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Flavor Tests

with Rare Decays

3 lectures

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Matter comes in generations

$$\psi \to \psi_i, i = 1, 2, 3$$

commonly labelled with increasing mass, distinguished by 'flavor'.

quarks:
$$\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}$$

leptons: $\begin{pmatrix} \nu_e \\ e \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$

Quark Spectrum



hierarchical! Spectrum spans five orders of magnitude.

Matter comes in generations

$$\psi \to \psi_i, i = 1, 2, 3$$

Complex phenomenology: wide range in spectra, $m_u/m_t \sim 10^{-5}$, CP violation, mixing; Flavor physics intimately linked to the making of the Standard Model. New questions with new physics.

These lectures cover:

- * Flavor in the Standard Model and beyond
- * Rare decays $|\Delta F| = 1 \text{theory}$
- * Rare decays points of recent interest for LHC(b), Belle II, ...

not directly covered: origin of flavor (very deep question)

Flavor Test w Rare Decays

lecture 1: Why Flavor Physics? (Sensitivity to beyond the Standard Model physics)

renormalizable QFT in 3+1 Minkowski space w. local symmetry

$$SU(3)_C \times SU(2)_L \times U(1)_Y \to SU(3)_C \times U(1)_{em}$$

$$\mathcal{L}_{SM} = -\frac{1}{4}F^2 + \bar{\psi}i\mathcal{D}\psi + \frac{1}{2}(D\Phi)^2 - \frac{1}{4}F^2 + \bar{\psi}i\mathcal{D}\psi + \frac{1}{2}(D\Phi)^2 - \frac{1}{4}F^2 + \frac{1}{4}F^2$$

$$\psi + \mu^2 \Phi^{\dagger} \Phi - \lambda (\Phi^{\dagger} \Phi)^2$$

Yukawa interact.

 ψ : fermions (quarks and leptons) $F_{\mu\nu}$: gauge bosons $g^a, \gamma, Z^0, W^{\pm}$

$$D_{\mu} = \partial_{\mu} - ieA_{\mu} + \dots$$

 Φ : Higgs doublet (1dof observation 2012 consistent with SM)

Known fundamental matter comes in generations $\psi \rightarrow \psi_i$, i = 1, 2, 3, subject to identical gauge transformations.

Flavor physics = investigations on generational structure of fermions and BSM partners.

The Standard Model of Particle Physics: Flavor

fields in representations under the SM group $SU(3)_C \times SU(2)_L \times U(1)_Y$ Higgs: $\Phi(1, 2, 1/2)$ hypercharge $Y = Q - T^3$ quarks: $Q_L(3, 2, 1/6)_i$, $D_R(3, 1, -1/3)_i$, $U_R(3, 1, 2/3)_i$ leptons: $L_L(1, 2, -1/2)_i$, $E_R(1, 1, -1)_i$ L: doublet, R:singlet under $SU(2)_L$

$$\mathcal{L}_{SM} = \sum_{\psi=Q,U,D,L,E} \bar{\psi}_i i \not D \psi_i$$
$$-\bar{Q}_{L_i} (Y_u)_{ij} \Phi^C U_{R_j} - \bar{Q}_{L_i} (Y_d)_{ij} \Phi D_{R_j} - \bar{L}_{L_i} (Y_e)_{ij} \Phi E_{R_j}$$
$$+\mathcal{L}_{higgs} + \mathcal{L}_{gauge}, \qquad \Phi^C = i\sigma^2 \Phi^*$$

 $Y_{u,d,e}$: Yukawa matrices (3 × 3, complex), off diagonal entries mix generations; sole sources of flavor in SM. In hypothetical limit $Y_{u,d,e} \rightarrow 0$ SM gains large "flavor-symmetry" $G_F = U(3)_{QL} \times U(3)_{UR} \times U(3)_{DR} \times U(3)_{LL} \times U(3)_{ER}$

The Standard Model of Particle Physics: Flavor

masses from spontaneous breaking of electroweak symmetry $\Phi^T(x) \rightarrow 1/\sqrt{2}(0, v + h(x))$, Higgs vev $\langle \Phi \rangle = v/\sqrt{2} \simeq 174 \text{ GeV}$ $\mathcal{L}_{SM}^{yukawa} = -\bar{Q}_L Y_u \Phi^C U_R - \bar{Q}_L Y_d \Phi D_R - \bar{L}_L Y_e \Phi E_R$

Want mass eigenstates rather than the above gauge eigenstates: perform unitary trafos on quark fields $Q_L = (U_L, D_L), U_R, D_R$ $q_A(gauge) \rightarrow \tilde{q}_A(mass) = V_{A,q}q_A$ with $V_{A,q}V_{A,q}^{\dagger} = 1$. $\mathcal{L}_{SM}^{yukawa} = -\bar{U}_L$ $\underbrace{V_{L,u}^{\dagger}V_{L,u}}_{=1}$ $Y_u\Phi^C$ $\underbrace{V_{R,u}^{\dagger}V_{R,u}}_{=1}$ U_R + down quarks discr(means $p_{A,q}(\Phi) = 1$ and $p_{A,q}(\Phi) = 1$

$$\operatorname{diag}(m_u, m_c, m_t) = \langle \Phi \rangle \cdot \operatorname{diag}(y_u, y_c, y_t) = \langle \Phi \rangle \cdot V_{L,u} Y_u V_{R,u}^{\dagger}$$
$$\operatorname{diag}(m_d, m_s, m_b) = \langle \Phi \rangle \cdot \operatorname{diag}(y_d, y_s, y_b) = \langle \Phi \rangle \cdot V_{L,d} Y_d V_{R,d}^{\dagger}$$

The Standard Model of Particle Physics: Flavor

unitary trafos: $\tilde{q}_A = V_{A,q}q_A$ with $V_{A,q}V_{A,q}^{\dagger} = 1$. $\mathcal{L}_{SM}^{yukawa} = -\bar{U}_L \qquad \underbrace{V_{L,u}^{\dagger}V_{L,u}}_{=1} \qquad Y_u\Phi^C \qquad \underbrace{V_{R,u}^{\dagger}V_{R,u}}_{=1} \qquad U_R + \text{down quarks.}$

$$\operatorname{diag}(m_u, m_c, m_t) = \langle \Phi \rangle \cdot V_{L,u} Y_u V_{R,u}^{\dagger}$$
$$\operatorname{diag}(m_d, m_s, m_b) = \langle \Phi \rangle \cdot V_{L,d} Y_d V_{R,d}^{\dagger}$$

$$\mathcal{L}_{\mathcal{SM}}^{up-mass} = -\underbrace{\bar{U}_L V_{L,u}^{\dagger}}_{\bar{\tilde{U}}_L} \quad \underbrace{V_{L,u} Y_u V_{R,u}^{\dagger}}_{diagonal} \Phi^C \quad \underbrace{V_{R,u} U_R}_{\equiv \tilde{U}_R} = -\overline{\tilde{U}}_{Li} m_{ui} \Phi^C \tilde{U}_{Ri}.$$

The tilde basis are mass eigenstates, down quarks analogously.

What else happened under the basis change in \mathcal{L}_{SM} ?

The SM higgs interactions are strictly flavor diagonal and neutral current gauge interactions γ , Z, g stay being flavor universal, since they dont mix the chiralities, for instance:

$$\begin{split} \bar{U}_L \gamma^{\mu} A_{\mu} U_L &= \bar{U}_L \quad (V_{L,u}^{\dagger} V_{L,u}) \quad \gamma^{\mu} A_{\mu} \quad (V_{L,u}^{\dagger} V_{L,u}) \quad U_L \\ &= \bar{\tilde{U}}_L \gamma^{\mu} A_{\mu} V_{L,u} V_{L,u}^{\dagger} \tilde{U}_L = \bar{\tilde{U}}_L \gamma^{\mu} A_{\mu} \tilde{U}_L \quad \text{nothing has happend!} \end{split}$$

However, lets look at the charged currents W^{\pm} :

$$\bar{U}_L \gamma^{\mu} W^+_{\mu} D_L = \bar{U}_L (V^{\dagger}_{L,u} V_{L,u}) \gamma^{\mu} W^+_{\mu} (V^{\dagger}_{L,d} V_{L,d}) D_L$$
$$= \bar{\tilde{U}}_L \gamma^{\mu} W^+_{\mu} \underbrace{V_{L,u} V^{\dagger}_{L,d}}_{\equiv V_{CKM} = V \neq 1} \tilde{D}_L$$

Since Y_u and Y_d dont diagonalize (as observed!) under same unitary transformations, there is one important net effect related to flavor.

The Standard Model of Particle Physics: CKM

The charged current interaction gets a flavor structure, encoded in the Cabibbo Kobayashi Maskawa (CKM) matrix V.

$$\mathcal{L}_{\rm CC} = -\frac{g}{\sqrt{2}} \left(\bar{\tilde{U}}_L \gamma^\mu W^+_\mu V \tilde{D}_L + \bar{\tilde{D}}_L \gamma^\mu W^-_\mu V^\dagger \tilde{U}_L \right).$$

 V_{ij} connects left-handed up-type quark of the *i*th gen. to left-handed down-type quark of *j*th gen. Intuitive labelling by flavor:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}, \quad V_{13} = V_{ub} \ etc$$

Via W exchange is the only way to change flavor in the SM.

V is unitary

- V is in general complex, and induces CP violation
- V has 4 physical parameters, 3 angles and 1 phase. Why?

unitary 3×3 matrix has 9 parameters. If V would be real, it would be orthogonal and contain 3 'Euler' angles $\Theta_{12}, \Theta_{13}, \Theta_{23}$. Then there should be 9-3=6 phases.

However, perform global, generation-dependent re-phasings of $(\tilde{q}_L)_k \to e^{i\alpha_{q_k}} (\tilde{q}_L)_k$, q = U, D, k = 1, 2, 3 in $\mathcal{L}_{CC} \sim \overline{\tilde{U}}_L \gamma^{\mu} W^+_{\mu} V \widetilde{D}_L$

(Rotate RH fields simultaneously $(\tilde{q}_R)_k \rightarrow e^{i\alpha_{q_k}} (\tilde{q}_R)_k$ to keep quark masses real.)

Removes 5 phases (6 fields have at most 5 indepednt rel. phases).

"PDG" parametrization (exact, fully general)

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

 $s_{ij} \equiv \sin \Theta_{ij}, c_{ij} \equiv \cos \Theta_{ij}$. δ is the CP violating phase.

In Nature, $\delta \sim O(1)$ and V is hierarchical $\Theta_{13} \ll \Theta_{23} \ll \Theta_{12} \ll 1$. Very different – large mixing angles for leptons (PMNS-Matrix): $\Theta_{23} \sim 45^\circ, \Theta_{12} \sim 35^\circ, \Theta_{13} \sim O(10^\circ)$ all O(1) – anarchy? Quark mixing matrix has 1 physical CP violating phase δ_{CKM} .

(with 3 generations)



The Nobel Prize in Physics 2008

"for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics"

"for the discovery of the origin of the broken symmetry which predicts the existence of at least three families of quarks in nature"





Photo: Kyoto University

Toshihide Maskawa

Kobayashi and Maskawa, Prog. Theor. Phys 49 (1973) 652

CP is violated!.. together with Quark Flavor

Quark mixing matrix has 1 physical CP violating phase δ_{CKM} . Verified in $B\bar{B}$ mixing $\sin 2\beta = 0.672 \pm 0.023$ HFAG Aug 2010



 δ_{CKM} is large, O(1)!

CPX also observed in *B*-decay $A_{CP}(B \rightarrow K^{\pm}\pi^{\mp}) = -0.098 \pm 0.013$

HFAG Aug 2010

$$\Gamma(B \to K^+ \pi^-) \neq \Gamma(\bar{B} \to K^- \pi^+)$$

V in Nature is hierarchical $\Theta_{13} \ll \Theta_{23} \ll \Theta_{12} \ll 1$. Wolfenstein parametrization; expansion in $\lambda = \sin \Theta_C$, $A, \rho, \eta \sim \mathcal{O}(1)$

$$V = \begin{pmatrix} 1 - \lambda^2/2 & +\lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & +A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

fits: $\lambda = 0.225$, A = 0.82, $\bar{\rho} = 0.13$, $\bar{\eta} = 0.34$ beyond lowest order $\bar{\rho} = \rho(1 - \lambda^2/2)$ and $\bar{\eta} = \eta(1 - \lambda^2/2)$ $\eta \neq 0$ signals CP violation; third gen. quarks decoupled at order λ^2 . There are in total 10 (known!) param. in quark flavor & CP sector:

6 masses, 3 angles and 1 phase in CKM-matrix

with accuracy: $|V_{us}| = 0.225$ (permille), $|V_{cb}| = 42 \cdot 10^{-3}$ (percent), $|V_{ub}| = 4 \cdot 10^{-3}$ (ten percent), $\sin 2\beta$ (measured) = 0.67 (percent)

PS: enormous progress from *B*-factories over past decade. PPS: still improving precision.

All hadronic flavor violation, including decays, productions rates at colliders and meson mixing effects should be described by these 10 parameters alone, if SM is correct. Since all parameters are known, this statement is very predictive and subject to numerous tests.

$$V$$
 is unitary $VV^{\dagger} = 1$ or, $\sum_{j} V_{ij}V_{kj}^{*} = \delta_{ik}$.

the unitarity triangle

 $V_{ub}V_{ud}^* + V_{cb}V_{cd}^* + V_{tb}V_{td}^* = 0$, all terms order λ^3 .



Its apex determines the Wolfenstein parameters $\bar{\rho}, \bar{\eta}$. In the absence of CP viol., the triangle would be squashed.

Information on the apex can come from various processes, measuring angles or sides.

SM tests with Quark flavor/CKM 1995 vs today

The CKM-picture of flavor and CP violation is currently consistent with all – and quite different – laboratory observations, although some tensions exist.



The quarks spectrum and mixings are hierarchical, and stem from the Yukawa matrices.

Numerically, we determined them as

$$Y_{u} \sim \begin{pmatrix} 10^{-5} & -0.002 & 0.008 + i \, 0.003 \\ 10^{-6} & 0.007 & -0.04 \\ 10^{-8} + i \, 10^{-7} & 0.0003 & 0.94 \end{pmatrix}$$

$$Y_{d} \sim \text{diag} \left(10^{-5}, 5 \cdot 10^{-4}, 0.025\right) \quad \left(\cdot \frac{\langle H_{u} \rangle}{\langle H_{d} \rangle}\right)$$

$$Y_{e} \sim \text{diag} \left(10^{-6}, 6 \cdot 10^{-4}, 0.01\right) \quad \left(\cdot \frac{\langle H_{u} \rangle}{\langle H_{d} \rangle}\right)$$

Very peculiar pattern. We dont know why it is this way.

So, Flavor and CP violation in SM is parameterized by 10 fundamental flavor parameters, which are (in part precisely) known. Lets go on and ask about flavor in generic SM extensions.

Exploring Physics at Highest Energies



In SM neutral currents conserve flavor. However, charged currents induce FCNCs through quantum loops.

 W^{\pm}

u, c, t



b



Different sectors and different couplings presently probed:

$$s \to d$$
: $K^0 - \bar{K}^0, K \to \pi \nu \bar{\nu}$
 $c \to u$: $D^0 - \bar{D}^0, \Delta A_{CP}$
 $b \to d$: $B^0 - \bar{B}^0, B \to \rho \gamma, b \to d \gamma, B \to \pi \mu \mu$
 $b \to s$: $B_s - \bar{B}_s, b \to s \gamma, B \to K_s \pi^0 \gamma, b \to sll, B \to K^{(*)}ll$ (precision,
angular analysis), $B_s \to \mu \mu$

 $t \rightarrow c, u, l \rightarrow l'$: not observed

in red: mentioned lated

Lets discuss a generic SM FCNC $b \rightarrow s$ amplitude ($\Delta B = \Delta S = 1$) W^{\pm} h = u.c.t = s

 $\mathcal{A}(b \to s)_{\rm SM} = V_{ub}V_{us}^*A_u + V_{cb}V_{cs}^*A_c + V_{tb}V_{ts}^*A_t$

quantum loop effect induced by the weak interaction. $A_q = A(m_q^2/m_W^2)$.

with CKM unitarity $VV^{\dagger} = 1$, specifically $\sum_{i} V_{ib}V_{is}^{*} = 0$: $\mathcal{A}(b \to s)_{SM} = V_{tb}V_{ts}^{*}(A_t - A_c) + V_{ub}V_{us}^{*}(A_u - A_c)$

 \mathcal{A} would vanish if *i* there wouldn't be a non-trivial CKM matrix, that is, one that allows for changes between different generations, and *ii* for identical up-type quark masses.

$$\mathcal{A}(b \to s)_{\rm SM} = \underbrace{V_{tb}V_{ts}^*}_{\lambda^2}(A_t - A_c) + \underbrace{V_{ub}V_{us}^*}_{\lambda^4}(A_u - A_c)$$

amplitude is dominated by first term because of lesser CKM suppression and because the GIM (Glashow Iliopoulos Maiani) suppression inactive for tops $\frac{m_t^2 - m_c^2}{m_W^2} \sim \mathcal{O}(1)$, whereas $\frac{m_u^2 - m_c^2}{m_W^2} \ll 1$.

We probe top properties with rare *b*-decays despite of $m_t \gg m_b$.

CP violation requires interference between the two terms with different phases; for $b \rightarrow s$, this is small, $O(\lambda^2)$.

The general features hold for any FCNC in the SM:

i FCNCs are induced by the weak interaction thru loops. *ii* FCNCs require $V \neq 1$.

iii FCNCs vanish for degenerate intermediate quarks. Since mass splitting among up-quarks is larger than for down quarks, GIM suppression is larger with external up-type than down-type quarks.

$$\mathcal{B}(b \to s\gamma) = 3 \cdot 10^{-4}$$
 ($E_{\gamma} > 1.6 \text{ GeV}$)
 $\mathcal{B}(b \to sl^+l^-) = 4 \cdot 10^{-6}$ ($m_{ll}^2 > 0.04 \text{ GeV}^2$)

$$\begin{split} & \mathsf{SM:}\ \mathcal{B}(t\to cg)\sim 10^{-10},\ \mathcal{B}(t\to c\gamma)\sim 10^{-12},\ \mathcal{B}(t\to cZ)\sim 10^{-13},\\ & \mathcal{B}(t\to ch)\lesssim 10^{-13}\ \text{Eilam, Hewett, Soni '91/99} \end{split}$$

We see that 3 mechanisms suppress FCNCs in SM: CKM, GIM and absence at tree level. New physics, which doesnt need to share these features, competes with small SM background!

FCNCs feel physics in the loops from energies much higher than the ones actually involved in the real process.

They are very useful to look for new physics, in fact, we already now a lot about new physics from FCNCs!

Probing Physics at Highest Energies with Flavor



$$\begin{split} \mathcal{A}(B^0_d \to \bar{B}^0_d)_{\rm SM} &\propto \frac{g^4}{16\pi^2} (V_{td} V^*_{tb})^2 \frac{\langle \bar{B}^0_d | (\bar{b}_L \gamma_\mu d_L) (\bar{b}_L \gamma^\mu d_L) | B^0_d \rangle}{m_W^2} S(m_t^2/m_W^2) \\ \mathcal{A}(B^0_d \to \bar{B}^0_d)_{\rm NP} &\propto \frac{\langle \bar{B}^0_d | (\bar{b}\Gamma d) (\bar{b}\Gamma' d) | B^0_d \rangle}{\Lambda_{\rm NP}^2} \qquad (tree) \\ \mathcal{A}(B^0_d \to \bar{B}^0_d)_{\rm NP} &\propto \frac{g^4}{16\pi^2} \frac{\langle \bar{B}^0_d | (\bar{b}\Gamma d) (\bar{b}\Gamma' d) | B^0_d \rangle}{\Lambda_{\rm NP}^2} \qquad (weak \ loop), \end{split}$$

where we allowed for generic new physics at the energies Λ_{NP} , either at tree level, or thru weak loops, as, e.g., in the MSSM.

Probing Physics at Highest Energies with Flavor

 $B^0_d - \bar{B}^0_d$ oscillation data ok with SM: $\mathcal{A}(B^0_d \to \bar{B}^0_d)_{\mathrm{NP}} \lesssim \mathcal{A}(B^0_d \to \bar{B}^0_d)_{\mathrm{SM}}$

 $\Lambda_{\rm NP} \gtrsim 4\pi m_W / (g^2 |V_{td}|) \sim \mathcal{O}(250)$ TeV (tree level) $\Lambda_{\rm NP} \gtrsim m_W / |V_{td}| \sim \mathcal{O}(10)$ TeV (weak loop)

In either case, the connection with the electroweak scale is lost!

	$K^0 \bar{K}^0$	$D^0 ar{D}^0$	$B^0_d \bar{B}^0_d$	$B^0_s \bar{B}^0_s$
$\Lambda_{ m NP}$ [TeV]	$2 \cdot 10^5$	$5 \cdot 10^3$	$2 \cdot 10^3$	$3 \cdot 10^2$

Table 1: The lower bounds on the scale of new physics from FCNC mixing data in TeV for arbitrary new physics at 95 % C.L.

Flavor Physics at the TeV-scale



The absence of O(1) New Physics observations in FCNCs implies that physics at the TeV-scale has non-generic flavor properties.

In particular, suppression mechanisms of similar power as CKM and GIM, which are built-in in the SM, need to be at work.



Connection to TeV-scale is lost, or TeV-scale flavor non-generic!

Effective field theory of flavor violation induced by Yukawas only:

SM has global symmetry for $Y_{u,d,l} = 0$: $G_F = U(3)_Q \times U(3)_{U_R} \times U(3)_{D_R} \times U(3)_L \times U(3)_{E_R} \to U(1)_B \times U(1)_L \times U(1)_Y$

A model is termed minimally flavor violating (MFV), if, as in the SM, all flavor symmetry breaking is induced by the Yukawa matrices.

More technically, assigning charges to the Yukawas under G_F a formally invariant effective Lagrangian can be obtained.

 $Y_u(3,\overline{3},1,1,1), \quad Y_d(3,1,\overline{3},1,1), \quad Y_e(1,1,1,3,\overline{3})$ "spurions" of G_F $\rightarrow B_s - \overline{B}_s$ mixing on blackboard; in MFV same suppression as SM * The superpotential (N = 1, unbroken R-parity) is MFV: $W_{MSSM} = Q_L Y_u H_u U_R + Q_L Y_d H_d D_R + L_L Y_e H_d E_R + \mu H_d H_u$

* Without further input there can be arbitrarily large and CP-violating intergenerational mixing among the scalar partners of the SM fermions from the SUSY breaking:

$$\mathcal{L}_{soft} = -\tilde{Q}_{Li}^{\dagger} (\tilde{m}_Q^2)_{ij} \tilde{Q}_{Lj} + \dots$$

This is ruled out by FCNC data for TeV-scale SUSY partners.



* The off-diagonal squark mass terms "mass insertions" $\delta_{ij}^Q = (\tilde{m}_Q^2)_{ij}/\tilde{m}_{ave}^2, i \neq j$, induce FCNCs, and are constrained by data.

	$\sqrt{ \Re(\delta^d_{12})^2_{ m LL} }$		$\sqrt{ \Im(\delta^d_{12})^2_{ m LL} }$		
x	TREE	NLO	TREE	NLO	
0.3	$1.4 imes10^{-2}$	$2.2 imes 10^{-2}$	$1.8 imes 10^{-3}$	$2.9 imes10^{-3}$	
1.0	$3.0 imes10^{-2}$	$4.6 imes10^{-2}$	$3.9 imes10^{-3}$	$6.1 imes10^{-3}$	
4.0	$7.0 imes 10^{-2}$	1.1×10^{-1}	$9.2 imes 10^{-3}$	$1.4 imes 10^{-2}$	
	$\sqrt{ \Re(\delta_{12}^d)_{\mathrm{LL}}(\delta_{12}^d)_{\mathrm{RR}} }$		$\sqrt{ \Im(\delta^d_{12})_{ ext{LL}}(\delta^d_{12})_{ ext{RR}} }$		
x	TREE	NLO	TREE	NLO	
0.3	1.8×10^{-3}	$8.6 imes10^{-4}$	$2.3 imes10^{-4}$	$1.1 imes 10^{-4}$	
1.0	$2.0 imes10^{-3}$	$9.6 imes10^{-4}$	$2.6 imes10^{-4}$	$1.3 imes 10^{-4}$	
4.0	$2.8 imes10^{-3}$	$1.3 imes10^{-3}$	$3.7 imes10^{-4}$	$1.8 imes 10^{-4}$	
	$\sqrt{ \Re(\delta_{12}^d)_{LR}^2 }$		$\sqrt{ \Im(\delta_{12}^d)_{LR}^2 }$		
x	TREE	NLO	TREE	NLO	
0.3	$3.1 imes10^{-3}$	$2.6 imes10^{-3}$	$4.1 imes 10^{-4}$	$3.4 imes10^{-4}$	
1.0	$3.4 imes 10^{-3}$	$2.8 imes 10^{-3}$	$4.6 imes 10^{-4}$	$3.7 imes 10^{-4}$	
4.0	$4.9 imes 10^{-3}$	$3.9 imes 10^{-3}$	$6.5 imes 10^{-4}$	$5.2 imes 10^{-4}$	

Table 1: Maximum allowed values for $|\Re \left(\delta_{12}^d \right)_{AB} |$ and $|\Im \left(\delta_{12}^d \right)_{AB} |$, with A, B = (L, R) for an average squark mass $m_{\tilde{q}} = 500 \text{ GeV}$ and for different values of $x = m_{\tilde{g}}^2/m_{\tilde{q}}^2$. The bounds are given at tree level in the effective Hamiltonian and at e.g., 0711.2903 NLO in QCD corrections as explained in the text. For different values of $m_{\tilde{q}}$ the bounds scale roughly as $m_{\tilde{q}}/500 \text{ GeV}$.

* MFV implies squark flavor-mixing given by quark-Yukawa matrices

$$\tilde{m}_Q^2 = \tilde{m}^2 (a_1 \mathbf{1} + b_1 Y_u Y_u^{\dagger} + b_2 Y_d Y_d^{\dagger})$$
 etc.

 $Y_u = \operatorname{diag}(y_u, y_c, y_t)$, $Y_d = V \cdot \operatorname{diag}(y_d, y_s, y_b)$ (up mass basis)

Controlled departure from flavor-blind SUSY breaking: e.g., s-top – s-charm mixing: $(\tilde{m}_Q^2)_{23}/\tilde{m}^2 \sim y_b^2 V_{cb} V_{tb}^* \sim 10^{-5} \tan \beta^2$ * $\mathcal{O}(1)$ deviations possible in MFV-MSSM from SM in rare processes if $\tan \beta$ is large. $\tan \beta = \langle H_u \rangle / \langle H_d \rangle$

* Anomaly mediation, gauge mediation and CMSSM/mSUGRA (by construction) are MFV.

* MFV coefficients also induced by renormalization group running.