

Lecture I

The SM and Beyond

What is the SM?

Like any other local, relativistic Quantum Field Theory, the SM is defined by its symmetries and by its field/particle content:

$$\text{"Symmetries" + "Fields" = SM}$$

In particular, the SM is not defined by its Lagrangian. The Lagrangian is a derived object, under certain additional assumptions as described below.

- Symmetries: Poincaré group + gauge invariance

$$G_{SM} = \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y$$

- Fields: ~~$G_\mu^\alpha, W_\mu^\alpha, B_\mu$~~ \rightarrow gauge fields

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \in \left\{ \begin{array}{l} (3, 2, 1/6) \\ (3, 1, 2/3) \end{array} \right\} \quad \text{quarks}$$

$$d_R \in (3, 1, -1/3)$$

$$l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \in (1, 2, -1/2) \quad \text{leptons}$$

$$e_R \in (1, 1, -1)$$

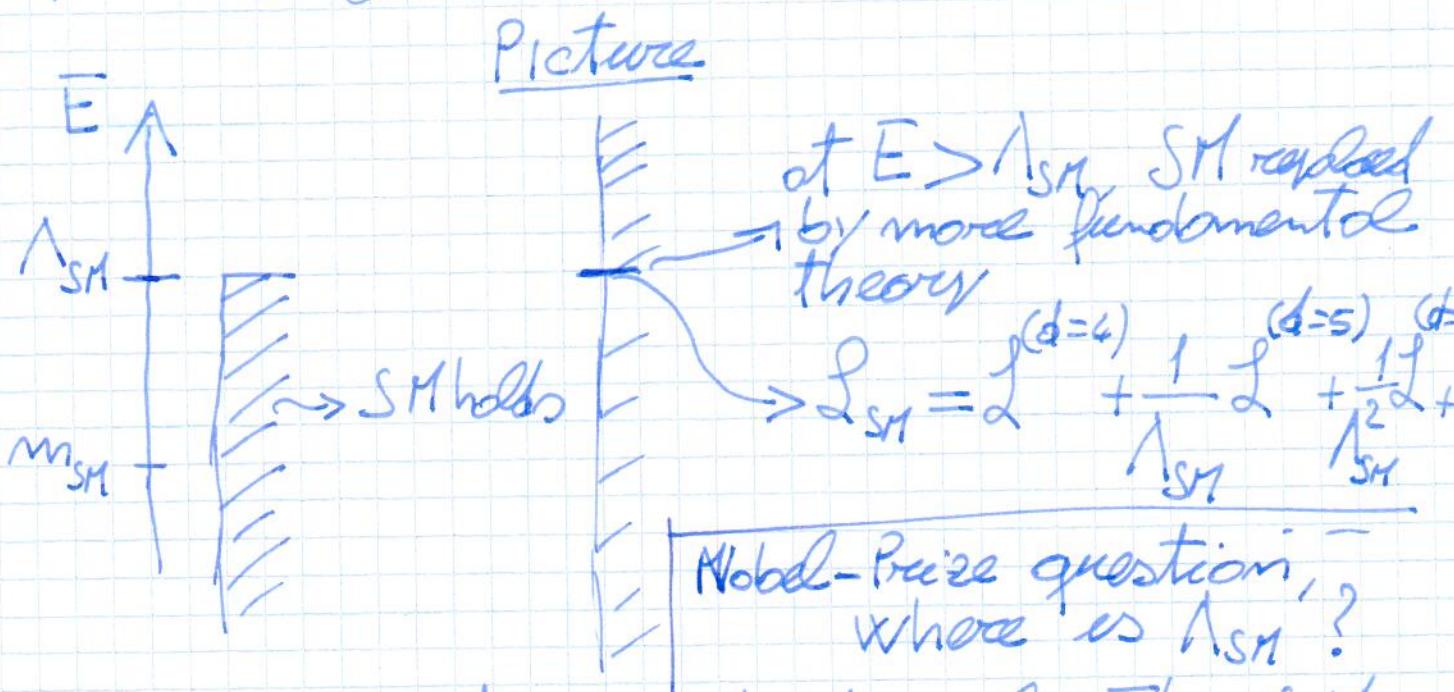
$$\phi \in (1, 2, 1/2) \rightarrow \text{Higgs}$$

Comments:

- Putting ν_R singlet or not putting it is a big thing! In my Lectures there are no ν_R 's in what I call "the SM"
- There is an interplay among local symmetries and fields in determining the physical particle content. Gauge symmetry generators are like "negative fields", they remove physical degrees of freedom rather than adding them? some!
- The SM requires no global internal symmetries, only gauged ones. Gauge invariance is not a symmetry, is a technical tool to describe spin one (or two) force carriers. Perhaps this is not a coincidence. Perhaps one might even say that the formulation of the SM requires no symmetries at all aside from Poincaré. Perhaps physical particle content is all what matters.
- The Lagrangian is derived from symmetries + fields:

$$L = \sum_{\mathcal{L}=1}^{\infty} \mathcal{L}(X) = \begin{array}{l} \text{"Sum of all local operators"} \\ \text{made of } \underline{\text{fields}} \text{ and} \\ \text{invariant under } \underline{\text{symmetries}} \end{array}$$

- If all the infinite set of operators matter the same the theory is not tractable in perturbation theory (but not useless, symmetries and particles remain)
- If all but few operators matter and the others are irrelevant, we can use perturbation theory.
- The SM falls in the second category, because of a mass gap:



New physics at some high scale. The SM cutoff, λ_{SM} , physically corresponds to the mass of BSM new particles

One day we will know the details of the microscopic BSM theory.
We will be able to recover the SM as a low-energy EFT.

We will compute its free parameters, which today are fixed by observations.

We saw this happening several times, for instance with the Fermi theory.

We classify the operators by their energy dimension. It is just ordinary dimensional analysis that sets the correct powers of Λ_{SM} .

whose powers of Λ_{SM} make a suppression, thus the most relevant (actually called marginal) operators are:

$$\begin{aligned} \mathcal{L}^{(d=4)} = & F^2 + \bar{\psi}_L i \not{D} \psi_L + |\not{D}\phi|^2 - |\not{D}\phi|^4 + \\ & + \gamma_L \bar{q}_L \not{D}^2 u_R + \gamma_R \bar{q}_R \not{D}^2 u_R + \gamma_L \bar{L}_L \not{D}^2 \ell_R + \text{h.c.} \end{aligned}$$

This is (almost) the SM Lagrangian written on the CERN T-shirts. Actually one important term is still missing.

It is a $d=4$ Lagrangian, but not because we advocate renormalizability.

After Wilson's work, renormalizability is not a principle, we do understand why Nature is so well described by a renormalizable Lagrangian. It is because $d=4$ is all what matters for E&N.

The success of $d=4$ Lagrangian in describing data is not (just) a collection of quantitatively solid predictions.

The most striking confirmations of the $d=4$ Lagrangian have a structural origin.

Accidental symmetries: Symmetries of the Lagrangian that are present without being imposed.

Stated differently: Accidental symmetries are present at $d=4$ even if they were badly violated in the microscopic BSM theory in the UV

1) Baryon Number, $U(1)_B$

$$\{q_L, u_R, d_R\} \rightarrow e^{\frac{i\theta}{3}} \{q_L, u_R, d_R\}$$

↓

The proton is absolutely stable

$$\Gamma_p^{ISM} = 0$$

experimentally: $\Gamma_p \gtrsim 10^{-32} \text{ yrs}$

$$\Gamma_p \lesssim 2 \cdot 10^{-64} \text{ GeV}$$

$\Gamma_p/m_p \lesssim 2 \cdot 10^{-64}$, imagine trying to explain this number without the help of a symmetry selection rule!

(6)

2) Lepton Family Numbers, $U(1)_e$, $U(1)_\mu$, $U(1)_\tau$

non-diagonal Yukawa's in the lepton sector can be rotated away.

If and only if there are no ν_R'

$$Y_L = V_L Y_L^d V_R^+$$

Field redefinition:

$$\left. \begin{array}{l} L_L \rightarrow V_L L_L \\ \ell_R \rightarrow V_R \ell_R \end{array} \right\}$$

In the new basis:

$$L_{Yuk}^{\text{new}} = y_e^{-1} \bar{L}_L \phi \ell_R + y_\mu^{-2} \bar{L}_L \phi \ell_R^2 + y_\tau^{-3} \bar{L}_L \phi \ell_R^3$$

Each lepton flavour (charged + neutrino) can be rotated independently

\Downarrow (for instance)

$$\text{BR}(\mu \rightarrow e\gamma) = 0$$

experimentally: $\text{BR}(\mu \rightarrow e\gamma) \lesssim 10^{-12}$

other big success!

- Those are the exact ~~approximate~~ accidental symmetries. There are also approximate ones.

$$3) U(3)^5 = U(3)_{q_L}^5 \times U(3)_{\mu_R} \times U(3)_{\tau_R} \times U(3)_L \times U(3)_{\ell_R}$$

broken only by the Yukawa matrices

This is called "Mirrored Flavor Violation",
 it explains several "coincidences" in the
 flavor Physics, among which the GIM
 mechanism

4) "Custodial $SU(4)$ ", under which the 4
 real Higgs components form one quartet.
 This explains why $\rho = \frac{m_W^2}{C_W^2 M_2^2} = 1$ at tree-level.

Accidental Symmetries are violated at $d > 4$
 We could avoid this only promoting them
to symmetries of the UV theory (this is what
 we typically do for low (TeV) scale BSM)

• Just one $d=5$ operator! conjugate spinor

$$\frac{1}{\Lambda_{SM}} \mathcal{L}^{(d=5)} = \frac{W_{eS}}{\Lambda_{SM}} \left(\begin{matrix} \bar{e} & H^c \\ \downarrow & \downarrow \\ \frac{1}{2} & -\frac{1}{2} \end{matrix} \right) \left(\begin{matrix} \bar{e} & H^c \\ \downarrow & \downarrow \\ \frac{1}{2} & -\frac{1}{2} \end{matrix} \right) = 0$$

the very famous Weinberg operator

For generic W coefficient matrix, this operator gives :

1) Mesons and Dimuons:

$$m_\nu \sim \frac{W}{\Lambda_{SM}} \cdot v^2$$

2) Lepton Family number violation:
 ν -oscillations

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ν masses at $d=5$. This could be the reason why neutrinos are so light:

$$m_\nu \sim 0.1 \text{ eV} \Rightarrow \Lambda_{\text{SM}} \approx W \cdot 10^{15} \text{ GeV} \\ \gg \text{EW scale}$$

LFFV ($\mu \rightarrow e\gamma$) induced by this operator are extremely small

Mesogenic neutrinos could be discovered by ν -less double B decay, this would be a striking confirmation of the picture.

We can not be completely quantitative here. W needs not to be of $O(1)$. W is a coupling square:

$$W \propto \gamma_e^2, g_W^2, \gamma_e^2 (\pm 10^{-5})^2 !$$

- Many $d=6$ operators. One interesting one is!

$$\frac{1}{\Lambda_{\text{SM}}^2} \mathcal{L}^{(d=6)} = \frac{1}{\Lambda_{\text{SM}}^2} \epsilon^{\alpha\beta\gamma} (\bar{q}_L^\alpha)_\alpha (\bar{q}_L^\beta)_\beta (\bar{q}_L^\gamma)_\gamma q_L^\alpha$$

$$-\frac{1}{6} - \frac{1}{6} - \frac{1}{6} + \frac{1}{2} = 0$$

$$\Gamma(P \rightarrow e^+ \pi^0) \simeq \frac{1}{8\pi} \left\{ \frac{M_P}{\Lambda_{\text{SM}}^4} \right\}^5$$

safe with bound of $1/\Lambda_{\text{SM}} \lesssim 4 \cdot 10^{15}$
still compatible with ν masses given long wait time

(9)

What if Baryon number was violated at $d=5$? You can check that

$$M_H^{B,d=5} > 10^{31} \text{ GeV}$$

The picture would have been invalidated...

Model Builders' Nightmare:

SM holds up to $\Lambda_{\text{SM}} \sim 10^{15} \text{ GeV} = \Gamma_{\text{GUT}}$

- Few indirect (very indirect!) signatures
- No hope for colliders

Many (not fully conclusive) arguments quantum ~~gravitational~~ against SM-only

1) Gravity: $\frac{G_N}{M_p^2} \sim \frac{E^2}{M_p^2} = 16\pi r^2$ \hookrightarrow perturbativity

$\Lambda_{\text{SM}} < 4\pi M_p \sim 10^{13} \text{ GeV}$ \hookrightarrow not very helpful

- 2) Strong CP problem
 3) Inflation
 4) Baryogenesis } $\left. \begin{array}{l} \text{some of them could have} \\ \text{to do with low energy.} \\ \text{All of them "early" explained} \\ \text{by very heavy and/or very} \\ \text{weakly coupled new particles} \end{array} \right\}$

- 5) Dark Matter: could be axions, but probably it could be a WIMP. Plausible but not conclusive option

Only one fundamental (not conducive...) objection:

The Naturalness (or Hierarchy) Problem

$$\Lambda_{\text{SM}} \xrightarrow{\quad} \mathcal{L} = \mathcal{L}^{(d=4)} + \frac{1}{\lambda} \mathcal{L}^{(d=5)} + \frac{1}{\lambda^2} \mathcal{L}^{(d=6)} + \dots \\ + \underbrace{\lambda^2 \mathcal{L}^{(d=2)}}_{\text{SM } \mathcal{L}}$$

by the same dimensional analysis which suppresses $d > 4$, $d < 4$ should be enhanced!

Only one $d < 4$ term is present in the SM:

$$\Lambda_{\text{SM}}^2 \mathcal{L}^{(d=2)} = + m_H^2 |\phi|^2 = c \Lambda_{\text{SM}}^2 |\phi|^2$$

Why then $m_H = 125 \text{ GeV} \ll M_{\text{GUT}}$?

If the estimate is badly violated for $d=2$, why it holds for $d=5, 6$ etc?

The Hierarchy Problem can be made more quantitative, following Wilson.

The pole Higgs mass m_H^{pole} is not the one in the Lagrangian.

By integrating out heavy modes we compute the UV Lagrangian:

$$(m_H^2)^{UV} = c \Lambda_{SM}^2$$

The pole Higgs mass, as computed in the full UV theory, is the sum of two pieces:

$$(m_H^2)^{\text{pole}} = \int_0^\infty dE \frac{dm_H^2}{dE} = \int_0^{\Lambda_{SM}} \dots + \int_{\Lambda_{SM}}^\infty \dots = Sm_H^2 + (m_H^2)^{UV}$$

"IR" term, from energy modes where the SM holds, calculable within the SM

UV contribution in the effective Lagrangian

$$Sm_H^2 = \text{---} \left(\begin{array}{c} \phi \\ \downarrow t \\ \phi \end{array} \right) \text{---} + \text{---} \left(\begin{array}{c} V \\ \text{---} \\ \text{---} \end{array} \right) \text{---} + \dots$$

$$= -\frac{Y_t^2}{t} \cdot 3 \cdot \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2} \sim -\frac{3 Y_t^2 \Lambda_{SM}^2}{2\pi^2} \sim -0.1 \Lambda_{SM}^2$$

The problem, thus, is not to explain why "c" is small. This might be due to the details of the UV theory.

The problem is why c is finely tuned and cancels Sm_H^2 so precisely:

$$(m_H^2)^{\text{pole}} = \left(c - \frac{3 Y_t^2}{2\pi^2} \right) \Lambda_{SM}^2 = (125 \text{ GeV})^2$$

The NakosLinos problem is a UV-IR cancellation happening for no known reason!

Comments :

- 1) A "solution" of the H.P. would be, strictly speaking, a mechanism ensuring this UV/IR cancellation automatically.
We don't know any!
- 2) The cancellation might just happen by accident, we quantify it by

$$\Delta = \frac{\max \left\{ S m_H^2, (m_H^2)^{av} \right\}}{m_H^2 \text{ pole}} \geq \frac{S m_H^2}{m_H^2 \text{ pole}}$$

$$= \left(\frac{125 \text{ GeV}}{m_H} \right)^2 \left(\frac{\Lambda_{SM}}{500 \text{ GeV}} \right)^2$$

New Physics
is close

$$\Delta < 100 \Rightarrow \boxed{\Lambda_{SM} \leq 5 \text{ TeV}}$$

- 3) If $\Delta \sim 10^{26}$, not only we would be surprised. Also, we will never be able to compute m_H , and in turn the EWSB scale v , starting from the UV theory

"The H.P. is an obstruction to ever understand the microscopic origin of EWSB!"

- 4) Naturalness is being tested at the LHC. Discovering, or not discovering low-scale new physics will radically change our perspective on fundamental interactions.

Two classes of "Unnatural Scenarios"

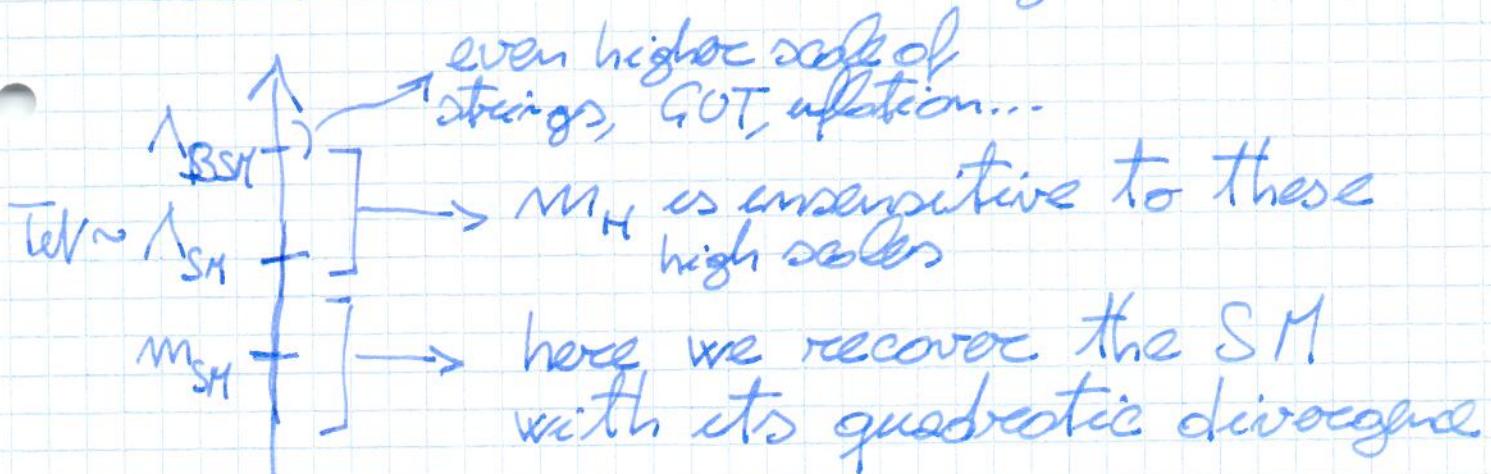
1) Anthropic Vacua selection:

- Could explain why M_H (and $\Lambda_{\text{cosmological}}$) is small
- Does not explain the origin of EWSB, M_H is effectively a free undecidable parameter

2) "No Scale Models": (no loops mass contributing to m_H)

- Please find one!
- No progress on calculability issue.

"Solutions" to the H.P.: light new physics.



$$\text{SUSY}: M_{\text{SM}} = M_{\text{SUSY}}$$

scalar mass \longleftrightarrow fermion mass (protected by chiral symmetry)

$$M_H^2 = \cancel{B} - \cancel{\frac{e}{\tau}} + \dots - \cancel{\frac{E}{\tau}} + \dots + \cancel{\frac{E}{\tau}}$$

$$= -\frac{3Y_E^2}{2\pi^2} M_H^2 + \left(\begin{array}{l} \text{effectively massless top loop} \\ \text{effectively + massless stop loop} \end{array} \right) + \text{"Ewm$_{\text{loop}}$ threshold"} \quad \overset{\text{"}}{O}$$

The contribution below Λ_{SM} is unavoidable

The one above Λ_{SM} is concealed

The result being natural requires $M_{\text{loop}} \sim \text{TeV}$

Composite Higgs: $\Lambda_{\text{SM}} = M_* =$ "confinement scale"

The Higgs is not an elementary point-like particle. Similar to a "QCD"-like hadron

$$\left\{ \begin{array}{l} \text{size} : \text{L}_H \sim 1/M_* \sim 1/\text{TeV} \sim 10^{-16} \text{ cm} \\ \text{mass} : M_H \sim 1/M_* \sim 1/\text{TeV} \sim 10^{-16} \text{ cm} \end{array} \right.$$

Transparent to waves with $\lambda \ll \text{L}_H$

More technically, ϕ is a composite field:

$$\phi \sim \bar{\psi}\psi \rightarrow \begin{array}{l} \dim[H] > 1 \\ \dim[H^2] > 2 \end{array}$$

↓
for instance

If $\dim[H^2] > 4$, $\text{deg} = 6$

$$M_H^2 = -\frac{3Y_E^2}{2\pi^2} M_*^2 + \text{"threshold from Ewm$_{\text{loop}}$"} + \frac{M_*^4}{\lambda^2} \overset{\text{O}}{}$$

from even higher energies BSM

This mechanism, according to which the field radically changes its dimension in the UV, is called

"Dimensional Transmutation"

it is at work also in "every-day life", namely in low-energy QCD, giving a small mass to pions, ρ mesons, σ , ϕ nucleons etc.