Froggatt-Nielsen Models with a Residual $\mathbb{Z}_4^R$ Symmetry

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Discrete symmetries

Use discrete symmetries to stabilise the proton

Exists many popular anomaly free discrete symmetries:

- R-parity R_p or Matter parity M_p

  G. Farrar & P. Fayet (1978); S. Dimopoulos, S. Raby & F. Wilczek (1981)

- Baryon triality B_3

  L. Ibáñez & G. Ross (1992)

- Proton hexality P_6

  K.S. Babu, L. Gogoladze & K. Wang (2002); H.K. Dreiner, C. Luhn & M. Thormeier (2006)

$\Longrightarrow$ These symmetries do not forbid the μ-term in the superpotential

Why a $\mathbb{Z}_4^R$ symmetry?

One convenient parametrisation of the $\mathbb{Z}_4^R$ symmetry is:

H. M. Lee, S. Raby, M. Ratz, G. G. Ross, et al. (2010)

	Q_i	$\bar{U}_i$	$\bar{D}_i$	L_i	$\bar{E}_i$	$\bar{N}_i$	H_u	H_d	θ
$\mathbb{Z}_4^R$	1	1	1	1	1	1	0	0	1

- Forbids μ-term in the superpotential
- Forbids dim-4 and 5 LNV and BNV terms
- Anomaly free
- Allows the Weinberg operator

The model

$$G_{\rm SM} \times \mathsf{U}(1)_R \xrightarrow[]{\text{family symmetry breaking}} G_{\rm SM} \times \mathbb{Z}_4^R \xrightarrow[]{\text{SUSY breaking}} G_{\rm SM} \times \mathbb{Z}_2$$

1. Provides a gauge symmetry origin of the $\mathbb{Z}_4^R$ symmetry
   $\hookrightarrow$ the $\mathsf{U}(1)_R$ must be gauged
2. Obtain correct charged fermion masses and mixings
   $\hookrightarrow$ Froggatt-Nielsen mechanism
3. Generate the μ-term at an acceptable scale
   $\hookrightarrow$ Giudice-Masiero mechanism
4. Ensure anomaly freedom
5. Allow a mechanism to generate neutrino masses

Outline

1. Motivation for BSM physics
2. A six step procedure for building a $\mathbb{Z}_4^R$ inspired flavour model
3. Results
4. Conclusion

Fermion Hierarchy

- Many free parameters in the SM unconstrained by theoretical arguments:

$\hookrightarrow$ 9 fermion masses containing huge hierarchy

$$m_t \simeq 172.7\,\text{GeV}$$
$$\vdots$$
$$m_e = 511\,\text{keV}$$

Further enhanced including neutrino masses $m_{\nu_i} \simeq 0.05\,\text{eV}$

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  Further enhanced including neutrino masses $m_{\nu_i} \simeq 0.05\,\text{eV}$

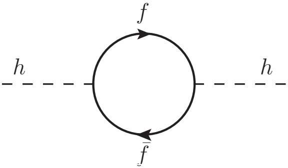
- Neutrino masses:

  Assuming G_{SM} & accidental global symmetries

  $\Longrightarrow$ No neutrino mass terms possible

Motivation for Supersymmetry

- Hierarchy Problem:



$$\Delta m_h^2 = -\frac{y_f^2}{8\pi^2}\Lambda^2 + \cdots$$

- Gauge coupling unification: Does not occur in the SM
- Dark matter: No possibilities within the SM

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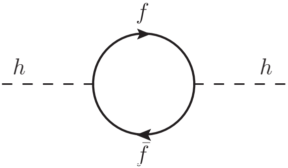

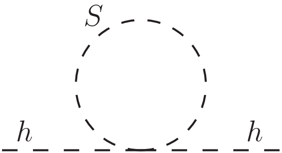

$$\Delta m_h^2 = -\frac{y_f^2}{8\pi^2}\Lambda^2 + \cdots \qquad \Delta m_h^2 = \frac{\lambda_S}{16\pi^2}\Lambda^2 + \cdots$$

- [illegible] unification: Does not occur in the SM
  - $\hookrightarrow$ Rectified with extra field content from supersymmetry
- Dark matter: No possibilities within the SM
  - $\hookrightarrow$ Numerous possibilities in supersymmetry

Superfield formalism

Left-chiral superfields

$$\Phi(y,\theta) = \varphi(y) + \sqrt{2}\theta\xi(y) + \theta^2 F(y)$$

Vector superfields (Wess-Zumino gauge)

$$V_{\mathrm{WZ}} = \theta\sigma^\mu\bar{\theta}A_\mu(x) + \theta\theta\bar{\theta}\bar{\lambda}(x) + \bar{\theta}\bar{\theta}\theta\lambda(x) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D(x)$$

Invariance under both supersymmetry and gauge transformations

$$\mathcal{L}_{\mathrm{SUSY}} = \int \mathrm{d}^2\theta \left[\mathcal{W}^A\mathcal{W}_A + W(\Phi_i) + h.c.\right] + \int \mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta}\, K(\Phi_i, \widetilde{\Phi}^{\dagger j})$$

Supersymmetric Standard Model

Superpotential terms invariant under G_{SM}:

$$W_{\text{ren}} = Y^e_{ij} L_i H_d \bar{E}_j + Y^d_{ij} Q_i H_d \bar{D}_j + Y^u_{ij} Q_i H_u \bar{U}_j + \mu H_u H_d$$
$$\frac{1}{2}\lambda_{ijk} L_i L_j \bar{E}_k + \lambda'_{ijk} L_i Q_j \bar{D}_k + \mu_i L_i H_u + \frac{1}{2}\lambda''_{ijk} \bar{U}_i \bar{D}_j \bar{D}_k$$

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$$W_{\text{dim 5}} = \kappa^{(0)}_{ij} L_i H_u L_j H_u + \kappa^{(1)}_{ijkl} Q_i Q_j Q_k L_l + \kappa^{(2)}_{ijkl} \bar{U}_i \bar{U}_j \bar{D}_k \bar{E}_l + \cdots$$

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Violates only lepton number

Supersymmetric Standard Model

Superpotential terms invariant under G_{SM}:

$$\begin{aligned} W_{\mathsf{ren}} =& Y^e_{ij} L_i H_d \bar{E}_j + Y^d_{ij} Q_i H_d \bar{D}_j + Y^u_{ij} Q_i H_u \bar{U}_j + \mu H_u H_d \\ & \frac{1}{2}\lambda_{ijk} L_i L_j \bar{E}_k + \lambda'_{ijk} L_i Q_j \bar{D}_k + \mu_i L_i H_u + \frac{1}{2}\lambda''_{ijk} \bar{U}_i \bar{D}_j \bar{D}_k \end{aligned}$$

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- Violates only baryon number

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- Violates only baryon number
- Violates both lepton and baryon number

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μ-term has mass dimension

The μ-problem

J.E. Kim & H.P. Nilles (1984)

Consider an SU(5) GUT

$$\bar{\mathbf{5}}_H \longrightarrow \overbrace{(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}}^{H_d} \oplus \overbrace{(\bar{\mathbf{3}},\mathbf{1})_{\frac{1}{3}}}^{H_d^T}$$

If SU(5) is broken at $M_{\mathrm{GUT}} \sim 10^{16}$ GeV

$$\mu\, \bar{\mathbf{5}}_H \mathbf{5}_H = M_{\mathrm{GUT}} \bar{\mathbf{5}}_H \mathbf{5}_H$$

- ✓ Stabilises proton namely H_d^T is heavy
- × $\mu H_u H_d$ now too large

The μ-problem

In the MSSM $\mu \simeq M_{\text{EW}}$

$\hookrightarrow$ Necessary to avoid miraculous cancellation between μ and soft squared-mass

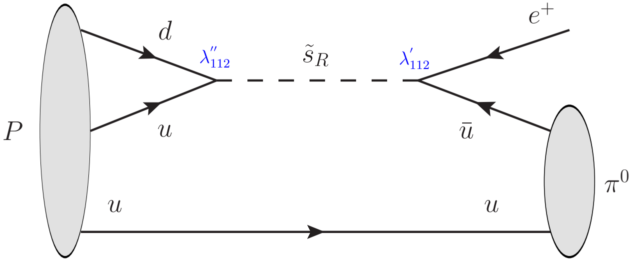
- Higgs obtains the correct VEV $v_u^2 + v_d^2 \simeq (174\,\text{GeV})^2$
- ential yields correct m_Z^2

$\Longrightarrow$ μ-term must be forbidden in the high energy theory

Proton Decay

If both lepton and baryon number hazardous for proton decay

S. Dimopoulos, S. Raby & F. Wilczek (1982)



$$|\lambda'_{112}\lambda''_{112}| \leq 2 \times 10^{-27} \left(\frac{M_{\tilde{s}_R}}{100\,\text{GeV}}\right)^2$$

R. Barbier et al. (2005)

Forbid terms allowing proton decay in the Lagrangian!

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The model

$$G_{\rm SM} \times \mathsf{U}(1)_R \xrightarrow[\text{breaking}]{\text{family symmetry}} G_{\rm SM} \times \mathbb{Z}_4^R \xrightarrow[\text{breaking}]{\text{SUSY}} G_{\rm SM} \times \mathbb{Z}_2$$

1. Forbid the μ-term
   $\hookrightarrow$ Possible with an R-symmetry
2. Obtain correct charged fermion masses and hierarchy
   $\hookrightarrow$ Froggatt-Nielsen mechanism
3. Generate the μ-term at an acceptable scale
   $\hookrightarrow$ Giudice-Masiero mechanism
4. Ensure anomaly freedom
5. Provides a gauge symmetry origin of the $\mathbb{Z}_4^R$ symmetry
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6. Allow a mechanism to generate neutrino masses

Step 1: Forbid the μ-term

Continuous R-symmetries:

Superspace coordinates are charged

$$\theta \to \theta' = e^{iq_\theta^R \alpha(x)} \theta$$
$$\int \mathrm{d}\theta \to \int \mathrm{d}\theta' = e^{-iq_\theta^R \alpha(x)} \int \mathrm{d}\theta$$

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Implications for model building:

- Superpotential must be charged under $\mathrm{U}(1)_R$

$$\int \mathrm{d}^2\theta\, W(\Phi) \to \int \mathrm{d}^2\theta\, e^{2iq_\theta^R \alpha(x)} W'(\Phi)$$

- Kähler potential unchanged

$$\int \mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta}\, K(\Phi_i, \widetilde{\Phi}^{\dagger j}) \to \int \mathrm{d}^2\theta\, \mathrm{d}^2\bar{\theta}\, e^{2iq_\theta^R \alpha(x)} e^{-2iq_\theta^R \alpha(x)} K(\Phi_i, \widetilde{\Phi}^{\dagger j})$$

Step 1: Forbid the μ-term

A gauged $\mathsf{U}(1)_R$ in supergravity includes:

- supergravity multiplet: $\left(e_\mu^a, \psi_\mu\right)$
- R-vector multiplet: $V^R = \left(A_\mu^R, \lambda^R, D^R\right)$
- usual MSSM vector multiplets: $V^k = \left(A_\mu^k, \lambda^k, D^k\right)$

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The spin-1/2 components transform under $\mathsf{U}(1)_R$ as:

$$\lambda(x) \to e^{iq_\theta^R \alpha(x)} \lambda(x), \qquad \xi(x) \to e^{i\left(q_\Phi^R - q_\theta^R\right)\alpha(x)} \xi(x)$$

$\hookrightarrow$ Expect a gaugino coupling to R-photon
$\hookrightarrow$ Contribution to anomalies from gauginos and gravitinos

Step 1: Forbid the μ-term

After the $\mathrm{U}(1)_R$ symmetry is spontaneously broken we require a $\mathbb{Z}_4^R$ symmetry of the form:

H. M. Lee, S. Raby, M. Ratz, G. G. Ross, et al. (2010)

	Q_i	$\bar{U}_i$	$\bar{D}_i$	L_i	$\bar{E}_i$	$\bar{N}_i$	H_u	H_d	θ
$\mathbb{Z}_4^R$	1	1	1	1	1	1	0	0	1

- Forbids μ-term in the superpotential
- Forbids dim-4 and 5 LNV and BNV terms
- Anomaly free

Step 2: Froggatt-Nielsen mechanism

C. Froggatt & H. B. Nielsen (1979)

Result: Free matrix textures → free charges under $\mathrm{U}(1)_R$

Step 2: Froggatt-Nielsen mechanism

Provides a symmetry argument to fix Yukawa textures

C. Froggatt & H. B. Nielsen (1979)

Requires:

- additional gauge symmetry
- flavon superfield Φ (G_{SM} singlet)

Result: Free matrix textures $\to$ free charges under $\text{U}(1)_R$

Charges under $\text{U}(1)_R$ constrained by:

- phenomenologically acceptable fermion masses
- anomaly cancellation
- requirement of a residual $\mathbb{Z}_4^R$ symmetry

Step 2: Froggatt-Nielsen mechanism

Start with standard Yukawa term in superpotential

$$Y_d^{ij} Q_i H_d \bar{D}_j$$

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Start with standard Yukawa term in superpotential

$$Y_d^{ij} Q_i H_d \bar{D}_j \xrightarrow{\text{step 1}} y_d^{ij} \left(\frac{\Phi}{M_P} \right)^n \Theta\left[n\right] Q_i H_d \bar{D}_j$$

1. multiply with powers of Φ introducing function $\Theta[n]$

$$\Theta\left[n\right] = \begin{cases} 1 & n \in \mathbb{N} \\ 0 & \text{otherwise} \end{cases}$$

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$$\xrightarrow{\text{step 2}} y_d^{ij} \left(\frac{\Phi}{M_P}\right)^{q^R_{Q_i}+q^R_{H_d}+q^R_{\bar{D}_j}-2q^R_\theta} \Theta\left[n\right] Q_i H_d \bar{D}_j$$

1. multiply with powers of Φ introducing function $\Theta[n]$
2.

This choice implies q_θ^R must be a free parameter!

Step 2: Froggatt-Nielsen mechanism

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$$\xrightarrow{\text{step 3}} y_d^{ij} \left(\frac{\langle\Phi\rangle}{M_P}\right)^{q^R_{Q_i} + q^R_{H_d} + q^R_{\bar{D}_j} - 2q^R_\theta} \Theta\left[n\right] Q_i H_d \bar{D}_j$$

1. multiply with powers of Φ introducing function $\Theta[n]$
2. choose $q_\Phi^R = -1$ and fix n through $\mathrm{U}(1)_R$ gauge invariance
3. flavon field acquires a VEV breaking $\mathrm{U}(1)_R$

Step 2: Froggatt-Nielsen mechanism

Define FN expansion parameter

$$\epsilon \equiv \left(\frac{\langle\Phi\rangle}{M_P}\right) \simeq \lambda_c = 0.23$$

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Fermion mass ratios and CKM matrix expressed as

$$\begin{aligned} m_d : m_s : m_b &\sim \lambda_c^4 : \lambda_c^2 : 1 \\ m_u : m_c : m_t &\sim \lambda_c^8 : \lambda_c^4 : 1 \\ m_e : m_\mu : m_\tau &\sim \lambda_c^{4+z} : \lambda_c^2 : 1 \\ m_b : m_t &\sim \lambda_c^x \cot\beta \\ m_\tau : m_b &\sim 1 \end{aligned}$$

$$U_{\text{CKM}} \sim \begin{pmatrix} 1 & \lambda_c^{1+y} & \lambda_c^{3+y} \\ \lambda_c^{1+y} & 1 & \lambda_c^2 \\ \lambda_c^{3+y} & \lambda_c^2 & 1 \end{pmatrix}$$

with parameters: $x = 0, 1, 2, 3$
$z = 0, 1 \quad y = -1, 0, -1$
$\Delta_{ij}^L = q_{L_i}^R - q_{L_j}^R$

$\hookrightarrow$ left with $q_{Q_1}^R$, $q_{L_1}^R$, $q_{H_u}^R$, $q_{H_d}^R$, q_θ^R & introduced parameters

Step 3: Giudice-Masiero mechanism

Used to effectively generate terms from Kähler potential

G.F. Giudice & A. Masiero (1988)

$$K_{\text{Higgs}} \supset \left(\mu H_u H_d^\dagger + h.c.\right) + \left(\frac{K Z^\dagger}{M_P} H_u H_d + h.c.\right) + \cdots$$

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If the F-term acquires a VEV $\langle F_{Z^\dagger} \rangle \sim m_{3/2} M_P$

$$\int \mathrm{d}^2\theta \, \mathrm{d}^2\bar{\theta} \left(\frac{\langle F_{Z^\dagger} \rangle \bar{\theta}^2}{M_P}\right) K H_u H_d \longrightarrow \int \mathrm{d}^2\theta m_{3/2} K H_u H_d$$

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Applying FN mechanism

$$m_{3/2}\kappa\, \epsilon^{|q^R_{H_u}+q^R_{H_d}|}\widetilde{\Theta}\left[q^R_{H_u}+q^R_{H_d}\right] H_u H_d$$

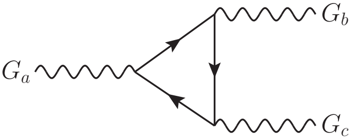
The important difference:

$$\widetilde{\Theta}\left[n\right] = \begin{cases} 1 & n \in \mathbb{Z} \\ 0 & \text{otherwise} \end{cases}$$

Step 4: Anomaly constraints

$$(\partial_\mu j^\mu(x))^a = \frac{-1}{32\pi^2} A^{abc} \epsilon^{\alpha\nu\beta\rho} F^b_{\alpha\nu} F^c_{\beta\rho}$$

$$A^{abc} = \frac{1}{2}\,\mathrm{tr}\{T^a, T^b\} T^c$$



Anomaly freedom requires either:

- anomaly coefficients cancel $A^{abc} = 0 \quad \forall a, b, c$

$\hookrightarrow$ too restrictive: fermion mass hierarchy not possible

Step 4: Anomaly constraints

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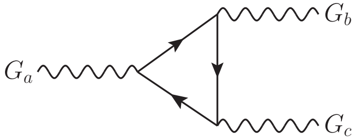
$T^b\}T^c$



Ano s either:

- cancel $A^{abc} = 0 \quad \forall a, b, c$

  $\hookrightarrow$ too restrictive: fermion mass hierarchy not possible

- anomaly cancellation through Green-Schwarz mechanism

M. B. Green & J. H. Schwarz (1984)

$$\frac{A_{\mathsf{SU}(3)_C^2-\mathsf{U}(1)_R}}{k_C} = \frac{A_{\mathsf{SU}(2)_W^2-\mathsf{U}(1)_R}}{k_W} = \frac{A_{\mathsf{U}(1)_Y^2-\mathsf{U}(1)_R}}{k_Y} = 2\pi^2\delta_{\mathsf{GS}}$$

$\hookrightarrow$ Can be used to fix $q^R_{Q_1}$ and $q^R_{H_u}$

Step 5: Residual discrete gauge origin of $\mathbb{Z}_4^R$

Why should discrete symmetries have gauge origins?

L.M. Krauss & F. Wilczek (1989)

- global discrete symmetries violated by quantum effects
- ensure domain walls are not present
- multiple $\mathrm{U}(1)$ symmetries present in many UV completions

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Simple U$(1)_X$ example:

$$\phi_i \longrightarrow \phi_i' = e^{i\alpha(x) q^X_{\phi_i}} \phi_i$$

If $q^X_{\phi_1} = N$ obtains a VEV

$$\sum_{i'} q^X_{\phi_{i'}} = 0 \xrightarrow{\langle \phi_1 \rangle \neq 0} \sum_{i'} q^X_{\phi_{i'}} = 0 \mod N$$

Remaining fields transform as

$$\phi_{i'} \longrightarrow \phi_{i'}' = e^{\frac{2\pi i}{N} q^X_{\phi_{i'}}} \phi_{i'}$$

Step 5: Residual discrete gauge origin of $\mathbb{Z}_4^R$

Ensure the $\mathbb{Z}_4^R$ symmetry results after Φ obtains a VEV
$\implies$ reverse engineer remaining unfixed $\mathrm{U}(1)_R$ charges
$\implies$ not transparent as $q_\Phi^R = -1$

Step 5: Residual discrete gauge origin of $\mathbb{Z}_4^R$

Ensure the $\mathbb{Z}_4^R$ symmetry results after Φ obtains a VEV
$\Longrightarrow$ reverse engineer remaining unfixed $\mathrm{U}(1)_R$ charges
$\Longrightarrow$ not transparent as $q_\Phi^R = -1$

Method:

- start with a completely general term

$$q_{\text{total}}^R = n_{H_u} q_{H_u}^R + n_{H_d} q_{H_d}^R + \sum_i (n_{Q_i} q_{Q_i}^R + n_{\bar{U}_i} q_{\bar{U}_i}^R + \cdots$$

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- eliminate parameters in above equation by:
  - enforcing SM gauge invariance
  - ensuring the minimum superpotential

Step 5: Residual discrete gauge origin of $\mathbb{Z}_4^R$

- The charge under the $\mathbb{Z}_4^R$ symmetry is expressed as

$$\sum_i \left(n_{Q_i} + n_{L_i} + n_{\bar{U}_i} + n_{\bar{D}_i} + n_{\bar{E}_i} + n_{\bar{N}_i}\right) = 4\mathcal{N}_{\mathbb{Z}_4^R} + 2S + C_4$$

$$S = \begin{cases} 1 & \text{for superpotential terms} \\ 0 & \text{for Kähler potential terms} \end{cases}$$

$\mathcal{N}_{\mathbb{Z}_4^R}$ implements mod 4 nature of $\mathbb{Z}_4^R$

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- Main idea: fix $\mathrm{U}(1)_R$ charges such that:

If $C_4 = 0$ then Eq. (1) must be fulfilled
If $C_4 \neq 0$ then Eq. (1) must not be fulfilled

$$q^R_{\text{total}} = \begin{cases} \mathbb{N} + 2q^R_\theta\,, & \text{for superpotential terms} \\ \mathbb{Z}\,, & \text{for Kähler potential terms} \end{cases} \tag{1}$$

Step 5: Residual discrete gauge origin of $\mathbb{Z}_4^R$

Performing above steps:
Fixes remaining charges and constrains q_θ^R

$$q_\theta^R = \frac{2\mathbb{Z}+1}{4}$$

Remaining integer parameters are x, y, z, Δ_{31}^L, Δ^H and ζ plus q_θ^R

Step 5: Residua [illegible] e gauge origin of $\mathbb{Z}_4^R$

Performing above step [illegible]
Fixes remaining charg [illegible] strains q_θ^R

$$q_\theta^R = \frac{2\mathbb{Z}+1}{4}$$

Remaining integer parameters are x, y, z, Δ_{31}^L, Δ^H and ζ plus q_θ^R
Previously $\epsilon \overset{!}{\simeq} \lambda_c = 0.23$

$$-2 \lesssim q_\theta^R \lesssim 1 \Longrightarrow q_\theta^R = \left\{-\frac{7}{4}, -\frac{5}{4}, -\frac{3}{4}, -\frac{1}{4}, \frac{1}{4}, \frac{3}{4}\right\}$$

Only two values lead to acceptable μ-term ($|n| \leq 5$)

$$m_{3/2}\kappa\epsilon^{|n|}H_uH_d \quad \Longrightarrow \quad q_\theta^R = \pm\frac{1}{4}$$

Step 6: Neutrino masses and mixing

Applying FN mechanism to generate masses:

- Non-renormalisable Weinberg term
  - × maximum obtainable mass scale 10^{-5} eV
- See-saw
  - ✓ phenomenologically acceptable mass scale possible
  - ✓ requires RH-neutrino fields $\bar{N}_i$
  - × $M_R^{ij}\bar{N}_i\bar{N}_j$ must be in superpotential

Via see-saw formula light neutrino mass matrix

$$M_\nu^{ij} \simeq \frac{\langle H_u \rangle^2}{M_P} \epsilon^{q_{L_i}^R + q_{L_j}^R + 2q_{H_u}^R - 2q_\theta^R}$$

$$\hookrightarrow \quad m_{\text{abs}}^\nu \simeq \frac{\langle H_u \rangle^2}{M_P} \epsilon^{\Delta^H - 2z + \frac{1}{2} + 2\Delta_{31}^L + 14q_\theta^R}$$

Step 6: Neutrino masses and mixing

Enforce suitable PMNS mixing matrix

$$U_{\text{PMNS}} = V_{e_L} \widetilde{V}^{\dagger}_{\nu_L} T \simeq \begin{pmatrix} 0.8 & 0.5 & 0.2 \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix} \simeq \begin{pmatrix} \epsilon^{0,1} & \epsilon^{0,1} & \epsilon^{0,1} \\ \epsilon^{0,1} & \epsilon^{0,1} & \epsilon^{0,1} \\ \epsilon^{0,1} & \epsilon^{0,1} & \epsilon^{0,1} \end{pmatrix}$$

FN structure $V_{\nu_L}^{ij} \simeq \epsilon^{|q^R_{L_i} - q^R_{L_j}|}$

$$\epsilon^{0,1} \simeq \epsilon^{|q^R_{L_i} - q^R_{L_j}|}$$

This sets the values of $(\Delta^L_{21}, \Delta^L_{31})$ where $\Delta^L_{ij} = q^R_{L_i} - q^R_{L_j}$:

- $(0, 0)$ allows for normal, inverted hierarchy and degenerate
- $(0, -1)$ allows for normal hierarchy
- $(-1, -1)$ allows for normal hierarchy and inverted hierarchy

Step 6: Neutrino masses and mixing

Fixing m^{ν}_{abs} depending on neutrino spectrum

$$m_1 < m_2 \ll m_3 \sim 0.05\,\text{eV} \qquad \text{(normal hierarchy)}$$
$$m_3 \ll m_1 < m_2 \sim 0.05\,\text{eV} \qquad \text{(inverted hierarchy)}$$
$$0.2\,\text{eV} < m_1 \sim m_2 \sim m_3 < 2.2\,\text{eV} \qquad \text{(degenerate)}$$

This determines all remaining charges and parameters except RH-neutrino charges

102 sets of unique charges and parameter values

Anomaly constraints

Three sets of promising charges:

Set 1

i	$q^R_{Q_i}$	$q^R_{\bar{U}_i}$	$q^R_{\bar{D}_i}$	$q^R_{L_i}$	$q^R_{\bar{E}_i}$
1	$\frac{69}{20}$	$\frac{109}{20}$	$\frac{13}{20}$	$-\frac{7}{20}$	$\frac{89}{20}$
2	$\frac{69}{20}$	$\frac{29}{20}$	$-\frac{27}{20}$	$-\frac{27}{20}$	$\frac{69}{20}$
3	$\frac{29}{20}$	$-\frac{11}{20}$	$-\frac{27}{20}$	$-\frac{27}{20}$	$\frac{29}{20}$

$q^R_{H_u}$	$q^R_{H_d}$	q^R_{θ}	q^R_{Φ}
$-\frac{7}{5}$	$-\frac{3}{5}$	$-\frac{1}{4}$	-1

- ✓ Accomodates both an inverted and normal neutrino hierarchy
- ✓ $\epsilon = 0.192$
- ✓ absolute neutrino mass scale $m^{\nu}_{\text{abs}} = 0.049\,\text{eV}$
- × μ-term has the size $m_{3/2}\epsilon^2$
- × $y = 1 \implies$ CKM matrix not optimal

Results

Three sets of promising charges:

Set 2

i	$q^R_{Q_i}$	$q^R_{\bar{U}_i}$	$q^R_{\bar{D}_i}$	$q^R_{L_i}$	$q^R_{\bar{E}_i}$
1	$\frac{65}{12}$	$\frac{67}{12}$	$-\frac{17}{12}$	$-\frac{5}{4}$	$\frac{25}{4}$
2	$\frac{41}{12}$	$\frac{43}{12}$	$-\frac{17}{12}$	$-\frac{5}{4}$	$\frac{13}{4}$
3	$\frac{17}{12}$	$-\frac{19}{12}$	$-\frac{17}{12}$	$-\frac{5}{4}$	$\frac{5}{4}$

$q^R_{H_u}$	$q^R_{H_d}$	q^R_{θ}	q^R_{Φ}
$-\frac{5}{2}$	$\frac{7}{2}$	$\frac{1}{4}$	-1

- ✓ Accomodates only a degenerate neutrino mass spectrum
- ✓ $\epsilon = 0.205$
- × absolute neutrino mass scale $m^{\nu}_{\text{abs}} = 4.158\,\text{eV}$
- ✓ μ-term has the size $m_{3/2}\epsilon$
- × $y = -1 \implies$ CKM matrix not optimal

Summary

Constructed a set of viable flavour models that using a gauged $\mathrm{U}(1)_R$ family symmetry

- stablise the proton
- generate correct μ-term
- result in a residual discrete $\mathbb{Z}_4^R$ gauge symmetry
- generate the necessary hierarchies in the Yukawa matrices

Thank you for your attention!

Discrete Symmetries

$$
= Y^e_{ij} L_i H_d \bar{E}_j + Y^d_{ij} Q_i H_d \bar{D}_j + Y^u_{ij} Q_i H_u \bar{U}_j + \mu H_u H_d
$$

$$
\frac{1}{2}\lambda_{ijk} L_i L_j \bar{E}_k + \lambda'_{ijk} L_i Q_j \bar{D}_k + \mu_i L_i H_u + \frac{1}{2}\lambda''_{ijk} \bar{U}_i \bar{D}_j \bar{D}_k
$$

$$
W_{\text{dim } 5} = \kappa^{(0)}_{ij} L_i H_u L_j H_u + \kappa^{(1)}_{ijkl} Q_i Q_j Q_k L_l + \kappa^{(2)}_{ijkl} \bar{U}_i \bar{U}_j \bar{D}_k \bar{E}_l + \cdots
$$

Forbidden by matter parity / R-parity

G. Farrar & P. Fayet (1978)

S. Dimopoulos, S. Raby & F. Wilczek (1981)

- ✓ Allows for neutrino masses from the Weinberg term
- × Allows dim 5 proton decay operators

Discrete Symmetries

$$W_{\text{ren}} = Y^e_{ij} L_i H_d \bar{E}_j + Y^d_{ij} Q_i H_d \bar{D}_j + Y^u_{ij} Q_i H_u \bar{U}_j + \mu H_u H_d$$
$$\frac{1}{2}\lambda_{ijk} L_i L_j \bar{E}_k + \lambda'_{ijk} L_i Q_j \bar{D}_k + \mu_i L_i H_u + \frac{1}{2}\lambda''_{ijk} \bar{U}_i \bar{D}_j \bar{D}_k$$

$$W_{\text{dim}} \quad \kappa^{(1)}_{ijkl} Q_i Q_j Q_k L_l + \kappa^{(2)}_{ijkl} \bar{U}_i \bar{U}_j \bar{D}_k \bar{E}_l + \cdots$$

Forbidden by baryon triality

L. Ibáñez & G. Ross (1992)

- ✓ Allows for neutrino masses from the Weinberg term
- ✓ Allows for neutrino masses from the LNV term LH_u
- ✓ Proton stability ensured as all BNV terms forbidden
- × Other $\not{M}_p$ terms strongly constrained
- × LSP decays ⇒ no dark matter candidate

Discrete Symmetries

$$W_{\text{ren}} = Y^e_{ij} L_i H_d \bar{E}_j + Y^d_{ij} Q_i H_d \bar{D}_j + Y^u_{ij} Q_i H_u \bar{U}_j + \mu H_u H_d$$

$$\frac{1}{2}\lambda_{ijk} L_i L_j \bar{E}_k + \lambda'_{ijk} L_i Q_j \bar{D}_k + \mu_i L_i H_u + \frac{1}{2}\lambda''_{ijk} \bar{U}_i \bar{D}_j \bar{D}_k$$

$$W_{\text{dim}} \quad \kappa^{(1)}_{ijkl} Q_i Q_j Q_k L_l + \kappa^{(2)}_{ijkl} \bar{U}_i \bar{U}_j \bar{D}_k \bar{E}_l + \cdots$$

Forbidden by proton hexality

K.S. Babu, L. Gogoladze & K. Wang (2002)

H.K. Dreiner, C. Luhn & M. Thormeier (2006)

- ✓ Allows for neutrino masses from the Weinberg terms
- ✓ Forbids all dim 4 and 5 proton decay inducing terms
- ✓ Forbids all LNV and BNV terms except the Weinberg term

Anomaly constraints

Green-Schwarz anomaly cancellation does not include $A_{\mathsf{U}(1)_R^2-\mathsf{U}(1)_Y}$

$\hookrightarrow \quad A_{\mathsf{U}(1)_R^2-\mathsf{U}(1)_Y} \stackrel{!}{=} 0$

Can be used to fix $q_{H_d}^R$

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Aside: anomalous nature of $\mathsf{U}(1)_R$ can be used to fix $\langle\Phi\rangle$

Dine-Seiberg-Wen-Witten mechanism induces Fayet-Iliopoulos term $\xi \sim \delta_{\mathsf{GS}}$

M. Dine, N. Seiberg, X.G. Wen & E. Witten (1986,1987)

For SUSY to be unbroken:

$$\Rightarrow \quad \epsilon = \frac{\langle\Phi\rangle}{M_P} = \frac{g_C}{4\pi}\sqrt{A_{\mathsf{SU}(3)_C^2-\mathsf{U}(1)_R}} = \frac{g_C}{4\pi}\sqrt{\frac{3}{2}\left(x+z-6q_\theta^R+6\right)}$$