Determination of the bottom quark mass from sum rules

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based on work in collaboration with
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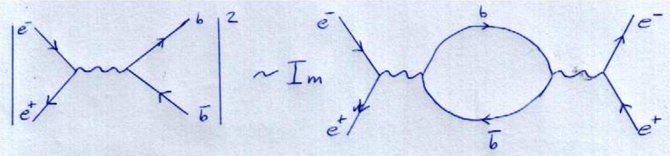
Consider $e^+e^- \to$ hadrons containing $b\bar{b}$ and apply the optical theorem:

$$R(s) \equiv \frac{\sigma(e^+e^- \to b\bar{b})}{\sigma(e^+e^- \to \mu^+\mu^-)} \sim \text{Im}\,\Pi(s)$$



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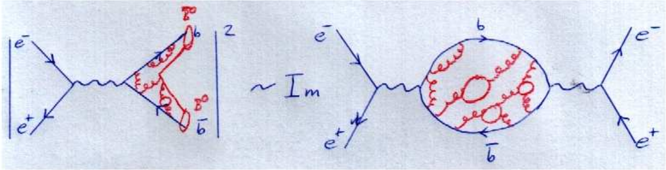

Compute with partons $\leftrightarrow$ Observe hadrons

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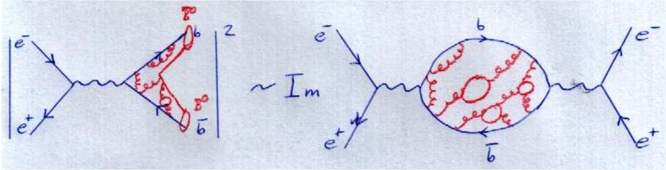
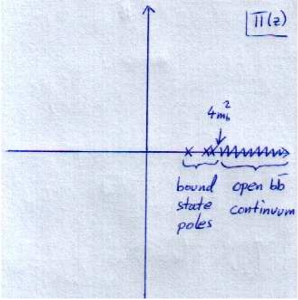


Compute with partons $\leftrightarrow$ Observe hadrons

How can we achieve a comparison including nonperturbative effects?

Generalize $\Pi(s)$ to complex momentum squared:



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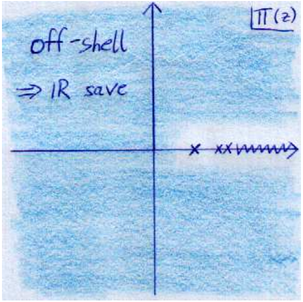

At large virtuality (blue): OPE, expansion in $\Lambda_{QCD}^2/\text{virtuality}$
$\Pi(z)$ can be computed including nonperturbative effects

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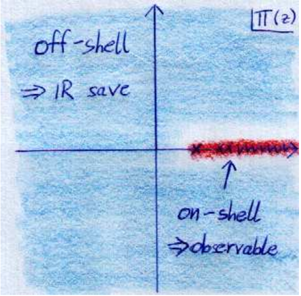
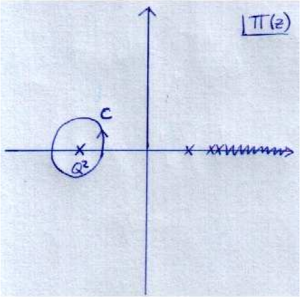


At large virtuality (blue): OPE, expansion in $\Lambda^2_{\mathrm{QCD}}/\text{virtuality}$
$\Pi(z)$ can be computed including nonperturbative effects

Observation only at physical on-shell z (red)

Make use of Cauchy's theorem:



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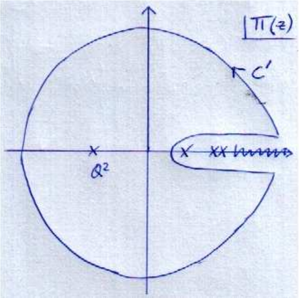

$$\Pi(Q^2) = \frac{1}{2\pi i} \oint_{\mathcal{C}'} dz \, \frac{\Pi(z)}{z - Q^2}$$

Deform contour and use $\Pi(s + i\epsilon) - \Pi(s - i\epsilon) = 2i \, \text{Im} \, \Pi(s)$:

$$\Pi(Q^2) = \frac{1}{\pi} \int_{s_0}^{\infty} ds \frac{\text{Im} \, \Pi(s)}{s - Q^2} + \frac{1}{2\pi i} \oint_{\bigcirc} dz \, \frac{\Pi(z)}{z - Q^2}$$

The integral over the circle at infinity is divergent. It can be removed by a subtraction or by taking a derivative in Q^2.

Use this to define so called moments as derivatives at $Q^2 = 0$:

$$\mathcal{M}_n \equiv \frac{12\pi^2}{n!} \left(\frac{d}{dq^2}\right)^n \Pi(q^2)\Big|_{q^2=0} = \int_{s_0}^{\infty} ds \frac{R(s)}{s^{n+1}}$$

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The left-hand side can be computed while the right-hand side can be determined from experiment.

Comparison yields a prediction for the bottom quark mass.

Our current project: Do this for large n (≈ 10) at NNNLO.
Dominated by threshold: Good experimental data, but need to resum Coulomb singularities $(\alpha_s/v)^n$.