

Lecture 0

The Standard Higgs Model

It is assumed that you are already familiar with the standard formulation. The aim here is to give you a less standard viewpoint on the subject.

Before Higgs discovery, EW interactions were well described by the so-called "EW Theory":

$$W_\mu^\alpha : \text{gauge fields of } SU(2)_L \Rightarrow W_\mu = W_\mu^a T_L^a$$

~~$\frac{1}{\sqrt{2}}(B_\mu - iE_\mu)$~~

$$B_\mu : \text{gauge field of } U(1)_Y$$

$$B_\mu \rightarrow B_\mu + \frac{1}{g'} \partial_\mu \phi_Y$$

$\gamma_L = e^{-i\beta_\alpha T_L^a}$

The couplings g and g' were measured from the SM fermion interactions with vector bosons:

$$l \rightarrow W, B \quad \Gamma \text{ for instance } G_F \sim \frac{1}{m_W^2}$$

very important: The fermion couplings respect the $G_{SM} = SU(2)_L \times U(1)_Y$ gauge group, defined by the transformations above

Γ for instance, ~~coupling universality!~~

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The electric charge is:

$$Q = T_L^3 + Y \Rightarrow \begin{cases} W_\mu^\pm = \frac{W_\mu^1 \mp i W_\mu^2}{\sqrt{2}} \rightarrow q = \pm 1 \\ W_\mu^3, B \rightarrow q = 0 \end{cases}$$

The photon and Z field are orthogonal combinations of W^3, B :

$$A = c_w B + s_w W^3$$

$$Z = c_w W^3 - s_w B$$

where $s_w = \sin \theta_w$, $c_w = \cos \theta_w$, θ_w is the Weinberg (or Wedek) angle

You all know the formulas for θ_w :

$$s_w = \frac{g'}{\sqrt{g^2 + g'^2}}$$

$$c_w = \frac{g}{\sqrt{g^2 + g'^2}}$$

it is interesting to derive θ_w , in order to show that these formulas have nothing to do with the Higgs model

The photon is defined to be the gauge field associated with Q. Thus it must transforms

$$A_\mu \xrightarrow{Q} A_\mu + \frac{1}{e} \partial_\mu \beta_q$$

\hookrightarrow with electric charge to be determined

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The Z is defined to be orthogonal to A and not to transform under Q :

$$Z_\mu \xrightarrow{Q} Z_\mu$$

the Q local transformation is:

$$\begin{aligned} g_Q &= e^{i\beta_q \cdot Q} = e^{i\beta_q T_L^3} \cdot e^{i\beta_q Y} \\ &= e^{i\beta_3 T_L^3} \cdot e^{i\beta_Y Y} \end{aligned}$$

Q = "combined action of $SU(2)_L$ and $(U(1)_Y$ with:

$$\beta_2 = \{\partial, 0, \beta_q\}; \quad \beta_Y = \beta_q$$

$$\Rightarrow W_\mu^3 \rightarrow W_\mu^3 + \frac{1}{g} \partial_\mu \beta_q$$

$$B_\mu \rightarrow B_\mu + \frac{1}{g'} \partial_\mu \beta_q$$

For the Z being invariant:

$$\frac{C_W}{g} - \frac{D_W}{g'} = 0 \Rightarrow \boxed{\tan \delta_W = \frac{D_W}{C_W} = \frac{g'}{g}}$$

The photon shifts by

$$A_\nu \rightarrow A_\nu + \left(\frac{C_W}{g'} + \frac{D_W}{g} \right) \partial_\nu \beta_q$$

$$\Rightarrow \frac{1}{e} = \frac{g C_W + g' D_W}{g g'} = \frac{1}{g g'} \left[g^2 + g'^2 \right] \cdot \frac{1}{\sqrt{g^2 + g'^2}} =$$

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$$= \sqrt{\frac{g^2 + g'^2}{g^2 g'^2}} = \sqrt{\frac{1}{g^2} + \frac{1}{g'^2}}$$

famous formula (general for $G_1 = G_2 + G_3$)

$$\frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2}$$

- Experiments tell us that $m_A = 0$, while $m_W, m_Z \neq 0$. The EW Lagrangian is:

$$\begin{aligned} \mathcal{L}^{EW} &= -\frac{1}{4} W_{\mu\nu}^+ W_{\mu\nu}^- - \frac{1}{2} B_{\mu\nu} B_{\mu\nu}^+ \xrightarrow[\text{rotated to } A, Z]{\text{scatdbly}} \\ &\quad + m_W^2 W_{\mu}^+ W_{\mu}^- + \frac{1}{2} m_Z^2 Z_{\mu} Z^{\mu} \\ &= \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{mass}} \end{aligned}$$

$\mathcal{L}_{\text{gauge}}$ is gauge-invariant. It leads to gauge field interactions like

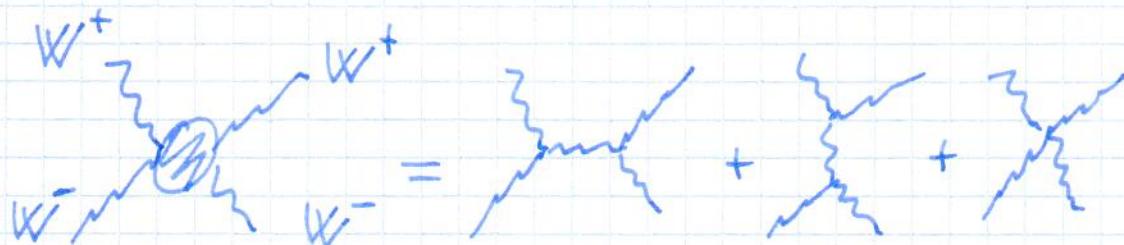
$$W_L^3 Z^2 \xrightarrow[\text{gauge inv.}]{} \frac{W_L^3}{\sim g^2} + \dots$$

Though not as precisely as for fermions, these interactions have been tested experimentally.

Therefore: gauge invariance confirmed
in couplings $\rightsquigarrow \mathcal{L}_{\text{gauge}}$
gauge invariance disproved
in mass $\rightsquigarrow \mathcal{L}_{\text{mass}}$ is not invariant

Why gauge invariance is good for interactions and not for the mass?

Not just a conceptual problem. Imagine computing the scattering of W bosons:



$$\lambda = \frac{g^2}{4m_W^2} (\sigma + t) + O(\frac{E}{m_W})$$

$\sigma, t \sim E^2$ = "Mandelstam Variables"

For $E \gg m_W$, the amplitude grows, and this is bad.

Any $2 \rightarrow 2$ amplitude must be below a critical threshold of $16\pi^2$:

$$X \sim \lambda \Rightarrow X \sim \frac{1}{16\pi^2} \Rightarrow \text{for } \lambda = 16\pi^2$$

For $\lambda > 16\pi^2$, theory is non-perturbative. We can not trust any calculation, since theory is not applicable any longer, we need another theory.

$$\Rightarrow A \not\ll 16\pi^2 \Rightarrow E < \frac{4\pi}{16\pi^2} \frac{2m_W}{3} \sim 3 \text{ TeV}$$

Maximal cutoff Λ_{MAX} : maximal energy at which the theory is applicable:

$$\Lambda_{MAX} = 4\pi \frac{2m_W}{g} \approx 3 \text{ TeV}$$

The EW theory needs a completion:

$$\begin{array}{c} \uparrow \Lambda_{MAX} \\ \vdash \Lambda \\ \dashv m_{\nu, \dots} \end{array}$$

"true cutoff Λ ", new physics scale
 $\Lambda < \Lambda_{MAX}$

This argument is the "No-Lose Theorem" by which we knew that something Beyond the EW Theory had to be discovered at the LHC!

- The problem is so important that it is worth discussing in more detail.

First, the growth comes from longitudinal W scattering, transverse are OK:

$$W_L \quad W_L \quad W_L \quad W_L$$

$$W_L = \overleftrightarrow{\downarrow} ; \quad W_T = \overrightarrow{\text{---}} \text{ or } \overleftarrow{\text{---}}$$

The reason is that W_L have a polarization vector

$$E_L^L \xrightarrow{E \gg m_W} \frac{p^L}{m_W} \rightarrow \text{grows like } E$$

There would be no problem for a massless particle, which has no "L" component

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WW scattering problem is deeply related with masses, which in turn are related with gauge symmetry "violation".

Second, gauge symmetry covariance is not a symmetry, and as such it can not be broken. Gauge transformations transform physical states into unphysical ones. Do not correspond to physical relations among physical quantities. Think to e.m. you always use charge conservation, which is a global element of $U(1)_q$. You never use any "local symmetry". Gauge invariance is a technical tool to describe spin one (and two) force carriers in a Local, Quantum and Relativistic way.

If this is true, then I can reformulate the EW theory in a perfectly gauge invariant way:

Stueckelberg trick (just a trick, but
"a deep one")

"Add extra scalar fields, which are unphysical as they disappear in the unitary gauge, in a way that restore gauge invariance"

⑧

Many ways to do this, all equivalent since the Stueckelberg field is unphysical. A convenient option in our case is to $e \chi_\alpha(x) \frac{\sigma^\alpha}{2}$ \rightarrow now, as a free parameter

$$\Sigma(x) = e \rightarrow 2 \times 2 \text{ Unitary matrix, } \Sigma^+ \Sigma = \mathbb{1}, \det[\Sigma] = 1$$

The Σ matrix transforms as:

$$\Sigma \rightarrow \gamma_L \Sigma \gamma_R^+ \quad \text{where} \quad \gamma_L = e^{e \beta_2 \frac{\sigma^2}{2}}$$

$\text{SO}(2)_L \quad \hookrightarrow \text{U}(1)_Y$

$$\gamma_Y = e^{e \beta_Y \frac{\sigma^3}{2}}$$

$$D_\mu \Sigma = \partial_\mu \Sigma - e g W_\mu^\alpha \frac{\sigma^\alpha}{2} \Sigma + e g' B_\mu \Sigma \frac{\sigma^3}{2}$$

You can check that this is the correct definition

- The Σ fields ~~are~~ unphysical, can be set to zero by a gauge transformation.

By gauge-fixing, we could always go to the

$$\text{Unitary Gauge : } \Sigma_{\text{Unit.}} = \mathbb{1}$$

- Consider the following object:

$$e \bar{\Sigma}^+ D_\mu \Sigma = e \bar{\Sigma}^+ J_\mu \Sigma + g_W W_\mu^\alpha \bar{\Sigma}^+ \frac{\sigma^\alpha}{2} \Sigma +$$

$$- g' B_\mu \frac{\sigma^3}{2}$$

It is a field-dependent matrix, transforming ⑨
as:

$$\epsilon \Sigma^+ D_\mu \Sigma \rightarrow \gamma_y (\epsilon \Sigma^+ D_\mu \Sigma) \gamma_y^+$$

In the unitary gauge,

$$\begin{aligned} \epsilon \Sigma^+ D_\mu \Sigma &|_{\text{U.G.}} = g W_\mu^2 \frac{\sigma^2}{2} + g' B_\mu \frac{\sigma^3}{2} \\ &= g W_\mu^{1,2} \frac{\sigma^{1,2}}{2} + \underbrace{(g W_\mu^3 - g' B_\mu)}_{= \sqrt{g^2 + g'^2} Z_\mu} \frac{\sigma^3}{2} \\ &= \frac{g}{c_W} Z_\mu \end{aligned}$$

Therefore :

$$\text{Tr} [k \Sigma^+ D_\mu \Sigma]^2 |_{\text{U.G.}} = \frac{g^2}{2} (W_\mu^{1,2})^2 + \frac{g^2}{2 c_W^2} Z_\mu^2$$

With a Lagrangian:

$$\begin{aligned} \mathcal{L}_\Sigma &= \frac{v^2}{4} \text{Tr} [(D_\mu \Sigma)^+ \Sigma^+ D_\mu \Sigma] \\ &\stackrel{\text{U.G.}}{=} \frac{g^2 v^2}{4} |W^\pm|^2 + \frac{g^2 v^2}{4 c_W^2} \cdot \frac{1}{2} Z^2 \end{aligned}$$

We reproduce the correct W boson mass by choosing

$$v = \frac{2 m_W}{g} = 246 \text{ GeV} \xrightarrow{\text{EWSSB, scale!}}$$

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We typically view " v " as the Higgs VEV, however we see now that its definition has nothing to do with the Higgs model!

By Σ , choosing v , we reproduced m_W , but what about m_Z ?

Ideally, Σ had two independent parameters for m_W and m_Z . Here we do get a Z mass term, but *a priori*, it has no reason to be the right one. We need a second parameter v' (*a priori*)

Consider another object: We will come back
on this point later

$$\epsilon \Sigma^+ D_\mu \cdot \sigma_3 \rightarrow \gamma_y (\epsilon \Sigma^+ D_\mu \sigma_3) \gamma_y^+$$

because $\gamma_y = e^{\frac{i \omega_y \sigma_3}{2}} \Rightarrow [\sigma_3, \gamma_y] = 0$

it transforms well, so I can use it in the Lagrangian. Consider:

$$\epsilon \Sigma^+ D_\mu \bar{\sigma}_\mu = g W_\mu \frac{1,2 \sigma^{1,2}}{2} \sigma_3 + \frac{g}{c_W} Z_\mu \frac{\sigma_3^2}{2}$$

$$\text{Tr} [\epsilon \Sigma^+ D_\mu \Sigma \cdot \sigma_3] = \frac{g}{c_W} Z_\mu \xrightarrow{\text{Tr} [\sigma^a \sigma^b]} \frac{g}{6} Z_\mu$$

We now understand that both Lagrange and Dirac are gauge invariant. The problem with the mass term, therefore, is not that it breaks gauge invariance.

In order to see what the problem is, let me state one important theory everybody should know (and use):

The Equivalence Theorem:

"high-energy ~~V_L~~ amplitudes are equivalent to those of the associated Goldstone scalars"

In this context, the "Goldstone" is just meant to be any scalar that mixes with the V fields.

In our specific case, the χ 's are Goldstones

$$\chi^{\pm} = \frac{\chi_1 \mp i\chi_2}{\sqrt{2}} \rightarrow W_{\pm}^{\pm} \text{ goldstones}$$

$$\chi^0 = \chi_3 \rightarrow 2 \text{ goldstone}$$

It is immediate to verify, expanding Σ :

$$\Sigma \approx \Gamma e^{\frac{e\chi_2 \sigma_2}{v}} \approx 1 + e \frac{e\chi_2}{v} \cdot \sigma_2 + \dots$$

that

$$e[\Sigma^+ D_\mu \Sigma] \approx e \left(2 \partial_\mu \frac{\chi^{1,2}}{v} + g W_\mu^{1,2} \right) \frac{\sigma^{1,2}}{2}$$

$$+ e \left(2 \frac{\partial_\mu \chi^0}{v} + \frac{g}{C_W} Z_\mu \right) \frac{\sigma^0}{2}$$

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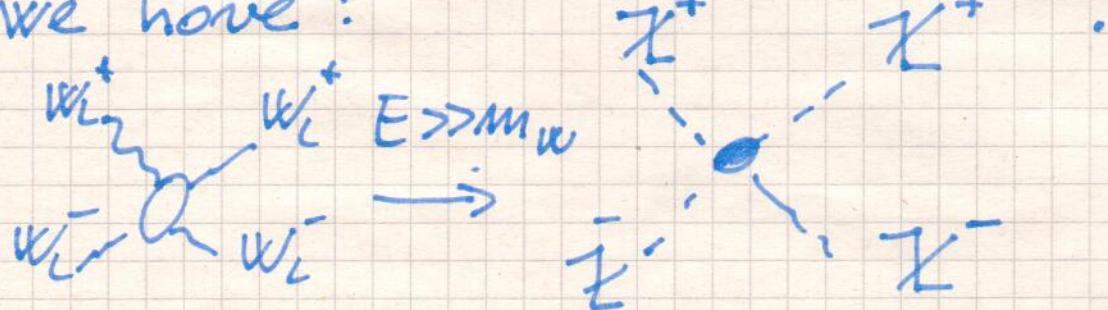
$$\Rightarrow \partial_\mu \chi = \frac{v^2}{4} \left[\frac{1}{2} \left(2 \partial_\mu \frac{\chi^{1,2}}{v} + g W_\mu^{1,2} \right)^2 + \frac{1}{2} \left(2 \frac{\partial_\mu \chi^0}{v} + \frac{g}{C_W} Z_\mu \right)^2 \right]$$

$$= [\partial_\mu \chi^+ + \frac{g v}{2} W_\mu^+]^2 + \frac{1}{2} \left(\partial_\mu \chi^0 + \frac{g v}{2 C_W} Z_\mu \right)^2$$

χ are canonically normalized modulus scale, mix with terms like

$$m_W \partial_\mu \chi^+ W_\mu^- + \text{h.c.}$$

thus we can apply the E.T. and we have:



The WW scattering problem comes from $\chi\chi$ scattering growing with energy.

The dangerous χ interactions, not surprisingly, come from \mathcal{L}_Σ :

$$\Sigma = e^{\chi \frac{\sigma^a}{v}} = \cos\left[\frac{|\vec{\chi}|}{v}\right] \cdot 1 + e^{\sin\left[\frac{|\vec{\chi}|}{v}\right]} \times \frac{1}{|\vec{\chi}|v} + \frac{\chi^2 \sigma^a}{v} \quad \text{to } O(\chi^2)$$

Σ is a highly non-linear combination of fields:

$\mathcal{L}_\Sigma \rightarrow$ relevant of $d > 4$ interactions

$v^2 \text{tr} [\bar{\chi} \Sigma^\dagger \bar{\chi}]$ couplings with negative energy dimension:

$$\mathcal{L}_\Sigma \sim v^2 \times J^2 \times \left(\frac{\chi}{v}\right)^{\text{ANY POWER}}$$

Two dots

In particular, one would find a 4-leg interaction:

$$v^2 \cdot \frac{1}{v^4} J^2 \chi^4$$

$$\therefore \sim \frac{P^2}{v^2} \Rightarrow \text{Amplitude grows!}$$

our result was:

$$A = \frac{g^2}{4m_W^2} (s+t) = \frac{1}{v^2} (s+t)$$

the result has the correct scaling

Exercise: check the coefficient.

Suggestion: use Mathematica to expand first

$$\langle I |^{+} D_{\mu} | I \rangle$$

The growth with the energy obliges us to abandon the EW theory and to add a new sector:

EWSB sector = "set of particles and interactions that solve the strong coupling issue"

The Higgs model is the minimal possibility:

h = real scalar, with mass m_h

h = "SM singlet", thus we can couple it to I as:

$$\frac{v^2}{2} \alpha h \text{tr} [D_\mu Z^+ D^\mu Z^-]$$

↳ arbitrary coefficient

This gives a vertex:

$$\frac{2\alpha^2 h |D_\mu Z^+|^2}{v} \rightarrow \begin{array}{c} + \\ \diagup \quad \diagdown \\ \text{---} \end{array} = \frac{2\alpha^2 P_+ \cdot P_-}{v}$$

and thus a contribution to the scattering:

$$\begin{array}{c} + \\ \diagup \quad \diagdown \\ \text{---} \end{array} \quad \begin{array}{c} + \\ \diagup \quad \diagdown \\ \text{---} \end{array} = t^2$$

$$= -\frac{1/v^2}{\sqrt{Q^2}} \frac{4(P_{out}^+ \cdot P_{in}^+) \cdot (P_{out}^- \cdot P_{in}^-)}{t - m_H^2} \frac{4f^2}{-\sqrt{Q^2} \frac{\Delta^2}{\Delta - m_H^2}}$$

$$A = \frac{1}{v^2} \left[\Delta + t - Q^2 \frac{\Delta^2}{\Delta - m_H^2} - Q^2 \frac{t^2}{t - m_H^2} \right]$$

~~$$= \frac{1-Q^2}{v^2} (\Delta + t) + O\left(\frac{E^2}{m_H^2}\right)$$~~

for $\alpha = 1$, the growth is canceled:

actually the ~~$\frac{1}{v^2}$~~ is a ~~weak~~ and so changes sign

$$|k^\infty| = \frac{M_H^2}{v^2} = \lambda_4^H$$

Provided m_H is low enough,

$$m_H < 4\pi v \sim 3 \text{ TeV} (= \Lambda_{\text{MSB}})$$

the theory stays perturbative

Back to unitary gauge, $\mathcal{I}=1$,
the Higgs interaction becomes

$$\alpha \frac{v}{2} h \left[g^2 |W|^2 + \frac{g^2}{2c_w^2} |Z|^2 \right]$$

$$\Rightarrow \begin{array}{c} h \\ \text{---} \end{array} \begin{array}{c} W \\ \text{---} \end{array} = \alpha \frac{g^2 v}{2}$$

$$\begin{array}{c} Z \\ \text{---} \end{array} = \alpha \frac{g^2 v}{2c_w^2}$$

$\alpha=1$ is the
only choice
of physical
Higgs coupling
to vector that
solves the WW
issue!

Let us finally go back to the standard formulation. Nobody would have believed the Higgs model if it had been introduced as I did... this is only partially the problem.

The model is defined by a doublet

$$H = \begin{pmatrix} h_u \\ h_d \end{pmatrix} \in \mathbb{Z}_{1/2} \text{ of } SU(2)_c \times U(1)_Y$$

with a potential $V(H) \rightarrow \langle H^\dagger H \rangle = \frac{v^2}{2}$

$$Q = T_3 + Y = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$\Rightarrow h_d$ is neutral and takes vev without breaking Q :

$$\langle H \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

The Lagrangian is:

$$\mathcal{L}_H = (D_\mu H)^+ D^\mu H - V[H]$$

$$D_\mu H = \partial_\mu H - e g_W \frac{\sigma^i}{2} H \pm e \frac{\sigma^i}{2} B_\mu H$$

In order to make contact with the previous notation, spontaneous fluctuations around the VEVs is follows?

$$K_{\alpha\beta} = \frac{1}{2} \left(\frac{\partial^2 V}{\partial h^\alpha \partial h^\beta} \right)$$

consider the following track:

- $H \in \mathbb{Z}_{1/2}$

$$H^c = (\cos \theta) H \in \mathbb{Z}_{-1/2}$$

$SU(2)_L$ is a pseudo-real group,
 $H^c = \begin{pmatrix} +h_d \\ -h_u \end{pmatrix}$

Define a 2×2 matrix:

$$\mathcal{H} = (H^c, H)$$

it transforms like the Σ field

$$\mathcal{H} \rightarrow \delta_L \mathcal{H} \delta_y^+$$

It can be shown that \mathcal{H} is a pseudo-real matrix:

$$\mathcal{H}' = \sigma^2 \mathcal{H} \sigma^2$$

which implies that it can be parametrized by:

$$\mathcal{H} = \underbrace{\xi}_\text{numbers} \cdot \underbrace{V}_\text{unitary matrix } \det=1$$

we immediately see that

$$\langle \mathcal{H} \rangle = \begin{pmatrix} \frac{v}{\sqrt{2}} & 0 \\ 0 & \frac{v}{\sqrt{2}} \end{pmatrix} = \frac{v}{\sqrt{2}} \cdot \mathbb{I}$$

We can thus parametrize fluctuations

as:

$$\mathcal{H} = \frac{v+h}{\sqrt{2}} \cdot I$$

by this field redefinition we get the same Lagrangian as before:

$$D_\mu \mathcal{H} = \partial_\mu \mathcal{H} - e g W_\mu \frac{g^2 \sigma_2}{2} \mathcal{H} + e g' B_\mu \mathcal{H} \frac{\sigma_3}{2}$$

$$\frac{1}{2} \text{Tr} [(D_\mu \mathcal{H})^\dagger D_\nu \mathcal{H}] = \text{Tr} [D_\mu H^\dagger D_\nu H] =$$

$$\frac{(v+h)^2}{4} \text{Tr} [D_\mu \Sigma^\dagger D_\nu \Sigma] + \frac{1}{4} (\partial_\mu h)^2 \text{Tr} [\bar{\Sigma}^\dagger \bar{\Sigma}]$$

Higgs kinetic term, mass from the potential

$$V[H^2] = V[\frac{1}{2} \text{Tr} [\partial \mathcal{H}^\dagger \partial \mathcal{H}]]$$

$$\Rightarrow \mathcal{L}_{h\Sigma} = \frac{v \cdot h}{2} \text{Tr} [-] \Rightarrow Q=1$$

$$\mathcal{L}_{hh\Sigma} = \frac{h^2}{4} \text{Tr} [-] \rightarrow \begin{array}{c} h \\ h \\ h \end{array} \xrightarrow{\text{W}} \text{vertices}$$

Now we understand why $\alpha=1$ leads to the cancellation:

For $\alpha=1$ the complicated non linear theory is equivalent to a simple quadratic $d=4$ Lagrangian. Amplitudes can not have power-low growth in a renormalizable theory

- Masses and coupling to fermions:

$$\mathcal{L}_Y = \gamma_L \bar{q}_L H \ell_R + \gamma_R \bar{q}_L H \ell_R + \gamma_L \bar{L}_L H \ell_R$$

$$m_f = \frac{\chi v}{\sqrt{2}}, \quad \text{---} \swarrow \text{---} \nearrow = - \frac{\gamma_f}{\sqrt{2}}$$

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \in (3, 2_{1/3})$$

$$u_R \in (3, 1_{2/3})$$

$$d_R \in (3, 1_{-1/3})$$

$$L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \in (1, 2_{-1/2})$$

$$e_R \in (1, 1_1)$$

- Basic phenomenology (slides)

Lecture 0'

Custodial Symmetry

- We wrote:

$$\mathcal{L}_\Sigma = \frac{v^2}{4} \text{tr} [(\bar{D}\Sigma)^+ D^* \Sigma]$$

and we chose

$$v = \frac{2m_\pi}{g}$$

in order to reproduce m_π , but what about m_2 ?

In general, we would need a second term to adjust:

$$\mathcal{L}'_\Sigma = \frac{v'^2}{8} \left| \text{tr} [\epsilon \Sigma^+ D_\mu \Sigma \cdot \sigma^3] \right|^2$$

$$\Sigma \rightarrow \gamma_L \Sigma \gamma_Y^+ ; \quad \gamma_Y = e^{i \beta_Y \sigma_2^3 / 2}$$

$\Rightarrow [\gamma_Y, \sigma_3] = 0$, the ^{new} term is perfectly allowed.

$$\mathcal{L}'_\Sigma|_{\text{UG}} = \frac{1}{2} \frac{g^2 v'^2}{4 C_W^2} Z^2 \quad \begin{matrix} \text{only a } Z \\ \text{mass} \end{matrix}$$

$$\Rightarrow (\mathcal{L}_\Sigma + \mathcal{L}'_\Sigma)|_{\text{UG}} = \frac{g^2 v^2}{4} |W|^2 + \frac{1}{2} \frac{g^2 v'^2}{4 C_W^2} Z^2 + \frac{1}{2} \frac{g^2 v'^2}{4 C_W^2} Z^2$$

$\hookrightarrow m_\pi^2$

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With v' (v'^2 could be negative) I can reproduce
 M_2 :

$$M_2^2 = \frac{g^2 v^2}{4C_W^2} + \frac{g' v'^2}{4C_W^2} = \frac{M_W^2}{C_W^2} \left[1 + \frac{v'^2}{v^2} \right]$$

$$\Rightarrow \left[1 + \frac{v'^2}{v^2} \right] = \frac{C_W^2 M_2^2}{M_W^2} = \frac{1}{\rho}$$

The "ρ parameter". Experimentally:

$$\rho = 1 + \delta(1\%)$$

\hookrightarrow easily explained by
radiative corrections

At our level of approximation:

$$\rho = 1 \Rightarrow v' = 0$$

This can be seen as the result of a symmetry:

- Take the limit $g' \rightarrow 0$:

$$D_\mu \Sigma = \partial_\mu \Sigma - g W_\mu \frac{\alpha \sigma_2}{2} \Sigma$$

- Define custodial group $SU(2)_L \times SU(2)_R$:

$$\Sigma \rightarrow g_L \Sigma g_R^+ \quad (\underline{\text{global group}})$$

$$W_\mu = W_\mu \frac{\alpha \sigma_2}{2} \rightarrow g_L W_\mu g_R^+$$

(3)

$$\Sigma \in (2, 2)$$

$W \in (3, 1) \rightarrow W$'s form a custodial triplet

• The same considerations hold for the Higgs model. In that case

$$\mathcal{L} = (H^c, H) \in (2, 2) \quad \boxed{1}$$

$$D_\mu I \rightarrow g_L D_\mu I g_R^+$$

L_I is custodial invariant

$L_{\Sigma'}$ is custodial breaking

$\rho = 0$ (i.e. $\rho = 1$) is an experimental evidence
of custodial symmetry

→ Custodial symmetry holds for the SM,
with \mathcal{L} matrix transforming as...

the h field is a custodial singlet

Theorem : Any custodial invariant theory has
 $\rho = 1$

Proof : Effective mass-Lagrangian ($I=1$)

$$m_W^2 |W|^2 + \frac{m_\phi^2}{2} \tilde{\chi}^2$$

Imagine this coming from complicated EWSB sector: 4

$$\frac{W/B}{m} \stackrel{h}{\cancel{\text{down}}} \frac{W/B}{1\rho_{co}} \textcircled{D} = \text{loops of } h, h', \rho \text{ of } t_c \dots, \underline{\text{NO } B}$$

but all custodial invariant

The photon will not get a mass because of $U(1)_q$, and \tilde{Z} is given, in full generality, by

$$\tilde{Z} = C_W W^3 \tilde{s} - \Delta_W B$$

$$\Rightarrow \frac{M_W^2}{2} [W_1^2 + W_2^2] + \frac{M_Z^2}{2} C_W^2 W_3^2 - \Delta_W^2 \Delta_W C_W W_3 B \\ + \frac{M_Z^2}{2} B^2$$

According to custodial, W 's are a triplet

Terms with B break custodial, as they should since g' breaks it

Terms without B , instead, must respect the symmetry:

$$\Delta_W \propto [W_1^2 + W_2^2 + W_3^2] \Rightarrow \boxed{M_W^2 = M_Z^2 C_W^2}$$

- $\rho \neq 1$ comes from B loops, but also from top quark loops.

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interesting to see how custodial works for fermions : Set $g' = 0$:

$$\bar{q}_L \not{D} q_L = \bar{q}_L \not{\partial} q_{L2} + \bar{q}_L \not{W} q_L$$

$$\bar{q}_R \not{D} q_R = \bar{q}_R \not{\partial} \bar{u}_R e \not{D} u_R + \bar{d}_R \not{\partial} \not{D} d_R$$

$$\Rightarrow q_L \in (2, 1) \\ q_R \in (1, 2)$$

Kinetic terms are invariant with this assignment

However the mass-terms (or similarly the Yukawa's in the Higgs model) are

$$m_C \bar{q}_L \sum q_R + m_{\bar{C}} \bar{q}_L \sum \cdot \sigma_3 q_R$$

$$= (m_C + m_{\bar{C}}) \bar{u}_L u_R + (m_e - m_{\bar{e}}) \bar{d}_L d_R$$

U.G.

$$\text{If } m_{\bar{e}} = 0 ; m_u = m_d$$

$$\text{Instead, } m_{\bar{e}} = \frac{m_u - m_d}{2} \Rightarrow \begin{array}{l} \text{custodial broken} \\ \text{by } u-d \text{ mass} \\ \text{difference} \end{array}$$

The top quark has the largest mass and thus gives the largest contribution to custodial breaking loop.

(6)

- Finally, an important technicity:

$$SU(2)_L \times SU(2)_R \cong SO(4)$$

Illustrate this by looking at the \mathcal{H} matrix:

$$\mathcal{H} = (H^c, H) \rightarrow g_L \mathcal{H} g_R^+$$

is a representation of the chiral group, we should be able to relate it to a rep

[of] $SO(4)$

$$\hookrightarrow \# \text{gen}[SO(4)] = \left(\begin{array}{cccc} \otimes & x & x & x \\ \otimes & x & x & x \\ \otimes & x & 0 & 0 \end{array} \right) \xrightarrow{\text{A.S. Imagen}} G = 3+3$$

A generic 2×2 pseudo-real matrix:

$$\mathcal{H}' = \sigma_2 \mathcal{H} \sigma_2$$

has 4 real components and it can be parametrized

\Rightarrow :

$$\mathcal{H} = \frac{1}{\sqrt{2}} \left[\vec{\Pi}^4 + i \vec{\Omega}_2 \cdot \vec{\Pi}^2 \right]$$

It is:

$$\text{Tr} [\mathcal{H}^\dagger \mathcal{H}] = \frac{1}{2} |\vec{\Pi}|^2$$

$\Rightarrow |\vec{\Pi}|$ is invariant under chiral transformation

$$\Rightarrow SU(2) \times SU(2)_L \subseteq SO(4)$$

\Downarrow because some $\vec{\Pi}$ of generators

$$\Leftrightarrow \vec{\Pi} \in \mathfrak{t} \text{ of } SO(4) \Rightarrow \underline{(2,2)} = \mathfrak{t}$$

It is often useful in explicit calculation to choose $SO(4)$ generators in $SU(2)_L \times SU(2)_R$ notation:

$$S_L \vec{\lambda} = -\delta_\alpha^L \frac{\sigma^\alpha}{2} \vec{\lambda}$$

$$S_L \vec{\Pi} = -\delta_\alpha^L t_L^\alpha \vec{\Pi}$$

$\downarrow \rightarrow$ compute explicitly

$$S_R \vec{\lambda} = -\delta_\alpha^R \lambda \frac{\sigma^\alpha}{2} \Rightarrow S_R \vec{\Pi} = -\delta_\alpha^R t_R^\alpha \vec{\Pi}$$

$$t_L^1 = \begin{pmatrix} 0 & 0 & 0 & -i/2 \\ 0 & 0 & -i/2 & 0 \\ 0 & i/2 & 0 & 0 \\ i/2 & 0 & 0 & 0 \end{pmatrix} \dots$$

$$t_R^1 = \begin{pmatrix} 0 & \dots & i/2 \\ -i/2 & -i/2 & \\ -i/2 & i/2 & \end{pmatrix} \dots$$

and so on. One can check that the $6t$ span the $SO(4)$ algebra and obey:

$$[t_{(L)}^\alpha, t_{(L)}^\beta] = -\epsilon^{\alpha\beta\gamma} t_{(L)}^\gamma$$

$$[t_{(L)}^\alpha, t_{(R)}^\beta] = 0$$