Very high precision theoretical challenges

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2nd Lecture

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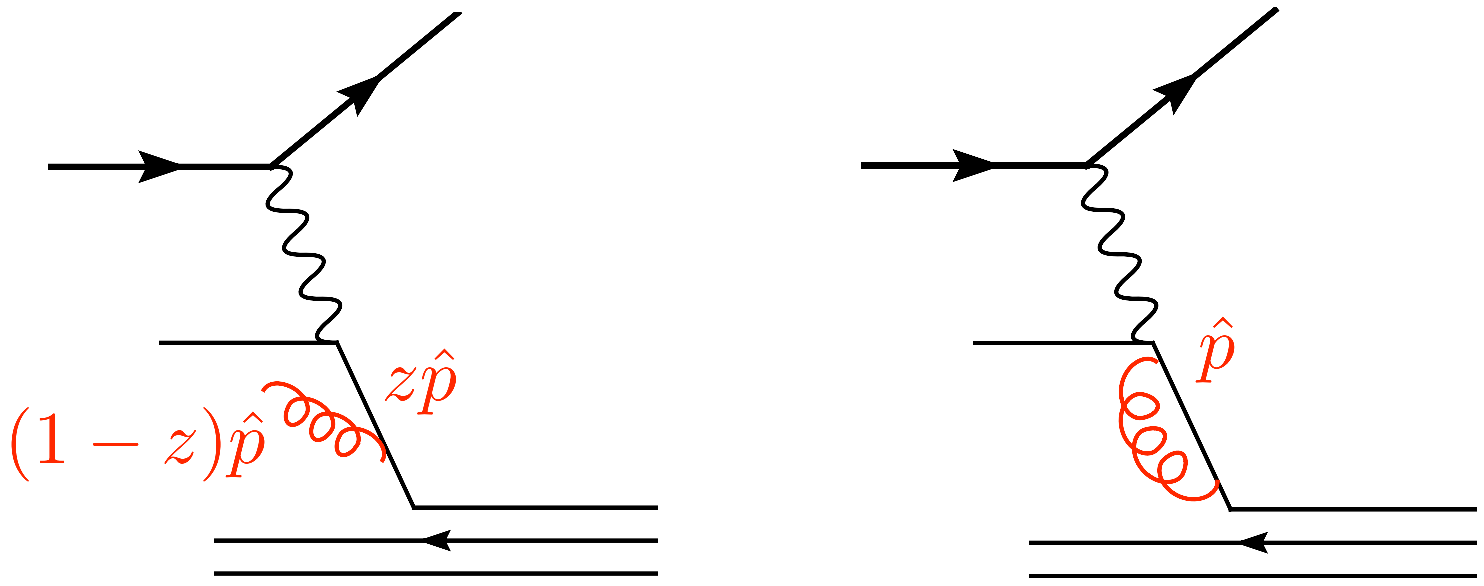
Multi-TeV Probes of Standard Model Physics with the LHC

Radiative corrections

To first order in the coupling:

need to consider the emission of one real gluon and a virtual one



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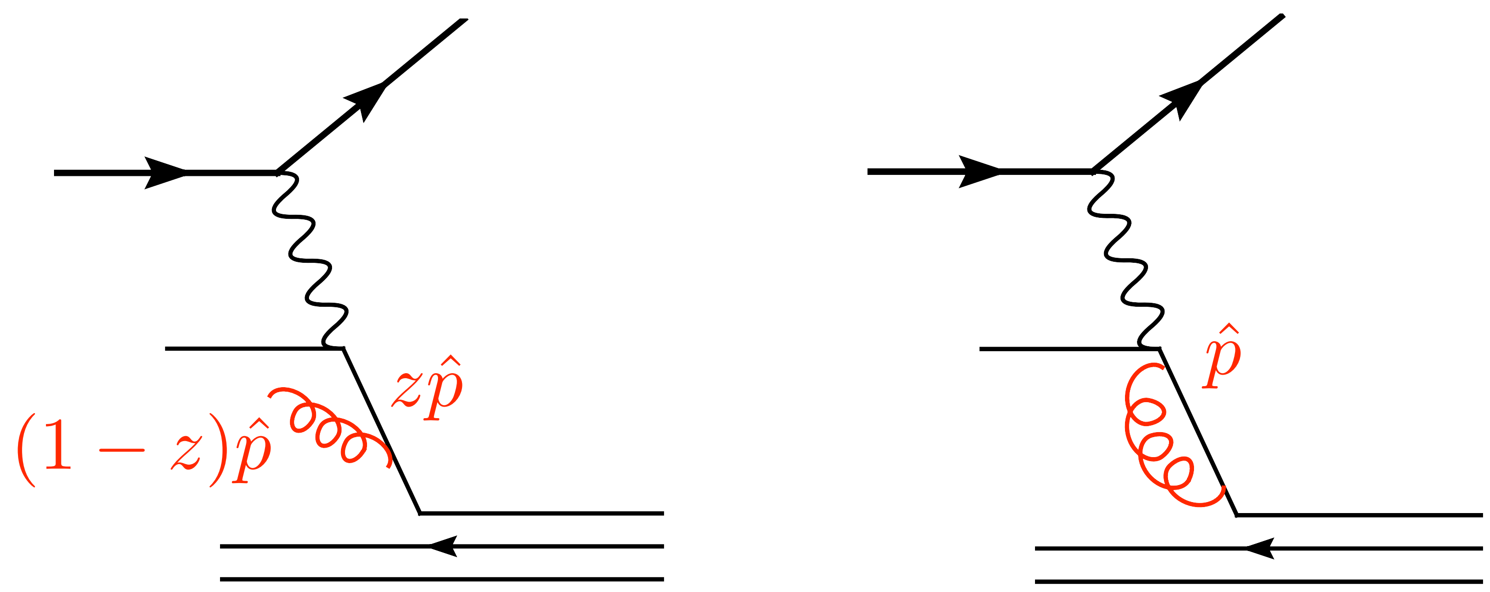

Adding real and virtual contributions, the partonic cross-section reads

$$\sigma^{(1)} = \frac{C_F \alpha_s}{2\pi} \int dz \frac{dk_\perp^2}{k_\perp^2} \frac{1+z^2}{1-z} \left(\sigma^{(0)}(z\hat{p}) - \sigma^{(0)}(\hat{p}) \right)$$

Partial cancellation between real (positive), virtual (negative), but real gluon changes the energy entering the scattering, the virtual does not

Radiative corrections

Partonic cross-section:

$$\sigma^{(1)} = \frac{\alpha_s}{2\pi}\int dz \int_{\lambda^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} P(z)\left(\sigma^{(0)}(z\hat{p}) - \sigma^{(0)}(\hat{p})\right), \quad P(z) = C_F \frac{1+z^2}{1-z}$$

Soft limit: singularity at z=1 cancels between real and virtual terms

Collinear singularity: $k_\perp \rightarrow 0$ with finite z. Collinear singularity does not cancel because partonic scatterings occur at different energies

Radiative corrections

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$\Rightarrow$ naive parton model does not survive radiative corrections

Radiative corrections

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$\Rightarrow$ naive parton model does not survive radiative corrections

Similarly to what is done when renormalizing UV divergences, collinear divergences from initial state emissions are absorbed into parton distribution functions

The plus prescription

Partonic cross-section:

$$\sigma^{(1)} = \frac{\alpha_s}{2\pi} \int_{\lambda^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \int_0^1 dz\, P(z) \left(\sigma^{(0)}(z\hat{p}) - \sigma^{(0)}(\hat{p}) \right)$$

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Plus prescription makes the universal cancelation of singularities explicit

$$\int_0^1 dz f_+(z) g(z) \equiv \int_0^1 f(z) \left(g(z) - g(1) \right)$$

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The partonic cross section becomes

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Collinear singularities still there, but they factorize.

Factorization scale

Schematically use
$$\ln\frac{Q}{\lambda^2} = \ln\frac{Q}{\mu_F^2} + \ln\frac{\mu_F}{\lambda^2}$$

$$\sigma = \sigma^{(0)} + \sigma^{(1)} = \left(1 + \frac{\alpha_s}{2\pi}\ln\frac{\mu_F^2}{\lambda^2}P_+\right) \times \left(1 + \frac{\alpha_s}{2\pi}\ln\frac{Q^2}{\mu_F^2}P_+\right)\sigma^{(0)}$$

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So we define

$$f_q(x, \mu_F) = f_q(x) \times \left(1 + \frac{\alpha_s}{2\pi} \ln \frac{\mu_F^2}{\lambda^2} P_{qq}^{(0)}\right) \qquad \hat{\sigma}(p, \mu_F) = \left(1 + \frac{\alpha_s}{2\pi} \ln \frac{Q^2}{\mu_F^2} P_{qq}^{(0)}\right) \sigma^{(0)}(p)$$

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NB:

- universality, i.e. the PDF redefinition does not depend on the process
- choice of $\mu_F \sim Q$ avoids large logarithms in partonic cross-sections
- PDFs and hard cross-sections don't evolve independently
- the factorization scale acts as a cut-off, it allows to move the divergent contribution into non-pertubative parton distribution functions

Improved parton model

Naive parton model:

$$\cancel{\sigma = \int dx_1 dx_2 f_1^{(P_1)}(x_1) f_2^{(P_2)}(x_2) \hat{\sigma}(x_1 x_2 s)} \qquad \hat{s} = x_1 x_2 s$$

After radiative corrections:

$$\sigma = \int dx_1 dx_2 f_1^{(P_1)}(x_1, \mu^2) f_2^{(P_2)}(x_2, \mu^2) \hat{\sigma}(x_1 x_2 s, \mu^2)$$

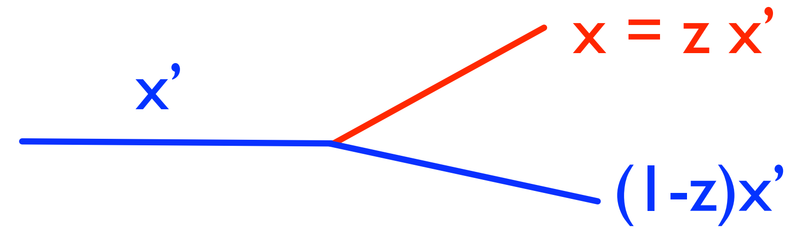
Intermediate recap

- With initial state parton collinear singularities don't cancel
- Initial state emissions with $k_\perp$ below a given scale are included in PDFs
- This procedure introduces a scale μ_F, the so-called factorization scale which factorizes the low energy (non-perturbative) dynamics from the perturbative hard cross-section
- As for the renormalization scale, the dependence of cross-sections on μ_F is due to the fact that the perturbative expansion has been truncated
- The dependence on μ_F becomes milder when including higher orders

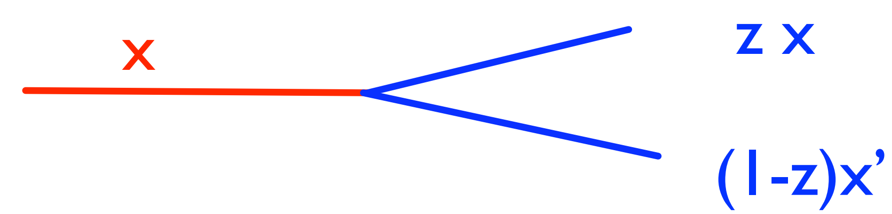
Evolution of PDFs

A parton distribution changes when

- a different parton splits and produces it



- the parton itself splits



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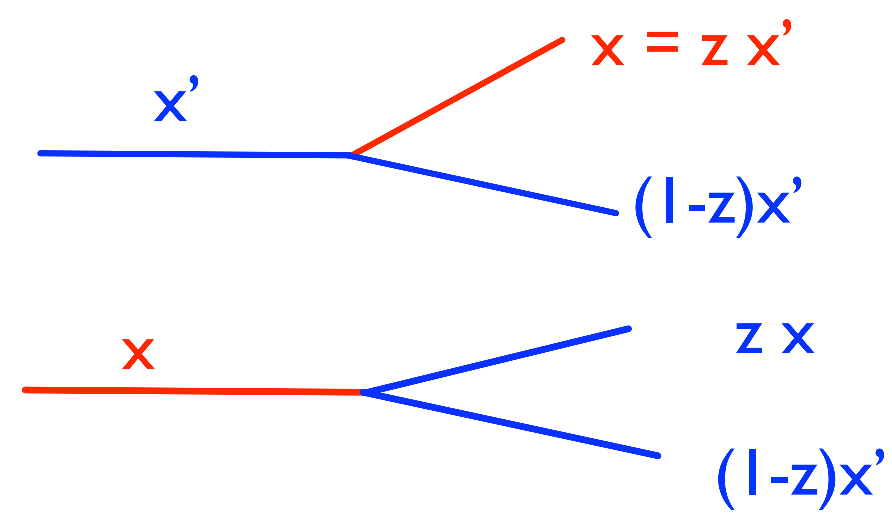


$$\mu^2 \frac{\partial f(x,\mu^2)}{\partial \mu^2} = \int_0^1 dx' \int_x^1 dz \frac{\alpha_s}{2\pi} P(z) f(x',\mu^2) \delta(zx'-x) - \int_0^1 dz \frac{\alpha_s}{2\pi} P(z) f(x,\mu^2)$$

$$= \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P(z) f\left(\frac{x}{z},\mu^2\right) - \int_0^1 dz \frac{\alpha_s}{2\pi} P(z) f\left(x,\mu^2\right)$$

$$= \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P_+(z) f\left(\frac{x}{z},\mu^2\right)$$

The plus prescription $\quad \int_0^1 dz f_+(z) g(z) \equiv \int_0^1 dz f(z) \left(g(z) - g(1)\right)$

DGLAP equation

$$\mu^2 \frac{\partial f(x,\mu^2)}{\partial \mu^2} = \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P(z) f\left(\frac{x}{z}, \mu^2\right)$$

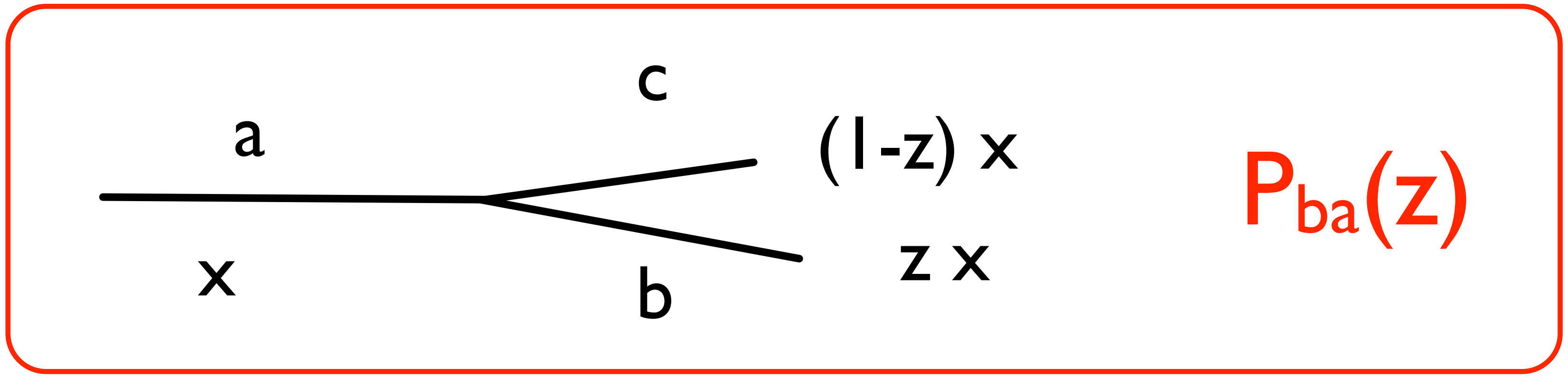
Altarelli, Parisi; Gribov-Lipatov; Dokshitzer '77

Master equation of QCD: we can not compute parton densities, but we can predict how they evolve from one scale to another

Universality of splitting functions: we can measure pdfs in one process and use them as an input for another process

Conventions for splitting functions

There are various partons flavours. Standard notation:



Accounting for the different species of partons the DGLAP equations become:

$$\mu^2 \frac{\partial f_i(x,\mu^2)}{\partial \mu^2} = \sum_j \int_x^1 \frac{dz}{z} P_{ij}(z) f_j\left(\frac{x}{z},\mu^2\right)$$

This is a system of coupled integro/differential equations

The above convolution in compact notation:

$$\mu^2 \frac{\partial f_i(x,\mu^2)}{\partial \mu^2} = \sum_j P_{ij} \otimes f_j(\mu^2)$$

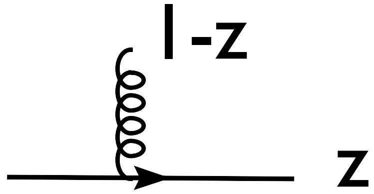
General DGLAP equation

Evolution equations in the general case:

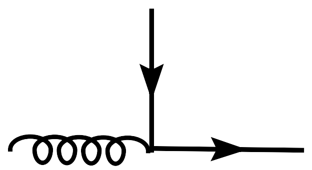
$$\mu^2 \frac{\partial f_i(z,\mu^2)}{\partial \mu^2} = \sum_j P_{ij} \otimes f_j(\mu^2) \qquad P_{ij}(x) = \frac{\alpha_s}{2\pi} P_{ij}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right)^2 P_{ij}^{(2)} + \dots$$

Leading order splitting functions:

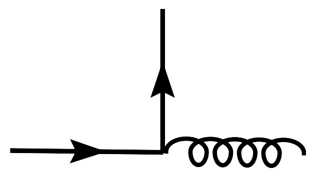
$$P_{qq}^{(0)} = P_{\bar{q}\bar{q}}^{(0)} = C_F \left(\frac{1+z^2}{1-z}\right)_+$$



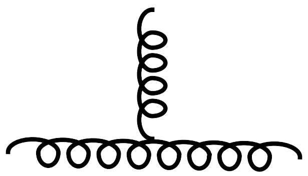
$$P_{qg}^{(0)} = P_{\bar{q}g}^{(0)} = T_R \left(z^2 + (1-z)^2\right)$$



$$P_{gq}^{(0)} = P_{g\bar{q}}^{(0)} = C_F \frac{1+(1-z)^2}{z}$$



$$P_{gg}^{(0)} = 2C_A \left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z)\right] + \frac{1}{6}(11C_A - 4n_f T_R)\delta(1-z)$$



NB: at higher orders $P_{q_i q_j}$ arise

History of splitting functions

☑ $P_{ab}^{(0)}$: Altarelly, Parisi; Gribov-Lipatov; Dokshitzer (1977)

☑ $P_{ab}^{(1)}$: Curci, Furmanski, Petronzio (1980)

☑ $P_{ab}^{(2)}$: Moch, Vermaseren, Vogt (2004)

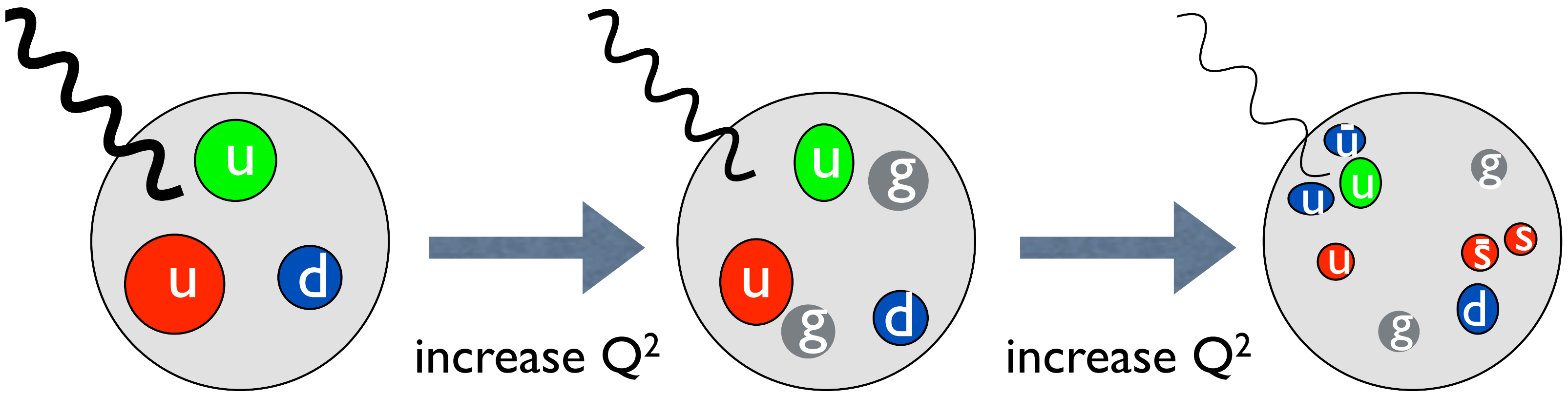
☛ $P_{ab}^{(2)}$: one of the hardest calculation ever performed in pQCD

☛ Essential input for NNLO pdfs determination (state of the art today)

Evolution

So, in perturbative QCD we can not predict values for

- the coupling
- the masses
- the parton densities
- ...



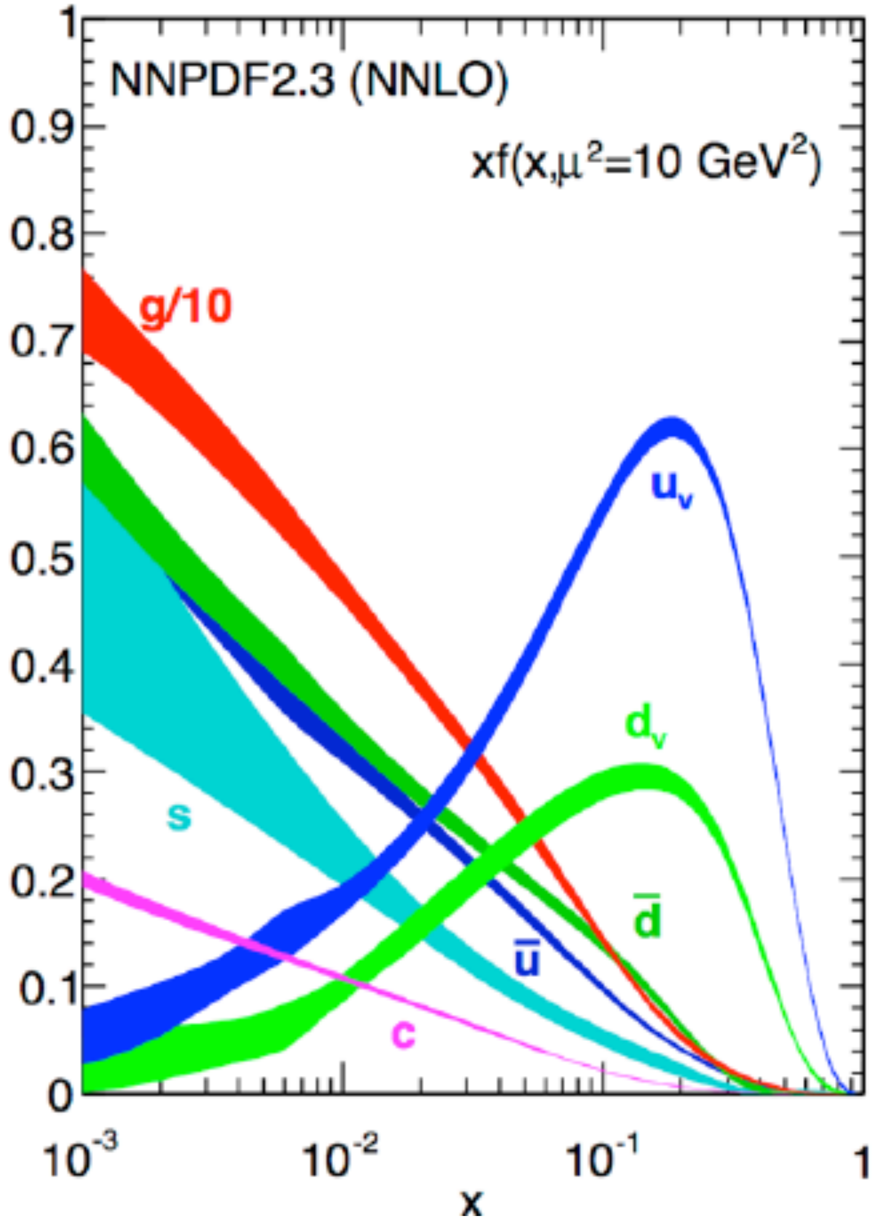
What we can predict is the evolution with the Q^2 of those quantities. These quantities must be extracted at some scale from data.

- not only is the coupling scale-dependent, but partons have a scale dependent sub-structure
- we started with the question of how one can access the gluon pdf: In DIS: because of the DGLAP evolution, we can access the gluon pdf indirectly, through the way it changes the evolution of quark pdfs. Today also direct measurements using Tevatron jet data and LHC tt production

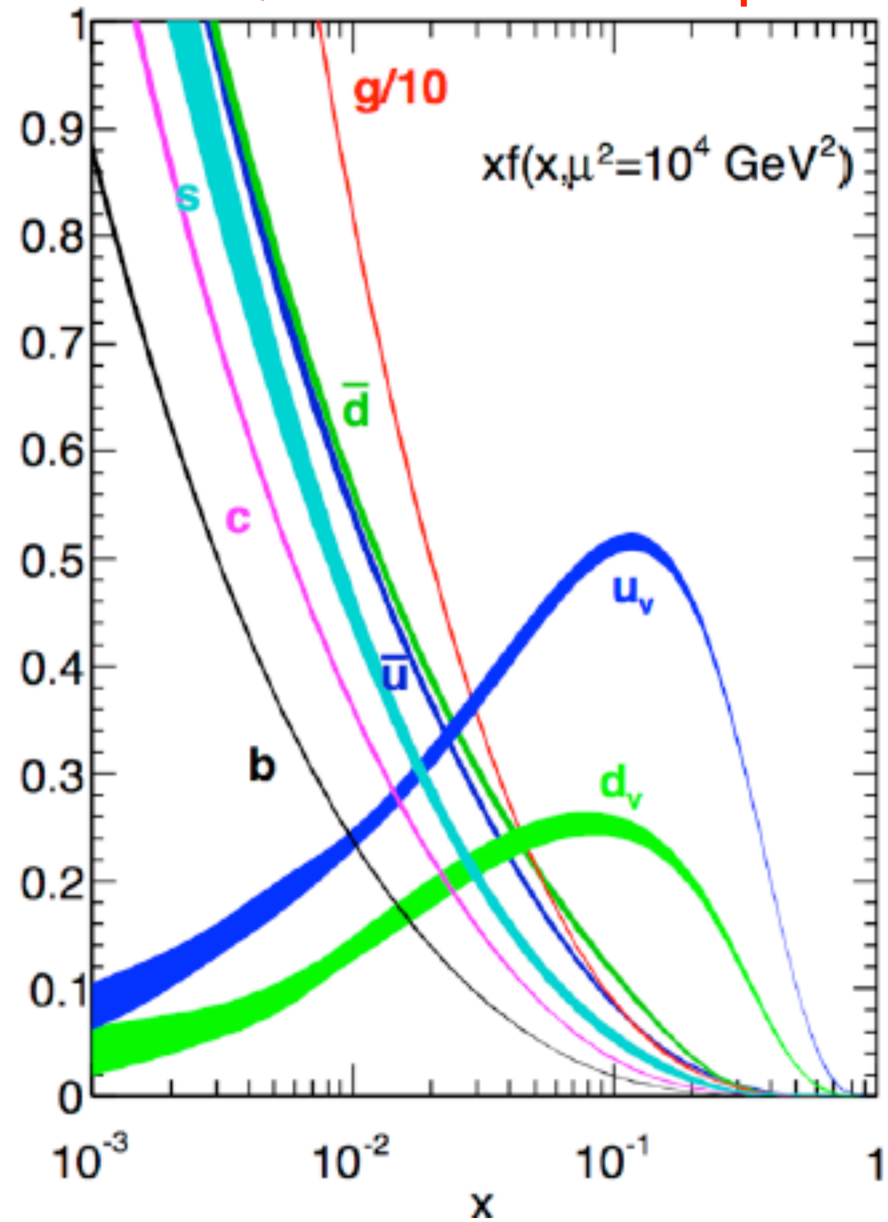
DGLAP Evolution

The DGLAP evolution is a key to precision LHC phenomenology: it allows to measure PDFs at some scale (say in DIS) and evolve upwards to make LHC (7, 8, 13, 14, 33, 100.... TeV) predictions

Measure PDFs at 10 GeV



Evolve in Q^2 and make LHC predictions

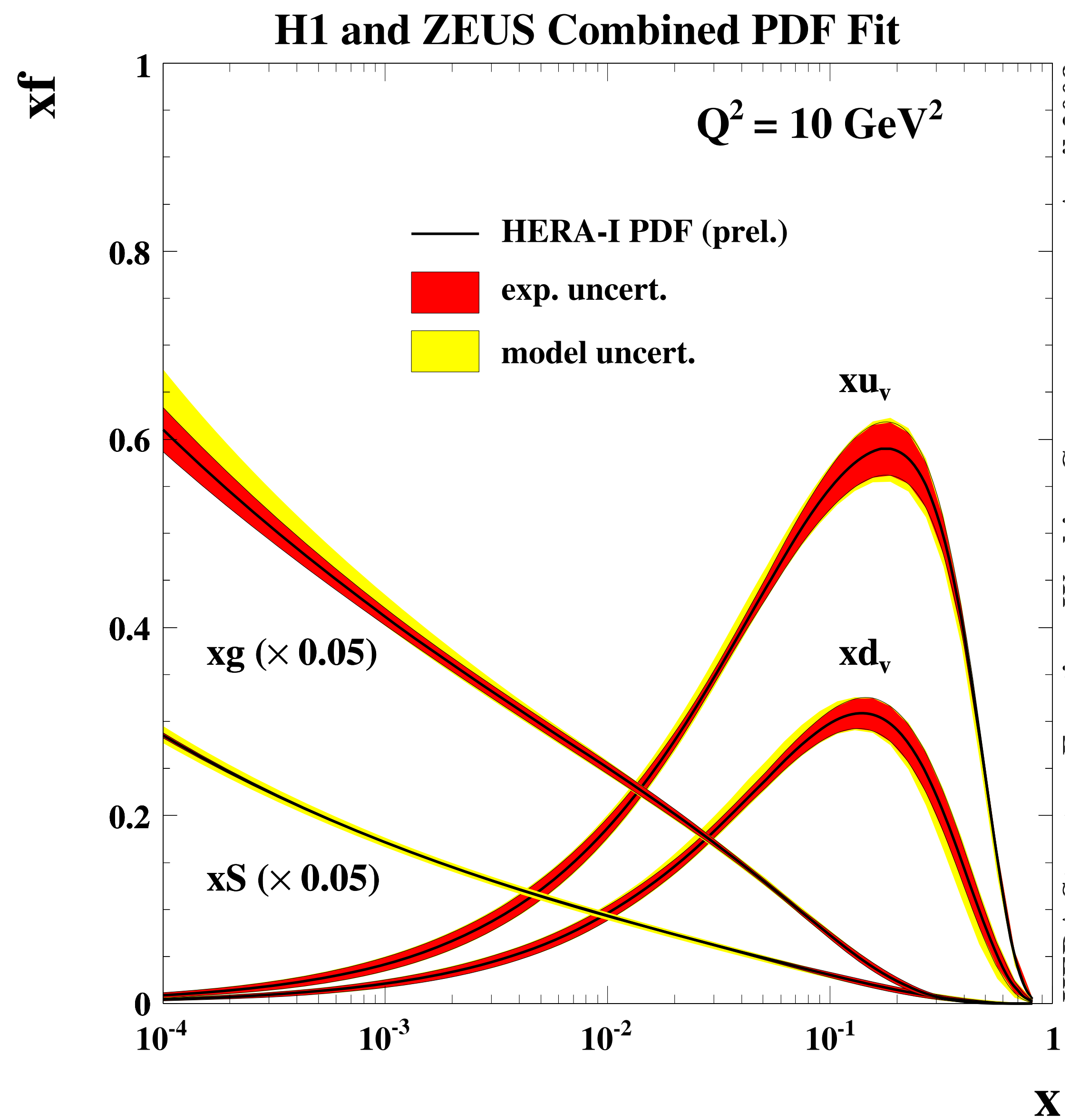


Different PDFs evolve in different ways (different equations + unitarity constraint)

Typical features of PDFs

<u>Typical features:</u>

- gluon distribution very large
- gluon and sea distributions grow at small x
- gluon dominates at small x
- valence distributions peak at x = 0.1 - 0.2
- largest uncertainties at very small or very large x

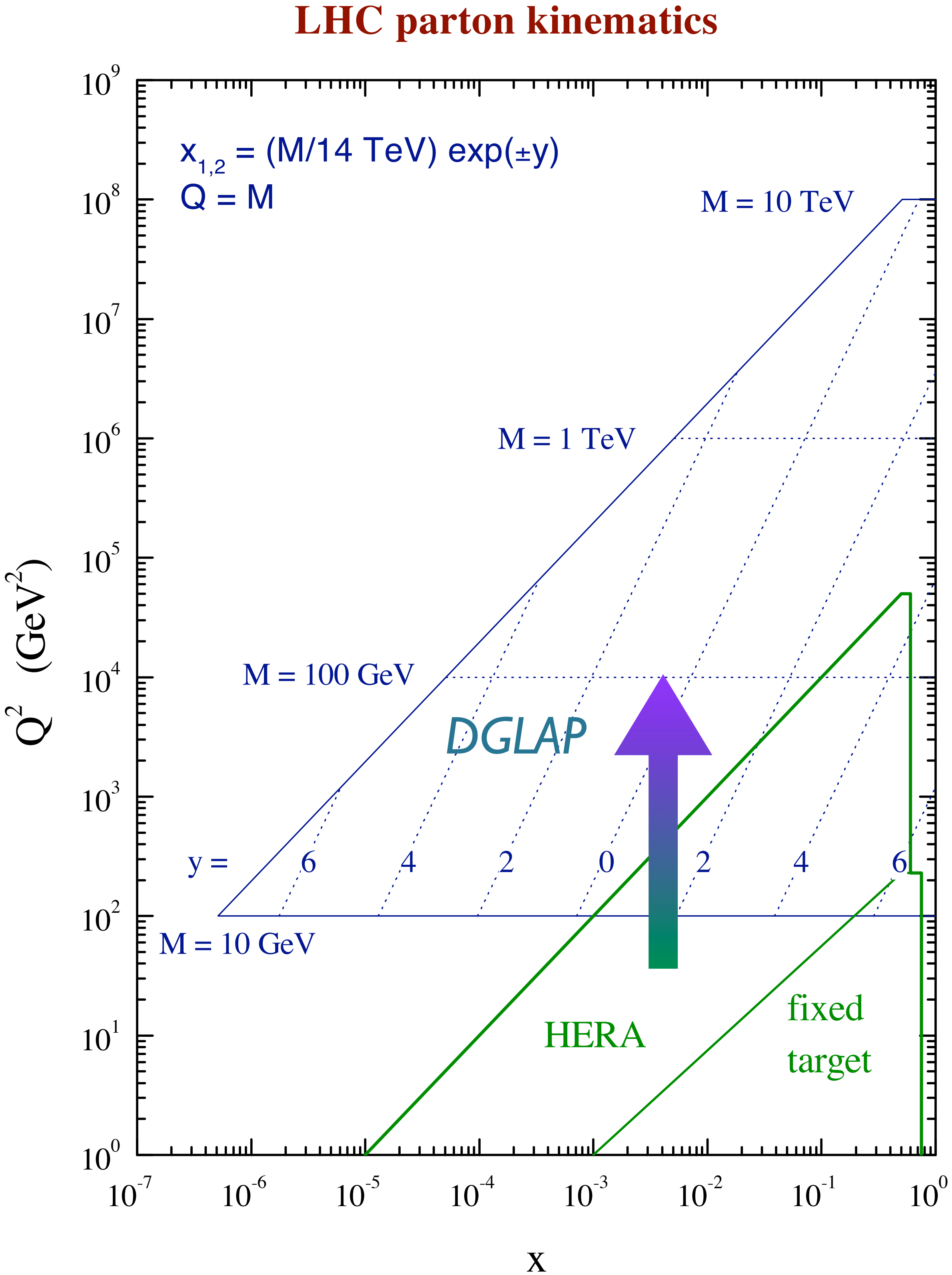


Crucial property: factorization!

PDFs extracted in DIS can be used at hadron colliders. This assumption can be checked against data (but often rigorous proof is missing)

Parton density coverage

- most of the LHC x-range covered by Hera
- need 2-3 orders of magnitude Q^2-evolution
- rapidity distributions probe extreme x-values
- 100 GeV physics at LHC: small-x, sea partons
- TeV physics: large x



Parton densities: recent progress

Recent major progress:

- full NNLO evolution (previous approximate NNLO)
- improved treatment of heavy flavors near the quark mass
  [Numerically: e.g. (6-7)% effect on Drell-Yan at LHC]
- more systematic use of uncertainties/correlations (e.g. dynamic tolerance, combinations of PDF + α_s uncertainty)
- Neural Network (NN) PDFs

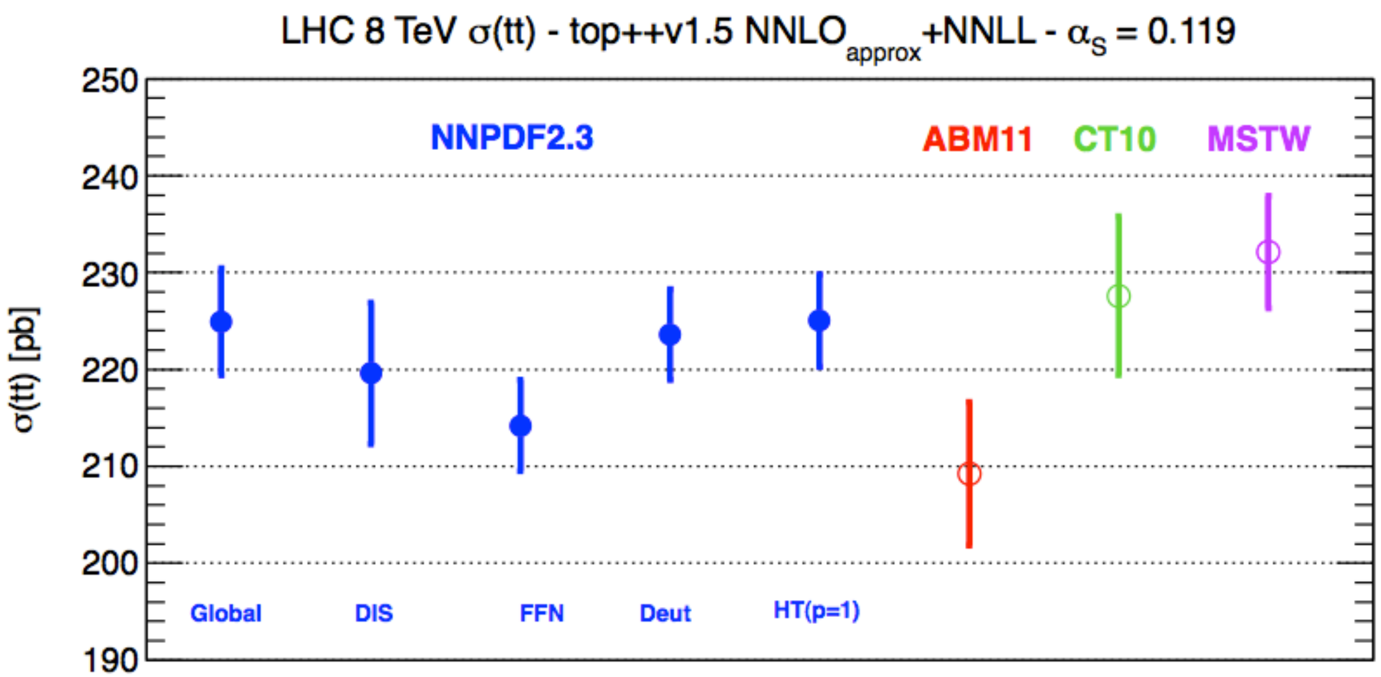
ABM, CTEQ, MSTW, NN collaboration

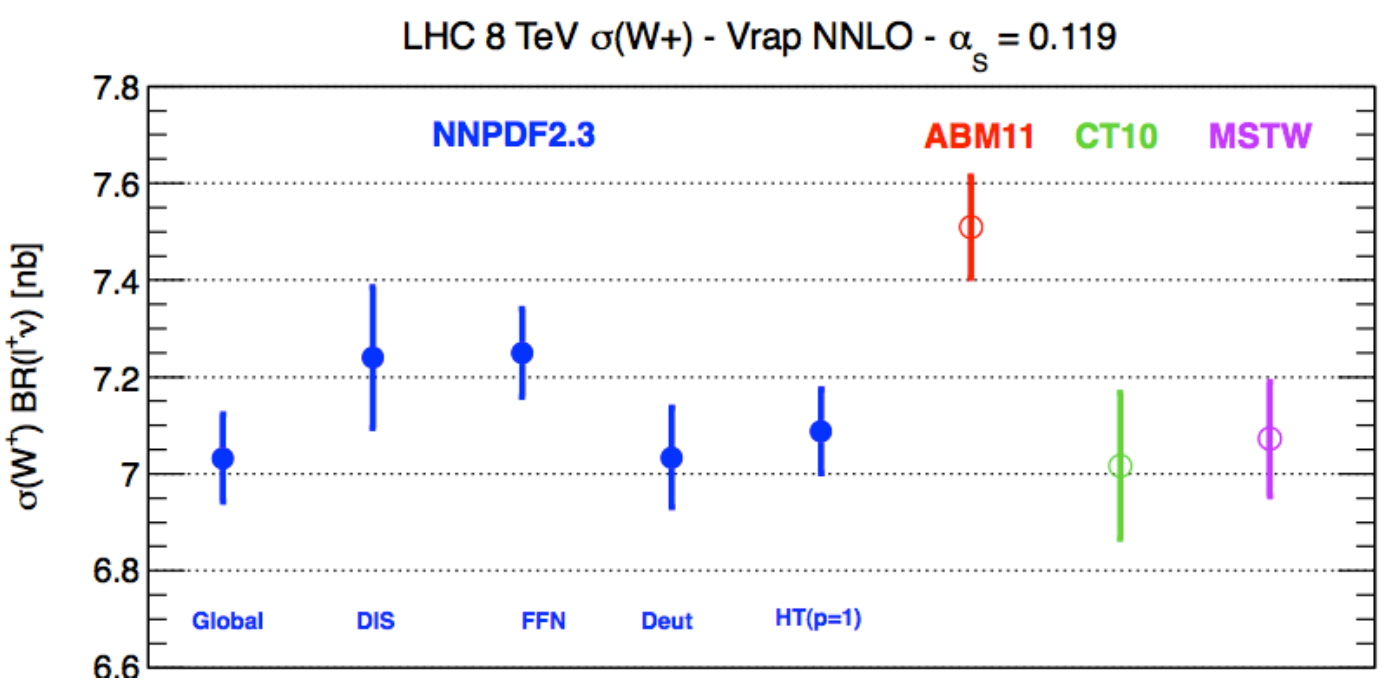
Still, considerable differences in predictions for benchmark process.

Parton densities: benchmark processes

Uncertainty from PDFs (no α_s) on benchmark processes

NN col. 1303.1189

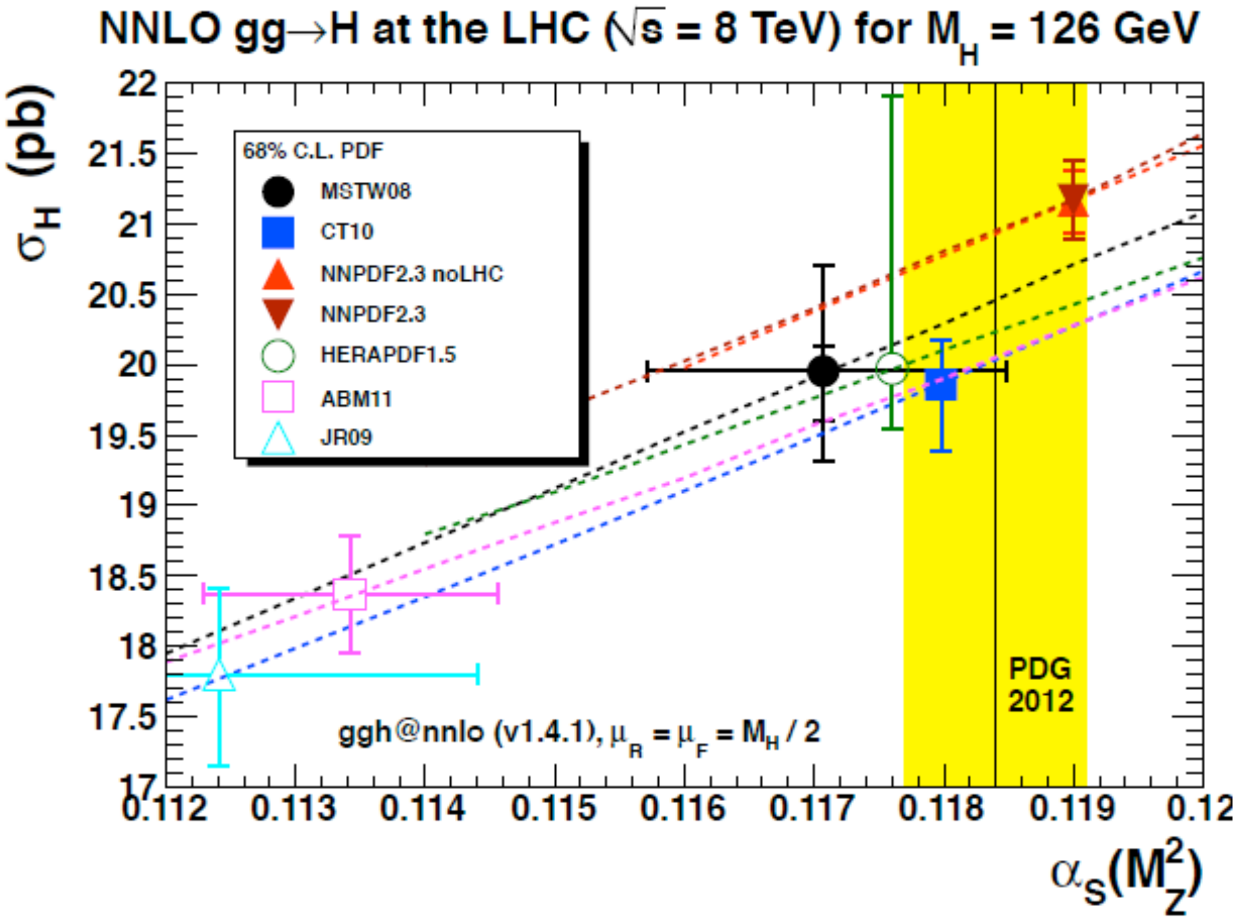




In general differences due to:

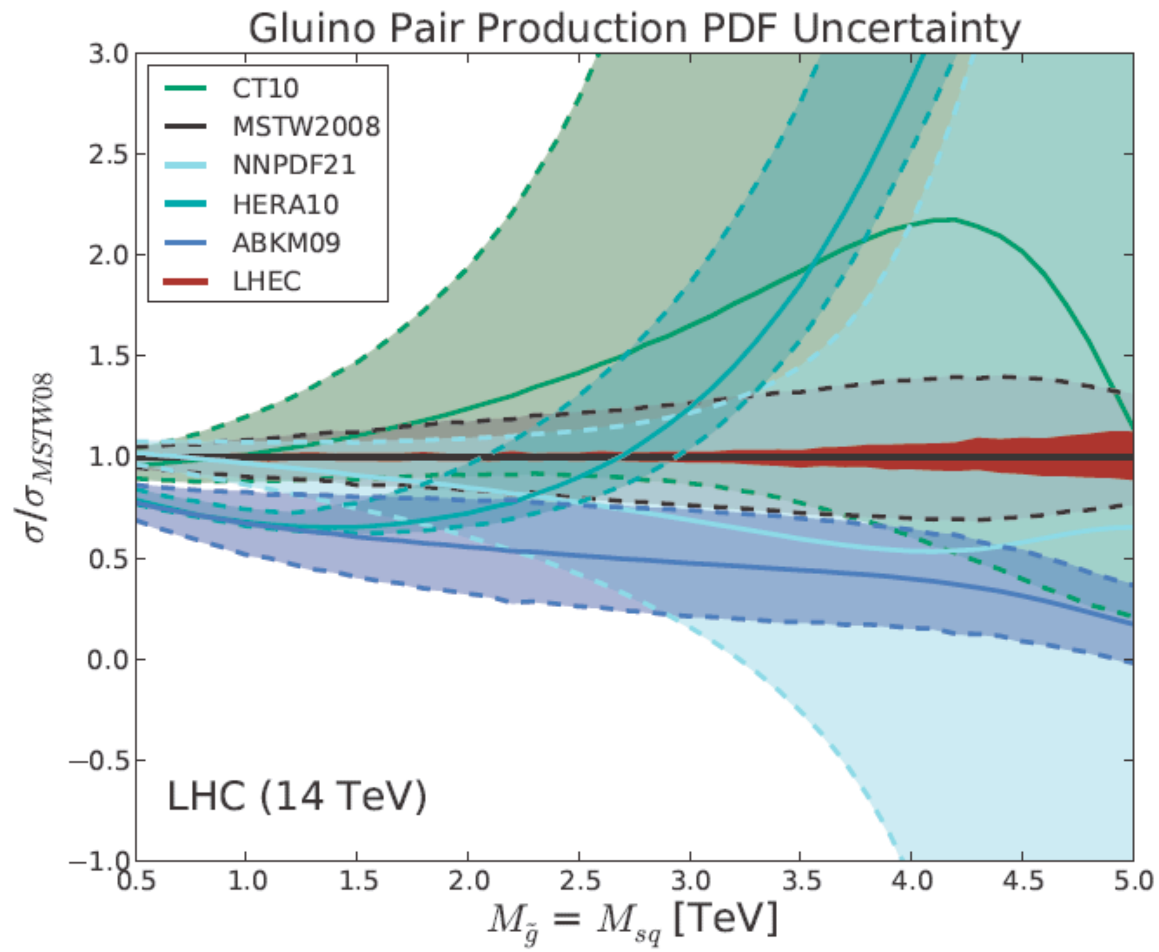
1) different data in fits
2) different methodology [parametrization, theory]
3) treatment of heavy quarks
4) different α_s

Parton densities and LHC phenomenology



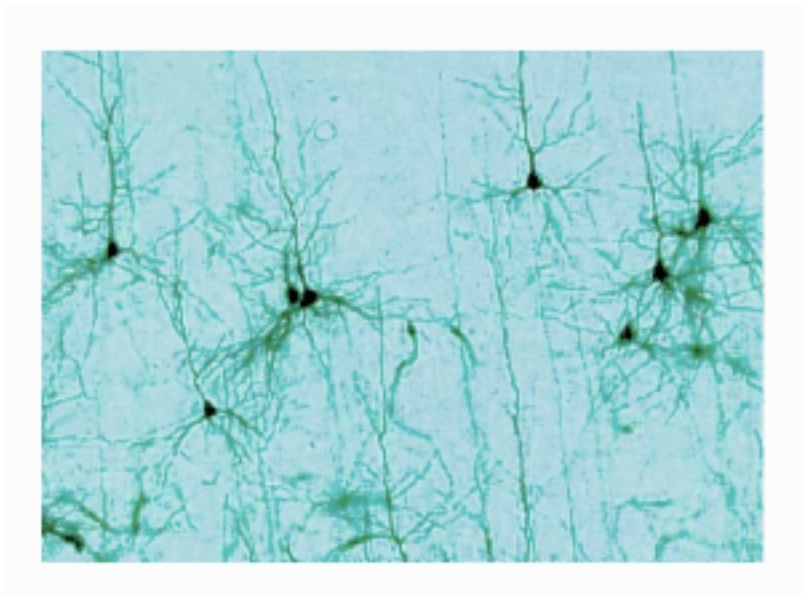
PDFs limit extraction of Higgs boson couplings. Crucial for Higgs boson characterization

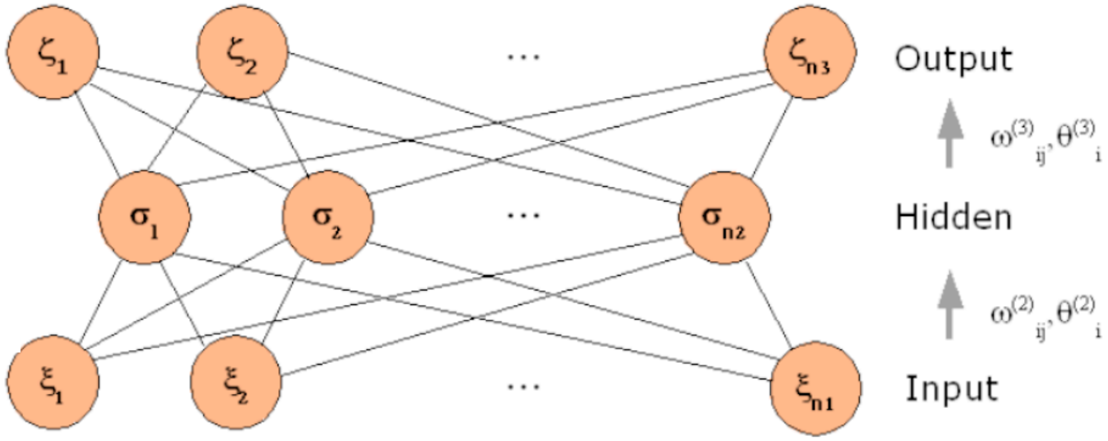
Very large (> 100%) uncertainties for new heavy particle production. Crucial in BSM searches



Artificial Neural Network PDFs

Novel PDF fitting methodology: Artificial Neural Network (ANN) PDFs





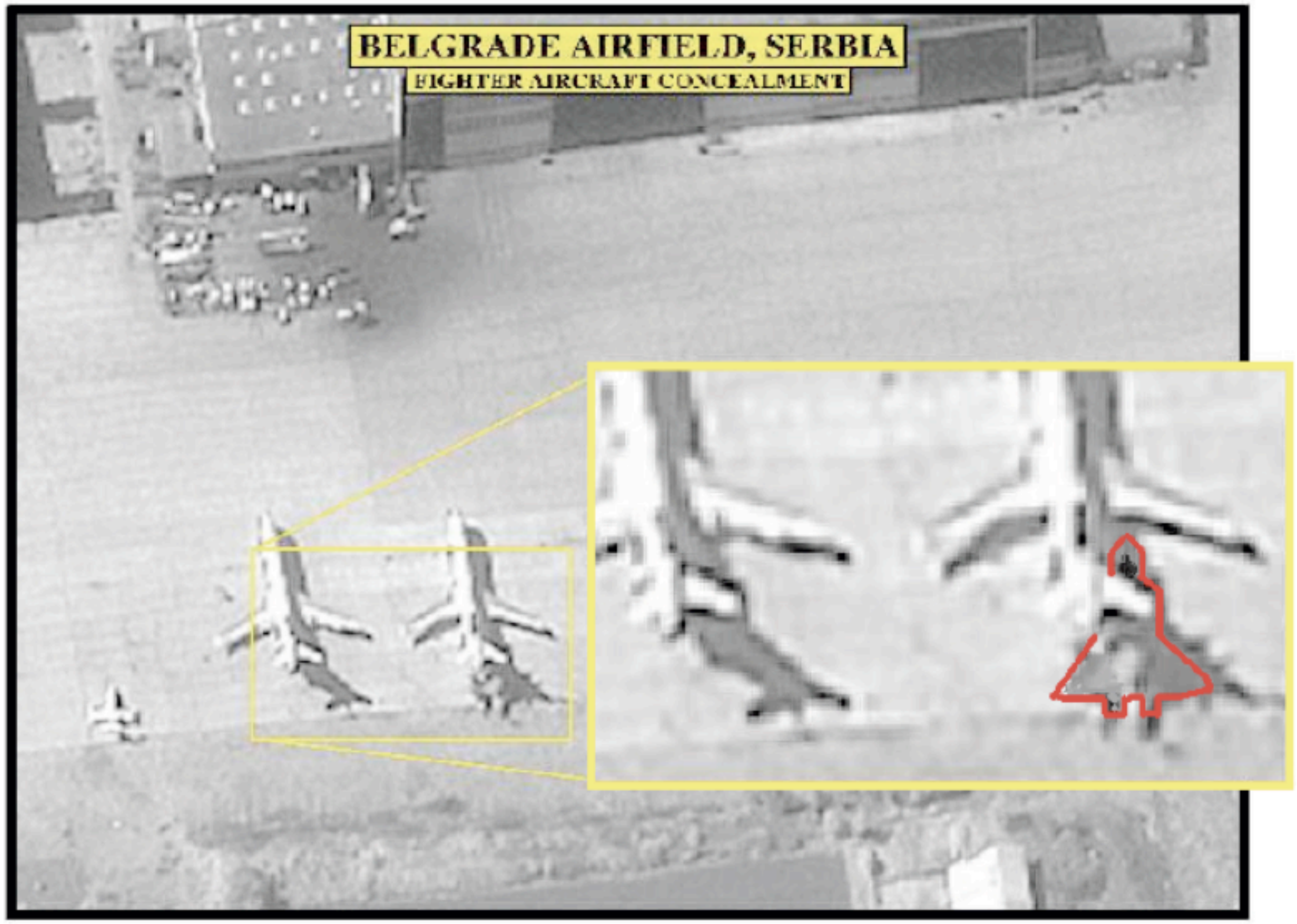
from biology ... ... to high-energy physics

Inspired by biological brains that excel in pattern recognition, classification, forecasting, etc. ANNs are mathematical algorithms widely used in a range of applications, from targeted marketing, finance forecasting, now to high-energy physics.

Artificial Neural Network PDFs

Example: pattern recognition During the Yugoslavian wars, the NATO used ANNs to recognize hidden military vehicles

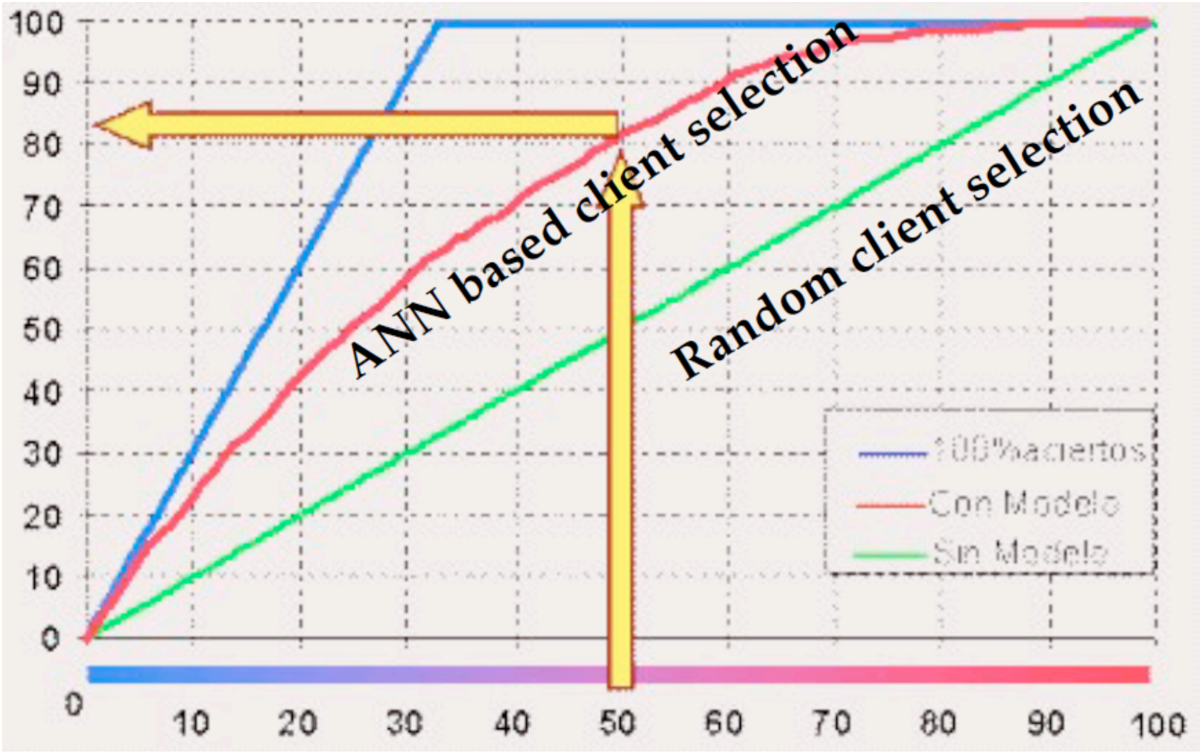
Military plane hidden below a commercial plane identified.



Artificial Neural Network PDFs

<u>Example: forecasting in marketing</u> A bank wants to offer a product to their client. Possible strategies:

1. contact all clients (slow, costly)
2. contact few percent of the clients and train ANN with the input (sex, income, family status...) and output (yes/no) from the clients.



<u>Cost effective method:</u>
Use this information to contact only clients likely to accept the offer

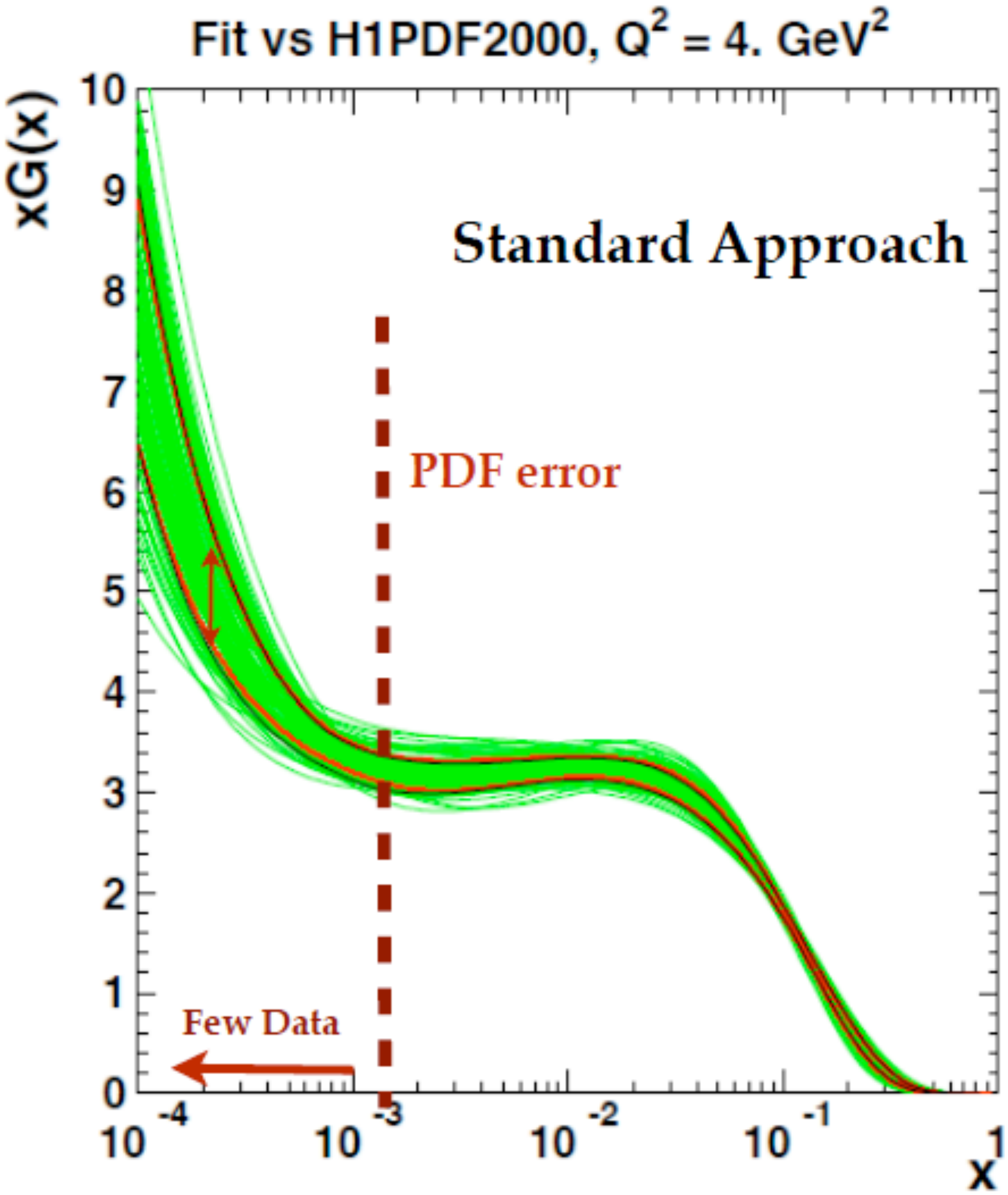
Artificial Neural Network PDFs

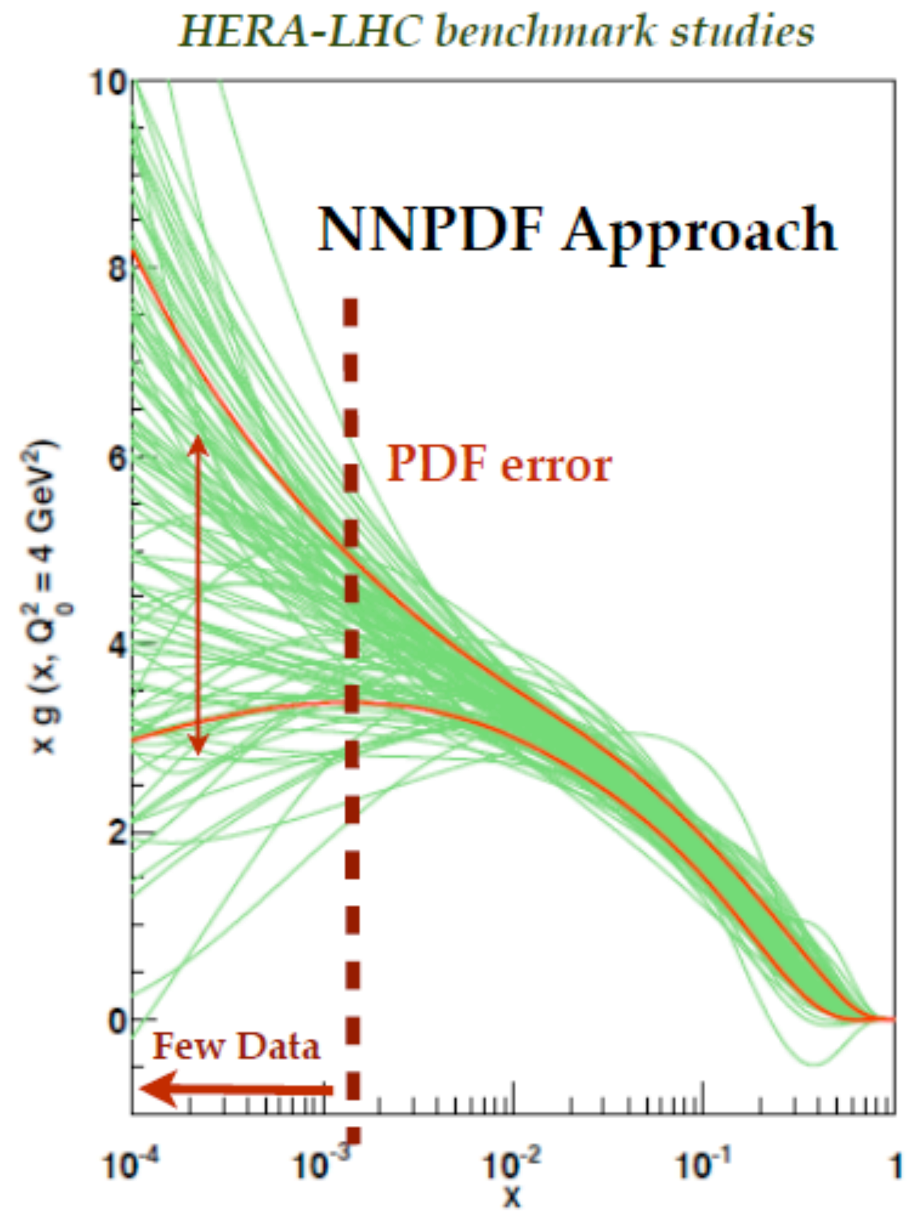
ANNs provide universal unbiased interpolants to parametrize non-perturbative dynamics
'Learn' the underlying physics laws from the experimental data using Genetic Algorithms (learn on ensemble of replica)

No theory bias introduced in the PDF determination by the choice of ad-hoc functional form

- NNpdfs approach: one ANN per PDF, O(500) parameters in total (PDFs identical if O(1000) parameters are used)
- traditional approach: one simple polynomial per PDF, O(10-25) parameters in total

Flexibility matters





- faithful error estimate: uncertainty blows up in region with no data
- crucial ingredient for reliable LHC searches at high mass

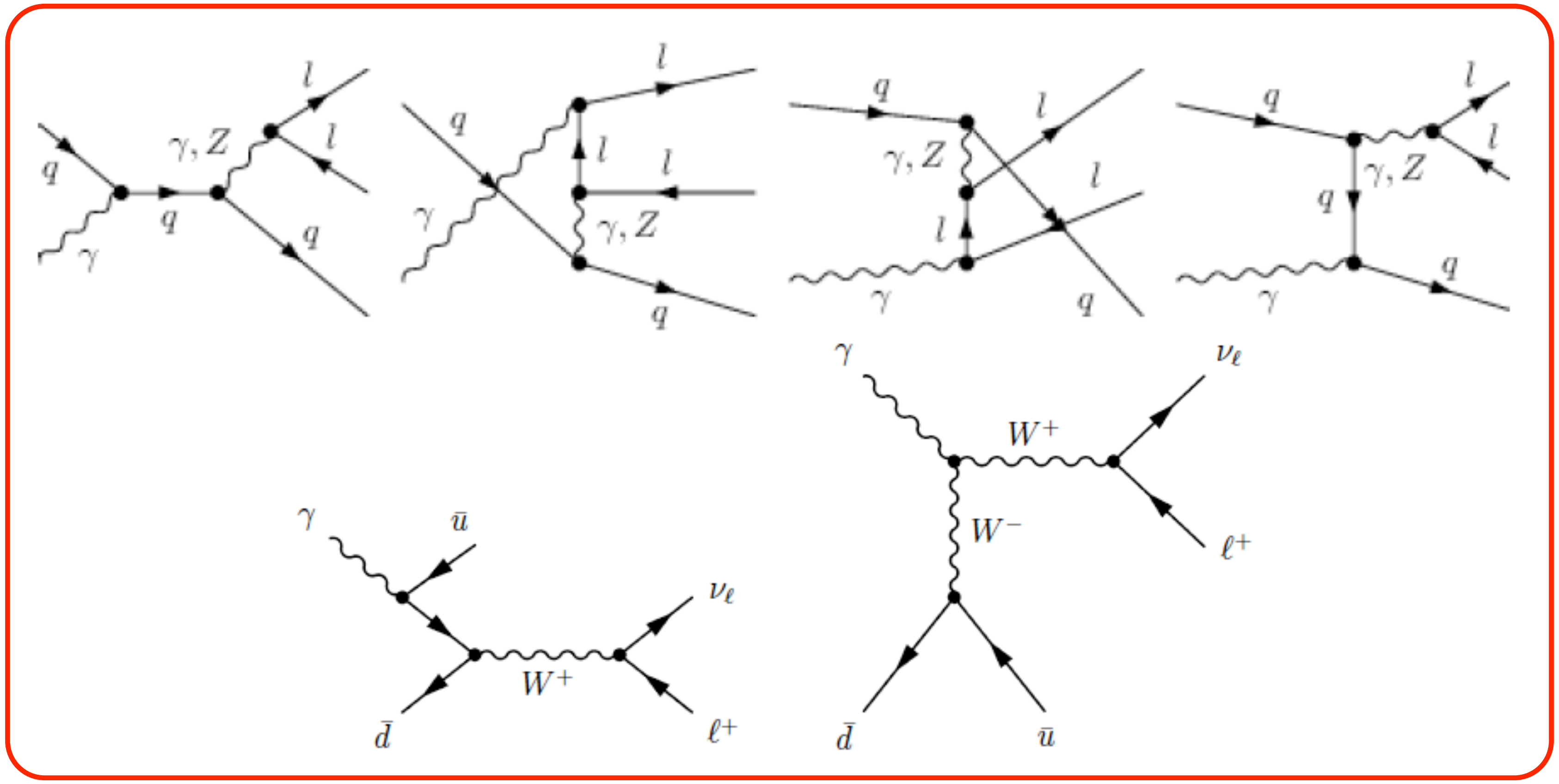
PDFs: future challenges

- PDFs have been fitting since many years ...
- the development of NN PDFs has been a huge step forward in the parameterization and reliable error estimate
- now is there any new challenge ahead? Or is it just a matter of adding more data to the fits?

When seeking for high precision electro-weak effects can not be neglected.

First step: QED corrections in PDFs

QED corrections



- photon initiated diagrams required for consistent EW calculation
- the DGLAP QCD equations can be modified to include QED corrections → NNPDF2.3: photon PDF from DIS and LHC data
- important for high precision phenomenology (M_W fits, WW production ...) and BSM searches (W', Z' ...)

PDFs: future challenges

At high energies (LHC Run II, 100 TeV machines) W/Z boson are "massless". Important to include photon/W/Z PDFs for precision measurements especially at high scales (BSM search regions)

Full EW DGLAP equations have been written down. But considerably more complicated structure than pure QCD

Crucial differences in spontaneously broken gauge theory: singularities structure complicated by the fact that initial state (e.g. electrons, protons)

- carry non-abelian (isospin) charges
- may be mixed charge states ⇒ Bloch-Nordsieck violation: double logs do not cancel in inclusive quantities

M Ciafaloni, P Ciafaloni, Comelli '01

Intermediate recap.

- Because of infrared and collinear divergences not all quantities can be computed in PT $\Rightarrow$ concept of IR-safety
- Parton model: incoherent sum of all partonic cross-sections, but failure of parton model when radiative corrections are included
- Factorization of initial state divergences into scale dependent parton densities
- DGLAP evolution of parton densities $\Rightarrow$ measure gluon PDF
- Determination of parton densities, electron & neutrino scattering in DIS, now also via new LHC data
- Recent progress in PDF: NNLO evolution, NNpdf, heavy quarks
- Future challenge: fully coupled QCD-EW evolution

Next: Perturbative calculations

Next, we will focus on perturbative calculations

- LO, NLO, NLO+MC, NNLO
- techniques, issues with divergences
- current status, sample results

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Perturbative calculations rely on the idea of an order-by-order expansion in the small coupling

$$\sigma \sim \underbrace{A}_{\text{LO}} + \underbrace{B\alpha_s}_{\text{NLO}} + \underbrace{C\alpha_s^2}_{\text{NNLO}} + \underbrace{D\alpha_s^3}_{\text{NNNLO}} + \ldots$$

Perturbative calculations

- Perturbative calculations = fixed-order expansion in the coupling constant, or more refined expansions that include terms to all orders
- Perturbative calculations are possible because the coupling is small at high energy
- In QCD (or in a generic QFT) the coupling depends on the energy (renormalization scale)
- So changing scale the result changes. By how much? What does this dependence mean?
- Let's consider some examples

Leading order n-jet cross-section

- Consider the cross-section to produce n jets. The leading order result at scale μ result will be

$$\sigma^{\mathrm{LO}}_{\mathrm{njets}}(\mu) = \alpha_s(\mu)^n A(p_i, \epsilon_i, \ldots)$$

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$$\sigma^{\mathrm{LO}}_{\mathrm{njets}}(\mu') = \alpha_s(\mu')^n A(p_i, \epsilon_i, \ldots) = \alpha_s(\mu)^n \left(1 + n\, b_0\, \alpha_s(\mu) \ln \frac{\mu^2}{\mu'^2} + \ldots \right) A(p_i, \epsilon_i, \ldots)$$

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- Notice that at Leading Order the normalization is not under control:

$$\frac{\sigma^{\rm LO}_{\rm njets}(\mu)}{\sigma^{\rm LO}_{\rm njets}(\mu')} = \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu')}\right)^n$$

NLO n-jet cross-section

Now consider n-jet cross-section at NLO. At scale μ the result reads

$$\sigma_{\text{njets}}^{\text{NLO}}(\mu) = \alpha_s(\mu)^n A(p_i, \epsilon_i, \dots) + \alpha_s(\mu)^{n+1} \left(B(p_i, \epsilon_i, \dots) - n b_0 \ln \frac{\mu^2}{Q_0^2} \right) + \dots$$

- So the NLO result compensates the LO scale dependence. The residual dependence is NNLO

- Scale dependence and normalization start being under control only at NLO, since a compensation mechanism kicks in

- Notice also that a good scale choice automatically resums large logarithms to all orders, while a bad one spuriously introduces large logs and ruins the PT expansion

- Scale variation is conventionally used to estimate the theory uncertainty, but the validity of this procedure should not be overrated (see later)

Leading order with Feynman diagrams

Get *any* LO cross-section from the Lagrangian

1. draw all Feynman diagrams
2. put in the explicit Feynman rules and get the amplitude
3. do some algebra, simplifications
4. square the amplitude
5. integrate over phase space + flux factor + sum/average over outgoing/ incoming states

Automated tools for (1-3): FeynArts/Qgraf, Mathematica/Form etc.

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Bottlenecks

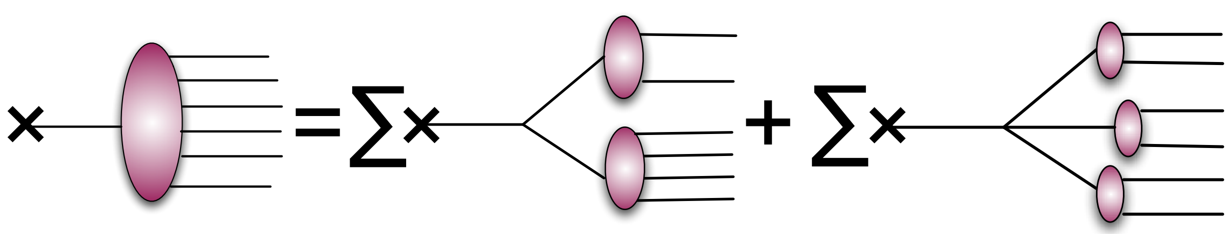
a) number of Feynman diagrams diverges factorially

b) algebra becomes more cumbersome with more particles

But given enough computer power everything can be computed at LO

Techniques beyond Feynman diagrams

✓ Berends-Giele relations: compute helicity amplitudes recursively using off-shell currents



Berends, Giele '88

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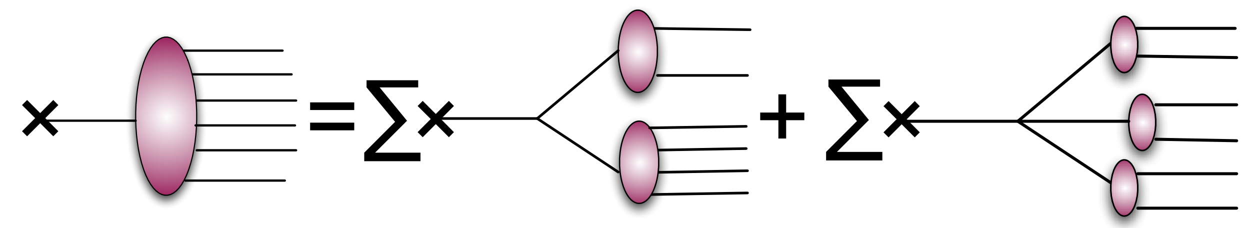

Berends, Giele '88

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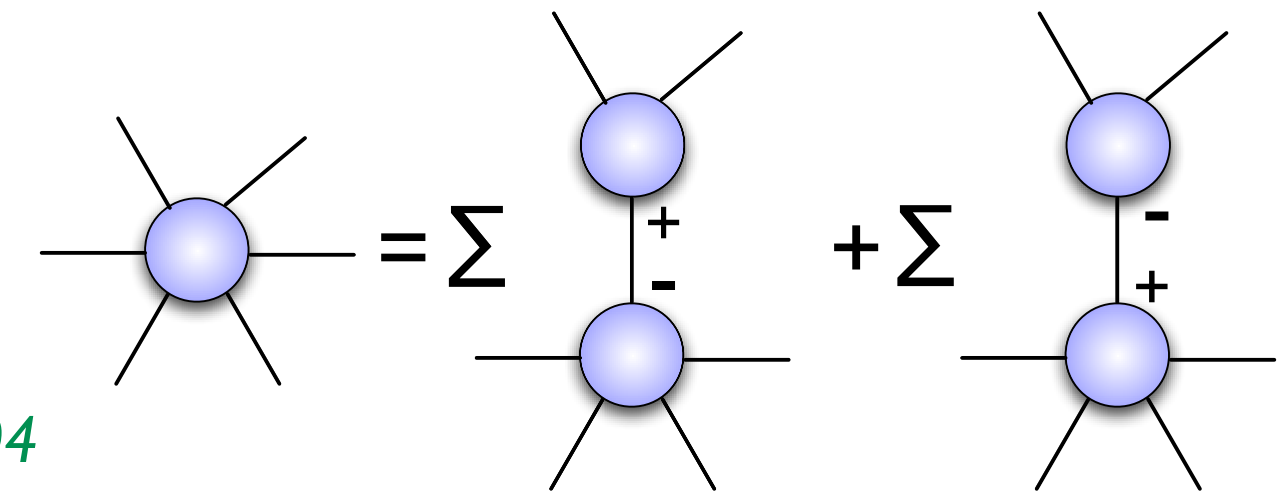

Britto, Cachazo, Feng '04

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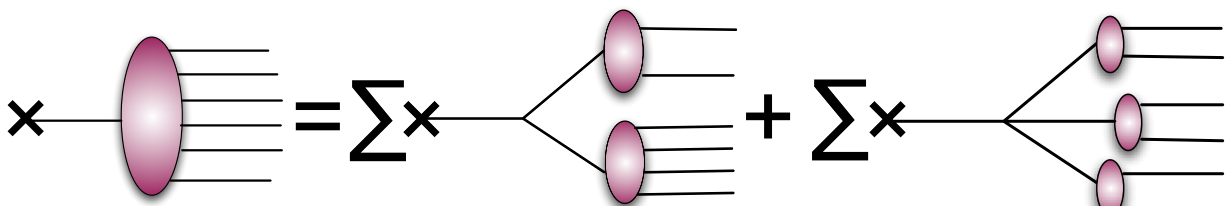

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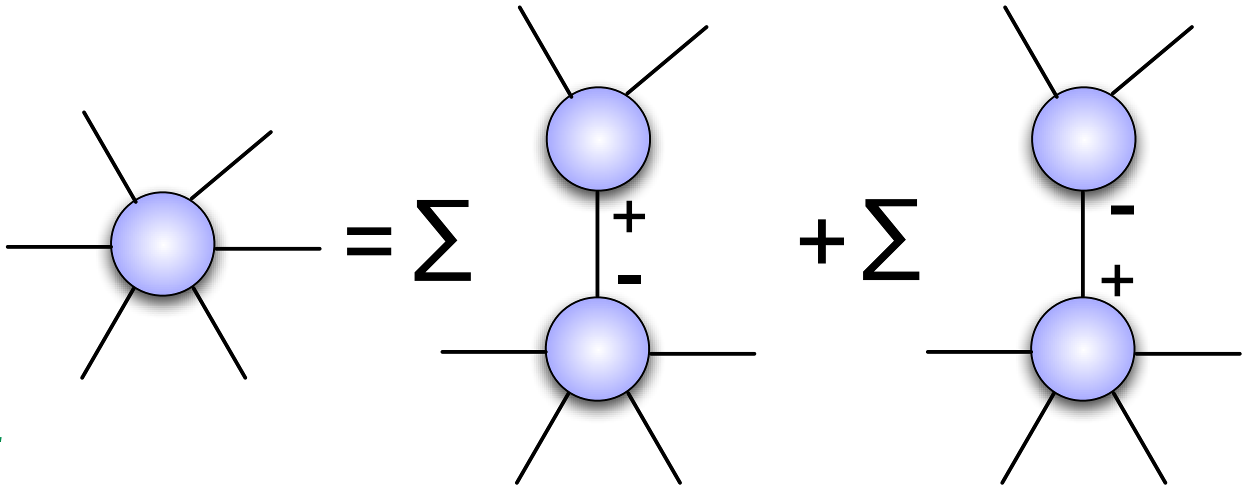
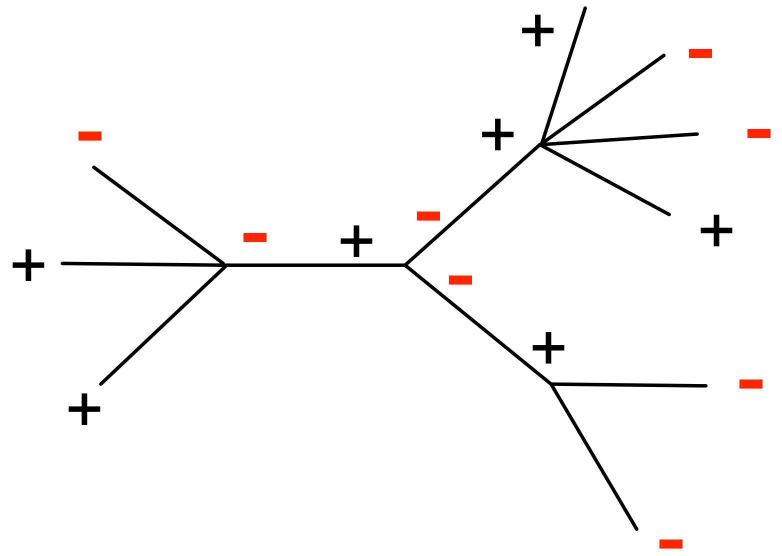


Britto, Cachazo, Feng '04

✓ CSW relations: compute helicity amplitudes by sewing together MHV amplitudes [- - + + ... +]



Cachazo, Svrcek, Witten '04

Matrix element generators

Fully automated calculation of leading-order cross-sections:

- generation of tree level matrix elements
  - Feynman diagrams [CompHEP/CalcHEP, Madgraph/Madevent, HELAS, Sherpa, ...]
  - Helicity amplitudes + off-shell Berends-Giele recursion [ALPHA/ALPGEN, Helac, Vecbos]
- phase space integration
- interface to parton showers

These codes are currently used extensively in many analysis of LHC data

Benefits and drawbacks of LO

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Drawbacks of LO:

- large scale dependences, reflecting large theory uncertainty
- no control on normalization
- poor control on shapes
- poor modeling of jets

Example: W+4 jet cross-section $\propto$ $\alpha_s(Q)^4$
Vary $\alpha_s(Q)$ by $\pm 10\%$ via change of Q $\Rightarrow$ cross-section varies by $\pm 40\%$

Next-to-leading order

<u>Benefits of next-to-leading order (NLO)</u>

- reduce dependence on unphysical scales

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We'll look at a few concrete examples in few minutes

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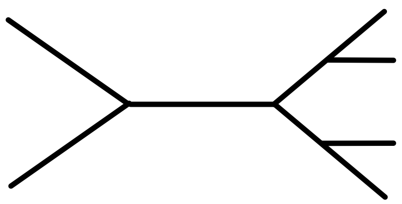

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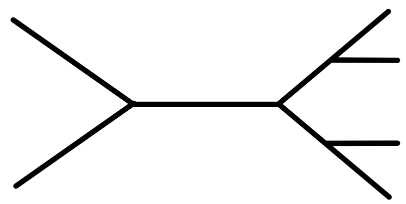

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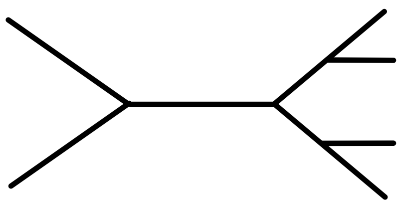

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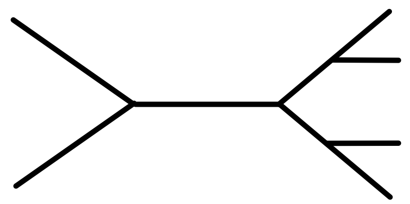

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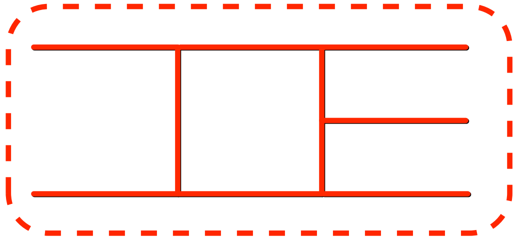

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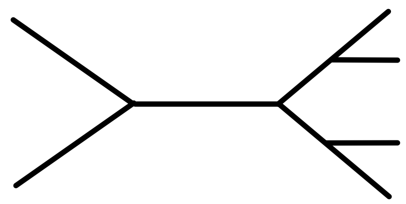

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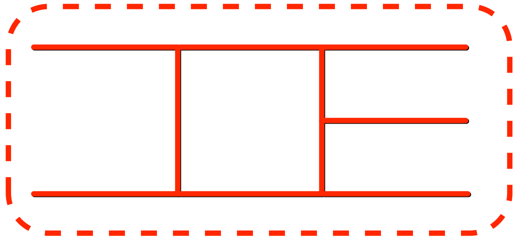

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- ☑ set of subtraction terms to cancel divergences

We won't have time to do detailed NLO calculations, but let's look a bit more in detail at the issue of divergences/subtraction

Regularization in QCD

Regularization: a way to make intermediate divergent quantities meaningful

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- In QCD dimensional regularization is today the standard procedure, based on the fact that d-dimensional integrals are more convergent if one reduces the number of dimensions.

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Alternative regularization schemes: photon mass (EW), cut-offs, Pauli-Villard ...
Compared to those methods, dimensional regularizatiom has the big virtue that it leaves the regularized theory Lorentz invariant, gauge invariant, unitary etc.

Renormalization schemes

Renormalization: a global redefinition of couplings and masses which absorbs all UV divergences. Several schemes are possible (MS, $\overline{\text{MS}}$, OS ...).

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- Note that as a consequence of this, the first two coefficients of the β-function do not change under such a transformation, i.e. they are scheme independent. This it not true for higher order coefficients.

The $\overline{\text{MS}}$ scheme

- Today's standard scheme: modified minimal subtraction scheme, $\overline{\text{MS}}$
- After regularizing integrals via the dimensional regularization, poles appear always in the combination

$$\frac{1}{\epsilon} + \ln(4\pi) - \gamma_E$$

- Therefore in the $\overline{\text{MS}}$-scheme, instead of subtracting poles minimally, one always subtracts that combination, and replaces the bare coupling with the renormalized one

- It is then standard to quote the coupling and Λ_{QCD} in this scheme, the current value is

$$206\text{MeV} < \Lambda_{\overline{\text{MS}}}(5) < 231\text{MeV}$$

- Uncertainties in this quantity propagate in the QCD cross-sections

Subtraction and slicing methods

- Consider e.g. an n-jet cross-section with some arbitrary infrared safe jet definition. At NLO, two divergent integrals, but the sum is finite

$$\sigma^J_{\mathrm{NLO}} = \int_{n+1} d\sigma^J_{\mathrm{R}} + \int_n d\sigma^J_{\mathrm{V}}$$

- Since one integrates over a different number of particles in the final state, real and virtual need to be evaluated first, and combined then

- This means that one needs to find a way of removing divergences before evaluating the phase space integrals

- Two main techniques to do this
  - *phase space slicing* $\Rightarrow$ obsolete because of practical/numerical issues
  - *subtraction method* $\Rightarrow$ most used in recent applications

Subtraction method

- The real cross-section can be written schematically as

$$d\sigma_R^J = d\phi_{n+1}|\mathcal{M}_{n+1}|^2 F_{n+1}^J(p_1, \ldots, p_{n+1})$$

where F^J is the arbitrary jet-definition

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- IR divergences in the loop integration regularized by taking D = 4-2ε

$$2\,\mathrm{Re}\{\mathcal{M}_V \cdot \mathcal{M}_0^*\} = \frac{1}{\epsilon}\mathcal{V}$$

Subtraction method

- The n-jet cross-section becomes

$$\sigma^J_{\mathrm{NLO}} = \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{M}(x) F^J_{n+1}(x) + \frac{1}{\epsilon} \mathcal{V} F^J_n$$

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- One can then add and subtract the analytically computed divergent part

$$\sigma_{\mathrm{NLO}}^{J} = \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{M}(x) F_{n+1}^{J}(x) - \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{V} F_n^J + \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{V} F_n^J + \frac{1}{\epsilon} \mathcal{V} F_n^J$$

Subtraction method

- This can be rewritten exactly as

$$\sigma^J_{\mathrm{NLO}} = \int_0^1 \frac{dx}{x^{1+\epsilon}} \mathcal{M}(x) \left(F^J_1(x) - \mathcal{V} F^J_0 \right) + \mathcal{O}(1) \mathcal{V} F^J_0$$

$\Rightarrow$ Now both terms are finite and can be evaluated numerically

- Subtracted cross-section must be calculated separately for each process (but mostly automated now). It must be valid everywhere in phase space
- Systematized in the seminal papers of Catani-Seymour (dipole subtraction, '96) and Frixione-Kunszt-Signer (FKS method, '96)
- Subtraction used in all recent NLO applications and public codes (Event2, Disent, MCFM, NLOjet++, MC@NLO, POWHEG ...)

Approaches to virtual (loop) part of NLO

Two complementary approaches:

- Numerical/traditional Feynman diagram methods:
  use robust computational methods [integration by parts, reduction techniques...], then let the computer do the work for you

  Bottleneck:
  factorial growth, $2 \to 4$ doable, difficult to go beyond

- Analytical approaches:
  improve understanding of field theory [e.g. generalized unitarity, recursions, OPP, Open Loops ...]

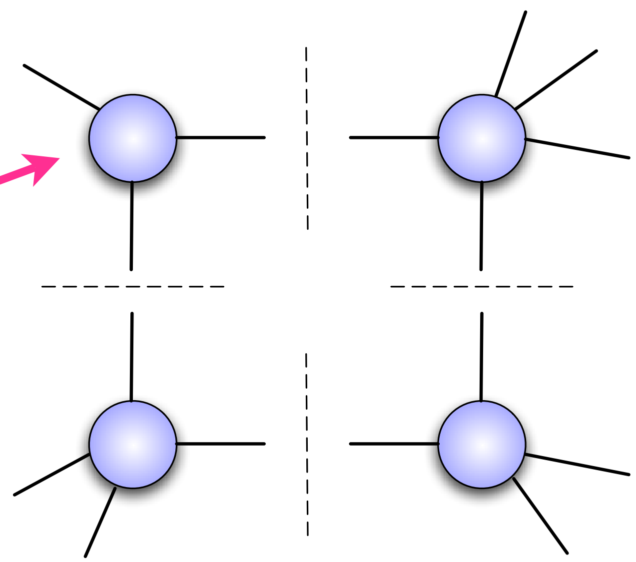
  Status:
  moving towards more legs (5 or 6 in the final state) + towards full automation [GoSam, MadLoop]

Two breakthrough ideas

Aim: NLO loop integral without doing the integration

1) "... we show how to use generalized unitarity to read off the (box) coefficients. The generalized cuts we use are quadrupole cuts ..."

NB: non-zero because cut gives complex momenta



Britto, Cachazo, Feng '04

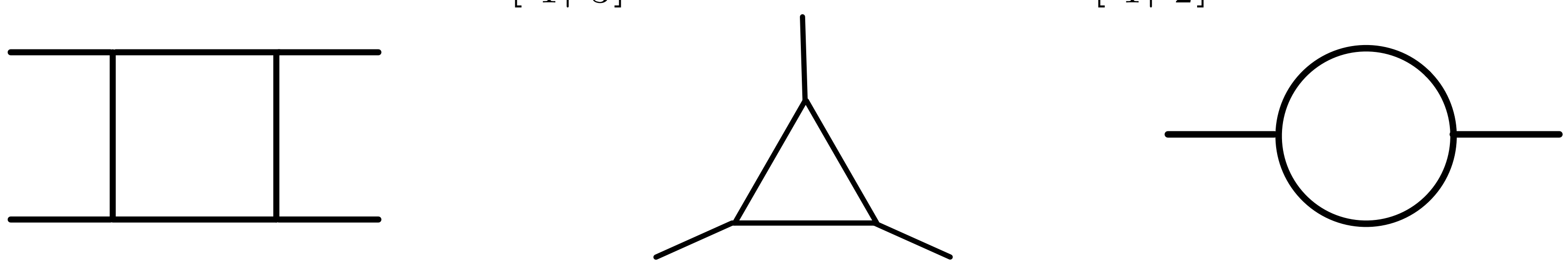
Quadrupole cuts: 4 on-shell conditions on 4 dimensional loop momentum) freezes the integration. But rational part of the amplitude, coming from D=4-2ε not 4, computed separately

Two breakthrough ideas

Aim: NLO loop integral without doing the integration

2) The OPP method: "We show how to extract the coefficients of 4-, 3-, 2- and 1-point one-loop scalar integrals...."

$$\mathcal{A}_N = \sum_{[i_1|i_4]} \left(d_{i_1 i_2 i_3 i_4} \; I^{(D)}_{i_1 i_2 i_3 i_4} \right) + \sum_{[i_1|i_3]} \left(c_{i_1 i_2 i_3} \; I^{(D)}_{i_1 i_2 i_3} \right) + \sum_{[i_1|i_2]} \left(b_{i_1 i_2} \; I^{(D)}_{i_1 i_2} \right)$$



Ossola, Pittau, Papadopolous '06

Coefficients can be determined by solving system of equations: no loops, no twistors, just algebra!

The 2007 Les Houches wishlist

Process ($V \in \{Z, W, \gamma\}$)	Comments
Calculations completed since Les Houches 2005	
1. $pp \to VV$jet	WWjet completed by Dittmaier/Kallweit/Uwer [3]; Campbell/Ellis/Zanderighi [4] and Binoth/Karg/Kauer/Sanguinetti (in progress)
2. $pp \to$ Higgs+2jets	NLO QCD to the gg channel completed by Campbell/Ellis/Zanderighi [5]; NLO QCD+EW to the VBF channel completed by Ciccolini/Denner/Dittmaier [6, 7]
3. $pp \to V\,V\,V$	ZZZ completed by Lazopoulos/Melnikov/Petriello [8] and WWZ by Hankele/Zeppenfeld [9]
Calculations remaining from Les Houches 2005	
4. $pp \to t\bar{t}\,b\bar{b}$	relevant for $t\bar{t}H$
5. $pp \to t\bar{t}$+2jets	relevant for $t\bar{t}H$
6. $pp \to V\,V\,b\bar{b}$,	relevant for VBF $\to H \to VV$, $t\bar{t}H$
7. $pp \to V\,V$+2jets	relevant for VBF $\to H \to VV$ VBF contributions calculated by (Bozzi/)Jäger/Oleari/Zeppenfeld [10–12]
8. $pp \to V$+3jets	various new physics signatures
NLO calculations added to list in 2007	
9. $pp \to b\bar{b}b\bar{b}$	Higgs and new physics signatures
Calculations beyond NLO added in 2007	
10. $gg \to W^*W^*\ \mathcal{O}(\alpha^2\alpha_s^3)$	backgrounds to Higgs
11. NNLO $pp \to t\bar{t}$	normalization of a benchmark process
12. NNLO to VBF and Z/γ+jet	Higgs couplings and SM benchmark
Calculations including electroweak effects	
13. NNLO QCD+NLO EW for W/Z	precision calculation of a SM benchmark

Table 1: The updated experimenter's wishlist for LHC processes

CLOSED

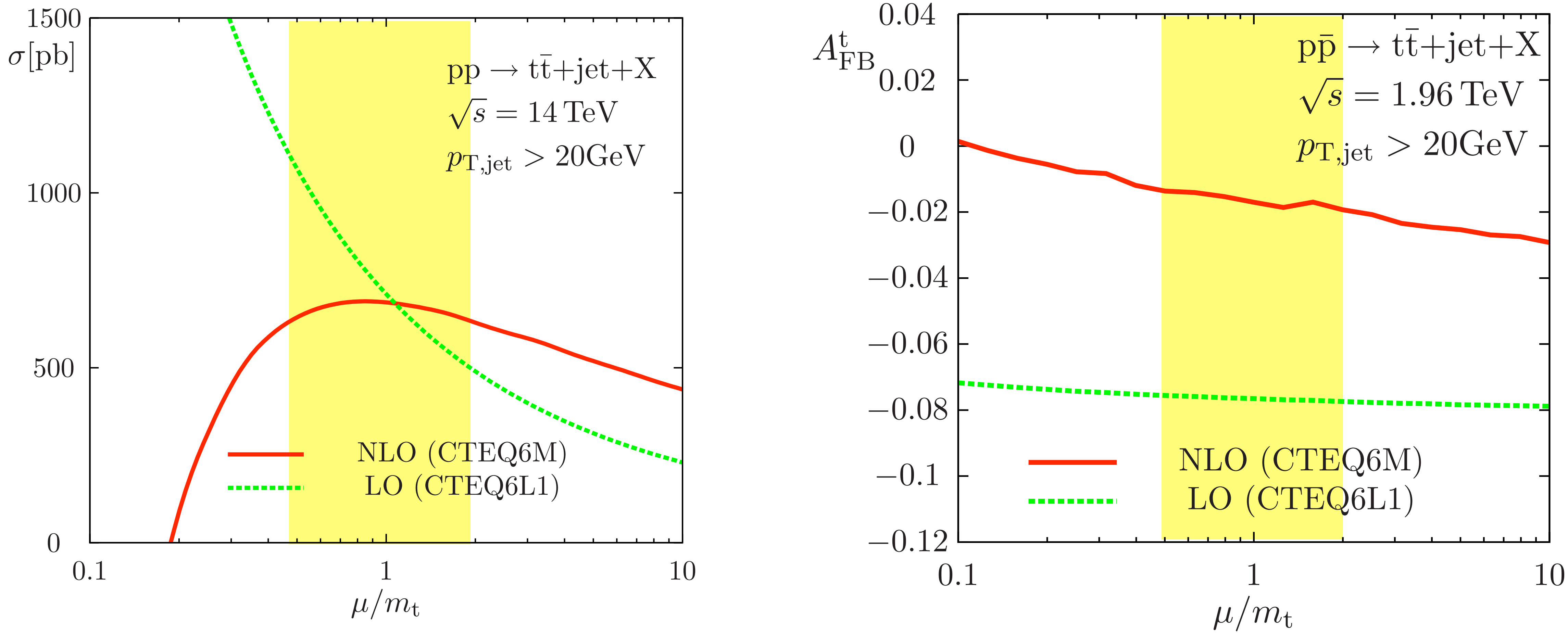
with Feynman diagrams

with Feynman diagrams or unitarity/onshell methods

The NLO multi-leg Working group report 0803.0494

Example of NLO result: tt+1jet

Dittmaier, Kallweit, Uwer '07-'08



- improved stability of NLO result [but no decays]
- forward-backward asymmetry at the Tevatron compatible with zero
- essential ingredient of NNLO tt production

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Process		Syntax	Cross section (pb)			
Vector boson +jets			LO 13 TeV		NLO 13 TeV	
a.1	$pp \to W^\pm$	`p p > wpm`	$1.375 \pm 0.002 \cdot 10^5$	$^{+15.4\%}_{-16.6\%}$ $^{+2.0\%}_{-1.6\%}$	$1.773 \pm 0.007 \cdot 10^5$	$^{+5.2\%}_{-9.4\%}$ $^{+1.9\%}_{-1.6\%}$
a.2	$pp \to W^\pm j$	`p p > wpm j`	$2.045 \pm 0.001 \cdot 10^4$	$^{+19.7\%}_{-17.2\%}$ $^{+1.4\%}_{-1.1\%}$	$2.843 \pm 0.010 \cdot 10^4$	$^{+5.9\%}_{-8.0\%}$ $^{+1.3\%}_{-1.1\%}$
a.3	$pp \to W^\pm jj$	`p p > wpm j j`	$6.805 \pm 0.015 \cdot 10^3$	$^{+24.5\%}_{-18.6\%}$ $^{+0.8\%}_{-0.7\%}$	$7.786 \pm 0.030 \cdot 10^3$	$^{+2.4\%}_{-6.0\%}$ $^{+0.9\%}_{-0.8\%}$
a.4	$pp \to W^\pm jjj$	`p p > wpm j j j`	$1.821 \pm 0.002 \cdot 10^3$	$^{+41.0\%}_{-27.1\%}$ $^{+0.5\%}_{-0.5\%}$	$2.005 \pm 0.008 \cdot 10^3$	$^{+0.9\%}_{-6.7\%}$ $^{+0.6\%}_{-0.5\%}$
a.5	$pp \to Z$	`p p > z`	$4.248 \pm 0.005 \cdot 10^4$	$^{+14.6\%}_{-15.8\%}$ $^{+2.0\%}_{-1.6\%}$	$5.410 \pm 0.022 \cdot 10^4$	$^{+4.6\%}_{-8.6\%}$ $^{+1.9\%}_{-1.5\%}$
a.6	$pp \to Zj$	`p p > z j`	$7.209 \pm 0.005 \cdot 10^3$	$^{+19.3\%}_{-17.0\%}$ $^{+1.2\%}_{-1.0\%}$	$9.742 \pm 0.035 \cdot 10^3$	$^{+5.8\%}_{-7.8\%}$ $^{+1.2\%}_{-1.0\%}$
a.7	$pp \to Zjj$	`p p > z j j`	$2.348 \pm 0.006 \cdot 10^3$	$^{+24.3\%}_{-18.5\%}$ $^{+0.6\%}_{-0.6\%}$	$2.665 \pm 0.010 \cdot 10^3$	$^{+2.5\%}_{-6.0\%}$ $^{+0.7\%}_{-0.7\%}$
a.8	$pp \to Zjjj$	`p p > z j j j`	$6.314 \pm 0.008 \cdot 10^2$	$^{+40.8\%}_{-27.0\%}$ $^{+0.5\%}_{-0.5\%}$	$6.996 \pm 0.028 \cdot 10^2$	$^{+1.1\%}_{-6.8\%}$ $^{+0.5\%}_{-0.5\%}$
a.9	$pp \to \gamma j$	`p p > a j`	$1.964 \pm 0.001 \cdot 10^4$	$^{+31.2\%}_{-26.0\%}$ $^{+1.7\%}_{-1.8\%}$	$5.218 \pm 0.025 \cdot 10^4$	$^{+24.5\%}_{-21.4\%}$ $^{+1.4\%}_{-1.6\%}$
a.10	$pp \to \gamma jj$	`p p > a j j`	$7.815 \pm 0.008 \cdot 10^3$	$^{+32.8\%}_{-24.2\%}$ $^{+0.9\%}_{-1.2\%}$	$1.004 \pm 0.004 \cdot 10^4$	$^{+5.9\%}_{-10.9\%}$ $^{+0.8\%}_{-1.2\%}$

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Process		Syntax	Cross section (pb)			
Vector-boson pair +jets			LO 13 TeV		NLO 13 TeV	
b.1	$pp \to W^+W^-$ (4f)	`p p > w+ w-`	$7.355 \pm 0.005 \cdot 10^1$	$^{+5.0\%}_{-6.1\%}$ $^{+2.0\%}_{-1.5\%}$	$1.028 \pm 0.003 \cdot 10^2$	$^{+4.0\%}_{-4.5\%}$ $^{+1.9\%}_{-1.4\%}$
b.2	$pp \to ZZ$	`p p > z z`	$1.097 \pm 0.002 \cdot 10^1$	$^{+4.5\%}_{-5.6\%}$ $^{+1.9\%}_{-1.5\%}$	$1.415 \pm 0.005 \cdot 10^1$	$^{+3.1\%}_{-3.7\%}$ $^{+1.8\%}_{-1.4\%}$
b.3	$pp \to ZW^\pm$	`p p > z wpm`	$2.777 \pm 0.003 \cdot 10^1$	$^{+3.6\%}_{-4.7\%}$ $^{+2.0\%}_{-1.5\%}$	$4.487 \pm 0.013 \cdot 10^1$	$^{+4.4\%}_{-4.4\%}$ $^{+1.7\%}_{-1.3\%}$
b.4	$pp \to \gamma\gamma$	`p p > a a`	$2.510 \pm 0.002 \cdot 10^1$	$^{+22.1\%}_{-22.4\%}$ $^{+2.4\%}_{-2.1\%}$	$6.593 \pm 0.021 \cdot 10^1$	$^{+17.6\%}_{-18.8\%}$ $^{+2.0\%}_{-1.9\%}$
b.5	$pp \to \gamma Z$	`p p > a z`	$2.523 \pm 0.004 \cdot 10^1$	$^{+9.9\%}_{-11.2\%}$ $^{+2.0\%}_{-1.6\%}$	$3.695 \pm 0.013 \cdot 10^1$	$^{+5.4\%}_{-7.1\%}$ $^{+1.8\%}_{-1.4\%}$
b.6	$pp \to \gamma W^\pm$	`p p > a wpm`	$2.954 \pm 0.005 \cdot 10^1$	$^{+9.5\%}_{-11.0\%}$ $^{+2.0\%}_{-1.7\%}$	$7.124 \pm 0.026 \cdot 10^1$	$^{+9.7\%}_{-9.9\%}$ $^{+1.5\%}_{-1.3\%}$
b.7	$pp \to W^+W^-j$ (4f)	`p p > w+ w- j`	$2.865 \pm 0.003 \cdot 10^1$	$^{+11.6\%}_{-10.0\%}$ $^{+1.0\%}_{-0.8\%}$	$3.730 \pm 0.013 \cdot 10^1$	$^{+4.9\%}_{-4.9\%}$ $^{+1.1\%}_{-0.8\%}$
b.8	$pp \to ZZj$	`p p > z z j`	$3.662 \pm 0.003 \cdot 10^0$	$^{+10.9\%}_{-9.3\%}$ $^{+1.0\%}_{-0.8\%}$	$4.830 \pm 0.016 \cdot 10^0$	$^{+5.0\%}_{-4.8\%}$ $^{+1.1\%}_{-0.9\%}$
b.9	$pp \to ZW^\pm j$	`p p > z wpm j`	$1.605 \pm 0.005 \cdot 10^1$	$^{+11.6\%}_{-10.0\%}$ $^{+0.9\%}_{-0.7\%}$	$2.086 \pm 0.007 \cdot 10^1$	$^{+4.9\%}_{-4.8\%}$ $^{+0.9\%}_{-0.7\%}$
b.10	$pp \to \gamma\gamma j$	`p p > a a j`	$1.022 \pm 0.001 \cdot 10^1$	$^{+20.3\%}_{-17.7\%}$ $^{+1.2\%}_{-1.5\%}$	$2.292 \pm 0.010 \cdot 10^1$	$^{+17.2\%}_{-15.1\%}$ $^{+1.0\%}_{-1.4\%}$
b.11*	$pp \to \gamma Zj$	`p p > a z j`	$8.310 \pm 0.017 \cdot 10^0$	$^{+14.5\%}_{-12.8\%}$ $^{+1.0\%}_{-1.0\%}$	$1.220 \pm 0.005 \cdot 10^1$	$^{+7.3\%}_{-7.4\%}$ $^{+0.9\%}_{-0.9\%}$
b.12*	$pp \to \gamma W^\pm j$	`p p > a wpm j`	$2.546 \pm 0.010 \cdot 10^1$	$^{+13.7\%}_{-12.1\%}$ $^{+0.9\%}_{-1.0\%}$	$3.713 \pm 0.015 \cdot 10^1$	$^{+7.2\%}_{-7.1\%}$ $^{+0.9\%}_{-1.0\%}$
b.13	$pp \to W^+W^+jj$	`p p > w+ w+ j j`	$1.484 \pm 0.006 \cdot 10^{-1}$	$^{+25.4\%}_{-18.9\%}$ $^{+2.1\%}_{-1.5\%}$	$2.251 \pm 0.011 \cdot 10^{-1}$	$^{+10.5\%}_{-10.6\%}$ $^{+2.2\%}_{-1.6\%}$
b.14	$pp \to W^-W^-jj$	`p p > w- w- j j`	$6.752 \pm 0.007 \cdot 10^{-2}$	$^{+25.4\%}_{-18.9\%}$ $^{+2.4\%}_{-1.7\%}$	$1.003 \pm 0.003 \cdot 10^{-1}$	$^{+10.1\%}_{-10.4\%}$ $^{+2.5\%}_{-1.8\%}$
b.15	$pp \to W^+W^-jj$ (4f)	`p p > w+ w- j j`	$1.144 \pm 0.002 \cdot 10^1$	$^{+27.2\%}_{-19.9\%}$ $^{+0.7\%}_{-0.5\%}$	$1.396 \pm 0.005 \cdot 10^1$	$^{+5.0\%}_{-6.8\%}$ $^{+0.7\%}_{-0.6\%}$
b.16	$pp \to ZZjj$	`p p > z z j j`	$1.344 \pm 0.002 \cdot 10^0$	$^{+26.6\%}_{-19.6\%}$ $^{+0.7\%}_{-0.6\%}$	$1.706 \pm 0.011 \cdot 10^0$	$^{+5.8\%}_{-7.2\%}$ $^{+0.8\%}_{-0.6\%}$
b.17	$pp \to ZW^\pm jj$	`p p > z wpm j j`	$8.038 \pm 0.009 \cdot 10^0$	$^{+26.7\%}_{-19.7\%}$ $^{+0.7\%}_{-0.5\%}$	$9.139 \pm 0.031 \cdot 10^0$	$^{+3.1\%}_{-5.1\%}$ $^{+0.7\%}_{-0.5\%}$
b.18	$pp \to \gamma\gamma jj$	`p p > a a j j`	$5.377 \pm 0.029 \cdot 10^0$	$^{+26.2\%}_{-19.8\%}$ $^{+0.6\%}_{-1.0\%}$	$7.501 \pm 0.032 \cdot 10^0$	$^{+8.8\%}_{-10.1\%}$ $^{+0.6\%}_{-1.0\%}$
b.19*	$pp \to \gamma Zjj$	`p p > a z j j`	$3.260 \pm 0.009 \cdot 10^0$	$^{+24.3\%}_{-18.4\%}$ $^{+0.6\%}_{-0.6\%}$	$4.242 \pm 0.016 \cdot 10^0$	$^{+6.5\%}_{-7.3\%}$ $^{+0.6\%}_{-0.6\%}$
b.20*	$pp \to \gamma W^\pm jj$	`p p > a wpm j j`	$1.233 \pm 0.002 \cdot 10^1$	$^{+24.7\%}_{-18.6\%}$ $^{+0.6\%}_{-0.6\%}$	$1.448 \pm 0.005 \cdot 10^1$	$^{+3.6\%}_{-5.4\%}$ $^{+0.6\%}_{-0.7\%}$

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Process		Syntax	Cross section (pb)					
Three vector bosons +jet			LO 13 TeV			NLO 13 TeV		
c.1	$pp \to W^+W^-W^\pm$ (4f)	`p p > w+ w- wpm`	$1.307 \pm 0.003 \cdot 10^{-1}$	${}^{+0.0\%}_{-0.3\%}$	${}^{+2.0\%}_{-1.5\%}$	$2.109 \pm 0.006 \cdot 10^{-1}$	${}^{+5.1\%}_{-4.1\%}$	${}^{+1.6\%}_{-1.2\%}$
c.2	$pp \to ZW^+W^-$ (4f)	`p p > z w+ w-`	$9.658 \pm 0.065 \cdot 10^{-2}$	${}^{+0.8\%}_{-1.1\%}$	${}^{+2.1\%}_{-1.6\%}$	$1.679 \pm 0.005 \cdot 10^{-1}$	${}^{+6.3\%}_{-5.1\%}$	${}^{+1.6\%}_{-1.2\%}$
c.3	$pp \to ZZW^\pm$	`p p > z z wpm`	$2.996 \pm 0.016 \cdot 10^{-2}$	${}^{+1.0\%}_{-1.4\%}$	${}^{+2.0\%}_{-1.6\%}$	$5.550 \pm 0.020 \cdot 10^{-2}$	${}^{+6.8\%}_{-5.5\%}$	${}^{+1.5\%}_{-1.1\%}$
c.4	$pp \to ZZZ$	`p p > z z z`	$1.085 \pm 0.002 \cdot 10^{-2}$	${}^{+0.0\%}_{-0.5\%}$	${}^{+1.9\%}_{-1.5\%}$	$1.417 \pm 0.005 \cdot 10^{-2}$	${}^{+2.7\%}_{-2.1\%}$	${}^{+1.9\%}_{-1.5\%}$
c.5	$pp \to \gamma W^+W^-$ (4f)	`p p > a w+ w-`	$1.427 \pm 0.011 \cdot 10^{-1}$	${}^{+1.9\%}_{-2.6\%}$	${}^{+2.0\%}_{-1.5\%}$	$2.581 \pm 0.008 \cdot 10^{-1}$	${}^{+5.4\%}_{-4.3\%}$	${}^{+1.4\%}_{-1.1\%}$
c.6	$pp \to \gamma\gamma W^\pm$	`p p > a a wpm`	$2.681 \pm 0.007 \cdot 10^{-2}$	${}^{+4.4\%}_{-5.6\%}$	${}^{+1.9\%}_{-1.6\%}$	$8.251 \pm 0.032 \cdot 10^{-2}$	${}^{+7.6\%}_{-7.0\%}$	${}^{+1.0\%}_{-1.0\%}$
c.7	$pp \to \gamma ZW^\pm$	`p p > a z wpm`	$4.994 \pm 0.011 \cdot 10^{-2}$	${}^{+0.8\%}_{-1.4\%}$	${}^{+1.9\%}_{-1.6\%}$	$1.117 \pm 0.004 \cdot 10^{-1}$	${}^{+7.2\%}_{-5.9\%}$	${}^{+1.2\%}_{-0.9\%}$
c.8	$pp \to \gamma ZZ$	`p p > a z z`	$2.320 \pm 0.005 \cdot 10^{-2}$	${}^{+2.0\%}_{-2.9\%}$	${}^{+1.9\%}_{-1.5\%}$	$3.118 \pm 0.012 \cdot 10^{-2}$	${}^{+2.8\%}_{-2.7\%}$	${}^{+1.8\%}_{-1.4\%}$
c.9	$pp \to \gamma\gamma Z$	`p p > a a z`	$3.078 \pm 0.007 \cdot 10^{-2}$	${}^{+5.6\%}_{-6.8\%}$	${}^{+1.9\%}_{-1.6\%}$	$4.634 \pm 0.020 \cdot 10^{-2}$	${}^{+4.5\%}_{-5.0\%}$	${}^{+1.7\%}_{-1.3\%}$
c.10	$pp \to \gamma\gamma\gamma$	`p p > a a a`	$1.269 \pm 0.003 \cdot 10^{-2}$	${}^{+9.8\%}_{-11.0\%}$	${}^{+2.0\%}_{-1.8\%}$	$3.441 \pm 0.012 \cdot 10^{-2}$	${}^{+11.8\%}_{-11.6\%}$	${}^{+1.4\%}_{-1.5\%}$
c.11	$pp \to W^+W^-W^\pm j$ (4f)	`p p > w+ w- wpm j`	$9.167 \pm 0.010 \cdot 10^{-2}$	${}^{+15.0\%}_{-12.2\%}$	${}^{+1.0\%}_{-0.7\%}$	$1.197 \pm 0.004 \cdot 10^{-1}$	${}^{+5.2\%}_{-5.6\%}$	${}^{+1.0\%}_{-0.8\%}$
c.12*	$pp \to ZW^+W^-j$ (4f)	`p p > z w+ w- j`	$8.340 \pm 0.010 \cdot 10^{-2}$	${}^{+15.6\%}_{-12.6\%}$	${}^{+1.0\%}_{-0.7\%}$	$1.066 \pm 0.003 \cdot 10^{-1}$	${}^{+4.5\%}_{-5.3\%}$	${}^{+1.0\%}_{-0.7\%}$
c.13*	$pp \to ZZW^\pm j$	`p p > z z wpm j`	$2.810 \pm 0.004 \cdot 10^{-2}$	${}^{+16.1\%}_{-13.0\%}$	${}^{+1.0\%}_{-0.7\%}$	$3.660 \pm 0.013 \cdot 10^{-2}$	${}^{+4.8\%}_{-5.6\%}$	${}^{+1.0\%}_{-0.7\%}$
c.14*	$pp \to ZZZj$	`p p > z z z j`	$4.823 \pm 0.011 \cdot 10^{-3}$	${}^{+14.3\%}_{-11.8\%}$	${}^{+1.4\%}_{-1.0\%}$	$6.341 \pm 0.025 \cdot 10^{-3}$	${}^{+4.9\%}_{-5.4\%}$	${}^{+1.4\%}_{-1.0\%}$
c.15*	$pp \to \gamma W^+W^-j$ (4f)	`p p > a w+ w- j`	$1.182 \pm 0.004 \cdot 10^{-1}$	${}^{+13.4\%}_{-11.2\%}$	${}^{+0.8\%}_{-0.7\%}$	$1.233 \pm 0.004 \cdot 10^{3}$	${}^{+18.9\%}_{-19.9\%}$	${}^{+1.0\%}_{-1.5\%}$
c.16	$pp \to \gamma\gamma W^\pm j$	`p p > a a wpm j`	$4.107 \pm 0.015 \cdot 10^{-2}$	${}^{+11.8\%}_{-10.2\%}$	${}^{+0.6\%}_{-0.8\%}$	$5.807 \pm 0.023 \cdot 10^{-2}$	${}^{+5.8\%}_{-5.5\%}$	${}^{+0.7\%}_{-0.7\%}$
c.17*	$pp \to \gamma ZW^\pm j$	`p p > a z wpm j`	$5.833 \pm 0.023 \cdot 10^{-2}$	${}^{+14.4\%}_{-12.0\%}$	${}^{+0.7\%}_{-0.6\%}$	$7.764 \pm 0.025 \cdot 10^{-2}$	${}^{+5.1\%}_{-5.5\%}$	${}^{+0.8\%}_{-0.6\%}$
c.18*	$pp \to \gamma ZZj$	`p p > a z z j`	$9.995 \pm 0.013 \cdot 10^{-3}$	${}^{+12.5\%}_{-10.6\%}$	${}^{+1.2\%}_{-0.9\%}$	$1.371 \pm 0.005 \cdot 10^{-2}$	${}^{+5.6\%}_{-5.5\%}$	${}^{+1.2\%}_{-0.9\%}$
c.19*	$pp \to \gamma\gamma Zj$	`p p > a a z j`	$1.372 \pm 0.003 \cdot 10^{-2}$	${}^{+10.9\%}_{-9.4\%}$	${}^{+1.0\%}_{-0.9\%}$	$2.051 \pm 0.011 \cdot 10^{-2}$	${}^{+7.0\%}_{-6.3\%}$	${}^{+1.0\%}_{-0.9\%}$
c.20*	$pp \to \gamma\gamma\gamma j$	`p p > a a a j`	$1.031 \pm 0.006 \cdot 10^{-2}$	${}^{+14.3\%}_{-12.6\%}$	${}^{+0.9\%}_{-1.2\%}$	$2.020 \pm 0.008 \cdot 10^{-2}$	${}^{+12.8\%}_{-11.0\%}$	${}^{+0.8\%}_{-1.2\%}$

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	Process	Syntax	Cross section (pb)			
	Four vector bosons		LO 13 TeV		NLO 13 TeV	
c.21*	$pp \to W^+W^-W^+W^-$ (4f)	`p p > w+ w- w+ w-`	$5.721 \pm 0.014 \cdot 10^{-4}$	$^{+3.7\%}_{-3.5\%}$ $^{+2.3\%}_{-1.7\%}$	$9.959 \pm 0.035 \cdot 10^{-4}$	$^{+7.4\%}_{-6.0\%}$ $^{+1.7\%}_{-1.2\%}$
c.22*	$pp \to W^+W^-W^\pm Z$ (4f)	`p p > w+ w- wpm z`	$6.391 \pm 0.076 \cdot 10^{-4}$	$^{+4.4\%}_{-4.1\%}$ $^{+2.4\%}_{-1.8\%}$	$1.188 \pm 0.004 \cdot 10^{-3}$	$^{+8.4\%}_{-6.8\%}$ $^{+1.7\%}_{-1.2\%}$
c.23*	$pp \to W^+W^-W^\pm \gamma$ (4f)	`p p > w+ w- wpm a`	$8.115 \pm 0.064 \cdot 10^{-4}$	$^{+2.5\%}_{-2.5\%}$ $^{+2.2\%}_{-1.7\%}$	$1.546 \pm 0.005 \cdot 10^{-3}$	$^{+7.9\%}_{-6.3\%}$ $^{+1.5\%}_{-1.1\%}$
c.24*	$pp \to W^+W^-ZZ$ (4f)	`p p > w+ w- z z`	$4.320 \pm 0.013 \cdot 10^{-4}$	$^{+4.4\%}_{-4.1\%}$ $^{+2.4\%}_{-1.7\%}$	$7.107 \pm 0.020 \cdot 10^{-4}$	$^{+7.0\%}_{-5.7\%}$ $^{+1.8\%}_{-1.3\%}$
c.25*	$pp \to W^+W^-Z\gamma$ (4f)	`p p > w+ w- z a`	$8.403 \pm 0.016 \cdot 10^{-4}$	$^{+3.0\%}_{-2.9\%}$ $^{+2.3\%}_{-1.7\%}$	$1.483 \pm 0.004 \cdot 10^{-3}$	$^{+7.2\%}_{-5.8\%}$ $^{+1.6\%}_{-1.2\%}$
c.26*	$pp \to W^+W^-\gamma\gamma$ (4f)	`p p > w+ w- a a`	$5.198 \pm 0.012 \cdot 10^{-4}$	$^{+0.6\%}_{-0.9\%}$ $^{+2.1\%}_{-1.6\%}$	$9.381 \pm 0.032 \cdot 10^{-4}$	$^{+6.7\%}_{-5.3\%}$ $^{+1.4\%}_{-1.1\%}$
c.27*	$pp \to W^\pm ZZZ$	`p p > wpm z z z`	$5.862 \pm 0.010 \cdot 10^{-5}$	$^{+5.1\%}_{-4.7\%}$ $^{+2.4\%}_{-1.8\%}$	$1.240 \pm 0.004 \cdot 10^{-4}$	$^{+9.9\%}_{-8.0\%}$ $^{+1.7\%}_{-1.2\%}$
c.28*	$pp \to W^\pm ZZ\gamma$	`p p > wpm z z a`	$1.148 \pm 0.003 \cdot 10^{-4}$	$^{+3.6\%}_{-3.5\%}$ $^{+2.2\%}_{-1.7\%}$	$2.945 \pm 0.008 \cdot 10^{-4}$	$^{+10.8\%}_{-8.7\%}$ $^{+1.3\%}_{-1.0\%}$
c.29*	$pp \to W^\pm Z\gamma\gamma$	`p p > wpm z a a`	$1.054 \pm 0.004 \cdot 10^{-4}$	$^{+1.7\%}_{-1.9\%}$ $^{+2.1\%}_{-1.7\%}$	$3.033 \pm 0.010 \cdot 10^{-4}$	$^{+10.6\%}_{-8.6\%}$ $^{+1.1\%}_{-0.8\%}$
c.30*	$pp \to W^\pm \gamma\gamma\gamma$	`p p > wpm a a a`	$3.600 \pm 0.013 \cdot 10^{-5}$	$^{+0.4\%}_{-1.0\%}$ $^{+2.0\%}_{-1.6\%}$	$1.246 \pm 0.005 \cdot 10^{-4}$	$^{+9.8\%}_{-8.1\%}$ $^{+0.9\%}_{-0.8\%}$
c.31*	$pp \to ZZZZ$	`p p > z z z z`	$1.989 \pm 0.002 \cdot 10^{-5}$	$^{+3.8\%}_{-3.6\%}$ $^{+2.2\%}_{-1.7\%}$	$2.629 \pm 0.008 \cdot 10^{-5}$	$^{+3.5\%}_{-3.0\%}$ $^{+2.2\%}_{-1.7\%}$
c.32*	$pp \to ZZZ\gamma$	`p p > z z z a`	$3.945 \pm 0.007 \cdot 10^{-5}$	$^{+1.9\%}_{-2.1\%}$ $^{+2.1\%}_{-1.6\%}$	$5.224 \pm 0.016 \cdot 10^{-5}$	$^{+3.3\%}_{-2.7\%}$ $^{+2.1\%}_{-1.6\%}$
c.33*	$pp \to ZZ\gamma\gamma$	`p p > z z a a`	$5.513 \pm 0.017 \cdot 10^{-5}$	$^{+0.0\%}_{-0.3\%}$ $^{+2.1\%}_{-1.6\%}$	$7.518 \pm 0.032 \cdot 10^{-5}$	$^{+3.4\%}_{-2.6\%}$ $^{+2.0\%}_{-1.5\%}$
c.34*	$pp \to Z\gamma\gamma\gamma$	`p p > z a a a`	$4.790 \pm 0.012 \cdot 10^{-5}$	$^{+2.3\%}_{-3.1\%}$ $^{+2.0\%}_{-1.6\%}$	$7.103 \pm 0.026 \cdot 10^{-5}$	$^{+3.4\%}_{-3.2\%}$ $^{+1.6\%}_{-1.5\%}$
c.35*	$pp \to \gamma\gamma\gamma\gamma$	`p p > a a a a`	$1.594 \pm 0.004 \cdot 10^{-5}$	$^{+4.7\%}_{-5.7\%}$ $^{+1.9\%}_{-1.7\%}$	$3.389 \pm 0.012 \cdot 10^{-5}$	$^{+7.0\%}_{-6.7\%}$ $^{+1.3\%}_{-1.3\%}$

Automated NLO

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Process		Syntax	Cross section (pb)	
Heavy quarks and jets			LO 13 TeV	NLO 13 TeV
d.1	$pp \to jj$	`p p > j j`	$1.162 \pm 0.001 \cdot 10^{6}\ ^{+24.9\%}_{-18.8\%}\ ^{+0.8\%}_{-0.9\%}$	$1.580 \pm 0.007 \cdot 10^{6}\ ^{+8.4\%}_{-9.0\%}\ ^{+0.7\%}_{-0.9\%}$
d.2	$pp \to jjj$	`p p > j j j`	$8.940 \pm 0.021 \cdot 10^{4}\ ^{+43.8\%}_{-28.4\%}\ ^{+1.2\%}_{-1.4\%}$	$7.791 \pm 0.037 \cdot 10^{4}\ ^{+2.1\%}_{-23.2\%}\ ^{+1.1\%}_{-1.3\%}$
d.3	$pp \to b\bar{b}$ (4f)	`p p > b b~`	$3.743 \pm 0.004 \cdot 10^{3}\ ^{+25.2\%}_{-18.9\%}\ ^{+1.5\%}_{-1.8\%}$	$6.438 \pm 0.028 \cdot 10^{3}\ ^{+15.9\%}_{-13.3\%}\ ^{+1.5\%}_{-1.7\%}$
d.4*	$pp \to b\bar{b}j$ (4f)	`p p > b b~ j`	$1.050 \pm 0.002 \cdot 10^{3}\ ^{+44.1\%}_{-28.5\%}\ ^{+1.6\%}_{-1.8\%}$	$1.327 \pm 0.007 \cdot 10^{3}\ ^{+6.8\%}_{-11.6\%}\ ^{+1.5\%}_{-1.8\%}$
d.5*	$pp \to b\bar{b}jj$ (4f)	`p p > b b~ j j`	$1.852 \pm 0.006 \cdot 10^{2}\ ^{+61.8\%}_{-35.6\%}\ ^{+2.1\%}_{-2.4\%}$	$2.471 \pm 0.012 \cdot 10^{2}\ ^{+8.2\%}_{-16.4\%}\ ^{+2.0\%}_{-2.3\%}$
d.6	$pp \to b\bar{b}b\bar{b}$ (4f)	`p p > b b~ b b~`	$5.050 \pm 0.007 \cdot 10^{-1}\ ^{+61.7\%}_{-35.6\%}\ ^{+2.9\%}_{-3.4\%}$	$8.736 \pm 0.034 \cdot 10^{-1}\ ^{+20.9\%}_{-22.0\%}\ ^{+2.9\%}_{-3.4\%}$
d.7	$pp \to t\bar{t}$	`p p > t t~`	$4.584 \pm 0.003 \cdot 10^{2}\ ^{+29.0\%}_{-21.1\%}\ ^{+1.8\%}_{-2.0\%}$	$6.741 \pm 0.023 \cdot 10^{2}\ ^{+9.8\%}_{-10.9\%}\ ^{+1.8\%}_{-2.1\%}$
d.8	$pp \to t\bar{t}j$	`p p > t t~ j`	$3.135 \pm 0.002 \cdot 10^{2}\ ^{+45.1\%}_{-29.0\%}\ ^{+2.2\%}_{-2.5\%}$	$4.106 \pm 0.015 \cdot 10^{2}\ ^{+8.1\%}_{-12.2\%}\ ^{+2.1\%}_{-2.5\%}$
d.9	$pp \to t\bar{t}jj$	`p p > t t~ j j`	$1.361 \pm 0.001 \cdot 10^{2}\ ^{+61.4\%}_{-35.6\%}\ ^{+2.6\%}_{-3.0\%}$	$1.795 \pm 0.006 \cdot 10^{2}\ ^{+9.3\%}_{-16.1\%}\ ^{+2.4\%}_{-2.9\%}$
d.10	$pp \to t\bar{t}t\bar{t}$	`p p > t t~ t t~`	$4.505 \pm 0.005 \cdot 10^{-3}\ ^{+63.8\%}_{-36.5\%}\ ^{+5.4\%}_{-5.7\%}$	$9.201 \pm 0.028 \cdot 10^{-3}\ ^{+30.8\%}_{-25.6\%}\ ^{+5.5\%}_{-5.9\%}$
d.11	$pp \to t\bar{t}b\bar{b}$ (4f)	`p p > t t~ b b~`	$6.119 \pm 0.004 \cdot 10^{0}\ ^{+62.1\%}_{-35.7\%}\ ^{+2.9\%}_{-3.5\%}$	$1.452 \pm 0.005 \cdot 10^{1}\ ^{+37.6\%}_{-27.5\%}\ ^{+2.9\%}_{-3.5\%}$

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Process		Syntax	Cross section (pb)			
Heavy quarks+vector bosons			LO 13 TeV		NLO 13 TeV	
e.1	$pp \to W^{\pm} b\bar{b}$ (4f)	`p p > wpm b b~`	$3.074 \pm 0.002 \cdot 10^{2}$	$^{+42.3\%}_{-29.2\%}$ $^{+2.0\%}_{-1.6\%}$	$8.162 \pm 0.034 \cdot 10^{2}$	$^{+29.8\%}_{-23.6\%}$ $^{+1.5\%}_{-1.2\%}$
e.2	$pp \to Z b\bar{b}$ (4f)	`p p > z b b~`	$6.993 \pm 0.003 \cdot 10^{2}$	$^{+33.5\%}_{-24.4\%}$ $^{+1.0\%}_{-1.4\%}$	$1.235 \pm 0.004 \cdot 10^{3}$	$^{+19.9\%}_{-17.4\%}$ $^{+1.0\%}_{-1.4\%}$
e.3	$pp \to \gamma b\bar{b}$ (4f)	`p p > a b b~`	$1.731 \pm 0.001 \cdot 10^{3}$	$^{+51.9\%}_{-34.8\%}$ $^{+1.6\%}_{-2.1\%}$	$4.171 \pm 0.015 \cdot 10^{3}$	$^{+33.7\%}_{-27.1\%}$ $^{+1.4\%}_{-1.9\%}$
e.4*	$pp \to W^{\pm} b\bar{b} j$ (4f)	`p p > wpm b b~ j`	$1.861 \pm 0.003 \cdot 10^{2}$	$^{+42.5\%}_{-27.7\%}$ $^{+0.7\%}_{-0.7\%}$	$3.957 \pm 0.013 \cdot 10^{2}$	$^{+27.0\%}_{-21.0\%}$ $^{+0.7\%}_{-0.6\%}$
e.5*	$pp \to Z b\bar{b} j$ (4f)	`p p > z b b~ j`	$1.604 \pm 0.001 \cdot 10^{2}$	$^{+42.4\%}_{-27.6\%}$ $^{+0.9\%}_{-1.1\%}$	$2.805 \pm 0.009 \cdot 10^{2}$	$^{+21.0\%}_{-17.6\%}$ $^{+0.8\%}_{-1.0\%}$
e.6*	$pp \to \gamma b\bar{b} j$ (4f)	`p p > a b b~ j`	$7.812 \pm 0.017 \cdot 10^{2}$	$^{+51.2\%}_{-32.0\%}$ $^{+1.0\%}_{-1.5\%}$	$1.233 \pm 0.004 \cdot 10^{3}$	$^{+18.9\%}_{-19.9\%}$ $^{+1.0\%}_{-1.5\%}$
e.7	$pp \to t\bar{t} W^{\pm}$	`p p > t t~ wpm`	$3.777 \pm 0.003 \cdot 10^{-1}$	$^{+23.9\%}_{-18.0\%}$ $^{+2.1\%}_{-1.6\%}$	$5.662 \pm 0.021 \cdot 10^{-1}$	$^{+11.2\%}_{-10.6\%}$ $^{+1.7\%}_{-1.3\%}$
e.8	$pp \to t\bar{t} Z$	`p p > t t~ z`	$5.273 \pm 0.004 \cdot 10^{-1}$	$^{+30.5\%}_{-21.8\%}$ $^{+1.8\%}_{-2.1\%}$	$7.598 \pm 0.026 \cdot 10^{-1}$	$^{+9.7\%}_{-11.1\%}$ $^{+1.9\%}_{-2.2\%}$
e.9	$pp \to t\bar{t} \gamma$	`p p > t t~ a`	$1.204 \pm 0.001 \cdot 10^{0}$	$^{+29.6\%}_{-21.3\%}$ $^{+1.6\%}_{-1.8\%}$	$1.744 \pm 0.005 \cdot 10^{0}$	$^{+9.8\%}_{-11.0\%}$ $^{+1.7\%}_{-2.0\%}$
e.10*	$pp \to t\bar{t} W^{\pm} j$	`p p > t t~ wpm j`	$2.352 \pm 0.002 \cdot 10^{-1}$	$^{+40.9\%}_{-27.1\%}$ $^{+1.3\%}_{-1.0\%}$	$3.404 \pm 0.011 \cdot 10^{-1}$	$^{+11.2\%}_{-14.0\%}$ $^{+1.2\%}_{-0.9\%}$
e.11*	$pp \to t\bar{t} Z j$	`p p > t t~ z j`	$3.953 \pm 0.004 \cdot 10^{-1}$	$^{+46.2\%}_{-29.5\%}$ $^{+2.7\%}_{-3.0\%}$	$5.074 \pm 0.016 \cdot 10^{-1}$	$^{+7.0\%}_{-12.3\%}$ $^{+2.5\%}_{-2.9\%}$
e.12*	$pp \to t\bar{t} \gamma j$	`p p > t t~ a j`	$8.726 \pm 0.010 \cdot 10^{-1}$	$^{+45.4\%}_{-29.1\%}$ $^{+2.3\%}_{-2.6\%}$	$1.135 \pm 0.004 \cdot 10^{0}$	$^{+7.5\%}_{-12.2\%}$ $^{+2.2\%}_{-2.5\%}$
e.13*	$pp \to t\bar{t} W^{-} W^{+}$ (4f)	`p p > t t~ w+ w-`	$6.675 \pm 0.006 \cdot 10^{-3}$	$^{+30.9\%}_{-21.9\%}$ $^{+2.1\%}_{-2.0\%}$	$9.904 \pm 0.026 \cdot 10^{-3}$	$^{+10.9\%}_{-11.8\%}$ $^{+2.1\%}_{-2.1\%}$
e.14*	$pp \to t\bar{t} W^{\pm} Z$	`p p > t t~ wpm z`	$2.404 \pm 0.002 \cdot 10^{-3}$	$^{+26.6\%}_{-19.6\%}$ $^{+2.5\%}_{-1.8\%}$	$3.525 \pm 0.010 \cdot 10^{-3}$	$^{+10.6\%}_{-10.8\%}$ $^{+2.3\%}_{-1.6\%}$
e.15*	$pp \to t\bar{t} W^{\pm} \gamma$	`p p > t t~ wpm a`	$2.718 \pm 0.003 \cdot 10^{-3}$	$^{+25.4\%}_{-18.9\%}$ $^{+2.3\%}_{-1.8\%}$	$3.927 \pm 0.013 \cdot 10^{-3}$	$^{+10.3\%}_{-10.4\%}$ $^{+2.0\%}_{-1.5\%}$
e.16*	$pp \to t\bar{t} Z Z$	`p p > t t~ z z`	$1.349 \pm 0.014 \cdot 10^{-3}$	$^{+29.3\%}_{-21.1\%}$ $^{+1.7\%}_{-1.5\%}$	$1.840 \pm 0.007 \cdot 10^{-3}$	$^{+7.9\%}_{-9.9\%}$ $^{+1.7\%}_{-1.5\%}$
e.17*	$pp \to t\bar{t} Z \gamma$	`p p > t t~ z a`	$2.548 \pm 0.003 \cdot 10^{-3}$	$^{+30.1\%}_{-21.5\%}$ $^{+1.7\%}_{-1.6\%}$	$3.656 \pm 0.012 \cdot 10^{-3}$	$^{+9.7\%}_{-11.0\%}$ $^{+1.8\%}_{-1.9\%}$
e.18*	$pp \to t\bar{t} \gamma\gamma$	`p p > t t~ a a`	$3.272 \pm 0.006 \cdot 10^{-3}$	$^{+28.4\%}_{-20.6\%}$ $^{+1.3\%}_{-1.1\%}$	$4.402 \pm 0.015 \cdot 10^{-3}$	$^{+7.8\%}_{-9.7\%}$ $^{+1.4\%}_{-1.4\%}$

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	Process	Syntax	Cross section (pb)	
	Single-top		LO 13 TeV	NLO 13 TeV
f.1	$pp \to tj$ (t-channel)	`p p > tt j $$ w+ w-`	$1.520 \pm 0.001 \cdot 10^{2}$ ${}^{+9.4\%}_{-11.9\%}$ ${}^{+0.4\%}_{-0.6\%}$	$1.563 \pm 0.005 \cdot 10^{2}$ ${}^{+1.4\%}_{-1.8\%}$ ${}^{+0.4\%}_{-0.6\%}$
f.2	$pp \to t\gamma j$ (t-channel)	`p p > tt a j $$ w+ w-`	$9.956 \pm 0.014 \cdot 10^{-1}$ ${}^{+6.4\%}_{-8.8\%}$ ${}^{+0.9\%}_{-1.0\%}$	$1.017 \pm 0.003 \cdot 10^{0}$ ${}^{+1.3\%}_{-1.2\%}$ ${}^{+0.8\%}_{-0.9\%}$
f.3	$pp \to tZj$ (t-channel)	`p p > tt z j $$ w+ w-`	$6.967 \pm 0.007 \cdot 10^{-1}$ ${}^{+3.5\%}_{-5.5\%}$ ${}^{+0.9\%}_{-1.0\%}$	$6.993 \pm 0.021 \cdot 10^{-1}$ ${}^{+1.6\%}_{-1.1\%}$ ${}^{+0.9\%}_{-1.0\%}$
f.4	$pp \to tbj$ (t-channel, 4f)	`p p > tt bb j $$ w+ w-`	$1.003 \pm 0.000 \cdot 10^{2}$ ${}^{+13.8\%}_{-11.5\%}$ ${}^{+0.4\%}_{-0.5\%}$	$1.319 \pm 0.003 \cdot 10^{2}$ ${}^{+5.8\%}_{-5.2\%}$ ${}^{+0.4\%}_{-0.5\%}$
f.5*	$pp \to tbj\gamma$ (t-channel, 4f)	`p p > tt bb j a $$ w+ w-`	$6.293 \pm 0.006 \cdot 10^{-1}$ ${}^{+16.8\%}_{-13.5\%}$ ${}^{+0.8\%}_{-0.9\%}$	$8.612 \pm 0.025 \cdot 10^{-1}$ ${}^{+6.2\%}_{-6.6\%}$ ${}^{+0.8\%}_{-0.9\%}$
f.6*	$pp \to tbjZ$ (t-channel, 4f)	`p p > tt bb j z $$ w+ w-`	$3.934 \pm 0.002 \cdot 10^{-1}$ ${}^{+18.7\%}_{-14.7\%}$ ${}^{+1.0\%}_{-0.9\%}$	$5.657 \pm 0.014 \cdot 10^{-1}$ ${}^{+7.7\%}_{-7.9\%}$ ${}^{+0.9\%}_{-0.9\%}$
f.7	$pp \to tb$ (s-channel, 4f)	`p p > w+ > t b~, p p > w- > t~ b`	$7.489 \pm 0.007 \cdot 10^{0}$ ${}^{+3.5\%}_{-4.4\%}$ ${}^{+1.9\%}_{-1.4\%}$	$1.001 \pm 0.004 \cdot 10^{1}$ ${}^{+3.7\%}_{-3.9\%}$ ${}^{+1.9\%}_{-1.5\%}$
f.8*	$pp \to tb\gamma$ (s-channel, 4f)	`p p > w+ > t b~ a, p p > w- > t~ b a`	$1.490 \pm 0.001 \cdot 10^{-2}$ ${}^{+1.2\%}_{-1.8\%}$ ${}^{+1.9\%}_{-1.5\%}$	$1.952 \pm 0.007 \cdot 10^{-2}$ ${}^{+2.6\%}_{-2.3\%}$ ${}^{+1.7\%}_{-1.4\%}$
f.9*	$pp \to tbZ$ (s-channel, 4f)	`p p > w+ > t b~ z, p p > w- > t~ b z`	$1.072 \pm 0.001 \cdot 10^{-2}$ ${}^{+1.3\%}_{-1.5\%}$ ${}^{+2.0\%}_{-1.6\%}$	$1.539 \pm 0.005 \cdot 10^{-2}$ ${}^{+3.9\%}_{-3.2\%}$ ${}^{+1.9\%}_{-1.5\%}$

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	Process	Syntax	Cross section (pb) LO 13 TeV		NLO 13 TeV	
Single Higgs production						
g.1	$pp \to H$ (HEFT)	`p p > h`	$1.593 \pm 0.003 \cdot 10^{1}$	$^{+34.8\%}_{-26.0\%}$ $^{+1.2\%}_{-1.7\%}$	$3.261 \pm 0.010 \cdot 10^{1}$	$^{+20.2\%}_{-17.9\%}$ $^{+1.1\%}_{-1.6\%}$
g.2	$pp \to Hj$ (HEFT)	`p p > h j`	$8.367 \pm 0.003 \cdot 10^{0}$	$^{+39.4\%}_{-26.4\%}$ $^{+1.2\%}_{-1.4\%}$	$1.422 \pm 0.006 \cdot 10^{1}$	$^{+18.5\%}_{-16.6\%}$ $^{+1.1\%}_{-1.4\%}$
g.3	$pp \to Hjj$ (HEFT)	`p p > h j j`	$3.020 \pm 0.002 \cdot 10^{0}$	$^{+59.1\%}_{-34.7\%}$ $^{+1.4\%}_{-1.7\%}$	$5.124 \pm 0.020 \cdot 10^{0}$	$^{+20.7\%}_{-21.0\%}$ $^{+1.3\%}_{-1.5\%}$
g.4	$pp \to Hjj$ (VBF)	`p p > h j j $$ w+ w- z`	$1.987 \pm 0.002 \cdot 10^{0}$	$^{+1.7\%}_{-2.0\%}$ $^{+1.9\%}_{-1.4\%}$	$1.900 \pm 0.006 \cdot 10^{0}$	$^{+0.8\%}_{-0.9\%}$ $^{+2.0\%}_{-1.5\%}$
g.5	$pp \to Hjjj$ (VBF)	`p p > h j j j $$ w+ w- z`	$2.824 \pm 0.005 \cdot 10^{-1}$	$^{+15.7\%}_{-12.7\%}$ $^{+1.5\%}_{-1.0\%}$	$3.085 \pm 0.010 \cdot 10^{-1}$	$^{+2.0\%}_{-3.0\%}$ $^{+1.5\%}_{-1.1\%}$
g.6	$pp \to HW^{\pm}$	`p p > h wpm`	$1.195 \pm 0.002 \cdot 10^{0}$	$^{+3.5\%}_{-4.5\%}$ $^{+1.9\%}_{-1.5\%}$	$1.419 \pm 0.005 \cdot 10^{0}$	$^{+2.1\%}_{-2.6\%}$ $^{+1.9\%}_{-1.4\%}$
g.7	$pp \to HW^{\pm}j$	`p p > h wpm j`	$4.018 \pm 0.003 \cdot 10^{-1}$	$^{+10.7\%}_{-9.3\%}$ $^{+1.2\%}_{-0.9\%}$	$4.842 \pm 0.017 \cdot 10^{-1}$	$^{+3.6\%}_{-3.7\%}$ $^{+1.2\%}_{-1.0\%}$
g.8*	$pp \to HW^{\pm}jj$	`p p > h wpm j j`	$1.198 \pm 0.016 \cdot 10^{-1}$	$^{+26.1\%}_{-19.4\%}$ $^{+0.8\%}_{-0.6\%}$	$1.574 \pm 0.014 \cdot 10^{-1}$	$^{+5.0\%}_{-6.5\%}$ $^{+0.9\%}_{-0.6\%}$
g.9	$pp \to HZ$	`p p > h z`	$6.468 \pm 0.008 \cdot 10^{-1}$	$^{+3.5\%}_{-4.5\%}$ $^{+1.9\%}_{-1.4\%}$	$7.674 \pm 0.027 \cdot 10^{-1}$	$^{+2.0\%}_{-2.5\%}$ $^{+1.9\%}_{-1.4\%}$
g.10	$pp \to HZj$	`p p > h z j`	$2.225 \pm 0.001 \cdot 10^{-1}$	$^{+10.6\%}_{-9.2\%}$ $^{+1.1\%}_{-0.8\%}$	$2.667 \pm 0.010 \cdot 10^{-1}$	$^{+3.5\%}_{-3.6\%}$ $^{+1.1\%}_{-0.9\%}$
g.11*	$pp \to HZjj$	`p p > h z j j`	$7.262 \pm 0.012 \cdot 10^{-2}$	$^{+26.2\%}_{-19.4\%}$ $^{+0.7\%}_{-0.6\%}$	$8.753 \pm 0.037 \cdot 10^{-2}$	$^{+4.8\%}_{-6.3\%}$ $^{+0.7\%}_{-0.6\%}$
g.12*	$pp \to HW^{+}W^{-}$ (4f)	`p p > h w+ w-`	$8.325 \pm 0.139 \cdot 10^{-3}$	$^{+0.0\%}_{-0.3\%}$ $^{+2.0\%}_{-1.6\%}$	$1.065 \pm 0.003 \cdot 10^{-2}$	$^{+2.5\%}_{-1.9\%}$ $^{+2.0\%}_{-1.5\%}$
g.13*	$pp \to HW^{\pm}\gamma$	`p p > h wpm a`	$2.518 \pm 0.006 \cdot 10^{-3}$	$^{+0.7\%}_{-1.4\%}$ $^{+1.9\%}_{-1.5\%}$	$3.309 \pm 0.011 \cdot 10^{-3}$	$^{+2.7\%}_{-2.0\%}$ $^{+1.7\%}_{-1.4\%}$
g.14*	$pp \to HZW^{\pm}$	`p p > h z wpm`	$3.763 \pm 0.007 \cdot 10^{-3}$	$^{+1.1\%}_{-1.5\%}$ $^{+2.0\%}_{-1.6\%}$	$5.292 \pm 0.015 \cdot 10^{-3}$	$^{+3.9\%}_{-3.1\%}$ $^{+1.8\%}_{-1.4\%}$
g.15*	$pp \to HZZ$	`p p > h z z`	$2.093 \pm 0.003 \cdot 10^{-3}$	$^{+0.1\%}_{-0.6\%}$ $^{+1.9\%}_{-1.5\%}$	$2.538 \pm 0.007 \cdot 10^{-3}$	$^{+1.9\%}_{-1.4\%}$ $^{+2.0\%}_{-1.5\%}$
g.16	$pp \to Ht\bar{t}$	`p p > h t t~`	$3.579 \pm 0.003 \cdot 10^{-1}$	$^{+30.0\%}_{-21.5\%}$ $^{+1.7\%}_{-2.0\%}$	$4.608 \pm 0.016 \cdot 10^{-1}$	$^{+5.7\%}_{-9.0\%}$ $^{+2.0\%}_{-2.3\%}$
g.17	$pp \to Htj$	`p p > h tt j`	$4.994 \pm 0.005 \cdot 10^{-2}$	$^{+2.4\%}_{-4.2\%}$ $^{+1.2\%}_{-1.3\%}$	$6.328 \pm 0.022 \cdot 10^{-2}$	$^{+2.9\%}_{-1.8\%}$ $^{+1.5\%}_{-1.6\%}$
g.18	$pp \to Hb\bar{b}$ (4f)	`p p > h b b~`	$4.983 \pm 0.002 \cdot 10^{-1}$	$^{+28.1\%}_{-21.0\%}$ $^{+1.5\%}_{-1.8\%}$	$6.085 \pm 0.026 \cdot 10^{-1}$	$^{+7.3\%}_{-9.6\%}$ $^{+1.6\%}_{-2.0\%}$
g.19	$pp \to Ht\bar{t}j$	`p p > h t t~ j`	$2.674 \pm 0.041 \cdot 10^{-1}$	$^{+45.6\%}_{-29.2\%}$ $^{+2.6\%}_{-2.9\%}$	$3.244 \pm 0.025 \cdot 10^{-1}$	$^{+3.5\%}_{-8.7\%}$ $^{+2.5\%}_{-2.9\%}$
g.20*	$pp \to Hb\bar{b}j$ (4f)	`p p > h b b~ j`	$7.367 \pm 0.002 \cdot 10^{-2}$	$^{+45.6\%}_{-29.1\%}$ $^{+1.8\%}_{-2.1\%}$	$9.034 \pm 0.032 \cdot 10^{-2}$	$^{+7.9\%}_{-11.0\%}$ $^{+1.8\%}_{-2.2\%}$

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	Process	Syntax	Cross section (pb)			
	Higgs pair production		LO 13 TeV		NLO 13 TeV	
h.1	$pp \to HH$ (Loop improved)	`p p > h h`	$1.772 \pm 0.006 \cdot 10^{-2}$	${}^{+29.5\%}_{-21.4\%}$ ${}^{+2.1\%}_{-2.6\%}$	$2.763 \pm 0.008 \cdot 10^{-2}$	${}^{+11.4\%}_{-11.8\%}$ ${}^{+2.1\%}_{-2.6\%}$
h.2	$pp \to HHjj$ (VBF)	`p p > h h j j $$ w+ w- z`	$6.503 \pm 0.019 \cdot 10^{-4}$	${}^{+7.2\%}_{-6.4\%}$ ${}^{+2.3\%}_{-1.6\%}$	$6.820 \pm 0.026 \cdot 10^{-4}$	${}^{+0.8\%}_{-1.0\%}$ ${}^{+2.4\%}_{-1.7\%}$
h.3	$pp \to HHW^{\pm}$	`p p > h h wpm`	$4.303 \pm 0.005 \cdot 10^{-4}$	${}^{+0.9\%}_{-1.3\%}$ ${}^{+2.0\%}_{-1.5\%}$	$5.002 \pm 0.014 \cdot 10^{-4}$	${}^{+1.5\%}_{-1.2\%}$ ${}^{+2.0\%}_{-1.6\%}$
h.4*	$pp \to HHW^{\pm}j$	`p p > h h wpm j`	$1.922 \pm 0.002 \cdot 10^{-4}$	${}^{+14.2\%}_{-11.7\%}$ ${}^{+1.5\%}_{-1.1\%}$	$2.218 \pm 0.009 \cdot 10^{-4}$	${}^{+2.7\%}_{-3.3\%}$ ${}^{+1.6\%}_{-1.1\%}$
h.5*	$pp \to HHW^{\pm}\gamma$	`p p > h h wpm a`	$1.952 \pm 0.004 \cdot 10^{-6}$	${}^{+3.0\%}_{-3.0\%}$ ${}^{+2.2\%}_{-1.6\%}$	$2.347 \pm 0.007 \cdot 10^{-6}$	${}^{+2.4\%}_{-2.0\%}$ ${}^{+2.1\%}_{-1.6\%}$
h.6	$pp \to HHZ$	`p p > h h z`	$2.701 \pm 0.007 \cdot 10^{-4}$	${}^{+0.9\%}_{-1.3\%}$ ${}^{+2.0\%}_{-1.5\%}$	$3.130 \pm 0.008 \cdot 10^{-4}$	${}^{+1.6\%}_{-1.2\%}$ ${}^{+2.0\%}_{-1.5\%}$
h.7*	$pp \to HHZj$	`p p > h h z j`	$1.211 \pm 0.001 \cdot 10^{-4}$	${}^{+14.1\%}_{-11.7\%}$ ${}^{+1.4\%}_{-1.1\%}$	$1.394 \pm 0.006 \cdot 10^{-4}$	${}^{+2.7\%}_{-3.2\%}$ ${}^{+1.5\%}_{-1.1\%}$
h.8*	$pp \to HHZ\gamma$	`p p > h h z a`	$1.397 \pm 0.003 \cdot 10^{-6}$	${}^{+2.4\%}_{-2.5\%}$ ${}^{+2.2\%}_{-1.7\%}$	$1.604 \pm 0.005 \cdot 10^{-6}$	${}^{+1.7\%}_{-1.4\%}$ ${}^{+2.3\%}_{-1.7\%}$
h.9*	$pp \to HHZZ$	`p p > h h z z`	$2.309 \pm 0.005 \cdot 10^{-6}$	${}^{+3.9\%}_{-3.8\%}$ ${}^{+2.2\%}_{-1.7\%}$	$2.754 \pm 0.009 \cdot 10^{-6}$	${}^{+2.3\%}_{-2.0\%}$ ${}^{+2.3\%}_{-1.7\%}$
h.10*	$pp \to HHZW^{\pm}$	`p p > h h z wpm`	$3.708 \pm 0.013 \cdot 10^{-6}$	${}^{+4.8\%}_{-4.5\%}$ ${}^{+2.3\%}_{-1.7\%}$	$4.904 \pm 0.029 \cdot 10^{-6}$	${}^{+3.7\%}_{-3.2\%}$ ${}^{+2.2\%}_{-1.6\%}$
h.11*	$pp \to HHW^{+}W^{-}$ (4f)	`p p > h h w+ w-`	$7.524 \pm 0.070 \cdot 10^{-6}$	${}^{+3.5\%}_{-3.4\%}$ ${}^{+2.3\%}_{-1.7\%}$	$9.268 \pm 0.030 \cdot 10^{-6}$	${}^{+2.3\%}_{-2.1\%}$ ${}^{+2.3\%}_{-1.7\%}$
h.12	$pp \to HHt\bar{t}$	`p p > h h t t~`	$6.756 \pm 0.007 \cdot 10^{-4}$	${}^{+30.2\%}_{-21.6\%}$ ${}^{+1.8\%}_{-1.8\%}$	$7.301 \pm 0.024 \cdot 10^{-4}$	${}^{+1.4\%}_{-5.7\%}$ ${}^{+2.2\%}_{-2.3\%}$
h.13	$pp \to HHtj$	`p p > h h tt j`	$1.844 \pm 0.008 \cdot 10^{-5}$	${}^{+0.0\%}_{-0.6\%}$ ${}^{+1.8\%}_{-1.8\%}$	$2.444 \pm 0.009 \cdot 10^{-5}$	${}^{+4.5\%}_{-3.1\%}$ ${}^{+2.8\%}_{-3.0\%}$
h.14*	$pp \to HHb\bar{b}$	`p p > h h b b~`	$7.849 \pm 0.022 \cdot 10^{-8}$	${}^{+34.3\%}_{-23.9\%}$ ${}^{+3.1\%}_{-3.7\%}$	$1.084 \pm 0.012 \cdot 10^{-7}$	${}^{+7.4\%}_{-10.8\%}$ ${}^{+3.1\%}_{-3.7\%}$

Automated NLO

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Process		Syntax	Cross section (pb)			
Heavy quarks and jets			LO 1 TeV		NLO 1 TeV	
i.1	$e^+e^- \to jj$	e+ e- > j j	$6.223 \pm 0.005 \cdot 10^{-1}$	$^{+0.0\%}_{-0.0\%}$	$6.389 \pm 0.013 \cdot 10^{-1}$	$^{+0.2\%}_{-0.2\%}$
i.2	$e^+e^- \to jjj$	e+ e- > j j j	$3.401 \pm 0.002 \cdot 10^{-1}$	$^{+9.6\%}_{-8.0\%}$	$3.166 \pm 0.019 \cdot 10^{-1}$	$^{+0.2\%}_{-2.1\%}$
i.3	$e^+e^- \to jjjj$	e+ e- > j j j j	$1.047 \pm 0.001 \cdot 10^{-1}$	$^{+20.0\%}_{-15.3\%}$	$1.090 \pm 0.006 \cdot 10^{-1}$	$^{+0.0\%}_{-2.8\%}$
i.4	$e^+e^- \to jjjjj$	e+ e- > j j j j j	$2.211 \pm 0.006 \cdot 10^{-2}$	$^{+31.4\%}_{-22.0\%}$	$2.771 \pm 0.021 \cdot 10^{-2}$	$^{+4.4\%}_{-8.6\%}$
i.5	$e^+e^- \to t\bar{t}$	e+ e- > t t~	$1.662 \pm 0.002 \cdot 10^{-1}$	$^{+0.0\%}_{-0.0\%}$	$1.745 \pm 0.006 \cdot 10^{-1}$	$^{+0.4\%}_{-0.4\%}$
i.6	$e^+e^- \to t\bar{t}j$	e+ e- > t t~ j	$4.813 \pm 0.005 \cdot 10^{-2}$	$^{+9.3\%}_{-7.8\%}$	$5.276 \pm 0.022 \cdot 10^{-2}$	$^{+1.3\%}_{-2.1\%}$
i.7*	$e^+e^- \to t\bar{t}jj$	e+ e- > t t~ j j	$8.614 \pm 0.009 \cdot 10^{-3}$	$^{+19.4\%}_{-15.0\%}$	$1.094 \pm 0.005 \cdot 10^{-2}$	$^{+5.0\%}_{-6.3\%}$
i.8*	$e^+e^- \to t\bar{t}jjj$	e+ e- > t t~ j j j	$1.044 \pm 0.002 \cdot 10^{-3}$	$^{+30.5\%}_{-21.6\%}$	$1.546 \pm 0.010 \cdot 10^{-3}$	$^{+10.6\%}_{-11.6\%}$
i.9*	$e^+e^- \to t\bar{t}t\bar{t}$	e+ e- > t t~ t t~	$6.456 \pm 0.016 \cdot 10^{-7}$	$^{+19.1\%}_{-14.8\%}$	$1.221 \pm 0.005 \cdot 10^{-6}$	$^{+13.2\%}_{-11.2\%}$
i.10*	$e^+e^- \to t\bar{t}t\bar{t}j$	e+ e- > t t~ t t~ j	$2.719 \pm 0.005 \cdot 10^{-8}$	$^{+29.9\%}_{-21.3\%}$	$5.338 \pm 0.027 \cdot 10^{-8}$	$^{+18.3\%}_{-15.4\%}$
i.11	$e^+e^- \to b\bar{b}$ (4f)	e+ e- > b b~	$9.198 \pm 0.004 \cdot 10^{-2}$	$^{+0.0\%}_{-0.0\%}$	$9.282 \pm 0.031 \cdot 10^{-2}$	$^{+0.0\%}_{-0.0\%}$
i.12	$e^+e^- \to b\bar{b}j$ (4f)	e+ e- > b b~ j	$5.029 \pm 0.003 \cdot 10^{-2}$	$^{+9.5\%}_{-8.0\%}$	$4.826 \pm 0.026 \cdot 10^{-2}$	$^{+0.5\%}_{-2.5\%}$
i.13*	$e^+e^- \to b\bar{b}jj$ (4f)	e+ e- > b b~ j j	$1.621 \pm 0.001 \cdot 10^{-2}$	$^{+20.0\%}_{-15.3\%}$	$1.817 \pm 0.009 \cdot 10^{-2}$	$^{+0.0\%}_{-3.1\%}$
i.14*	$e^+e^- \to b\bar{b}jjj$ (4f)	e+ e- > b b~ j j j	$3.641 \pm 0.009 \cdot 10^{-3}$	$^{+31.4\%}_{-22.1\%}$	$4.936 \pm 0.038 \cdot 10^{-3}$	$^{+4.8\%}_{-8.9\%}$
i.15*	$e^+e^- \to b\bar{b}b\bar{b}$ (4f)	e+ e- > b b~ b b~	$1.644 \pm 0.003 \cdot 10^{-4}$	$^{+19.9\%}_{-15.3\%}$	$3.601 \pm 0.017 \cdot 10^{-4}$	$^{+15.2\%}_{-12.5\%}$
i.16*	$e^+e^- \to b\bar{b}b\bar{b}j$ (4f)	e+ e- > b b~ b b~ j	$7.660 \pm 0.022 \cdot 10^{-5}$	$^{+31.3\%}_{-22.0\%}$	$1.537 \pm 0.011 \cdot 10^{-4}$	$^{+17.9\%}_{-15.3\%}$
i.17*	$e^+e^- \to t\bar{t}b\bar{b}$ (4f)	e+ e- > t t~ b b~	$1.819 \pm 0.003 \cdot 10^{-4}$	$^{+19.5\%}_{-15.0\%}$	$2.923 \pm 0.011 \cdot 10^{-4}$	$^{+9.2\%}_{-8.9\%}$
i.18*	$e^+e^- \to t\bar{t}b\bar{b}j$ (4f)	e+ e- > t t~ b b~ j	$4.045 \pm 0.011 \cdot 10^{-5}$	$^{+30.5\%}_{-21.6\%}$	$7.049 \pm 0.052 \cdot 10^{-5}$	$^{+13.7\%}_{-13.1\%}$

Automated NLO

Alwall et al '14

Process		Syntax	Cross section (pb)			
Top quarks +bosons			LO 1 TeV		NLO 1 TeV	
j.1	$e^+e^-\to t\bar{t}H$	`e+ e- > t t~ h`	$2.018 \pm 0.003 \cdot 10^{-3}$	$^{+0.0\%}_{-0.0\%}$	$1.911 \pm 0.006 \cdot 10^{-3}$	$^{+0.4\%}_{-0.5\%}$
j.2*	$e^+e^-\to t\bar{t}Hj$	`e+ e- > t t~ h j`	$2.533 \pm 0.003 \cdot 10^{-4}$	$^{+9.2\%}_{-7.8\%}$	$2.658 \pm 0.009 \cdot 10^{-4}$	$^{+0.5\%}_{-1.5\%}$
j.3*	$e^+e^-\to t\bar{t}Hjj$	`e+ e- > t t~ h j j`	$2.663 \pm 0.004 \cdot 10^{-5}$	$^{+19.3\%}_{-14.9\%}$	$3.278 \pm 0.017 \cdot 10^{-5}$	$^{+4.0\%}_{-5.7\%}$
j.4*	$e^+e^-\to t\bar{t}\gamma$	`e+ e- > t t~ a`	$1.270 \pm 0.002 \cdot 10^{-2}$	$^{+0.0\%}_{-0.0\%}$	$1.335 \pm 0.004 \cdot 10^{-2}$	$^{+0.5\%}_{-0.4\%}$
j.5*	$e^+e^-\to t\bar{t}\gamma j$	`e+ e- > t t~ a j`	$2.355 \pm 0.002 \cdot 10^{-3}$	$^{+9.3\%}_{-7.9\%}$	$2.617 \pm 0.010 \cdot 10^{-3}$	$^{+1.6\%}_{-2.4\%}$
j.6*	$e^+e^-\to t\bar{t}\gamma jj$	`e+ e- > t t~ a j j`	$3.103 \pm 0.005 \cdot 10^{-4}$	$^{+19.5\%}_{-15.0\%}$	$4.002 \pm 0.021 \cdot 10^{-4}$	$^{+5.4\%}_{-6.6\%}$
j.7*	$e^+e^-\to t\bar{t}Z$	`e+ e- > t t~ z`	$4.642 \pm 0.006 \cdot 10^{-3}$	$^{+0.0\%}_{-0.0\%}$	$4.949 \pm 0.014 \cdot 10^{-3}$	$^{+0.6\%}_{-0.5\%}$
j.8*	$e^+e^-\to t\bar{t}Zj$	`e+ e- > t t~ z j`	$6.059 \pm 0.006 \cdot 10^{-4}$	$^{+9.3\%}_{-7.8\%}$	$6.940 \pm 0.028 \cdot 10^{-4}$	$^{+2.0\%}_{-2.6\%}$
j.9*	$e^+e^-\to t\bar{t}Zjj$	`e+ e- > t t~ z j j`	$6.351 \pm 0.028 \cdot 10^{-5}$	$^{+19.4\%}_{-15.0\%}$	$8.439 \pm 0.051 \cdot 10^{-5}$	$^{+5.8\%}_{-6.8\%}$
j.10*	$e^+e^-\to t\bar{t}W^{\pm}jj$	`e+ e- > t t~ wpm j j`	$2.400 \pm 0.004 \cdot 10^{-7}$	$^{+19.3\%}_{-14.9\%}$	$3.723 \pm 0.012 \cdot 10^{-7}$	$^{+9.6\%}_{-9.1\%}$
j.11*	$e^+e^-\to t\bar{t}HZ$	`e+ e- > t t~ h z`	$3.600 \pm 0.006 \cdot 10^{-5}$	$^{+0.0\%}_{-0.0\%}$	$3.579 \pm 0.013 \cdot 10^{-5}$	$^{+0.1\%}_{-0.0\%}$
j.12*	$e^+e^-\to t\bar{t}\gamma Z$	`e+ e- > t t~ a z`	$2.212 \pm 0.003 \cdot 10^{-4}$	$^{+0.0\%}_{-0.0\%}$	$2.364 \pm 0.006 \cdot 10^{-4}$	$^{+0.6\%}_{-0.5\%}$
j.13*	$e^+e^-\to t\bar{t}\gamma H$	`e+ e- > t t~ a h`	$9.756 \pm 0.016 \cdot 10^{-5}$	$^{+0.0\%}_{-0.0\%}$	$9.423 \pm 0.032 \cdot 10^{-5}$	$^{+0.3\%}_{-0.4\%}$
j.14*	$e^+e^-\to t\bar{t}\gamma\gamma$	`e+ e- > t t~ a a`	$3.650 \pm 0.008 \cdot 10^{-4}$	$^{+0.0\%}_{-0.0\%}$	$3.833 \pm 0.013 \cdot 10^{-4}$	$^{+0.4\%}_{-0.4\%}$
j.15*	$e^+e^-\to t\bar{t}ZZ$	`e+ e- > t t~ z z`	$3.788 \pm 0.004 \cdot 10^{-5}$	$^{+0.0\%}_{-0.0\%}$	$4.007 \pm 0.013 \cdot 10^{-5}$	$^{+0.5\%}_{-0.5\%}$
j.16*	$e^+e^-\to t\bar{t}HH$	`e+ e- > t t~ h h`	$1.358 \pm 0.001 \cdot 10^{-5}$	$^{+0.0\%}_{-0.0\%}$	$1.206 \pm 0.003 \cdot 10^{-5}$	$^{+0.9\%}_{-1.1\%}$
j.17*	$e^+e^-\to t\bar{t}W^+W^-$	`e+ e- > t t~ w+ w-`	$1.372 \pm 0.003 \cdot 10^{-4}$	$^{+0.0\%}_{-0.0\%}$	$1.540 \pm 0.006 \cdot 10^{-4}$	$^{+1.0\%}_{-0.9\%}$

Automated NLO

- few years ago: each item in each table resulted in a paper. Now, as for leading order, just run a code and get the results (also for distributions)
- possibility to do precise studies of signal and backgrounds using the same tool (very practical + avoid errors)
- what lead to this remarkable progress? the fact that

1. leading order can be computed automatically and efficiently (e.g. via recursion relations)
2. one can reduce the one-loop to product of tree-level amplitudes
3. it was well understood how to subtract singularities
4. the basis of master integrals was known

But for item 2. everything was there since the time of Passarino-Veltman (even item 2. was understood, but no efficient/practical method exited).
We will now compare this to the current status of NNLO

NNLO: when is NLO not good enough?

- when NLO corrections are large (NLO correction ~ LO)

  This may happens when

  - process involve very different scales → large logarithms of ratio of scales appear
  - new channels open up at NLO (at NLO they are effectively LO)
  - paramount example: Higgs production

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- when high precision is needed to match small experimental error

  - W/Z hadro-production, heavy-quark hadro-production, α_s from event shapes in e^+e^- ...

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  - W/Z hadro-production, heavy-quark hadro-production, α_s from event shapes in e^+e^- ...

- when a reliable error estimate is needed

Some history of NNLO

- first NNLO computation of a collider process was inclusive Drell-Yan production by Hamberg, van Neerven and Matsuura in ’91
- second NNLO calculation: Higgs production in gluon-gluon fusion by Harlander and Kilgore in ’02

  Both calculations refer to inclusive, total cross-sections that are not measurable

- first exclusive NNLO computation (for fiducial volume cross-sections) was Higgs $\to \gamma\gamma$ in ’04 by Anastasiou, Melnikov and Petriello, followed by other exclusive calculations of Higgs and Drell-Yan processes
- only last year NNLO corrections to $2 \to 2$ processes also with QCD partons in the final state started to appear. This indicates a more complete understanding of NNLO

  Many things at NNLO are new and took a while to understand. Today’s technology is likely not to be finalized yet

Ingredients for NNLO

Remember crucial steps for automated NLO were

1. leading order can be computed automatically and efficiently (e.g. via recursion relations)
2. one can reduce the one-loop to product of tree-level amplitudes
3. it was well understood how to subtract singularities
4. the basis of master integrals was known

At NNLO the situation is very different

1. leading order of very limited importance
2. no procedure to reduce two-loop to tree-level (unitarity approaches at two face still many outstanding issues)
3. subtraction of singularities far from trivial
4. basis set of master integrals not known, integrals not all/always known analytically

And all this for simple processes (no result exist, or has been attempted, for any $2 \to 3$ scattering process)

Ingredients for NNLO

What changed in the last years

1. technology to compute integrals
2. extension of systematic FKS subtraction to NNLO

Two-loop virtual

Complexity increases with

1. number external particles (topology)
2. number of kinematic invariants
   ($p_i p_j$ and masses, however more singularities for massless particles)

Standard procedure

1. start from Feynman diagrams
2. reduce integrals by integration by parts identities (IPB)
3. compute master integrals

While at one-loop there is a well-defined algebraic procedure to reduce tensor integrals to master integrals (e.g. Passarino-Veltman reduction), at two-loop integrals must be brought in the right form first, so that scalar products involving the loop momentum can be canceled with propagators

⇒ process-specific algebra on the Feynman diagrams (cumbersome)

Two-loop virtual

Unlike one-loop, the reduction of two-loop integrals to master is non-trivial

Last ten years: large effort devoted to automate IBP. Now public code exist [FIRE, REDUZE]

Also calculation of master integrals not straightforward. Still done on a case-by-case basis.

Recently, many interesting developments. Let me mention a new approach based on Henn's conjecture (strategy: compute Feynman integrals using differential equations, crucial to choose basis where eqs are simple)

Henn '13

Collider processes known at NNLO

1. Drell-Yan (Z,W) (inclusive) — van Neerven '90
2. Higgs (inclusive) — Harlander et al '02; Anastasiou et al '02; Ravindran et al '03
3. Higgs differential — Anastasiou et al '04; Catani et al '07
4. WH/ZH total cross-section — Brein et al '04; Ferrera et al '11
5. di-photon production — Catani et al '11
6. H+1jet — Boughezal et al. '13
7. top-pair production — Czakon et al '13
8. inclusive jets — Currie et al. '13
9. Z/W + photon — Grazzini et al. '13-14
10. ZZ — Cascioli et al. '14
11. t-channel single top — Bruscherseifer '14

NB: this list is growing really quickly now ...

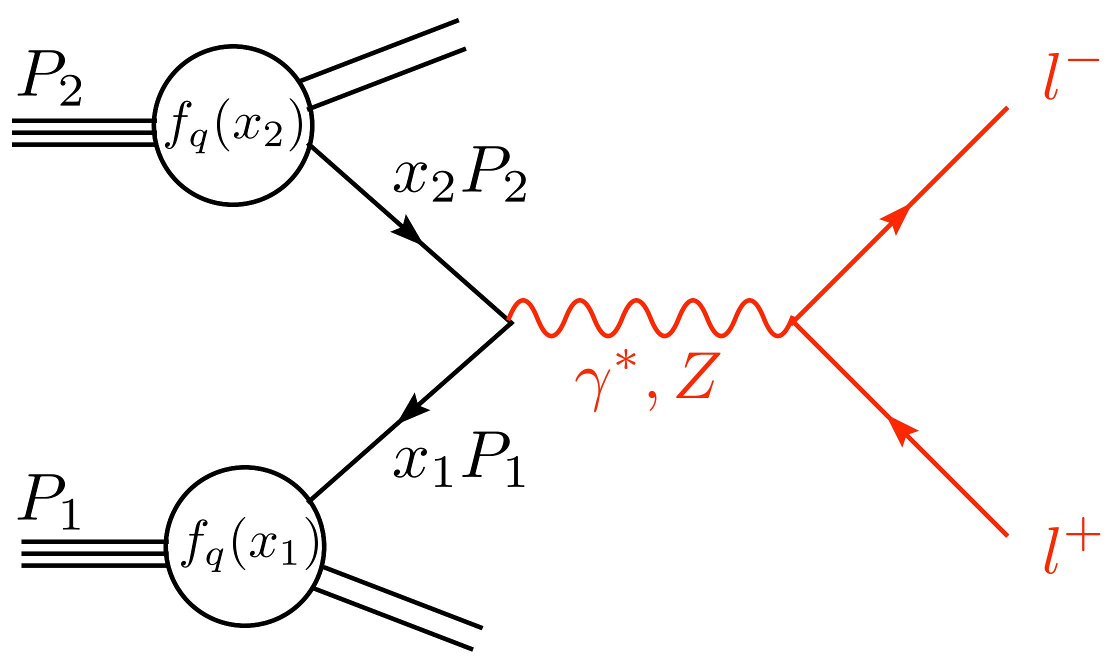
Drell-Yan processes

Drell-Yan processes: Z/W production ($W \to l\nu$, $Z \to l^+l^-$)

Very clean, golden-processes in QCD because

- ✓ dominated by quarks in the initial state
- ✓ no gluons or quarks in the final state (QCD corrections small)
- ✓ leptons easier experimentally (clear signature)

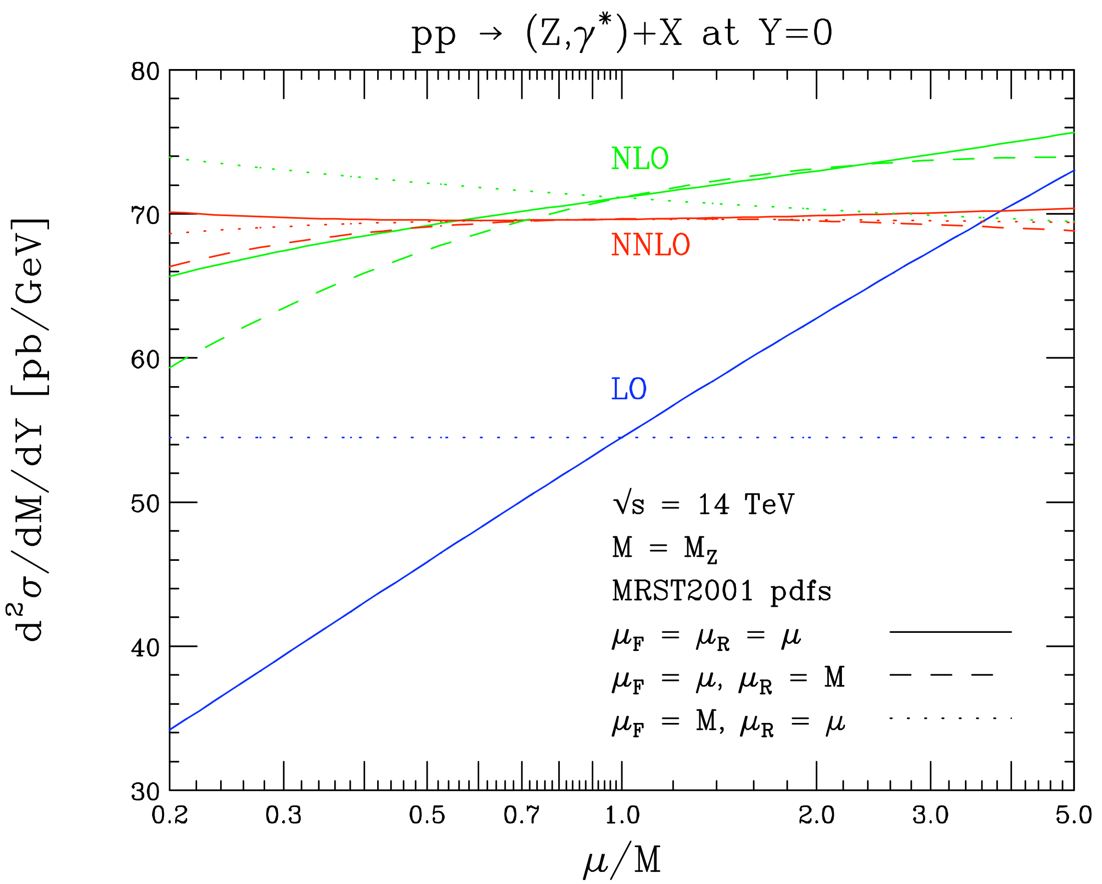
$\Rightarrow$ as clean as it gets at a hadron collider

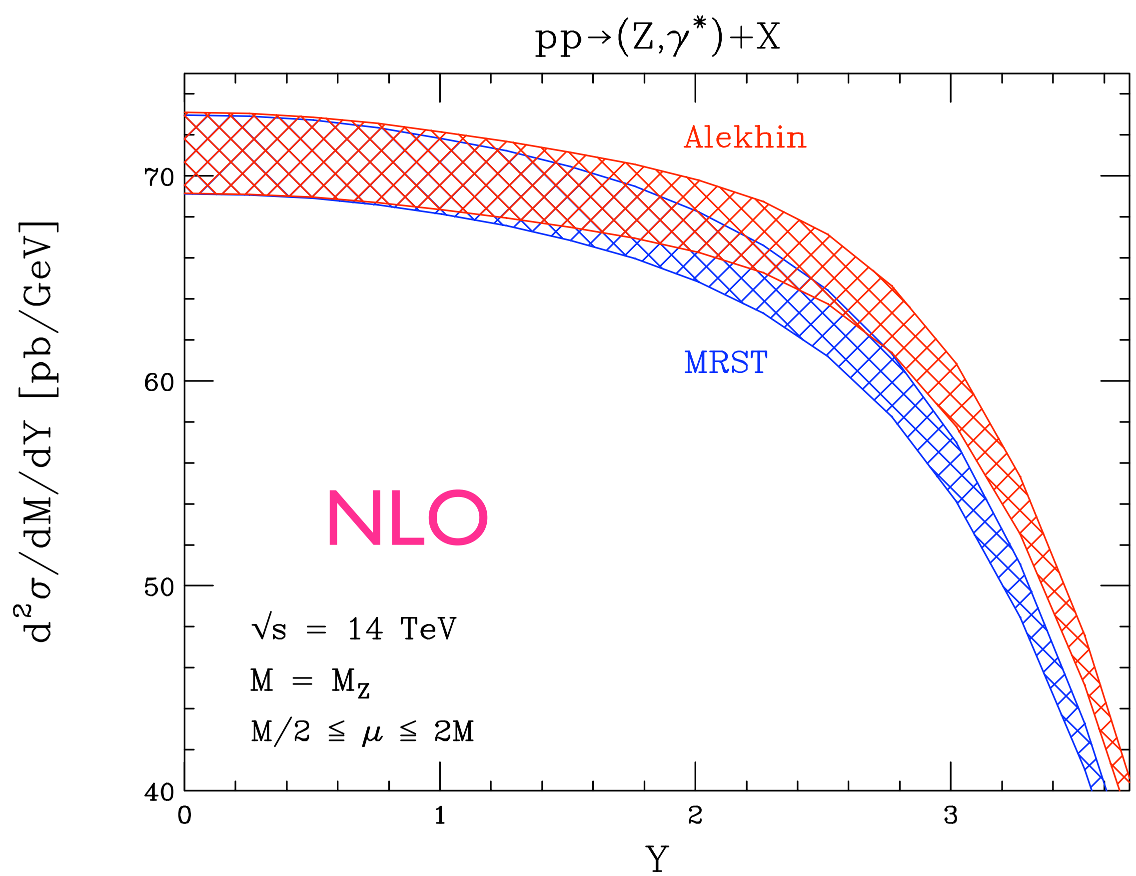


Drell-Yan

- most important and precise test of the SM at the LHC
- with spin-correlations, finite-width effects, γ-Z interference, fully differential in lepton momenta

<u>Scale stability and sensitivity to PDFs</u>





Anastasiou, Dixon, Melnikov, Petriello ’03, ’05; Melnikov, Petriello ’06

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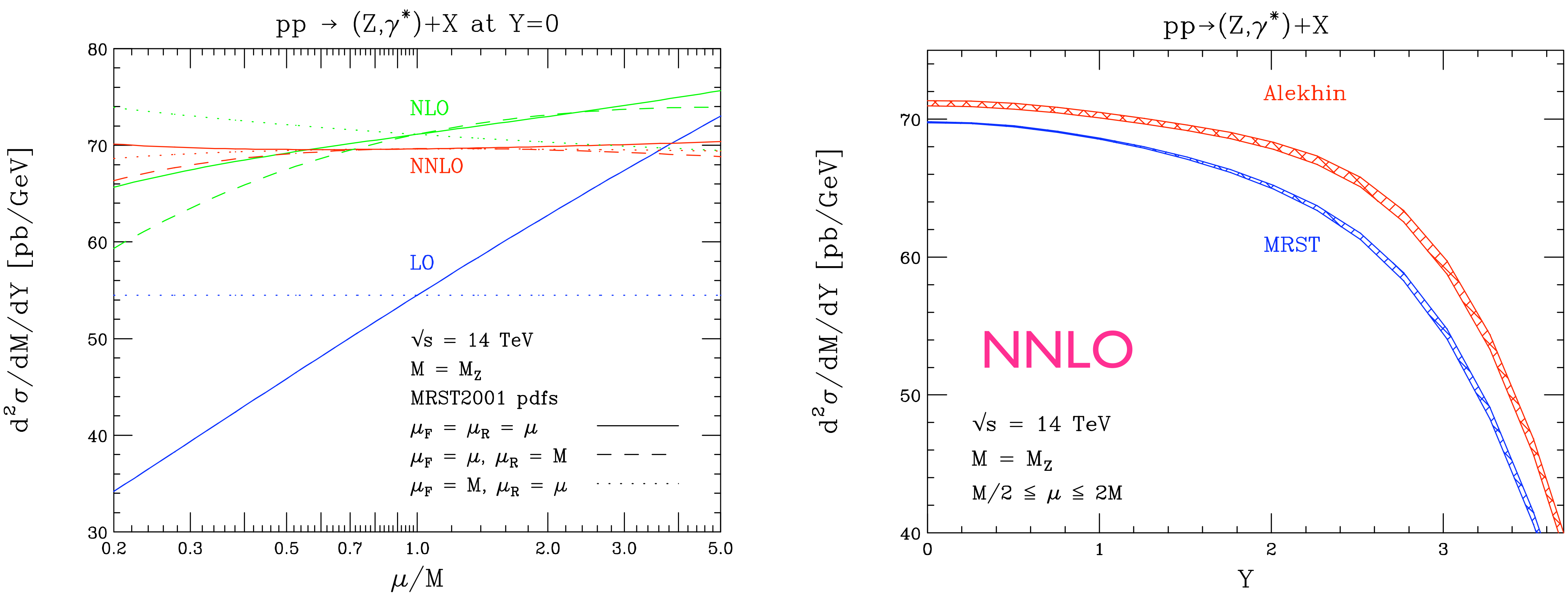
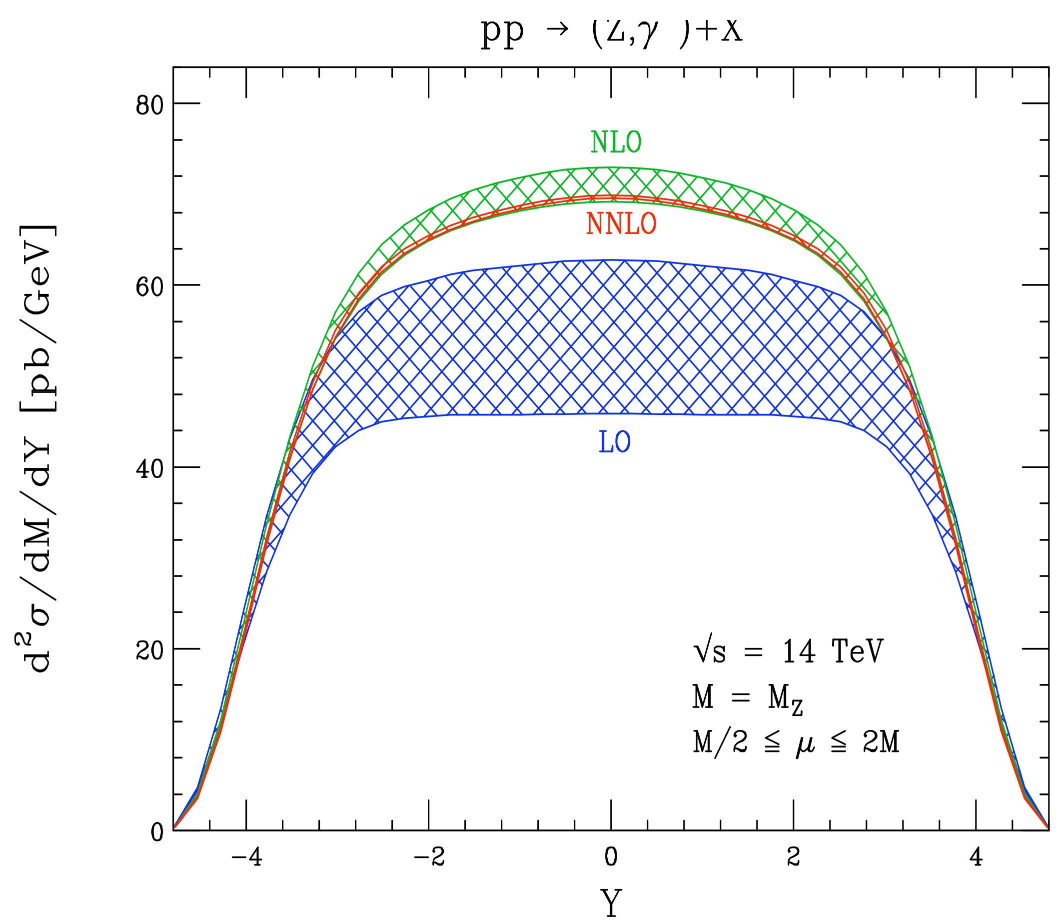
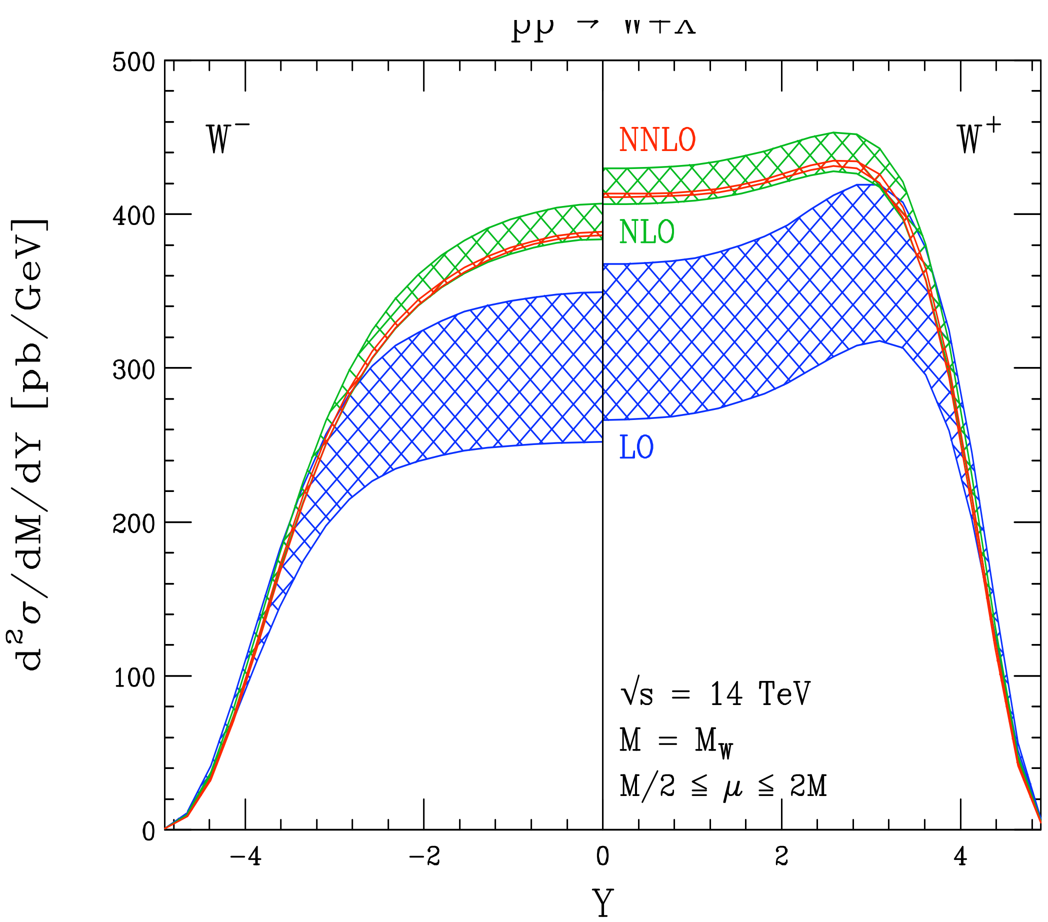


Anastasiou, Dixon, Melnikov, Petriello ’03, ’05; Melnikov, Petriello ’06

Drell-Yan: rapidity distributions





Anastasiou, Dixon, Melnikov, Petriello '03, '05; Melnikov, Petriello '06

☛ at the LHC: perturbative accuracy of the order of 1%

NNLO vs LHC data

Impressive agreement between experiment and NNLO theory

CMS-PAS-SMP-14-003