

Lecture III

(2)

$d=6$ Operators Classified (EW sector) (with a logic!)

If new particles are heavy, it will be hard to produce them. But one can still study their virtual effects on the low-energy phenomena. These are called indirect effects, as opposite to direct particle production.

Virtues of indirect searches:

- a) Studied in model-independent fashion: any heavy N.P., when integrated out, leads to one particular set of $d > 4$ effective operators. We can study a generic EFT Lagrangian

\mathcal{L}_{EFT}

and derive results which apply to a large class of theories

~~cannot test of the SM-only hypothesis. If only the SM is there up to high scale~~

Limitations of indirect searches:

- 1) Often not the optimal way to test a given scenario

2) If deviations are discovered, hard to tell from which BSM scenario they come from (hope one day we will have to worry about this!)

• The poor man's Effective Field Theory:

$$\mathcal{L}^{EFT} = \mathcal{L}^{SM} + \sum_k \frac{1}{\Lambda^{d_k-4}} \mathcal{O}^{(d_k)}$$

new particles at Λ give $1/\Lambda^n$ effects

But why "1" / Λ ?

In basically NO known BSM theory we get "1". Actually "1" = "product of couplings"

Stated differently, in no plausible BSM theory all the operators of equal d matter the same! Coupling factor in front makes a lot of difference.

For instance, consider a flavor breaking $d=6$:

$$\frac{1}{\Lambda^2} (\bar{d}_R \delta^m \gamma_\mu \gamma_5 d_R) \left(\bar{d}_R \gamma_\mu \gamma_5 d_R \right)$$


$$K_0 \leftrightarrow K_0$$

$$\Lambda \gtrsim 10^3 \text{ TeV}$$

versus another $d=6$

$$\frac{1}{\Lambda^2} (\partial_\mu |H|^2) (\partial^\mu |H|^2)$$

$\Lambda \gtrsim$ few hundred GeV

The "true" BSM theory, of $\Lambda \sim \text{TeV}$, must be endowed with a coupling suppression for the first one.

• In conclusion, it is wrong to classify the importance of EFT operators only by dimension.

• We need a plausible power-counting rule, SILH (hep-ph/0403164) is one option:

$$\mathcal{L}_{\text{SILH}} = \frac{m_*^4}{g_*^2} \mathcal{L} \left[\frac{g_* H}{m_*}, \frac{eA}{m_*}, \frac{Z}{m_*} \right] + \mathcal{O} \left(\frac{g_*^2}{16\pi^2} \right)$$

it is the appropriate power-counting for the Goldstone Boson Higgs, we will further justify it by large- N_c considerations.

• Important: power-counting is not a convention, ~~it~~ is fixed by authority principle.
It is the reflection of physical assumptions on the underlying BSM theory!

operator dimension is boring but useful

(4)

Class I: operators with $2D + 4H$

dimension = 6.

$$\text{SIH coefficient} = \frac{m_x^4}{g_x^2} \frac{1}{m_x^2} \frac{g_x^4}{m_x^4} = \frac{1}{f^2}$$

SKIP

Clorifying operators (not missing any) require patience and one strategy.

In this case, proceed as follows:

Step one: because of $U(1)_Y$, $2H + 2H^*$

$$H_\alpha \quad H^{*\beta} \quad H_\gamma \quad H^{*\delta}$$

Step two: $SU(2)_L$ invariants formed either with δ or ϵ , but

$$\epsilon^{\alpha\beta} \epsilon_{\gamma\delta} = \delta_\beta^\alpha \delta_\delta^\gamma - \delta_\delta^\alpha \delta_\beta^\gamma$$

only one gauge invariant contraction:

$$H_\alpha \quad H^{*\alpha} \quad H_\beta \quad H^{*\beta}$$

Step three: add derivatives in all possible ways. Any $\square H$ can be eliminated by integration by parts. We have

$$(D_\mu H^\dagger \overleftrightarrow{D}^\mu H) |H|^2$$

$$(H^\dagger \overleftrightarrow{D}^\mu H) \cdot (H^\dagger D_\mu H)$$

$$(H^\dagger \overleftrightarrow{D}^\mu H) \cdot (D_\mu H^\dagger H)$$

$$(D_\mu H^\dagger H) \cdot (D_\mu H^\dagger H)$$

related by complex conjugation

Step four (requires experience): rewrite the operators in the most convenient form:

$$\mathcal{O}_2 \equiv |H|^2 (D_\mu H^\dagger \overleftrightarrow{D}^\mu H)$$

this one we keep. Next, notice that

$$H^\dagger \overleftrightarrow{D}_\mu H + D_\mu H^\dagger \cdot H = \partial_\mu (|H|^2)$$

$$H^\dagger \overleftrightarrow{D}_\mu H - D_\mu H^\dagger H = H^\dagger \overleftrightarrow{D}_\mu H \equiv \mathcal{J}_{\mu,H}$$

$\mathcal{J}_{\mu,H}$ is the $U(1)_Y$ Higgs current in the SM (check it by the Noether theorem)

The remaining 3 operators can thus be written in terms of the linear combinations:

$$\mathcal{O}_H = \partial_\mu (H^\dagger H) \overleftrightarrow{D}^\mu (H^\dagger H)$$

$$\text{all Hermitian } \mathcal{O}_T = \mathcal{J}_{\mu,H} \overleftrightarrow{D}^{\mu,H} = (H^\dagger \overleftrightarrow{D}_\mu H)^2$$

$$\mathcal{O}' = \mathcal{J}_{\mu,H} \overleftrightarrow{D}^\mu (|H|^2) = -e |H|^2 \overleftrightarrow{D}_\mu \mathcal{J}_{\mu,H}$$

Step five (related with step four): Think hard (6)
to what you have found!

\mathcal{O}_T involves H and gauge fields, when H takes a VEV it contributes to the EW bosons mass:

$$\langle H^\dagger \rangle \overleftrightarrow{D}_\mu \langle H \rangle = \frac{2i}{4} v^2 \frac{g}{C_W} Z_\mu$$

$$\Rightarrow \frac{1}{2} \frac{C_T}{f^2} (-) \frac{g^2 v^2}{C_W^2} \cdot v^2 Z^2$$

$$\Rightarrow M_Z^2 = (M_Z^2)_{SM} + 4C_T \mathcal{O} (M_Z^2)_{SM}$$

$$\Rightarrow \rho - 1 = \frac{M_W^2}{M_Z^2} - 1 \approx -4C_T \mathcal{O} \quad \left(\begin{array}{l} \ll 1 \\ \text{chance it} \\ \text{not zero...} \end{array} \right)$$

From LEP we know:

$$\rho = 1 + \mathcal{O}(1\%) \pm \mathcal{O}(1\%)$$

From SM
Loop

from uncertainties

$$\Rightarrow C_T \mathcal{O} \lesssim 10^{-3}; \text{ strong bound on this operator}$$

However \mathcal{O}_T does not arise in a model with custodial symmetry, indeed it was absent in the $SO(5)$ model.

$\rightarrow SO(6)$

Exercise: Check that \mathcal{O}_T breaks custodial (7)
 while \mathcal{O}_κ and \mathcal{O}_H are custodial invariant.
 To do this, notice that

$$\mathcal{O}_H = \text{Tr} [\sigma^3 \mathcal{L}^\dagger \hat{D}_\mu \mathcal{L}] \xrightarrow{\text{NOT } \text{SO}(2)_R \text{ invariant}}$$

while

$$\mathcal{O}_\kappa \propto \text{Tr} [\mathcal{L}^\dagger \mathcal{L}] \cdot \text{Tr} [\hat{D}_\mu \mathcal{L}^\dagger \hat{D}^\mu \mathcal{L}]$$

$$\mathcal{O}_H \propto (\mathcal{O}_\kappa [\text{Tr} [\mathcal{L}^\dagger \mathcal{L}]])^2$$

Consider now \mathcal{O}_H and \mathcal{O}_κ .

\mathcal{O}_H , in the Unitary gauge, becomes

$$\begin{aligned} \frac{C_H}{f^2} \frac{1}{4} (\mathcal{O}_\kappa (v+h)^2)^2 &= \frac{C_H}{f^2} \frac{1}{4} (2v_\mu h + 2h_\mu h)^2 \\ &= \frac{C_H}{f^2} [v^2 (\partial h)^2 + 2v h (\partial h)^2 + h^2 (\partial h)^2] \end{aligned}$$

main effect is a renormalization of the h kinetic term:

$$(\partial h)^2 \rightarrow (1 + \frac{3}{2} C_H) (\partial h)^2$$

reabsorbed by $h \rightarrow h / \sqrt{1 + \frac{3}{2} C_H} \approx h (1 - \frac{1}{2} \frac{3}{2} C_H)$

All Higgs couplings rescaled $\Rightarrow C_H$ drops
 from the BR's of H and appears in total
 production rate and in total width

\mathcal{O}_Σ happens to have the same effect as \mathcal{O}_H on Higgs + gauge physics. Its effect can be disentangled only by using fermion couplings. (8)

But why so? This kind of surprising facts often come from reparametrization invariance. Consider a field redefinition:

$$H^\alpha \rightarrow H^\alpha + \frac{\gamma}{f^2} (H^\dagger H) H^\alpha$$

The $d=4$ kinetic term gives:

$$\begin{aligned} D_\mu H^\dagger \hat{D}^\mu H &\rightarrow D_\mu H^\dagger \hat{D}^\mu H + \frac{\gamma}{f^2} \left[\partial_\mu (H^\dagger H) H^\dagger + H^\dagger \partial_\mu H \right] \\ &\times \hat{D}^\mu H + \frac{\gamma}{f^2} D_\mu H^\dagger \left[\partial^\mu (H^\dagger H) H^\alpha + H^\dagger H \partial^\mu H^\alpha \right] + \\ &+ \mathcal{O}\left(\frac{1}{f^4}\right) \rightarrow d=8 \text{ operators not included} \\ &\text{in our analysis:} \end{aligned}$$

we get a shift:

$$\begin{aligned} &\frac{\gamma}{f^2} \partial_\mu (H^\dagger H) \hat{D}^\mu (H^\dagger H) + \frac{2\gamma}{f^2} |H|^2 (D_\mu H)^2 = \\ &= \frac{\gamma}{f^2} \mathcal{O}_H + 2 \frac{\gamma}{f^2} \mathcal{O}_\Sigma \end{aligned}$$

By setting $\gamma = -C_\Sigma/2$ we get

$$C'_H = C_H - C_\Sigma/2$$

Equivalent operators, up to $d > 6$, of measuring bosonic couplings only.

However from the shift it also happens that

$$(Y \bar{l} H l \dots) \rightarrow \frac{1}{f^2} |H|^2 (Y \bar{l} H l)$$

New universal $d=6$ fermion-Higgs coupling

$$C_Y = -\frac{C_{\tau}}{2}$$

Therefore C_{τ} also induces modifications of the Higgs coupling to fermions, unrelated with the coupling to vector bosons. C_{τ} thus affects the relative BR among bosons and fermions Higgs decays

• Last operator in this class:

$$\mathcal{O} = -c |H|^2 \partial_{\mu} \mathcal{J}_{\mu H}$$

Has no effect, equivalent to $d=8$. Why?

• short answer:

$$\partial_{\mu} \mathcal{J}_{\mu H} = 0 \rightarrow \text{is a conserved current}$$

• more precisely, $\partial_{\mu} \mathcal{J}_{\mu} = 0$ only on the Equations of Motion, but why the fields should obey the EOM?

- It is customary to use EOM to simplify the EFT operators, let us see why: (10)

$$S^{\text{EFT}} = \int \mathcal{L}^{\text{EFT}} = \int \mathcal{L}_{\text{SM}}^{d=4} + \epsilon \int \mathcal{L}^{d>4} = S_{\text{SM}} + \epsilon S^{d>4}$$

$\epsilon = 1/\Lambda^2$ in this case, in general is a small expansion parameter and we work at the $\mathcal{O}(\epsilon)$ order.

The $\mathcal{O}(\epsilon^0)$ EOM (SM) are defined by:

$$\frac{\delta S_{\text{SM}}}{\delta \Phi_k} = 0 \quad \forall \Phi_k \text{ of our theory}$$

Perform a generic field redefinition:

$$\Phi_k \rightarrow \Phi_k + \epsilon \mathcal{O}_k \quad \begin{matrix} \searrow \\ \text{in previous case,} \\ \mathbb{H}^1 \quad \mathbb{H}^d \end{matrix}$$

$$S_{\text{SM}}[\Phi] \rightarrow S_{\text{SM}}[\Phi] + \epsilon \int \frac{\delta S_{\text{SM}}}{\delta \Phi_k(x)} \mathcal{O}_k(x) + \mathcal{O}(\epsilon^2)$$

$$\epsilon S^{d>4}[\Phi] \rightarrow \epsilon S^{d>4}[\Phi] + \mathcal{O}(\epsilon^2)$$

$$S^{\text{EFT}} \rightarrow S_{\text{SM}} + \epsilon \int dx \left[\mathcal{L}^{d>4} + \frac{\delta S_{\text{SM}}}{\delta \Phi_k(x)} \mathcal{O}_k(x) \right]$$

$\forall \mathcal{O}_k(x)$ local operators.

Therefore, any term proportional to the eom, can be added or subtracted from $\mathcal{L}^{(d>4)}$.

This is why we can use SMEFT to simplify 12
 the operators. Up to $\mathcal{O}(E^2)$ (higher dimensional)
 operators.

Our $SO(5)/SO(4)$ composite Higgs model
 gives definite predictions for the c 's
 From the Lagrangian worked out in the previous
 lecture:

$$\frac{f^2}{2|H|^2} \sin^2 \frac{\sqrt{2}}{f} |H| (D_\mu H)^2 + \frac{f^2}{8|H|^6} (\dots) (D_\mu H)^2 \simeq$$

$$D_\mu H^\dagger D^\mu H - \frac{2}{3f^2} |H|^2 |D_\mu H|^2 + \frac{1}{6f^2} \partial_\mu (H^\dagger H) \partial^\mu (H^\dagger H)$$

Therefore:

$$c_T = 0 \quad (\text{custodial } SO(4))$$

$$c_H = \frac{1}{6} \quad (\text{up to redel. of } f)$$

$$c_\pi = -\frac{2}{3}$$

Or, after the field redefinition:

$$c'_H = \frac{1}{6} + \frac{1}{3} = 2$$

$$c'_\pi = 0$$

$$c'_\gamma = \frac{1}{3}$$

} different convention
 with respect to
 the SILH paper

- Other cosets might obviously give other c 's,
 but with the same power-counting.

- There exist many other classes of operators, and some of them require special care. A simple example:

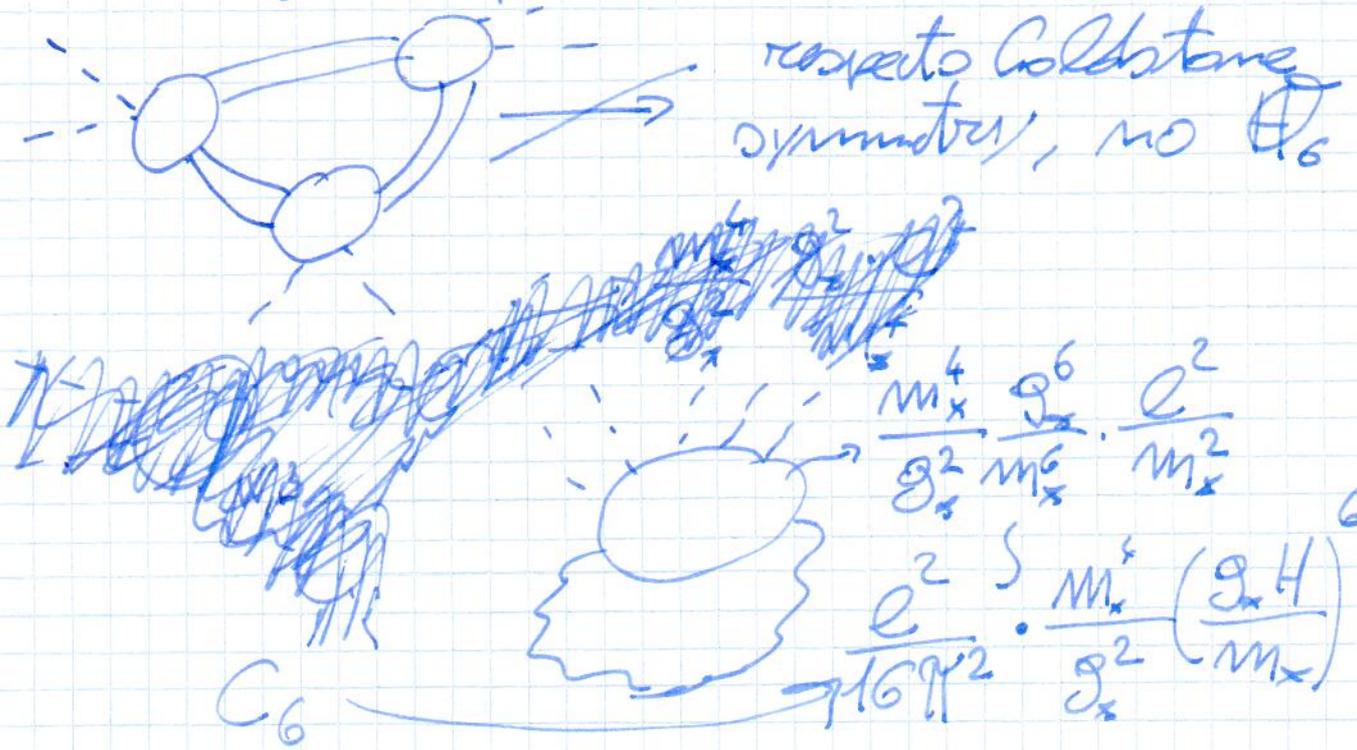
$$\mathcal{O}_6 = |H|^6$$

what is its coefficient?

From the Strong sector alone $\mathcal{O}_6 = 0$, because the Goldstones do not have any potential. Operators without derivatives can not be invariant under the shift symmetry

$$\pi \rightarrow \pi + \alpha + \dots$$

However \mathcal{O}_6 will be generated by loops of the elementary fields, whose couplings break the Goldstone symmetry. For instance, in the language of the previous lecture:



In the SILH power-counting, Goldstone symmetry breaking operators are weighted by

$$\frac{e^2}{16\pi^2} \rightarrow e = g, g' \text{ or } Y_{\text{top}},$$

when fermion couplings will be introduced.

Being a 1-loop correction, one further e^2 coupling is needed to match 1/2 dimensionality.

Class II: the second class of important operators, with unsuppressed coefficients, are those with

$$1 \frac{W_{\mu\nu}^2}{B_{\mu\nu}}, 2 D, 2 H$$

Those are d=6, called " $\mathcal{O}(P^4)$ " because in analogy with QCD literature we count as "P" either D or gauge field.

The classification is left as an exercise. One can start from $SU(2)_L \times U(1)_Y$ irreps:

$$B_{\mu\nu} \quad H^{\alpha} \quad H_{\alpha}$$

$$W_{\mu\nu}^2 \quad H^{\alpha} \quad \sigma_{\alpha}^{\beta} \quad H_{\beta}$$

plus derivatives inserted in all possible ways.

σ_{HW}, σ_{HB} are less constrained. Furthermore they happen not to arise in the weakly coupled models of CH we know today. For instance, in holographic CH.

Why? not known, but it is a very interesting theoretical question.

σ_{HW}, σ_{HB} are called "non-minimally coupled":

- Induce $\delta h \neq 2$ while in minimal coupling δ does not couple to neutrals
- Induce gyromagnetic ratio $g \neq 2$ for the W

In these theories, σ_{HW} and σ_{HB} have coefficient

$$\frac{g_*^2}{16\pi^2} \frac{e}{m_*^2}$$

→ arise at one loop order from the strong sector.

Since we do not understand its origin deeply, we can ignore the suppression

• None of $\mathcal{O}_{WB, HW, HB}$ induce on-shell $h \rightarrow \gamma\gamma$

This is easily checked by inspection

However \mathcal{O}_{WW} does:

$$\mathcal{O}_{WW} \xrightarrow{\text{unitary gauge}} 2v h \frac{g^2}{16\pi^2} \gamma_{\mu\nu} \gamma^{\mu\nu}$$

Therefore \mathcal{O}_{WW} violates the Goldstone symmetry, thus it is suppressed by

$$\frac{g^2}{16\pi^2}, \text{ like } C_6$$

The reason is the following: The photon couples to

$$Q = T_L^3 + T_R^3 \quad \text{via } \gamma_{\mu} J^{\mu Q}$$

current operator \leftarrow
in the strong sector

But h is the neutral Goldstone, π_4 , which means that

$$[Q, \hat{T}^4] = 0$$

\Rightarrow photon coupling does not break the shift symmetry in the direction of the higgs, therefore

$$h \rightarrow h + \alpha^4$$

is still a symmetry, also in the presence 18
of the γ coupling.

$h\gamma\gamma$ will be generated by W and Z couplings
but only at the loop level.

Some considerations for hgg . These
operators, which are very important
as they control a sizable channel of
Higgs production and decay

$gg \rightarrow H \rightarrow \gamma\gamma$
are suppressed in CH (SILH) scenarios!