Optimization of Luminosity of a 100 km Circumference e+/e- Collider as a Function of Lattice Parameters, Especially β_y at Collision Point

Richard Talman

Laboratory of Elementary-Particle Physics
Cornell University

FCC Workshop,
Lepton Collider Breakout Session,
University of Geneva, 14 February, 2014

Outline

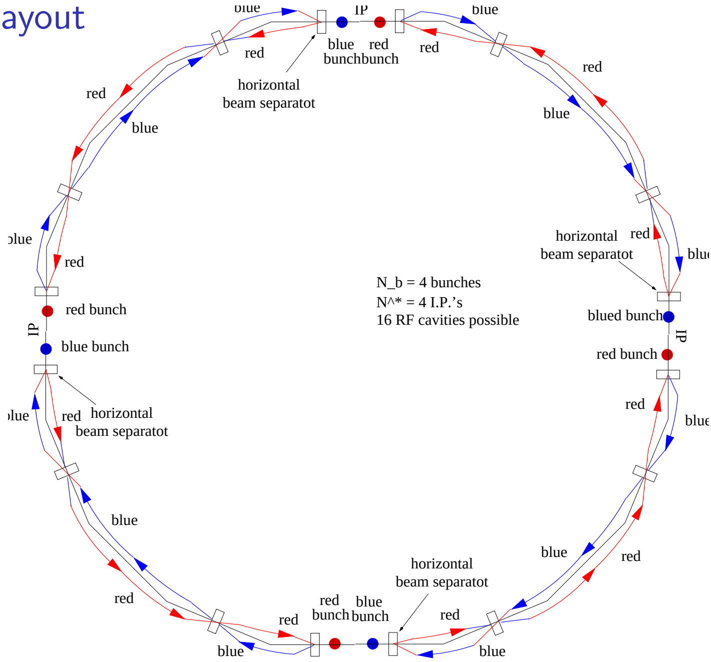
Ring Layout

Beam Height Equilibrium: Beam-Beam Heating vs. Radiation Cooling

Unique Reconciliation of Luminosity and Beamstrahlung

Optimized Performance vs β_y at Collision Point

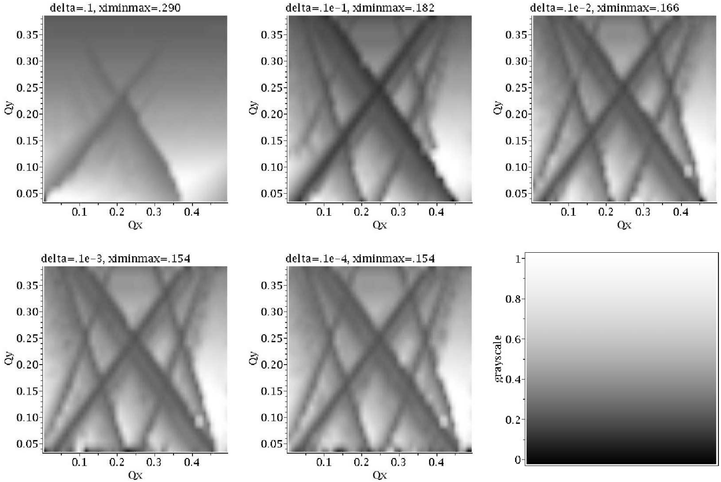
Ring Layout



IP
blue
red
blue bunch
red bunch
horizontal beam separatot
red
blue
blue
red
red
blue
horizontal beam separatot
red
blued bunch
IP
red bunch
red
blue
N_b = 4 bunches
N^* = 4 I.P.'s
16 RF cavities possible
red
blue
red bunch
IP
blue bunch
red
horizontal beam separatot
blue
red
blue
red
red
red bunch
blue bunch
horizontal beam separatot
red
blue
blue
blue
red

- The design orbit spirals in significantly; this requires the RF acceleration to be distributed quite uniformly.
- Basically the ring is a "curved linac".
- Horizontal electric fields separate the beams in the arcs; the electrodes are separated enough to be masked from synchrotron radiation.
- Beams cross over at RF locations.
- "Topping-off" injection is essential; especially to permit large tune shifts summed over multiple I.P.s. To avoid a nearby resonance the *change in coherent tune over the time between fills* has to be small.

Saturated Tune Shift $\xi^{\text{sat.}}$ in (Q_x, Q_y) Plane, for 5 Orders of Magnitude Range of Damping Decrement δ



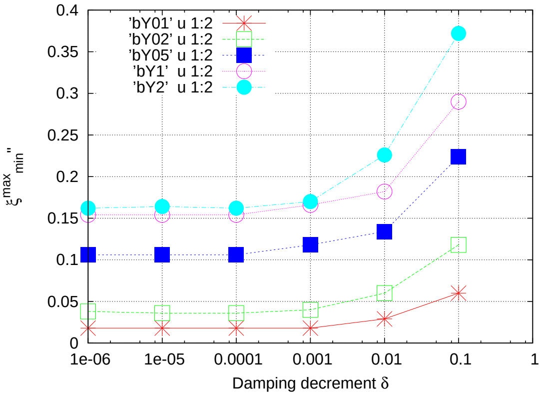


Figure: Plot of saturation tune shift, $\xi^{\text{sat.}}$ versus damping decrement δ, for $\beta_y = 1{,}2{,}5{,}10$, and 20 mm. In all cases $\sigma_z = 0.01$ m, $Q_s = 0.03$.

- **Note:** As well as depending on damping decrement δ, the saturation tune shift depends strongly on other parameters, especially vertical beta function β_y and bunch length σ_z.

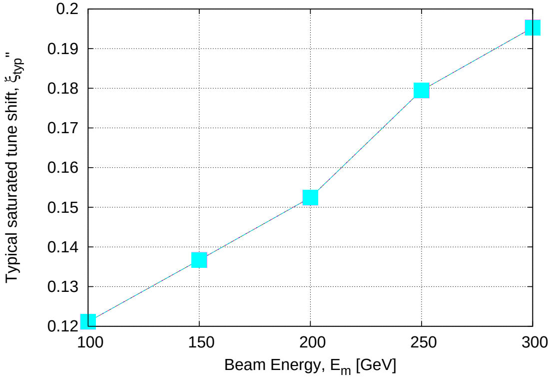


Figure: Plot of “typical” saturated tune shift $\xi_{\rm typ}$ as a function of maximum beam energy E_m for ring radius R scaling as $E_m^{1.25}$.
$\beta_y = \sigma_z = 5\,$mm.

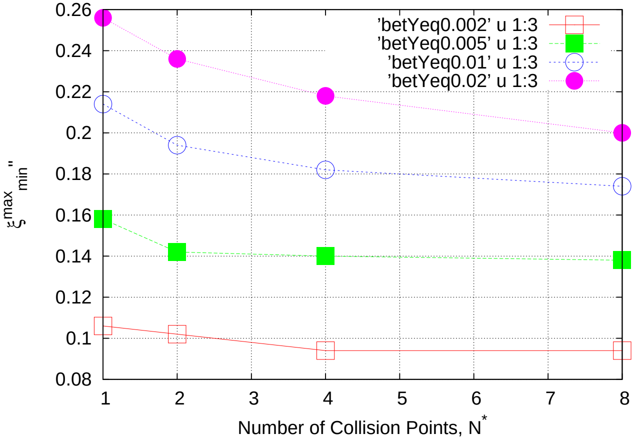


Figure: Plot of saturation tune shift value $\xi^{\mathrm{sat.}}$ versus number of collision points N^*, for $\beta_y = 2,5,10$, and 20 mm.

Beam Height Equilibrium: Beam-Beam Heating vs. Betatron Cooling

- Under ideal single beam conditions beam height $\sigma_y \approx 0$. This would give infinite luminosity which is unphysical.
- In fact beam-beam forces cause the beam height to grow into a new equilibrium with normal radiation damping.
- The parametric modulation provides a force with resonance driving strength proportional to $1/\sigma_y =$ infinite ?
- Nature "abhors" both zero and infinity and plays off beam-beam emittance growth against radiation dampling.
- Amplitude dependent detuning limits the growth, so there is no particle loss.
- The simulation accounts for whatever resonances are nearby.

- For Higgs factory design, scan the tune plane, for various vertical beta function values (as well as other, less influential, parameters.)
- Read the ratio $\xi^{\mathrm{sat.}}/\beta_y$ from the figure.

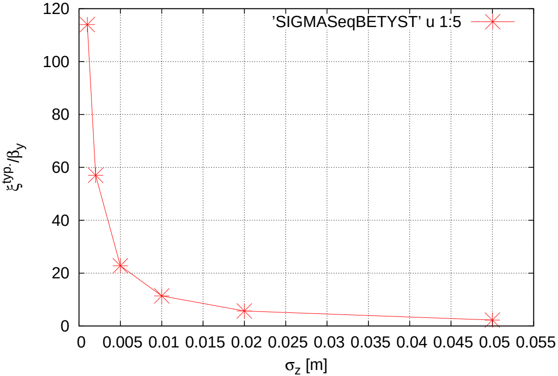


Figure: Plot of $\xi^{\mathrm{typ.}}/\beta_y$ as a function of σ_z, with $\beta_y = \sigma_z$, $\delta = 0.00764$, and synchrotron tune advance between collisions $Q_s = 0.0075$.

- The ratio $\xi^{\text{typ.}}/\beta_y$ determines the beam area just sufficient for saturation A_{β_y} according to the formula,

$$A_{\beta_y} = \pi\sigma_x\sigma_y = \frac{N_p r_e}{2\gamma}\frac{1}{(\xi^{\text{sat.}}/\beta_y)}. \tag{1}$$

- It is only the product $\sigma_x\sigma_y$ that is fixed but the aspect ratio $a_{xy} = \sigma_x/\sigma_y \approx 15$ is good enough. To within this ambiguity all transverse betatron parameters are then fixed.
- The number of electrons per bunch N_p itself is fixed by the available RF power and the number of bunches N_b. For increasing the luminosity N_b wants to be **reduced**.
- To keep beamstrahlung acceptably small N_b has to be **increased**.
- The maximum achievable luminosity is determined by this compromise between beamstrahlung and available power.

$\underline{R}$: bend radius

$\mathcal{C}$: circumference $= 3\pi R$ is good enough

$\underline{N^*}$: number of I.P.′s

$\underline{N_p}$: particles per bunch, $N_{\rm tot.} = N_b N_p$, fixes RF power, $P_{\rm rf}$

β_x : horizontal beta function in arc, fixed by arc design

ϵ_x : horizontal emittance, fixed by arc design

δ : betatron damping decrement, known from R and E

$\underline{\beta_y^*}$: vertical beta function at I.P.

$\underline{\sigma_y^*}$: r.m.s. bunch height at I.P. is to be calculated

ϵ_y : vertical emittance $= {\sigma_y^*}^2/\beta_y^*$ is then known

σ_x^* : r.m.s. bunch width at I.P. $\equiv a_{xy}\sigma_y^* = 15\sigma_y^*$ is good enough

β_x^* : horz. beta function at I.P. $= {\sigma_x^*}^2/\epsilon_x$

σ_z : r.m.s. bunch length $\equiv \beta_y^*/r_{yz} = \beta_y^*/0.6$ is good enough

Q_x, Q_y : transverse tunes (unimportant in simulation)

$\underline{Q_s}$: synchrotron tune (important in simulation)

Reconciling Luminosity and Beamstrahlung

- $\mathcal{L}^{\rm RF}_{\rm pow}$ is the RF power limited luminosity
- $\mathcal{L}^{\rm bb}_{\rm sat}$ is the beam-beam saturated luminosity
- $\mathcal{L}^{\rm bs}_{\rm trans}$ is the beamstrahlung transverse-limited luminosity

$$\mathcal{L}^{\rm RF}_{\rm pow} = \frac{N^*}{N_b}\, H(r_{yz})\, \frac{1}{a_{xy}}\, \frac{f}{4\pi}\, \Big(\frac{n_1 P_{\rm rf}[\rm MW]}{\sigma_y}\Big)^2,$$
$$N_{\rm tot} = n_1 P_{\rm rf}[\rm MW]$$

- Single beam dynamics gives $\sigma_y = 0$, $\Longrightarrow \mathcal{L}^{\rm RF}_{\rm pow} = \infty$? Nonsense. Resonance drive force $\propto 1/\sigma_y \Longrightarrow \infty$, also.
- Beam-beam force expands $\sigma_y = 0$ as necessary. **Saturation is automatic** (unless the single beam emittance is already too great for the beam-beam force to take control).

$$\mathcal{L}^{\mathrm{RF}}_{\mathrm{pow}} = \frac{N^*}{N_b}\, H(r_{yz})\, \frac{1}{a_{xy}}\, \frac{f}{4\pi} \left(\frac{n_1 P_{\mathrm{rf}}[\mathrm{MW}]}{\sigma_y}\right)^2,$$

$$\mathcal{L}^{\mathrm{bb}}_{\mathrm{sat}} = N^* N_{\mathrm{tot.}}\, H(r_{yz})\, f\, \frac{\gamma}{2 r_e}\, (\xi^{\mathrm{sat.}}/\beta_y),$$

$$\mathcal{L}^{\mathrm{bs}}_{\mathrm{trans}} = N^* N_b H(r_{yz})\, a_{xy} \sigma_z^2\, f \left(\frac{\sqrt{\pi}\ 1.96 \times 10^5}{28.0\,\mathrm{m}\ \sqrt{2/\pi}}\right)^2 \frac{1}{r_e^2 \widetilde{E}^2} \left(\frac{91\eta}{\ln\left(\frac{1/\tau_{\mathrm{bs}}}{f\ n^*_{\gamma,1}\ \mathcal{R}^{\mathrm{Gauss}}_{\mathrm{unif.}}}\right)}\right.$$

- If $\mathcal{L}^{\mathrm{bs}}_{\mathrm{trans}} < \mathcal{L}^{\mathrm{bb}}_{\mathrm{sat}}$ we must increase N_b ! $\mathcal{L}^{\mathrm{bs}}_{\mathrm{trans}} \propto N_b$, $\mathcal{L}^{\mathrm{RF}}_{\mathrm{pow}} \propto 1/N_b$,

$$N_b = \frac{\mathcal{L}^{\mathrm{bb}}_{\mathrm{sat}}}{\mathcal{L}^{\mathrm{bs}}_{\mathrm{trans}}} \text{ is good enough.}$$

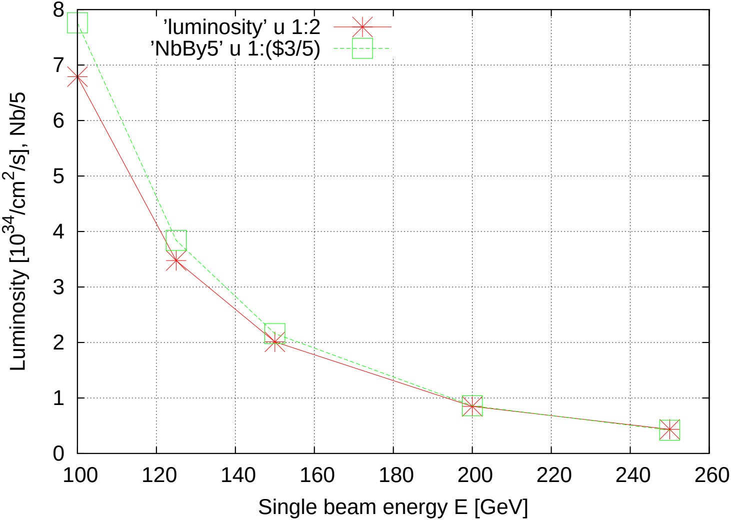


Figure: Dependence of luminosity on single beam energy. (scaled) number of bunches $N_B/5$ is also shown.

$P_{\mathrm{RF}} = 100\,\mathrm{MW}$

E GeV	$\mathcal{C}$ km	R km	f KHz	U_1 GeV	eV_{excess} GeV	n_1 elec./MW	$U_1/(D/2)$ MV/m	$\delta = \alpha_4$	u_c GeV	ϵ_x nm	σ_x^{arc} mm
100	100	10.6	3.00	0.8	64	2.50e+12	0.050	0.0010	0.00021	1.797	0.278
125	100	10.6	3.00	2.0	63	1.02e+12	0.122	0.0020	0.00041	2.807	0.347
150	100	10.6	3.00	4.2	61	4.93e+11	0.253	0.0035	0.00071	4.043	0.417
200	100	10.6	3.00	13.3	52	1.56e+11	0.801	0.0083	0.00167	7.187	0.556
250	100	10.6	3.00	32.6	32	6.39e+10	1.955	0.0163	0.00326	11.229	0.695

Nst= 4 , BETYST= 0.002 m , XITYPbyBY= 30.000 , taubs=600.000 s , RGauUnif= 0.300 , Prf=100.000 MW , eVrf= 65.000 GeV , OVreq= 20.000 GV , axy= 15.000 , ryz= 0.600 m , bxarcmax= 43.000 m ,

Nst= 4 , BETYST= 0.003 m , XITYPbyBY= 22.800 , taubs=600.000 s , RGauUnif= 0.300 , Prf=100.000 MW , eVrf= 65.000 GeV , OVreq= 20.000 GV , axy= 15.000 , ryz= 0.600 m , bxarcmax= 43.000 m ,

Nst= 4 , BETYST= 0.005 m , XITYPbyBY= 17.800 , taubs=600.000 s , RGauUnif= 0.300 , Prf=100.000 MW , eVrf= 65.000 GeV , OVreq= 20.000 GV , axy= 15.000 , ryz= 0.600 m , bxarcmax= 43.000 m ,

Nst= 4 , BETYST= 0.007 m , XITYPbyBY= 15.000 , taubs=600.000 s , RGauUnif= 0.300 , Prf=100.000 MW , eVrf= 65.000 GeV , OVreq= 20.000 GV , axy= 15.000 , ryz= 0.600 m , bxarcmax= 43.000 m ,

Nst= 4 , BETYST= 0.010 m , XITYPbyBY= 13.000 , taubs=600.000 s , RGauUnif= 0.300 , Prf=100.000 MW , eVrf= 65.000 GeV , OVreq= 20.000 GV , axy= 15.000 , ryz= 0.600 m , bxarcmax= 43.000 m ,

$P_{\mathrm{RF}} = 100\,\mathrm{MW}$

E GeV	R km	β_y^* m	ϵ_y m	ξ_{sat}	N_{tot}	σ_y μm	σ_x μm	u_c^* GeV	$n_{\gamma,1}^*$	$\mathcal{L}^{\mathrm{RF}}$ 10^{34}	$\mathcal{L}_{\mathrm{trans}}^{\mathrm{bs}}$ 10^{34}	$\mathcal{L}^{\mathrm{bb}}$ 10^{34}	N_b	β_x^* m
100	10.6	0.002	4.11e-10	0.060	2.5e+14	0.907	13.61	0.006	23.06	24.3	36.4	24.285	1489.9	0.1
125	10.6	0.002	2.72e-10	0.060	1.0e+14	0.738	11.07	0.010	23.45	12.4	18	12.434	737.5	0.044
150	10.6	0.002	1.95e-10	0.060	4.9e+13	0.624	9.36	0.014	23.78	7.2	10.1	7.195	415.0	0.022
200	10.6	0.002	1.14e-10	0.060	1.6e+13	0.478	7.18	0.026	24.32	3.04	4.06	3.036	167.4	0.0072
250	10.6	0.002	7.59e-11	0.060	6.4e+12	0.39	5.84	0.041	24.76	1.55	2	1.554	82.7	0.003
100	10.6	0.003	1.11e-09	0.068	2.5e+14	1.83	27.42	0.008	26.84	18.5	26	18.456	482.8	0.42
125	10.6	0.003	7.38e-10	0.068	1.0e+14	1.49	22.32	0.013	27.31	9.45	12.8	9.450	238.8	0.18
150	10.6	0.003	5.27e-10	0.068	4.9e+13	1.26	18.87	0.019	27.70	5.47	7.21	5.469	134.3	0.088
200	10.6	0.003	3.11e-10	0.068	1.6e+13	0.965	14.48	0.035	28.35	2.31	2.9	2.307	54.1	0.029
250	10.6	0.003	2.06e-10	0.068	6.4e+12	0.786	11.80	0.055	28.87	1.18	1.43	1.181	26.7	0.012
100	10.6	0.005	3.16e-09	0.089	2.5e+14	3.98	59.67	0.011	35.60	14.4	18.9	14.409	130.6	2
125	10.6	0.005	2.10e-09	0.089	1.0e+14	3.24	48.58	0.017	36.23	7.38	9.32	7.377	64.5	0.84
150	10.6	0.005	1.50e-09	0.089	4.9e+13	2.74	41.08	0.025	36.76	4.27	5.23	4.269	36.3	0.42
200	10.6	0.005	8.84e-10	0.089	1.6e+13	2.1	31.54	0.045	37.64	1.8	2.1	1.801	14.6	0.14
250	10.6	0.005	5.87e-10	0.089	6.4e+12	1.71	25.71	0.072	38.34	0.922	1.03	0.922	7.2	0.059
100	10.6	0.007	6.41e-09	0.105	2.5e+14	6.7	100.47	0.013	42.57	12.1	15.2	12.142	54.7	5.6
125	10.6	0.007	4.25e-09	0.105	1.0e+14	5.46	81.83	0.020	43.33	6.22	7.48	6.217	27.0	2.4
150	10.6	0.007	3.04e-09	0.105	4.9e+13	4.61	69.20	0.030	43.98	3.6	4.2	3.598	15.2	1.2
200	10.6	0.007	1.79e-09	0.105	1.6e+13	3.54	53.15	0.054	45.04	1.52	1.68	1.518	6.1	0.39
250	10.6	0.007	1.19e-09	0.105	6.4e+12	2.89	43.33	0.086	45.90	0.777	0.828	0.777	3.0	0.17
100	10.6	0.010	1.25e-08	0.130	2.5e+14	11.2	167.52	0.015	53.31	10.5	12.5	10.523	22.7	16
125	10.6	0.010	8.28e-09	0.130	1.0e+14	9.1	136.45	0.024	54.28	5.39	6.17	5.388	11.2	6.6
150	10.6	0.010	5.92e-09	0.130	4.9e+13	7.69	115.42	0.035	55.10	3.12	3.46	3.118	6.3	3.3
200	10.6	0.010	3.50e-09	0.130	1.6e+13	5.91	88.68	0.063	56.44	1.32	1.39	1.315	2.5	1.1
250	10.6	0.010	1.45e-09	0.130	6.4e+12	3.81	57.09	0.080	45.42	0.673	1.12	0.673	2.0	0.29

$P_{\mathrm{RF}} = 100\,\mathrm{MW}$

E GeV	β_y^* m	ξ_{sat}	$\mathcal{L}^{\mathrm{bb}}$ 10^{34}	N_b
100	0.002	0.06	24.285	1489.9
125	**0.002**	**0.06**	**12.434**	**7.4e+02**
150	0.002	0.06	7.195	4.2e+02
200	0.002	0.06	3.036	1.7e+02
250	0.002	0.06	1.554	83
100	0.003	0.068	18.456	482.8
125	**0.003**	**0.068**	**9.450**	**2.4e+02**
150	0.003	0.068	5.469	1.3e+02
200	0.003	0.068	2.307	54
250	0.003	0.068	1.181	27
100	0.005	0.089	14.409	130.6
125	**0.005**	**0.089**	**7.377**	**65**
150	0.005	0.089	4.269	36
200	0.005	0.089	1.801	15
250	0.005	0.089	0.922	7.2
100	0.007	0.1	12.142	54.7
125	**0.007**	**0.1**	**6.217**	**27**
150	0.007	0.1	3.598	15
200	0.007	0.1	1.518	6.1
250	0.007	0.1	0.777	3
100	0.010	0.13	10.523	22.7
125	**0.010**	**0.13**	**5.388**	**11**
150	0.010	0.13	3.118	6.3
200	0.010	0.13	1.315	2.5
250	0.010	0.13	0.673	2

$P_{\mathrm{RF}} = 100\,\mathrm{MW}$ at Maximum Higgs Production Energy

E GeV	β_y^* m	ξ_{sat}	$\mathcal{L}^{\mathrm{bb}}$ 10^{34}	N_b
125	0.002	0.06	12.434	7.4e+02
125	0.003	0.068	9.450	2.4e+02
125	0.005	0.089	7.377	65
125	0.007	0.1	6.217	27
125	0.010	0.13	5.388	11