Inflation from Higher-Dimensional Gauge and Gravity Theory

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What we are doing

We study higher-dimensional gauge plus gravity theory and its application to inflationary cosmology.

In our model, two scalar fields, gauge-scalar $\boldsymbol{\theta}$ and radion $\boldsymbol{\phi}$, are obtained from the five dimensional gravity field and the five dimensional gauge field with the S^1 compactification.

We investigate the stability of the one-loop effective potential for $\boldsymbol{\theta}$ and $\boldsymbol{\phi}$. As the potential $V(\boldsymbol{\theta}, \boldsymbol{\phi})$ can be identified with the inflaton potential, we have to check if $V(\boldsymbol{\theta}, \boldsymbol{\phi})$ satisfies all constraints for inflation.

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Why inflation?

The Hot Big-Bang scenario has explained how our universe was made.

But the Hot Big-Bang scenario has a few problems.

- Flatness problem
- Horizon problem
- monopole problem   etc.

If inflation occurred before the Hot Big-Bang,

these problems are solved.

What is inflation?

Inflation is the rapidly accelerated expansion of space at the early stage of the universe.

$$a(t) \sim e^{Ht} \qquad \left(H = \frac{\dot{a}}{a} \right)$$

a : scale factor

Inflation occurs if the universe is filled with $\underbrace{\text{a scalar field } \varphi}_{\text{inflaton}}$.

We need to take the inflaton potential $V(\varphi)$ into consideration.

Constraints for inflation

1) slow-roll conditions $\epsilon \equiv \frac{1}{2} M_P^2 \left(\frac{V'}{V}\right)^2 \ll 1, \quad |\eta| \equiv M_P^2 \left|\frac{V''}{V}\right| \ll 1$

2) spectral index $n_s \equiv 1 - 6\epsilon + 2\eta, \quad 0.948 \leq n_s \leq 0.977$

3) number of e-folding $N \equiv \int_{t_i}^{t_f} H dt \simeq \frac{1}{M_P^2} \left| \int_{\phi_i}^{\phi_f} \frac{V}{V'} d\phi \right| \simeq 50 \sim 60$

4) curvature perturbation $\mathcal{P}_\zeta^{\frac{1}{2}} = \frac{1}{2\sqrt{6}\pi M_P^2} \left(\frac{V(\phi)}{\epsilon}\right)^{\frac{1}{2}} \Bigg|_* = (4.9 \pm 0.2) \times 10^{-5}$

($*$: at the horizon exit)

5) Quantum gravity correction is negligible.

6) tensor to scalar ratio

$$r = \frac{\mathcal{P}_h}{\mathcal{P}_\zeta} = 16\epsilon$$

(restriction for the energy scale where inflation occurs)

Higher-dimensional theory

Higher-dimensional fields unify some 4D fields.

- 5D gauge field includes 4D gauge fields and 4D scalar fields.
- 5D gravity field includes 4D gravity fields, 4D U(1) gauge fields and 4D scalar fields.

Higher-dimensional theory can be regarded as 4D theory after compactification.

We concentrate on the extra space scalar components of higher-dimensional fields B_M and $\hat{g}_{\mathrm{MN}}$.

S^1 compactification

5D theory $\longrightarrow$ Effective 4D theory

5D gauge field

4D gauge field

$$B_M(x_M) = \begin{pmatrix} B_0 \\ B_1 \\ B_2 \\ B_3 \\ B_5 \end{pmatrix} \longrightarrow B_\mu^{(n)}(x_\mu) = \begin{pmatrix} B_0 \\ B_1 \\ B_2 \\ B_3 \end{pmatrix}$$

4D scalar field

$B_5^{(n)}(x_\mu)$: gauge-scalar

(n: integer $n = -\infty \sim \infty$)

Can the zero mode of gauge-scalar $B_5^{(0)}$ be identified with inflaton?

Extranatural Inflation (Arkani-Hamed et al. '03)

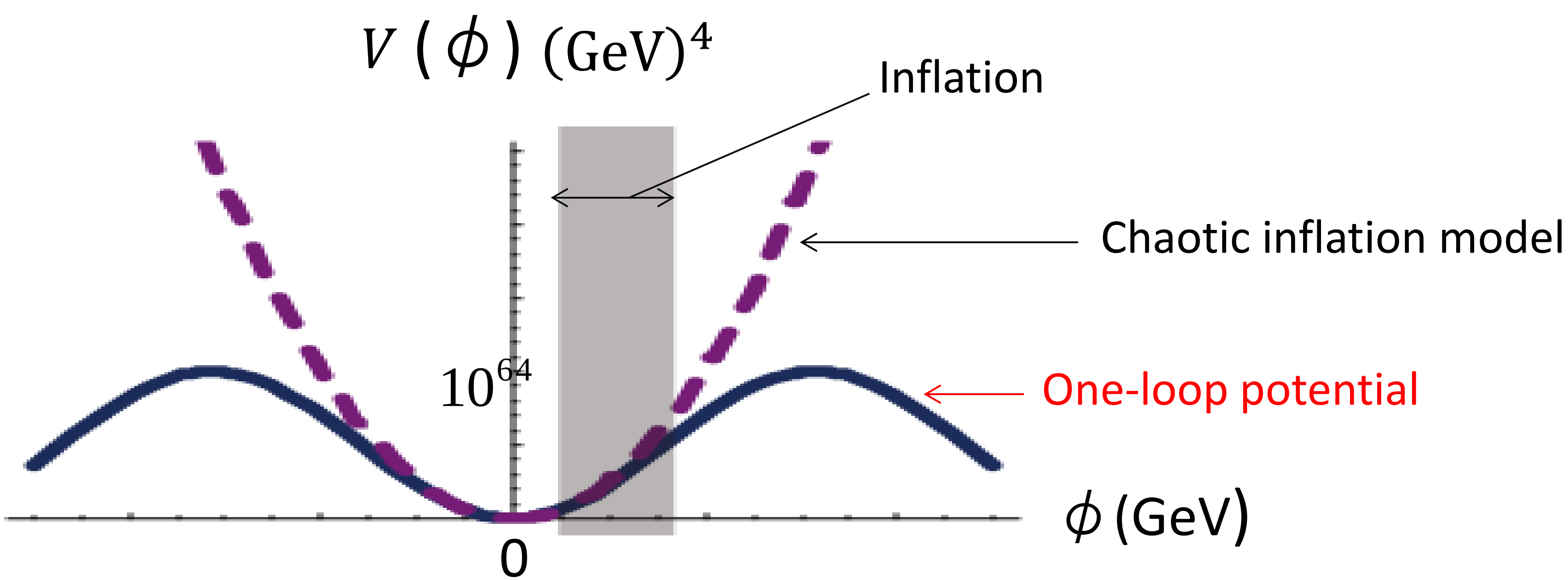
5D-action (SU(N)gauge + matter)

$$S_5 = \int d^5x \left[-\frac{1}{2}\mathrm{tr}(F^{MN}F_{MN}) + \bar{\psi}(i\gamma^M D_M - \mu)\psi \right]$$

$$F_{MN} = \partial_M B_N - \partial_N B_M + i\,\lambda_5 [B_M, B_N]$$

(λ_5: 5D gauge coupling constant)

The one-loop effective potential is finite and periodic.



5D-metric can be parametrized as

$$\hat{g}_{\mathrm{MN}} = \boldsymbol{\phi}^{-\frac{1}{3}} \begin{pmatrix} g_{\mu\nu} + A_\mu A_\nu g_{55} & A_\mu \boldsymbol{\phi} \\ A_\nu \boldsymbol{\phi} & \boldsymbol{\phi} \end{pmatrix}$$

4D scalar field

$\boldsymbol{\phi}^{(n)}(x_\mu)$: radion

S^1 compactification

(Appelquist & Chodos '83)

Can the zero mode of radion $\boldsymbol{\phi}^{(0)}$ be identified with inflaton as well as gauge-scalar?

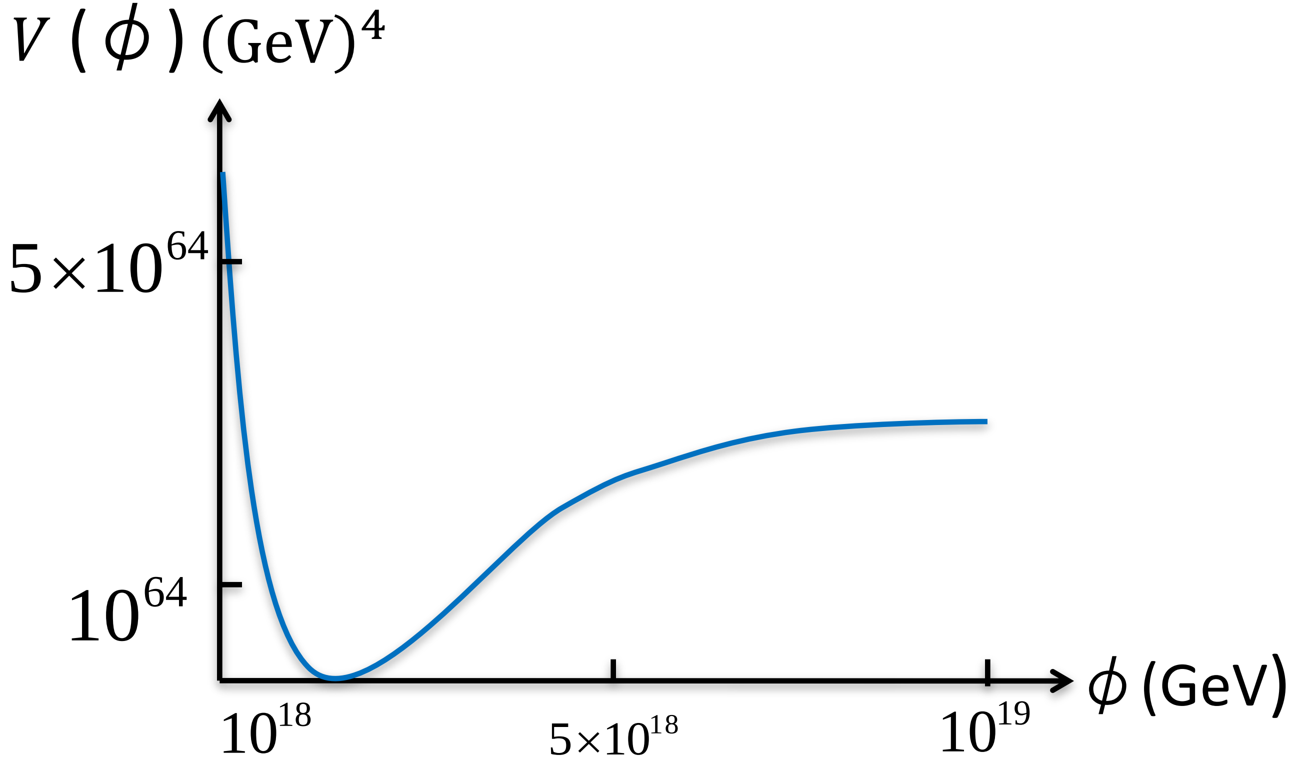
Radion Inflation (Inami et al. '12)

5D-action (gravity + matter)

$$S_5 = \int d^5x \sqrt{\det g_{AB}} \, [R + \bar{\psi}(i\gamma^M D_M - \mu)\psi]$$

(R : scalar curvature, μ : matter mass)

The one-loop effective potential is finite.



Constraint for inflation

$\varepsilon \ll 1$

$\eta \ll 1$

$0.948 < n_s < 0.977$

$N = 50 \sim 60$

$\delta_H = 1.91 \times 10^{-5}$

Our Motivation

How about the model which includes both gauge-scalar and radion?

5D-gauge field ＋ 5D-gravity field

⬇

inflation

?

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Model

5D-action (Gravity + U(1)-Gauge + Matter)

$$S_5 = \int d^5x \sqrt{-\det \hat{g}_{\mathrm{MN}}} \left[\mathcal{L}_{Gravity} + \mathcal{L}_{Gauge} + \mathcal{L}_{Matter} \right]$$

5D-metric $\quad \hat{g}_{\mathrm{MN}} = \boldsymbol{\phi}^{-\frac{1}{3}} \begin{pmatrix} g_{\mu\nu} + A_\mu A_\nu g_{55} & A_\mu \boldsymbol{\phi} \\ A_\nu \boldsymbol{\phi} & \boldsymbol{\phi} \end{pmatrix}$

$$\mathcal{L}_{Gravity} = \frac{1}{16\pi G_5} \hat{R}_{(5)}$$ ($\hat{R}_{(5)}$: 5D Ricci scalar)

$$\mathcal{L}_{Gauge} = -\frac{1}{4} \hat{g}^{\mathrm{MP}} \hat{g}^{\mathrm{NL}} F_{\mathrm{MN}} F_{\mathrm{PL}} \qquad F_{\mathrm{MN}} = \partial_{\mathrm{M}} B_{\mathrm{N}} - \partial_{\mathrm{N}} B_M$$

$$\mathcal{L}_{Matter} = \overline{\Psi}_i \left(-i \hat{g}^{\mathrm{MN}} \Gamma_{\mathrm{M}} D_{\mathrm{N}} - m \right) \Psi_i \qquad D_{\mathrm{N}} = \partial_{\mathrm{N}} - i g_5' B_{\mathrm{N}}$$

(m: mass of charged fermions) (g_5': 5D gauge coupling const.)

Kaluza-Klein expansions

$$B_5\,(x_\mu,y) = \left\langle B_5^{(0)} \right\rangle + \frac{1}{\sqrt{L}} \sum_{n=-\infty}^{\infty} B_5^{(n)}(x_\mu) e^{i\frac{2\pi n}{L}y}$$

$$\boldsymbol{\phi}(x_\mu,y) = \langle \boldsymbol{\phi}^{\,(0)} \rangle + \sum_{n=-\infty}^{\infty} \boldsymbol{\phi}^{\,(n)}(x_\mu) e^{i\frac{2\pi n}{L}y}$$

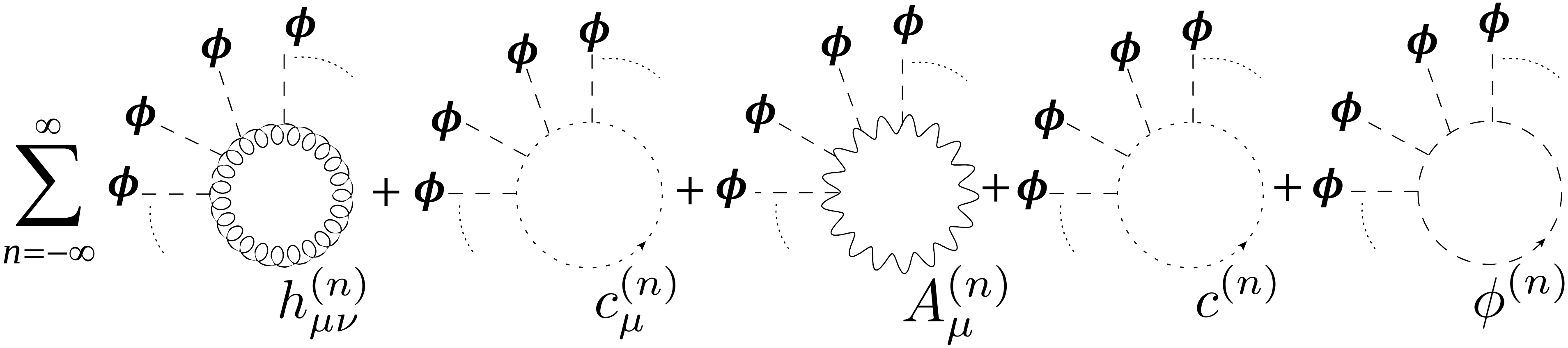
We define

$$\boldsymbol{\theta} \equiv \frac{g'_4 L \left\langle B_5^{(0)} \right\rangle}{2\pi}, \quad \boldsymbol{\phi} \equiv \langle \boldsymbol{\phi}^{\,(0)} \rangle$$

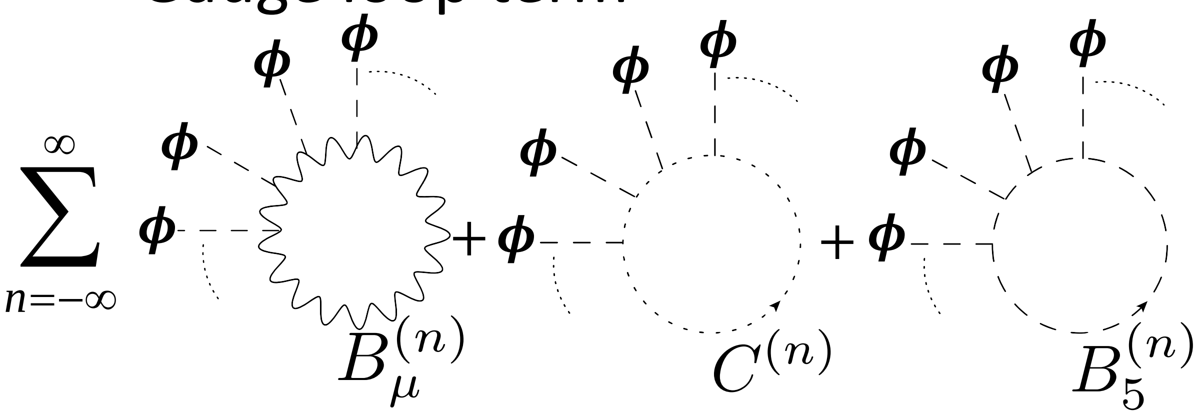
where g'_4 is the 4D gauge coupling constant and L is the compactification circumference.

One-loop diagram

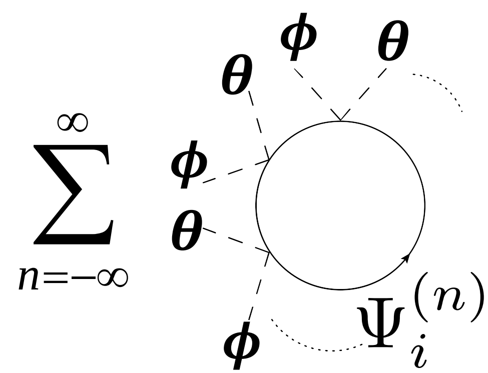
Gravity loop term



Gauge loop term



Matter loop term



One-loop effective potential

$$V(\boldsymbol{\theta},\boldsymbol{\phi}) = V_{Gravity}(\boldsymbol{\phi}) + V_{Gauge}(\boldsymbol{\phi}) + V_{Matter}(\boldsymbol{\theta},\boldsymbol{\phi})$$

$$= -\frac{15}{4\pi^2}\frac{1}{\boldsymbol{\phi}^{2}L^4}\boldsymbol{\zeta}(5) - \frac{9}{4\pi^2}\frac{1}{\boldsymbol{\phi}^{2}L^4}\boldsymbol{\zeta}(5)$$

$$+\frac{3}{\pi^2}\frac{1}{\boldsymbol{\phi}^{2}L^4}\mathrm{Re}[c_1\boldsymbol{L_{i_5}}(e^{-Lm\boldsymbol{\phi}^{\frac{1}{3}}}e^{i\boldsymbol{\theta}})+c_1 Lm\boldsymbol{\phi}^{\frac{1}{3}}\boldsymbol{L_{i_4}}(e^{-Lm\boldsymbol{\phi}^{\frac{1}{3}}}e^{i\boldsymbol{\theta}})$$

$$+c_1\frac{1}{3}L^2m^2\boldsymbol{\phi}^{\frac{2}{3}}\boldsymbol{L_{i_3}}(e^{-Lm\boldsymbol{\phi}^{\frac{1}{3}}}e^{i\boldsymbol{\theta}})]$$

c_1 : number of charged fermions

$\boldsymbol{\zeta}(n) = \sum_{k=1}^{\infty}\frac{1}{k^n}$: zeta function

$\boldsymbol{L_{i_n}}(x) = \sum_{k=1}^{\infty}\frac{x^k}{k^n}$: polylogarithm function

$$\boldsymbol{\theta} \equiv \frac{g'_4 L\left\langle B_5^{(0)}\right\rangle}{2\pi}, \quad \boldsymbol{\phi} \equiv \left\langle \boldsymbol{\phi}^{(0)}\right\rangle$$

Approximation of the potential for some values of $\boldsymbol{\phi}$ and $\boldsymbol{\theta}$

- $V(\boldsymbol{\theta}, \boldsymbol{\phi})$ goes to zero at large value of $\boldsymbol{\phi}$

$$\boldsymbol{L}_{\boldsymbol{i}_{\boldsymbol{n}}}(e^{-Lm\boldsymbol{\phi}^{\frac{1}{3}}}) \sim e^{-Lm\boldsymbol{\phi}^{\frac{1}{3}}} \to 0$$

$$\therefore V(\boldsymbol{\theta}, \boldsymbol{\phi}) \approx \frac{1}{\boldsymbol{\phi}^{2} L^4}[-8\boldsymbol{\zeta}(5)] \to 0 \quad (\boldsymbol{\phi} \to 0)$$

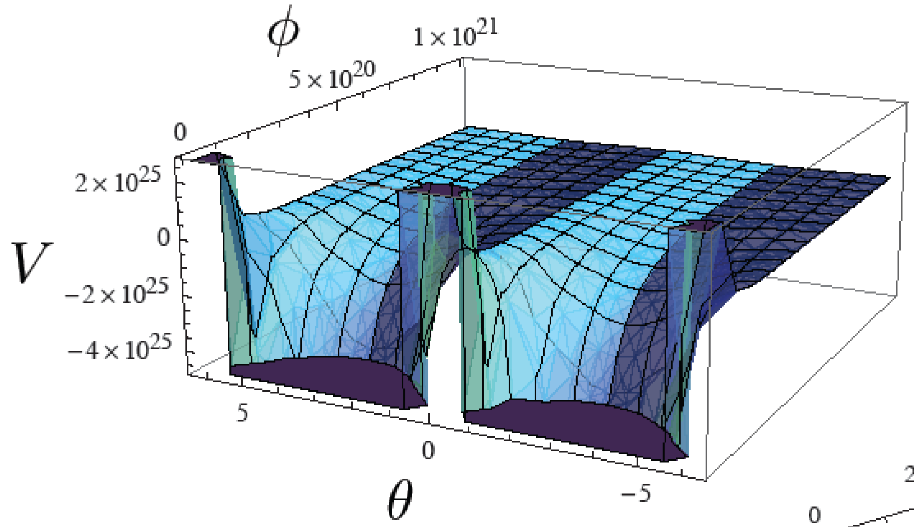
- $V(\boldsymbol{\theta}, \boldsymbol{\phi})$ at small $\boldsymbol{\phi}$

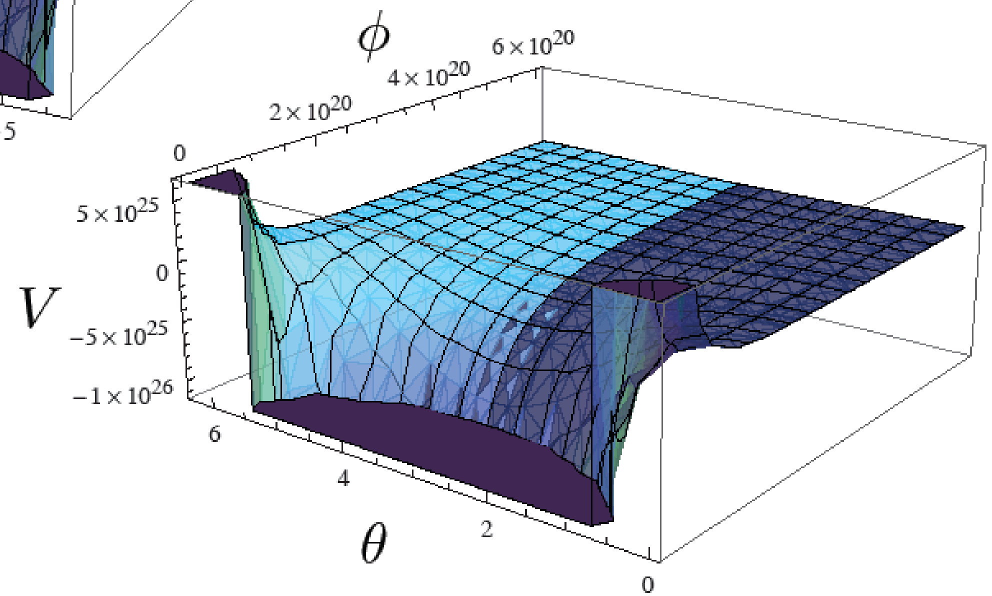
$$V(\boldsymbol{\theta}, \boldsymbol{\phi}) = \frac{3}{4\pi^2}\frac{1}{\boldsymbol{\phi}^{2} L^4}\Big[-8\boldsymbol{\zeta}(5) + 4c_1 \boldsymbol{L}_{\boldsymbol{i}_{\boldsymbol{5}}}\left(e^{i\boldsymbol{\theta}}\right) - \frac{8}{3}c_1 Lm\boldsymbol{\phi}^{\frac{1}{3}}\boldsymbol{L}_{\boldsymbol{i}_{\boldsymbol{3}}}\left(e^{i\boldsymbol{\theta}}\right) - \frac{4}{3}c_1 L^2 m^2 \boldsymbol{\phi}^{\frac{2}{3}}\boldsymbol{L}_{\boldsymbol{i}_{\boldsymbol{2}}}\left(e^{i\boldsymbol{\theta}}\right)\Big]$$

Note that the sign of the matter contribution is positive or negative depending on the value of $\boldsymbol{\theta}$ while the graviton and gauge boson contributions are always negative sign(attractive).

3D-Graph

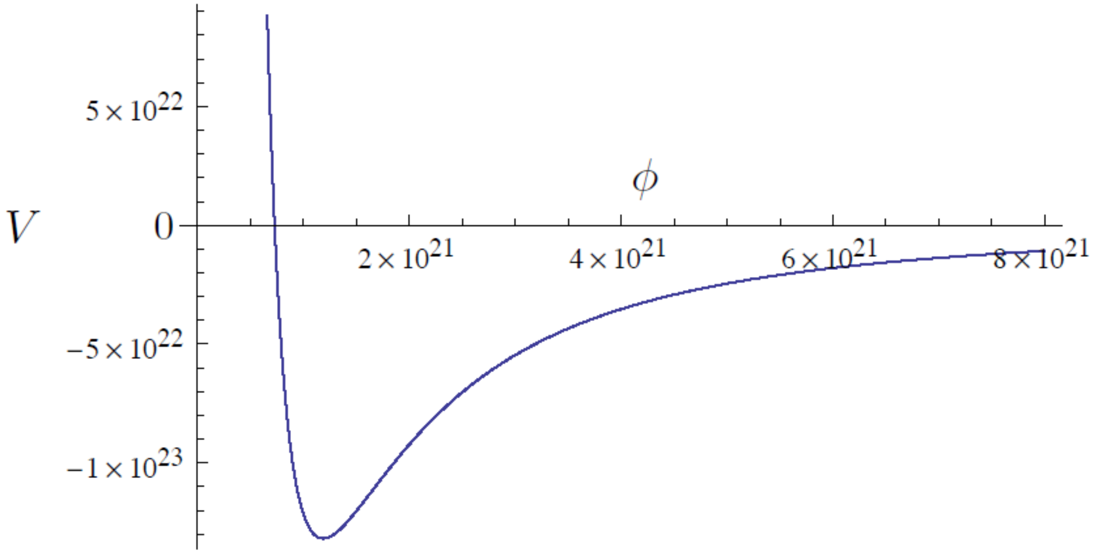
Gauge-scalar $\boldsymbol{\theta}$ contribution makes the potential unstable.



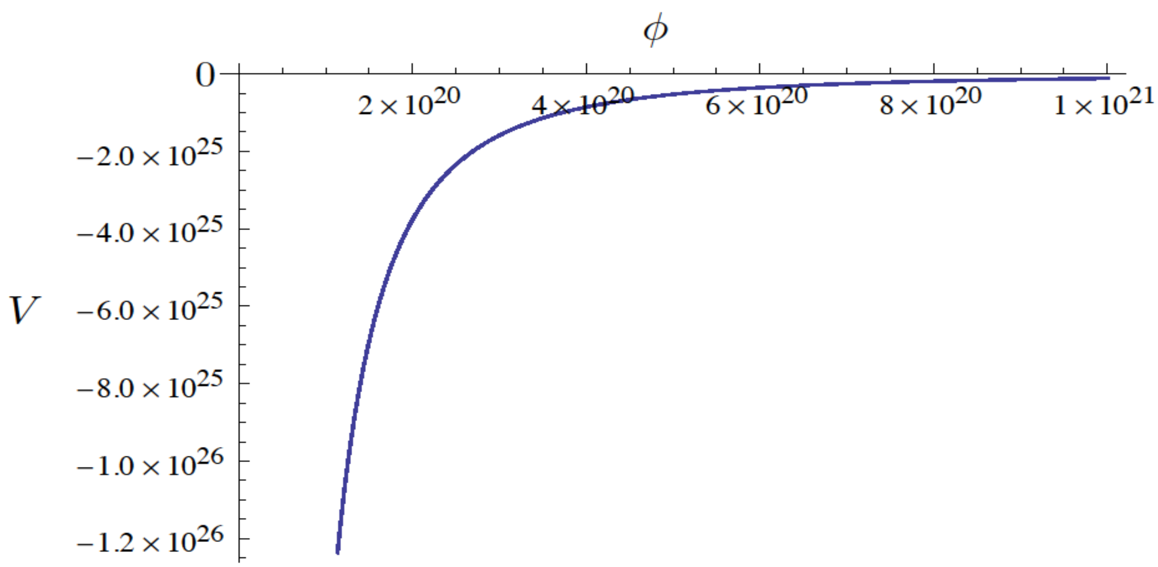


$\boldsymbol{\theta}$-dependence

$\boldsymbol{\theta} \neq \pi$ case



$\boldsymbol{\theta} = \pi$ case



Adding the neutral fermions

But inflation needs a stable minimum in scalar potential for reheating.

The charged fermions are coupled with gauge-scalar $\boldsymbol{\theta}$. That is why gauge-scalar $\boldsymbol{\theta}$ contribution makes the potential unstable.

We believe that adding the contribution caused by neutral fermions is a good way to obtain stable potential.

Modified potential

$$V(\boldsymbol{\theta},\boldsymbol{\phi}) = V_{Gravity}(\boldsymbol{\phi}) + V_{Gauge}(\boldsymbol{\phi}) + V_{Matter}(\boldsymbol{\theta},\boldsymbol{\phi})$$

$$= -\frac{15}{4\pi^2}\frac{1}{\boldsymbol{\phi}^{\ 2}L^4}\boldsymbol{\zeta}(5) - \frac{9}{4\pi^2}\frac{1}{\boldsymbol{\phi}^{\ 2}L^4}\boldsymbol{\zeta}(5)$$

$$+\frac{3}{\pi^2}\frac{1}{\boldsymbol{\phi}^{\ 2}L^4}\mathrm{Re}[c_2\boldsymbol{L}_{\boldsymbol{i_5}}(e^{-L\mu\boldsymbol{\phi}^{\frac{1}{3}}})+c_2L\mu\boldsymbol{\phi}^{\frac{1}{3}}\boldsymbol{L}_{\boldsymbol{i_4}}(e^{-L\mu\boldsymbol{\phi}^{\frac{1}{3}}})$$

$$+c_2\frac{1}{3}L^2\mu^2\boldsymbol{\phi}^{\frac{2}{3}}\boldsymbol{L}_{\boldsymbol{i_3}}(e^{-L\mu\boldsymbol{\phi}^{\frac{1}{3}}})$$

$$+c_1\boldsymbol{L}_{\boldsymbol{i_5}}(e^{-Lm\boldsymbol{\phi}^{\frac{1}{3}}}e^{i\boldsymbol{\theta}})+c_1Lm\boldsymbol{\phi}^{\frac{1}{3}}\boldsymbol{L}_{\boldsymbol{i_4}}(e^{-Lm\boldsymbol{\phi}^{\frac{1}{3}}}e^{i\boldsymbol{\theta}})$$

$$+c_1\frac{1}{3}L^2m^2\boldsymbol{\phi}^{\frac{2}{3}}\boldsymbol{L}_{\boldsymbol{i_3}}(e^{-Lm\boldsymbol{\phi}^{\frac{1}{3}}}e^{i\boldsymbol{\theta}})]$$

c_2 : number of neutral fermions

μ : mass of neutral fermions

Approximation of the potential for some values of $\boldsymbol{\phi}$ and $\boldsymbol{\theta}$

- $V(\boldsymbol{\theta},\boldsymbol{\phi})$ goes to zero at large value of $\boldsymbol{\phi}$

$$\boldsymbol{L}_{\boldsymbol{i}_{\boldsymbol{n}}}\left(e^{-Lm\boldsymbol{\phi}^{\frac{1}{3}}}\right)\sim e^{-Lm\boldsymbol{\phi}^{\frac{1}{3}}} \to 0$$

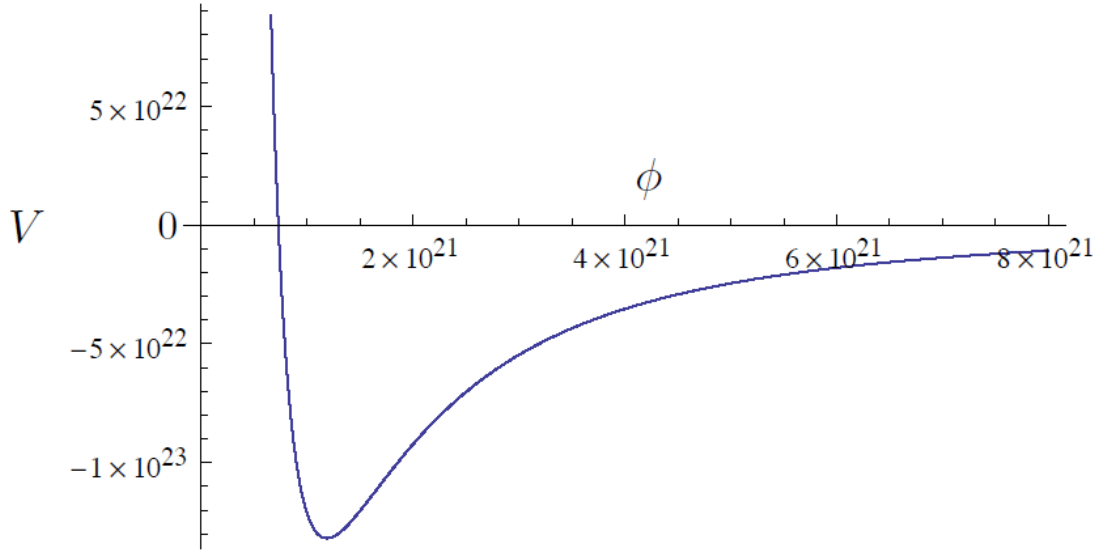
$$\therefore V(\boldsymbol{\theta},\boldsymbol{\phi}) \approx \frac{1}{\boldsymbol{\phi}^{2}L^{4}}[-8\boldsymbol{\zeta}(5)] \to 0 \quad (\boldsymbol{\phi} \to 0)$$

- $V(\boldsymbol{\theta},\boldsymbol{\phi})$ at small $\boldsymbol{\phi}$

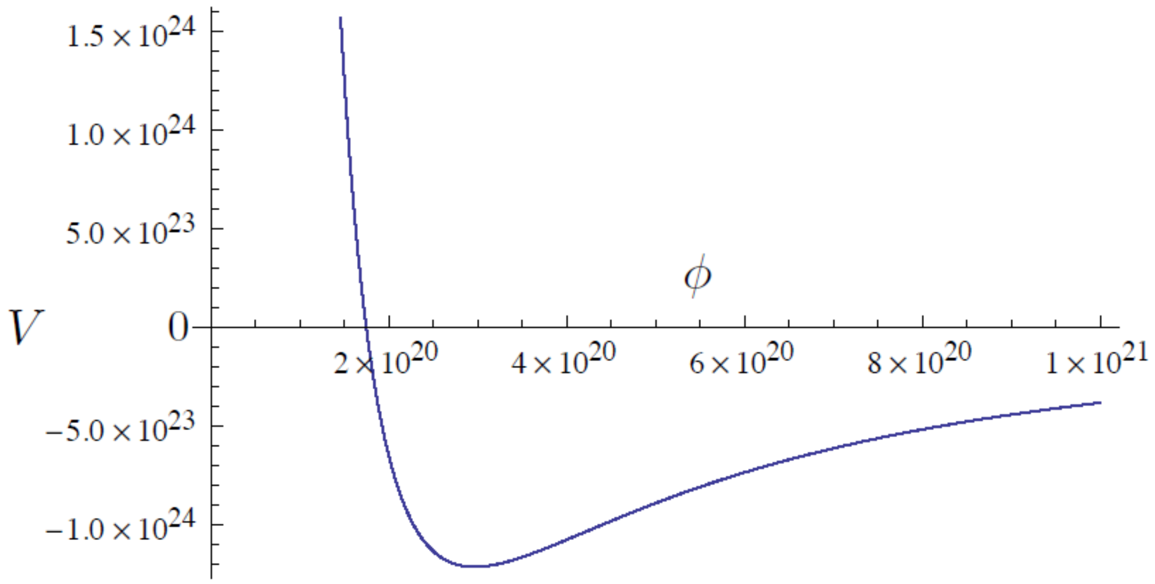
$$V(\boldsymbol{\theta},\boldsymbol{\phi}) = \frac{3}{4\pi^{2}}\frac{1}{\boldsymbol{\phi}^{2}L^{4}}\Big[\{-8+4c_{2}\}\boldsymbol{\zeta}(5) - \frac{8}{3}c_{2}L\mu\boldsymbol{\phi}^{\frac{1}{3}}\boldsymbol{\zeta}(3) - \frac{4}{3}c_{2}L^{2}\mu^{2}\boldsymbol{\phi}^{\frac{2}{3}}\boldsymbol{\zeta}(2) + 4c_{1}\boldsymbol{L}_{\boldsymbol{i}_{\boldsymbol{5}}}\left(e^{i\boldsymbol{\theta}}\right) - \frac{8}{3}c_{1}Lm\boldsymbol{\phi}^{\frac{1}{3}}\boldsymbol{L}_{\boldsymbol{i}_{\boldsymbol{3}}}\left(e^{i\boldsymbol{\theta}}\right) - \frac{4}{3}c_{1}L^{2}m^{2}\boldsymbol{\phi}^{\frac{2}{3}}\boldsymbol{L}_{\boldsymbol{i}_{\boldsymbol{2}}}\left(e^{i\boldsymbol{\theta}}\right)\Big]$$

$\boldsymbol{\theta}$-dependence

$\boldsymbol{\theta} \neq \pi$ case



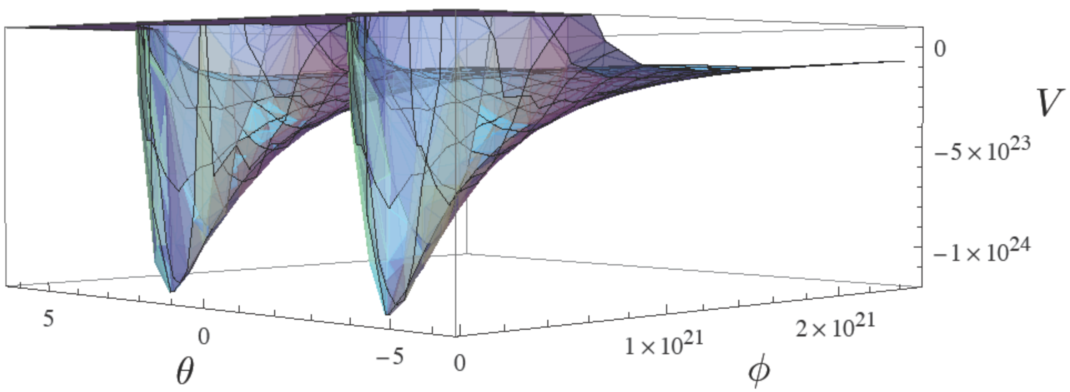
$\boldsymbol{\theta} = \pi$ case

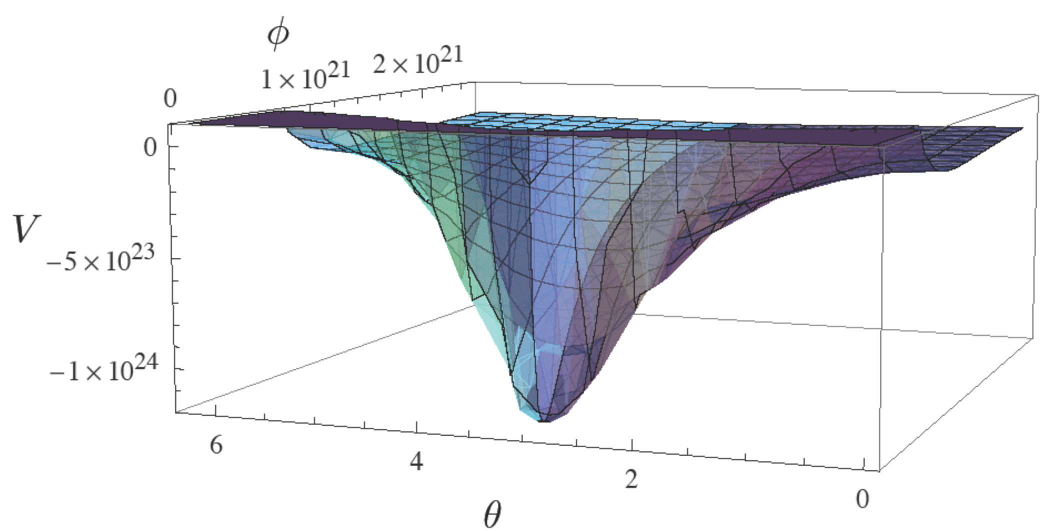


3D-Graph

Neutral fermions is not coupled with gauge-scalar $\boldsymbol{\theta}$ and contribute as the repulsive potential.

We achieve a stable minimum in the potential when $c_2 \geq 2 + c_1$.





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Conclusion

- We calculate the one-loop effective potential which is dominated by both radion $\boldsymbol{\phi}$ and gauge-scalar $\boldsymbol{\theta}$.

- The stable effective potential can be obtained by the 5D U(1) gauge plus the 5D gravity theory.

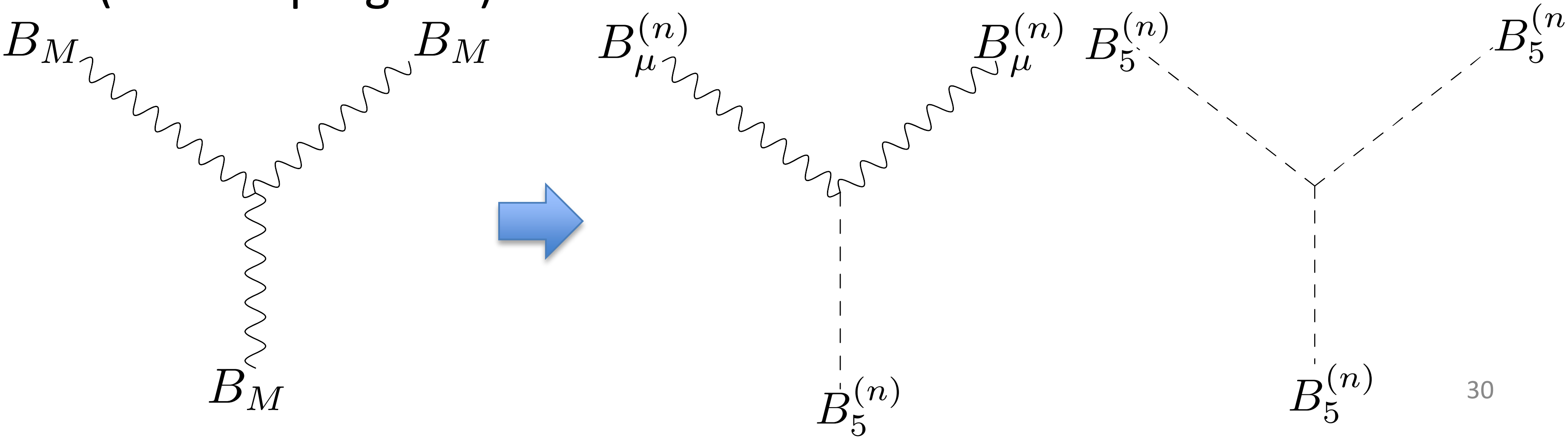
- Both gravity and gauge loops are attractive. The charged fermions contribution can become repulsive or attractive depending on the value of gauge-scalar $\boldsymbol{\theta}$. We need to add the neutral fermions to achieve a stable minimum in the effective potential $V(\boldsymbol{\theta}, \boldsymbol{\phi})$ for all values of gauge-scalar $\boldsymbol{\theta}$.

Contents

Discussion

- We have to check if $V(\boldsymbol{\theta}, \boldsymbol{\phi})$ satisfies all constraints for inflation and the possibility of hybrid inflation model. (work in progress)

- We progress in the non-abelian gauge theory. We investigate the 5D SU(2) gauge plus the 5D gravity theory. How effects is given by the self-interaction of the 5D gauge field? (work in progress)



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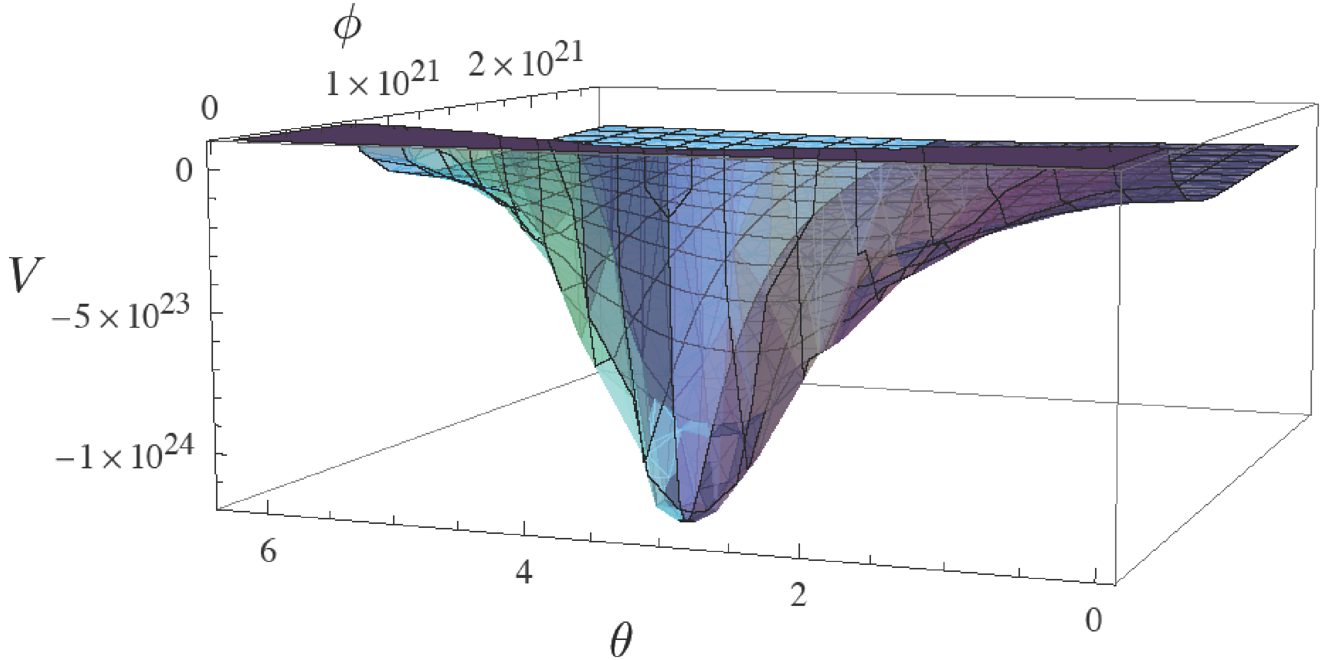

the compactification circumference $L = 3 \times 10^{-17}\ \mathrm{Gev}^{-1}$

the number of the charged fermions $c_1 = 6$

the mass of the charged fermions $\mathrm{m} = 1 \times 10^{10}\ \mathrm{Gev}$

the number of the neutral fermions $c_2 = 3$

the mass of the neutral fermions $\mu = 1 \times 10^{10}\ \mathrm{Gev}$