * $M_c \approx 1.5$ GeV

Understanding the origin of mass is a major open problem

- CKM elements relevant for top: Vtb ≈ 1.
  - Top coupling to non-b down-type quarks must be very small (CKM suppression)
  - Top couplings to other up-type quarks is non-zero at loop-level but tiny.

Any significant top coupling to non-b quarks might be a sign of bSM physics

Top quark: the basics

Top's very large mass* dictates its properties (both intrinsic and production ones)

- $M_{top} >> M_W$
  Implication: top readily decays; not true for the other quarks.
- $\Gamma_{top} \approx 1.5$ GeV $>> \Lambda_{QCD} \approx 0.3$ GeV
  Implication: top's lifetime ($\sim 1/\Gamma_{top}$) is much smaller than the typical hadronization time ($\sim 1/\Lambda_{QCD}$).
  Profound consequence: top decays before forming strongly interacting bound states (i.e. mesons).

Top is the only quark that decays as a *bare* particle.

✓ This is of major importance. For the other quarks we have to make conclusions based on modeling of non-perturbative physics. This can be done but can be extremely tricky. In certain cases even beyond our ability to model QCD (not even speaking of solving it).

✓ The fact that top decays (largely*) free of non-perturbative effects gives us added confidence that we know what we are doing regarding SM physics (it really matters in the grand scheme of things…).

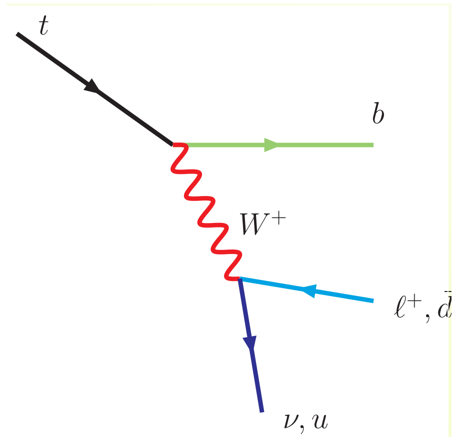
* To be elaborated upon later.

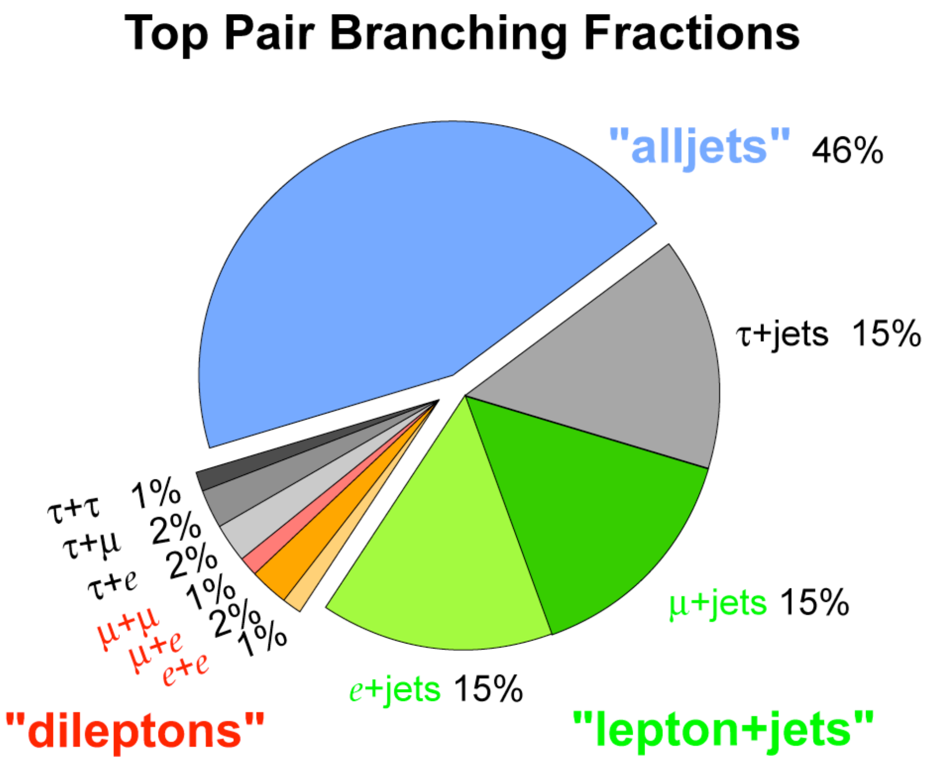
Top quark: the basics

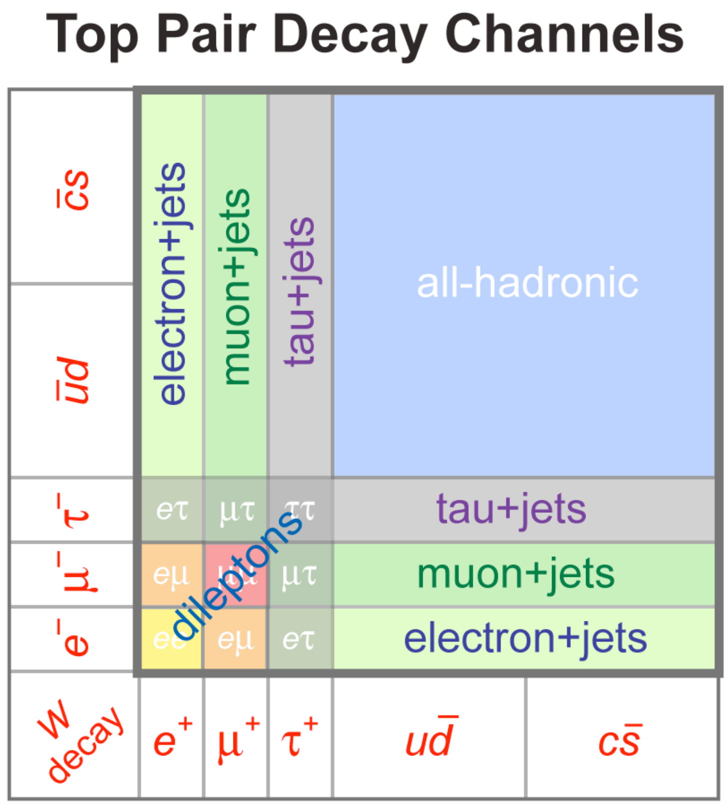
- We refer to the top mode based on the measured final state. Here are the SM options:

$$t \rightarrow W^+ + b$$
$$\quad \hookrightarrow l^+ + \nu$$

$$t \rightarrow W^+ + b$$
$$\quad \hookrightarrow q + \bar{q}$$

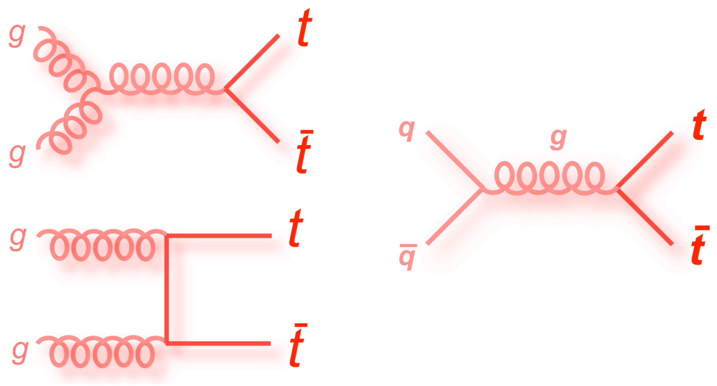


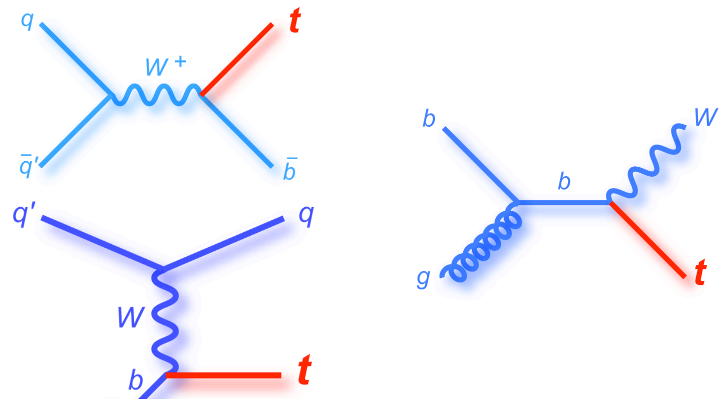




Top quark: the basics

- At hadron colliders top quarks are produced in pairs (dominant) or singly.





- Top quark production rates, for various initial states and colliders:

Top pairs only

	TeVatron	LHC 7 TeV	LHC 8 TeV	LHC 14 TeV
gg	15.4%	84.8%	86.2%	90.2%
$qg + \bar{q}g$	-1.7%	-1.6%	-1.1%	0.5%
qq	86.3%	16.8%	14.9%	9.3%

From W. Bernreuther '08

$t\bar{t}$ pairs	dominant reaction	$N_{t\bar{t}}$
Tevatron: $p\bar{p}$ (1.96 TeV)	$q\bar{q} \to t\bar{t}$	$\sim 7 \cdot 10^4 \times L$
LHC: pp (14 TeV)	$gg \to t\bar{t}$	$\sim 9 \cdot 10^5 \times L$
ILC: e^+e^- (400 GeV)	$e^+e^- \to t\bar{t}$	$\sim 800 \times L$
single top	dominant reaction	$(N_t + N_{\bar{t}})$
Tevatron:	$u + b \xrightarrow{W} d + t$	$\sim 3 \cdot 10^3 \times L$
LHC:	$u + b \xrightarrow{W} d + t$	$\sim 3.3 \cdot 10^5 \times L$

Question: any guesses why the rate for the qg reaction (starts at NLO) is negative? Is this OK?

Top quark: the basics

Top quark quantum numbers

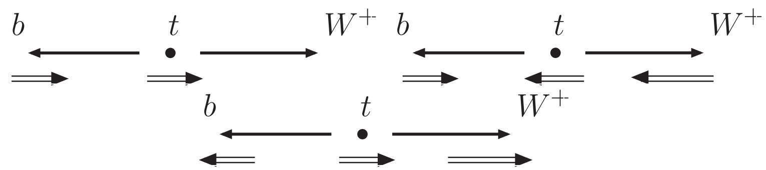
- Electric charge = +2/3|e|.
- Because tops are mostly pair produced, it was only recently shown that the exotic charge -4/3 (i.e. decay to bW^-) is unlikely.

- CKM: from weak decays it follows that:

$$B(t \to bW^+) = 0.998, \qquad B(t \to sW^+) \simeq 1.9 \times 10^{-3}, \qquad B(t \to dW^+) \simeq 10^{-4}.$$

- Limits from measurements of top decays are much weaker.

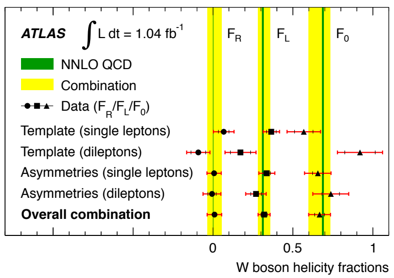
- Top spin: strongly correlated with the helicity of the W



SM predictions for the W helicity fractions:

$$F_0 = 0.99 \times F_0^B, \qquad F_- = 1.02 \times F_-^B, \qquad F_+ = 0.001$$

F_0^B ≈ 0.7 $\qquad$ F_-^B ≈ 0.3

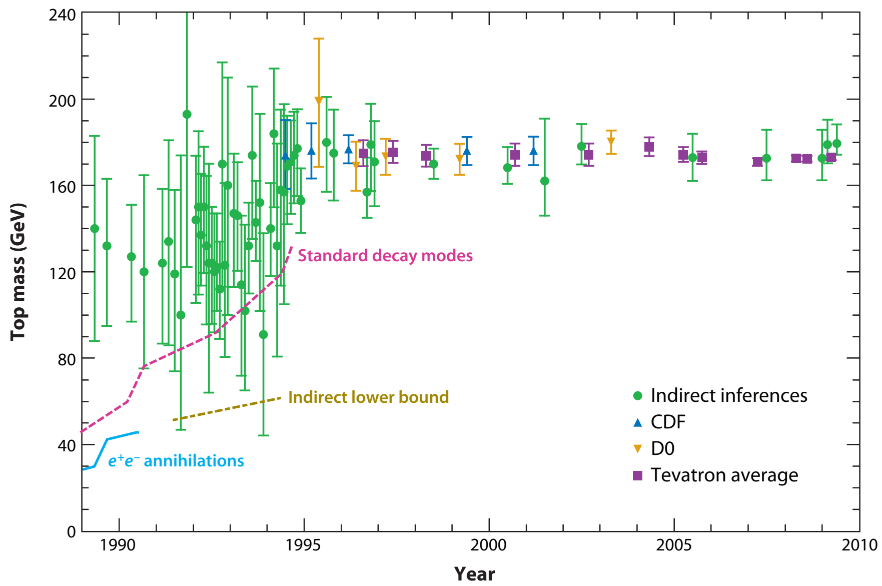


Very sensitive to V-A structure of the tbW vertex

Top quark and EW precision fits

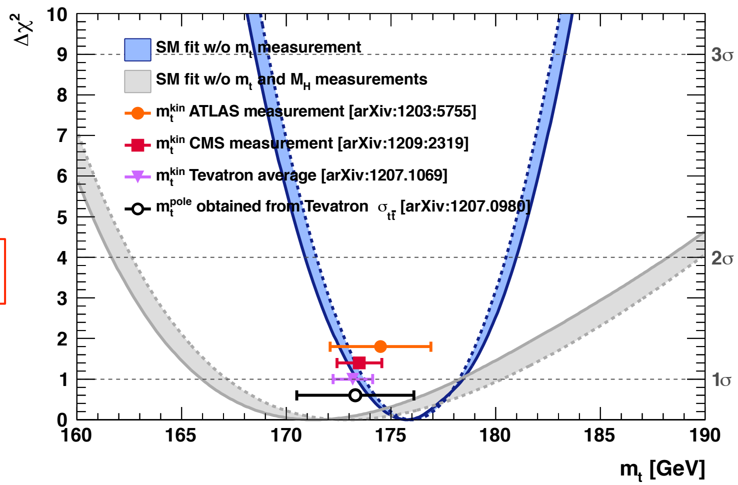
The run-up to the discovery of the top quark is an important lesson in today searches.

- We had an idea about M_{top} before top quarks were first seen:



- Using the known Higgs and W masses one can again indirectly "rediscover" the top. The returned mass is $m_t = 175.8^{+2.7}_{-2.4}\,\mathrm{GeV}$ in impressive agreement with direct determinations.

arXiv:1209.2716



Top pair production at hadron colliders

- Impressive agreement between theory and experiment across colliders and collider energies

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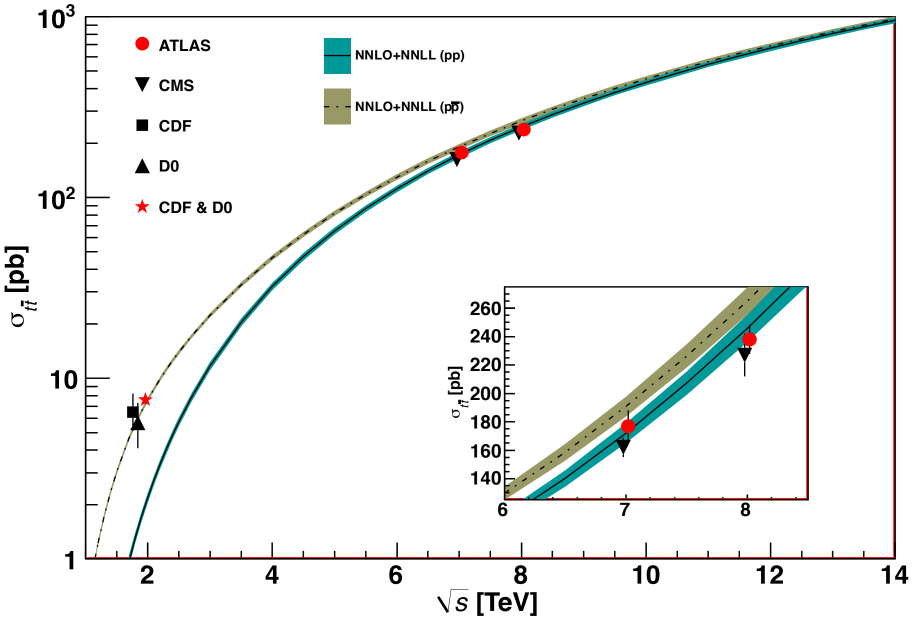


- Theory includes NNLO + NNLL; let's discuss what this means in the rest of this lecture.

- Let's begin with the simplest (theoretically) observable: the total cross-section.

$$\sigma_{\rm tot} = \sum_{i,j} \int_0^{\beta_{\rm max}} d\beta\, \Phi_{ij}(\beta, \mu_F^2)\, \hat{\sigma}_{ij}(\alpha_S(\mu_R^2), \beta, m^2, \mu_F^2, \mu_R^2)$$

- A dimensional quantity (typically in [pb]). Tells us the rate for producing top pairs.
  The total number of produced pairs is obtained after multiplying by the collider Luminocity.

- In reality, the total x-section cannot be measured because of the presence of *cuts.*
- Cuts represent the basic fact that any detector has a finite size. What is actually measured are the number of events within the detector.

- The part of phase space which is covered by the detector is called *fiducial* volume (or fiducial x-section).

Note: we pair produce top quarks, but due to the finiteness of the fiducial volume, it may happen that one top from the pair goes inside the detector (and is thus detected) while the second goes outside of the detector.

- This way we detect events containing a single top quark, not a pair.
  Yet this is the same process. Is this OK? What about single top?

$$\sigma_{\rm tot} = \sum_{i,j} \int_0^{\beta_{\rm max}} d\beta\, \Phi_{ij}(\beta,\mu_F^2)\, \hat{\sigma}_{ij}(\alpha_S(\mu_R^2),\beta,m^2,\mu_F^2,\mu_R^2)$$

<u>The computed x-section is a convolution of a partonic flux and a perturbative x-section</u>

$$\Phi_{ij}(\beta,\mu_F^2) = \frac{2\beta}{1-\beta^2}\, \mathcal{L}_{ij}\left(\frac{1-\beta^2_{\rm max}}{1-\beta^2},\mu_F^2\right)$$ where: $$\mathcal{L}_{ij}(x,\mu_F^2) = x\,(f_i \otimes f_j)\,(x,\mu_F^2) = x \int_0^1 dy \int_0^1 dz\, \delta(x-yz) f_i(y,\mu_F^2) f_j(z,\mu_F^2)$$

Jacobian factor

Partonic luminocity

- We sum over all possible pairs of partons. The details depend on the perturbative order
  - LO: qqbar, gg
  - NLO: LO ones + qg
  - NNLO: NLO ones + qq,qq', qqbar'

- Thus, starting with NNLO, all possible partonic channels contribute!
- Note: q, qbar, etc. run over all quark flavors lighter than top (u,d,s,c,b).
- This is known as a scheme with 5 active flavors (i.e. N_F=5).
- We do not have to work in this scheme (it is a matter of choice!).
  A natural choice for top at TEV/LHC
- An alternative would be to work with 6 flavor scheme (i.e. we need top pdf).
- This is not needed for top @LHC but at a future higher energy collider (100TeV for example).
- Total x-section is dominated by scales ~ Mtop, thus even for future collider N_F=5 scheme is OK
- For differential x-section, P_T distribution at high P_T, N_F=6 scheme would start to be needed.
- Note: same situation for bottom production.

$$\sigma_{\rm tot} = \sum_{i,j} \int_0^{\beta_{\rm max}} d\beta\, \Phi_{ij}(\beta, \mu_F^2)\, \hat{\sigma}_{ij}(\alpha_S(\mu_R^2), \beta, m^2, \mu_F^2, \mu_R^2)$$

The computed x-section is a convolution of a partonic flux and a perturbative x-section

- The above equation is not exact (due to factorization breaking effects)
- Factorization breaking effects for inclusive observables are small ~ $\Lambda_{QCD}/\mu_F << 1$
- For less inclusive observables, these effects start to grow.
- If we try to imagine fully exclusive final states then these correction become O(1), i.e. factorization breaks down and the above equation is not applicable any more.

To remember (for non-experts):

- Factorization breaking effects are usually not a worry
- But we cannot ignore them altogether
- Every time we do something kinematically extreme, it is worth checking if it is safe.

$$\sigma_{\text{tot}} = \sum_{i,j} \int_0^{\beta_{\max}} d\beta \, \Phi_{ij}(\beta, \mu_F^2) \, \hat{\sigma}_{ij}(\alpha_S(\mu_R^2), \beta, m^2, \mu_F^2, \mu_R^2)$$

<u>Meaning of the various arguments above.</u>

- m – mass of the heavy quark. It is a scheme dependent quantity. Usually the pole mass.
- $\beta = \sqrt{1-\rho}$, with $\rho \equiv 4m^2/s$ $\quad 0 \le \beta < 1$.
- s – partonic c.m. energy. $s=x_1 x_2 S_{\text{collider}}$; $x_{1,2}$: partonic fractions; we integrate over them.
- β is the only kinematical variable (the only dimensionless one!).
  It has a meaning of a relative velocity of the two tops.
- β≈0 is called partonic threshold (i.e. $4m^2 \approx s$). All the energy in the system is taken by the masses; ==> no energy left for additional radiation.
- Therefore, only soft radiation is possible (soft == vanishing energy).
  Important: This is the basis of the soft gluon resummation which we will discuss later.
- β≈1 is the high-energy limit (i.e. $4m^2 \ll s$, or simply put, m → 0).

continue…

$$\sigma_{\rm tot} = \sum_{i,j} \int_0^{\beta_{\rm max}} d\beta \, \Phi_{ij}(\beta, \mu_F^2) \, \hat{\sigma}_{ij}(\alpha_S(\mu_R^2), \beta, m^2, \mu_F^2, \mu_R^2)$$

- Important:

Recall: β≈1 is the high-energy limit (i.e. $4m^2 << s$, or simply put, m → 0).

- In high energy collisions, the mass of a parton is not its intrinsic property anymore.
- All that matters is if the mass is much smaller than the relevant kinematic scale Q.
- If m<<Q we can simply set m=0.
- If x-section diverges due to collinear singularities, then we need to use *collinearly safe observables,* like jets, or introduce PDF / fragmentation functions that "absorb" the collinear singularities.
- If we leave small but non-zero mass then we have an artificially finite result.
- It contains terms like Log(m/Q) >>1.
- Moreover at all orders in perturbation theory we have terms like $\alpha_S^n \, \mathrm{Log}^k(m/Q)$.
- This breaks the convergence of the perturbative expansion by effectively changing

$$\alpha_S^n \rightarrow \alpha_S^n \, \mathrm{Log}^k(m/Q) >> \alpha_S^n$$

- Such term then need to be resummed to all orders (different kind of resummation w/r to the soft gluon resummation mentioned above!).

Mele, Nason '91; natural generalization to amplitudes see Mitov, Moch '06 '07

- Above is true for all massive partons t,b,c but also e in the context of QED.

$$\sigma_{\rm tot} = \sum_{i,j} \int_0^{\beta_{\max}} d\beta \, \Phi_{ij}(\beta, \mu_F^2) \, \hat{\sigma}_{ij}(\alpha_S(\mu_R^2), \beta, m^2, \mu_F^2, \mu_R^2)$$

<u>The computed x-section is a convolution of a partonic flux and a perturbative x-section</u>

- μ_F : factorization scale; separates long distance (pdf) from short distance (partonic x-section) effects
- It is unphysical. It appears as a result of the approximations we make (i.e. the factorized form of the above equation).
- There is no "first principles" idea about how to choose its value.
- In top physics we set it equal to the relevant hard scale:
  - For the total x-section only one such scale exists: M_{top}.
  - For more differential observables better values are $H_T/2$ or $m_{T,top}$ with: $m_{T,i} = \sqrt{p_{T,i}^2 + m_i^2}$
  - For even more extreme kinematics, even the rapidity might have to be included (not yet)

- Because there is no unique (or "best") choice for the value of this scale, the philosophy is: we pick one reasonable value (called central value) and then vary μ_F around it.
- How much to vary is anybody's guess (more later)

- μ_R - Renormalization scale. This is the scale at which the running coupling is evaluated.
- Natural choice: $\mu_F = \mu_R = Q$ (=M_{top}). Has to be varied, too.
- One idea exists about how to fix its value (BLM). Comes from the requirement for restoring conformal invariance of the QCD Lagrangian. Caution: could work very well, but not always!

Brodsky, Lepage, Mackenzie '83

$$\sigma_{\text{tot}} = \sum_{i,j} \int_0^{\beta_{\max}} d\beta\, \Phi_{ij}(\beta, \mu_F^2)\, \hat{\sigma}_{ij}(\alpha_S(\mu_R^2), \beta, m^2, \mu_F^2, \mu_R^2)$$

<u>The computed x-section is a convolution of a partonic flux and a perturbative x-section</u>

- α_S: the MS_{bar} renormalized strong coupling through the same perturbative order as the partonic crosss-section.
- It is running with N_F=5 active flavors (same as pdf). This could be chosen differently.
- Note: the strong coupling α_S appears in two places: explicitly in the partonic cross-section but also implicitly in the partonic fluxes (through their DGLAP evolution).
- Note: nowadays we typically use the LHAPDF library for pdf evolution. It provides us with the evolved strong coupling. This way we ensure we use the same coupling (i.e. defined the same way) in both places.

- More on the choice of scales:

- If we use a running scale (also called dynamic) we, in essence, include certain higher order terms. For example, at NLO ($b_{0,1}$ is the beta function of QCD):

$$\begin{aligned} \alpha_S(k^2) &= \frac{\alpha_S(\mu^2)}{1 + b_0 \alpha_S(\mu^2) \ln(k^2/\mu^2)} \left(1 - \frac{b_1}{b_0} \frac{\alpha_S(\mu^2)}{1 + b_0 \alpha_S(\mu^2) \ln(k^2/\mu^2)} \right. \\ &\times \left. \ln(1 + b_0 \alpha_S(\mu^2) \ln(k^2/\mu^2)) + \mathcal{O}\left(\alpha_S^2(\mu^2)[\alpha_S(\mu^2) \ln(k^2/\mu^2)]^n\right) \right) \end{aligned} \quad (\text{:}$$

$$\sigma_{\rm tot} = \sum_{i,j} \int_0^{\beta_{\rm max}} d\beta\, \Phi_{ij}(\beta, \mu_F^2)\, \hat{\sigma}_{ij}(\alpha_S(\mu_R^2), \beta, m^2, \mu_F^2, \mu_R^2)$$

<u>The computed x-section is a convolution of a partonic flux and a perturbative x-section</u>

Setting $\mu_F = \mu_R = \mu$, the partonic cross-section reads through NNLO:

$$\hat{\sigma}_{ij} = \frac{\alpha_S^2}{m^2}\left\{\sigma_{ij}^{(0)} + \alpha_S\left[\sigma_{ij}^{(1)} + L\,\sigma_{ij}^{(1,1)}\right] + \alpha_S^2\left[\sigma_{ij}^{(2)} + L\,\sigma_{ij}^{(2,1)} + L^2\sigma_{ij}^{(2,2)}\right]\right\} \quad \text{and} \quad L = \ln\left(\mu^2/m^2\right)$$

LO NLO NNLO

- The LO and NLO x-sections are known fully differentially (as well as the inclusive ones) since the late 1980's.
- The NNLO total x-sections are now known, too. Differential ones are not yet known through NNLO, but this will change very soon.
- Various approximations are known; we will discuss them later.
- The functions ~ L are of one order lower, i.e. the NNLO ones $\sigma^{(2,1)}$ and $\sigma^{(2,2)}$ can be derived from the NLO x-sections $\sigma^{(1)}$.

$$\sigma_{\rm tot} = \sum_{i,j} \int_0^{\beta_{\rm max}} d\beta\, \Phi_{ij}(\beta, \mu_F^2)\, \hat{\sigma}_{ij}(\alpha_S(\mu_R^2), \beta, m^2, \mu_F^2, \mu_R^2)$$

$$\hat{\sigma}_{ij} = \frac{\alpha_S^2}{m^2} \left\{ \sigma_{ij}^{(0)} + \alpha_S \left[\sigma_{ij}^{(1)} + L\, \sigma_{ij}^{(1,1)} \right] + \alpha_S^2 \left[\sigma_{ij}^{(2)} + L\, \sigma_{ij}^{(2,1)} + L^2 \sigma_{ij}^{(2,2)} \right] \right\}$$

How to derive the scale dependence for $\mu_F = \mu_R = \mu$?

- Recall that the LHS is formally independent of μ
- Take a log-derivative w/r to μ of both sides. LHS vanishes.
- The pdf's satisfy DGLAP evolution equation, i.e. the log-derivative w/r to μ of the pdf is the pdf convoluted with the splitting function.

The remaining procedure is straightforward, and we get:

$$\begin{aligned} s_{ij}^{(1,1)} &= \frac{1}{2\pi} \left[2\beta_0 s_{ij}^{(0)} - P_{ki}^{(0)} \otimes s_{kj}^{(0)} - s_{ik}^{(0)} \otimes P_{kj}^{(0)} \right] , \qquad (5.1 \\ s_{ij}^{(2,2)} &= \frac{1}{(2\pi)^2} \left[3\beta_0^2 s_{ij}^{(0)} - \frac{5}{2}\beta_0 P_{ki}^{(0)} \otimes s_{kj}^{(0)} - \frac{5}{2}\beta_0 s_{ik}^{(0)} \otimes P_{kj}^{(0)} \right. \\ &\quad \left. + \frac{1}{2} P_{ki}^{(0)} \otimes P_{lk}^{(0)} \otimes s_{lj}^{(0)} + \frac{1}{2} s_{il}^{(0)} \otimes P_{lk}^{(0)} \otimes P_{kj}^{(0)} + P_{ki}^{(0)} \otimes s_{kl}^{(0)} \otimes P_{lj}^{(0)} \right] , \\ s_{ij}^{(2,1)} &= \frac{1}{(2\pi)^2} \left[2\beta_1 s_{ij}^{(0)} - P_{ki}^{(1)} \otimes s_{kj}^{(0)} - s_{ik}^{(0)} \otimes P_{kj}^{(1)} \right] + \frac{1}{2\pi} \left[3\beta_0 s_{ij}^{(1)} - P_{ki}^{(0)} \otimes s_{kj}^{(1)} - s_{ik}^{(1)} \otimes P_{kj}^{(0)} \right] \end{aligned}$$

- To derive the case $\mu_F \neq \mu_R$ we only need to substitute the running coupling discussed before.

Collinear singularities.

$$\hat{\sigma}_{ij} = \frac{\alpha_S^2}{m^2}\left\{\sigma_{ij}^{(0)} + \alpha_S\left[\sigma_{ij}^{(1)} + L\,\sigma_{ij}^{(1,1)}\right] + \alpha_S^2\left[\sigma_{ij}^{(2)} + L\,\sigma_{ij}^{(2,1)} + L^2\sigma_{ij}^{(2,2)}\right]\right\}$$

- Tops are massive. But due to emission of collinear radiation from the massless initial state partons (i,j) the resulting partonic cross-sections are not finite.
- For example, in dim reg, they contain a single pole per loop:

$$\tilde{\sigma}_{ij}(\epsilon,\rho) = \frac{\alpha_S^2}{m^2}\left\{\tilde{\sigma}_{ij}^{(0)}(\epsilon,\rho) + \alpha_S\tilde{\sigma}_{ij}^{(1)}(\epsilon,\rho) + \alpha_S^2\tilde{\sigma}_{ij}^{(2)}(\epsilon,\rho) + \dots\right\}$$

- This collinear singularity has to be factored out into the initial state PDF's.

$$\frac{\tilde{\sigma}_{ij}(\epsilon,\rho)}{\rho} = \sum_{k,l}\left[\frac{\hat{\sigma}_{kl}(x)}{x}\otimes\Gamma_{ki}\otimes\Gamma_{lj}\right](\rho)$$

- Through NNLO the collinear counterterms read:

- They are process independent!
- They are scheme dependent!

$$\Gamma_{ij}(\epsilon,x) = \delta_{ij}\delta(1-x) + \alpha_S\Gamma_{ij}^{(1)}(\epsilon,x) + \alpha_S^2\Gamma_{ij}^{(2)}(\epsilon,x)\,,$$

$$\Gamma_{ij}^{(1)}(\epsilon,x) = -\frac{1}{2\pi}\,\frac{P_{ij}^{(0)}(x)}{\epsilon}\,,$$

$$\Gamma_{ij}^{(2)}(\epsilon,x) = \left(\frac{1}{2\pi}\right)^2\left\{\frac{1}{2\epsilon^2}\left[P_{ik}^{(0)}\otimes P_{kj}^{(0)}(x) + \beta_0 P_{ij}^{(0)}(x)\right] - \frac{1}{2\epsilon}P_{ij}^{(1)}(x)\right\}$$

- Note hat if we consider something like single inclusive top production, then there will also be collinear singularities in the final state. Top fragmentation functions might be needed if the top P_T become $P_T \gg M_{top}$ (never done so far, but might be needed for the LHC13)

Calculation of the partonic cross-sections:

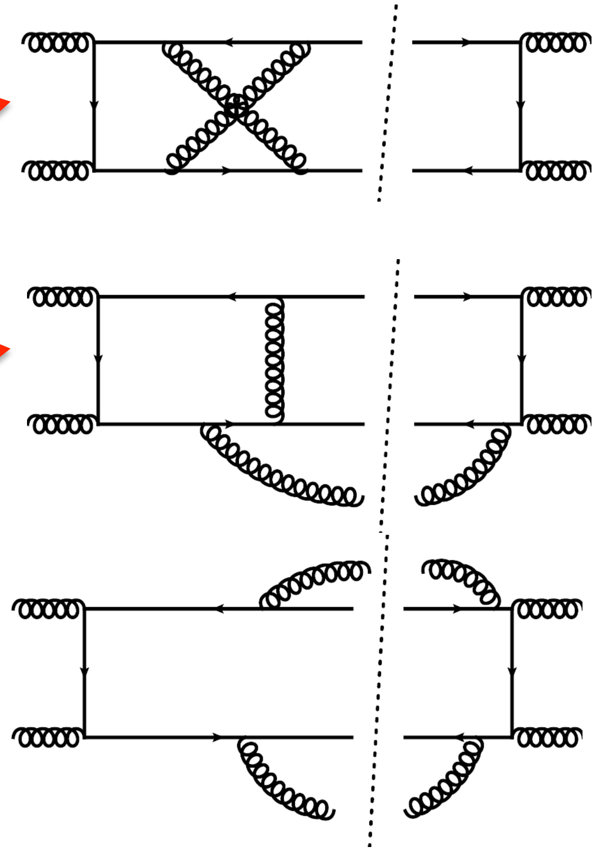
$$\tilde{\sigma}_{ij}(\epsilon,\rho) = \frac{\alpha_S^2}{m^2}\left\{\tilde{\sigma}_{ij}^{(0)}(\epsilon,\rho) + \alpha_S\tilde{\sigma}_{ij}^{(1)}(\epsilon,\rho) + \alpha_S^2\tilde{\sigma}_{ij}^{(2)}(\epsilon,\rho) + \dots\right\}$$

No fully developed methods exist that can handle in unified way all aspects of any massive calculation through NNLO. The working approach of today is:

- Compute all pieces numerically, and separately, as IR divergent quantities.
- Then add them up.
- Make all checks, especially verify the cancellation of all singularities (after collinear factorization)

As any NNLO calculation, there are 3 principle contributions:

There are 3 principle contributions:



✓ 2-loop virtual corrections (V-V)

✓ 1-loop virtual with one extra parton (R-V)

✓ 2 extra emitted partons at tree level (R-R)

And 2 secondary contributions:

✓ Collinear subtraction for the initial state

✓ One-loop squared amplitudes (analytic)

Known, in principle. Done numerically.

Korner, Merebashvili, Rogal `07
Anastasiou, Mert-Aybot `08

Weinzierl `11

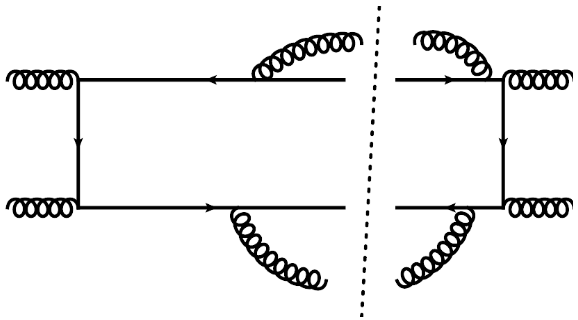
May be avoided?

- The single most important piece is how to handle the double-real contributions (i.e. to integrate over the phase space of the two unobserved partons)

Recall: the matrix elements are tree-level (and thus finite)

phase space integration is singular (soft and/or collinear logs) and produces terms ~$1/\varepsilon^4$

✓ A wonderful result By M. Czakon Czakon `10-11



✓ The method is general (also to other processes, differential kinematics, etc).

✓ Explicit contribution to the total cross-section given.

✓ Used now in a number of NNLO processes: ttbar, single top, Higgs+jet

Czakon, Fiedler,Mitov
Boughezal, Caola, Melnikov, Petriello, Schulze '13
Brucherseifer, Caola, Melnikov '14

Threshold approximations and resummations in top pair production

Threshold approximations and resummations in top pair production

- This has been an extremely fertile and useful field.
- Helps in our understanding of QCD at higher orders and non-perturbative phenomena.
- Very limited kinematical applicability: certain phase-space regions need it, most do not.

What is threshold?

- Kinematical configuration where all the partonic energy is taken by the top pair and very little, if any, energy is left for radiation.

- Distinguish "absolute threshold" and "threshold":

- Absolute threshold is a particular case of a threshold, where almost all the partonic energy is used to produce the tops at rest.

- Absolute threshold kinematics is relevant for the resummation of the total x-section.

- General threshold resummation is needed for differential observables

- The two types are related; absolute threshold resummation can be obtained from the differential case by integrating over the phase space and then taking the limit $\beta \to 0$.

See Czakon, Mitov, Sterman '09 for the detailed procedure

Threshold approximations and resummations in top pair production

- Important subtle point: if a resummation is done at the differential level, and then numerically integrate over the phase space, the result will differ from the one done in absolute threshold resummation due to subleading terms. In other words, the leading terms in the threshold limit will be correctly resummed, but in one case subleading terms would also come along.

- This is a an important issue since these subleading terms are typically not small numerically so their inclusion affects the results significantly.

- This has led to many discussions in the past, sometimes quite animated; no consensus has been reached in the literature.

- The good news: with NNLO + resummation precision, the effects of the resummation becomes less important, so these differences become more marginal.

Threshold approximations and resummations in top pair production

- Threshold resummation, in top or otherwise, has traditionally been done in the so-called Mellin space approach (since mid-1990's)

Kidonakis, Sterman
Bonciani, Catani, Mangano, Nason

- In the last 5-6 years SCET has also been extensively used for performing soft-gluon resummation in top pair production.

Beneke, Falgari, Scwinn
Ferroglia, Neubert, Pecjak, Yang

There are many differences; almost all are technical. The essentials are:

- One of them works in N-space, the other in x-space.
- To leading power both approaches agree; differences at subleading terms (could be large)
- There is a belief (unclear if it is correct) that the x-space approach, unlike the N-space one, allows MC implementation of resummation. The idea is to eventually have a fully differential soft gluon resummation.

- If this can be done, or not, is for the future to show. It is a nice open problem.

See recent work: Broggio, Papanastasiou, Signer '14

Threshold approximations and resummations in top pair production

Here is how soft-gluon resummation is done (in the case of N-space)

✓ Identify the threshold kinematics. Introduce a variable z, such that the threshold is z→1.
Examples:
- total cross-section: $z = 4m^2/s$
- Differential case: $z = M^2_{t\bar{t}}/s$

✓ Introduce a dual Mellin variable (threshold is now the limit N→∞). The leading power behavior is simple:

$$\begin{aligned}\sigma(N) &= \int_0^1 dz z^{N-1}\sigma(z) \\ &= \int_0^1 dz e^{-(N-1)(1-z)}\sigma(z) + \mathcal{O}(1/N)\end{aligned}$$

✓ In the soft limit N→∞ the partonic cross-section factorizes

$$\begin{aligned}\omega_P\left(N, \hat{\eta}, \frac{M^2}{\mu^2}, \frac{m^2}{\mu^2}, \alpha_s(\mu^2)\right) &= J_1(N, \alpha_s(\mu^2)) \ldots J_k(N, M/\mu, m/\mu, \alpha_s(\mu^2)) \\ &\times \mathrm{Tr}\left[\mathbf{H}^P\left(\frac{M^2}{\mu^2}, \frac{m^2}{\mu^2}, \hat{\eta}, \alpha_s(\mu^2)\right)\mathbf{S}^P\left(\frac{N^2\mu^2}{M^2}, \frac{M^2}{m^2}, \hat{\eta}, \alpha_s(\mu^2)\right)\right] + \mathcal{O}(1/N)\end{aligned}$$

Threshold approximations and resummations in top pair production

$$\omega_P\left(N,\hat{\eta},\frac{M^2}{\mu^2},\frac{m^2}{\mu^2},\alpha_s(\mu^2)\right) = J_1(N,\alpha_s(\mu^2))\ldots J_k(N,M/\mu,m/\mu,\alpha_s(\mu^2)) \times \mathrm{Tr}\left[\mathbf{H}^P\left(\frac{M^2}{\mu^2},\frac{m^2}{\mu^2},\hat{\eta},\alpha_s(\mu^2)\right)\mathbf{S}^P\left(\frac{N^2\mu^2}{M^2},\frac{M^2}{m^2},\hat{\eta},\alpha_s(\mu^2)\right)\right] + \mathcal{O}(1/N)$$

- The leading power behavior close to threshold can now be made explicit to all orders in the coupling. This is why the soft resummation is needed in the first place: to resum terms like

$$\alpha_S^n \rightarrow \alpha_S^n \, \mathrm{Log}^k(N) >> \alpha_S^n$$

and restore the convergence of perturbation theory close to the partonic threshold!

- The all-order (in α_S) behavior of the above functions ($\mathbf{J}_i$ and **S**; **H** is a constant) is derived by solving their evolution equations.
- The evolution equations are driven by anomalous dimensions that can be computed in fixed order perturbation theory. The deeper the expansion, the higher the logarithmic accuracy of the resummation.
- Nowadays, NNLL "comes standard".
- Note: same story in x-space: same factorization (looks more complicated there) same evolution equations. Same physics.

- One possible difference between the N- and x-space approaches: subleading terms have better behavior in N-space (lack of factorial growth). See

Catani, Mangano, Nason, Trentadue '96

Threshold approximations and resummations in top pair production

- The Jet functions are process independent
- The hard function is derived from FO process dependent calculation. Numerically important.
- The soft function is complicated but now known to two loops (NNLL) in massive case:

$$\begin{aligned}
\mathbf{S}\left(\frac{N^2\mu^2}{M^2},\beta_i\cdot\beta_j,\alpha_s(\mu^2)\right)\Big|_{\mu=M} &= \overline{\mathcal{P}}\exp\left\{-\int_{M/\bar{N}}^{M}\frac{d\mu'}{\mu'}\mathbf{\Gamma}_S^\dagger\left(\beta_i\cdot\beta_j,\alpha_s\left(\mu'^2\right)\right)\right\} \\
&\quad\times\mathbf{S}\left(1,\beta_i\cdot\beta_j,\alpha_s\left(M^2/\bar{N}^2\right)\right) \\
&\quad\times\mathcal{P}\exp\left\{-\int_{M/\bar{N}}^{M}\frac{d\mu'}{\mu'}\mathbf{\Gamma}_S\left(\beta_i\cdot\beta_j,\alpha_s\left(\mu'^2\right)\right)\right\} \\
&= \overline{\mathcal{P}}\exp\left\{\int_0^1 dx\frac{x^{N-1}-1}{1-x}\,\mathbf{\Gamma}_S^\dagger\left(\beta_i\cdot\beta_j,\alpha_s\left((1-x)^2M^2\right)\right)\right\} \\
&\quad\times\mathbf{S}\left(1,\beta_i\cdot\beta_j,\alpha_s\left(M^2/N^2\right)\right) \\
&\quad\times\mathcal{P}\exp\left\{\int_0^1 dx\frac{x^{N-1}-1}{1-x}\,\mathbf{\Gamma}_S\left(\beta_i\cdot\beta_j,\alpha_s\left((1-x)^2M^2\right)\right)\right\},
\end{aligned}$$

- All the N-dependence is now explicit, to all orders in the coupling!

- The needed anomalous dimensions all come from the IR singularities of the corresponding amplitudes. There is a very deep connection between the IR behavior of gauge theory amplitudes and soft gluon resummation in the corresponding cross-sections.

Threshold approximations from resummations in top pair production

- Imagine we do not know the NNLO result for top pair production
- From the resummed result, once it is expanded in powers of the strong coupling, one can *predict* the leading power threshold behavior of the cross-section!

- This has been done analytically for the total cross-section as well as implemented in numerical programs for the differential cross-section.

- The result for the total cross-section reads:

$$\sigma_{ij,\mathbf{I}}(\beta,\mu,m) \;=\; \sigma^{(0)}_{ij,\mathbf{I}}\left\{1+\frac{\alpha_s(\mu^2)}{4\pi}\left[\sigma^{(1,0)}_{ij,\mathbf{I}}+\sigma^{(1,1)}_{ij,\mathbf{I}}\ln\left(\frac{\mu^2}{m^2}\right)\right]\right.$$
$$\left.+\left(\frac{\alpha_s(\mu^2)}{4\pi}\right)^2\left[\sigma^{(2,0)}_{ij,\mathbf{I}}+\sigma^{(2,1)}_{ij,\mathbf{I}}\ln\left(\frac{\mu^2}{m^2}\right)+\sigma^{(2,2)}_{ij,\mathbf{I}}\ln^2\left(\frac{\mu^2}{m^2}\right)\right]+\mathcal{O}(\alpha_s^3)\right\}$$

where:

$$\sigma^{(2)}_{q\bar{q}} = \frac{3.60774}{\beta^2}+\frac{1}{\beta}\Big(-140.368\ln^2\beta+32.106\ln\beta+3.95105\Big)$$
$$+910.222\ln^4\beta-1315.53\ln^3\beta+592.292\ln^2\beta+528.557\ln\beta+C^{(2)}_{q\bar{q}}\,,$$

$$\sigma^{(2)}_{gg} = \frac{68.5471}{\beta^2}+\frac{1}{\beta}\Big(496.3\ln^2\beta+321.137\ln\beta-8.62261\Big)$$
$$+4608\ln^4\beta-1894.91\ln^3\beta-912.349\ln^2\beta+2456.74\ln\beta+C^{(2)}_{gg}\,,$$

The analytic result:

$$\sigma^{(2,0)}_{q\bar{q},\mathbf{8}} = \frac{(2C_F-C_A)^2\pi^4}{3\beta^2}+\frac{(2C_F-C_A)\pi^2}{9\beta}\Big[288C_F\ln^2\beta+6\big(48C_F\ln 2-23C_A+2n_l\big)\ln\beta$$
$$+12C_F\big(-24+9\ln 2+\pi^2\big)+3C_A\big(89-58\ln 2-3\pi^2\big)+6n_l\big(-5+6\ln 2\big)-32\Big]$$
$$+512C_F^2\ln^4\beta+\frac{128}{9}C_F\Big[72C_F\big(-2+3\ln 2\big)-29C_A+2n_l\Big]\ln^3\beta$$
$$+\frac{16}{9}\Big[2C_F\big(12C_F(120-207\ln 2+156\ln^2 2-7\pi^2)+3C_A(217-198\ln 2-4\pi^2)$$
$$+6n_l(-9+10\ln 2)-32\big)+3C_A(17C_A-2n_l)\Big]\ln^2\beta$$
$$+\frac{8}{27}\Big[2C_F\big(18C_F(-960+\ln 2(1368-84\pi^2)-1140\ln^2 2+576\ln^3 2+55\pi^2+336\zeta_3)$$
$$+C_A(-7582+108\ln 2(115-2\pi^2)-5886\ln^2 2+360\pi^2+189\zeta_3)$$
$$+2n_l(338-630\ln 2+378\ln^2 2-9\pi^2)+192(2-3\ln 2)\big)$$
$$+3C_A\big(3C_A(-185+126\ln 2+6\pi^2-6\zeta_3)+6n_l(11-10\ln 2)+32\big)\Big]\ln\beta+C^{(2)}_{q\bar{q}}\,, \quad (5)$$

$$\sigma^{(2,0)}_{gg,\mathbf{1}} = \frac{4C_F^2\pi^4}{3\beta^2}+\frac{2C_F\pi^2}{9\beta}\Big[288C_A\ln^2\beta+6\big(C_A(-11+48\ln 2)+2n_l\big)\ln\beta$$
$$+9C_F\big(-20+\pi^2\big)+C_A\big(67-66\ln 2+3\pi^2\big)+2n_l\big(-5+6\ln 2\big)\Big]+512C_A^2\ln^4\beta$$
$$+\frac{128}{9}C_A\Big[C_A\big(-155+216\ln 2\big)+2n_l\Big]\ln^3\beta+\frac{32}{9}C_A\Big[9C_F\big(-20+\pi^2\big)$$
$$+C_A\big(1963-2790\ln 2+1872\ln^2 2-96\pi^2\big)+2n_l\big(-17+18\ln 2\big)\Big]\ln^2\beta$$
$$+\frac{16}{27}\Big[27C_F\big(-2C_F\pi^2+C_A(80+6\ln 2(-20+\pi^2)-5\pi^2)\big)+C_A\big(C_A(-23758$$
$$+18\ln 2(1963-96\pi^2)-24246\ln^2 2+10368\ln^3 2+1251\pi^2+6237\zeta_3)$$
$$+2n_l(218-306\ln 2+162\ln^2 2-9\pi^2)\big)\Big]\ln\beta+C^{(2)}_{gg,\mathbf{1}}\,, \quad (6)$$

$$\sigma^{(2,0)}_{gg,\mathbf{8}} = \frac{(2C_F-C_A)^2\pi^4}{3\beta^2}+\frac{(2C_F-C_A)\pi^2}{18\beta}\Big[576C_A\ln^2\beta+12\big(C_A(-23+48\ln 2)+2n_l\big)\ln\beta$$
$$+18C_F\big(-20+\pi^2\big)+C_A\big(278-132\ln 2-3\pi^2\big)+4n_l\big(-5+6\ln 2\big)\Big]+512C_A^2\ln^4\beta$$
$$+\frac{128}{9}C_A\Big[C_A\big(-173+216\ln 2\big)+2n_l\Big]\ln^3\beta+\frac{16}{9}C_A\Big[18C_F\big(-20+\pi^2\big)$$
$$+C_A\big(4553-6156\ln 2+3744\ln^2 2-201\pi^2\big)+2n_l\big(-37+36\ln 2\big)\Big]\ln^2\beta$$
$$+\frac{4}{27}\Big[54C_F\big(-4C_F\pi^2+C_A(180+12\ln 2(-20+\pi^2)-7\pi^2)\big)+C_A\big(C_A(-111418$$
$$+36\ln 2(4499-201\pi^2)-105624\ln^2 2+41472\ln^3 2+5823\pi^2+24840\zeta_3)$$
$$+4n_l(505-666\ln 2+324\ln^2 2-18\pi^2)\big)\Big]\ln\beta+C^{(2)}_{gg,\mathbf{8}}\,. \quad (7)$$

Threshold approximations from resummations in top pair production

- Replacing the (unknown) exact NNLO result with its soft approximation (prev. page) became known as $NNLO_{approx}$ approaches.

Warning: the reliability of such approaches in approximating the full result is questionable. Comparisons with exact results show that subleading terms could indeed be numerically large.

- Another subtlety: in top production, there is another effect that lives close to threshold (i.e. same kinematics, different physics): bound state formation.

- These produce additional power singularities in the threshold region (i.e. soft resummation leads to $Log^n[\beta]$ terms, while Coulombic interactions lead mostly to $1/\beta^n$ terms.
- Interestingly, the numerical impact of these additional(and stronger) singularities is small. we need to be very close to threshold to notice their effect (which is very rare).

Top pair production: pheno

Convergence of perturbation theory: excellent for the total x-section through NNLO

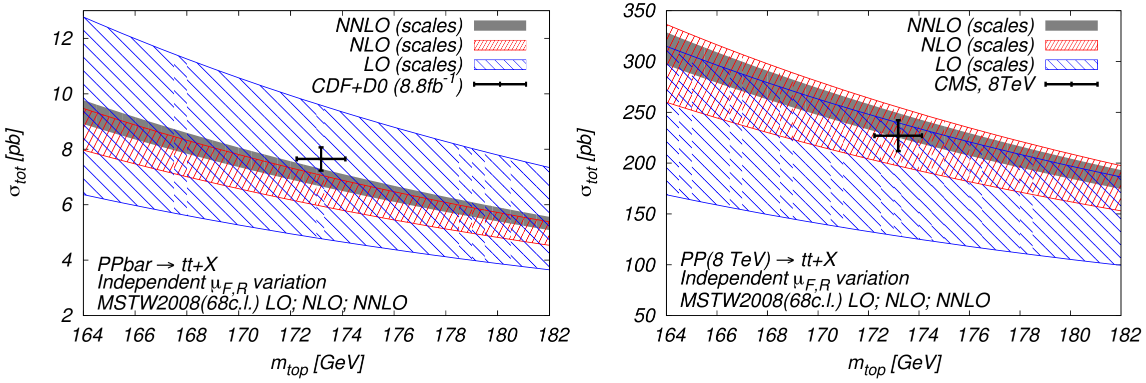


Fig. 1. – Scale dependence of the total cross-section at LO (blue), NLO (red) and NNLO (black) as a function of $m_{\rm top}$ at the Tevatron (left) and the LHC 8 TeV (right). No soft gluon resummation is included. For reference the most precise experimental measurements are also shown.

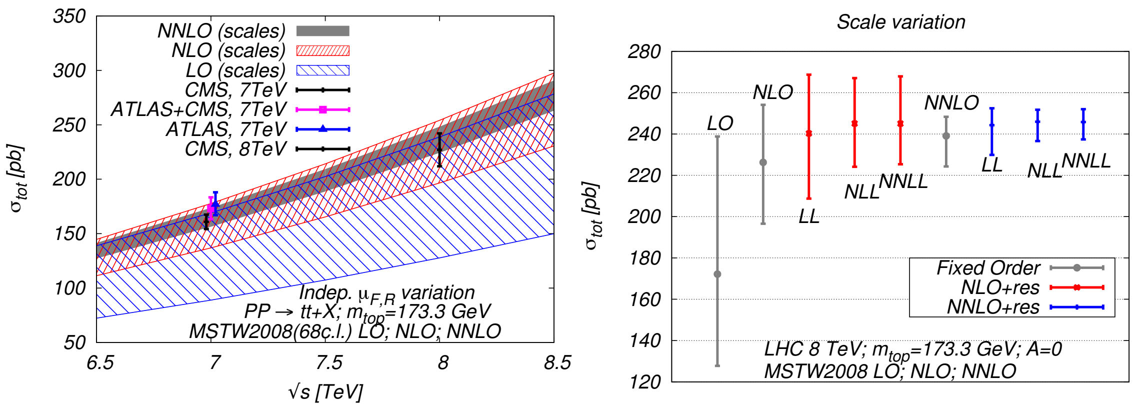


Fig. 2. – Scale dependence of the predicted cross-section at LO, NLO and NNLO at the LHC as a function of $\sqrt{s}$ (left). On the right plot: detailed breakdown of scale uncertainty for LHC 8 TeV at LO, NLO and NNLO including also soft-gluon resummation at LL, NLL and NNLL.

- The best proof the soft gluon resummation matters exactly where it is needed:

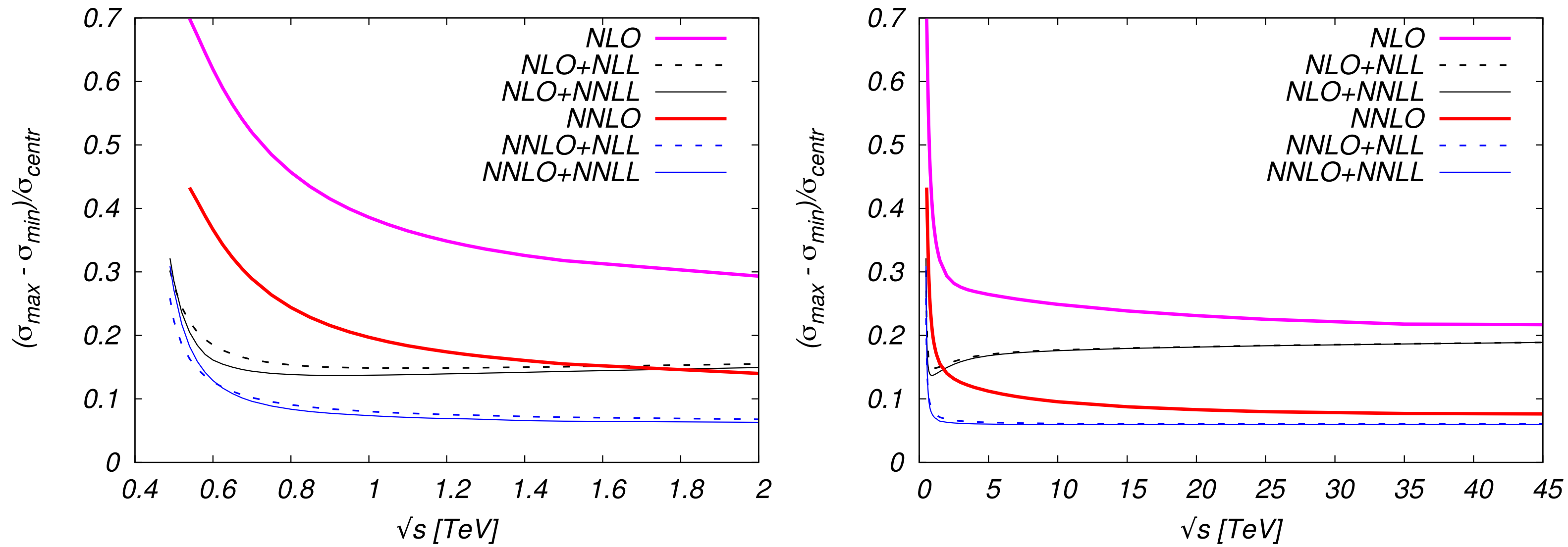


Fig. 3. – The relative scale uncertainty of the $t\bar{t}$ cross-section, computed as a function of the LHC collider energy at fixed order (NLO and NNLO) and including with soft-gluon resummation (NLL and NNLL).

- Resummed results better behaving closer to threshold; Fixed Order perturbation theory starts to fail close to threshold (as it should)

- A note on scale variation:
- To estimate the error from missing higher-order terms we use the size of scale variation.
- This can be done in a number of ways, and it does make a difference:
    - $\mu_F = \mu_R$ and the two are varied together
    - μ_F =/= μ_R and the two are varied independently, but restricted to their ratio in (0.5 , 2)
- The difference is yet higher order terms and the two approaches are formally equivalent
- Yet it is noticed than the second yields much larger variation (could be up to a factor of 3)
- The previous plots demonstrates that he second approach produces scale variation which could be interpreted as due to missing higher order terms.

Effect of running scales:

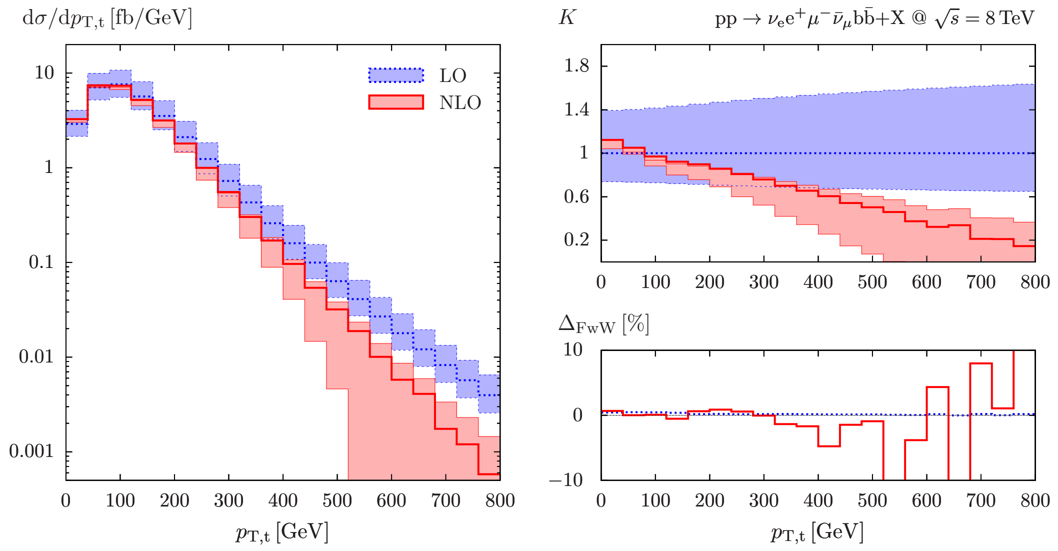


Figure 9: Transverse-momentum distribution of the top quark with standard cuts for the LHC at $\sqrt{s} = 8$ TeV for fixed scale $\mu_0 = m_t/2$.

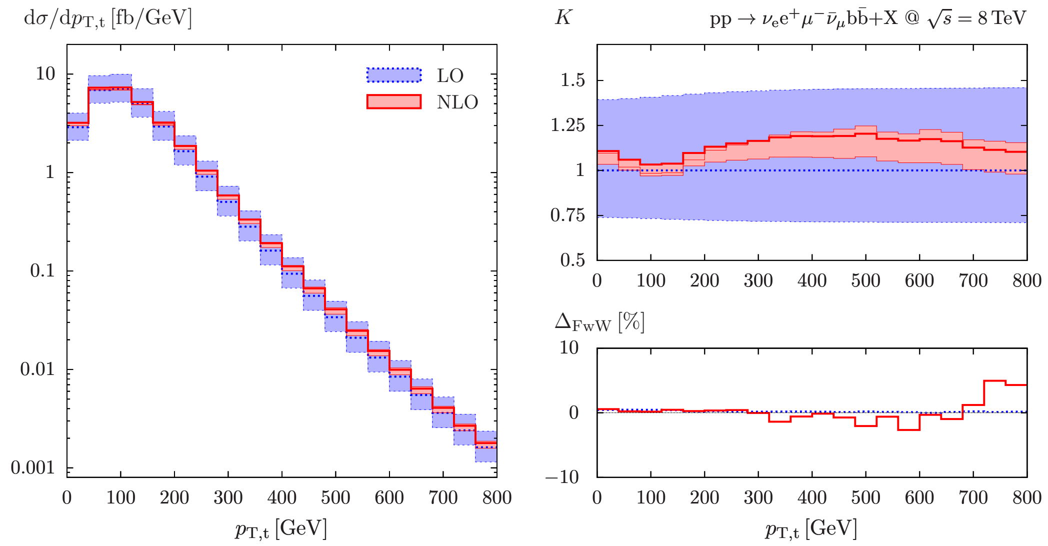
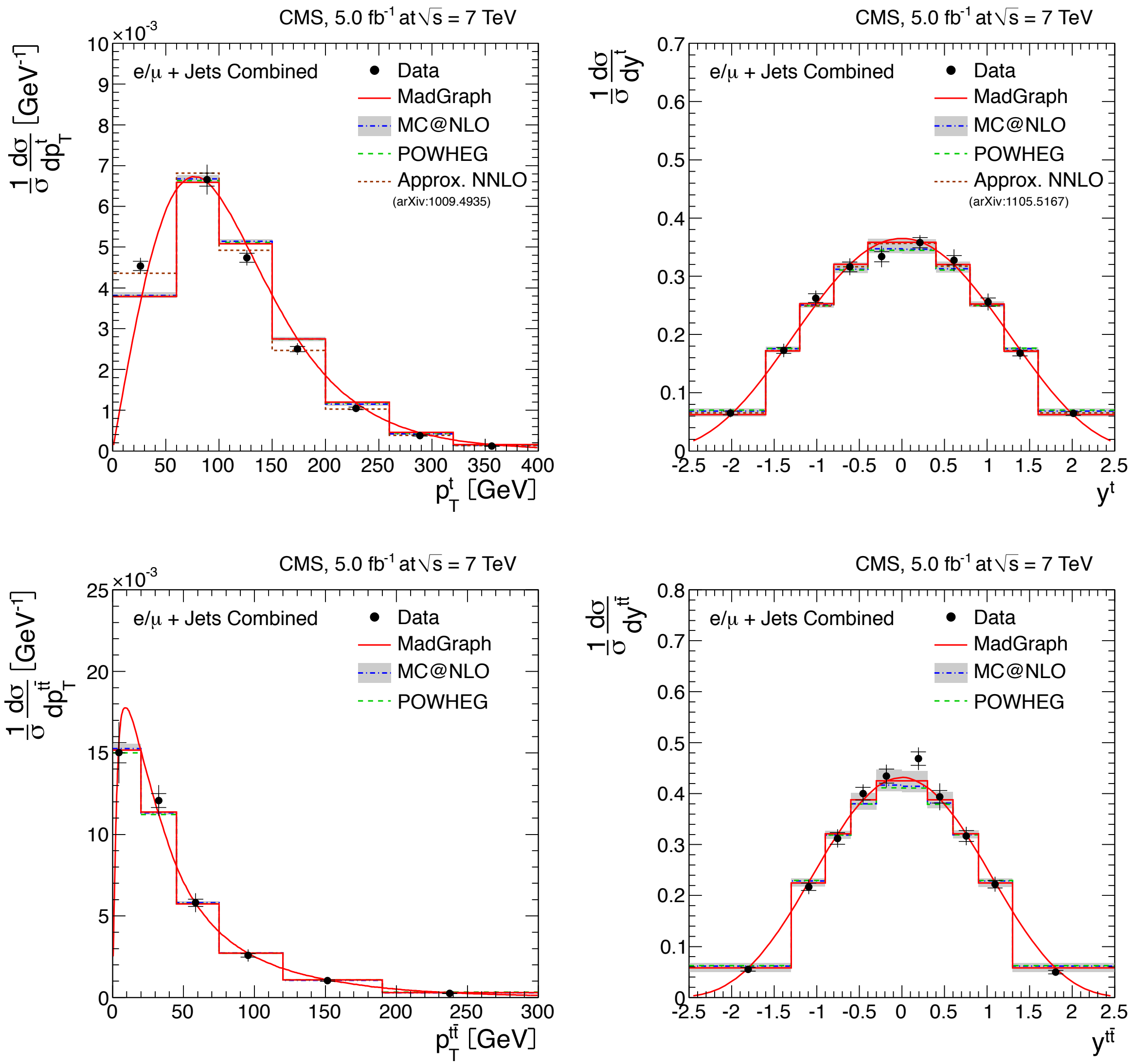


Figure 11: Transverse-momentum distribution of the top quark with standard cuts for the LHC at $\sqrt{s} = 8$ TeV for dynamical scale $\mu_0 = E_T/2$.

From Arxiv:1207.5018

Top differential spectra (LHC 7 TeV, CMS)



Interesting discrepancy; is it from higher order terms; or top reconstruction?