Causal structure in (i) Teleparallel gravity and (ii) Massive gravity

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Y.C. Ong, K.I. J. M. Nester, P. Chen
arXiv:1309.6461 [gr-qc]
K.I., J.-A. Gu, Y.C. Ong

Class.Quant.Grav. 30 (2013) 184008
K.I., Y.C. Ong
Physics Letters B 726 (2013), 544
S. Deser, K.I., Y.C. Ong, A. Waldron

Keisuke Izumi (LeCosPA)

Modification of Gravity

- General relativity

  Singularity problem

- Quantum gravity

  Problem of renormalization

- Cosmology

  Dark energy, Dark matter

⇒ **Modified Gravity**

- Modification of Lagrangian : f(R)
- Modification of vacuum state : ghost condensation
- Modification of concept of geometry

  higher dimension : Braneworld

  other manifold : Teleparallel gravity

- Introducing mass of graviton : massive gravity

Consistency Check of modified gravity

0-th order (of cosmology) : FLRW universe without perturbation

Consistency with standard cosmology
DM and DE??

1-th order : perturbation on FLRW background

Consistency with standard cosmology
Consistency with solar system physics
Stability

Nonlinear property

Causal structure ← Main topic in this talk
Nonlinear stability

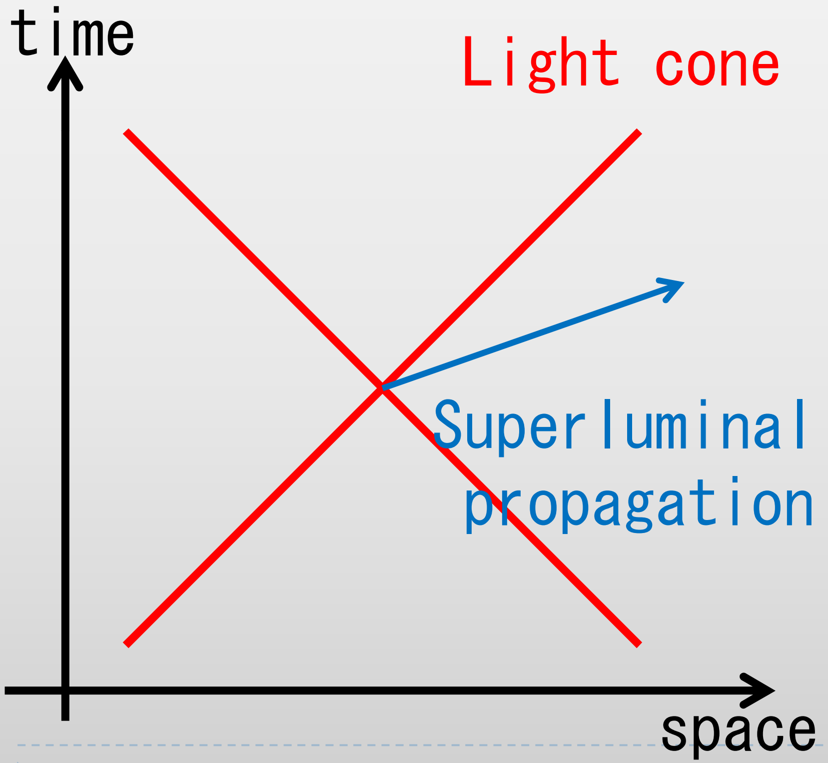
Quantization

⋮

Superluminal mode and Acausality

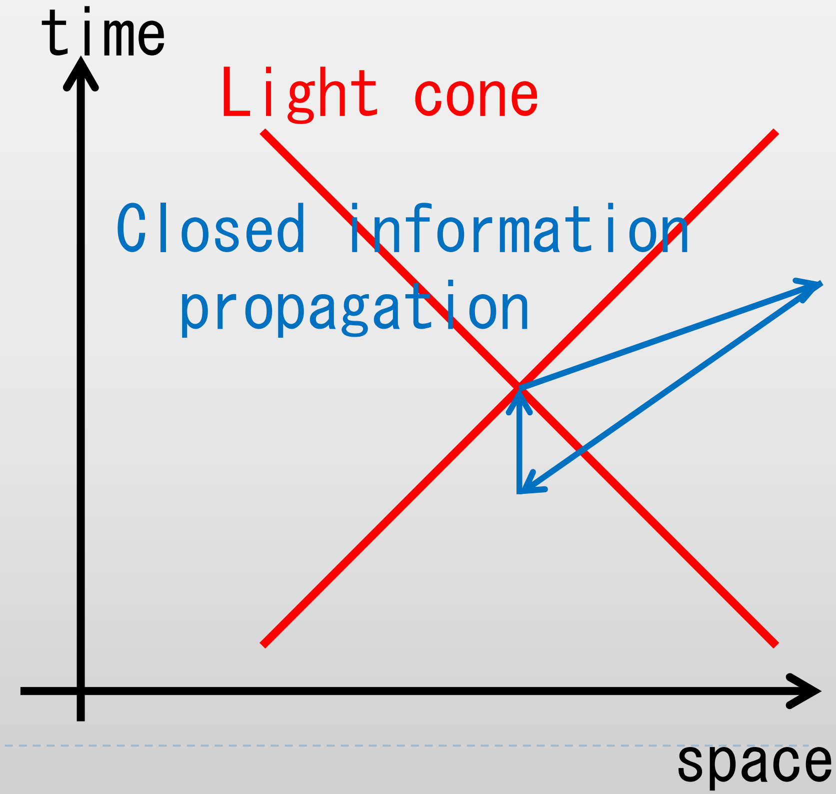
Superluminal mode

Propagation the speed of which is higher than that of light



Acausality

Pathological causal structure



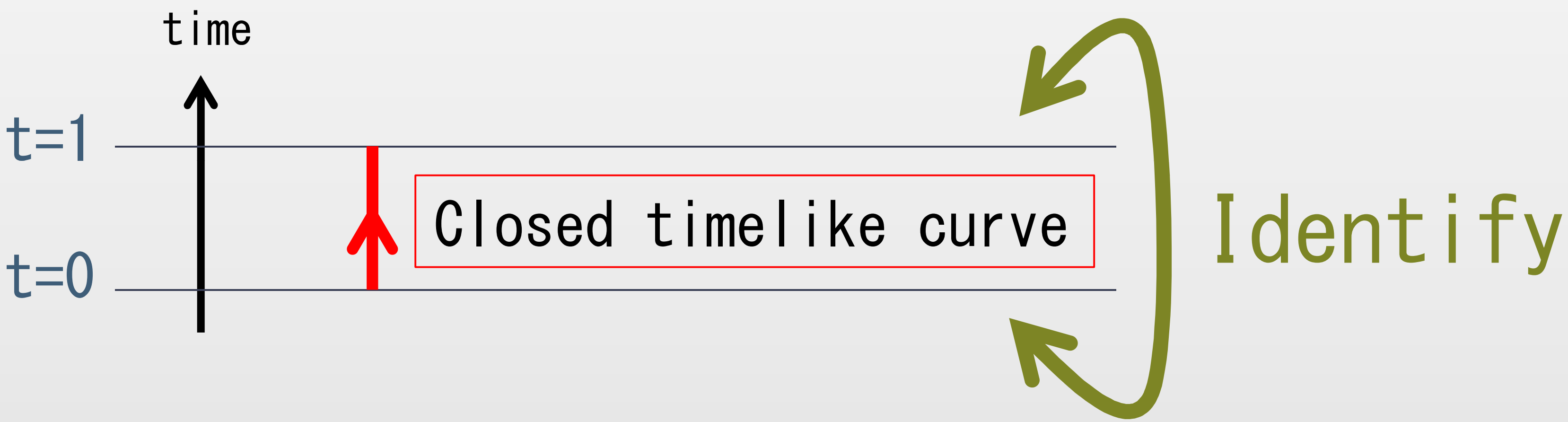
Locality of Acausality

GR: Solution with closed timelike curve

Gödel solution

Simplest example:

Identification of t=0 and t=1 in Minkowski



While closed time like curve on these solution cannot be infinitesimal,

Acausality here we discuss is local (i.e. infinitesimal acausality).

Teleparallel Gravity

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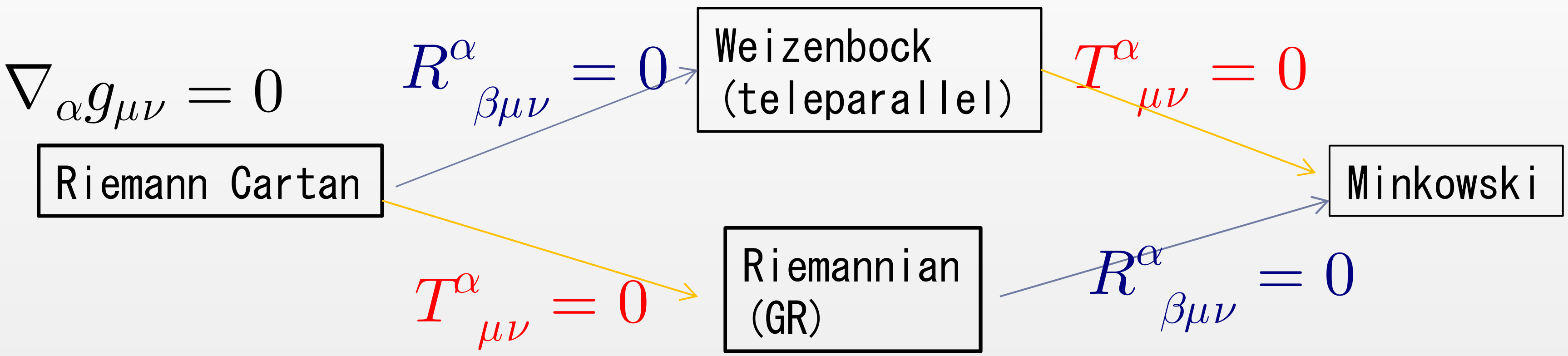
Teleparallel Gravity I



Teleparallel gravity ⟷ General Relativity

Equivalence

if

- Action is constructed from squared $T^\alpha{}_{\mu\nu}$.
- Local Lorentz symmetry is imposed.

Teleparallel Equivalent of General Relativity (TEGR)

Extension of TEGR and Its Issue

TEGR $T = \frac{1}{4} T^{\rho\mu\nu} T_{\rho\mu\nu} + \frac{1}{2} T^{\rho\mu\nu} T_{\nu\mu\rho} - T^{\mu} T_{\mu}$

$$S_{TEGR} = -\frac{1}{2\kappa} \int d^4x |e| T$$

$$T = -R^{(Levi-Civita)} - 2\nabla_{\mu}^{(Levi-civita)} T^{\mu}$$

Break Local Lorentz invariance

B. Li et al. (2011)

Motivation of extension

Dark energy

Extension of GR → f(R), Brans-Dicke theory

Extension of TEGR → f(T), Brans-Dicke extension of Teleparallel

Additional three degrees of freedom!!

Linear D.o.F. VS Non-linear D.o.F.

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Linear analysis

No additional D. o. F. (Totally 2 D. o. F.)

Non-Linear analysis

3 additional D. o. F. (Totally 5 D. o. F.)

Additional 3 D. o. F. are extremely non-linear!

e. c. $\mathcal{L} = -f(\phi)\dot{\phi}^2 + g(\phi)(\partial_i\phi)^2$

$$\Rightarrow \mathcal{L}_2 = -f(\phi)\dot{\delta\phi}^2 + g(\phi)(\partial_i\delta\phi)^2 + \cdots$$

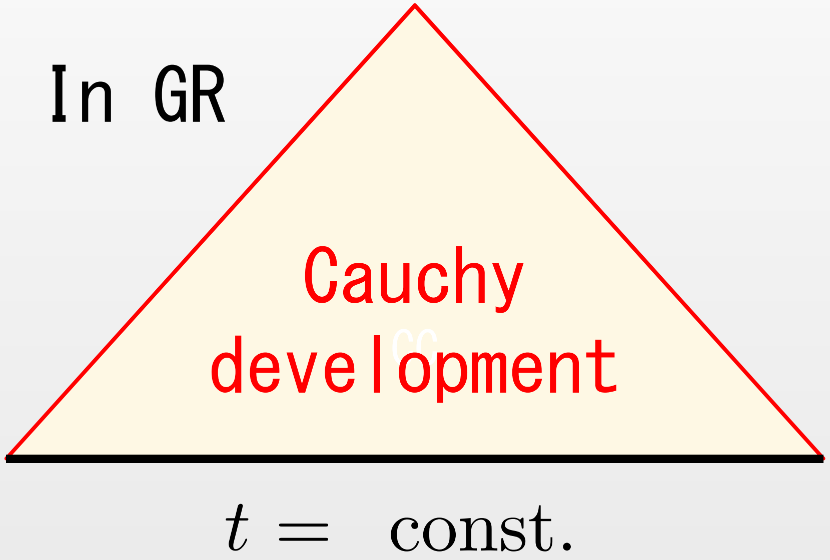
No D. o. F. in linear analysis on $f(\phi) = 0$

Speed of sound : $c_s^2 = \frac{g(\phi)}{f(\phi)} \longrightarrow \infty$ for $f(\phi) \longrightarrow 0$

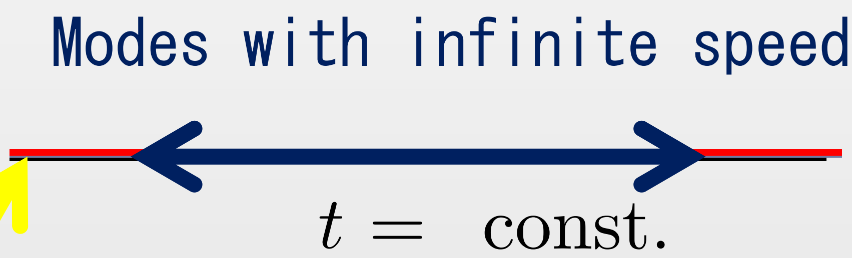
Non-linear analysis is needed!

Causal structure of FLRW in f(T)

In GR



In f(T) and BDT



Couple with gauge invariant metric

⬇

Acausal modes are observable

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Summary of teleparallel gravity

Linear analysis

2 DoF of graviton

$$(2\kappa)^{-1} \Rightarrow -f'_0$$

The theory seems to be healthy,
but indeed not!
The extra modes become nonlinear.

Causal structure

In f(T) gravity and in BDT gravity

FLRW solutions are **acausal !!**

Massive Gravity

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Nonlinear massive gravity de Rham, Gabadadze 2010

・Relation between $\Pi_{\mu\nu}$ and $H_{\mu\nu}$

For $h_{\mu\nu} = 0, A_\mu = 0$ ⇨ $H_{\mu\nu} = 2\Pi_{\mu\nu} - \Pi_{\mu\alpha} g^{\alpha\beta} \Pi_{\beta\nu}$

⇨ $g^{\mu\alpha}\Pi_{\alpha\nu} = \delta^\mu_\nu - \left(\sqrt{g^{-1}(g-H)}\right)^\mu{}_\nu$

・Replace $\Pi_{\mu\nu}$ with $K^\mu{}_\nu \equiv \delta^\mu_\nu - \left(\sqrt{g^{-1}(g-H)}\right)^\mu{}_\nu$

$$L_2 = [K]^2 - [K^2]$$

$$L_3 = [K]^3 - 3[K][K^2] + 2[K^3]$$

$$L_4 = [K]^4 - 6[K^2][K^2] + 8[K^3][K] + 3[K^2]^2 - 6[K^4]$$

No BD ghost even for $h_{\mu\nu} \neq 0, A_\mu \neq 0$ (Hassan&Rosen)

$$S_{mass} = M_{pl}^2 m_g^2 \int d^4x \sqrt{-g}\,(L_2 + \alpha_3 L_3 + \alpha_4 L_4)$$

Superluminal modes and Acausality

Superluminal modes

$$-\frac{3m^2}{2}+l^{\mu}_{o}[\bar{R}_{\mu\nu}{}^{\nu}{}_{o}+K_{\mu\nu\rho}K^{\nu\rho}{}_{o}]-\frac{1}{2}K_i l^{ij}[f\times Kl]_j=0$$

Acausality

$$f_1+f_2=0$$

f_i is the eigenvalue of f_{ij}

$$f_{\mu\nu}=e_{a\mu}f^{a}{}_{\nu}$$

$e^{a}{}_{\mu}$: dynamical tetrad

$f^{a}{}_{\nu}$: fiducial tetrad

MG cannot be a fundamental.
It should be effective theory

Superluminal modes and Acausality

Superluminal modes

$$-\frac{3m^2}{2}+l^{\mu}_{o}[\bar{R}_{\mu\nu}{}^{\nu}{}_{o}+K_{\mu\nu\rho}K^{\nu\rho}{}_{o}]-\frac{1}{2}K_i l^{ij}[f\times Kl]_j=0$$

Acausality

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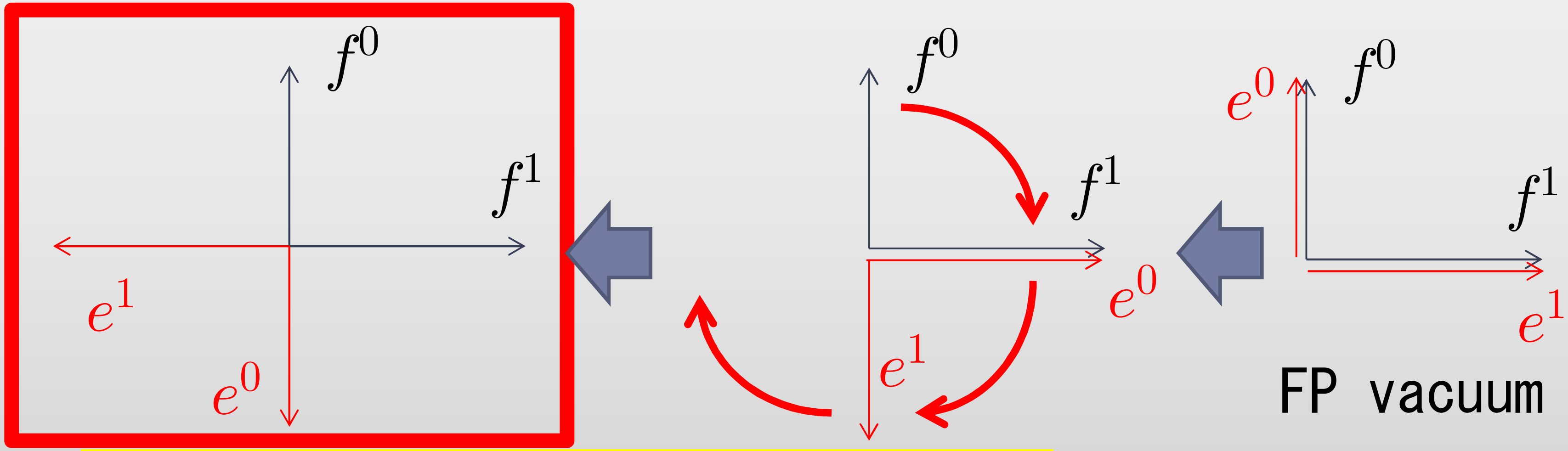

$\sqrt{M_{pl}m_{grav}}\sim 10^{-3}eV$ for $m_{grav}\sim H_0$

Summary of massive gravity

$\partial_t f_{00}$ joins in the Characteristic equations, and then we have different result from Deser and Waldron's

(Phys. Rev. Lett. 110, 111101)

➡ Not every hypersuface can be characteristics

Some of solutions have local acausal structure.

Massive gravity can not be UV-complete fundamental theory of gravity,

but can be an effective theory with cutoff scale

Rough estimation ➡ $\sqrt{M_{pl} m_{grav}} \sim 10^{-3} eV$

Thank you!

謝謝

ありがとうございました。

Cosmological Perturbation in f(T) gravity Revisited

JCAP 1306 (2013) 029

K. I., Yen Chin Ong

Model and Background Solution

Model

Gravity $S_g = \int d^4x \;\; ef(T)$

Matter $S_g = \int d^4x \;\; e\Big(-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\Big)$

Background cotedrads and scalar field

Our Background is FLRW universe

$$\bar{e}^0{}_\mu dx^\mu = dt, \;\; \bar{e}^a{}_\mu dx^\mu = a(t)\delta_{ai}dx^i \qquad a = 1,2,3$$

$$\Rightarrow \quad ds^2 = -dt + a^2\delta_{ij}dx^i dx^j$$

$$\bar{\phi} = \bar{\phi}(t)$$

Background EOM

$$f_0 - 12H^2 f_0' - \frac{1}{2}\dot{\phi}_0^2 - V = 0,$$

$$f_0 - 12H^2 f_0' - 4\dot{H} f_0' - 48H^2\dot{H} f_0'' + \frac{1}{2}\dot{\phi}_0^2 - V = 0,$$

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 + V' = 0.$$

Perturbative cotetrads and scalar field

Perturbative cotetrads

$$\delta e^{0}{}_{t} = \Phi,$$
$$\delta e^{0}{}_{i} = a\left(\partial_i \beta + u_i\right),$$
$$\delta e^{a}{}_{t} = \delta_{ai}\left(\partial_i B + v_i\right),$$
$$\delta e^{a}{}_{i} = a\delta_{aj}\left[\delta_{ij}\psi + \partial_i\partial_j E + \partial_i w_j + \partial_j w_i + h_{ij} + \epsilon_{ijk}\left(\partial_k \tilde{\sigma} + \tilde{V}_k\right)\right],$$

5 scalars Φ, β, B, ψ, E

3 vectors u_i, v_i, w_i

1 tensor h_{ij}

1 pseudoscalar $\tilde{\sigma}$

1 pseudovector $\tilde{V}_i$

Totally $5 + 1 + 3 \times 2 + 1 \times 2 + 1 \times 2 = 16$ degrees of freedom

Perturbation of scalar field

$$\phi = \bar{\phi} + \delta\phi$$

Gauge degrees of freedom

~~Local Lorentz symmetry~~ B. Li et al. (2010)

Invariance under coordinate transformation

$$x^\mu \rightarrow x'^\mu = x^\mu + \xi^\mu(x)$$

$$\xi^i = a^{-1}(\partial_i \xi + \xi_i^{(v)})$$

We can fix two scalars by D.o.F of ξ^t and ξ

one vector by D.o.F. of $\xi_i^{(v)}$

Gauge Transformation and Gauge Fixing

$$\Phi \to \Phi' = \Phi - \dot{\xi}^t.$$
$$\beta \to \beta' = \beta - \frac{1}{a}\xi^t,$$
$$B \to B' = B - \left(\dot{\xi} - \frac{\dot{a}}{a}\xi\right),$$
$$\psi \to \psi' = \psi - \frac{\dot{a}}{a}\xi^t,$$
$$E \to E' = E - \frac{1}{a}\xi,$$
$$\tilde{\sigma} \to \tilde{\sigma}' = \tilde{\sigma},$$
$$\delta\phi \to \delta\phi' = \delta\phi - \dot{\bar{\phi}} H^{-1}\xi^t$$

$$u_i \to u_i' = u_i',$$
$$v_i \to v'^i = v^i - \left(\dot{\xi}_i^{(v)} - \frac{\dot{a}}{a}\xi_i^{(v)}\right),$$
$$w_i \to w_i' = w_i - \frac{1}{2a}\xi_i^{(v)},$$
$$\tilde{V}_i \to \tilde{V}_i' = \tilde{V}_i - \frac{1}{a}\epsilon_{ijk}\partial_j\xi_k^{(v)},$$
$$h_{ij} \to h_{ij}' = h_{ij},$$

Gauge fixing

$$\beta = 0, \quad E = 0, \quad \tilde{V}_i = 0$$

Note:
Vectors and pseudovector can be coupled with each other in the following form

$$\epsilon_{ijk}\left(\partial_i u_j\right)\tilde{V}_k.$$

Scalar D.o.F

$$S_2^S = \int dt d^3x\, a^3$$

$$\times \Big[\frac{\dot\phi_0^2}{\cdots}(\dot H + 3H^2)\Big\{ \Phi + \frac{\dot H}{\cdots}\Big(-\frac{\dot\phi_0^2}{\cdots}(\dot H + 3H^2)\dot\psi + \frac{\dot\phi_0^2}{\cdots}(\dot H + 3H^2)\psi \cdots$$

Only difference from in GR appears as replacement of $(2\kappa)^{-1}$ by effective gravitational coupling $-f_0'$

$$+\frac{1}{2}\dot\alpha^2 + \frac{1}{2a^2}\alpha\Delta\alpha - \frac{1}{2}V''\alpha^2 + \left(\frac{\dot\phi_0^4}{16 f_0'^2 H^2} + \frac{3\dot\phi_0^2}{4f_0'} + \frac{\dot\phi_0 V'}{2H f_0'}\right)\alpha^2\Big], \qquad (3.47)$$

$$\alpha \equiv \delta\phi - \frac{\dot\phi_0}{H}\psi.$$

Gauge invariant form in “GR”

Overfixing Gauge D.o.F (wrong way!!)

Kinetic term of ψ appears!!

$$S_2^S = \int dt d^3x \, a^3$$

$$\times \left[\frac{\dot\phi_0^2}{2\dot H}(\dot H + 3H^2) \cdots \frac{\dot H}{\dot\phi_0^2(\dot H + 3H^2)} \left(-\frac{\dot\phi_0^2}{\dot H H}(\dot H + 3H^2)\dot\psi + \frac{\dot\phi_0^2}{H^2}(\dot H + 3H^2)\psi \right. \right.$$

$$\left. \left. + 4a^{-2} f_0' \Delta\psi \cdots V'\alpha + \frac{H}{\cdots} \dot\phi^2 a \Delta B \right) \right\}^2$$

false result!

$$\cdots \frac{\cdots}{\dot H + 3H^2} a\Delta B + \frac{\cdots}{H} \alpha + \frac{\cdots}{\dot\phi_0^2(\dot H + 3H^2)} \alpha \Big) \Big\}$$

$$+8H^2 f_0'' \left(a\Delta B - \frac{3}{4 f_0'}\dot\phi_0 \alpha \right)^2$$

$$\left. +\frac{1}{2}\dot\alpha^2 + \frac{1}{2a^2}\alpha\Delta\alpha - \frac{1}{2}V''\alpha^2 + \left(\frac{\dot\phi_0^4}{16 f_0'^2 H^2} + \frac{3\dot\phi_0^2}{4 f_0'} + \frac{\dot\phi_0 V'}{2H f_0'} \right)\alpha^2 \right], \quad (3.47)$$

Overfixing of D.o.F $\quad \Phi = 0$

Other modes

Pseudoscalar

$$S^{PS} = 0$$

Vector and Pseudovector

$$S_2^V = \int dtd^3x \; \frac{1}{2} a f_0' (v_i - u_i - 2a\dot{w}_i)\Delta(v_i - u_i - 2a\dot{w}_i).$$

Tensor

$$S_2^T = \int dtd^3x \; a^3(-f_0') \left[\dot{h}_{ij}^2 - a^{-2}(\partial_i h_{jk})^2\right].$$

Only difference from in GR appears as replacement of $(2\kappa)^{-1}$ by effective gravitational coupling $-f_0'$

Summary So far

One scalar mode which comes from D.o.F. of scalar field.
One tensor graviton.

Only difference from in GR appears as replacement of $(2\kappa)^{-1}$ by effective gravitational coupling $-f'_0$

No problem in f(T) gravity??

Causal structure in teleparallel gravity

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Characteristics on FLRW metric

FLRW tetrad and metric

$$e^0{}_\mu dx^\mu = dt \qquad e^a{}_\mu dx^\mu = a(t)\delta^a_i dx^i$$

$$g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$$

Action

f(T): $S = \int d^4x|e|f(T)$

or

BDT: $S = \int d^4x|e|\left[F(\phi)T - (\partial_\mu\phi)^2 - V(\phi)\right]$

⇩

Characteristic direction and hypersurface

$$\xi^\mu = \left(\frac{\partial}{\partial t}\right)^\mu \qquad t = \text{const.}$$

Massive Gravity

Linear theory of Massive Gravity

(Fierz and Pauli 1939)

$$L = L_{EH}^{lin}[h] + m_g^2(a_1 h_{\mu\nu}h^{\mu\nu} + a_2 h^2)$$

$$h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}$$

$$h \equiv h_\mu{}^\mu$$

- Broken diffeomorphism ⇨ Additional 4 D.o.F

  6 D.o.F ⇨ 5 D.o.F of massive spin-2

  1 D.o.F of spin-0 ⇦ ghost

- Unique linear theory without ghosts

  $a_1 + a_2 = 0$ ➡ Spin-0 mode disappears

van Dam, Veltman, Zakharov discontinuity

(1970)

Massless limit of linear massive theory
does not correspond to linear massless theory

Propagator of massless spin-2 (2 D.o.F)

$$D_{\alpha\beta,\sigma\lambda} = \frac{-i}{p^2}\left[\frac{1}{2}(\eta_{\alpha\sigma}\eta_{\beta\lambda} + \eta_{\alpha\lambda}\eta_{\beta\sigma}) - \frac{1}{2}\eta_{\alpha\beta}\eta_{\sigma\lambda}\right]$$

mismatch

Propagator of massive spin-2 (5 D.o.F)

$$D_{\alpha\beta,\sigma\lambda} = \frac{-i}{p^2+m^2}\left[\frac{1}{2}(\eta_{\alpha\sigma}\eta_{\beta\lambda} + \eta_{\alpha\lambda}\eta_{\beta\sigma}) - \frac{1}{3}\eta_{\alpha\beta}\eta_{\sigma\lambda}\right]$$

This mismatch leads to 25% off of light bending.

Vainshtein effects (1972)

Can non-linear effect resolve vDVZ discontinuity?

$$L = L_{EH}[h] + m_g^2(h_{\mu\nu}h^{\mu\nu} - h^2)$$

Non-linear terms

Spherical symmetry $ds^2 = -e^{\nu}dt^2 + e^{\sigma}dr^2 + r^2 e^{\mu}(d\theta^2 + \sin^2\theta d\phi^2)$

Week field expansion

$$r_m \equiv 2G_N M$$

$$\nu = -\frac{r_m}{r}\left[1 + O\left(\frac{r_m}{m^4 r^5}\right)\right],$$

$$\lambda = \frac{1}{2}\frac{r_m}{r}\left[1 + O\left(\frac{r_m}{m^4 r^5}\right)\right],$$

$$\mu = \frac{1}{2}\frac{r_m}{m^2 r^3}\left[1 + O\left(\frac{r_m}{m^4 r^5}\right)\right],$$

Non-linear effect becomes dominant for

$$r < r_V \equiv \left(\frac{r_m}{m^4}\right)^{\frac{1}{5}} \longrightarrow \infty$$

$$m \longrightarrow 0$$

Vainshtein effects (1972)

Can non-linear effect resolve vDVZ discontinuity?

$$L = L_{EH}[h] + m_g^2(h_{\mu\nu}h^{\mu\nu} - h^2)$$

Non-linear terms

Spherical symmetry $ds^2 = -e^{\nu}dt^2 + e^{\sigma}dr^2 + r^2 e^{\mu}(d\theta^2 + \sin^2\theta d\phi^2)$

Expansion w.r.t graviton mass $r_m \equiv 2G_N M$

$$\nu = -\frac{r_m}{r}\left[1 + O\left(\sqrt{\frac{m^4 r^5}{r_m}}\right)\right],$$

$$\lambda = \frac{r_m}{r}\left[1 + O\left(\sqrt{\frac{m^4 r^5}{r_m}}\right)\right],$$

$$\mu = \sqrt{\frac{8r_m}{13r}}\left[1 + O\left(\sqrt{\frac{m^4 r^5}{r_m}}\right)\right],$$

correspond to leading order of Schwarzschild

This expansion is valid for

$$r < r_V \equiv \left(\frac{r_m}{m^4}\right)^{\frac{1}{5}}$$

Boulware Deser (BD) ghost (1972)

$$L = L_{EH}[h] + m_g^2(a_1 h_{\mu\nu}h^{\mu\nu} + a_2 h^2)$$

lapse N and shift Ni are quadratic in the action
Equations from deviation w.r.t lapse and shift are
not like constraints for 3-dimensional metric
but fixed lapse and shift themselves ⇒ 6 D.o.F

$a_1 + a_2 = 0$ ⇐ lapse N becomes linear in linearized action

⇓ Non-linear

lapse N becomes quadratic or higher order

⇒ Spin-0 can survive

⇕

BD ghost (unbounded Hamiltonian)

Stuckelberg fields

Arkani-Hamed, Georgi & Schwarz (2003)

Covariant form of mass term

Stuckelberg scalar fields $\phi^a \ (a = 0, 1, 2, 3)$

$$g_{\mu\nu} = \eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b + H_{\mu\nu} \qquad \phi^a = x^a + \bar{A}^a$$

fiducial metric (gauge fixing $\bar{A}^a = 0 \ \rightarrow \ H_{\mu\nu} = h_{\mu\nu}$)

Spin-0 transverse

extract the spin-0 mode ⇒ $\eta_{ab}\bar{A}^b = \partial_a\pi + A_a$

$$H_{\mu\nu} = h_{\mu\nu} + \partial_\mu A_\nu + \partial_\nu A_\mu - \partial_\mu A^\alpha \partial_\nu A_\alpha$$
$$+ 2\partial_\mu\partial_\nu\pi - \partial_\mu A^\alpha\partial_\nu\partial_\alpha\pi - \partial_\nu A^\alpha\partial_\mu\partial_\alpha\pi - \partial_\mu\partial^\alpha\pi\partial_\nu\partial_\alpha\pi$$

π always appears with the second derivatives

⇒ origin of BD ghost

Avoidance of BD ghost

In order to avoid BD ghost, π should not appear → π appears only in the form of total derivatives

Concentrate only on π

$$\Pi_{\mu\nu} \equiv \partial_\mu \partial_\nu \pi$$

Three possible forms

$$L_2 = [\Pi]^2 - [\Pi^2]$$
$$L_3 = [\Pi]^3 - 3[\Pi][\Pi^2] + 2[\Pi^3]$$
$$L_4 = [\Pi]^4 - 6[\Pi^2][\Pi^2] + 8[\Pi^3][\Pi] + 3[\Pi^2]^2 - 6[\Pi^4]$$

Bracket $[T]$ means trace of T

$$([\Pi] \equiv g^{\mu\nu}\Pi_{\mu\nu})$$

Nonlinear massive gravity de Rham, Gabadadze 2010

・Relation between $\Pi_{\mu\nu}$ and $H_{\mu\nu}$

For $h_{\mu\nu} = 0, A_\mu = 0$ ➡ $H_{\mu\nu} = 2\Pi_{\mu\nu} - \Pi_{\mu\alpha} g^{\alpha\beta} \Pi_{\beta\nu}$

➡ $g^{\mu\alpha}\Pi_{\alpha\nu} = \delta^\mu_\nu - \left(\sqrt{g^{-1}(g-H)}\right)^\mu{}_\nu$

・Replace $\Pi_{\mu\nu}$ with $K^\mu{}_\nu \equiv \delta^\mu_\nu - \left(\sqrt{g^{-1}(g-H)}\right)^\mu{}_\nu$

$$L_2 = [K]^2 - [K^2]$$

$$L_3 = [K]^3 - 3[K][K^2] + 2[K^3]$$

$$L_4 = [K]^4 - 6[K^2][K^2] + 8[K^3][K] + 3[K^2]^2 - 6[K^4]$$

No BD ghost even for $h_{\mu\nu} \neq 0, A_\mu \neq 0$ (Hassan&Rosen)

$$S_{mass} = M_{pl}^2 m_g^2 \int d^4x \sqrt{-g}\,(L_2 + \alpha_3 L_3 + \alpha_4 L_4)$$

Spherically symmetric analysis on open FLRW solution in non-linear massive gravity

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C.-I. Chiang, K.I., P. Chen

FLRW universe

・No additional flat FLRW solution　　D’Amico, et al. (2011)

・No additional closed FLRW solution
・two additional open FLRW solutions　　Gumrukcuoglu, Lin, Mukohyama (2011)

$Z_{\mu\nu} \equiv \eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b$:　fiducial metric

$$\begin{cases} \phi^0 = f(t)\sqrt{1+|K|(x^2+y^2+z^2)} \\ \phi^1 = \sqrt{|K|}\,f(t)x \\ \phi^2 = \sqrt{|K|}\,f(t)y \\ \phi^3 = \sqrt{|K|}\,f(t)z \end{cases}$$

$$Z_{\mu\nu}dx^\mu dx^\nu = -\dot{f}(t)^2dt^2 + |K|f(t)^2\Omega_{ij}dx^idx^j$$

$$g_{\mu\nu}dx^\mu dx^\nu = -N(t)^2dt^2 + a(t)^2\Omega_{ij}dx^idx^j$$

$$\Omega_{ij}dx^idx^j = dx^2+dy^2+dz^2 - \frac{|K|(xdx+ydy+zdz)^2}{1+|K|(x^2+y^2+z^2)}$$

E.o.M for metric

$$3H^2 + \frac{3K}{a^2} = \Lambda_\pm + M_{pl}^{-2}\rho$$

$$\Lambda_\pm \equiv -\frac{m_g^2}{(\alpha_3+\alpha_4)^2}\left[(1+\alpha_3)(2+\alpha_3+2\alpha_3^2-3\alpha_4) \pm 2(1+\alpha_3+\alpha_3^2-\alpha_4)^{3/2}\right]$$

Cosmological perturbation

Gumrukcuoglu, Lin, Mukohyama (2011)

Gauge invariant variables in GR

$$S^{(2)} = \tilde{S}^{(2)}[Q_I, \Phi, \Psi, B_i, \gamma_{ij}] + \tilde{S}^{(2)}_{mass}[\psi^\pi, E^\pi, F^\pi_i, \gamma_{ij}]$$

$$\tilde{S} \equiv S_{EH+\tilde{\Lambda}}[g_{\mu\nu}] + S_{matter}[g_{\mu\nu}, \sigma_I] \qquad \tilde{\Lambda} \equiv \Lambda + \Lambda_\pm$$

$$\tilde{S}^{(2)}_{mass} \equiv M^2_{pl} \int d^4x N a^3 \sqrt{\Omega}\, M^2_{GW}(t) \times \left[3(\psi^\pi)^2 - \frac{1}{12}E^\pi \Delta(\Delta + 3K)E^\pi + \frac{1}{16}F^\pi_i(\Delta + 2K)F^\pi_i - \frac{1}{8}\gamma^2_{ij}\right]$$

$$M^2_{GW}(t) \equiv \pm (r-1) m^2_g X^2_\pm \sqrt{1 + \alpha_3 + \alpha_3^2 - \alpha_4} \qquad r \equiv \frac{na}{N\alpha}$$

Integrate out ψ^π, E^π and F^π_i ⇒ $S^{(2)}_{s,v} = S^{(2)}_{GR\ s,v}$

Only massive helicity-2 mode of graviton can propagate

Motivation

Massive graviton on Minkowski 5 D.o.F

Mismatch

Massive graviton on open FLRW 2 D.o.F

Where do 3 D.o.F disappear to?

2 Helicity-1 modes
1 Helicity-0 mode

In order to catch these 3 D.o.F., we look the detail.

considering only spherically symmetric case

Metrics and gauge fixing

Gauge fixing

No perturbation of Stuckelberg fields $\delta\phi^a = 0$

fiducial metric

$$Z_{\mu\nu}dx^\mu dx^\nu = -(\dot{f}(t))^2 dt^2 + \frac{|K|(f(t))^2}{1-Kr^2}dr^2 + |K|(f(t))^2 r^2 d\Omega_{(2)}^2,$$

Physical metric

$$ds^2 = -e^{2\Phi}dt^2 + \frac{a^2}{1-Kr^2}e^{2\Psi}\left(dr + \beta dt\right)^2 + a^2 r^2 e^{2E} d\Omega_{(2)}^2,$$

Result ①

$$ds^2 = -e^{2\Phi}dt^2 + \frac{a^2}{1-Kr^2}e^{2\Psi}(dr+\beta dt)^2 + a^2r^2e^{2E}d\Omega^2_{(2)},$$

Non-linear form of effective energy tensor

$$T^{eff}_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta S_{mass}}{\delta g^{\mu\nu}}$$

$$\begin{aligned}T^{\rm eff}_{mn} = -2M^2_{Pl}m^2_g\Bigg(\Bigg[\Big\{&-3+3X_\pm e^{-E}-\frac{1}{2}X^2_\pm e^{-2E}\\ &+\alpha_3\left(-2+3X_\pm e^{-E}-X^2_\pm e^{-2E}\right)-\frac{\alpha_4}{2}\left(1-X_\pm e^{-E}\right)^2\Big\}\\ &+\mathbb{W}_T\left\{\left(\frac{3}{2}-X_\pm e^{-E}\right)+\alpha_3\left(\frac{3}{2}-2X_\pm e^{-E}+\frac{1}{2}X^2_\pm e^{-2E}\right)+\frac{\alpha_4}{2}\left(1-X_\pm e^{-E}\right)^2\right\}\Bigg]g_{mn}\\ &+\left\{\left(-3+2X_\pm e^{-E}\right)+\alpha_3\left(-3+4X_\pm e^{-E}-X^2_\pm e^{-2E}\right)-\alpha_4\left(1-X_\pm e^{-E}\right)^2\right\}\mathcal{W}_{mn}\Bigg), \quad (3.13)\end{aligned}$$

$$\begin{aligned}T^{\rm eff}_{ij} = -2M^2_{Pl}m^2_g\Bigg[&\left\{-3+\frac{3}{2}X_\pm e^{-E}+\alpha_3\left(-2+\frac{3}{2}X_\pm e^{-E}\right)+\frac{\alpha_4}{2}\left(-1+X_\pm e^{-E}\right)\right\}\\ &+\mathbb{W}_T\left\{\frac{3}{2}-\frac{1}{2}X_\pm e^{-E}+\alpha_3\left(\frac{3}{2}-X_\pm e^{-E}\right)+\frac{\alpha_4}{2}\left(1-X_\pm e^{-E}\right)\right\}\\ &+\sqrt{\det(\mathbb{Z})}\left\{-\frac{1}{2}+\alpha_3\left(-1+\frac{1}{2}X_\pm e^{-E}\right)+\frac{\alpha_4}{2}\left(-1+X_\pm e^{-E}\right)\right\}\Bigg]g_{ij}, \quad (3.14)\end{aligned}$$

Only if E=0, $T^{eff}_{\mu\nu} = -M^2_{pl}\Lambda_\pm g_{\mu\nu}$

Result ②

$$ds^2 = -e^{2\Phi}dt^2 + \frac{a^2}{1-Kr^2}e^{2\Psi}\left(dr+\beta dt\right)^2 + a^2r^2e^{2E}d\Omega_{(2)}^2,$$

Even in linear, we can see the difference from in GR

Linearized E.o.M

$$\frac{2(1-Kr^2)}{a^2r^2}A + \frac{2(1-Kr^2)}{a^2r}A' + 2H^2B = 0,$$

$$\frac{2}{r}\left(\frac{\dot{H}}{H}A + HB\right) = 0,$$

$$\frac{2}{r^2}\left[\left(\frac{\dot{H}}{H^2}-1\right)A - \frac{1}{H}\left(\dot{A}-HB\right)\right] = -2m_g^2C_\pm\left(\frac{a^2}{1-Kr^2}\right)\left(1-\frac{aH}{\sqrt{|K|}}\right)E,$$

$$r(1-Kr^2)B' - a^2r^2(\dot{H}B + H\dot{B}) - (2Kr^2+3a^2r^2H^2)B - \frac{r(1-Kr^2)}{H}\dot{A}' + \frac{Kr^2}{H}\dot{A} - r(1-Kr^2)\left(1-\frac{\dot{H}}{H^2}\right)A' = -m_g^2C_\pm a^2r^2\left(1-\frac{aH}{\sqrt{|K|}}\right)(\Psi+E).$$

A, B ⇧ Gauge invariant combination in GR

eliminate B, Ψ

$$A' = -\frac{(1-3Kr^2)}{r(1-Kr^2)}A. \quad \Rightarrow \quad A = \frac{1}{8\pi M_{Pl}^2}\frac{M(t)}{ar(1-Kr^2)}$$

$$-\frac{2}{H^2r^2}\left(H^2A + H\dot{A}\right) = -2m_g^2C_\pm\left(\frac{a^2}{1-Kr^2}\right)\left(1-\frac{aH}{\sqrt{|K|}}\right)E,$$

While in GR ($m_g = 0$) we can fix M(t),

- in MG we cannot fix M(t)

Result ③ & Discussion

$$A = \frac{1}{8\pi M_{Pl}^2} \frac{M(t)}{ar(1-Kr^2)},$$

$$B = -\frac{1}{8\pi M_{Pl}^2} \frac{K}{a^2 H^2} \frac{M(t)}{ar(1-Kr^2)},$$

Gauge invariant variables in GR

$$E = \frac{1}{8\pi M_{Pl}^2} \frac{1}{m_g^2 C_\pm} \left(1 - \frac{aH}{\sqrt{|K|}}\right)^{-1} \frac{\dot{M}(t)}{a^3 r^3},$$

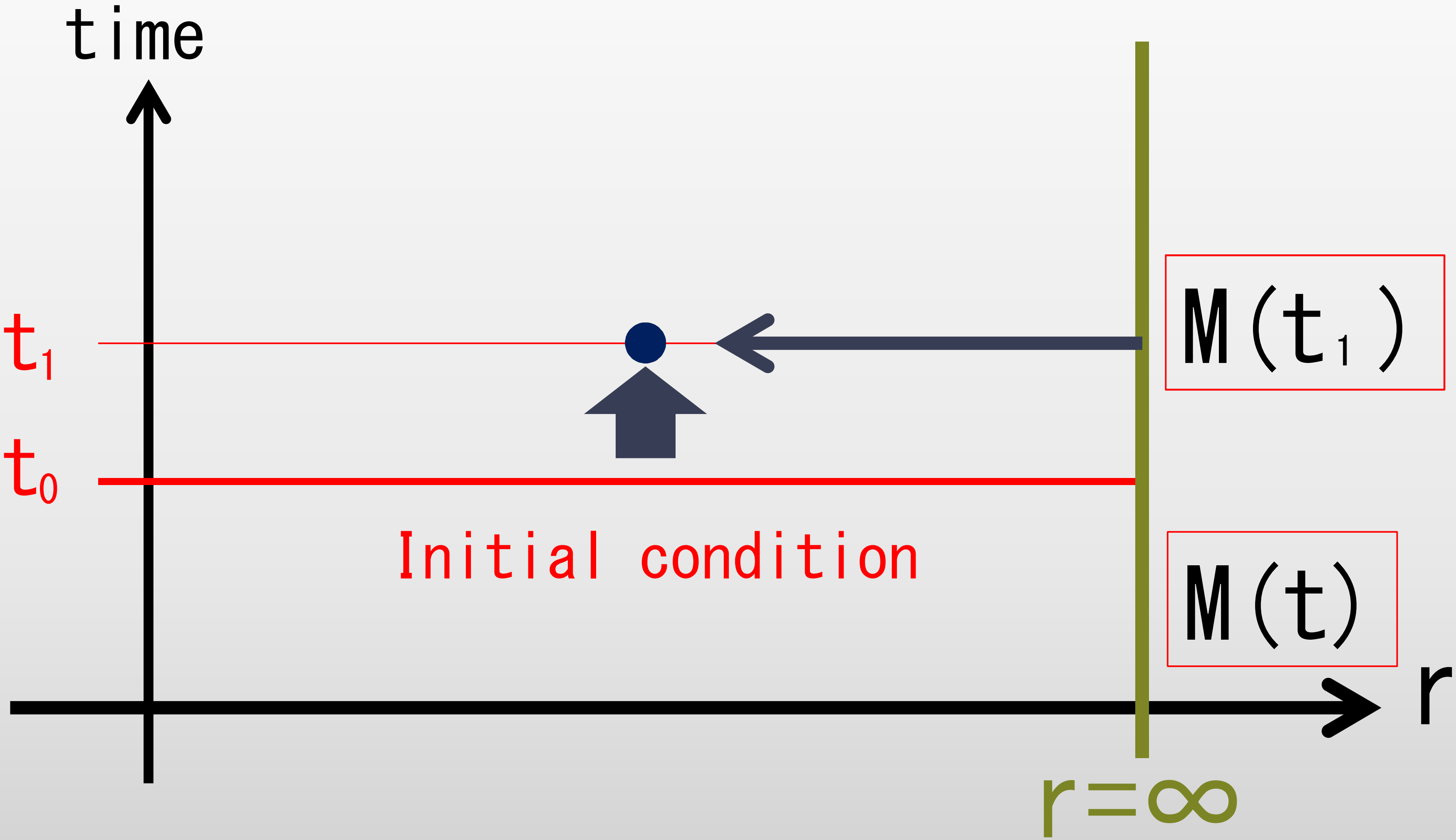
$$\Psi = -\frac{1}{8\pi M_{Pl}^2} \frac{2}{m_g^2 C_\pm} \left(1 - \frac{aH}{\sqrt{|K|}}\right)^{-1} \frac{\dot{M}(t)}{a^3 r^3},$$

Non-gauge-invariant variables in GR

- Only when $\dot{M} = 0 \ (E = 0)$ solution becomes the same as that in GR
- D.o.F. of M(t) is probably related to helicity-0 mode
- M(t) depends only on time

➡ Infinite speed of propagation??

Meaning of Time-Dependent Mass



Causal structure in massive gravity

Class.Quant.Grav. 30 (2013) 184008
K. I., Y.-C. Ong
Physics Letters B 726 (2013), 544
S. Deser, K. I., Y.-C. Ong, A. Waldron

Motivation

Class.Quant.Grav. 30 (2013) 184008
K.I., Y.-C. Ong

In **Phys. Rev. Lett. 110, 111101 (Deser and Waldron)**

Superluminal propagation
Acausality

Based on Characteristics analysis

not every

On ~~any~~ hypersurface
Characteristics equation is satisfied
and thus information can propagate to any direction
even if it is spacelike.

We revisited this problem
and obtain different result

Our model of Massive gravity

Action

$$S = (2\kappa)^{-1} \int d^4 x e (R + 2 \sum_{i=0}^{4} \alpha_i \mathcal{L}_i)$$

$$\mathcal{L}_0 = 1$$
$$\mathcal{L}_1 = f$$
$$\mathcal{L}_2 = f^2 - f_{\mu\nu} f^{\mu\nu}$$
$$\vdots$$

$$f_{\mu\nu} = e_{a\mu} f^a{}_\nu$$

$e^a{}_\mu$: dynamical tetrad

$f^a{}_\nu$: fiducial tetrad

$$f = f_{\mu\nu} g^{\mu\nu}$$

For simplicity, consider the case where $\alpha_2 = \alpha_3 = \alpha_4 = 0$

EoM

$$G_{\mu\nu} - \alpha_0 g_{\mu\nu} + \alpha_1 (f_{\nu\mu} + f g_{\mu\nu}) = 0$$

⇒ $f_{\mu\nu} = f_{\nu\mu}$

Chara

Introd

D & W

$$\partial_t\partial_t f_{00}, \partial_t\partial_t f, \partial_t\partial_t f$$

$$\mathbf{M} = \begin{pmatrix} 0 & B & C \\ 0 & E & F \\ 0 & H & I \end{pmatrix}$$

$M_{0i0} =$ ... f_{0i}

$M_{0ij} = -M_{i0j} - \frac{1}{2}(\partial_0 f_{ij} + \partial_i f_{0j})$

$\partial M = \partial\partial f$

$\partial_0 M_{ij0}$ ... M_{ij0}

$\partial_0 M_{ijk}$ ... M_{ijk}

$$\partial_t f_{00}, \partial_t M_{0i0}, \partial_t M_{0ij}$$

$$\mathbf{M} = \begin{pmatrix} A & B & C \\ D & E & F \\ G & H & I \end{pmatrix}$$

EoM

$G_{\mu\nu}$

Characteristic equation: $\det \mathbf{M}[\mathbf{U}] = 0$