Effect of reduced focus coil current on MICE steps IV and VI

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Abstract

During the commissioning of MICE focus coil 1, it was found that the focus coil failed to reach the specified currents. In particular, the focus coil failed to reach the current required for nominal running at 200 MeV/c, considered to be the MICE baseline. In this note, the consequences of operating the focus coil with reduced current are studied.

1 Emittance Reduction in MICE

In MICE, muons are passed through an absorbing material in order to demonstrate the reduction of muon beam emittance through ionisation cooling. In ionisation cooling, a beam of particles has momentum reduced in an ionising medium, which reduces the normalised beam emittance; and energy is replaced by RF cavities. The effect is ruined by multiple scattering, which puts additional transverse momentum in the beam. In order to reduce the effect of multiple scattering, the beam is focussed onto the absorber. At a focus, the spread in transverse momentum is larger, so the relative effect of any scattering is smaller.

The equation for transverse emittance change in an absorber is given by [1]

$$\frac{d\epsilon_n}{dz} = \frac{-1}{\beta_{rel}^2 E}\left\langle\frac{dE}{dz}\right\rangle\epsilon_n + \frac{1}{2m}\frac{13.6^2}{L_R}\frac{\beta_\perp}{\beta_{rel}^3 E}. \tag{1}$$

For low emittance beams, the left hand, cooling term, is relatively small and the right hand, heating term, is relatively larger. The beam has a sufficiently small emittance that even for tight focussing, the beam has a small momentum spread and multiple scattering has a relatively large effect. There exists an equilibrium emittance where the heating and cooling effects are equal, given by

$$\epsilon_n(eqm) = \frac{1}{2m}\frac{13.6^2}{L_R}\frac{\beta_\perp}{\beta_{rel} < dE/dz >} \tag{2}$$

MICE will measure the emittance reduction and equilibrium emittance for a number of different magnet, beam and absorber configurations.

Coil Name	Centre z [mm]	Inner radius [mm]	Radial thickness [mm]	Length [mm]	Current Density [A/mm^2]
End 2	3201	258	68.2	110	135.18 fixed
Centre	2451	258	22	1294	152.44 fixed
End 1	1701	258	60.9	110.6	127.37 fixed
Match 2	1301	258	30.9	199.5	148.09 maximum
Match 1	861	258	46.2	201.3	145.94 maximum
Focus	205	263	84	210	113.95 nominal

Tab. 1: Coil pack used for simulation of MICE Step IV. The field has odd symmetry about z=0; upstream magnets have opposite polarity. Field values and parameters for the Spectrometer Solenoid were sourced from [2].

2 Focussing from the Focus Coils

Focussing at the MICE absorbers is primarily provided by three focus coil modules. These three coil pairs are designed to operate either in non-flip mode, where the two coils operate with the same polarity, or in flip mode, where the two coils operate with opposite polarity.

When the coils operate with the same polarity, stronger focussing fields can be achieved with lower currents. However, the beam holds kinetic angular momentum as it passes through the absorber. The absorber tends to reduce the amount of kinetic angular momentum held by the beam, and over a number of cells this effect builds up resulting in a beam that is mismatched to the lattice. For this reason, most cooling lattices are designed to have a lattice that flips field polarity every half cell.

MICE will measure the effect of this angular momentum build up, but the baseline lattice is one that flips the polarity every half cell. This is the lattice that places the greatest demands on the focus coil modules. Unfortunately, recent measurements of the first focus coil module indicate that the coil will be unable to reach its design current before quenching [3]. The focus coil module has surpassed the requirements for operation in non-flip mode by some margin, but has not reached the required current for stable operation at 200 MeV/c in flip mode. In this note, it is assumed that the focus coil can operate reliably at a level 10 % below the nominal 200 MeV/c value. In the original specification, the coils were required to operate reliably at a level 20 % above the nominal 200 MeV/c value, for operation at 240 MeV/c.

3 Step IV

The effect on Step IV beam optics is investigated below. The coil geometry used is listed in Table 1.

Momentum	Focus coil current [A/mm^2]		
MeV/c	102.56	113.95	136.74
140	104-332, 700-925	75-252, 456-979	32-92, 269-849
160	146-365	116-298	64-168, 469-601
180	186-407	155-342	100-229
200	224-450	192-385	136-278
220	257-493	226-425	169-319
240	356-535	306-463	215-356

Tab. 2: Range of matched β functions [mm] that can be achieved for different momenta and focus coil currents. Note that for low momentum settings there are two distinct β ranges that can be accessed.

3.1 Calculation of Matched Coil Currents

In this note matching parameters were deduced by means of linear beam optics, as calculated by MAUS 0.7.3. Transfer matrices were calculated by numerically calculating the first derivative of phase space parameters, corrected for errors due to the second derivative.

Beam envelopes were projected through the transfer maps, assuming a constant optical β in the constant field region where the tracker sits. A coil current was selected for coil match 2 and coil match 1 was scanned in 20 steps between 0 and the nominal maximum current in order to find solutions with optical α nearly zero at the focus ($z = 0$ in coordinate system used in this note). The final match was achieved by using Minuit to find a solution with absolute value of $\alpha < 1e - 4$.

A sample matched β function is shown in Figure 3.1 along with the on-axis magnetic field that was used to generate it. As required, the β function is symmetric about $z = 0$ while the field has odd symmetry.

3.2 Available Range of Matched Beams

A further scan was performed over match 2 to determine the range of optics solutions that could be achieved for a given Focus coil current. The resultant β at the absorber is shown as a function of momentum in Figure 3.2. The range of matched β available for each focus coil current is listed in table 3.2.

It should be noted that at 140 MeV/c, for some values of match 2, there were two values of match 1 that created a matched solution. It is thought that at this low momentum there are solutions with a phase advance $\phi < \pi$ and $\pi < \phi < 2\pi$ between the solenoid and the focus. This leads to two families of solutions.

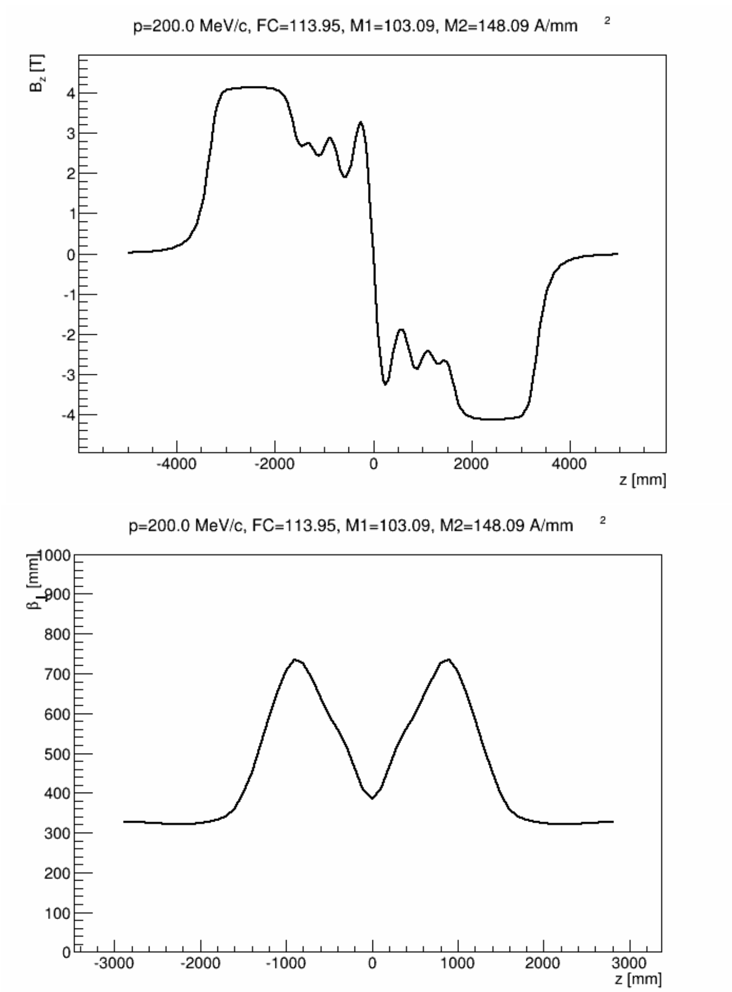

Fig. 1: (Top) Example magnetic field on axis and (bottom) corresponding matched β function

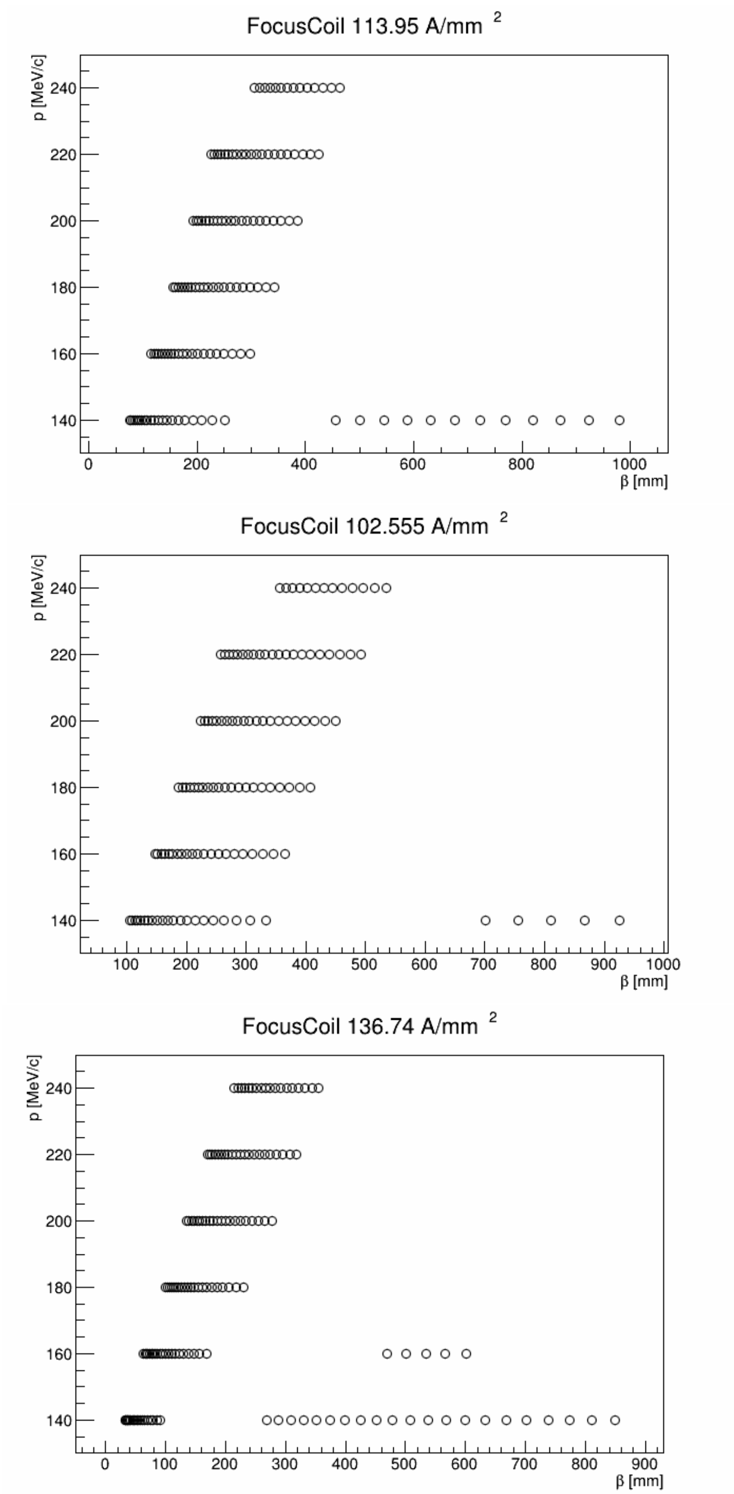

Fig. 2: Available optical β functions at the MICE step IV focus for (top) nominal focus coil current, (middle) 90 % of nominal focus coil current and (bottom) 120 % of nominal focus coil current, as a function of momentum.

Coil Name	Centre z [mm]	Inner radius [mm]	Radial thickness [mm]	Length [mm]	Current Density [A/mm^2]
Focus	205	263	84	210	113.95 nominal
Coupling	1375	725	116	250	96.21 nominal
Focus	2545	263	84	210	113.95 nominal

Tab. 3: SFoFo lattice used for simulation of MICE Step VI. The lattice was repeated with a cell period of 5500 mm and adjacent half cells having opposite polarity (i.e. flip mode). 3 half cells were placed on either side of the test cell to ensure correct application of fringe fields.

4 Step VI

The Step VI lattice consists of an SFoFo cell, with a matching section at either end. The SFoFo cell has an absorber at each of the three focusses. In this note, the optics of the SFoFo cell is studied for the nominal focus coil current and for the focus coil with a 10 % reduction in available current. Coil parameters are listed in table 3. The lattice is taken to be a repeating SFoFo lattice extending to infinity upstream and downstream of $z = 0$. It is assumed that a match from the spectrometer solenoid to the SFoFo lattice is possible for all optics.

4.1 Calculation of Matched Beams

In the repeating lattice outlined above, there exists a well defined lattice β function with optical parameters that is periodic with the periodicity of the magnetic field. This was found numerically using a Newton-Raphson search with several different seeds. Where no solution could be found, it was assumed that the lattice was on a linear resonance.

A sample matched β function is shown in Figure 4.1 for the nominal current settings, together with the on-axis field. The solution is periodic with a focus at the focus coil, and a anti-focus at the coupling coil. β at the focus determines the rate of cooling while β at the anti-focus determines the acceptance.

4.2 Momentum dependence of β, 200 MeV/c settings

The momentum dependence of β at the focus and anti-focus as a function of momentum is shown in Figure 4.2 for the nominal setting.

There are a number of momenta where no matched solution can be found, corresponding to linear resonances. Here phase advance is complex, there is no lattice β function, particles take hyperbolic trajectories in phase space and there is considerable emittance growth.

MICE operates in the region with phase advance between 2π and 4π. For the nominal settings, there is a large stable region with relatively flat β function at around 400 mm.

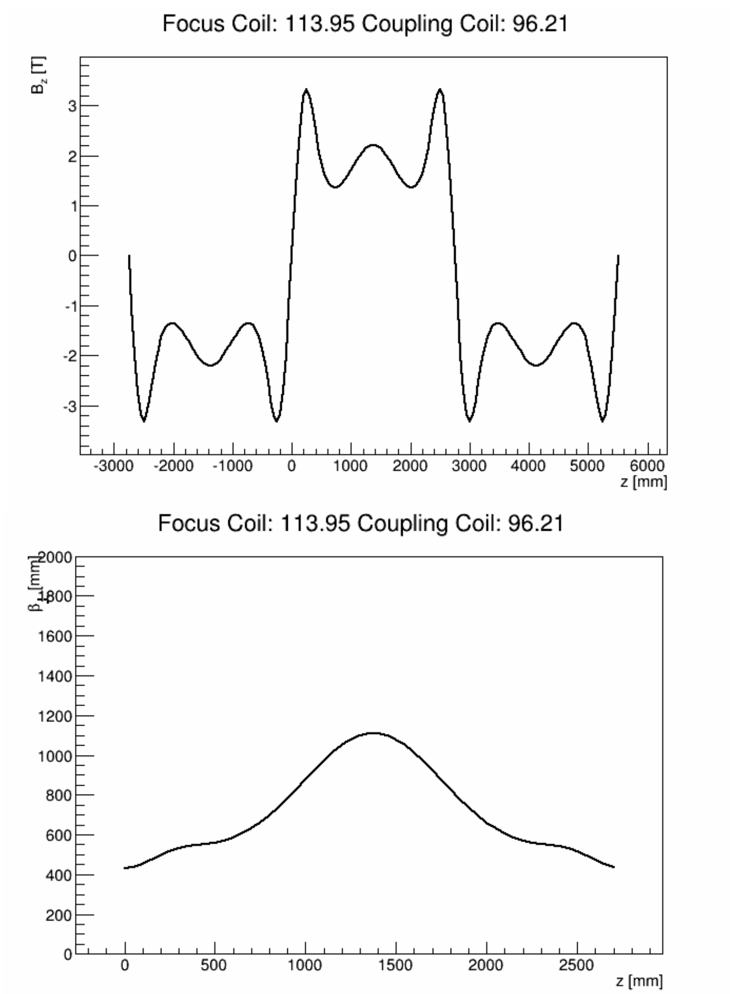

Fig. 3: (Top) Example magnetic field on axis and (bottom) corresponding matched β function

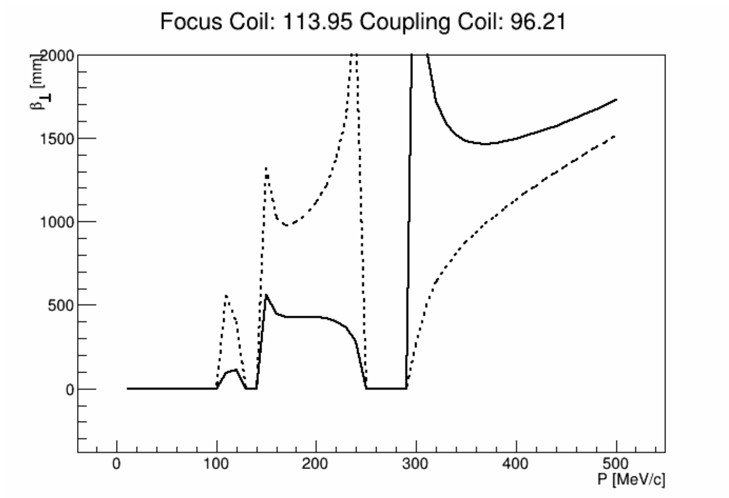

Fig. 4: β function dependence on momentum, for nominal magnet settings. β is shown at the focus (full line) and at the antifocus (dashed line).

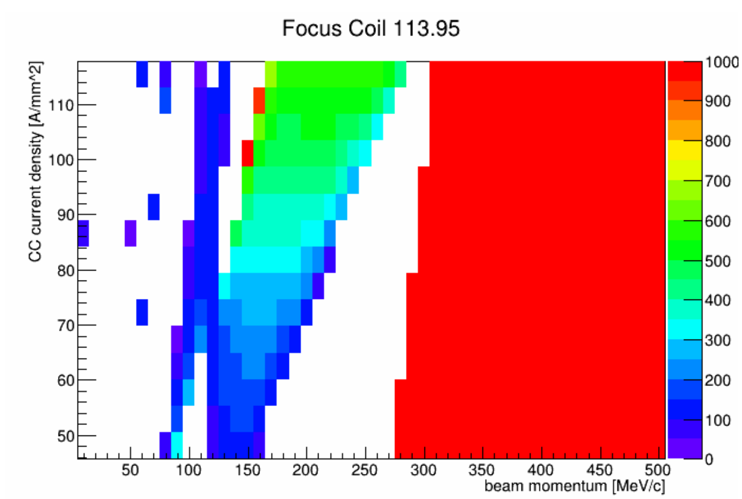

Fig. 5: β function dependence on momentum and coupling coil current. Colours indicate the β function at the focus for each value of momentum and coupling coil current. The focus coil is held constant at 113.95 A/mm^2.

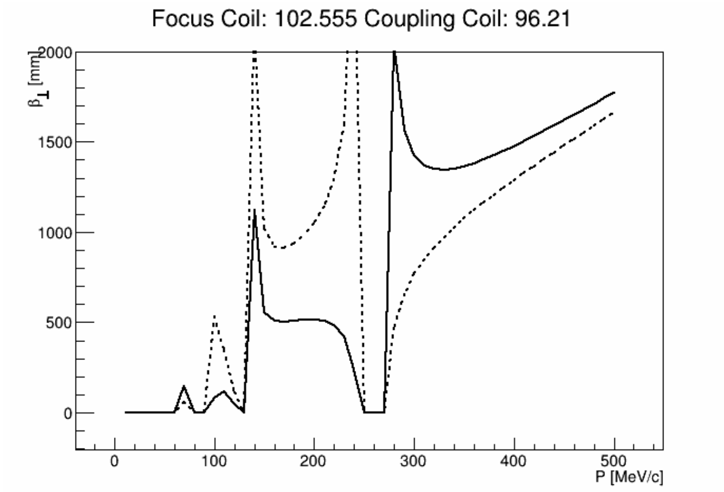

Fig. 6: β function dependence on momentum, for nominal coupling coil setting and focus coil reduced by 10 %. β is shown at the focus (full line) and at the antifocus (dashed line).

The dependence of β on coupling coil current is shown in Figure 4.2 for the nominal settings. It can be seen that adjusting the coupling coil current moves the stable region, with an increase in current resulting in a slightly higher momentum in the acceptance band and an increase in the matched β function.

The equivalent plots for the case where the Focus coil current is reduced by 10 % is shown in Figures 4.2 and 4.2. For the nominal coupling coil current, the β function at the focus is increased from around 400 mm to around 500 mm, while the momentum acceptance shows no change at the resolution of this analysis. Changing the coupling coil current reveals broadly the same behaviour as with nominal focus coil currents, although β at the focus is consistently higher.

4.3 Momentum dependence of β, 240 MeV/c settings

Also of interest is the performance of the lattice at 240 MeV/c settings, such that the coupling coil operates at 20 % above the nominal current. The β function dependence on momentum for this situation is shown in Figure 4.3. Where the focus coil also operates at 20 % above the nominal current, the β function shows identical behaviour to the nominal settings, but with momenta scaled by 20%, as expected.

In the reduced focus coil current situation, the 2 π resonance is partially suppressed as the field produced around the focus coil is quite similar to the field produced around the coupling coil. In affect, the focussing produced by the lattice looks like it has a periodicity of 2.75/2 m, rather than the nominal 2.75 m. The β function in this case is more like 700-800 mm rather than the

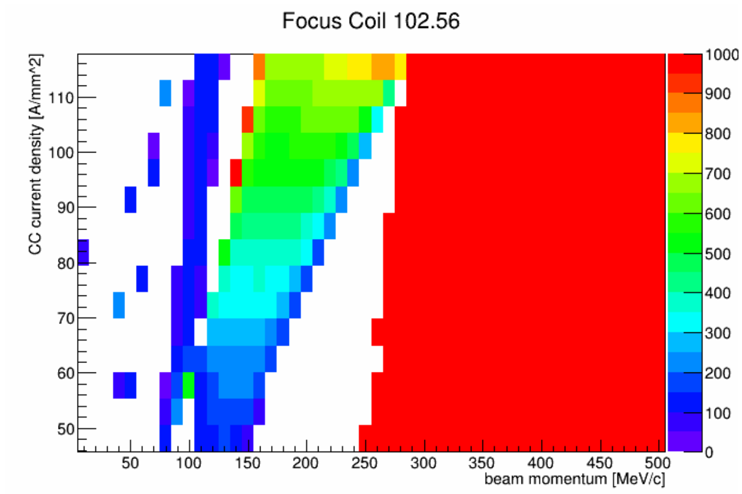

Fig. 7: β function dependence on momentum and coupling coil current. Colours indicate the β function at the focus for each value of momentum and coupling coil current. The focus coil is held constant at 102.555 A/mm^2.

baseline 420 mm.

5 Discussion

The performance of MICE at Step IV and Step VI has been studied with a focus coil that does not perform to the specification and compared with the MICE baseline situation. The consequences can be summarised as follows:

- The minimum β function that can be achieved in flip mode at Step IV is slightly limited. The minimum β function is increased by up to a factor 3, ruling out some of the exotic low β function lattices. In the approximation of equation 2, this would result in a proportional increase in equilibrium emittance. However, for all momenta the baseline β function of 420 mm is accessible.
- The β function that can be achieved in flip mode at Step VI is limited. The β function is increased by around 20 % at 200 MeV/c, and around 100 % at 240 MeV/c. This would result in a proportional increase in equilibrium emittance.

It should be noted that the baseline Neutrino Factory front end, at the time of writing, operates with equilibrium emittance of around 6 mm (800 mm β) and central momentum around 230 MeV/c, so the higher momentum case is certainly of interest to the muon accelerator community. Even with the reduced currents

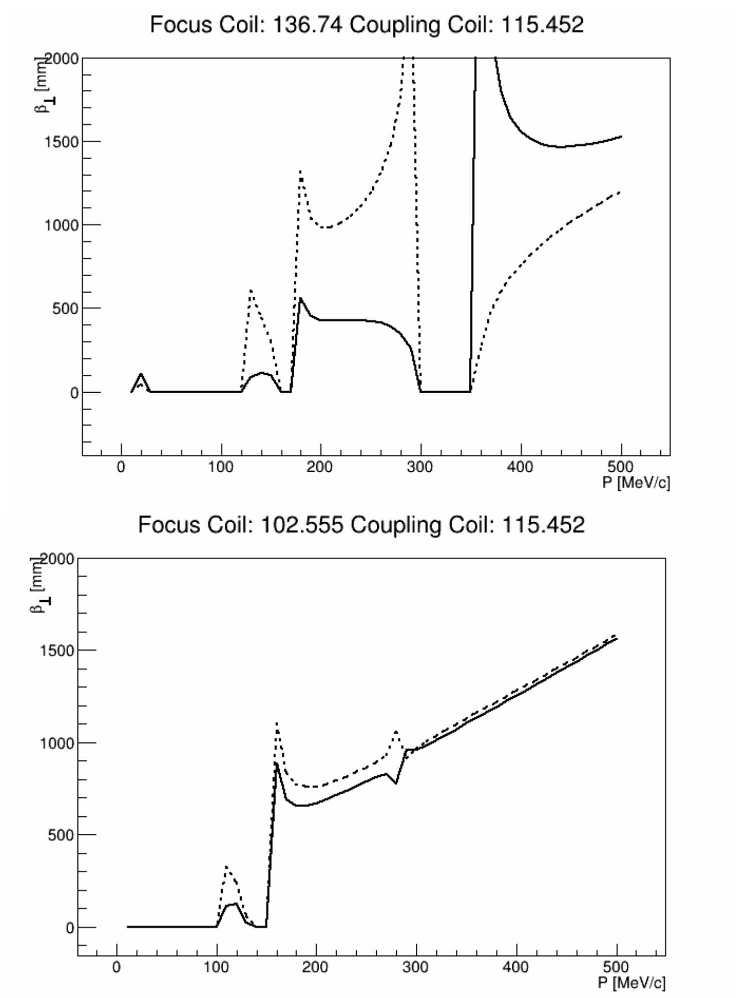

Fig. 8: β function dependence on momentum, for coupling coil operating at 20 % above nominal, (top) focus coil operating at 20 % above nominal and (bottom) focus coil operating at 10 % below nominal.

discussed herein, MICE can produce a quite satisfactory physics result at this higher momentum, comparable to the current baseline Neutrino Factory. At the baseline momentum, the cooling potential of the lattice would be relatively unchanged.

5.1 Alternatives and Further Work

The discussion in this note is intended as guidance only. If further work on the focus coil fails to generate a satisfactory outcome, it is expected that a full set of optics solutions would be generated and a full tracking study performed.

Especially, it is noted that the β functions which the matching system can produce and the β functions available within the SFoFo lattice proper seem to not quite correspond.

The option to operate the focus coils with an asymmetric current set has not been explored in this note. No lattice has been proposed with such a set of currents in the literature, and so it is considered that it would not be of benefit to the community to test such a lattice.

References

[1] D. Neuffer, Principles and Applications of Muon Cooling, Fermilab Note FN-378, 1983.

[2] P. Hanlet et al., MICE Spectrometer Solenoid Controls and Monitoring System Review, MICE-NOTE-MAGN-401, 2012. Coil parameters were taken from Table 1, 6 and 7; match coil current limits were taken as the maximum of the various currents listed in those tables.

[3] M. Courthold and J. Cobb, Focus Coils, MICE Collaboration Meeting 36, 2013