

Higgs Vacuum Stability and Physics Beyond the Standard Model

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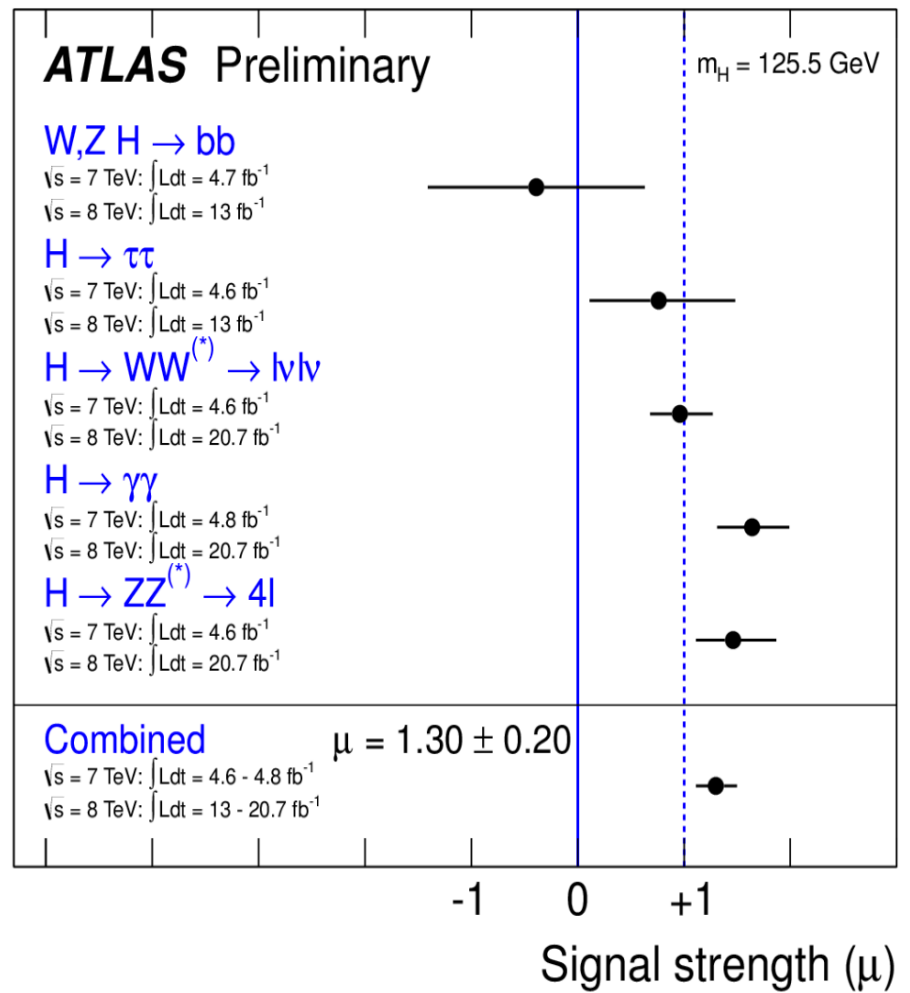
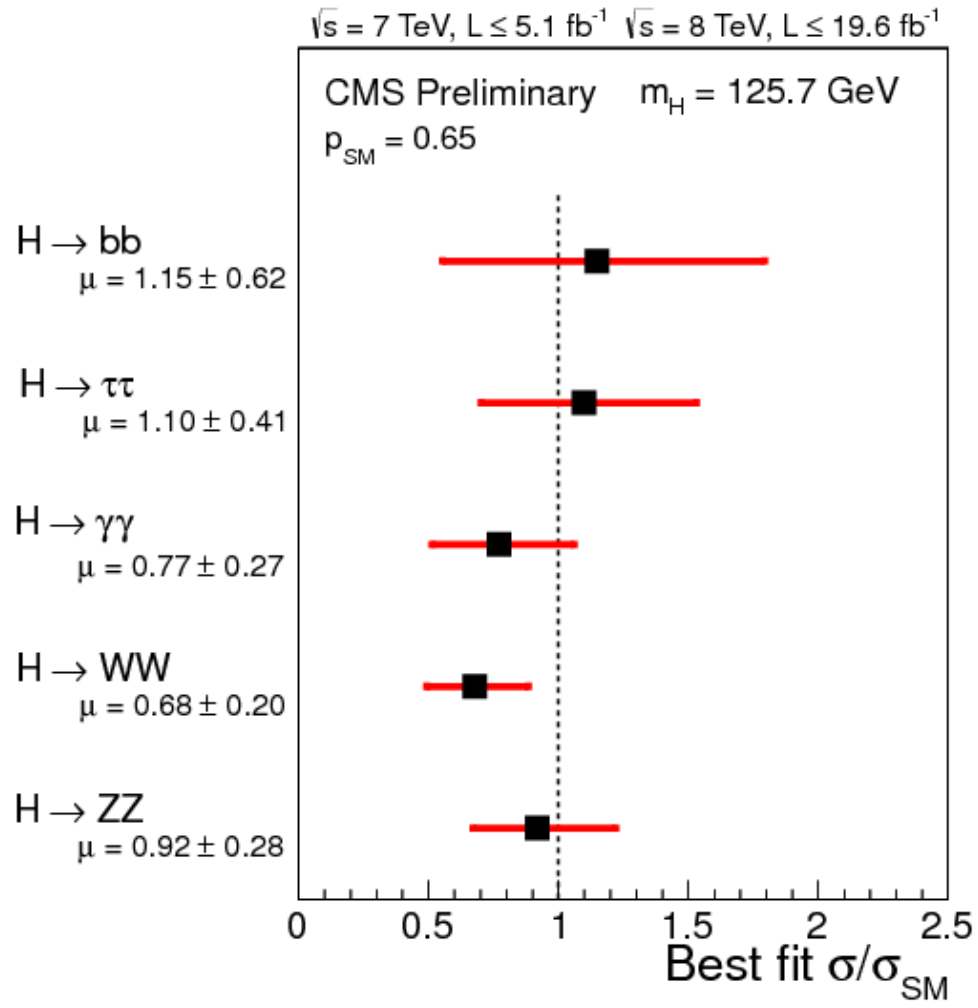
AK & A. Spencer-Smith, Phys Lett B 722 (2013) 130 [arXiv:1301.2846]

AK & A. Spencer-Smith, JHEP 1308 (2013) 036 [arXiv:1305.7283]

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Combined 2012-13 data: It is a Higgs boson, maybe even the Higgs boson



The Standard Model Higgs with $m_h=125-126$ GeV

- **Naturalness problem:** somewhat heavy than typical prediction of the supersymmetric models and somewhat light than typical prediction of technicolour models.
- More notably, the Standard Model vacuum state $|0\rangle_{EW}$

$${}_{EW}\langle 0|h|0\rangle_{EW} = v_{EW} \approx 246 \text{ GeV}$$

is a false (local) vacuum. The true vacuum state

$$\langle 0|h|0\rangle \sim M_P \approx 10^{18} \text{ GeV} ,$$

and it carries large negative energy density $\sim - (M_P)^4$.

- **How long does the electroweak vacuum live?**

EW vacuum lifetime: flat spacetime estimate

- Electroweak Higgs doublet (in the unitary gauge): $H = \begin{pmatrix} 0 \\ h(x)/\sqrt{2} \end{pmatrix}$

$$V_H^{(0)}(h) = \frac{\lambda}{8} (h^2 - v_{EW})^2$$

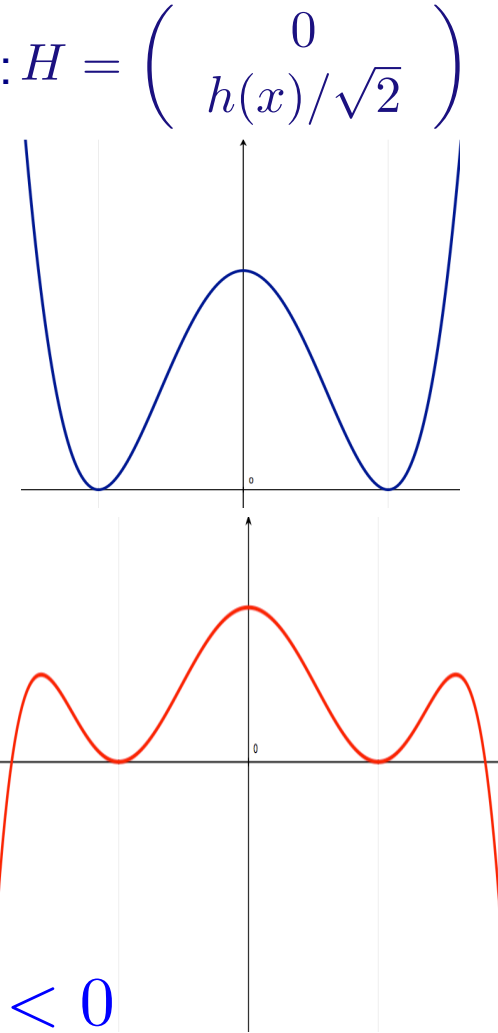
- Effective (quantum-corrected) potential

$$V_H^{(1\text{-loop})}(h) = \frac{\lambda(h)}{8} (h^2 - v_{EW})^2 ,$$

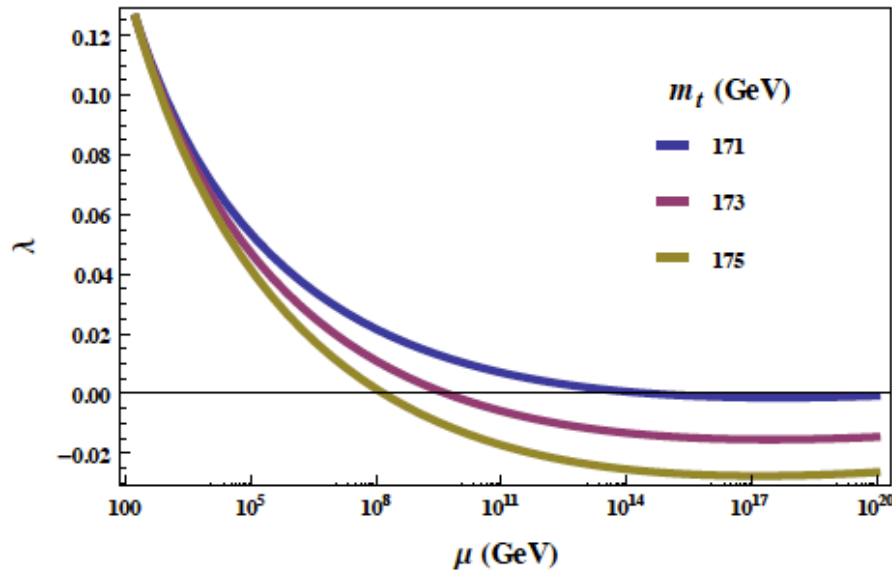
$$\lambda(h) = \lambda(\mu) + \beta_\lambda \ln(h/\mu)$$

$$(4\pi)^2 \beta_\lambda = -6y_t^4 + 24\lambda^2 + \dots$$

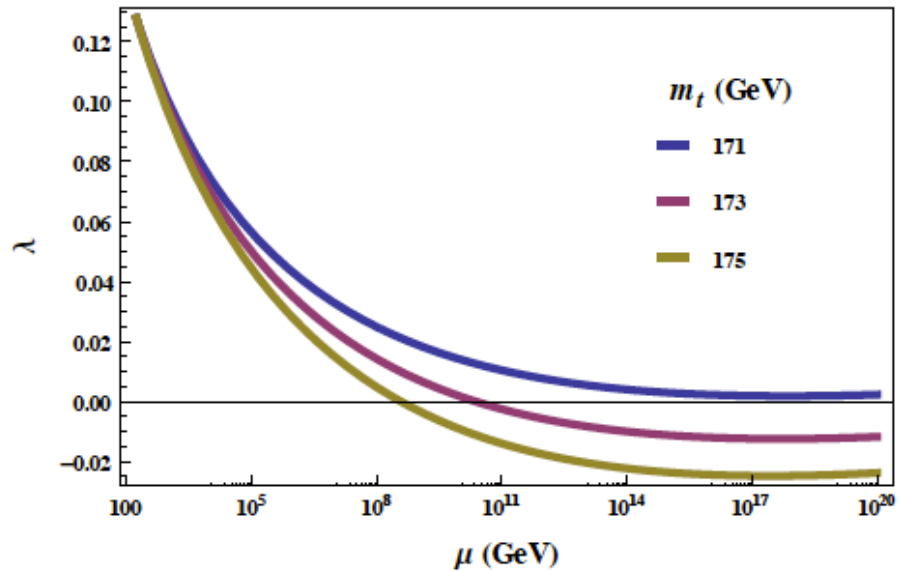
$$y_t(m_t) \approx 1 , \quad \lambda(m_h) \approx 0.13 \quad \longrightarrow \quad \beta_\lambda < 0$$



EW vacuum lifetime: flat spacetime estimate



(a) $m_h = 125$ GeV



(b) $m_h = 126$ GeV

Figure 1: Two loop running of the Higgs quartic coupling in the SM.

AK & A. Spencer-Smith, arXiv:1305.7283

Instability scale $\lambda(\mu_i) = 0, \mu_i \approx 10^{10}$ GeV

EW vacuum lifetime: flat spacetime estimate

- Large field limit:

$$V_H = \frac{\bar{\beta}_\lambda \ln(h/\mu_i)}{4} h^4, \quad \bar{\beta}_\lambda = \beta_\lambda|_{\mu=\mu_i}$$

$$h_* = \mu_i e^{-1/4}$$

- Using Coleman's prescription, one can calculate that the decay of electroweak vacuum is dominated by small size Lee-Wick bounce solution,

$$R \sim 1/\mu_m \approx 10^{-17}/\text{GeV}, \quad \beta_\lambda|_{\mu=\mu_m} = 0$$

$$S_{\text{LW}} = \frac{8\pi^2}{3|\lambda(\mu_m)|}, \quad |\lambda(\mu_m)| \approx 0.01 - 0.02$$

$$P_{\text{EW}} = e^{-p} \approx 1, \quad p = (\mu_m/H_0)^4 \exp(-S_{\text{LW}}) \ll 1$$

Electroweak vacuum in the Standard Model is metastable!

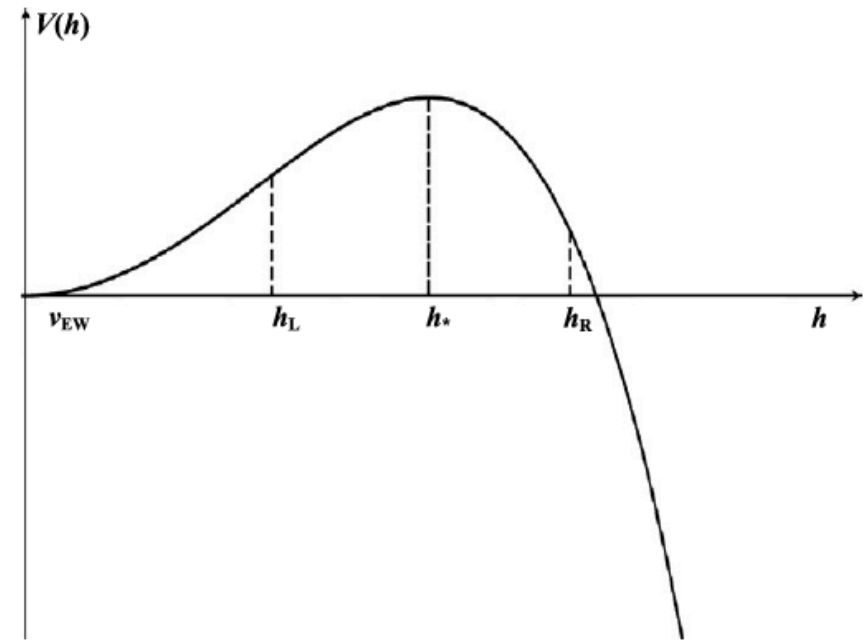


Fig. 1. The Higgs potential. For large values of the Higgs field h , the electroweak vacuum configuration is regarded as trivial, $v_{\text{EW}} \approx 0$.

EW vacuum in flat spacetime

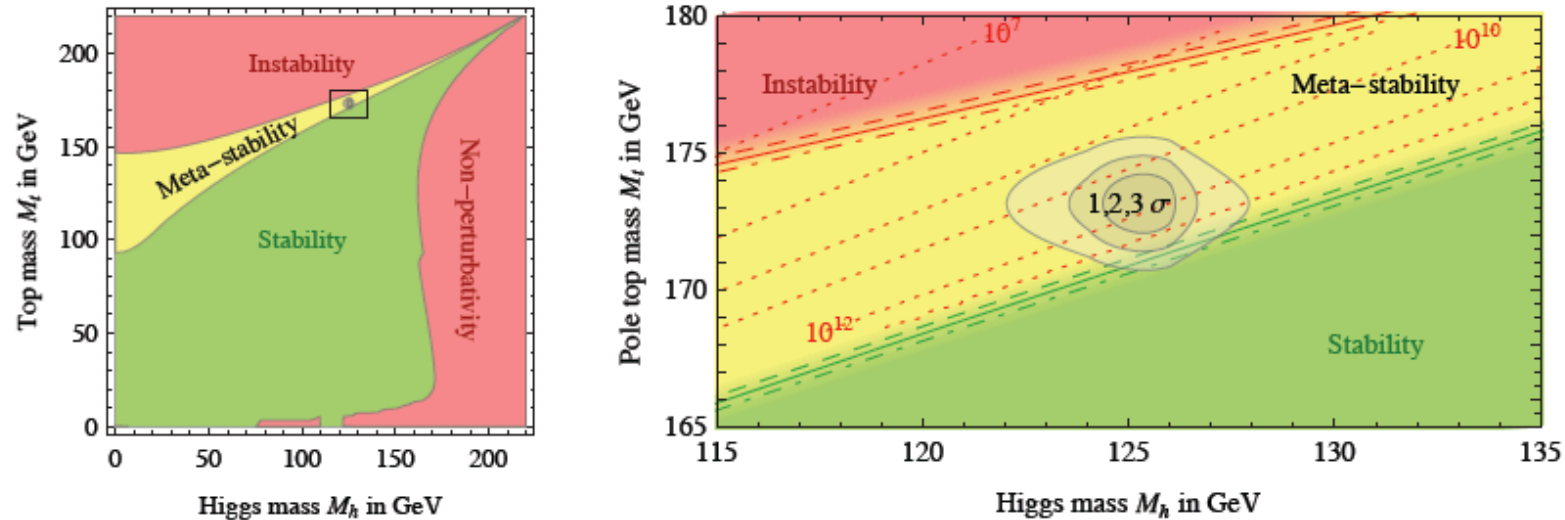


Figure 5: *Regions of absolute stability, meta-stability and instability of the SM vacuum in the M_t – M_h plane. Right: Zoom in the region of the preferred experimental range of M_h and M_t (the gray areas denote the allowed region at 1, 2, and 3 σ). The three boundaries lines correspond to $\alpha_s(M_Z) = 0.1184 \pm 0.0007$, and the grading of the colors indicates the size of the theoretical error. The dotted contour-lines show the instability scale Λ in GeV assuming $\alpha_s(M_Z) = 0.1184$.*

For $m_h < 126$ GeV, stability up to the Planck mass is excluded at 98% C.L. – J. Elias-Miro, et al, JHEP 1206 (2012) 031 [arXiv:126497]

EW vacuum in inflationary universe

- Electroweak vacuum decay may qualitatively differ in cosmological spacetimes:

(i) Thermal activation of a decay process, $T_r < \mu_i$

(ii) Production of large amplitude Higgs perturbations during inflation,

$$H_{\text{inf}} < \mu_i \quad [\text{J.R. Espinosa, G.F. Giudice, A. Riotto, JCAP 0805 (2008) 002}]$$

The bound that follows from the above consideration can be avoided, e.g., in curvaton models, or when $m_h^{\text{eff}} > H_{\text{inf}}$

- Actually, the dominant decay processes are due to instantons, (Hawking-Moss, or more generic CdL) [AK & A. Spencer-Smith, Phys Lett B 722 (2013) 130 [arXiv:1301.2846]]

$$V(h, \phi) = V_{\text{H}}(h) + V_{\text{inf}}(\phi) + V_{\text{H-inf}}$$

$$V_{\text{inf}}(\phi) = \mathcal{V}_{\text{inf}} + V'_*(\phi - \phi_{\text{inf}}) + 1/2V''_*(\phi - \phi_{\text{inf}})^2 + \dots$$

$$\epsilon = \frac{M_P^2}{2} \left(\frac{V'_*}{\mathcal{V}_{\text{inf}}} \right)^2 \ll 1, \quad -1 \ll \eta = M_P^2 \frac{V''_*}{\mathcal{V}_{\text{inf}}} \ll 1$$

EW vacuum in inflationary universe

- Fixed background approximation: $\phi = \phi_{\text{inf}}, ds^2 = d\chi^2 + \rho^2(\chi)d\Omega_3,$

$$\rho(\chi) = H_{\text{inf}}^{-1} \sin(H_{\text{inf}}\chi), \quad \chi = t^2 + r^2, \quad \chi \in [0, \pi/H_{\text{inf}}], \quad H_{\text{inf}}^2 = \mathcal{V}_{\text{inf}}/3M_P^2$$

- EoM for Higgs field:

$$\ddot{h} + 3H_{\text{inf}} \cot(H_{\text{inf}}\chi)\dot{h} = \frac{\partial V(\phi_{\text{inf}}, h)}{\partial h}$$

$$\dot{h}(0) = \dot{h}(\pi/H_{\text{inf}}) = 0$$

$$h_L(x_*) = h_R(x_*)$$

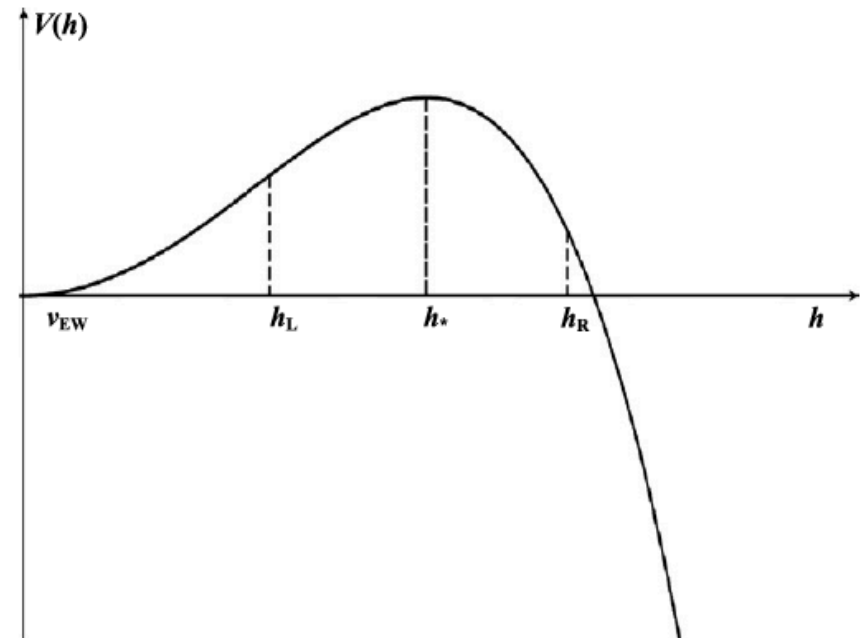


Fig. 1. The Higgs potential. For large values of the Higgs field h , the electroweak vacuum configuration is regarded as trivial, $v_{\text{EW}} \approx 0$.

EW vacuum in inflationary universe

- Hawking-Moss instanton: $\frac{\partial V}{\partial h} = 0$, $h(x) = h_*$,

$$p \approx \exp \left\{ -\frac{8\pi^2}{3} \frac{V_H(h_*) + V_{H-\text{inf}}(\phi_{\text{inf}}, h_*)}{H_{\text{inf}}^4} \right\}$$

- For $V_{H-\text{inf}}(\phi_{\text{inf}}, h_*) \ll V(h_*)$,

HM transition generates a fast decay of the electroweak vacuum, unless

$$H_{\text{inf}} < 10^9 (10^{12}) \text{ GeV}$$

$$m_h = 126 \text{ GeV}, \quad m_t = 174(172) \text{ GeV}$$

- Together with $n_s < 1$, this implies that only small-field inflationary models are allowed with a negligible tensor/scalar:

$$r < 10^{-11} (10^{-5})$$

EW vacuum in inflationary universe

- Consider, $V_{H-\text{inf}} = \frac{\alpha}{2} h^2 \phi^2$ ($\alpha > 0$), $m_h^{\text{eff}} = \alpha^{1/2} \phi_{\text{inf}} > H_{\text{inf}}$

[O. Lebedev & A. Westphal Phys.Lett. B719 (2013) 415]

[similar consideration applies $\frac{\xi}{2} R^2 h^2$]

$$h_* = (-\alpha/\lambda)^{1/2} \phi_{\text{inf}} > \mu_i, \quad (\lambda(h_*) < 0)$$

- Large-field chaotic inflation [$V_{\text{inf}} = 1/2 m_\phi^2 \phi^2$, $m_\phi = 10^{-5} M_{\text{P}}$], with

$$\alpha > 1.4 \sqrt{|\lambda|} (H_{\text{inf}}/\phi_{\text{inf}})^2 > 6 \cdot 10^{-12} .$$

- Naturalness constraint:

$$\alpha < 64\pi^2 (m_\phi/m_h)^2 \approx 2 \cdot 10^{-20}$$

Tuning is needed!

EW vacuum in inflationary universe

- In the limit $m_h^{\text{eff}} \gg H_{\text{inf}}$

$$h'' + 3h'\chi = \frac{\partial V(\phi_{\text{inf}}, h)}{\partial h}, \quad [x = m_h^{\text{eff}} \chi]$$

$$h(x) = \begin{cases} 8h_R \left(8 + \left(\frac{h_R}{h_*} \right)^2 x^2 \right)^{-1}, & 0 \leq x < x_* \\ \frac{x_* h_*}{x(J_1(ix_*) + iY_1(-ix_*))} (J_1(ix) + iY_1(-ix)), & x_* < x < \infty \end{cases},$$

$$x_* = \frac{2\sqrt{2}h_*}{h_R} \left(\frac{h_R}{h_*} - 1 \right)^{1/2}.$$

$$B_{\text{CdL}} = -\frac{2\pi^2}{\lambda} I < 0, \quad I = \int_0^\infty x^3 dx \left[h^2(x) \left(1 - \frac{h^2(x)}{2h_*^2} \right) \right] < 0, \quad \lambda(\mu > \mu_i) < 0.$$

$p \propto \exp\{-B_{\text{CdL}}\} \gg 1$ EW vacuum is unstable!

EW vacuum in inflationary universe

- Fast decay of EW ceases inflation globally (no eternal inflation)

$$e^{3H_{\text{inf}}\tau} e^{-(\tau H_{\text{inf}})^4 p}$$

$$\tau_{\text{stop}} \approx (3/p)^{1/3} H_{\text{inf}}^{-1} < 1.4 H_{\text{inf}}^{-1}$$

- The above considerations applies to models with curvaton

Conclusions:

- For the ‘unperturbed’ SM potential the condition of vacuum metastability rules out all large scale models. All models with sizeable Higgs-inflaton interactions are ruled out.
- Observation of tensor perturbations in the CMB by the Planck satellite would provide a strong indication of new physics beyond the Standard Model.

Neutrino masses and vacuum stability

AK & A. Spencer-Smith, JHEP 1308 (2013) 036 [arXiv:1305.7283]

$$\beta_\lambda^{(1)} = 24\lambda^2 - 6y_t^4 + \frac{3}{4}g_2^4 + \frac{3}{8}(g_1^2 + g_2^2)^2 + \lambda(-9g_2^2 - 3g_1^2 + 12y_t^2)$$

- ① Extension of the electroweak gauge sector
 - ② Extension of a scalar sector
 - ③ Extension of the fermionic sector
-
- Working with MS-bar couplings:
 - ① Modification of beta-functions above the particle mass threshold
 - ② Finite threshold correction due to the matching of low and high energy theories

 - Neutrino oscillations (= masses) provide the most compelling evidence for the physics beyond the Standard Model.

Neutrino masses and vacuum stability

AK & A. Spencer-Smith, JHEP 1308 (2013) 036 [arXiv:1305.7283]

- Type I see-saw models: additional massive sterile neutrinos – not capable to solve the vacuum stability problem
- Type III see-saw models: additional electroweak-triplet fermions – may solve the problem for very specific range of parameters

More promising candidates:

- Type III see-saw: additional electroweak-triplet scalar
- Left-right symmetric models: additional gauge bosons, scalars and fermions

Type II see-saw models and vacuum stability

Scalar potential:

$$V(\phi, \Delta) = -m_\phi^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 + m_\Delta^2 \text{tr}(\Delta^\dagger \Delta) + \frac{\lambda_1}{2} (\text{tr}(\Delta^\dagger \Delta))^2 + \frac{\lambda_2}{2} [(\text{tr}(\Delta^\dagger \Delta))^2 - \text{tr}(\Delta^\dagger \Delta)^2] + \lambda_4 (\phi^\dagger \phi) \text{tr}(\Delta^\dagger \Delta) + \lambda_5 \phi^\dagger [\Delta^\dagger, \Delta] \phi + \left[\frac{\lambda_6}{\sqrt{2}} \phi^T i\sigma_2 \Delta^\dagger \phi + \text{h.c.} \right].$$

Neutrino masses:

$$\frac{1}{\sqrt{2}} (y_\Delta)_{fg} l_L^{Tf} C i\sigma_2 \Delta l_L^g + \text{h.c.}$$

Type II see-saw models and vacuum stability

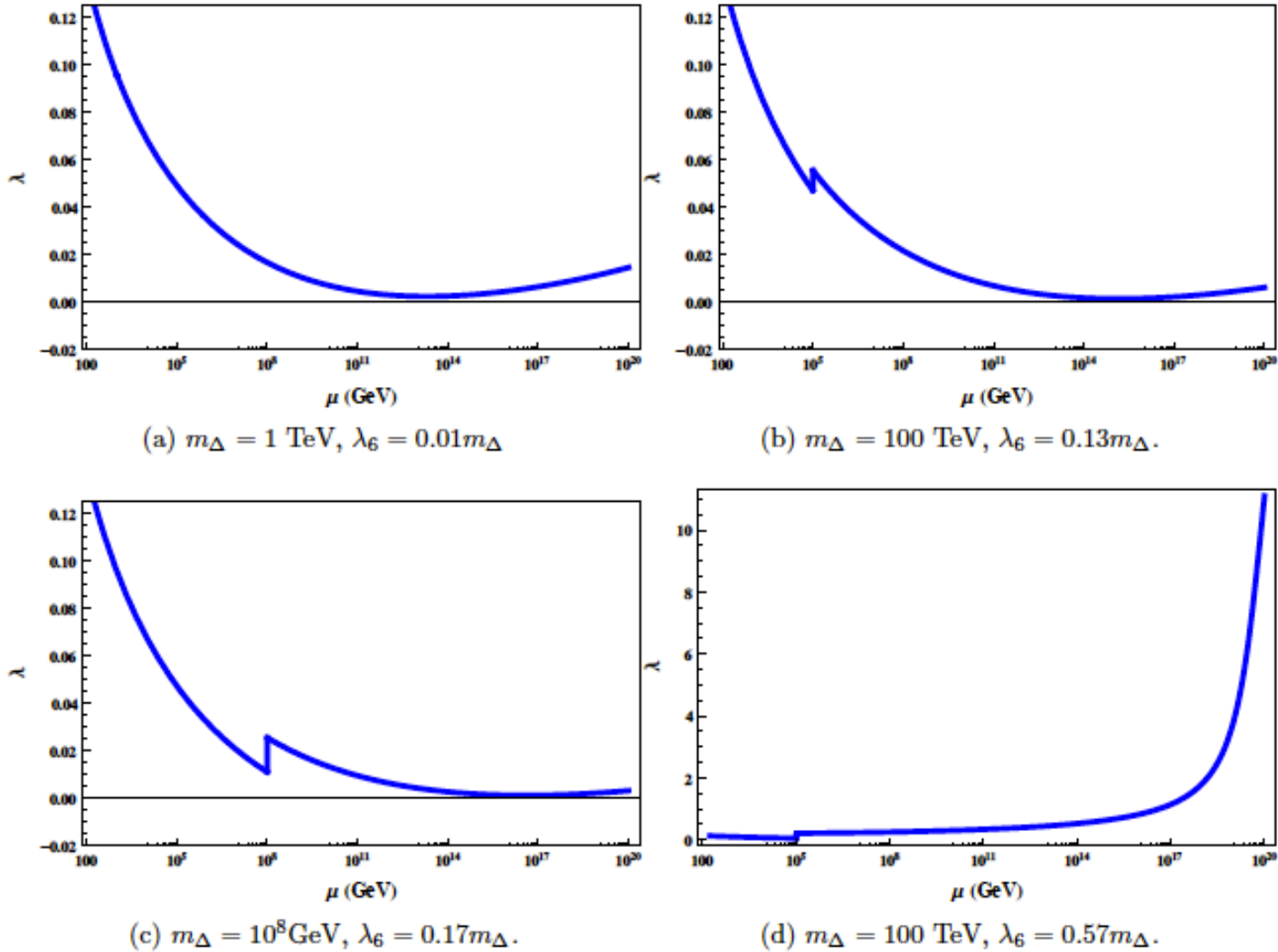
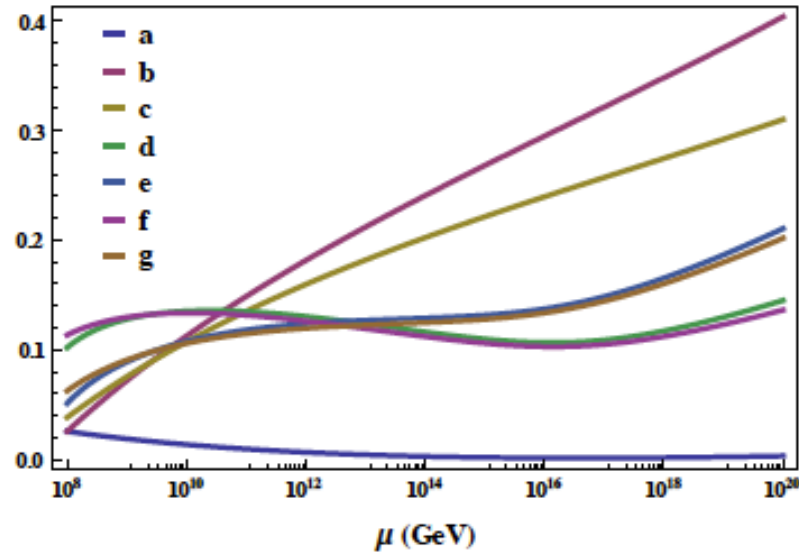
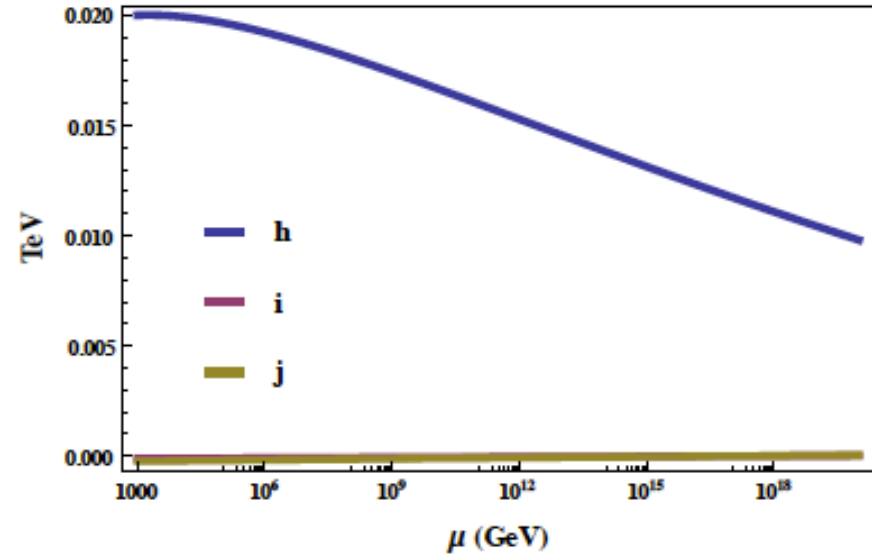


Figure 2: One loop running of the Higgs quartic coupling in the type-II seesaw model, with $m_h = 125 \text{ GeV}$ and $m_t = 173 \text{ GeV}$.

Type II see-saw models and vacuum stability



(a) $m_\Delta = 10^8 \text{ GeV}$, $\lambda_6 = 0.17 m_\Delta$.



(b) $m_\Delta = 1 \text{ TeV}$, $\lambda_6 = 0.01 m_\Delta$.

Figure 3: Conditions for stability and absence of tachyonic modes in the scalar potential of the type-II seesaw model, with $m_h = 125 \text{ GeV}$ and $m_t = 175 \text{ GeV}$. Line labels correspond to the stability conditions (3.9)-(3.16) with $a \equiv \lambda$, $b \equiv \lambda_1$, $c \equiv \lambda_1 + \frac{\lambda_2}{2}$, $d/e \equiv \lambda_4 \pm \lambda_5 + 2\sqrt{\lambda\lambda_1}$, $f/g \equiv \lambda_4 \pm \lambda_5 + 2\sqrt{\lambda(\lambda_1 + \frac{\lambda_2}{2})}$, $h \equiv \lambda_6$, $i \equiv -\lambda_5 v_\Delta$, $j \equiv -2\lambda_5 v_\Delta - \frac{\lambda_2 v_\Delta^3}{v_{EW}^2}$.

Alternating LR-symmetric model with universal see-saw for all fermion masses

Scalar sector:

$$\phi_L \in (\mathbf{1}, \mathbf{1}, \mathbf{2}, 1/2), \quad \phi_R \in (\mathbf{1}, \mathbf{2}, \mathbf{1}, 1/2).$$

$$V(\phi_L, \phi_R) = -m^2 (\phi_L^\dagger \phi_L + \phi_R^\dagger \phi_R) + \frac{\lambda}{2} (\phi_L^\dagger \phi_L + \phi_R^\dagger \phi_R)^2 + \sigma \phi_L^\dagger \phi_L \phi_R^\dagger \phi_R.$$

New fermions:

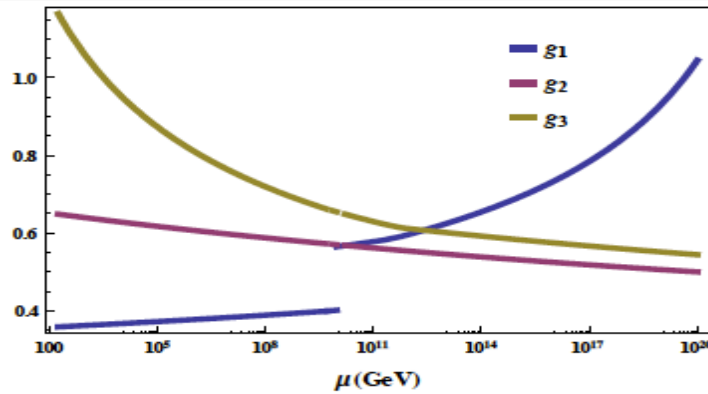
$$N_L, N_R \in (\mathbf{1}, \mathbf{1}, \mathbf{1}, 0),$$

$$E_L, E_R \in (\mathbf{1}, \mathbf{1}, \mathbf{1}, -1),$$

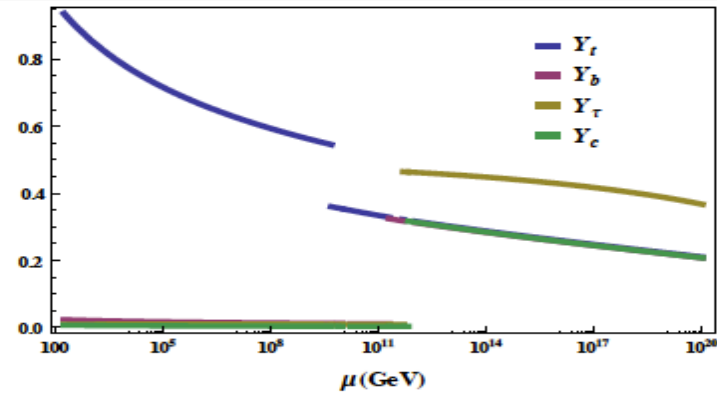
$$U_{iL}, U_{iR} \in (\mathbf{3}, \mathbf{1}, \mathbf{1}, 2/3),$$

$$D_{iL}, D_{iR} \in (\mathbf{3}, \mathbf{1}, \mathbf{1}, -1/3),$$

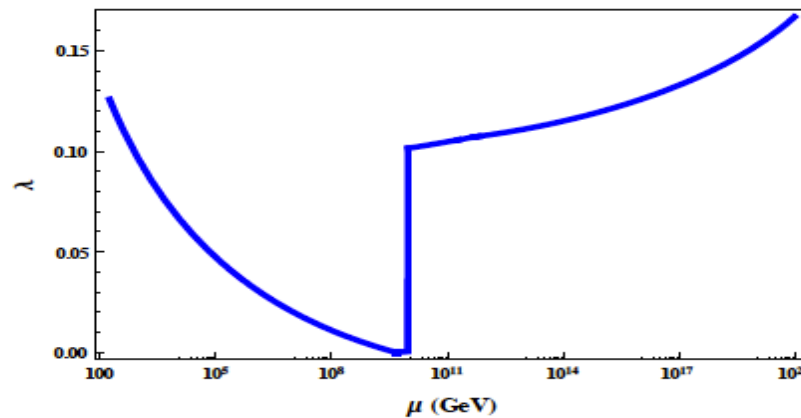
Alternating LR-symmetric model with universal see-saw for all fermion masses



(a) Running gauge couplings.



(b) Running Yukawa couplings.



(c) Running Higgs quartic coupling

Figure 4: Running couplings in the ALRSM with $M_T = 4.7 \times 10^9$ GeV and $m = 1 \times 10^{10}$ GeV.

Conclusions:

- Electroweak vacuum instability may already provide a hint for physics beyond the Standard Model
- Interesting interplay with cosmological inflation: observation of gravitational waves will provide a strong hint for a new physics that is responsible for the stabilization of the vacuum
- The most promising models of neutrino masses with a stable electroweak vacuum are: type II see-saw and LR models. They may potentially be tested at the LHC