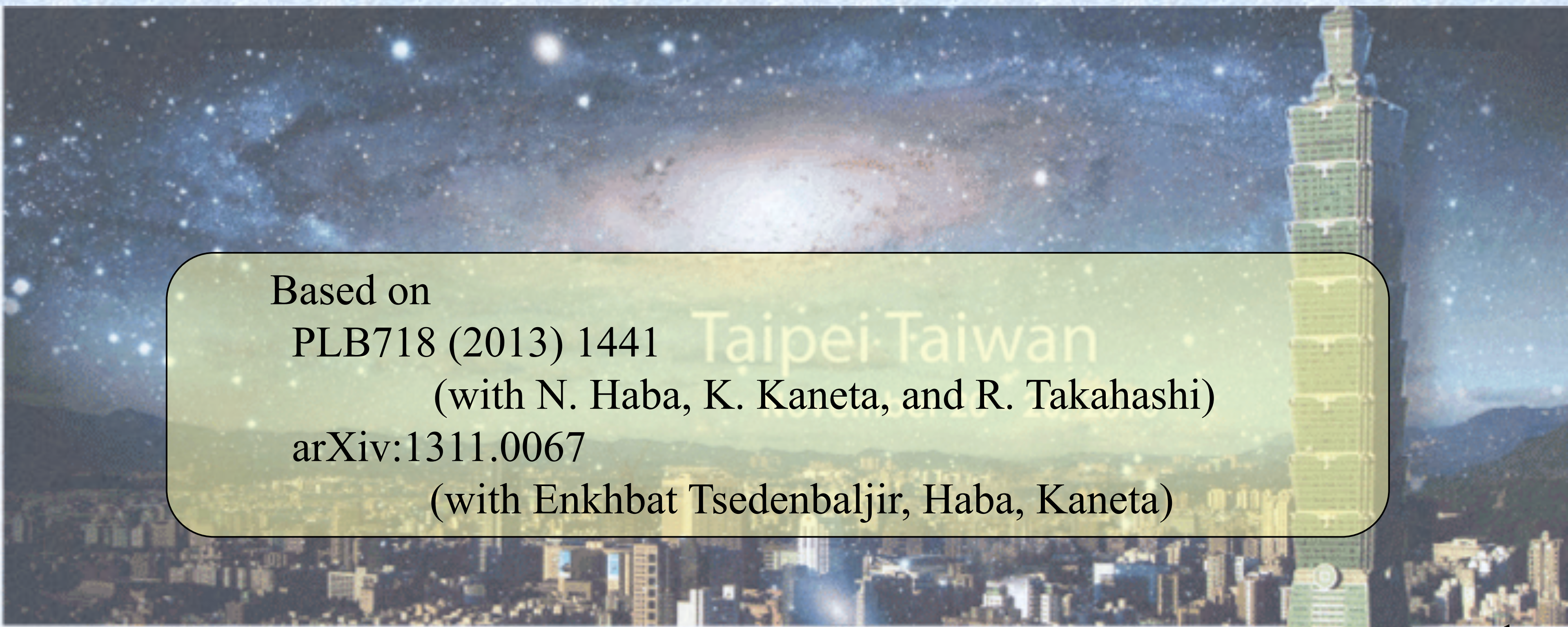
Higgs pair production at the LHC & ILC from general potential

Yukihiro Mimura
(National Taiwan University)



Based on
PLB718 (2013) 1441
(with N. Haba, K. Kaneta, and R. Takahashi)
arXiv:1311.0067
(with Enkhbat Tsedenbaljir, Haba, Kaneta)

PASCOS 2013 at NTU (2013.11.21)

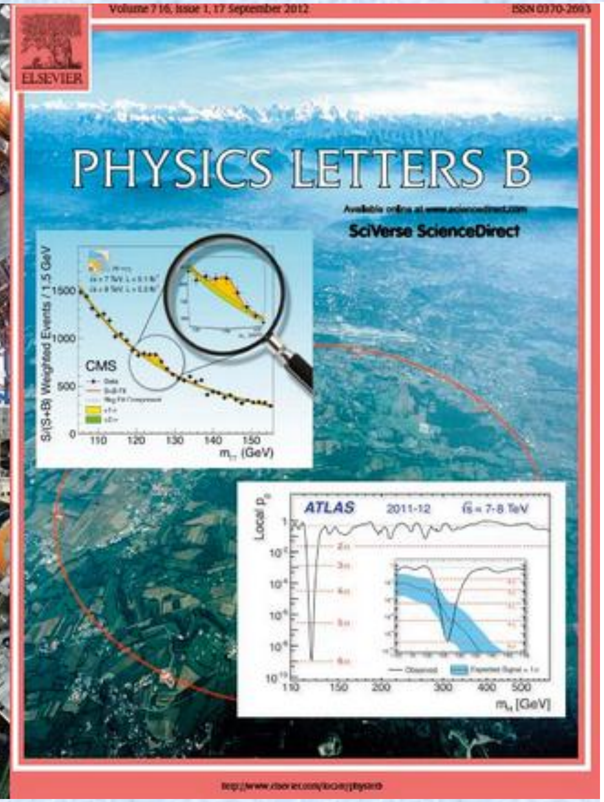
Menu

- Introduction (Higgs forces)
- Higgs potential and the cubic Higgs coupling
- Non-perturbative Higgs model in SUSY QCD
- Pair-Higgs production

  $pp \to gg \to hh$

  $e^+e^- \to hh\bar{\nu}\nu \quad e^+e^- \to Zhh$
- Conclusion

Discovery of the Higgs boson



The Nobel Prize in Physics 2013
François Englert, Peter Higgs

The Nobel Prize in Physics 2013

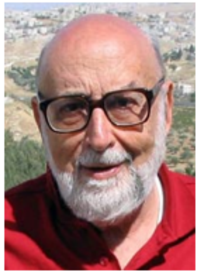
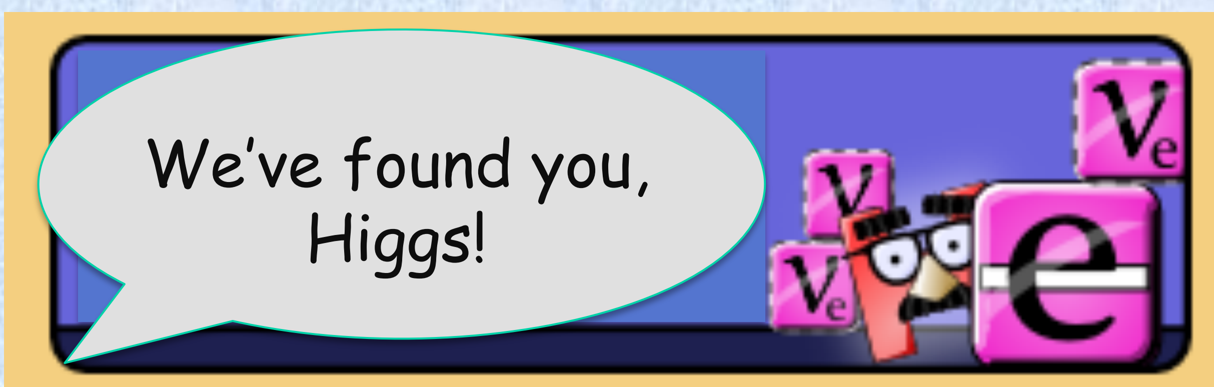


Photo: Pnicolet via Wikimedia Commons
François Englert

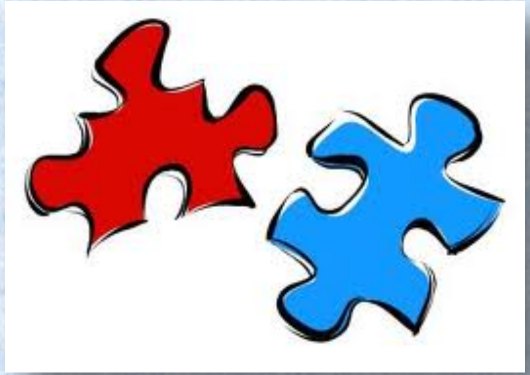
Photo: G-M Greuel via Wikimedia Commons
Peter W. Higgs

The Nobel Prize in Physics 2013 was awarded jointly to François Englert and Peter W. Higgs *"for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, and which recently was confirmed through the discovery of the predicted fundamental particle, by the ATLAS and CMS experiments at CERN's Large Hadron Collider"*



SM Higgs?

We need to look at it carefully.



“Higgs Force?”

There are 4 forces :

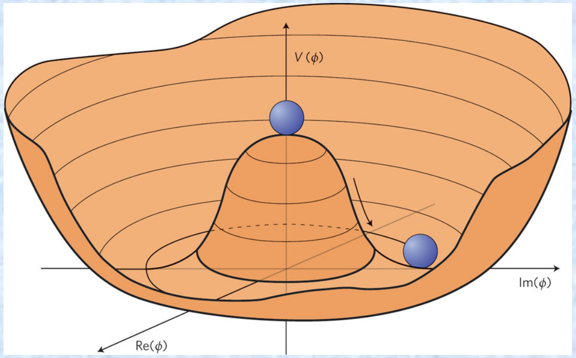
ElectroMagnetic, Weak, Strong, and Gravity

(➔Gauge interaction : successful!)

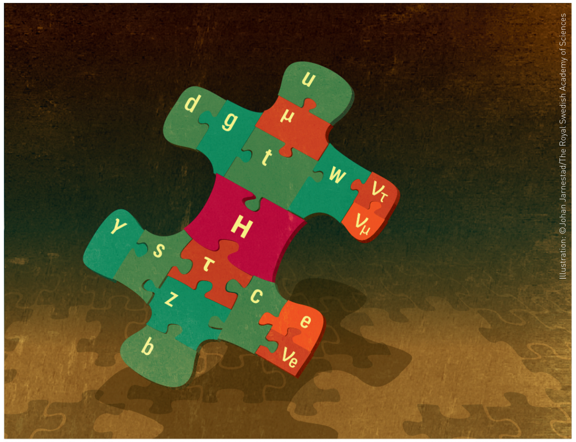
What is “Higgs force”?

"Higgs Force?"

1. Self-interaction
   To stabilize the Higgs potential
   and give a vacuum expectation value.



2. Yukawa Interaction
   To give masses to fermions



3. Electroweak Interaction
   To give masses to W/Z bosons

"Higgs Forces"

1. Higgs self-coupling

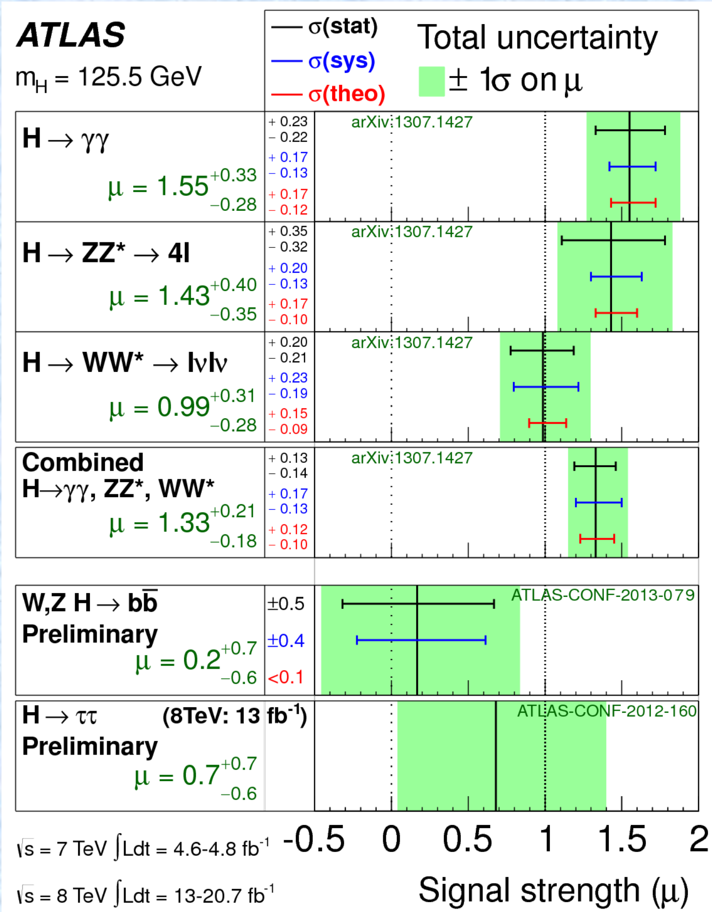
   How does the Higgs field acquire a VEV ?

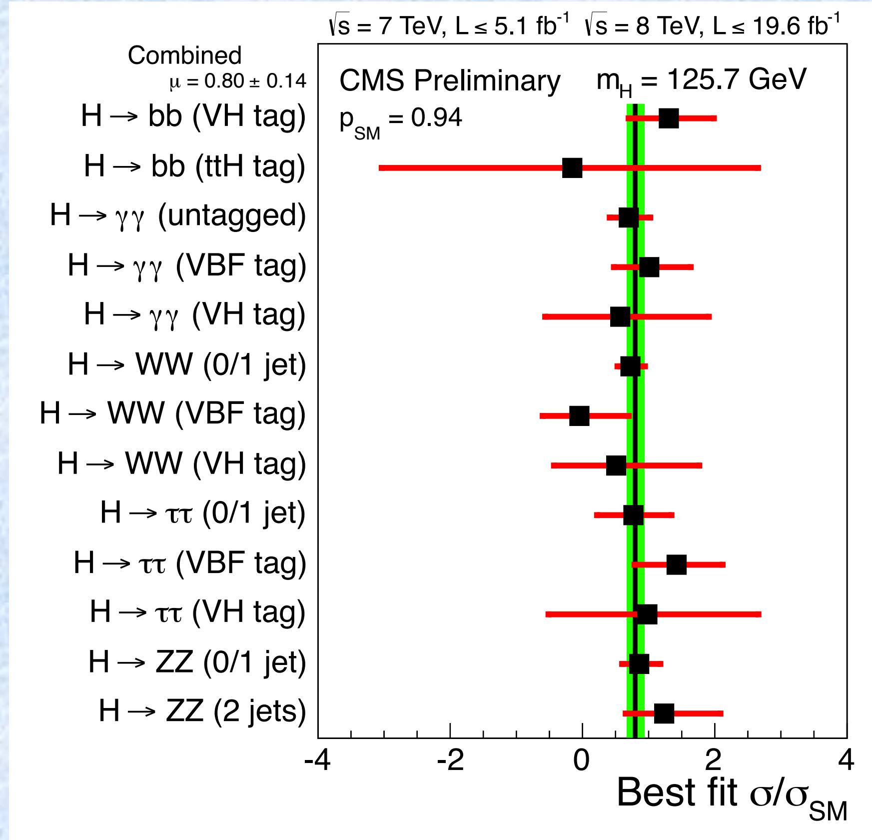
   $$V = m_H^2 |H|^2 + \lambda |H|^4 \qquad m_H^2 < 0$$

2. Couplings to fermions (Yukawa coupling) $Y_t \overline{q_{3L}} t_R H$

   How does the Higgs VEV give masses to fermions?

3. Couplings to gauge bosons $\mathcal{L} = \left| (\partial - i\frac{g}{2} W^a \tau^a - i\frac{g'}{2} B) H \right|^2$





- Combined μ → Best accuracy but no strong physics motivation:
  - ATLAS (γγ, WW* and ZZ*) μ = (1.33 ± 0.20) (1.23±0.18 including bb and ττ)
  - CMS (γγ, ττ, bb, WW* and ZZ*) μ = (0.80 ± 0.14)
  - TEVATRON (bb, γγ, ττ, WW*) μ = (1.44 ± 0.60)

Compatible with SM Higgs boson expectation: Accuracy ~ 15%

“Higgs Forces”

1. Higgs self-coupling

   How does the Higgs field acquire a VEV ?

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2. Couplings to fermions (Yukawa coupling) $Y_t \overline{q_{3L}} t_R H$

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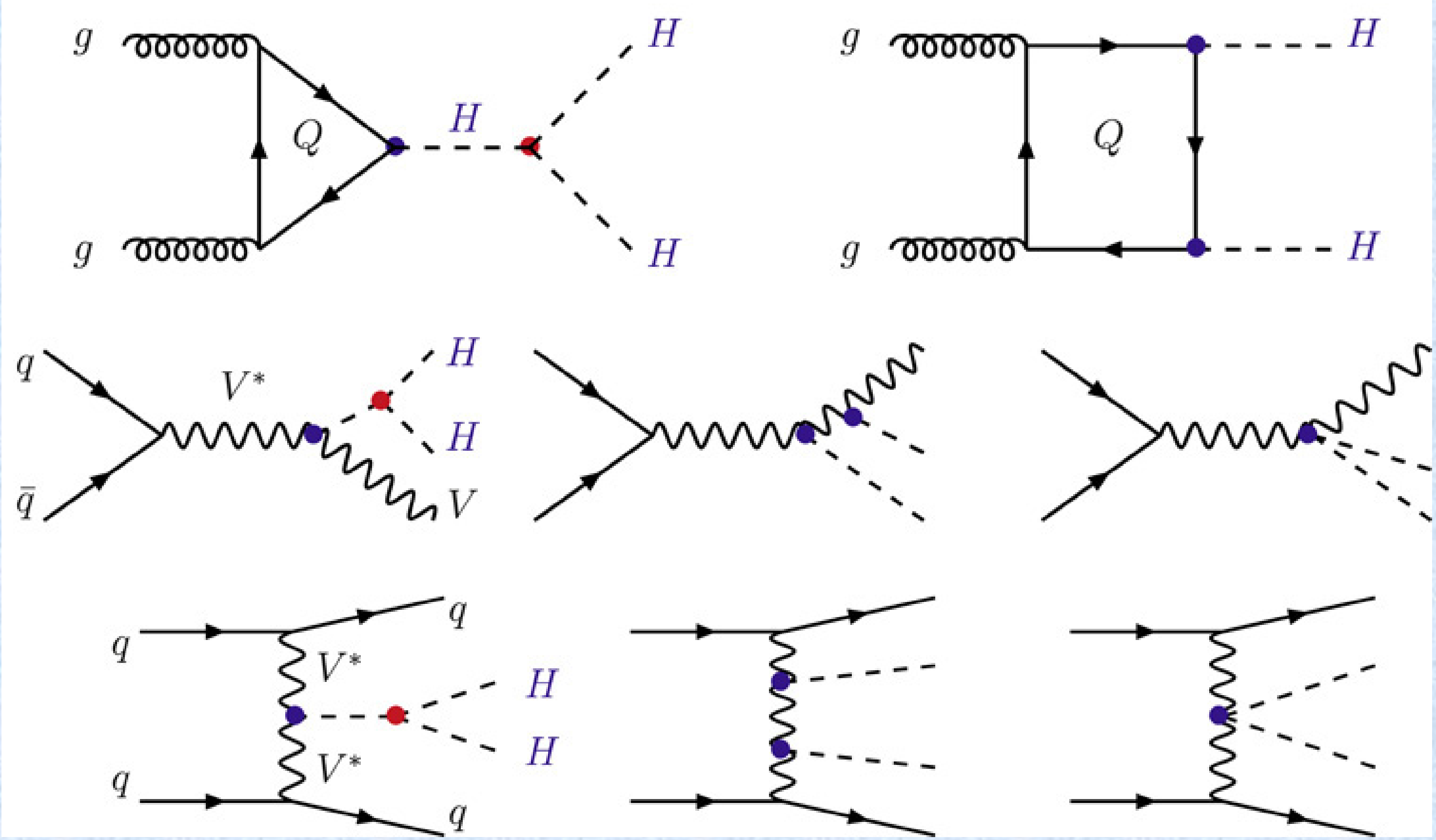
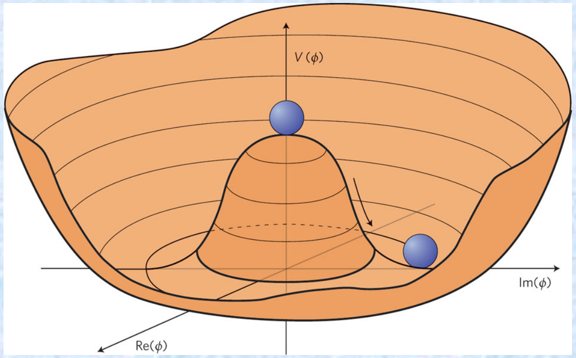
Probing the Higgs self-interaction



Pair Production of the Higgs boson at the LHC

Probing the Higgs self-interaction

$$V = m_H^2 |H|^2 + \lambda |H|^4$$



$$m_H^2 < 0$$

$$\langle H \rangle = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} \qquad v = 246\,\mathrm{GeV}$$

$$\langle V \rangle = \frac{m_H^2}{2} v^2 + \frac{\lambda}{4} v^4$$

Minimization condition:

$$\frac{\partial \langle V \rangle}{\partial v} = m_H^2 v + \lambda v^3 = 0$$

$$V = V(|H|^2) \qquad H = \begin{pmatrix} (v+h+i\chi)/\sqrt{2} \\ \chi^- \end{pmatrix}$$

$$|H|^2 = \frac{v^2}{2} + vh + \frac{h^2+\chi^2}{2} + \chi^+\chi^-$$

$$V = V\left(\frac{v^2}{2}\right) + V'\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2+\chi^2}{2} + \chi^+\chi^-\right)$$
$$+\frac{1}{2}V''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2+\chi^2}{2} + \chi^+\chi^-\right)^2 + \cdots$$

Stationary condition : $V'\left(v^2/2\right) = 0$

Mass of physical Higgs (h) : $m_h^2 = v^2 V''\left(v^2/2\right)$

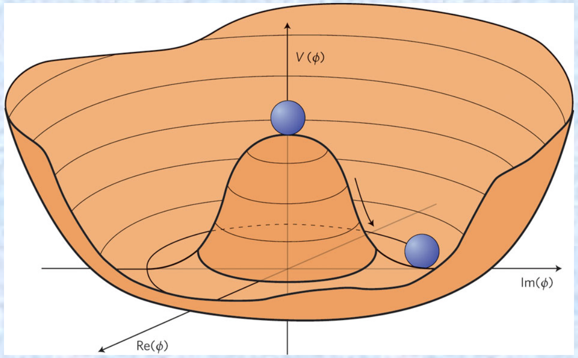
$$V = V(|H|^2) \Longrightarrow m_h^2 = v^2 V''$$

$$\begin{aligned} V &= V\left(\frac{v^2}{2}\right) + V'\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right) \\ &+ \frac{1}{2}V''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^2 \\ &+ \frac{1}{6}V'''\left(\frac{v^2}{2}\right)\left(vh + \frac{h^2}{2} + \frac{\chi^2}{2} + \chi^+\chi^-\right)^3 + \cdots \end{aligned}$$

$$\begin{aligned} -\mathcal{L} \supset \quad & V''vh\left(\frac{\chi^2}{2} + \chi^+\chi^-\right) + \frac{1}{2}V''\left(\frac{\chi^2}{2} + \chi^+\chi^-\right)^2 \\ & + \frac{1}{2}\left(V'' + \frac{1}{3}v^2V'''\right)vh^3 \\ & + \frac{1}{2}\left(V'' + v^2V'''\right)\left(\frac{\chi^2}{2} + \chi^+\chi^-\right)h^2. \end{aligned}$$

(Chivukula-Koulovassilopoulos, Boudjema-Chopin, …)

$$V = V(|H|^2)$$



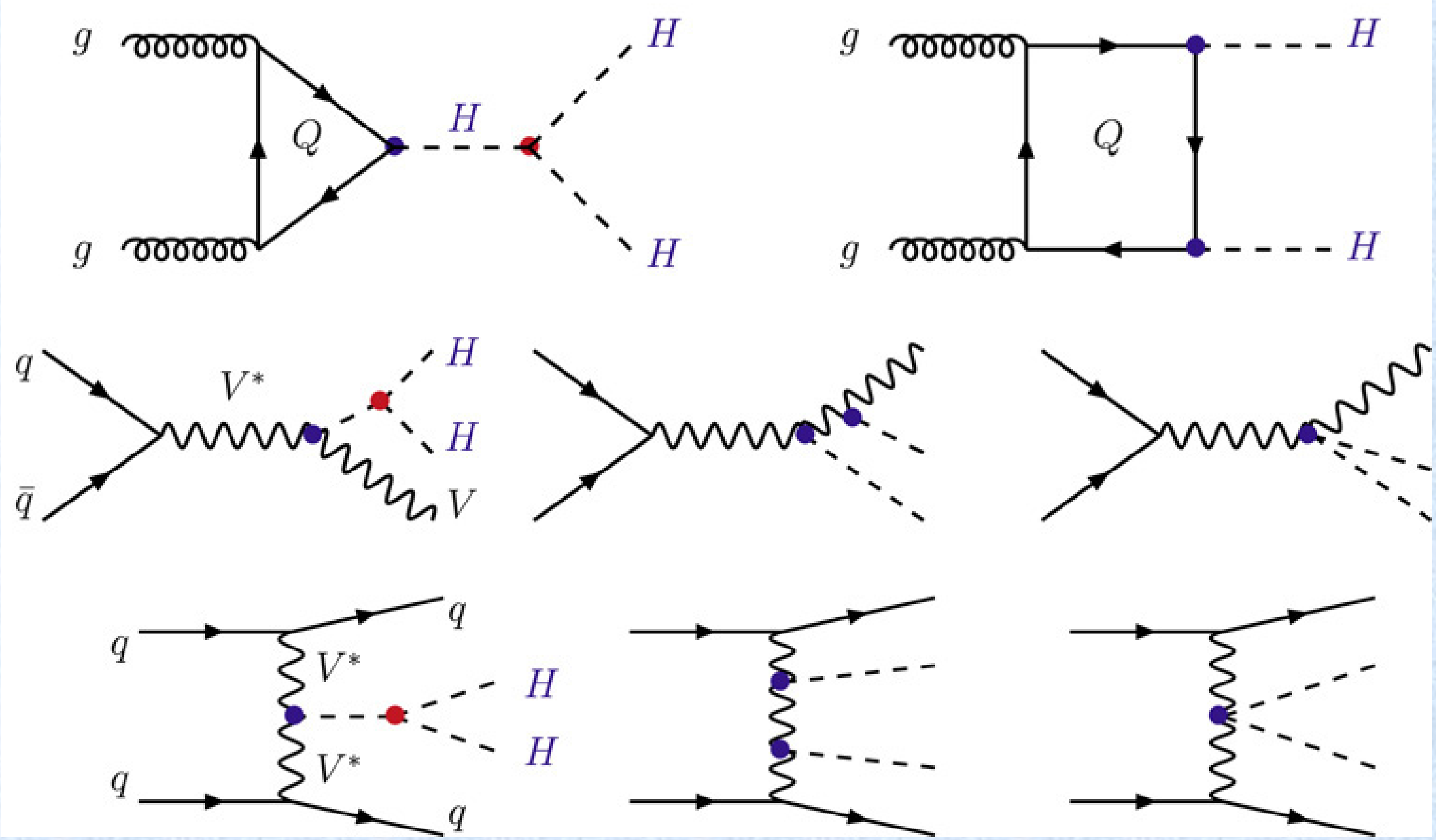
Mass of physical Higgs (h) : $m_h^2 = v^2 V''(v^2/2)$

Cubic Higgs coupling : λ_{hhh}

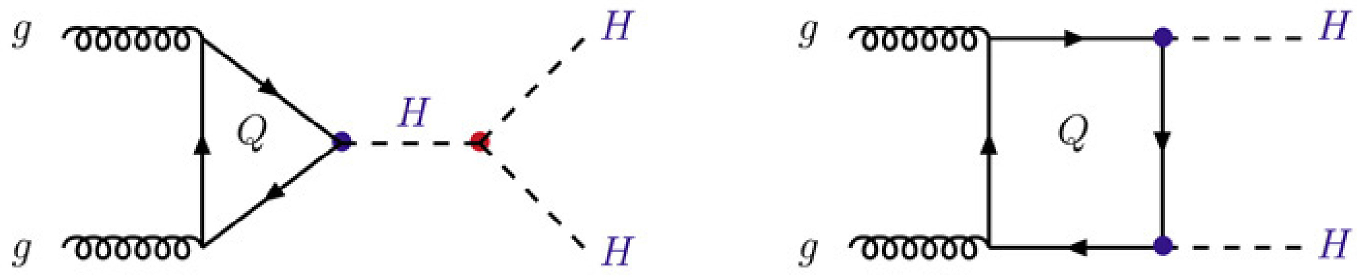
$$\lambda_{hhh} = 3vV'' + v^2 V''' = \frac{3m_h^2}{v}\left(1 + v^2 \frac{V'''}{3V''}\right)$$

$\equiv C_h$

Pair production of the Higgs boson at the LHC

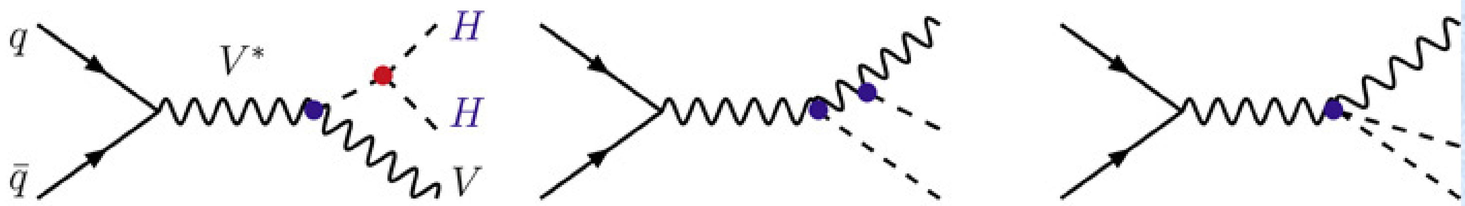


Pair production of the Higgs boson at the LHC



Gluon fusion

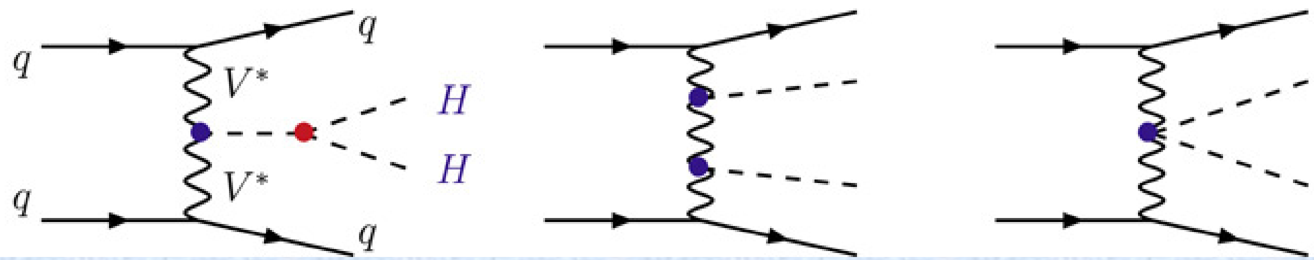
Pair production of the Higgs boson at the LHC



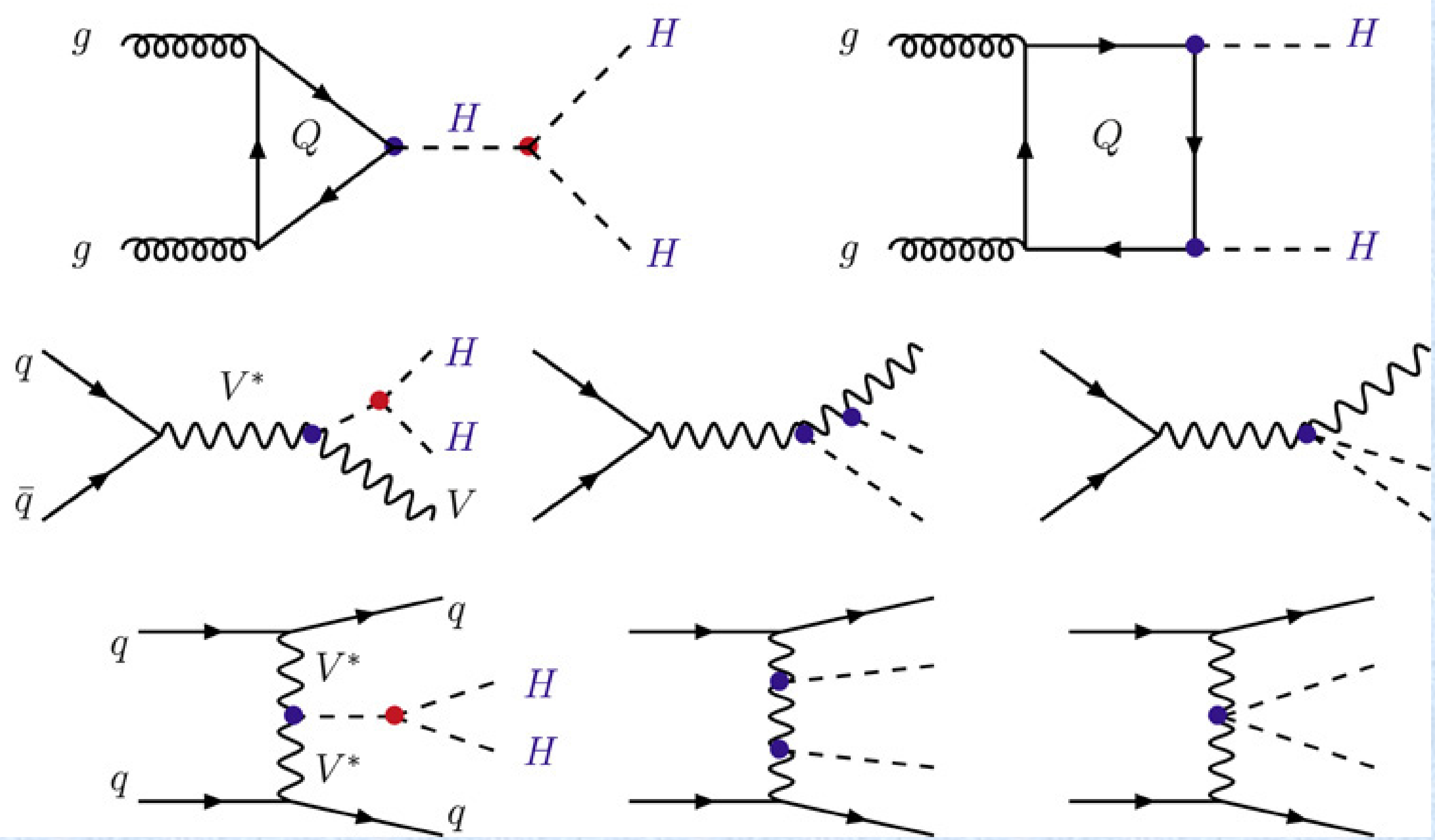
(double) Higgs-strahlung

Pair production of the Higgs boson at the LHC

vector boson fusion



Pair production of the Higgs boson at the LHC

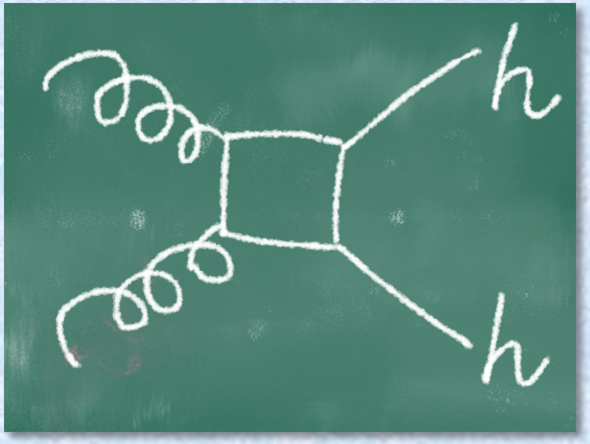


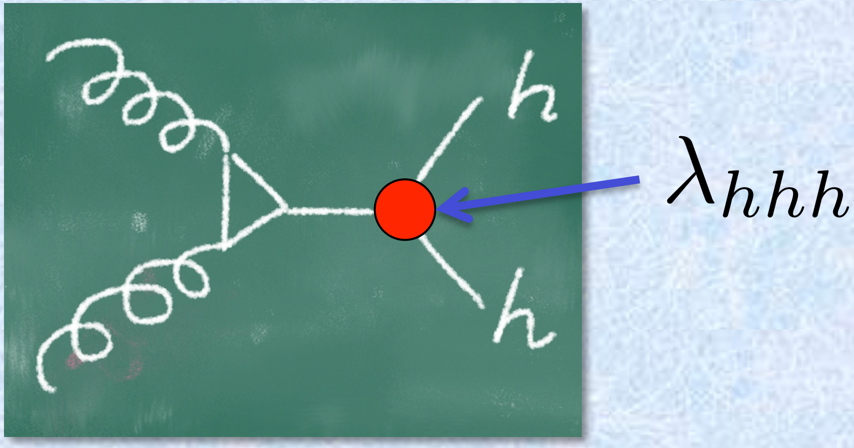
$$\sigma(pp \to gg \to hh)^{\mathrm{NLO}}_{\mathrm{SM,14\ TeV}} = 30 - 40\ (\mathrm{fb})$$

$$\lambda_{hhh} = 3\frac{m_h^2}{v}\left(1 + \frac{1}{3}v^2\frac{V'''}{V''}\right) \qquad C_h = \frac{1}{3}v^2\frac{V'''}{V''}$$

(Model-independent parameter)

$gg \to hh$

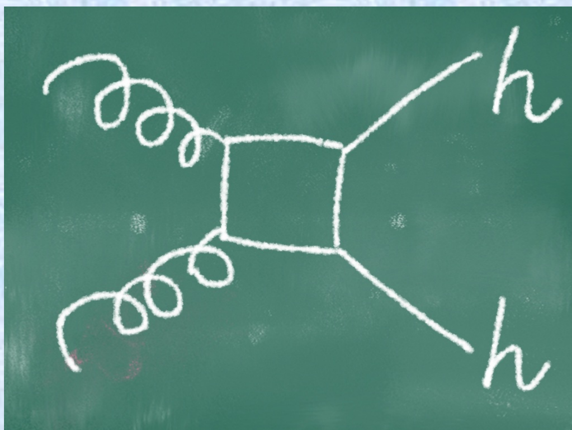


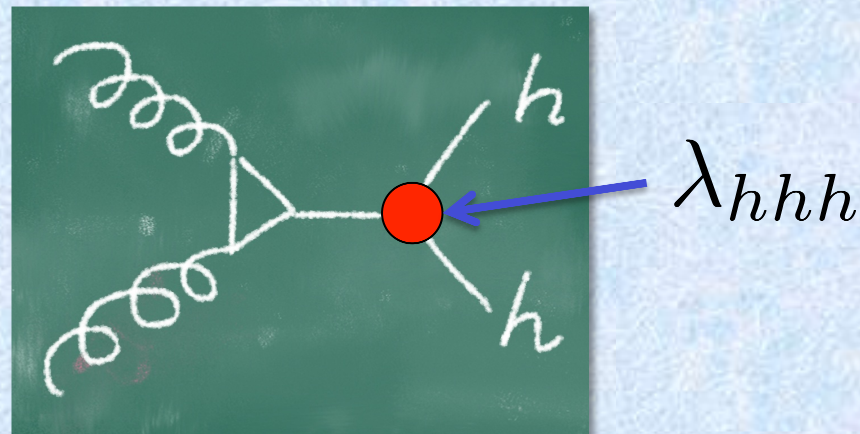


Let us understand the C_h dependence!

Gluon-Gluon-Higgses effective interactions (Hagiwara-Murayama):

$$
\begin{aligned}
\mathcal{L}_{\text{eff}} &= \frac{\alpha_s}{12\pi}(\log H) G^a_{\mu\nu} G^{a\,\mu\nu} \\
&= \frac{\alpha_s}{12\pi}\left(\frac{h}{v} - \frac{h^2}{2v^2} + \frac{h^3}{3v^3} - \cdots\right) G^a_{\mu\nu} G^{a\,\mu\nu}
\end{aligned}
$$





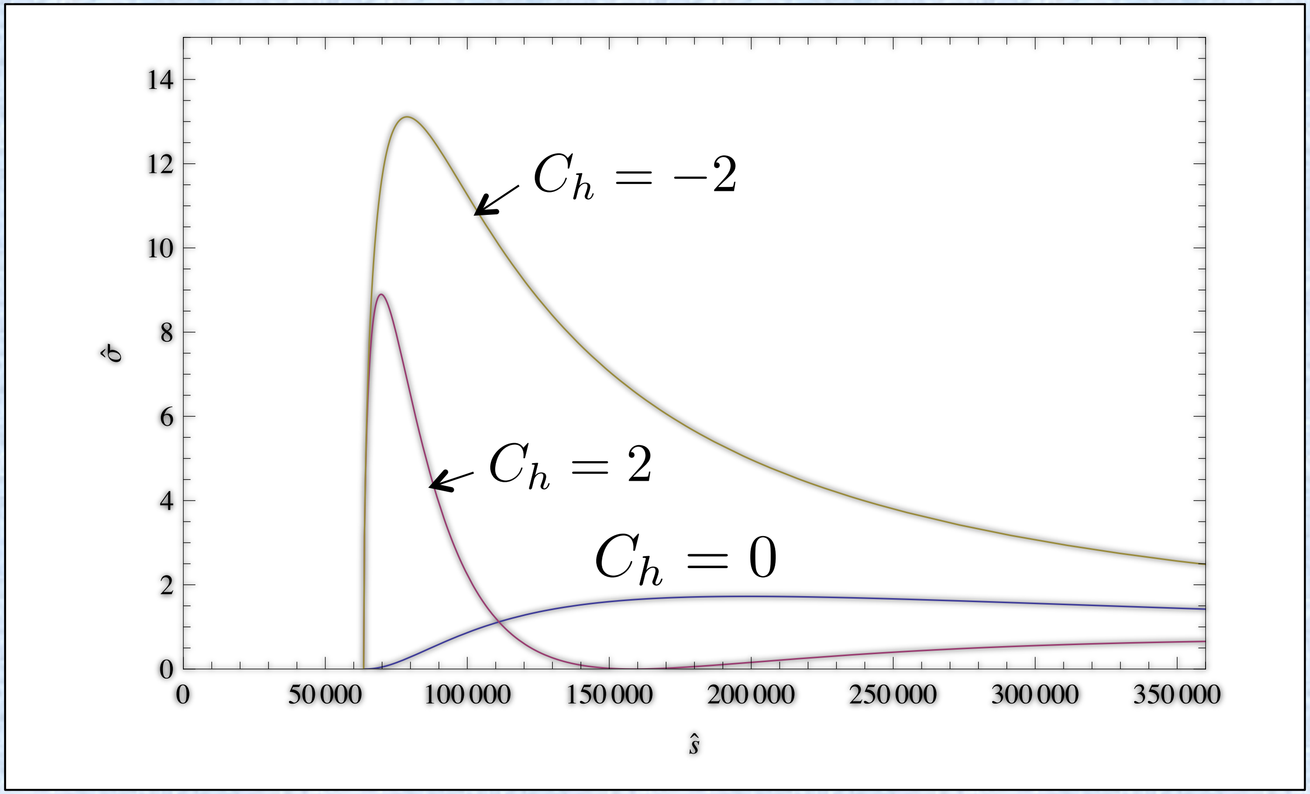
$$
\mathcal{M}(gg \to hh) = \frac{\alpha_s}{3\pi v^2}\left(-1 + \frac{3m_h^2(1+C_h)}{\hat{s} - m_h^2}\right)
$$

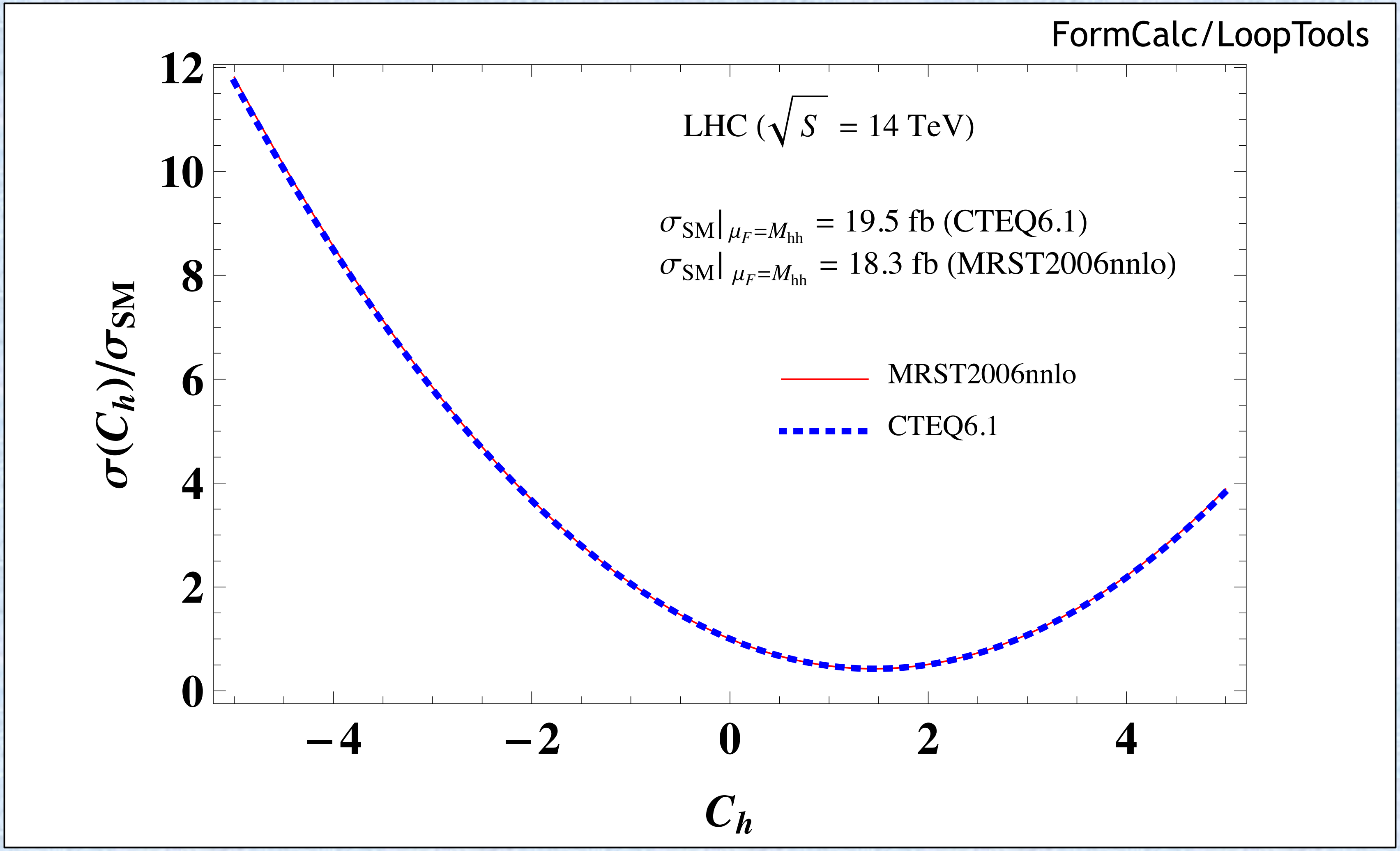
(neglecting top momentum)

$$\sigma(pp \to hh) = \int_{4m_h^2/s}^{1} d\tau \frac{d\mathcal{L}^{gg}}{d\tau} \hat{\sigma}(gg \to hh, \hat{s} = \tau s)$$

$$\mathcal{M}(gg \to hh) = \frac{\alpha_s}{3\pi v^2}\left(-1 + \frac{3m_h^2(1+C_h)}{\hat{s} - m_h^2}\right) \qquad \hat{s} \geq (2m_h)^2$$

$\mathcal{M} = 0$ at $\hat{s} = (4 + 3C_h)m_h^2$





$$\lambda_{hhh} = 3\frac{m_h^2}{v}\left(1 + \frac{1}{3}v^2\frac{V'''}{V''}\right) \qquad C_h = \frac{1}{3}v^2\frac{V'''}{V''}$$

(Plehn-Spira-Zerwas, Djouadi-Kilian-Muhlleitner-Zerwas, …)

(For 125 GeV Higgs, Shao-Li-Li-Wang, Goertz-Papaefstathiou-Yang-Zurita, …)

Want a negative C_h?

Toy potential to enlarge the cross section :

$$V = V(|H|^2) = m^2|H|^2 + \Lambda^{4-2a}(|H|^2)^a.$$

$$\Rightarrow \quad \frac{v^2}{2}\frac{V'''}{V''} = a - 2$$

$$C_h = \frac{1}{3}v^2\frac{V'''}{V''} = \frac{2}{3}(a-2)$$

Run-away potential $(a < 0)$ makes C_h negative.

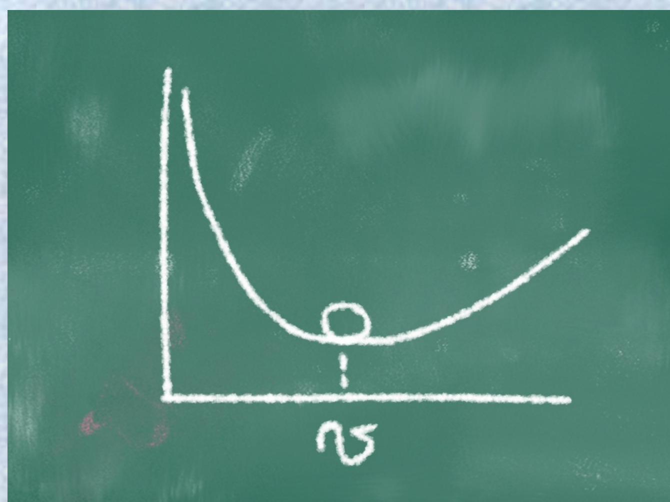
$\Rightarrow$ Pair-Higgs production is enlarged.

Chiral symmetry breaking via non-perturbative potential

NP potential

Quadratic mass

(D'Hoker-YM-Sakai)



SUSY QCD (Seiberg et al, 90's)

$$SU(N_c) \times SU(N_f) \times SU(N_f) \times U(1)_B$$

$$Q : (\mathbf{N_c}, \mathbf{N_f}, \mathbf{1}, +1), \quad \bar{Q} : (\bar{\mathbf{N}}_\mathbf{c}, \mathbf{1}, \mathbf{N_f}, -1).$$

$$W \propto \frac{\Lambda^{3+\frac{2N_f}{N_c - N_f}}}{(\det \bar{Q}Q)^{\frac{1}{N_c - N_f}}}.$$

for $N_c > N_f$

Non-perturbative Higgs model

(Haba-Okada)

Higgs fields are moduli of SUSY QCD.

$$SU(N_c) \times SU(2)_L \times U(1)_Y \times SU(3)_c$$

Hypercolor

$$\Lambda H_1^a = \bar{Q}_1 Q^a, \quad \Lambda H_2^a = \bar{Q}_2 Q^a.$$

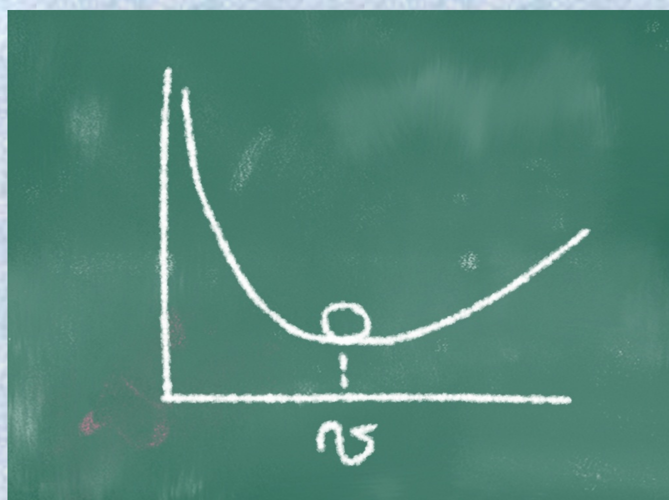
$$W = \frac{\Lambda^{3+2\kappa}}{(H_1 \cdot H_2)^\kappa} \qquad \kappa = \frac{1}{N_c - 2}$$

$$C_h \simeq -\frac{5}{3} - \frac{4}{3}\kappa$$

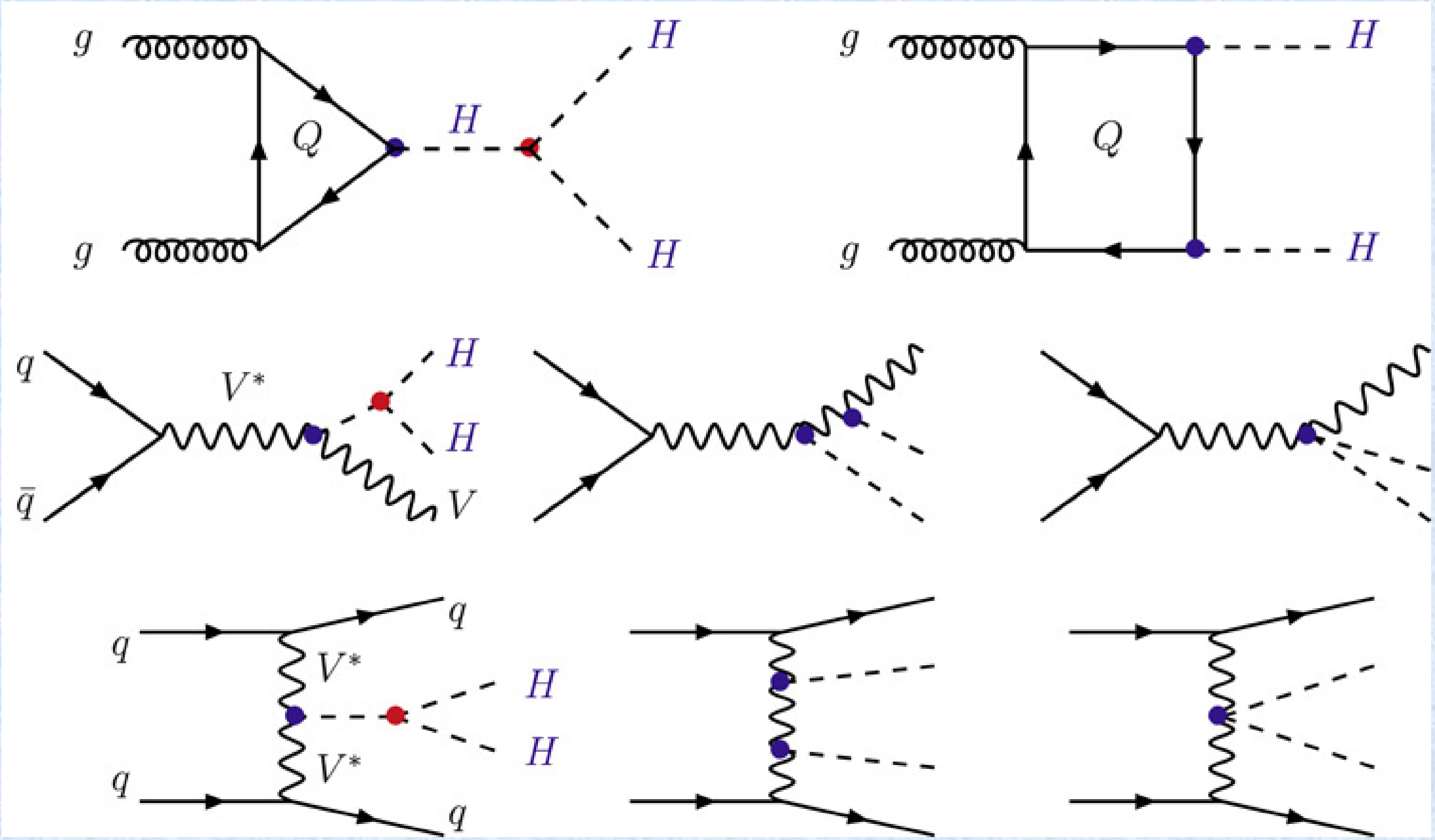
NP potential

SUSY breaking mass

(D'Hoker-YM-Sakai)



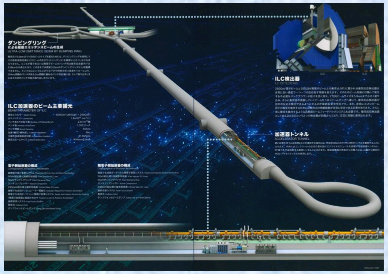
Pair production of the Higgs boson at the LHC

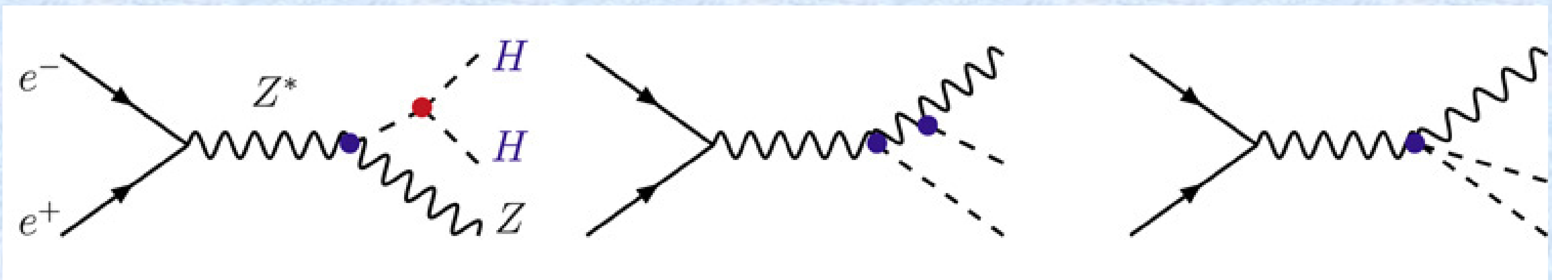


$$\sigma(pp \to gg \to hh)^{\rm NLO}_{\rm SM,14\ TeV} = 30 - 40\ ({\rm fb})$$

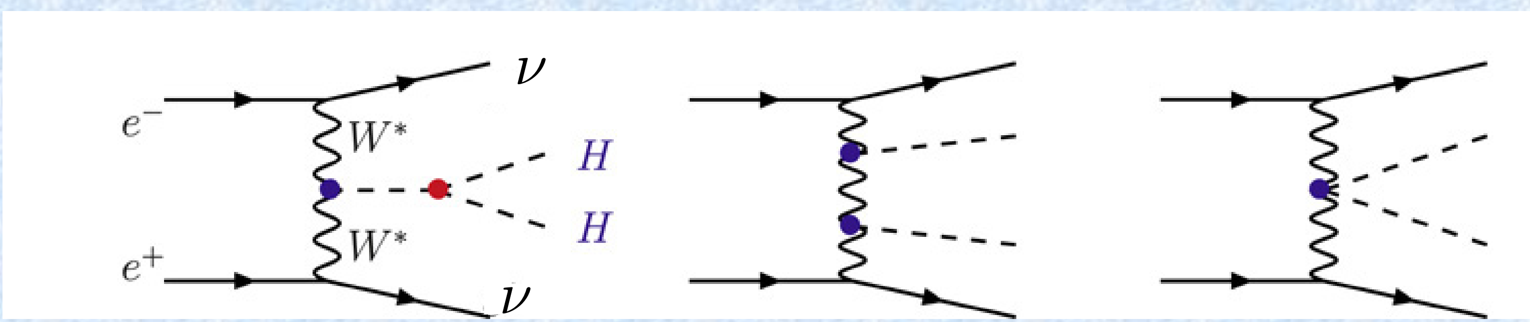
$$\sigma(pp \to hhjj)_{\rm SM,14\ TeV} = 1.6\ ({\rm fb})$$

Pair production of the Higgs boson at the ILC





(double Higgs-strahlung)



(WW fusion)

$$V = V(|H|^2) \quad \Longrightarrow \quad m_h^2 = v^2 V''$$

$$-\mathcal{L} \supset \quad V'' v h \left(\frac{\chi^2}{2} + \chi^+\chi^-\right) + \frac{1}{2} V'' \left(\frac{\chi^2}{2} + \chi^+\chi^-\right)^2$$
$$+\frac{1}{2}\left(V'' + \frac{1}{3} v^2 V'''\right) v h^3$$
$$+\frac{1}{2}\left(V'' + v^2 V'''\right)\left(\frac{\chi^2}{2} + \chi^+\chi^-\right) h^2.$$

Higgs-NG interaction:

$$-\mathcal{L} \supset \frac{m_h^2}{v} h \left(\frac{\chi^2}{2} + \chi^+\chi^-\right) + \frac{m_h^2}{2v^2}(1 + 3C_h) h^2 \left(\frac{\chi^2}{2} + \chi^+\chi^-\right)$$

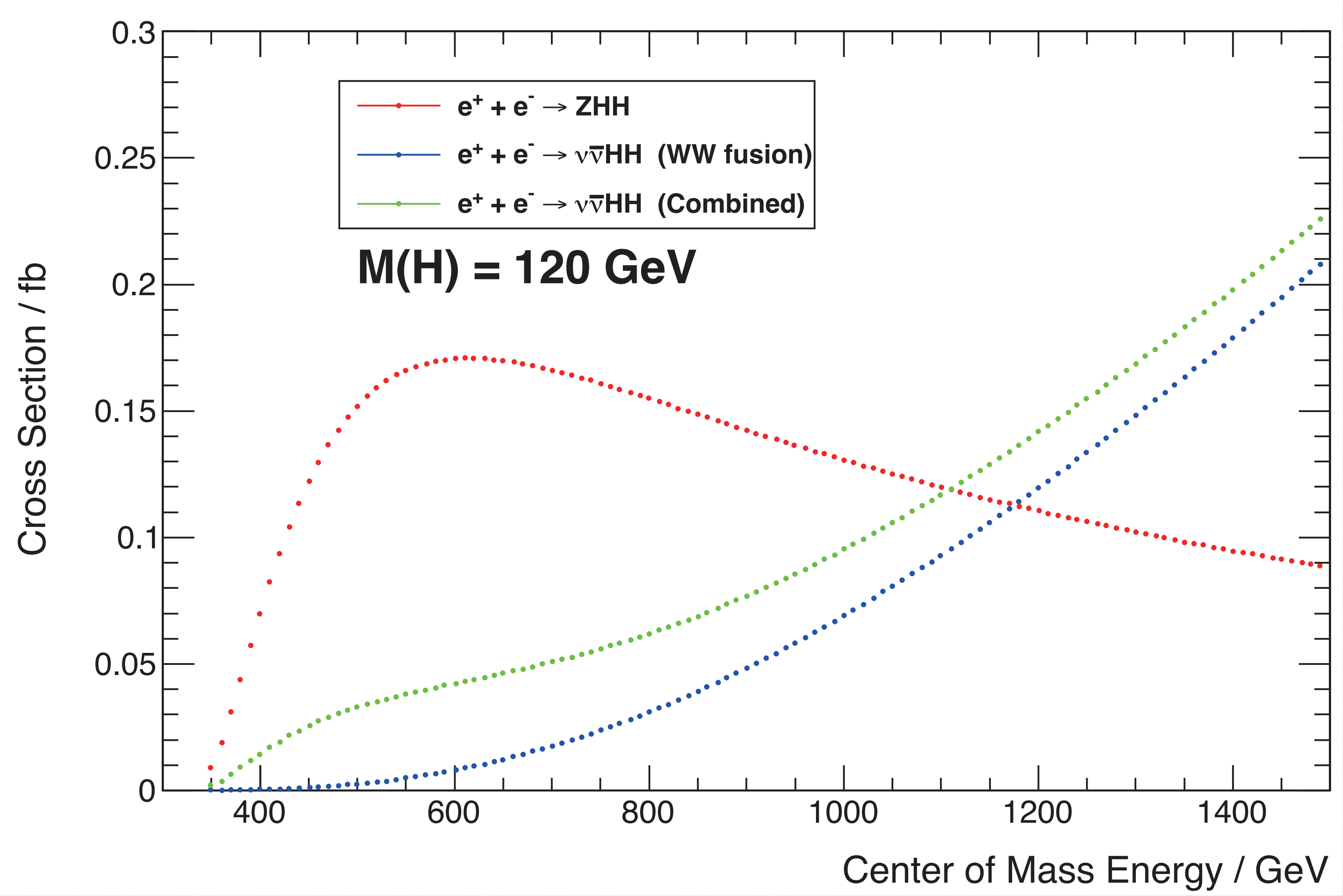
$$-\mathcal{L} \supset \frac{m_h^2}{v} h \left(\frac{\chi^2}{2} + \chi^+ \chi^- \right) + \frac{m_h^2}{2v^2} (1 + 3C_h) h^2 \left(\frac{\chi^2}{2} + \chi^+ \chi^- \right)$$

$$\mathcal{M}(\chi^+ \chi^- \to hh) = \frac{m_h^2}{v^2} \left(1 + 3C_h + \frac{3(1 + C_h) m_h^2}{s - m_h^2} + \frac{m_h^2}{t - M_W^2} + \frac{m_h^2}{u - M_W^2} \right).$$

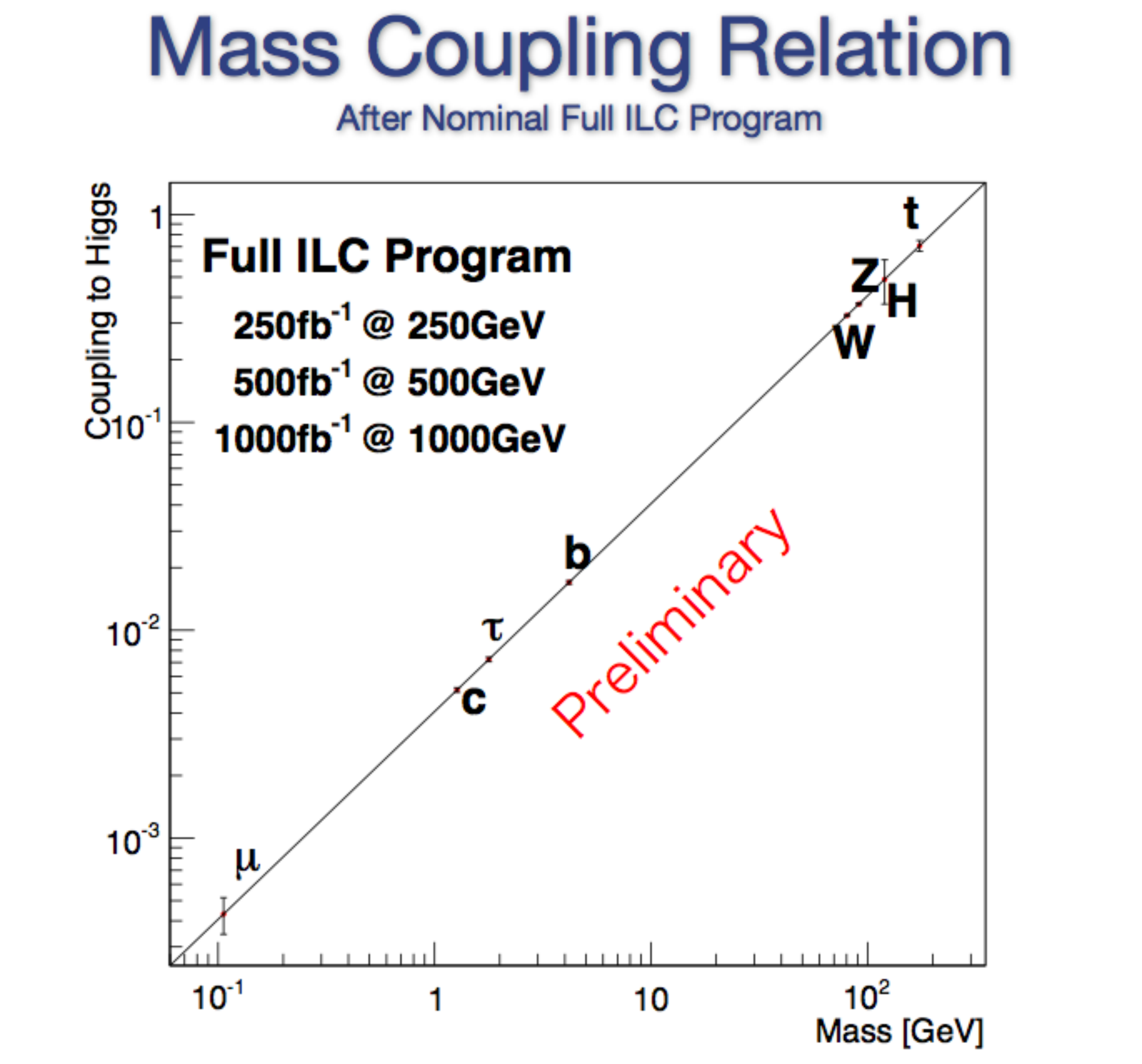
Equivalence theorem:

$$\mathcal{M}(W_L^+ W_L^- \to hh) = \mathcal{M}(\chi^+ \chi^- \to hh) + O(M_W^2 / s)$$

$$\lambda_{hhh} = \frac{3 m_h^2}{v} (1 + C_h)$$



H. Baer et al, Physics Chapter of the ILC Detailed Baseline Design Report



$$\delta\lambda_{hhh}/\lambda_{hhh} = 20\%$$

"Higgs Forces"

1. Higgs self-coupling

   How does the Higgs field acquire a VEV ?

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2. Couplings to fermions (Yukawa coupling) $Y_t \overline{q_{3L}} t_R H$

   How does the Higgs VEV give masses to fermions?

3. Couplings to gauge bosons $\mathcal{L} = \left| (\partial - i\frac{g}{2} W^a \tau^a - i\frac{g'}{2} B) H \right|^2$

Non-canonical kinetic term

(Chivukula-Koulovassilopoulos,…)

$$\mathcal{L}_{\rm kin} = F\left(\frac{|H|^2}{v^2/2}\right) D_\mu H^\dagger D^\mu H$$

In SM, $F(x) = 1.$

$$(M_W^2 W^+ W^- + \frac{M_Z^2}{2} Z^2)\left(1 + G'(1)\frac{2h}{v} + (G'(1) + 2G''(1))\frac{h^2}{v^2} + \cdots\right)$$

$$G(x) = xF(x)$$

In SM, $G' = 1$, $G'' = 0$.

Non-canonical kinetic term

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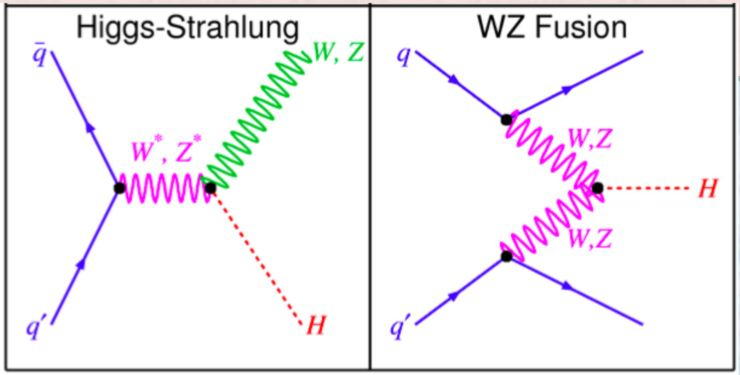


In SM, $G' = 1$, $G'' = 0$.

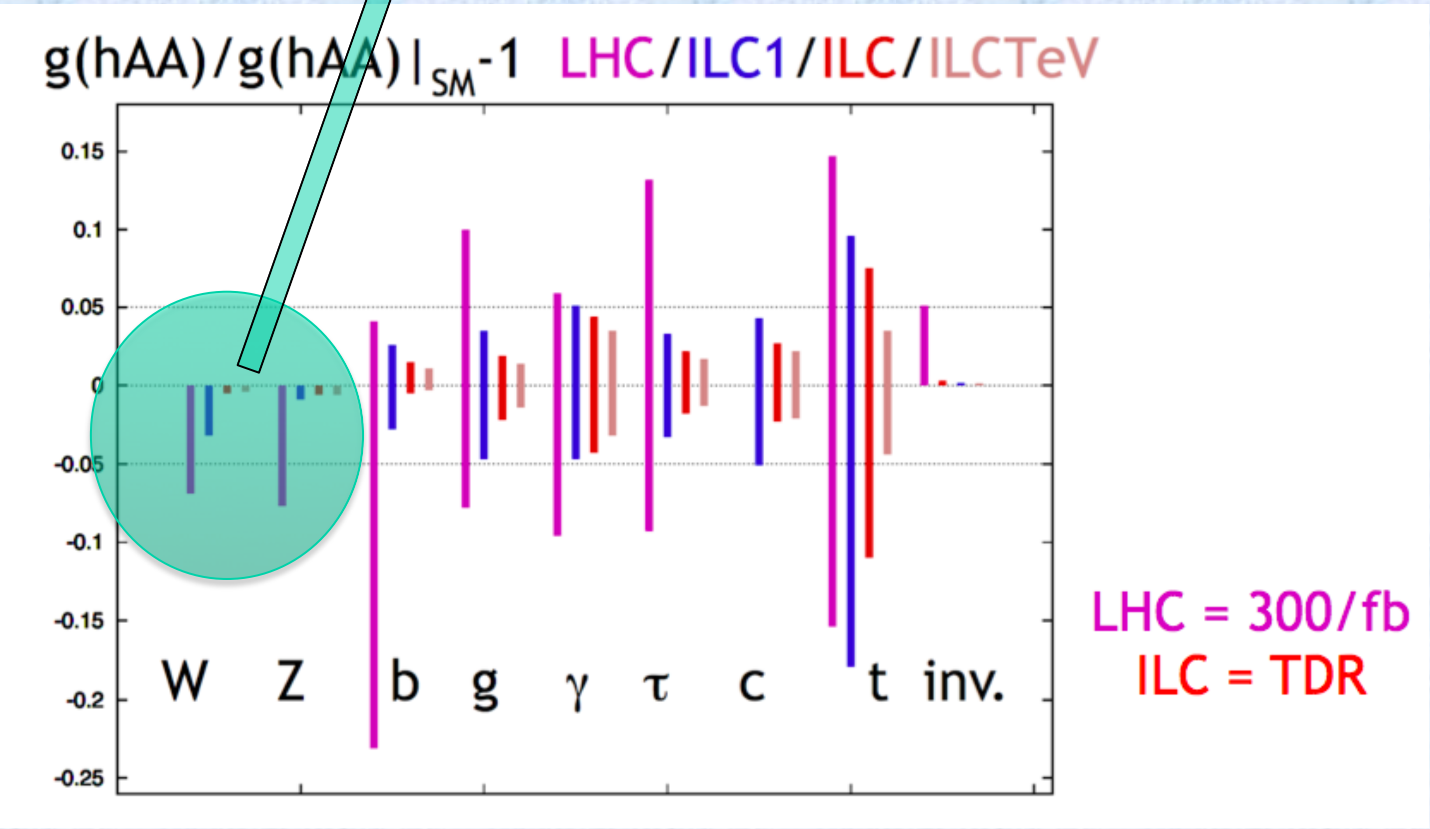
CMS: Evidence for V-boson mediated production 3.2σ

ATLAS: Evidence for VBF production (VH "profiled") 3.3σ

$\Rightarrow$ $G'(1) \sim 1$ (or -1)

$$\left(M_W^2 W^+ W^- + \frac{M_Z^2}{2} Z^2\right)\left(1 + G'(1)\frac{2h}{v} + (G'(1) + 2G''(1))\frac{h^2}{v^2} + \cdots\right)$$

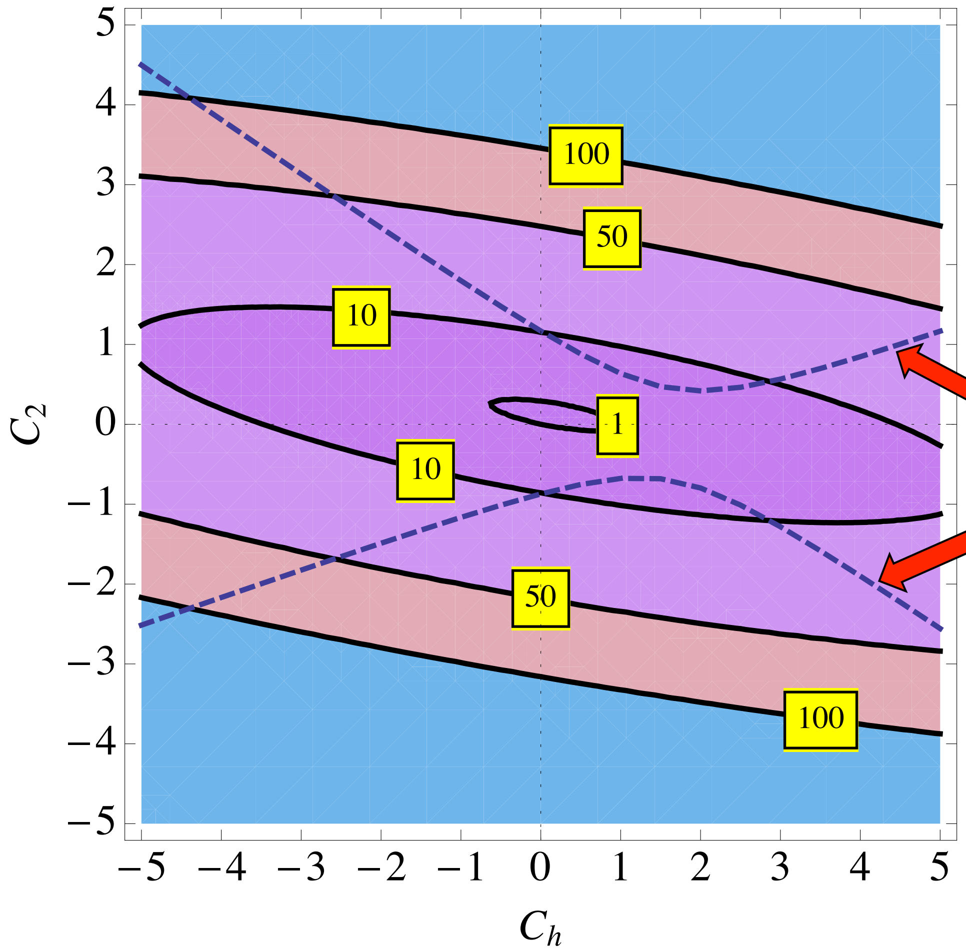
hVV coupling will be measured accureately.



$$(M_W^2 W^+W^- + \frac{M_Z^2}{2} Z^2)\left(1 + G'(1)\frac{2h}{v} + (G'(1) + 2G''(1))\frac{h^2}{v^2} + \cdots\right)$$

$= 1$ $\quad \equiv 1 + C_2$

Ratio of cross sections



MadGraph v5

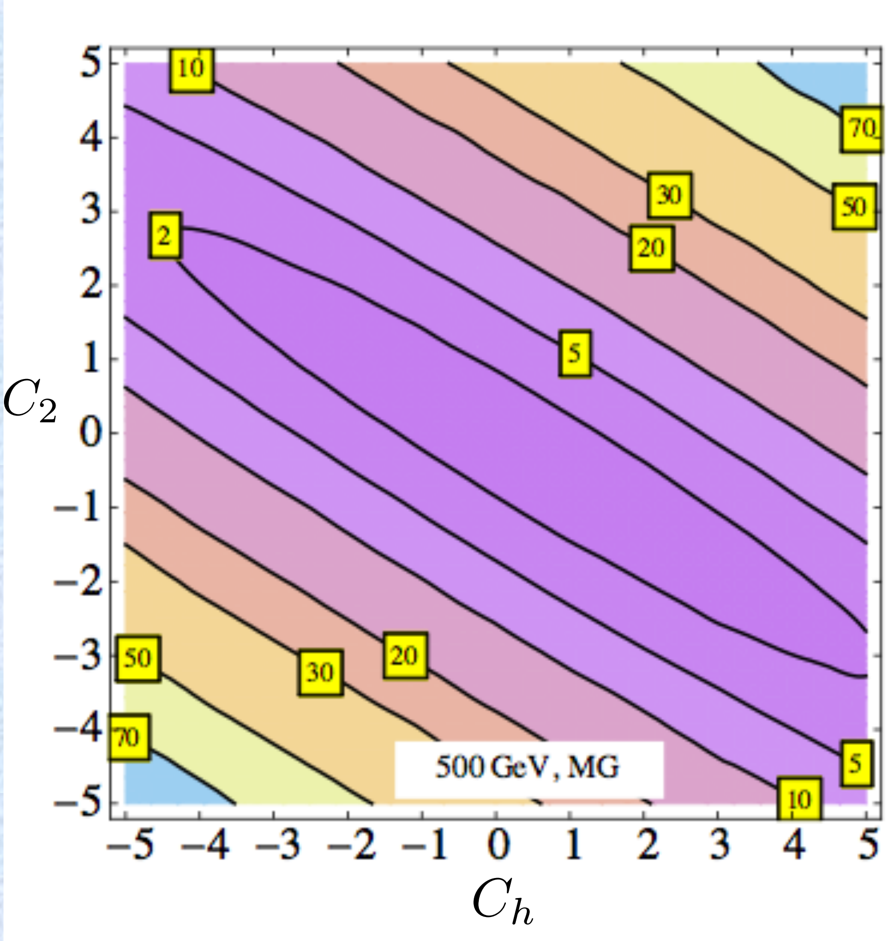
$\sigma(pp \to gg \to hh)$
$= \sigma(pp \to hhjj)$

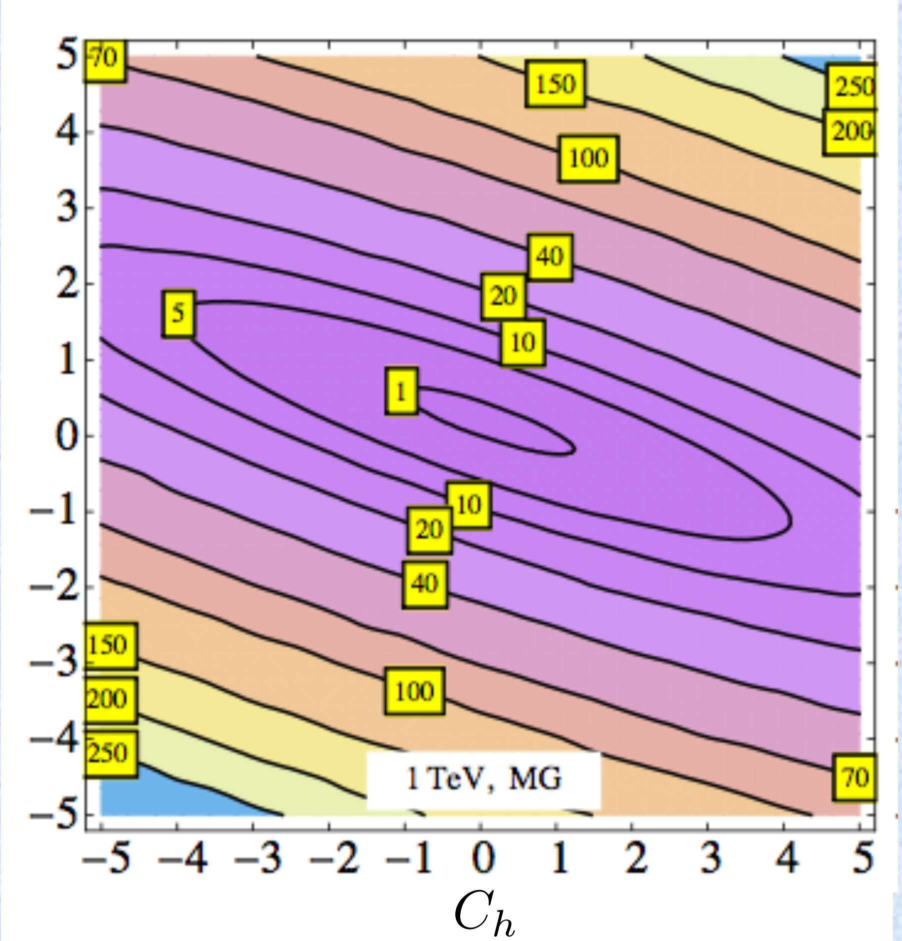
$\sqrt{s} = 14\,\text{TeV}$

$\sigma(pp \to hhjj, C_2, C_h)/\sigma(C_2 = C_h = 0)$

Ratio of cross sections

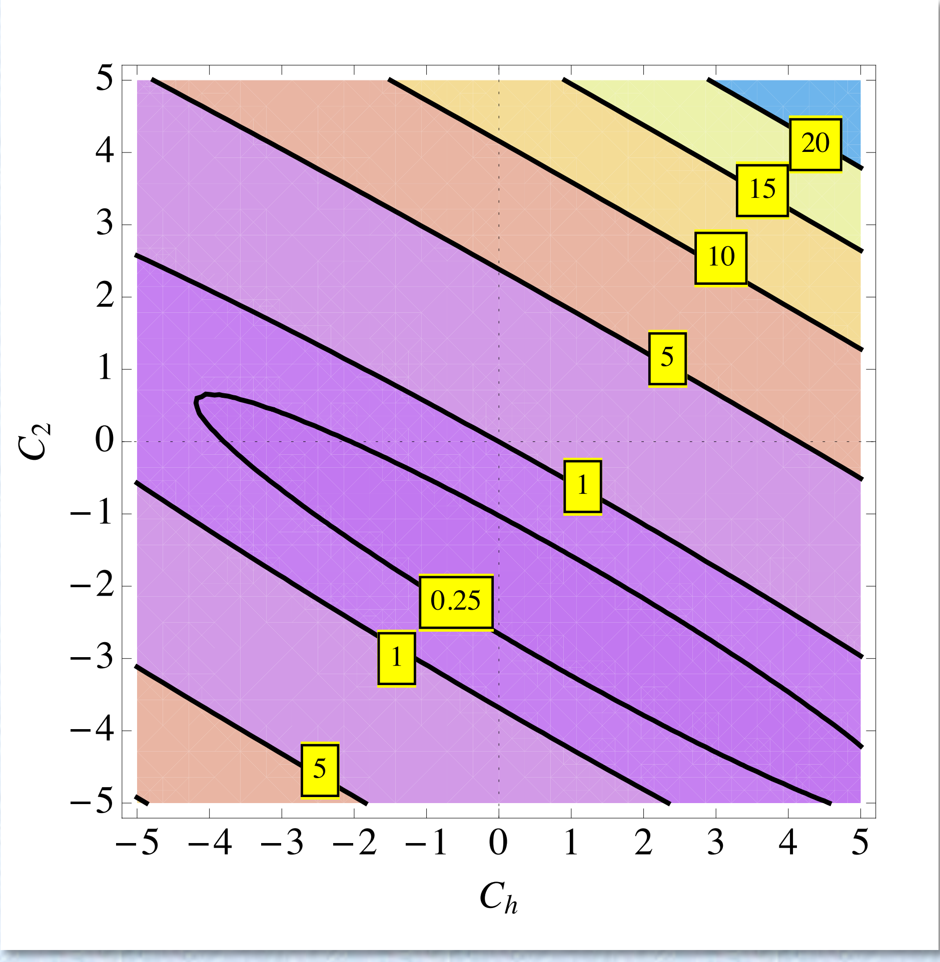
$$\sigma(e^+e^- \to hh\nu\bar{\nu})/\sigma(\mathrm{SM})$$



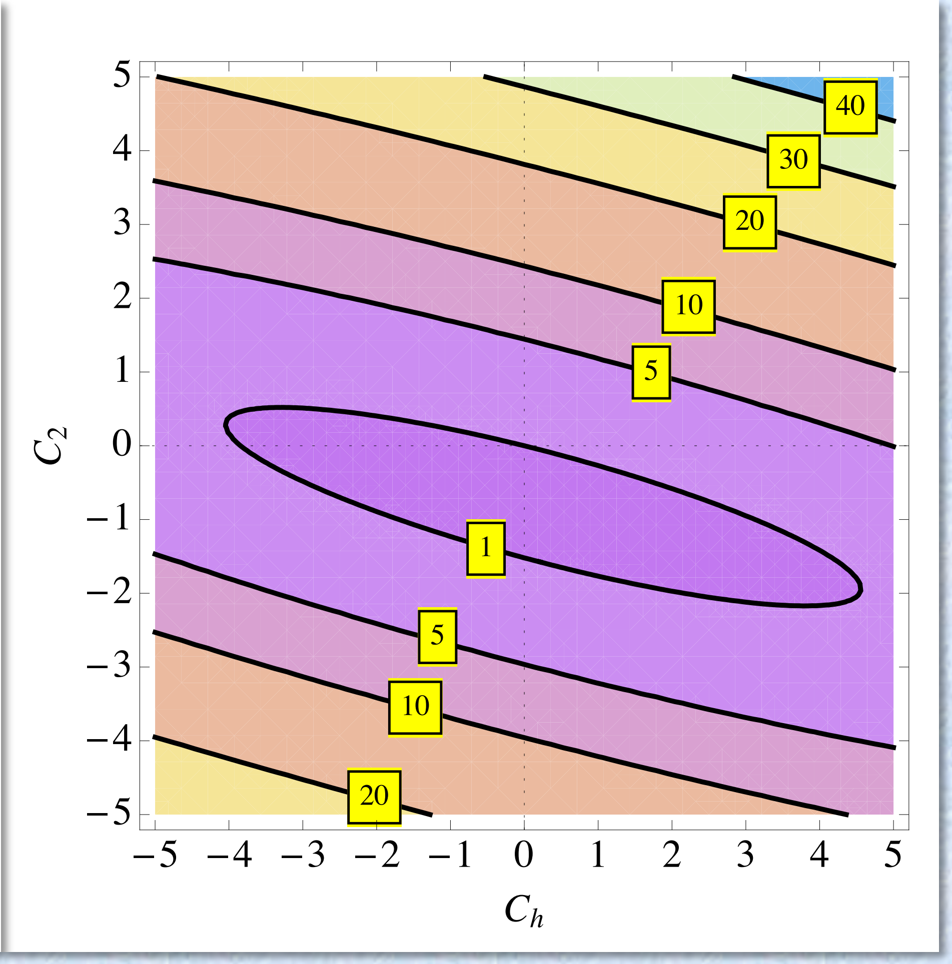


Ratio of cross sections

$$\sigma(e^+e^- \to hhZ)/\sigma(\mathrm{SM})$$



$\sqrt{s} = 500\,\mathrm{GeV}$



$\sqrt{s} = 1\,\mathrm{TeV}$

$$H = \bar{Q}Q \quad \Longrightarrow \quad K = \mathrm{tr}\sqrt{H^\dagger H}$$

(Affleck-Dine-Seiberg)

We obtain: $K = 2\sqrt{|H_1|^2 + |H_2|^2 + 2\sqrt{H_1 \cdot H_2}}$

$$\mathcal{L}_{\mathrm{kin}} = \frac{K}{2} DH_i^* DH_i + \frac{2}{K}((H_i DH_i^*)(H_j^* DH_j) - (H \cdot DH)(H \cdot DH)^*)$$

$$\mathcal{L}_Z = \frac{M_Z^2}{2} Z_\mu Z^\mu \left(1 + 3\sin(\beta - \alpha)\frac{h}{v} + 3\frac{h^2}{v^2} + \cdots\right)$$

Cf. In 2HDM,

$$\mathcal{L}_Z = \frac{M_Z^2}{2} Z_\mu Z^\mu \left(1 + 2\sin(\beta - \alpha)\frac{h}{v} + \frac{h^2}{v^2}\right)$$

Summary

- We still have missing pieces for the “Higgs forces”.
- It is important to probe the self-Higgs coupling.
- Non-perturbative Higgs model in SUSY QCD is proposed.
- Possible enhancement of pair-Higgs production is discussed.
- We look forward to more data to probe the “Higgs forces”.

  $O(100)\,\mathrm{fb}^{-1}$ at the LHC; ILC (at Tohoku?)

Dynamical Higgsino mass term

(VEV-dependent)

$$\frac{\partial^2 W}{\partial H_u^+ \partial H_d^-} \tilde{H}_u^+ \tilde{H}_d^-$$

$$= -\kappa \Lambda^{3+2\kappa} (H_u^0 H_d^0)^{-\kappa-1} \tilde{H}_u^+ \tilde{H}_d^-$$

$$= -\kappa \Lambda \left(\frac{\Lambda^2}{v_u v_d} \right)^{1+\kappa} \left(1 - (\kappa+1) \frac{2}{v} \frac{\cos(\alpha+\beta)}{\sin 2\beta} h + \cdots \right) \tilde{H}_u^+ \tilde{H}_d^-$$

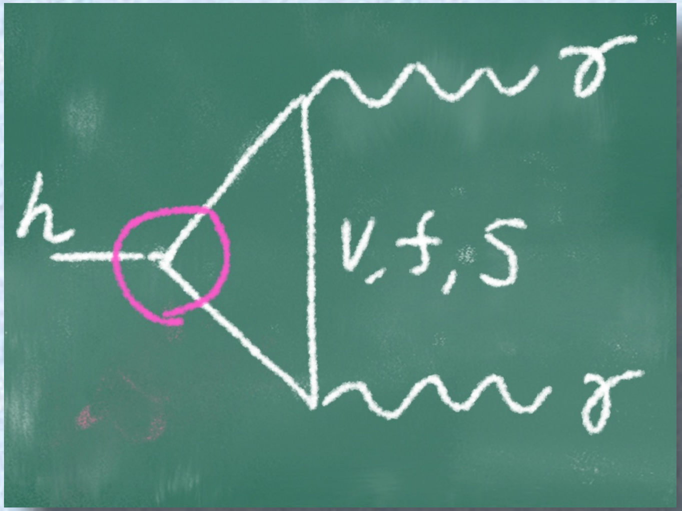
$$\frac{g_{h\tilde{H}^+\tilde{H}^-}}{m_{\tilde{H}^+}} = -(\kappa+1) \frac{2}{v} \boxed{\frac{\cos(\alpha+\beta)}{\sin 2\beta}}$$

$\simeq 1$ (for $\sin(\beta-\alpha) \simeq 1$)

Higgs to diphoton decay width

$$\Gamma(h \to \gamma\gamma) = \frac{\alpha^2 m_h^3}{1024\pi^3} \left| \frac{g_{hVV}}{m_V^2} Q_V^2 A_1(\tau_V) + \frac{2g_{hf\bar{f}}}{m_f} N_{c,f} Q_f^2 A_{1/2}(\tau_f) + N_{c,S} Q_S^2 \frac{g_{hSS}}{m_S^2} A_0(\tau_S) \right|^2 .$$

(Carena-Low-Wagner)



$$\tau_i = \left(\frac{2m_i}{m_h}\right)^2$$

$$A_1 = -8.32, \quad N_c Q_t^2 A_{1/2} = 1.84$$

(W loop) (Top loop)

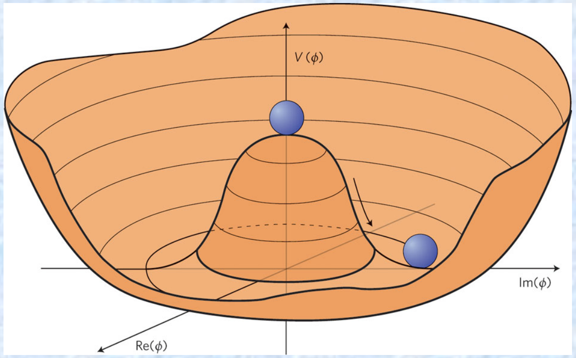
$$\frac{g_{hWW}}{m_W^2} = 2\frac{g_{ht\bar{t}}}{m_t} = \frac{2}{v}$$

For chiral fermions:

$$g_{hf\bar{f}} = (\text{Yukawa})/\sqrt{2}$$
$$m_f = (\text{Yukawa})v/\sqrt{2}$$

Tree-level Higgs potential in SM:

$$V = m_H^2 |H|^2 + \lambda |H|^4$$



$$V(x) = m_H^2 x + \lambda x^2$$

$$V'' = 2\lambda, \quad V''' = 0$$

$$\Rightarrow \quad m_h^2 = 2\lambda v^2, \qquad C_h = 0$$

$$\lambda_{hhh} = 3vV'' = \frac{3m_h^2}{v}$$

$$V(x) = m_H^2 x + \lambda x^2 - \frac{3}{16\pi^2} y_t^4 x^2 \left(\ln \frac{y_t^2 x}{Q^2} - \frac{3}{2} \right)$$

One-loop effective potential from top quark loop

$$V'''(v^2/2) = -3y_t^2/(4\pi^2 v^2)$$

$$\lambda_{hhh} = \frac{3m_h^2}{v}\left(1 + v^2 \frac{V'''}{3V''}\right)$$

$$\equiv C_h$$

$$= -\frac{m_t^4}{\pi^2 v^2 m_h^2} \simeq -0.1$$

$$(M_W^2 W^+ W^- + \frac{M_Z^2}{2} Z^2) \left(1 + G'(1) \frac{2h}{v} + (G'(1) + 2G''(1)) \frac{h^2}{v^2} + \cdots \right)$$

Toy examples

$$F(x) = 1 + a \ln x$$

$\Rightarrow$ $G'(1) = 1 + a$ $\qquad$ $G'(1) + 2G'' = 1 + 3a$

$$F(x) = x^n$$

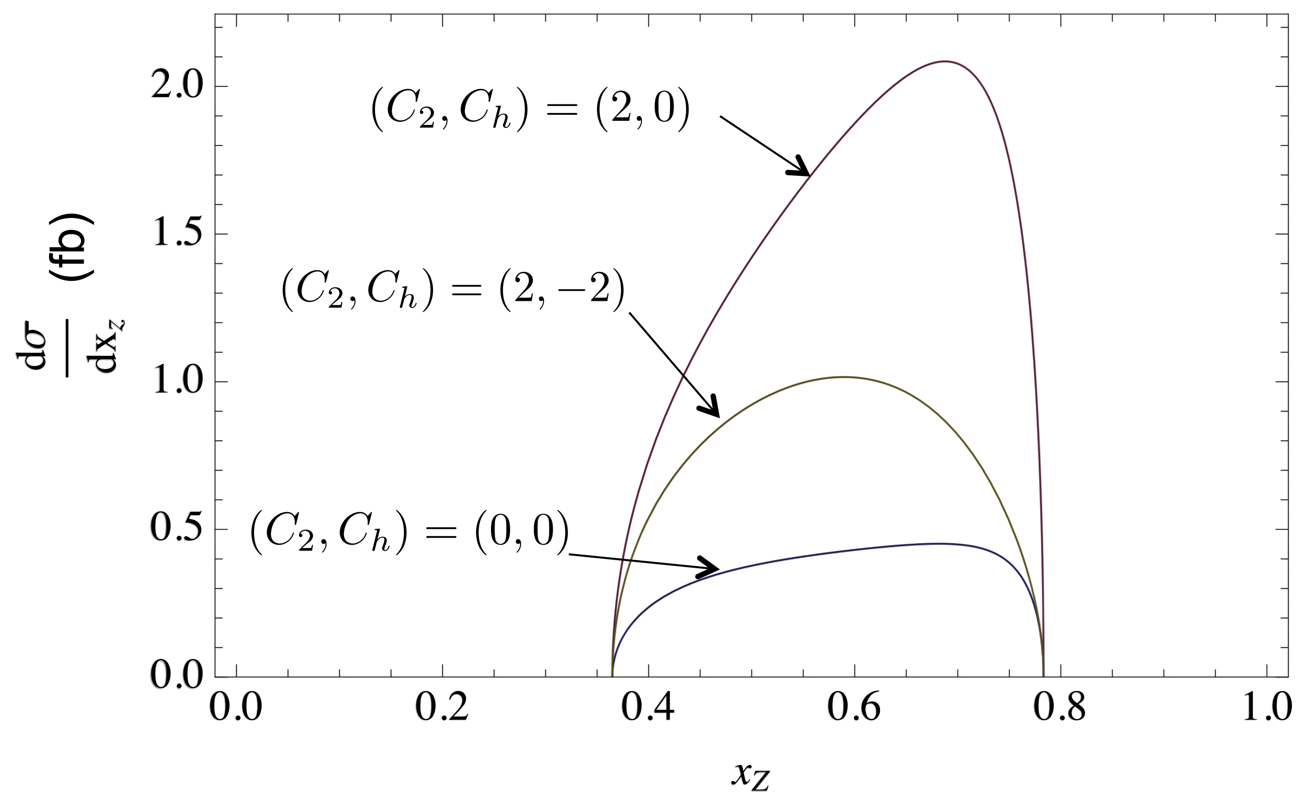
$\Rightarrow$ $G'(1) = 1 + n$ $\qquad$ $G'(1) + 2G'' = 1 + n(2n+3)$

$n = -2$ $\qquad$ $G'(1) = -1$ $\qquad$ $G'(1) + 2G'' = 3$

$h \to -h$ $\Rightarrow$ Single production is same as SM

Energy distribution of Z in $e^+e^- \to Zhh$

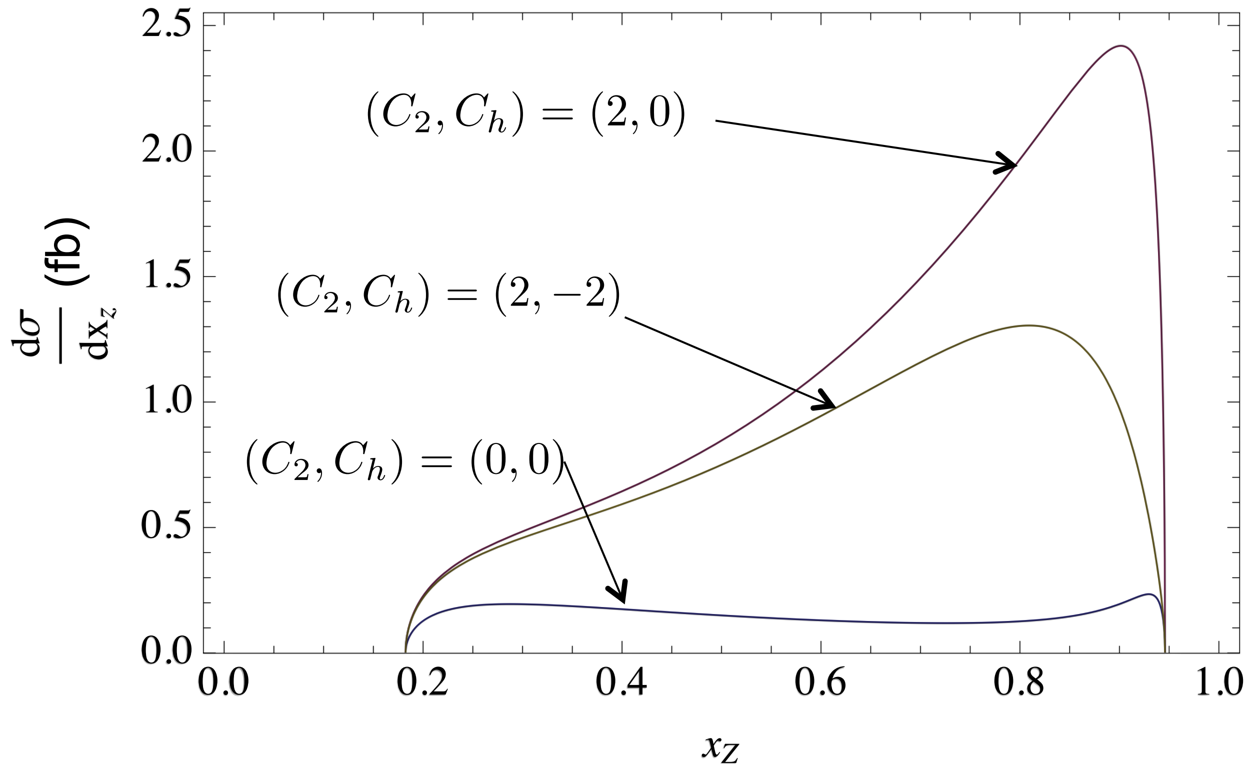
$$x_z \equiv 2E_Z/\sqrt{s}$$



$$\sqrt{s} = 500\,\text{GeV}$$

Energy distribution of Z in $e^+e^- \to Zhh$

$$x_z \equiv 2E_Z/\sqrt{s}$$



$$\sqrt{s} = 1\,\mathrm{TeV}$$