

Interference Effects of neutral MSSM Higgs Bosons

with a Generalised Narrow-Width Approximation.



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DESY/ CERN

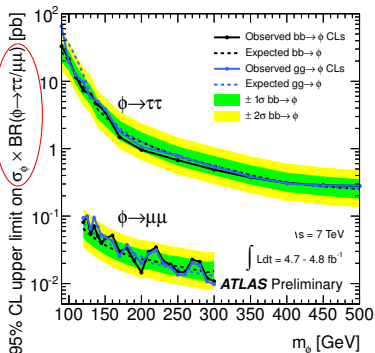
in collaboration with
Silja Thewes and Georg Weiglein

Cargèse, July 2014

Student talk

Useful approximation for New Physics searches

- ▶ NP: many-particle final state difficult at higher order
- ▶ \rightsquigarrow simplified by factorisation into **production** \times **decay**



Narrow-width approximation (NWA)

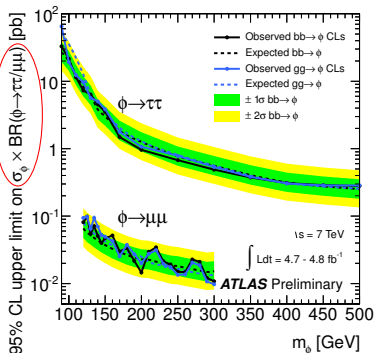
$$\sigma \approx \sigma_{\text{prod}}(q^2 = M^2) \cdot \text{BR}_{\text{dec}}$$

- ▶ narrow width $\Gamma \ll M$
- ▶ kinematically open, away from thresholds
- ▶ non-factorisable corrections small
- ▶ no interference with other processes



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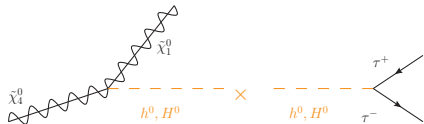
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Mass degeneracy: interference term significant

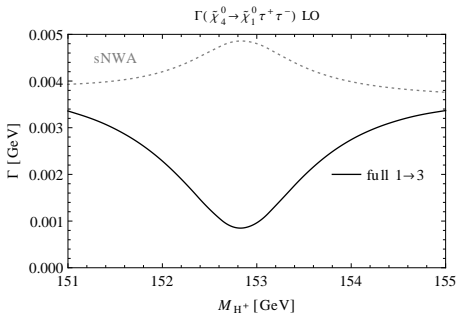
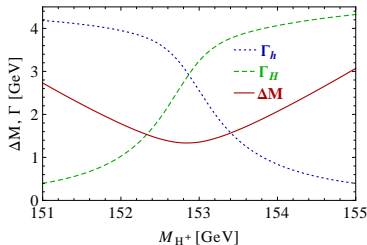
- ▶ NWA not applicable for $|M_i - M_j| \lesssim \Gamma_i, \Gamma_j \rightarrow$ generalised NWA



Example process: $\Gamma(\tilde{\chi}_4^0 \rightarrow \tilde{\chi}_1^0 \tau^+ \tau^-)$ at leading order



Scenario: small $\Delta M = M_H - M_h$

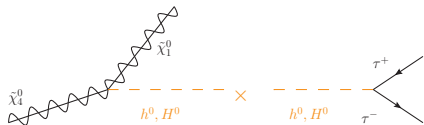


large discrepancy between sNWA and full 3-body decay width

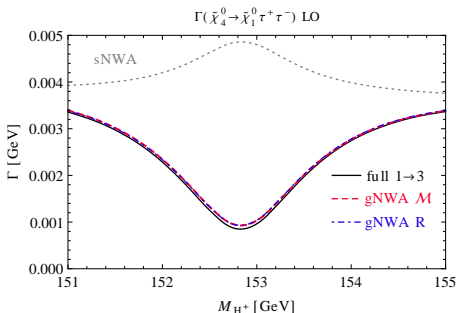
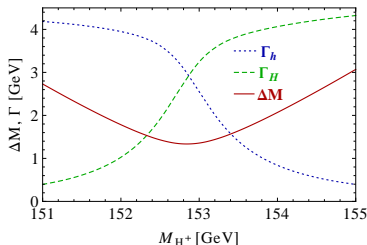
On-shell interference term

- ▶ 'full': $1 \rightarrow 3$ with $h, H + \text{Int}$
- ▶ sNWA: $\Gamma_{P_h} \text{BR}_h + \Gamma_{P_H} \text{BR}_H$

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On-shell interference term

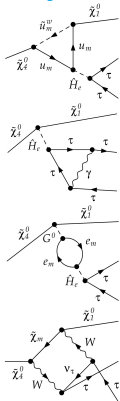
- ▶ 'full': 1 → 3 with $h, H + \text{Int}$
- ▶ sNWA: $\Gamma_{P_h} \text{BR}_h + \Gamma_{P_H} \text{BR}_H$
- ▶ **gNWA**: sNWA + Int_{on-shell}

large negative interference effect well approximated by **gNWA** (\mathcal{M}/\mathcal{R})

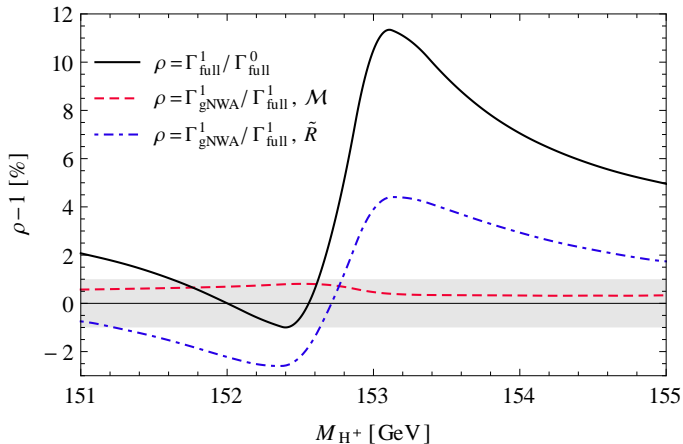


Accuracy of gNWA at 1-loop order

Comparison with 3-body decay:



gNWA precision vs. full loop size



uncertainty: $\mathcal{M} < 1\% \sim$ estimated full uncertainty; $\tilde{R} < 4\%$

Summary: interference and NLO effects in generalised NWA

- ▶ example: decay $\tilde{\chi}_4^0 \xrightarrow{h^0, H^0} \tilde{\chi}_1^0 \tau^+ \tau^-$ with interference of Higgs bosons
- ▶ demonstrated how gNWA can be applied at the loop level: inclusion of virtual and real corrections, cancellations of IR-divergences preserved
 - on-shell matrix elements: 1% agreement with full result
 - R-factor: 4% agreement with full result, but technically easier
- ▶ gNWA enables factorisation into production and decay with interference and NLO effects → useful for various BSM models



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Ongoing: \mathcal{CP} -violating mixing

- ▶ $\text{MSSM}_{\mathcal{C}}$: $\mathcal{CP} \Rightarrow H^0 - A^0$ interference
 - $b\bar{b} (h/H/A \rightarrow \tau^+ \tau^- / b\bar{b})$
 - $gg \rightarrow h/H/A \rightarrow \tau^+ \tau^- / \mu^+ \mu^-$
- ▶ negative interference terms could relax limits on σ
- ▶ impact on experimental parameter limits?



Thank you!



Do you have any questions?



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A Graduate Education Program
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in Cooperation with DESY



Factorisation of the n -particle phase space $d\Phi_n$

- ▶ $d\Phi_n \equiv dlips(P; p_1, \dots, p_f) = (2\pi)^4 \delta^{(4)}(P - \sum_{f=1}^n p_f) \prod_{f=1}^n \frac{d^3 p_f}{(2\pi)^3 2E_f}$
- ▶ here: kinematics of 3-body decay \rightarrow 2-body
 $d\Phi = dlips(\sqrt{s}; p_c, p_e, p_f) = dlips(\sqrt{s}; p_c, q) \frac{dq^2}{2\pi} dlips(q; p_e, p_f)$

Production \times decay

- ▶ instead of BREIT-WIGNER propagator $\frac{1}{q^2 - M^2 + iM\Gamma}$
- ▶ **on-shell** production of particle with mass M , and subsequent decay:

$$\sigma_{ab \rightarrow cef} \approx \sigma_{ab \rightarrow cd}(q^2 = M^2) \cdot BR_{d \rightarrow e f}$$

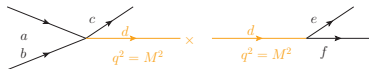
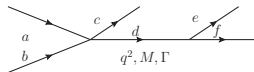
- ▶ uncertainty of $\mathcal{O}\left(\frac{\Gamma}{M}\right)$



Standard NWA: Conditions and limitations

generic example:

$$ab \xrightarrow{d} cef$$



Validity

- ▶ narrow width $\Gamma \ll M$, otherwise off-shell effects e.g. [Gigg, Richardson '08]
- ▶ kinematically open, away from thresholds e.g. [Kauer '08]
- ▶ non-factorisable corrections small e.g. [Denner, Dittmaier, Roth '98]
- ▶ no interference with other processes e.g. [Reuter '07] [Berdine, Kauer, Rainwater '07]

[Kalinowski, Kilian, Reuter, Robens, Rolbiecki '08]



2 steps for on-shell approximation of interference term

- ▶ matrix elements on-shell $\mathcal{M}(q^2 = M^2)$
 - pro close to full result
 - con no automated evaluation of squared matrix elements

- ▶ 'interference weight factor' R : $\sigma \approx \sum_i \sigma_{P_i} B R_i \cdot (1 + R_i)$
 - pro building blocks available as in sNWA: $\sigma_P, \Gamma_D, \Gamma^{tot}, g_P, g_D$
 - con **additional approximation** $M_h \approx M_H$

accuracy vs. technical simplification of approximation



Cross section with full interference term

$$\sigma(ab \rightarrow cef) = \frac{1}{F} \int d\Phi \left(\frac{|\mathcal{M}(ab \rightarrow ch)|^2 |\mathcal{M}(h \rightarrow ef)|^2}{(q^2 - M_h^2)^2 + M_h^2 \Gamma_h^2} + \frac{|\mathcal{M}(ab \rightarrow cH)|^2 |\mathcal{M}(H \rightarrow ef)|^2}{(q^2 - M_H^2)^2 + M_H^2 \Gamma_H^2} \right. \\ \left. + 2\text{Re} \left\{ \frac{\mathcal{M}(ab \rightarrow ch) \mathcal{M}^*(ab \rightarrow cH) \mathcal{M}(h \rightarrow ef) \mathcal{M}^*(H \rightarrow ef)}{(q^2 - M_h^2 + iM_h \Gamma_h)(q^2 - M_H^2 - iM_H \Gamma_H)} \right\} \right)$$

Mass degeneracy: interference term significant [Fowler, PhD Thesis '10]

- ▶ NWA not applicable for $|M_i - M_j| \lesssim \Gamma_i, \Gamma_j$ (BREIT-WIGNER overlap)
- ▶ e.g. MSSM: for some parameters, h^0, H^0, A^0 have similar masses
- ▶ also relevant for other models



Generalised NWA with interference term

$$\sigma(ab \rightarrow cef) = \frac{1}{F} \int d\Phi \left(\frac{|\mathcal{M}(ab \rightarrow ch)|^2 |\mathcal{M}(h \rightarrow ef)|^2}{(q^2 - M_h^2)^2 + M_h^2 \Gamma_h^2} + \frac{|\mathcal{M}(ab \rightarrow cH)|^2 |\mathcal{M}(H \rightarrow ef)|^2}{(q^2 - M_H^2)^2 + M_H^2 \Gamma_H^2} \right. \\ \left. + 2\text{Re} \left\{ \frac{\mathcal{M}(ab \rightarrow ch) \mathcal{M}^*(ab \rightarrow cH) \mathcal{M}(h \rightarrow ef) \mathcal{M}^*(H \rightarrow ef)}{(q^2 - M_h^2 + iM_h \Gamma_h)(q^2 - M_H^2 - iM_H \Gamma_H)} \right\} \right)$$

$$\mathcal{M} \text{ on-shell} \approx \sigma_{ab \rightarrow ch} BR_{h \rightarrow ef} + \sigma_{ab \rightarrow cH} BR_{H \rightarrow ef}$$

$$+ \frac{2}{F} \text{Re} \left\{ \int \frac{dq^2}{2\pi} \left(\Delta_1^{BW}(q^2) \Delta_2^{*BW}(q^2) \left[\int d\Phi_P(q^2) \mathcal{M}_{P_1}(M_1^2) \mathcal{M}_{P_2}^*(M_2^2) \right] \right. \right. \\ \left. \left. \left[\int d\Phi_D(q^2) \mathcal{M}_{D_1}(M_1^2) \mathcal{M}_{D_2}^*(M_2^2) \right] \right) \right\}$$

$$M_h \simeq M_H \approx \sigma_{P_1} BR_1 \cdot (1 + R_1) + \sigma_{P_2} BR_2 \cdot (1 + R_2)$$

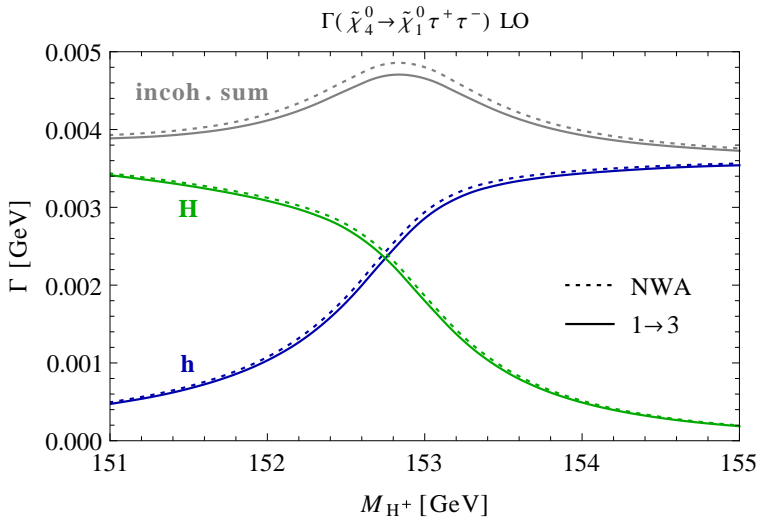
$$R_i := 2M_i \Gamma_i w_i \cdot 2\text{Re} \{x_i I\}$$

$$w_i := \frac{\sigma_{P_i} BR_i}{\sigma_{P_1} BR_1 + \sigma_{P_2} BR_2}$$

$$x_i := \frac{g_{P_i} g_{P_j}^* g_{D_i} g_{D_j}^*}{|g_{P_i}|^2 |g_{D_i}|^2} \quad (g_{P/D} : \text{couplings in production/ decay})$$

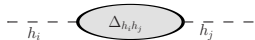


Uncertainty of the sNWA at tree level



3×3 mixing (approximation of 6×6) $\rightarrow 2 \times 2$ for \mathcal{CP}

- ▶ mixing self-energies \Rightarrow mass matrix (m_{tree}, M_{loop})



- ▶ masses and widths from complex pole $\mathcal{M}_i^2 = M_i^2 - iM_i\Gamma_i$

- ▶ diagonal propagator $\Delta_{ii} = \frac{i}{p^2 - m_i^2 + \hat{\Sigma}_{ii}^{\text{eff}}(p^2)} \xrightarrow{p^2 \simeq \mathcal{M}_i^2} \Delta_i^{BW} \cdot \hat{\mathbf{Z}}$

Finite wave function normalisation factors (Z-factors)

- ▶ correct on-shell properties of external Higgs bosons with mixing: $\hat{\mathbf{Z}}_{ij}$

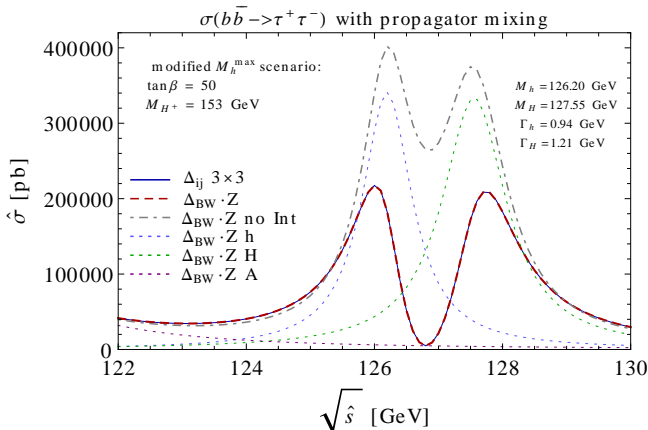


$$\Gamma_{h_i}^{(Z)} = \hat{\mathbf{Z}}_{h_i h} \Gamma_h + \hat{\mathbf{Z}}_{h_i H} \Gamma_H + \dots$$

\rightarrow FeynHiggs

Higgs propagator mixing: BW approximation

- ▶ simple example process $b\bar{b} \rightarrow \tau^+\tau^-$ to study propagator mixing effects



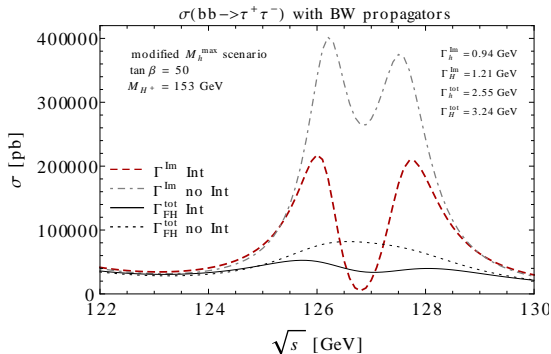
- ▶ mixing propagators well approximated by $\hat{Z} \cdot \Delta^{\text{BW}}$
- ▶ sensitive to exact values of total widths



Breit-Wigner propagators: total width

- ▶ Higgs mass matrix with mixing self-energies

$$\mathbf{M}_0(p^2) = \begin{pmatrix} m_h^2 - \hat{\Sigma}_{hh}(p^2) & -\hat{\Sigma}_{hH}(p^2) & -\hat{\Sigma}_{hA}(p^2) \\ \hat{\Sigma}_{Hh}(p^2) & m_H^2 - \hat{\Sigma}_{HH}(p^2) & -\hat{\Sigma}_{HA}(p^2) \\ \hat{\Sigma}_{Ah}(p^2) & -\hat{\Sigma}_{AH}(p^2) & m_A^2 - \hat{\Sigma}_{AA}(p^2) \end{pmatrix}$$



- ▶ larger **total widths** from FeynHiggs reduce cross section
- ▶ 2 resonances **overlap** in 1 broad peak



Neutralino and chargino matrices

$$Y = \begin{pmatrix} M_1 & 0 & -M_Z c_\beta s_W & M_Z s_\beta s_W \\ 0 & M_2 & M_Z c_\beta c_W & -M_Z s_\beta c_W \\ -M_Z c_\beta s_W & M_Z c_\beta c_W & 0 & -\mu \\ M_Z s_\beta s_W & -M_Z s_\beta c_W & -\mu & 0 \end{pmatrix}, \quad X = \begin{pmatrix} M_2 & \sqrt{2} M_W s_\beta \\ \sqrt{2} M_W c_\beta & \mu \end{pmatrix}$$

On-shell conditions [Fowler, Weiglein '09] [Bharucha, Fowler, Moortgat-Pick, Weiglein '12] [Chatterjee,

Drees, Kulkarni, Xu '11] [Bharucha, Heinemeyer, Pahlen, Schappacher '12],...

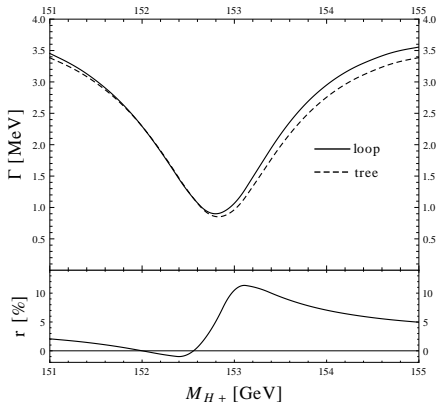
- ▶ 3 out of 6 $\tilde{\chi}^0, \tilde{\chi}^\pm$ masses on-shell
- ▶ choose most bino-, wino- and higgsino-like states as input
→ 3 parameters $|M_1|, |M_2|, |\mu|$ properly fixed

stable scheme choice depends on scenario

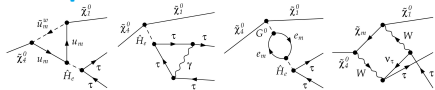


1 → 3 decay at NLO

$$\Gamma(\chi_4^0 \rightarrow \chi_1^0 \tau\tau), r = (\Gamma^{\text{loop}} - \Gamma^{\text{tree}}) / \Gamma^{\text{tree}}$$



1-loop calculation



- ▶ diagrams
 - vertices
 - self-energy
 - box
 - soft photon radiation
- ▶ Higgs mixing by \hat{Z} -factors (finite wave function renormalisation factors)
- ▶ manageable at 1-loop level

use process to **validate gNWA** at 1-loop level

Loop corrections to sub-processes of $\tilde{\chi}_4^0 \rightarrow \tilde{\chi}_1^0 \tau^+ \tau^-$

- ▶ **Production:** full vertex corrections $\mathcal{O}(15\%)$
- ▶ **Decay:** virtual contributions and real γ -emission $\mathcal{O}(-1\%)$
- ▶ **Higgs propagator:** self-energy mixing by $\hat{\mathbf{Z}}$ -factors

Strategy: combination of precise partial results

> separate calculation of loop corrections to **production** and **decay**

> approximation of **interference term** based on NLO matrix elements

> **IR-cancellations** between on-shell matrix elements with virtual + real soft γ

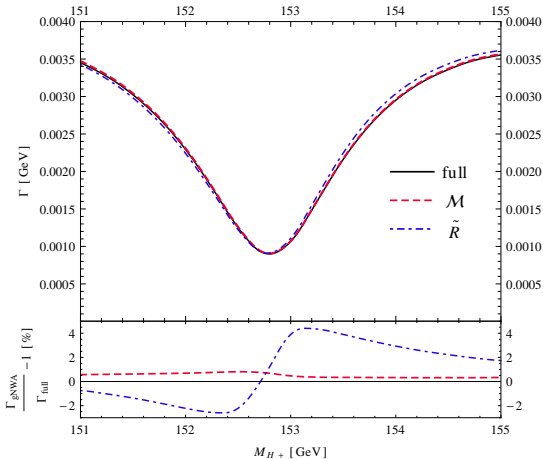
> precise Γ, M, Z, BR (FeynHiggs)



combination of higher-order corrections to sub-processes in **generalised NWA**

$1 \rightarrow 3$ decay vs. gNWA at NLO

$$\Gamma(\tilde{\chi}_4^0 \rightarrow \tilde{\chi}_1^0 \tau^+ \tau^-) \text{ gNWA NLO}$$



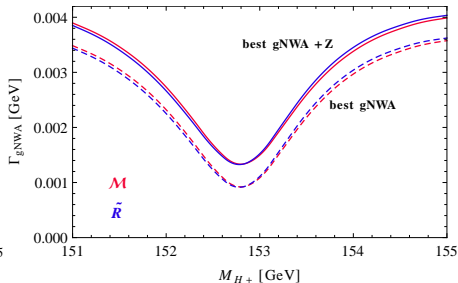
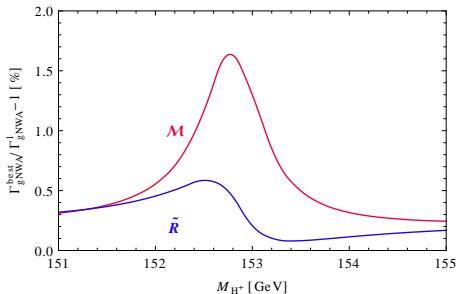
1-loop gNWA

- ▶ 1-loop expansion of matrix elements
- ▶ Higgs-sector: $M, \Gamma, \hat{\mathbf{Z}}$ at leading 2-loop level from FeynHiggs



gNWA with most precise subprocesses

$$\sigma_{\text{gNWA}}^{\text{best}} = \sigma_{\text{full}}^0 + \sum_{i=h,H} \left(\sigma_{P_i}^{\text{best}} \text{BR}_i^{\text{best}} - \sigma_{P_i}^0 \text{BR}_i^0 \right) + \sigma_{\text{gNWA}}^{\text{int}1} + \sigma_{\text{gNWA}}^{\text{int}+}$$



use **factorisation**: include σ_P and BR at **highest available precision** in gNWA



Relation of masses and total widths

