

Plan of lectures: $L_{SM} + L_{BSM}$ $\xrightarrow{\text{data/BSM}}$ $\xrightarrow{\text{Coyere 14 time}}$

- * Flavor in SM: $Y_u, Y_d \rightarrow 6$ masses } Standard
3 angles } Flavor
1 CP phase } parameters
- * FCNCs suppressed (in SM) i loop ii mixing
iii degeneracy
ii / iii : flavor suppressions
- * ~~observation~~ non-observation of SM
breakdown in FCNCs implies
flavor suppression at similar power of
weak or $\Lambda_{NP} \gg \text{TeV}$.

Symmetry-based

One framework for flavor suppression: MFV
Y_{u,d} are the only sources of flavor breaking
"spurious"

$$G_{\text{flavor}} = U(3)^3 \supset \underbrace{SU(3)_Q}_{\text{QF}} \times \underbrace{SU(3)_D}_{\text{DF}} \times \underbrace{SU(3)_U}_{\text{UF}}$$

G_{QF} quark flavor

The SM would be formally G_{QF} -invariant
if the Yukawa-matrices would be "fields"
carrying flavor-charges as.

$$Y_u (3, 1, \bar{3})$$

$$Y_d (\bar{3}, \bar{3}, 1)$$

$$[L_{SM} \supset -\bar{Q}_L \gamma^\mu \not{U}_R - \bar{Q}_D \gamma^\mu \not{D}]$$

Allows to construct G_{QF} -invariant theories BSM.

In MFV, all flavor violation is due to Y_u, Y_d as in SM. Since they are measured, this is very predictive, ta

(dim 6)
Example: [soft operator] (e.g. ~~meron~~-mixing) X_{ij} : flavor matrix

$$\frac{1}{\Lambda^2} (\bar{Q}_L \underset{i,j}{X} Q_L) (\bar{Q}_L \underset{i,j}{X} Q_L)$$

NP

$X = X^+$ (hermitian)

in MFV: $X = a_0 \mathbb{1} + b_1 Y_u Y_u^+ + b_2 Y_d Y_d^+ + \dots$

$$\left(\frac{|b_i|}{|a_0|} \approx 1 \right)$$

higher power

$B_s - \bar{B}_s$ -mixing: $i=3 \quad j=2$

go to down-quark mass basis (useful choice)

$$Y_d = \underbrace{\text{diag}(m_d, m_s, m_b)}_{V} ; \quad Y_u = V_{CKM}^T \underbrace{\text{diag}}_V$$

[check: $V \equiv V_{CKM} = V_u V_d^+ \text{ and } V_u V_d V_d^+ V_u^+ = \text{diag}^2$]

$$\begin{aligned} \Rightarrow X_{32} &= b_1 (V^+ \text{diag}^2 V)_{32} + \dots \\ &= b_1 \left[V_{33}^* \frac{m_t^2}{v^2} V_{32} + V_{23}^* \frac{m_c^2}{v^2} V_{22} + \dots \right] + \dots \\ &= b_1 \left[V_{ts}^* V_{ts} \frac{m_t^2}{v^2} + V_{cb}^* V_{cb} \frac{m_c^2}{v^2} + \dots \right] + \dots \\ &\approx b_1 V_{ts}^* V_{ts} \left(\frac{m_t^2 - m_c^2}{v^2} \right) + \dots \end{aligned}$$

C_{KM} and G_{KM} suppressed as in SM!

Example: flavor in SUSY

superpotential ($N=1$, unbroken R-parity) in MFV:

$$W_{\text{MSSM}} = Q_L H_u H_u^{\dagger} + Q_L Y_d H_d D_R + L_C Y_E H_e E_R + \mu H_u$$

without further model building

Soft-SUSY breaking induces generically $O(1)$ intergenerational flavor mixing and CP-violation and is muddied out ($\tilde{m} \sim 0$ TeVish).

$$\mathcal{L}_{\text{soft}} = - \tilde{Q}_L^+ \tilde{m} \tilde{Q}_L + \dots$$

$$\tilde{m}^2 \tilde{Q} = \tilde{m}^2 \begin{pmatrix} a & \delta_{11} & \delta_{13} \\ b & & \delta_{23} \\ \text{h.c.} & & c \end{pmatrix}; \quad \delta_{ij} \in \mathbb{C}$$

$a, b, c \in \mathbb{R}$

tree level FCNC

$$q_i \bar{q}_j$$

contributes to FCNC-processes

$$\begin{array}{l} \text{flavor} \\ \text{basis} \\ \text{picture} \end{array} \quad \begin{array}{c} \tilde{b}_L \tilde{b}_L^* X \tilde{s}_L \tilde{s}_L^* \\ \tilde{m} \tilde{m}^* \\ \tilde{g} \end{array} \quad \sim \frac{g_S^2}{16\pi^2} \left(\begin{array}{c} \overset{D}{S_{23}} \\ S_{23} \end{array} \right)_{LL} \quad \Rightarrow \left(\begin{array}{c} 0 \\ S_{23} \end{array} \right) \underset{LL}{\sim} \frac{14s}{g_S}$$

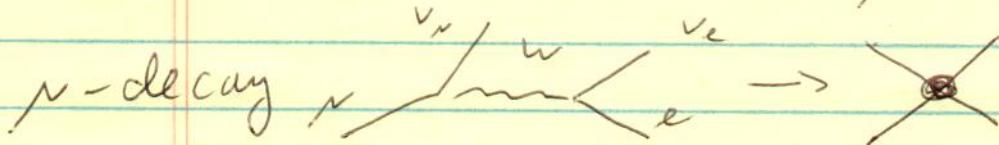
$$SM: \quad \sim \frac{g_W^2}{16\pi^2} V_{tb} V_{ts}^* \quad \left. \begin{array}{l} \text{Details depend} \\ \text{on process and} \end{array} \right.$$

$$\frac{m_{\tilde{g}}}{\tilde{m}}. \quad \text{MFV: } S_{23} \sim V_{ts}$$

bounds very dep. on ij , as strong as 10^{-4} .
 \rightarrow SUSY-flavor problem.

Connecting \mathcal{L} to data: construct \mathcal{L}_{eff}
 extend masses, momenta $\ll m_W$, $\Lambda_{\cancel{\text{NP}}}$; $\Lambda = \Lambda_{\text{NP}}$

Low energy description for $\nu \tilde{\nu} m_W, \Lambda_{\cancel{\text{NP}}}$.



$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 m_W^2}$$

$$\frac{G_F}{\sqrt{2}} (\bar{\nu}_n \gamma_\mu (1 - \delta_S) \nu_n) (\bar{e} \gamma^\mu (1 - \delta_S) e) \nu_e$$

\uparrow^N $\mathcal{L}_{\text{full}}$

Λ matching
 model-independent
 description

$$\mathcal{L}_{\text{eff}} = \sum_i C_i(N) \frac{\mathcal{O}_i(\nu)}{\Lambda^2} + \mathcal{O}\left(\left(\frac{p_{\text{ext}}}{\Lambda^2}\right)^2\right)$$

logs-relevant
 $\log\left(\frac{1}{m_W}\right)$

operators

consistent with symmetries (gauge, Lorentz-inv.)

\mathcal{O}_i : dim 6 out of light d.o.f. ($u, d, s, v, \nu, e, \dots$, δ, g, \dots)

"effective vertices"

C_i : Wilson coefficients; contain info on $\nu \gtrsim \Lambda$.

Full model ($S_N, Z_{\text{HDM}}, \text{flavor mixing}, \dots$) given:

calculate $C_i(N=\Lambda) = C_i(\text{electro weak parameters})$
 + RGE (S_N) to relevant scale, e.g. $\nu = m_b$

$C_i(\text{mb}) \rightarrow$ calculate observables

\leftarrow ~~fit~~ extract experimentally

(Keep $C_i(\text{mb})$ as ~~fit~~ parameters)

Explicitly : $|\Delta B| = 1$ FCNCs :

$b \rightarrow s \gamma$

\rightarrow insert <

$F_{\nu\nu}$: photon field strength tensor

$$b \rightarrow s \gamma : \frac{b_R t_L}{s_L} \gamma^\mu s_L + \cancel{\text{higher order terms}}$$

Calculate the amplitude in full model, e.g. SM

$$A_{S\bar{n}} = \frac{V_{tb} V_{ts} g^2}{m_W^2 16\pi^2} f\left(\frac{m_t}{m_W^2}\right) \bar{s}_L \tilde{g}_{\nu\nu} F^{\nu\nu} b_R$$

$$+ \mathcal{O}\left(\frac{V_{ub} V_{us}}{V_{tb} V_{ts}}, \frac{m_s}{m_b}\right) \cancel{\text{dust}}$$

$$b_R \cdot s_L$$

~~matching~~ $O_2 \equiv \frac{e m_b}{16\pi^2} \bar{s}_L \tilde{g}_{\nu\nu} F^{\nu\nu} b_R$

matching ~~full~~ full and EFT at $\nu = m_W$:

~~check~~ $L_{\text{eff}} = 4 G_F V_{tb} V_{ts} \sum C_i O_i$

$$\Rightarrow A_{S\bar{n}} = \frac{4 G_F}{\sqrt{2}} V_{tb} V_{ts} C_2 \cdot O_2 = \frac{g^2}{2 m_W^2} V_{tb} V_{ts} C_2 O_2$$

$$\Rightarrow C_2(S\bar{n}) (\nu = m_W) = \frac{1}{2} \cdot f\left(\frac{m_t}{m_W^2}\right)$$

e.g. in MSSM $C_2 = C_{\tilde{t}}^{W^\pm} + C_{\tilde{t}}^{H^\pm} + C_{\tilde{t}}^0 + \dots$

depends on model parameters

generically: $C_i = C_i^{S\bar{n}} + C_i^{NP}$ || scale of process

RG evolve C_i from $\nu = m_W$ to $\nu = m_b$ (process).

$$\nu \frac{dC_i}{d\nu} = \delta_{ji} C_j(\nu) \rightarrow \boxed{C_i(m_b)} \quad \begin{array}{l} \text{use} \\ \text{to} \\ \text{calculate} \\ \text{observables} \end{array}$$

operator mixing in general

$$O_2 \propto \bar{s}_L \gamma^\mu b_L \bar{s}_L \gamma^\mu c_L \xrightarrow{f_{mix}} \cancel{\text{mix}} \rightarrow \cancel{\text{X}} \rightarrow \cancel{\text{O}}_{\text{obs}}$$

mixes onto O_2

strategy: calculate Observables in terms of C_i

→ and fit to data / and/or compare to your model

example: $\text{Br}(b \rightarrow s \gamma) \propto |C_7(\text{Lb})|^2$ at LO

how many dim 6 operators are there ~~possible~~ consistent with Lorentz-invariance and SU-gauge invariance?

$\mathcal{O}(100)$ (Wyler, Buchmuller)

for $b \rightarrow s l^+ l^-$ alone, the number is > 20

$$O_9 \propto \bar{s}_L \delta_{\mu\nu} b_R \bar{l} \delta^{\mu\nu} l$$

$$O_{10} \propto \bar{s}_L \delta_{\mu\nu} b_R \bar{l} \delta^{\mu\nu} l \quad + \text{flipped} \quad \begin{matrix} & \\ & L \leftrightarrow R \end{matrix}$$

$$\bar{s}_L b_R \bar{l} l$$

$$\bar{s}_L b_R \bar{l} \delta_{\mu\nu} l$$

$$O_7 \propto \bar{s}_L \delta_{\mu\nu} F^{\mu\nu} b_R$$

$$O_8 \propto \bar{s}_L \delta_{\mu\nu} G^{\mu\nu} b_R$$

$$O_3 - O_6 \propto \bar{s}_L \delta_{\mu\nu} b \sum_{udsccb} \bar{q}_i \delta_{\mu\nu}^N q_j \quad \begin{matrix} \text{color} \\ i \neq j \\ \alpha \neq \beta \end{matrix}$$

$$O_{27-30} \propto \bar{s}_L \delta_{\mu\nu} b \sum_{udsccb} \bar{q}_i \delta_{\mu\nu}^N q_j \quad \begin{matrix} \text{color} \\ i \neq j \\ \alpha \neq \beta \end{matrix}$$

$$\bar{s}_L \delta_{\mu\nu} b \bar{l} \delta^{\mu\nu} e$$

$$\bar{s}_L \delta_{\mu\nu} b \bar{l} \delta^{\mu\nu} l$$

$$\left| \begin{array}{l} \text{MFV:} \\ \bar{s}_L Y_u Y_u^* Y_d b_R \\ \propto m_b V_{ub} V_{us}^* m_L^2 \\ \bar{s}_R Y_d^* Y_u Y_u^* b_L \\ \propto m_s V_{tb} V_{ts}^* m_t^2 \\ \Rightarrow \frac{C_{\text{flipped}}}{C} \propto \frac{m_s}{m_b} \text{ in MFV} \end{array} \right.$$

+ factor 2 if C_i complex

+ lepton flavor dependent effects

C_i^e , and lepton

flavor violation $e^+ \nu^-$

~~A fully model-independent analysis is not tractable.~~ ~~Analytically, we usually, make assumptions such as MFV, or those driven by models (BSM).~~

Connecting \mathcal{L} to rare decay data

i) we $\mathcal{L}_{\text{eff}} = \sum C_i(\mu) O_i(\mu)$ for $N \gtrsim 1$

↑ effective vertices
their couplings

out of light degrees of freedom ($u, d, s, c, b, \gamma, g, \dots$)

ii) R^b - evolve $C_i(\mu \approx 1)$ to $C_i(\mu \approx m_b)$
using SM-RG. (Known $(N^*) \subset 0$)

iii) calculate rare decay amplitudes, observables, distributions, asymmetries as functions of $C_i(m_b)$

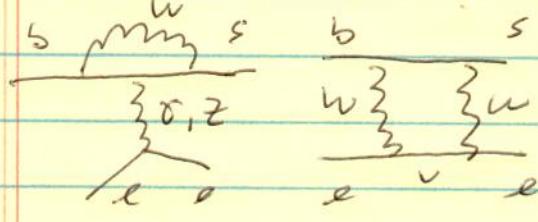
iv) compare to data.

today: step iii) and iv).

$$m_b \gg 1_{\text{QCD}} ; \chi_s(m_b) \approx 0.2 \quad \smile$$

$$m_c \gg 1_{\text{QCD}} ; \chi_s(m_c) \approx 0.4 \quad \frown$$

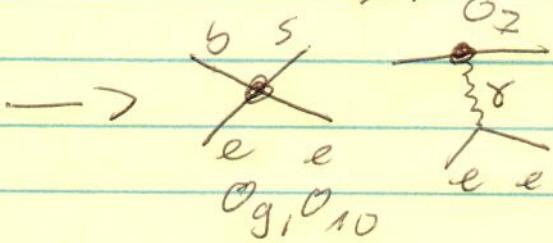
$b \rightarrow s \ell \bar{\ell}$ in LO SM



$$O_9 \sim \bar{s}_L \sigma_N b_L \bar{e} \sigma^\mu e$$

$$O_{10} \sim \bar{s}_L \sigma_N b_L \bar{e} \sigma^\mu \bar{e} s_L$$

in \mathcal{L}_{eff}



$$O_7 \sim m_b \bar{s}_L \sigma_\mu \nu b_R F^{\mu\nu}$$

exclusive processes: $B \rightarrow K^{(*)} \ell \bar{\ell}$, $B_s \rightarrow \ell \bar{\ell}$, $B_s \rightarrow \phi \ell \bar{\ell}$,
rare $|\Delta B| = |\Delta D| = 1$

question: What about $|\Delta B| = |\Delta D| = 1$?

$$R_{ds} = \frac{Br(B_d \rightarrow \ell \bar{\ell})}{Br(B_s \rightarrow \ell \bar{\ell})} \approx ?$$

$$R_{ds} \stackrel{SM, M=V}{\approx} \frac{|V_{td}|^2}{|V_{ts}|^2} \times SU(3)_C \times \text{phase space}$$

$$A(B \rightarrow K^{(*)} ll) = \langle K^{(*)} ll | \mathcal{L}_{eff} | B \rangle$$

$$= -\frac{4 G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i C_i \underbrace{\langle K^{(*)} ll | O_i | B \rangle}_{\text{hadronic matrix element}}$$

$$\langle K^{(*)} ll | O_i | B \rangle = \underbrace{\langle e e | \bar{s} \gamma^\mu s | 0 \rangle}_{\substack{\text{factorization} \\ \text{analytic}}} \underbrace{\langle K^{(*)} | \bar{s} \gamma^\mu b | B \rangle}_{\substack{\text{standard} \\ \text{techniques}}} \underbrace{\langle \bar{s} \gamma^\mu b | 0 \rangle}_{\substack{\text{form-factor}}}$$

form factor = fact of momentum transfer ;
coefficient in Lorentz decomposition

$$\text{example : } \langle K(p_K) | \bar{s} \gamma^\mu b | B(p_B) \rangle = a p_B^\mu + b p_K^\mu$$

$$a, b = \text{function of } q^2 = (p_B - p_K)^2 \quad (= p_{e^+} + p_{e^-})$$

check parity (conserved by QCD)

$$(-1) (-1) (-1) = (-1) \quad \checkmark$$

$$\Rightarrow 1. \quad \langle K | \bar{s} \gamma^\mu s | B \rangle \equiv 0 \quad !$$

$$2. \quad A(B \rightarrow K l e) \text{ involves } (C_{10} + C'_{10})$$

$$V-A \quad V+A$$

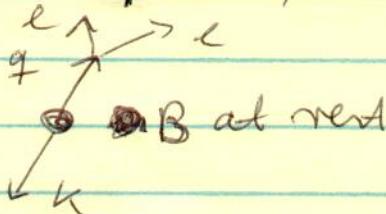
$$\text{rewrite } a p_B^\mu + b p_K^\mu = f_+^{(q^2)} (p_B + p_K)^\mu + f_-^{(q^2)} (p_B - p_K)^\mu$$

$f_-(q^2)$ does not contribute for $m_e = 0$ (use Dirac equation).

O_2 -piece:

$$\langle k | \bar{s} g_{\mu\nu} q^\mu b | B \rangle \rightarrow \text{additive form factor } f_T(q^2)$$

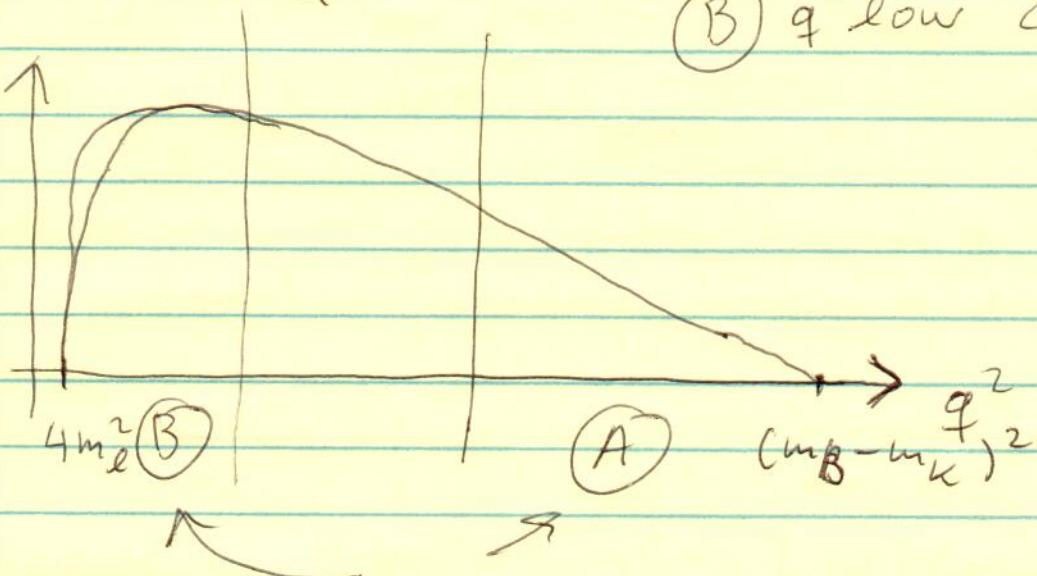
$m_e = 0$: need $f_+^{(q^2)}$, $f_T^{(q^2)}$ [related at LO in γ/mb !]



(A) $q^2 \sim O(\text{mb}^2) \rightarrow E_K \sim "low \text{ recoil"} O(1)$

(B) $q^2 \text{ low} \rightarrow E_K \gg 1$

$$\frac{dBr}{dq^2}$$

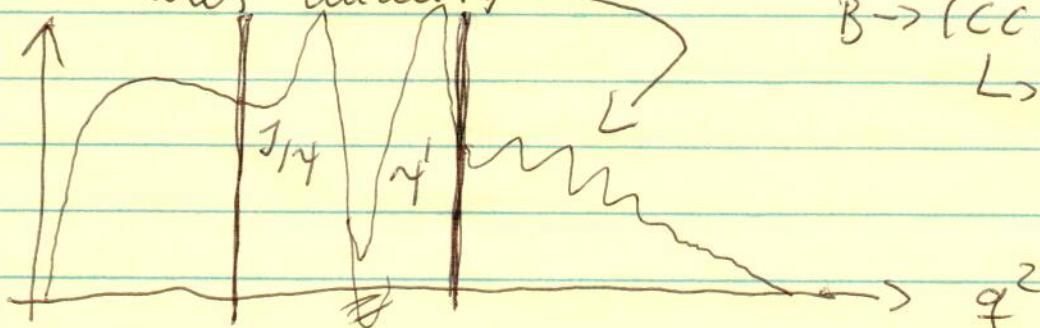


different th description

(B) : QCdf = factorization at LO in γ/mb
form factors from sum rules "color transparency"

(A) OPE ; uses hard momentum transfer $q^2 \sim O(\text{mb}^2)$
factorization at NLO in γ/mb
 γ/mb -corrections parametrically suppressed
form factors from lattice QCD
uses duality

what is measured:



$$B \rightarrow (c\bar{c}) K$$

BGD
L \rightarrow ll

model-independent analysis (ongoing as we speak)

assume NP in $O_{7,9,10}$ only, no NP CPV (working assumption)

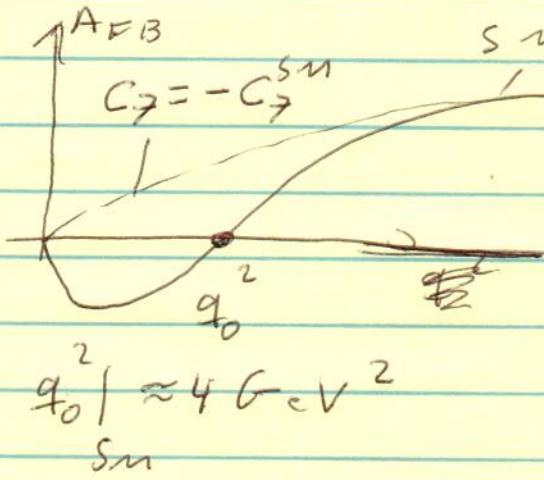
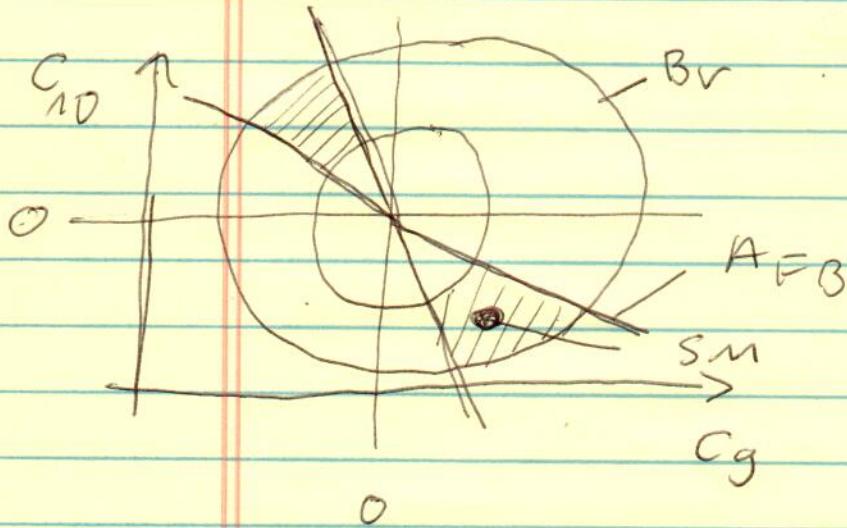
From b-s-s data: $|C_7|$ fixed near $|C_7^{\text{SM}}|$; both signs
 $B_C(B \rightarrow K^{(*)} \ell \ell) \propto |C_g|^2 + |C_{10}|^2$
 large q^2

$$A_{FB}(B \rightarrow K^{(*)} \ell \ell) \propto -\underbrace{C_{10}\left(C_g + 2 \frac{m_b}{q^2} C_7\right)}_{\text{dominant large } q^2}$$

forward-backward asymmetry

$$= \frac{\# \text{ forward } e^+ - \# \text{ backward } e^+}{+}$$

vs \bar{B} meson in dilepton cuts



there are plenty of further, in part "optimized" observables (regarding the uncertainties) available through angular analysis:

$$B \rightarrow K^*(\rightarrow K\pi) ll$$

constell decay

quasi
4-body

$$d^4 P = \sum_{i=1}^N f_i(\theta_e, \theta_K, \varphi) J_i(q^2) dq^2 d\theta_e d\theta_K d\varphi$$

trigonometric ↗ angular
 $\sim Y_m$, Known coefficients
 "observables"

$\theta_e = \delta(\vec{e}, \vec{B})$ in dilepton CUS

$\theta_K = \delta(K, \vec{B})$ in $(K\pi)$ CUS

φ : the angle between the (ll) and $(K\pi)$ decay planes.

$N=11$ if $m_e=0$ and no scalar operators.

discuss R_K , P_5' & global fits
 (slides)