NIKHEF

Soft gluons: a new level of precision

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Based on arXiv:0811.2067 arXiv:1010.1860 arXiv:1406.7184

Threshold resummation

Soft gluon emissions generate Large Logs that spoil perturbation theory.
Close to the threshold $\xi \longrightarrow 0$:

$$\frac{d\sigma}{d\xi} = \sum_{n=0}^{\infty} \sum_{m=0}^{2n-1} \alpha_s^n \left[a_{nm} \frac{\log^m(\xi)}{\xi} + b_{nm} \log^m(\xi) + \mathcal{O}(\xi) \right]$$

Plus Distributions,

leading with respect to simple Logs (large N in Mellin space),

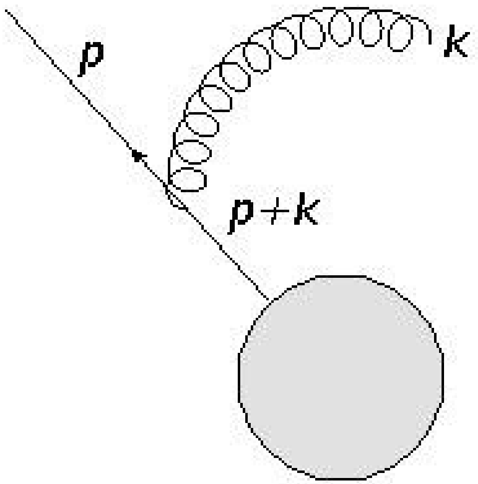
they require "**Eikonal approximation**"

Simple Logs,

subleading in the threashold (order 1/N in Mellin space),

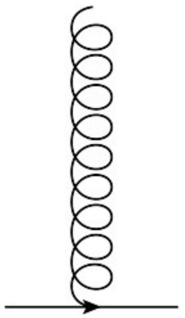
they require " **Next-to-Eikonal approximation**",

their contribution might be very important for some processes (e.g. Higgs production)



Eikonal approximation

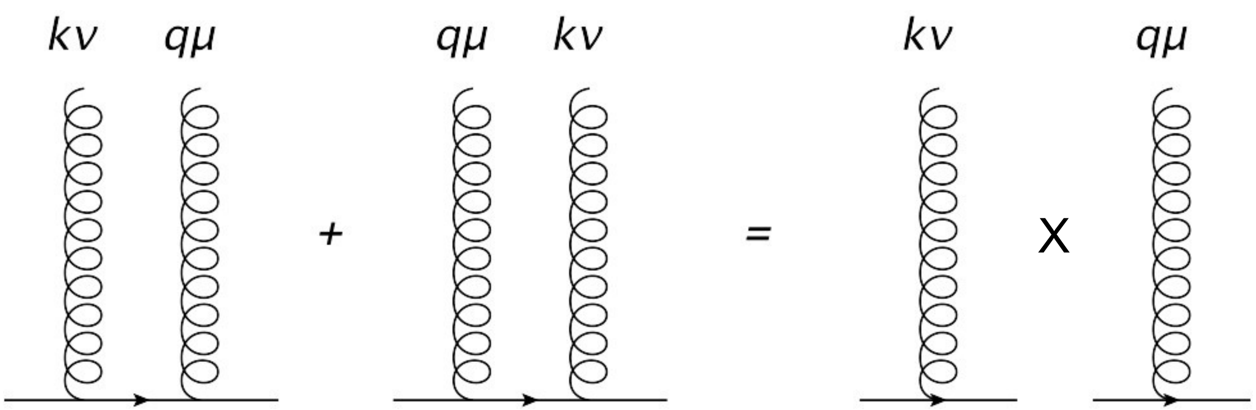
Effective Feynman rule:



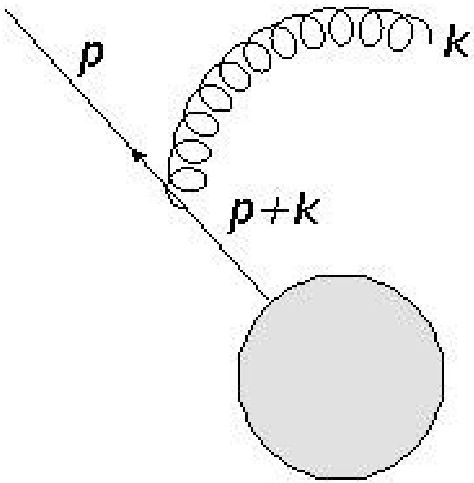
$$= \frac{p^\mu}{p \cdot k}$$

- *No **recoil**: hard legs become Wilson Lines*
- *No **spin** interactions (scalar vertex)*

Eikonal Identity



→ Eikonal emissions are independent → **Eikonal Factorization**

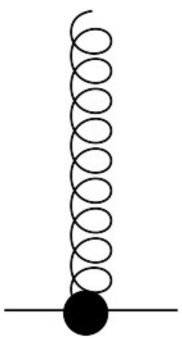


Next-to-eikonal

$$\frac{1}{(p+k)^2} \to \frac{1}{2(p\cdot k)} - \frac{k^2}{4(p\cdot k)^2} + \frac{k^4}{8(p\cdot k)^3} + \ldots$$

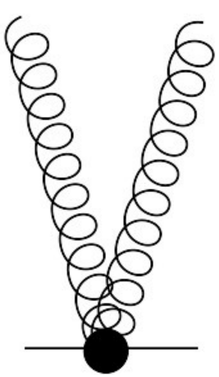
E ***NE*** ***NNE***

Effective Feynman rules:



$$\frac{\gamma^\mu \not{k}}{2p\cdot k} - k^2 \frac{p^\mu}{2(p\cdot k)^2}$$

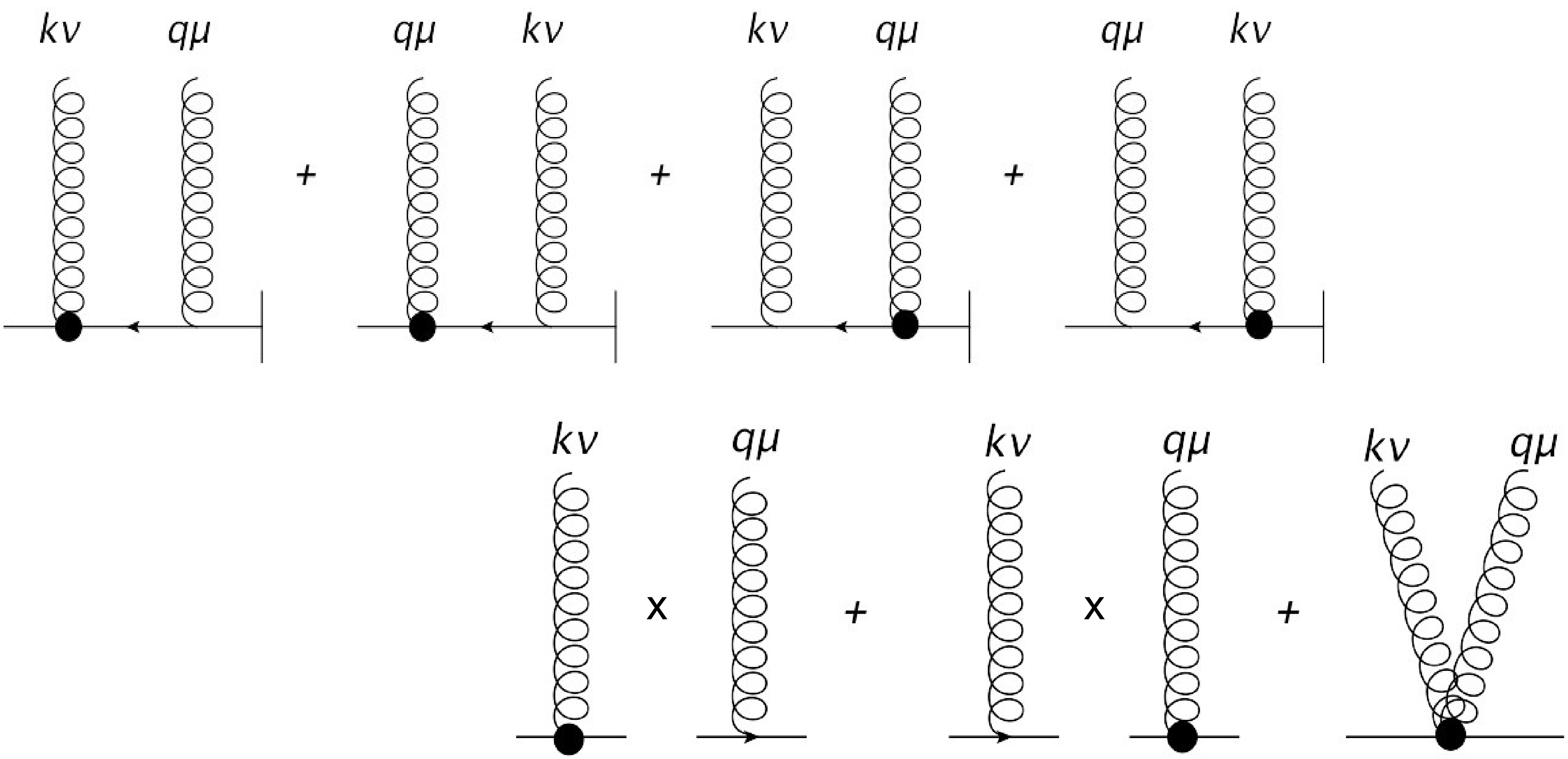
- ***Recoil***: *fluctuations along the semiclassical straight path (arXiv:0811.2067)*
- ***Spin*** *interactions (spinorial vertex)*



$$-\frac{(p\cdot k_2)p^\mu k_1^\nu + (p\cdot k_1)k_2^\mu p^\nu - (p\cdot k_1)(p\cdot k_2)\, g^{\mu\nu} - (k_1\cdot k_2)p^\mu p^\nu}{p\cdot(k_1+k_2)};$$

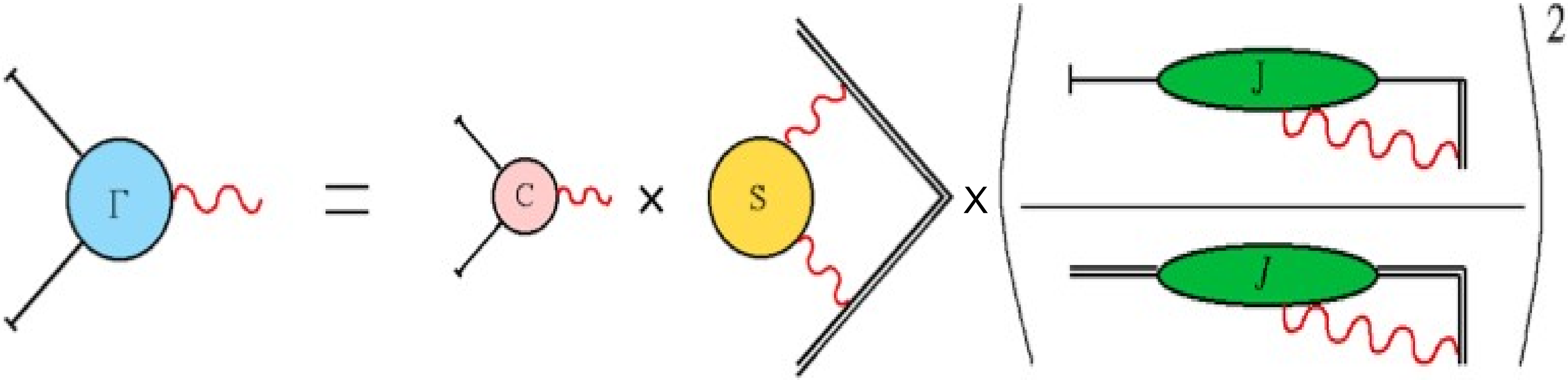
Next-to-eikonal

Next-to-Eikonal Identity



Naïve factorization is broken at Next-to-Eikonal Level.
Is it still possible to factorize (i.e. resum)?

Soft Collinear Factorization *(Collins, 1989)*



Finite

Soft singularities

Collinear singularities

Soft function $\mathcal{S}\left(\beta_1\cdot\beta_2,\alpha_s(\mu^2),\epsilon\right)=\langle 0|\Phi_{\beta_2}(\infty,0)\,\Phi_{\beta_1}(0,-\infty)\,|0\rangle$

Jet function $J\left(\frac{(p\cdot n)^2}{n^2\mu^2},\alpha_s(\mu^2),\epsilon\right)u(p)=\langle 0\,|\Phi_n(\infty,0)\,\psi(0)\,|p\rangle$

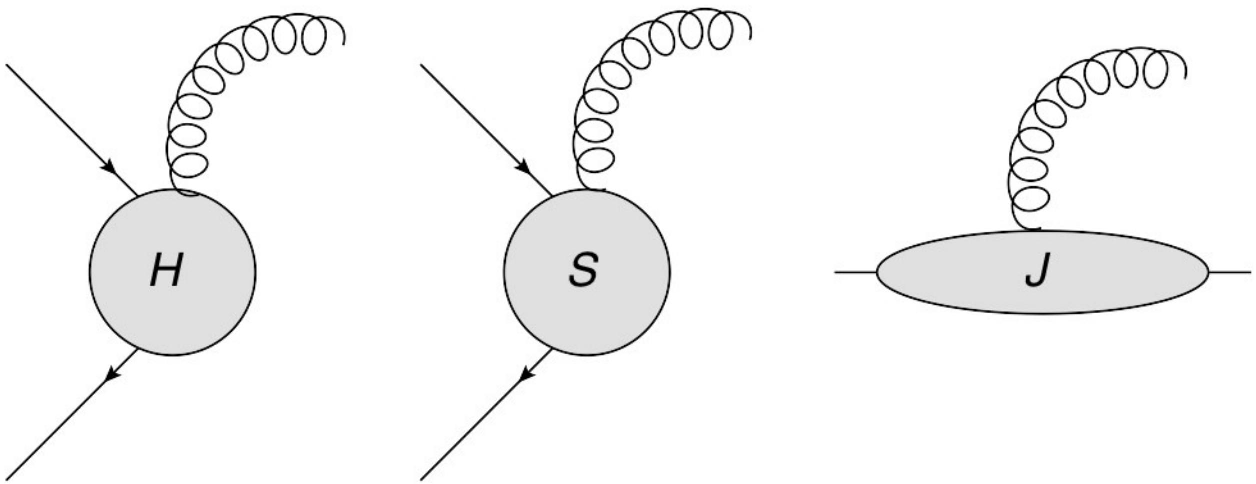
Eikonal Jet function $\mathcal{J}\left(\frac{(\beta_1\cdot n_1)^2}{n_1^2},\alpha_s(\mu^2),\epsilon\right)=\langle 0|\Phi_{n_1}(\infty,0)\,\Phi_{\beta_1}(0,-\infty)\,|0\rangle$

See arXiv:0805.3515 for more details

Low-Burnett-Kroll theorem + Del Duca modifications

(Low 1958, Burnett-Kroll 1968, Del Duca 1990: Nuclear Physics B345 369—388)

Express the radiative amplitude in terms of the non radiative one (Ward Identity)



It is possible to rearrange the emissions in 4 contributions:

- E,NE,.. emission x **Full Form Factor** (Factorizable contribution)
- Derivative of the **Full Form Factor** (NON Factorizable contribution)
- Derivative of the **Jet Function** (NON Factorizable contribution)
- Emission from inside the **Jet function** (NON Factorizable contribution)

Log structure of NNLO Drell-Yan

Exact result in the soft limit

$$\frac{256-256\mathcal{D}_0}{\epsilon^3}+\frac{192\mathcal{D}_0-256\mathcal{D}_1+256L_1-256}{\epsilon^2}+$$
$$\frac{128L_2-288L_1-256\mathcal{D}_0+192\mathcal{D}_1-128\mathcal{D}_2+368}{\epsilon}+$$
$$-\frac{128}{3}\mathcal{D}_3+96\mathcal{D}_2-256\mathcal{D}_1+256\mathcal{D}_0+\frac{128}{3}L_3-168L_2+368L_1-376$$

We want to reproduce this result with 2 different approaches:

- **expansion by regions**:

divide space of loop momenta into soft, hard, collinear regions;

- it works, very powerful, but no resummation;

- **factorization approach**:

use the effective Feynman rules and Low theorem;

- factorization implies resummation (work in progress).