Localization on round three-sphere revisited

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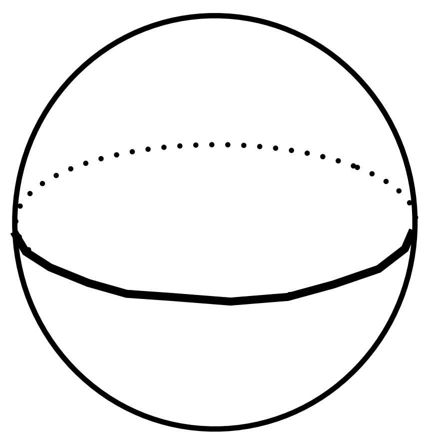
@GIS conference center
Taipei, Taiwan

arXiv:1309.4992

History of Localization on 3-sphere

2009: Kapustin, Willett, Yaakov arXiv:0909.4559

SUSY on  → Z, W

= integral over Coulomb branch σ

History of Localization on 3-sphere

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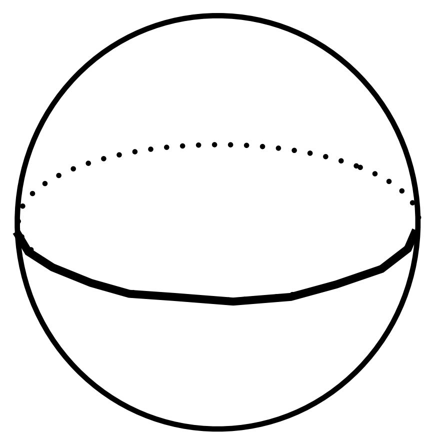
SUSY on  $\longrightarrow$ Z, W

= integral over Coulomb branch σ

$$\mathcal{N}=2 \text{ Vector} \rightarrow \int d\sigma \prod_{\alpha>0} \sinh^2(\pi\alpha\cdot\sigma)$$

History of Localization on 3-sphere

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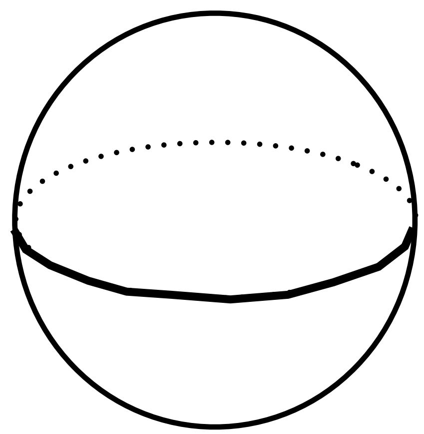
SUSY on  $\longrightarrow$ Z, W

= integral over Coulomb branch σ

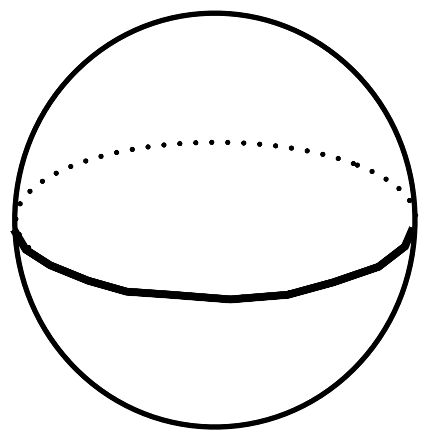
$\mathcal{N}=2$ Vector $\rightarrow \displaystyle\int d\sigma \prod_{\alpha>0} \sinh^2(\pi\alpha\cdot\sigma)$

$\mathcal{N}=2$ Canonical Matter $\rightarrow \displaystyle\prod_{\rho:\mathrm{weight}} \left(\frac{1}{\cosh\pi\rho\cdot\sigma}\right)^{\frac{1}{2}}$

History of Localization on 3-sphere

2010: Hama, Hosomichi, Lee arXiv:1012.3512

Jafferis arXiv:1012.3210

SUSY on  $\longrightarrow$ Z, W

= integral over Coulomb branch σ

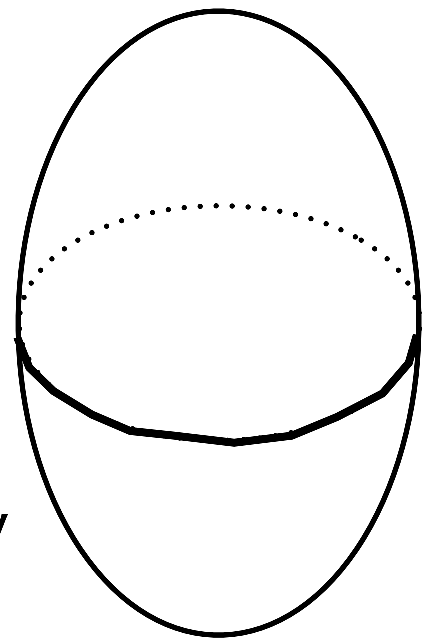
$\mathcal{N}=2$ Vector $\rightarrow \int d\sigma \prod_{\alpha>0} \sinh^2(\pi\alpha\cdot\sigma)$

$\mathcal{N}=2$ Anomalous Matter $\rightarrow \prod_{\rho:\text{weight}} s_{b=1}(i - i\Delta - \rho\cdot\sigma)$

History of Localization on 3-sphere

2011: Hama, Hosomichi, Lee arXiv:1102.4716

SUSY on



+Background field V

$\longrightarrow$ Z, W

= integral over Coulomb branch σ

$\mathcal{N}=2$ Vector$\rightarrow \int d\sigma \prod_{\alpha>0} \sinh\left(\sqrt{\tilde{l}/l}\pi\alpha\cdot\sigma\right)\sinh(\sqrt{l/\tilde{l}}\pi\alpha\cdot\sigma\Big)$

$\mathcal{N}=2$ Anomalous Matter$\rightarrow \prod_{\rho\,:\,\mathrm{weight}} s_b\Big(i\frac{\sqrt{\tilde{l}/l}+\sqrt{l/\tilde{l}}}{2}[1-\Delta]-\sqrt{l\tilde{l}}\rho\cdot\sigma\Big)$

History of Localization on 3-sphere

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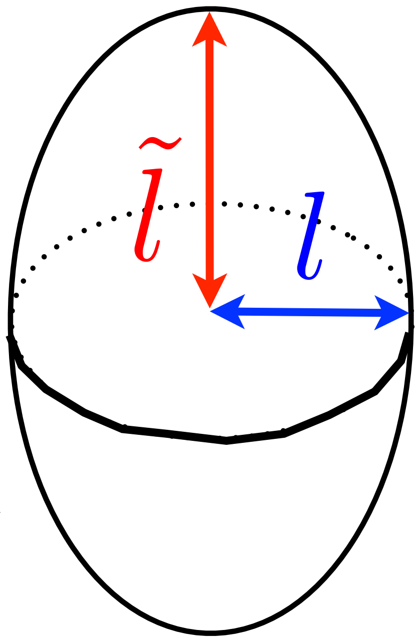
SUSY on  $\longrightarrow$ Z, W

+Background field V

= integral over Coulomb branch σ

$\mathcal{N}=2$ Vector$\rightarrow$ $\displaystyle\int d\sigma \prod_{\alpha>0} \sinh\left(\sqrt{\tilde{l}/l}\pi\alpha\cdot\sigma\right)\sinh(\sqrt{l/\tilde{l}}\pi\alpha\cdot\sigma)$

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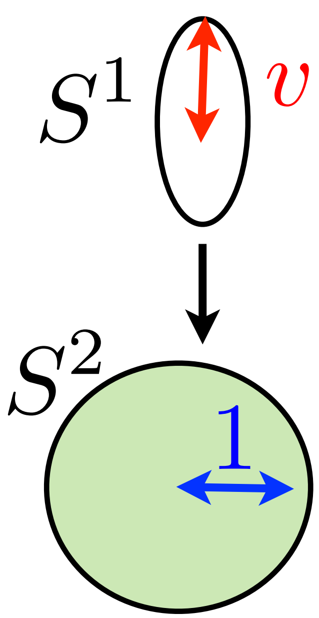
History of Localization on 3-sphere

2011: Imamura, Yokoyama arXiv:1109.4734



$v \to \tilde{l}/l$

SUSY on S^2 $\longrightarrow$ Z, W

+Background field V

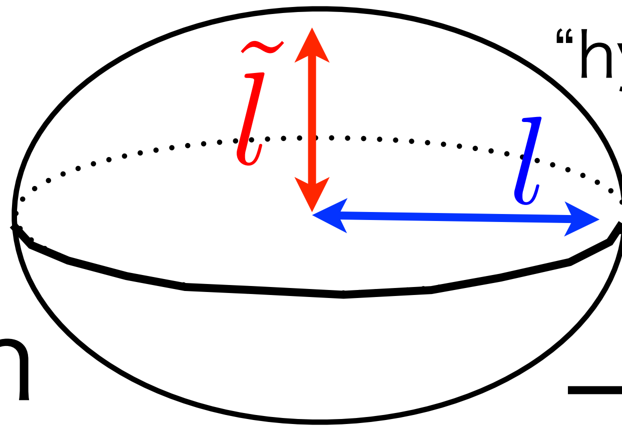
= integral over Coulomb branch σ

$\mathcal{N}=2$ Vector$\rightarrow$ $\displaystyle\int d\sigma \prod_{\alpha>0} \sinh\left(\sqrt{\tilde{l}/l}\pi\alpha\cdot\sigma\right)\sinh\left(\sqrt{l/\tilde{l}}\pi\alpha\cdot\sigma\right)$

$\mathcal{N}=2$ Anomalous Matter$\rightarrow$ $\displaystyle\prod_{\rho\,:\,\text{weight}} s_b\left(i\frac{\sqrt{\tilde{l}/l}+\sqrt{l/\tilde{l}}}{2}[1-\Delta]-\sqrt{l\tilde{l}}\rho\cdot\sigma\right)$

History of Localization on 3-sphere

2011: Martelli, Passias, Sparks arXiv:1100.6400



“hyperbolic” squashed sphere

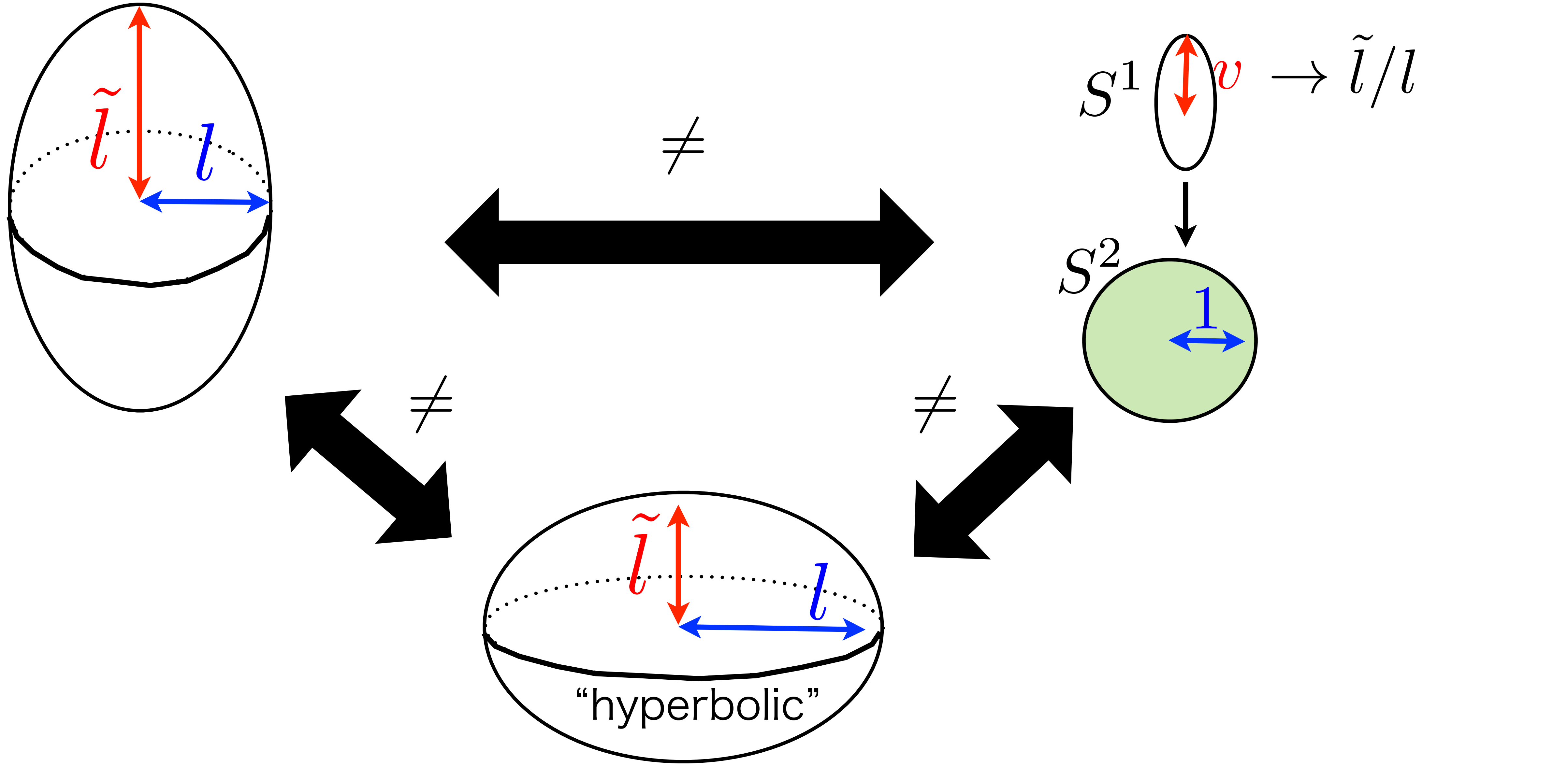
SUSY on $\longrightarrow$ Z, W

= integral over Coulomb branch σ

+Background field V

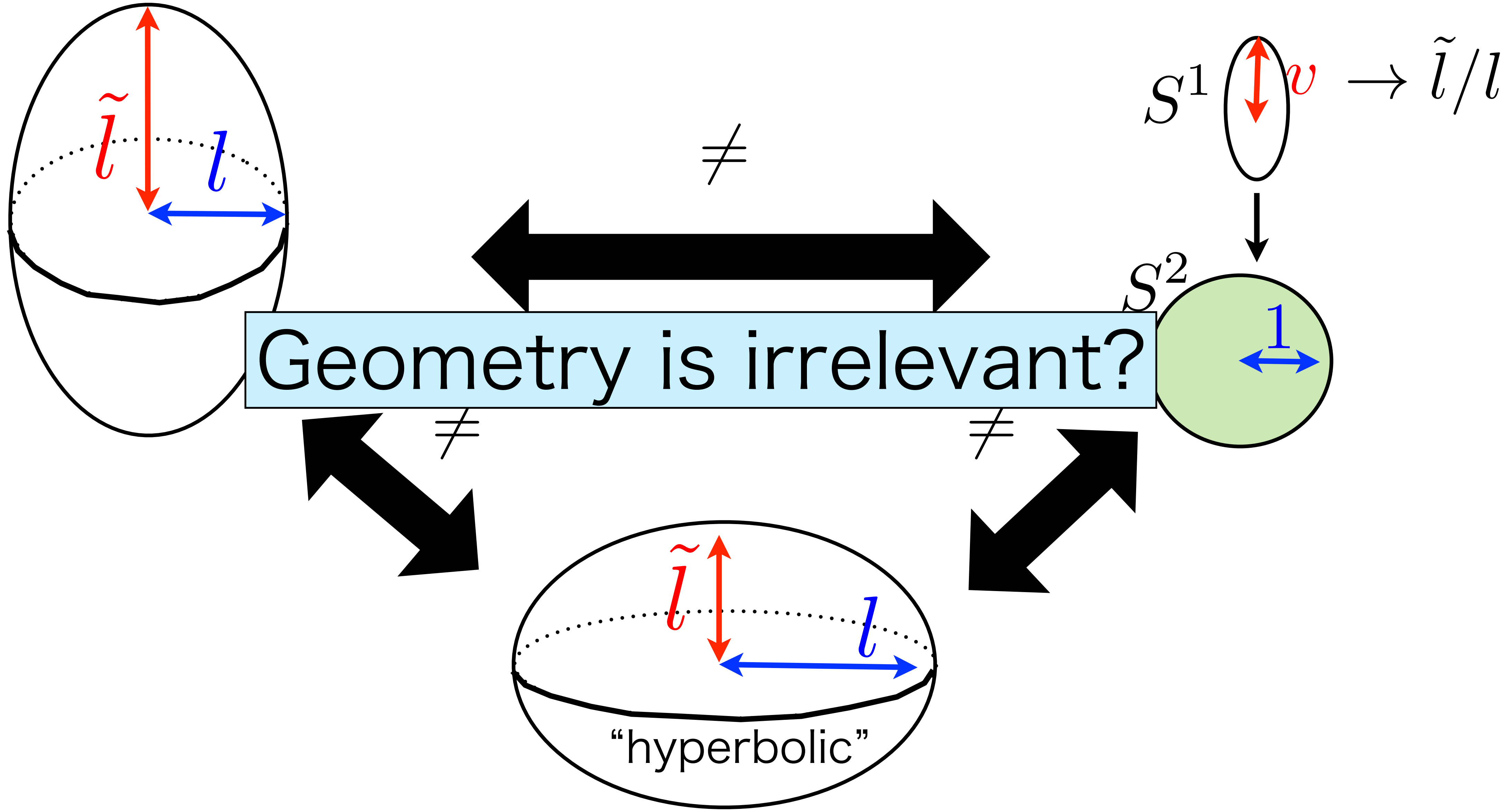
$$\mathcal{N}=2 \text{ Vector} \rightarrow \int d\sigma \prod_{\alpha>0} \sinh\left(\sqrt{\tilde{l}/l}\pi\alpha\cdot\sigma\right)\sinh\left(\sqrt{l/\tilde{l}}\pi\alpha\cdot\sigma\right)$$

$$\mathcal{N}=2 \text{ Anomalous Matter} \rightarrow \prod_{\rho\,:\,\text{weight}} s_b\left(i\frac{\sqrt{\tilde{l}/l}+\sqrt{l/\tilde{l}}}{2}[1-\Delta] - \sqrt{l\tilde{l}}\rho\cdot\sigma\right)$$



$$\mathcal{N}=2 \text{ Vector} \to \int d\sigma \prod_{\alpha>0} \sinh\left(\sqrt{\tilde{l}/l}\,\pi\alpha\cdot\sigma\right)\sinh\left(\sqrt{l/\tilde{l}}\,\pi\alpha\cdot\sigma\right)$$

$$\mathcal{N}=2 \text{ Anomalous Matter} \to \prod_{\rho\,:\,\text{weight}} s_b\left(i\frac{\sqrt{\tilde{l}/l}+\sqrt{l/\tilde{l}}}{2}[1-\Delta]-\sqrt{l\tilde{l}}\,\rho\cdot\sigma\right)$$



$\mathcal{N}=2$ Vector $\to \displaystyle\int d\sigma \prod_{\alpha>0} \sinh\left(\sqrt{\tilde{l}/l}\,\pi\alpha\cdot\sigma\right)\sinh\left(\sqrt{l/\tilde{l}}\,\pi\alpha\cdot\sigma\right)$

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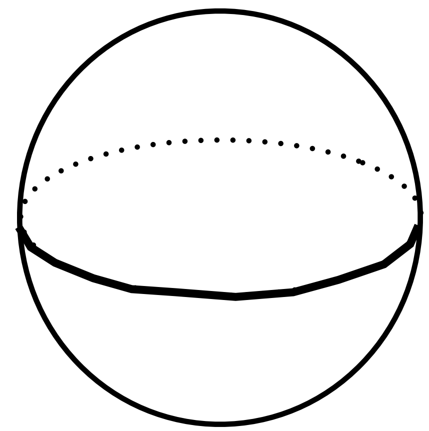

?

$$\mathcal{N}=2 \text{ Vector} \rightarrow \int d\sigma \prod_{\alpha>0} \sinh\left(\sqrt{\tilde{l}/l}\,\pi\alpha\cdot\sigma\right)\sinh\left(\sqrt{l/\tilde{l}}\,\pi\alpha\cdot\sigma\right)$$

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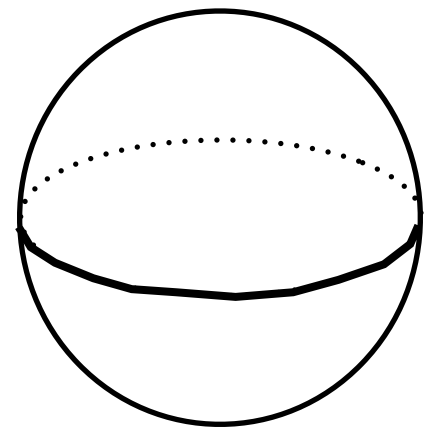

+Background field V

?

$\mathcal{N}=2$ Vector$\rightarrow$ $\int d\sigma \prod_{\alpha>0} \sinh\left(\sqrt{\tilde{l}/l}\pi\alpha\cdot\sigma\right)\sinh(\sqrt{l/\tilde{l}}\pi\alpha\cdot\sigma\Big)$

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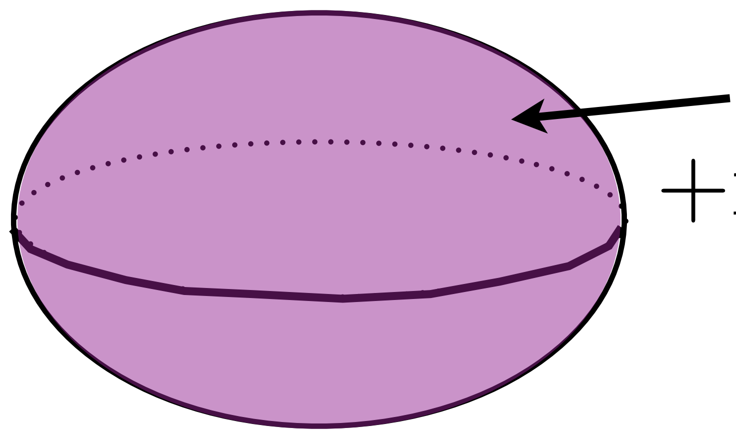
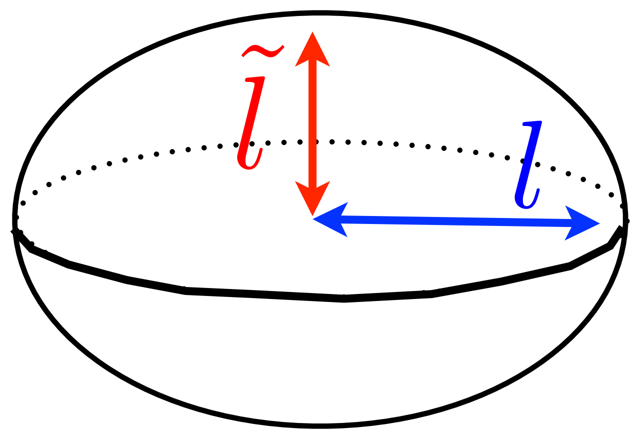
Applications

D Martelli, A Passias & J Sparks arXiv:1110.6400





"Hyperbolic" squashed sphere

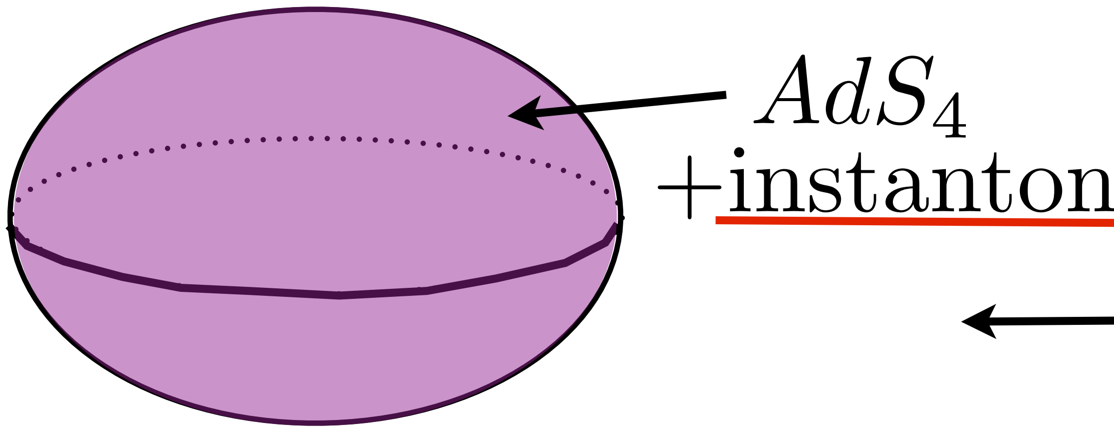
+Background U(1) vector

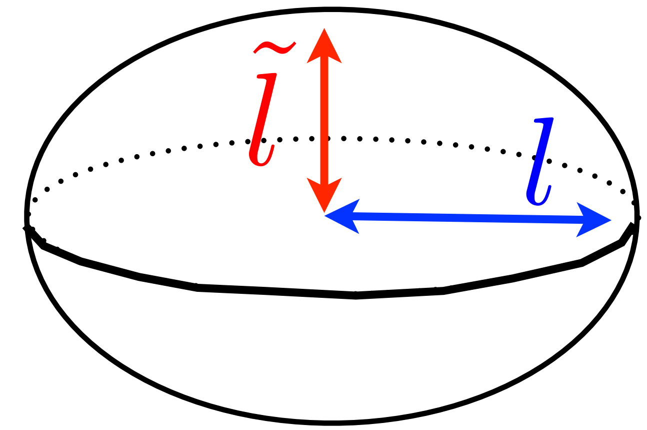
Killing spinors on 3D

4D Gravity

Field theory

D Martelli, A Passias & J Sparks arXiv:1110.6400





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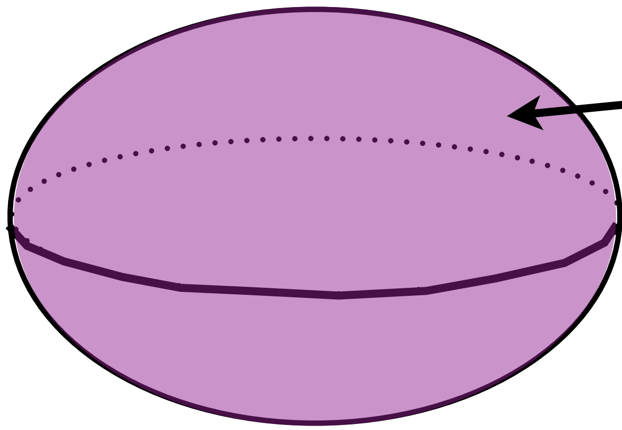
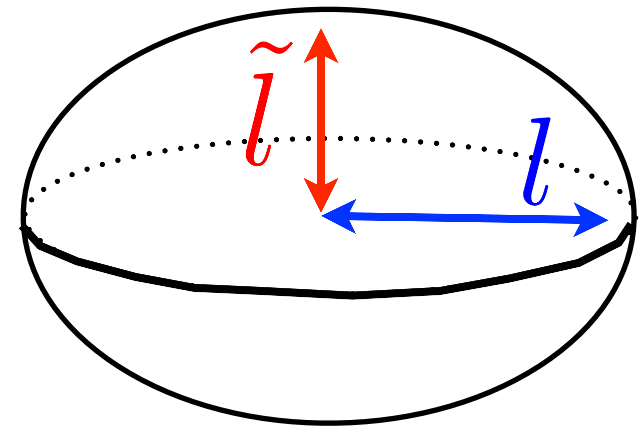


AdS_4
+instanton



“Hyperbolic” squashed sphere
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Killing spinors on 3D

Einstein-Maxwell type action

$$-\frac{1}{16\pi G_4}\int d^4x\sqrt{g}(R-2\Lambda-F_{\mu\nu}F^{\mu\nu}$$
$$+\ \mathrm{gravitino})$$

4D Gravity

Field theory

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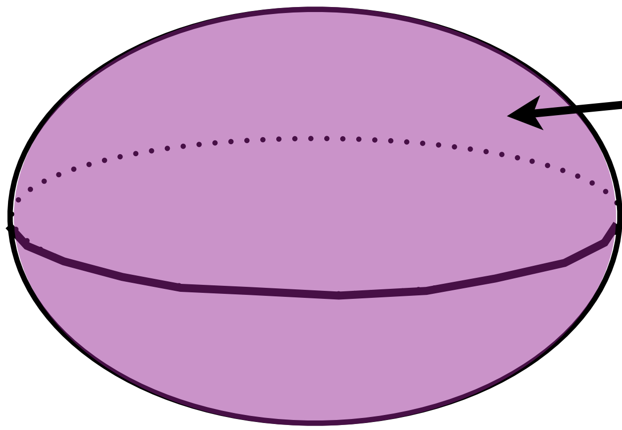

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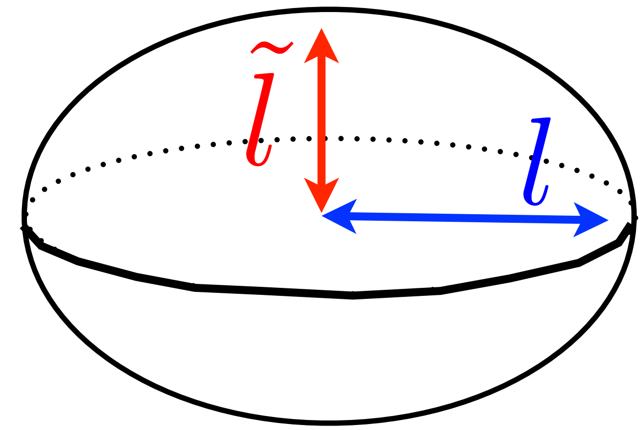

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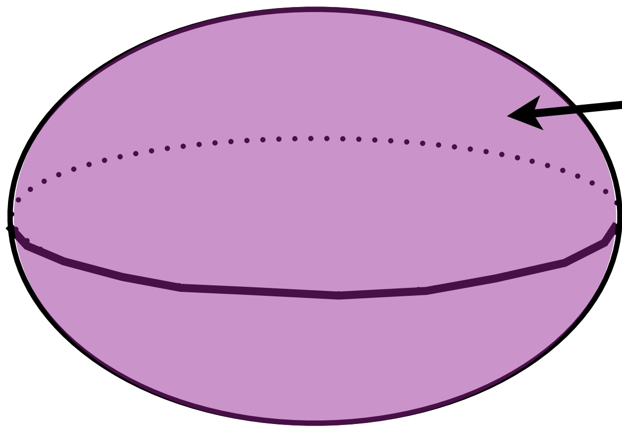

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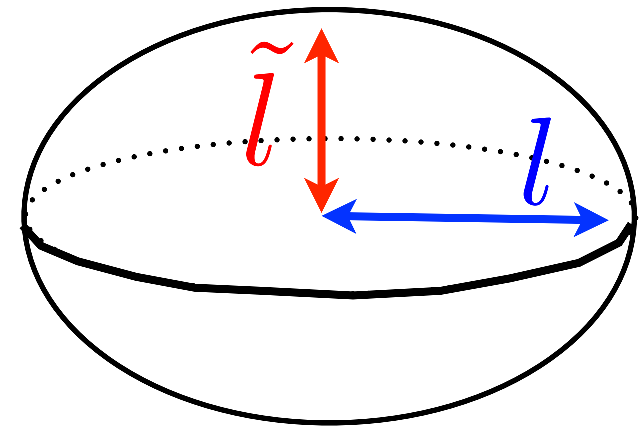

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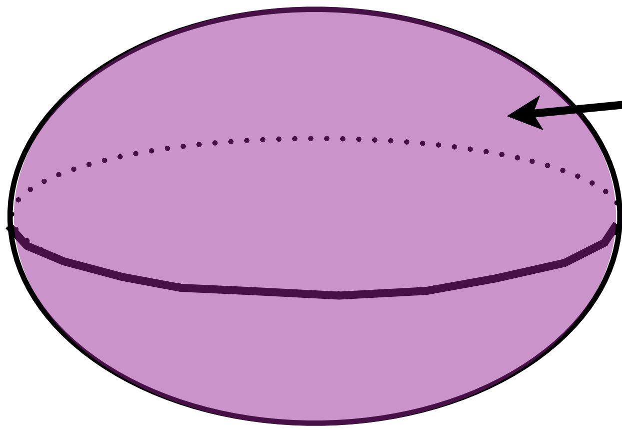

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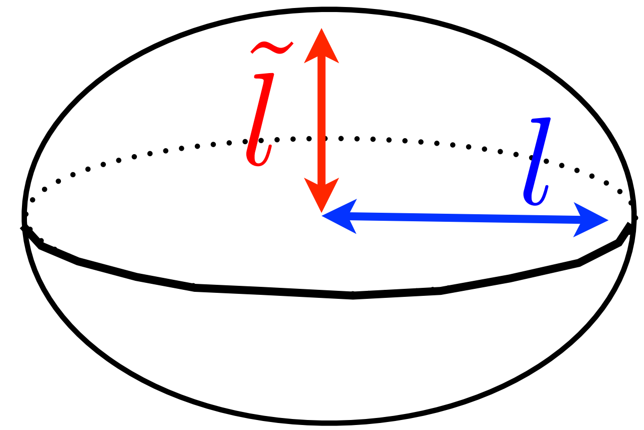

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$$-\frac{1}{16\pi G_4}\int d^4x\sqrt{g}\big(\underline{R-2\Lambda}-\underline{F_{\mu\nu}F^{\mu\nu}} + \text{gravitino}\big)$$

4D Gravity

Field theory

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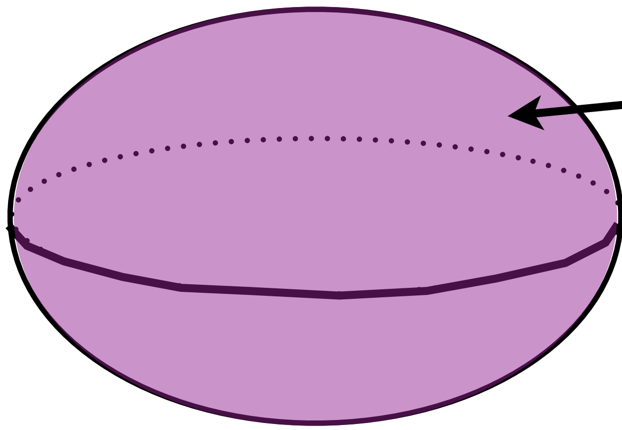
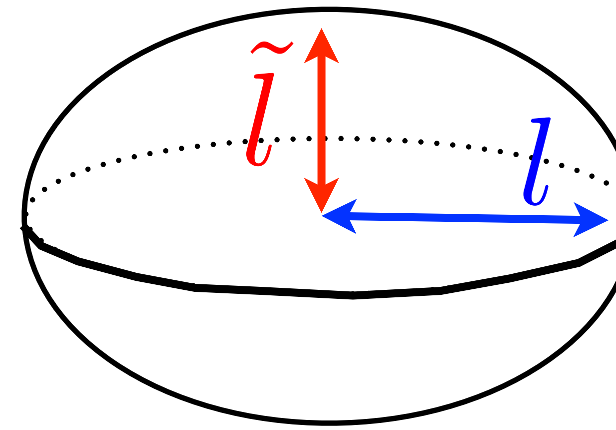


AdS_4

$+\text{instanton}$



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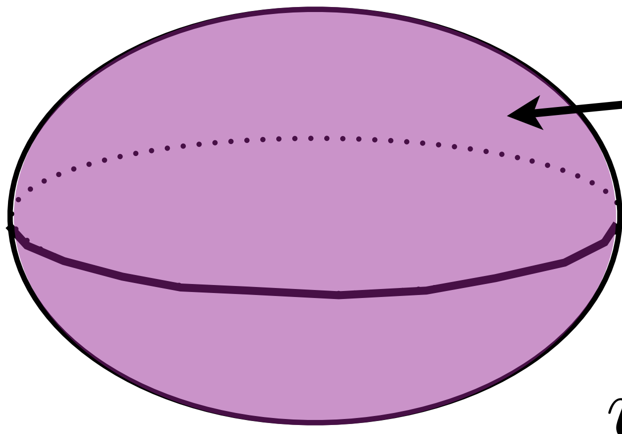


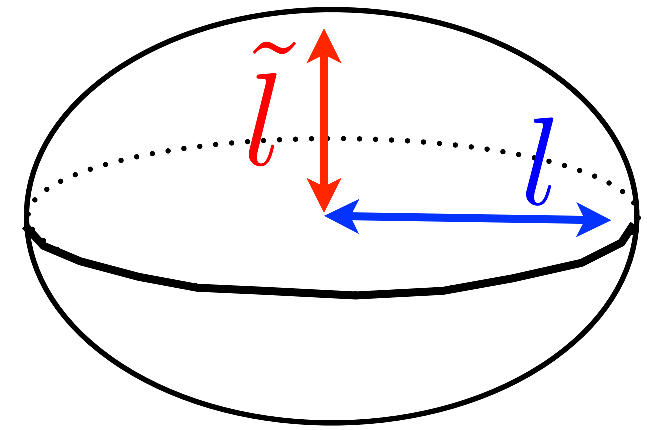
AdS_4
+instanton

$\psi_\mu = 0$

Einstein-Maxwell type action

$$-\frac{1}{16\pi G_4}\int d^4x\sqrt{g}(R-2\Lambda+F_{\mu\nu}F^{\mu\nu}$$
$$+\ \text{gravitino})$$

4D Gravity



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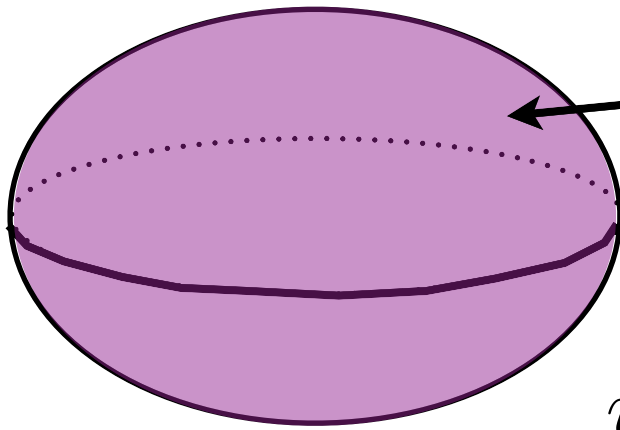

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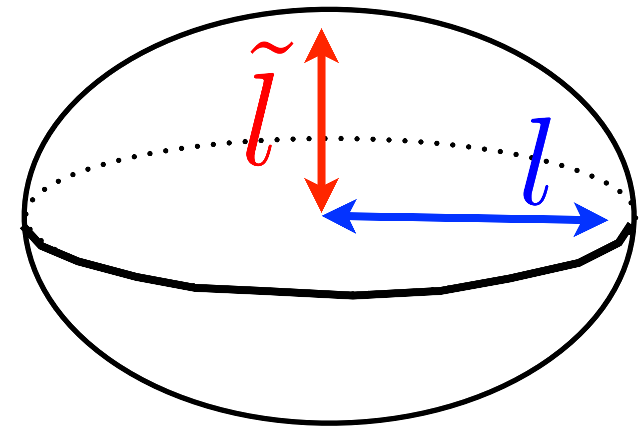

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Killing spinors on 3D

Einstein-Maxwell type action

$$-\frac{1}{16\pi G_4}\int d^4x\sqrt{g}\left(\underline{R-2\Lambda}-\underline{F_{\mu\nu}F^{\mu\nu}}\right.$$
$$\left.+\,\underline{\text{gravitino}}\right)$$

4D Gravity

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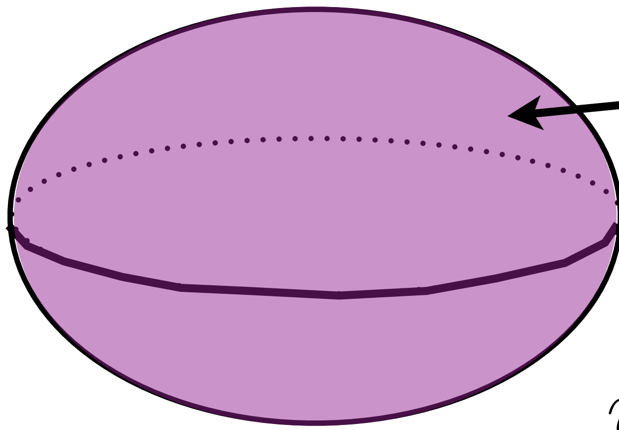

AdS_4

$+$instanton

$\psi_\mu = 0$

BPS cond: $\delta\psi_\mu = 0$

Einstein-Maxwell type action

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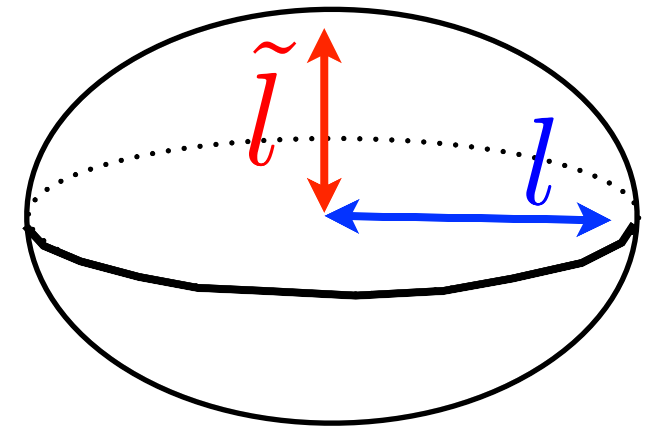
4D Gravity



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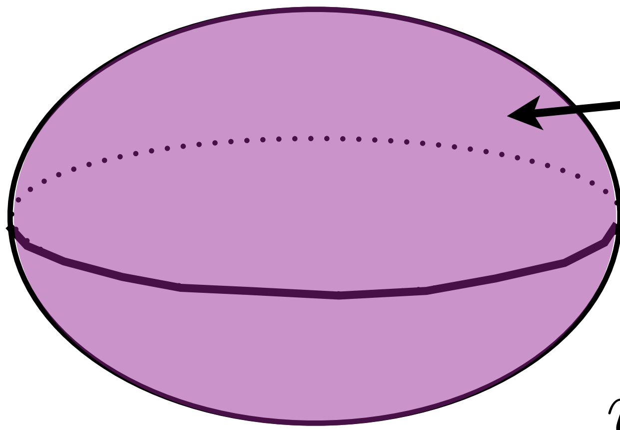

AdS_4
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$\psi_\mu = 0$

BPS cond: $\delta\psi_\mu = 0$

Killing spinor on 4D

Einstein-Maxwell type action

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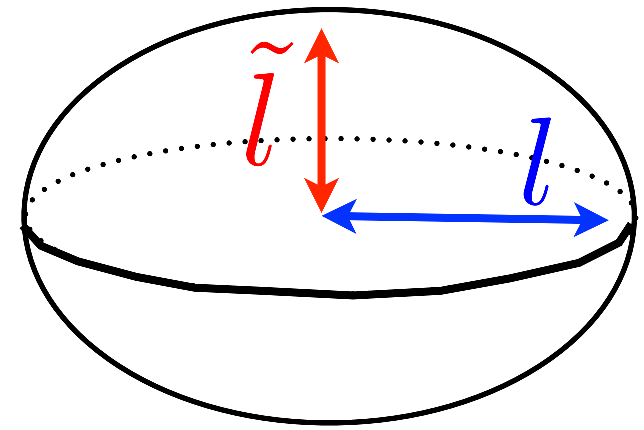
4D Gravity



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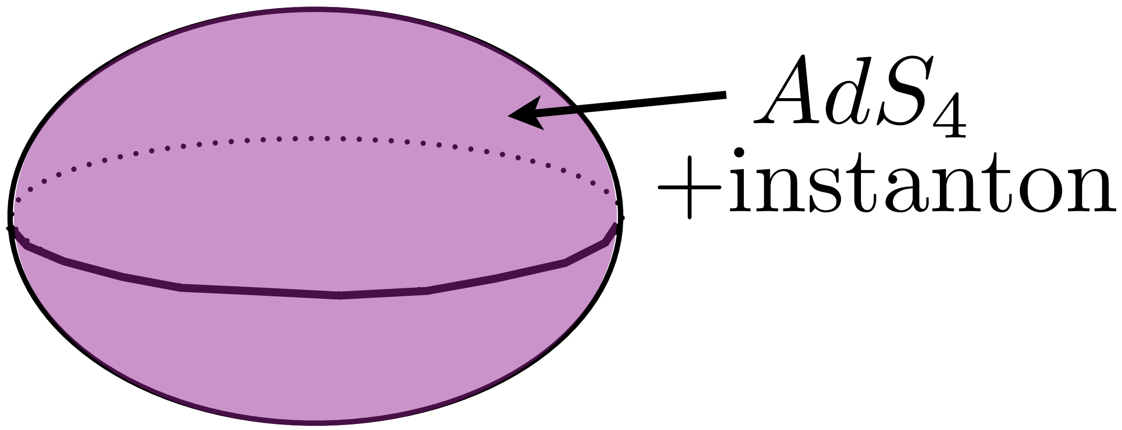
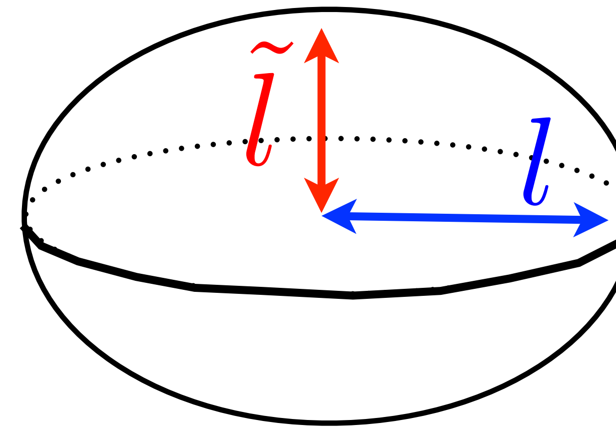


BPS cond: $\delta\psi_\mu = 0$

Killing spinor on 4D



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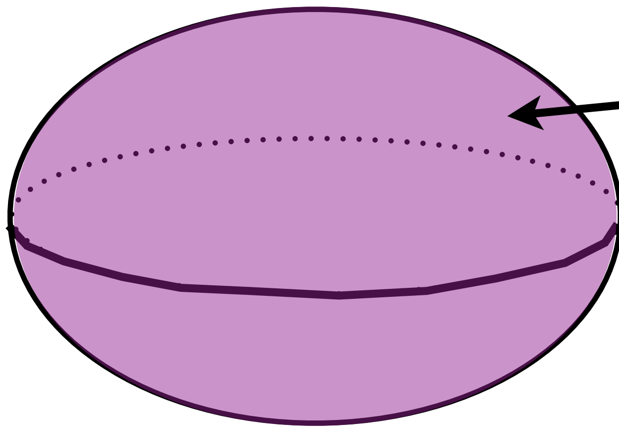

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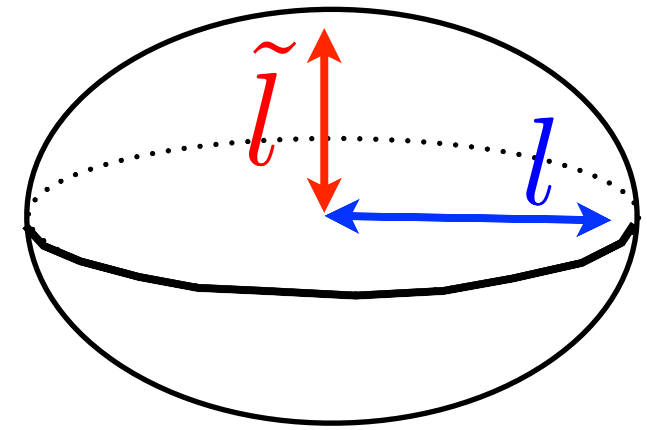

"Hyperbolic" squashed sphere
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BPS cond: $\delta\psi_\mu = 0$

Killing spinor on 4D

Killing spinors on 3D

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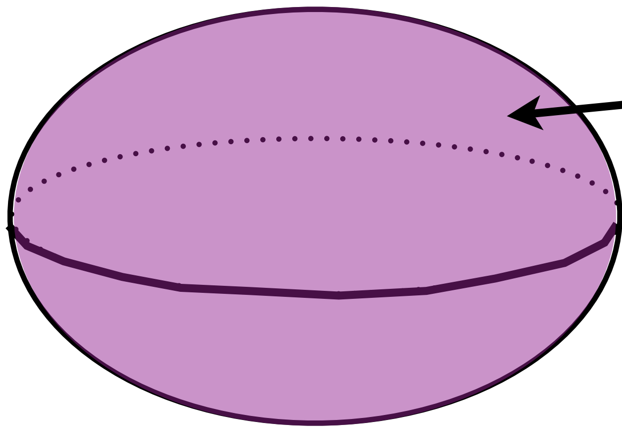

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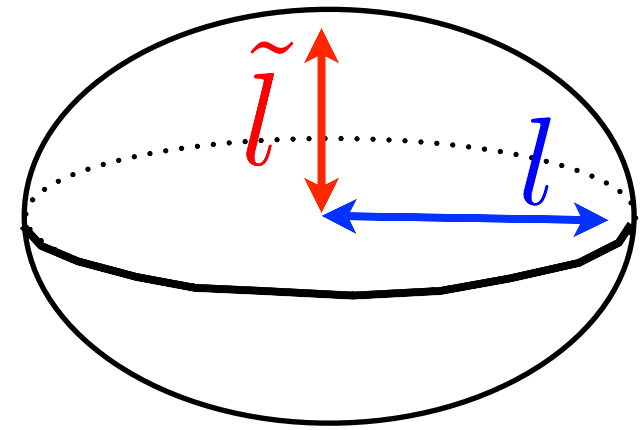

"Hyperbolic" squashed sphere
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BPS cond: $\delta\psi_\mu = 0$

➡ Killing spinor on 4D

Killing spinors on 3D

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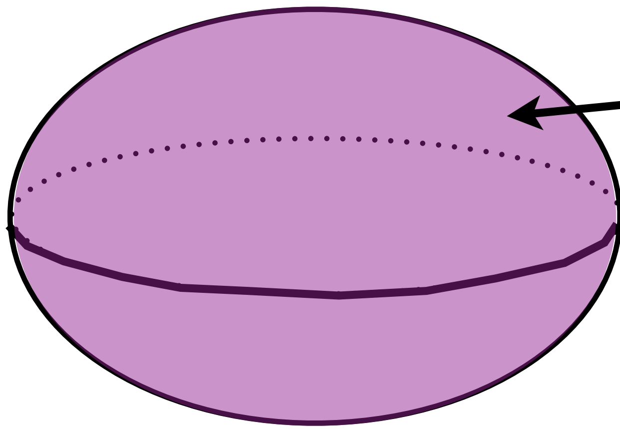

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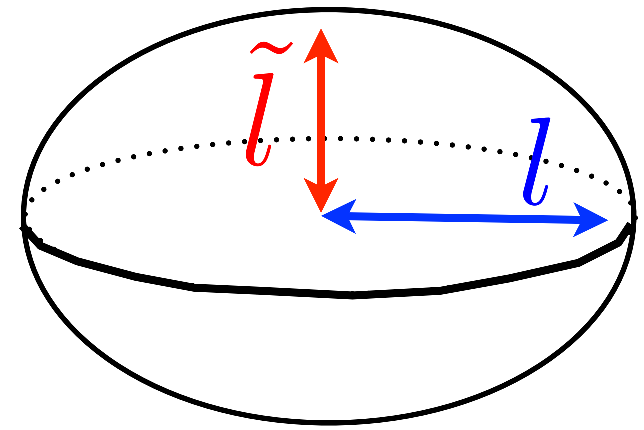

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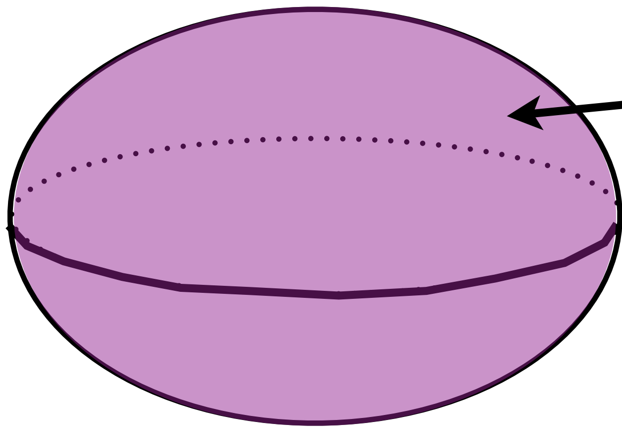
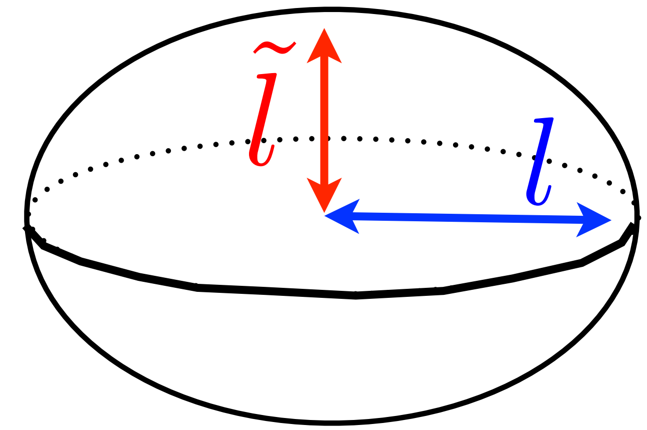
Field theory

D Martelli, A Passias & J Sparks arXiv:1110.6400



AdS_4
+instanton



"Hyperbolic" squashed sphere
+Background U(1) vector

BPS cond: $\delta\psi_\mu = 0$

Killing spinor on 4D

Killing spinors on 3D

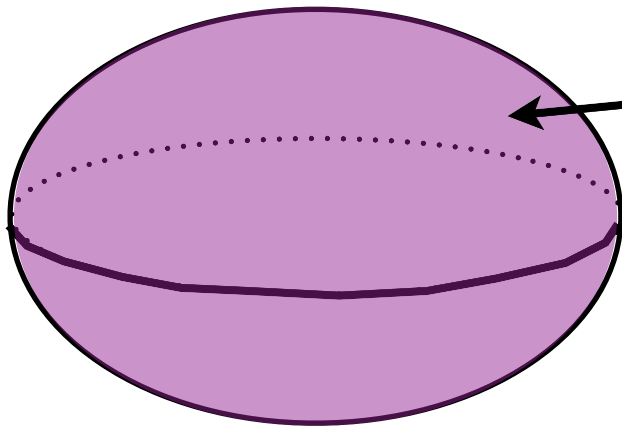
$$\log Z_{grav}$$

$$\log Z_{field}$$

4D Gravity

Field theory

D Martelli, A Passias & J Sparks arXiv:1110.6400



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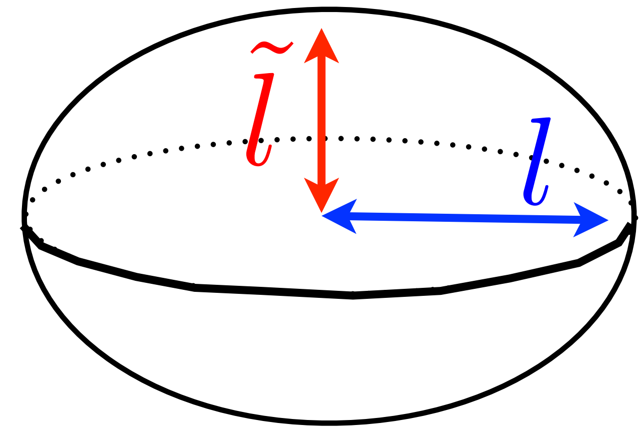


"Hyperbolic" squashed sphere
+Background U(1) vector

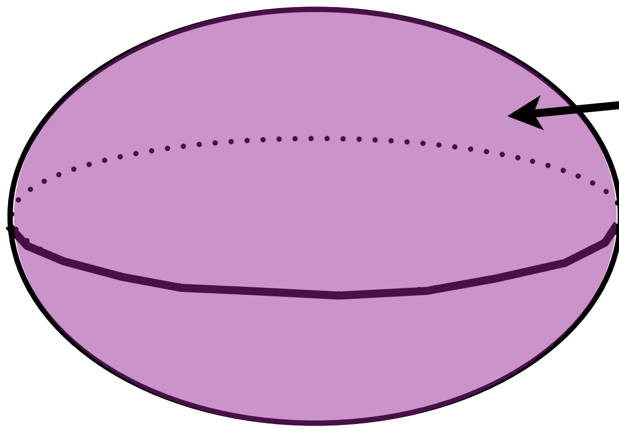
BPS cond: $\delta\psi_\mu = 0$

➡ Killing spinor on 4D → Killing spinors on 3D

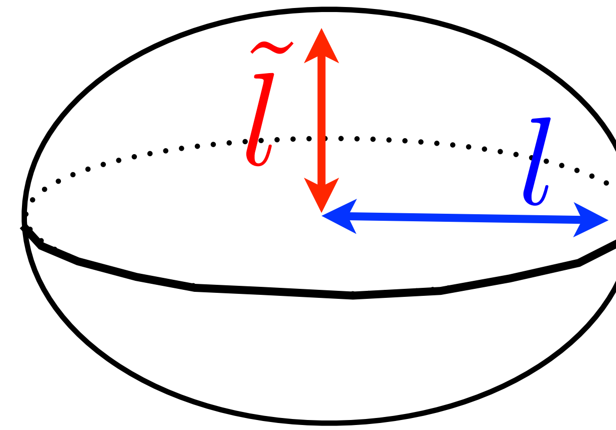
$$\log Z_{grav} \xleftarrow{\boxed{\text{Large N}}} \log Z_{field}$$

4D Gravity | Field theory

D Martelli, A Passias & J Sparks arXiv:1110.6400



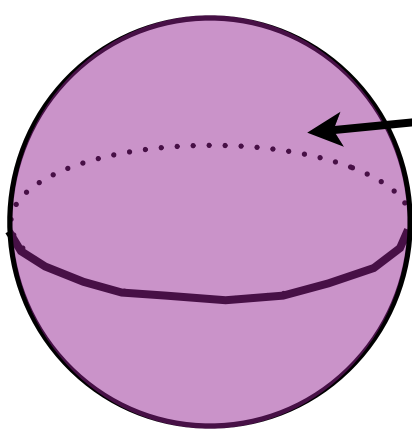
AdS_4
+instanton



changing "slices"

"Hyperbolic" squashed sphere
+Background U(1) vector

Killing spinors on 3D



AdS_4
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4D Gravity

Field theory

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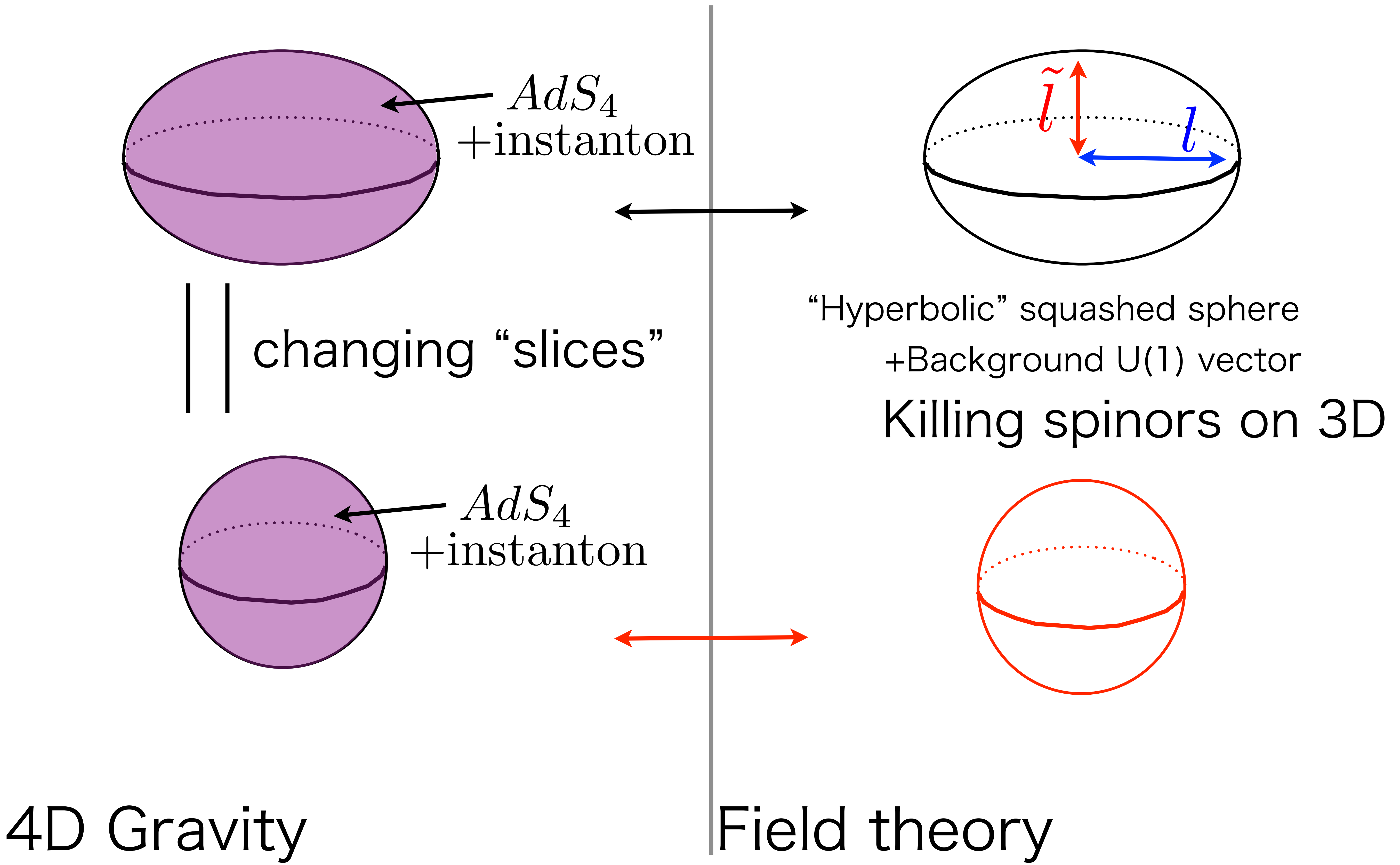
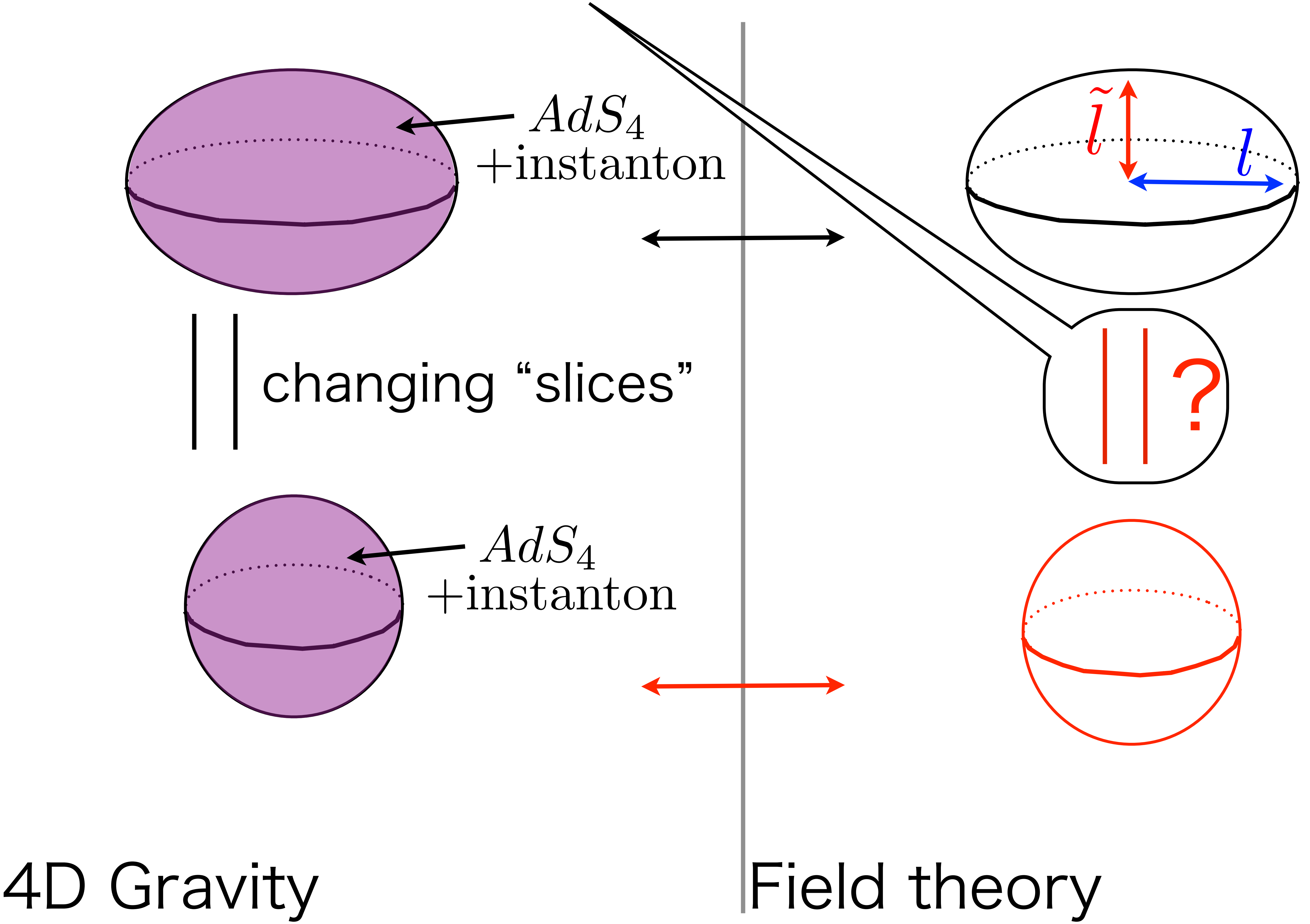


D Martelli, A Passias & J Sparks arXiv:1110.6400



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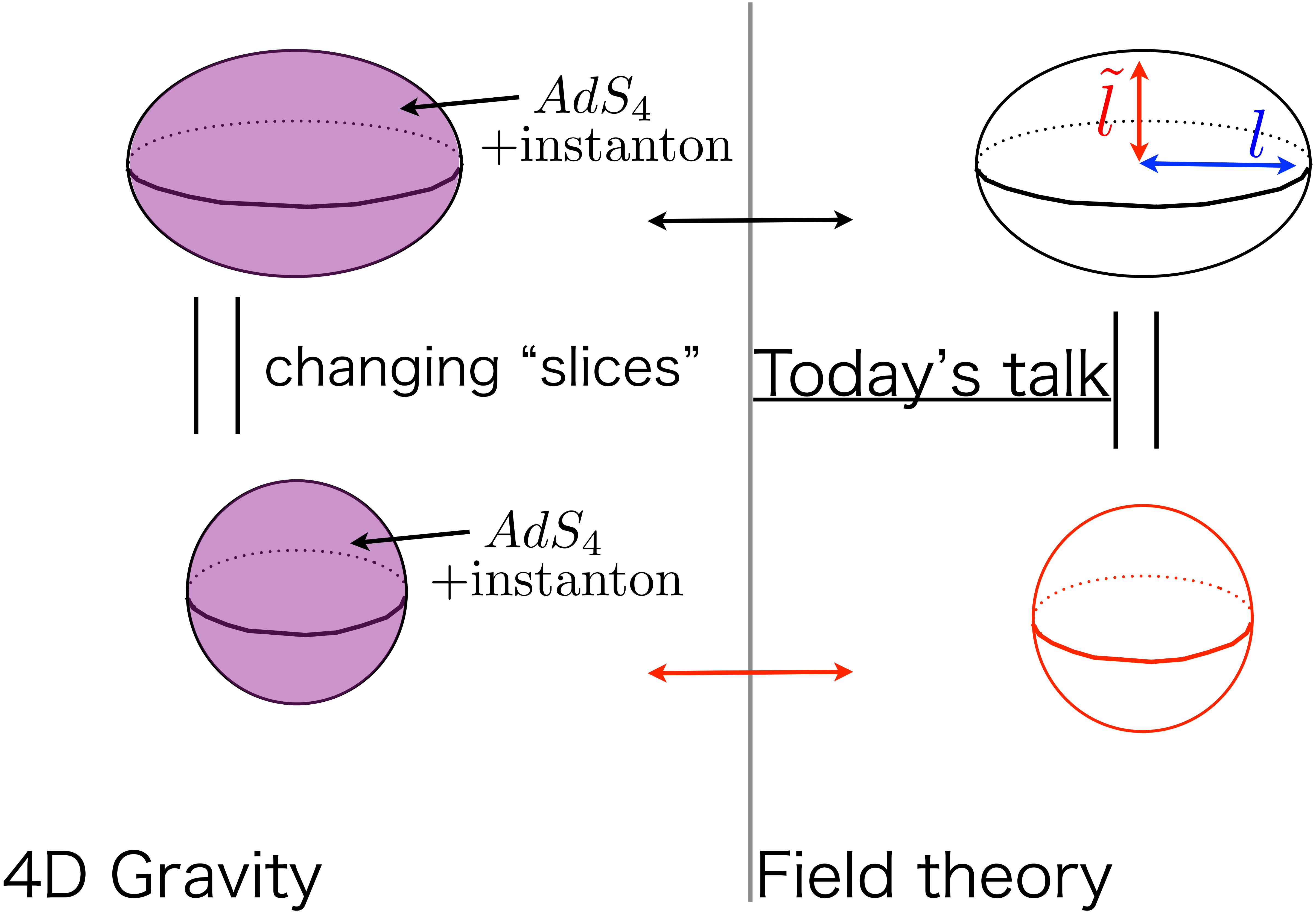

Contents

- ✔History & Intro
- ✔How to construct?
- ○SUSY on round sphere
- Applications

Metric

$$ds^2 = d\theta^2 + \cos^2\theta d\phi^2 + \sin^2\theta d\chi^2$$

Killing spinor

$$\epsilon = \begin{pmatrix} -(\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

SUSY

$$\delta_\epsilon A_\mu = -\frac{i}{2}\bar{\lambda}\gamma_\mu\epsilon, \ \text{etc}$$

$$\delta_\epsilon \phi = 0, \ \text{etc}$$

Squashed sphere's case

Metric

$$ds^2 = d\theta^2 + \cos^2\theta d\phi^2 + \sin^2\theta d\chi^2$$

$$\leftrightarrow ds^2 = f^2(\theta)d\theta^2 + l^2\cos^2\theta d\phi^2 + \tilde{l}^2\sin^2\theta d\chi^2$$

Killing spinor

$$\epsilon = \begin{pmatrix} -(\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$$\leftrightarrow \epsilon = \begin{pmatrix} -(\cos\theta + i\sin\theta)^{1/2} \\ (\cos\theta - i\sin\theta)^{1/2} \end{pmatrix}$$

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$$\delta_\epsilon A_\mu = -\frac{i}{2}\bar{\lambda}\gamma_\mu\epsilon, \text{ etc}$$

$$\delta_\epsilon\phi = 0, \text{ etc}$$

Action $\left(f^2(\theta) = \sin^2\theta + \frac{1}{s^2}\cos^2\theta\right)$

$$\mathcal{L}_{\rm CS} = {\rm Tr}\Big[\frac{1}{\sqrt{g}}\varepsilon^{\mu\nu\lambda}(A_\mu\partial_\nu A_\lambda - \frac{2i}{3}A_\mu A_\nu A_\lambda) - \overline{\lambda}\lambda + 2D\sigma\Big].$$

$$\mathcal{L}_{\rm YM} = (2sf){\rm Tr}\Big(\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\mathcal{D}^i\sigma\mathcal{D}_i\sigma + \frac{1}{2}(\mathcal{D}_\theta\sigma + \frac{f'}{f}\sigma)^2 + \frac{1}{2}(D + \frac{1}{sf^2}\sigma)^2$$
$$+ \frac{i}{2}\overline{\lambda}\gamma^i\mathcal{D}_i\lambda + \frac{i}{2}\overline{\lambda}\gamma^\theta(\mathcal{D}_\theta\lambda + \frac{1}{2}\frac{f'}{f}\lambda) + \frac{i}{2}\overline{\lambda}[\sigma,\lambda] - \frac{1}{4}\frac{1}{sf^2}\overline{\lambda}\lambda\Big)$$

$$\mathcal{L}_{\rm mat} = (2sf^{2\Delta-1})\Big(\mathcal{D}_\mu\overline{\phi}\mathcal{D}^\mu\phi + \overline{\phi}\sigma^2\phi + i\overline{\phi}D\phi + \overline{\phi}\sigma^2\phi + i\frac{2\Delta-1}{sf^2}\overline{\phi}\sigma\phi$$
$$- \frac{\Delta(2\Delta-1)}{2(sf^2)^2}\overline{\phi}\phi - \frac{\Delta(2\Delta-1)}{2}(\frac{f'}{f})^2\overline{\phi}\phi + \frac{\Delta}{4}R\overline{\phi}\phi$$
$$- i\overline{\psi}\gamma^\mu\mathcal{D}_\mu\psi - i\frac{f'}{f}(\Delta - \frac{1}{2})\overline{\psi}\gamma^3\psi + i\overline{\psi}\sigma\psi - \frac{2\Delta-1}{2sf^2}(\overline{\psi}\psi)$$
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$$\begin{aligned}\mathcal{L}_{\rm YM} = (2sf){\rm Tr}\Big(&\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\mathcal{D}^i\sigma\mathcal{D}_i\sigma + \frac{1}{2}(\mathcal{D}_\theta\sigma + \frac{f'}{f}\sigma)^2 + \frac{1}{2}(D + \frac{1}{sf^2}\sigma)^2\\ &+ \frac{i}{2}\overline{\lambda}\gamma^i\mathcal{D}_i\lambda + \frac{i}{2}\overline{\lambda}\gamma^\theta(\mathcal{D}_\theta\lambda + \frac{1}{2}\frac{f'}{f}\lambda) + \frac{i}{2}\overline{\lambda}[\sigma,\lambda] - \frac{1}{4}\frac{1}{sf^2}\overline{\lambda}\lambda\Big)\end{aligned}$$

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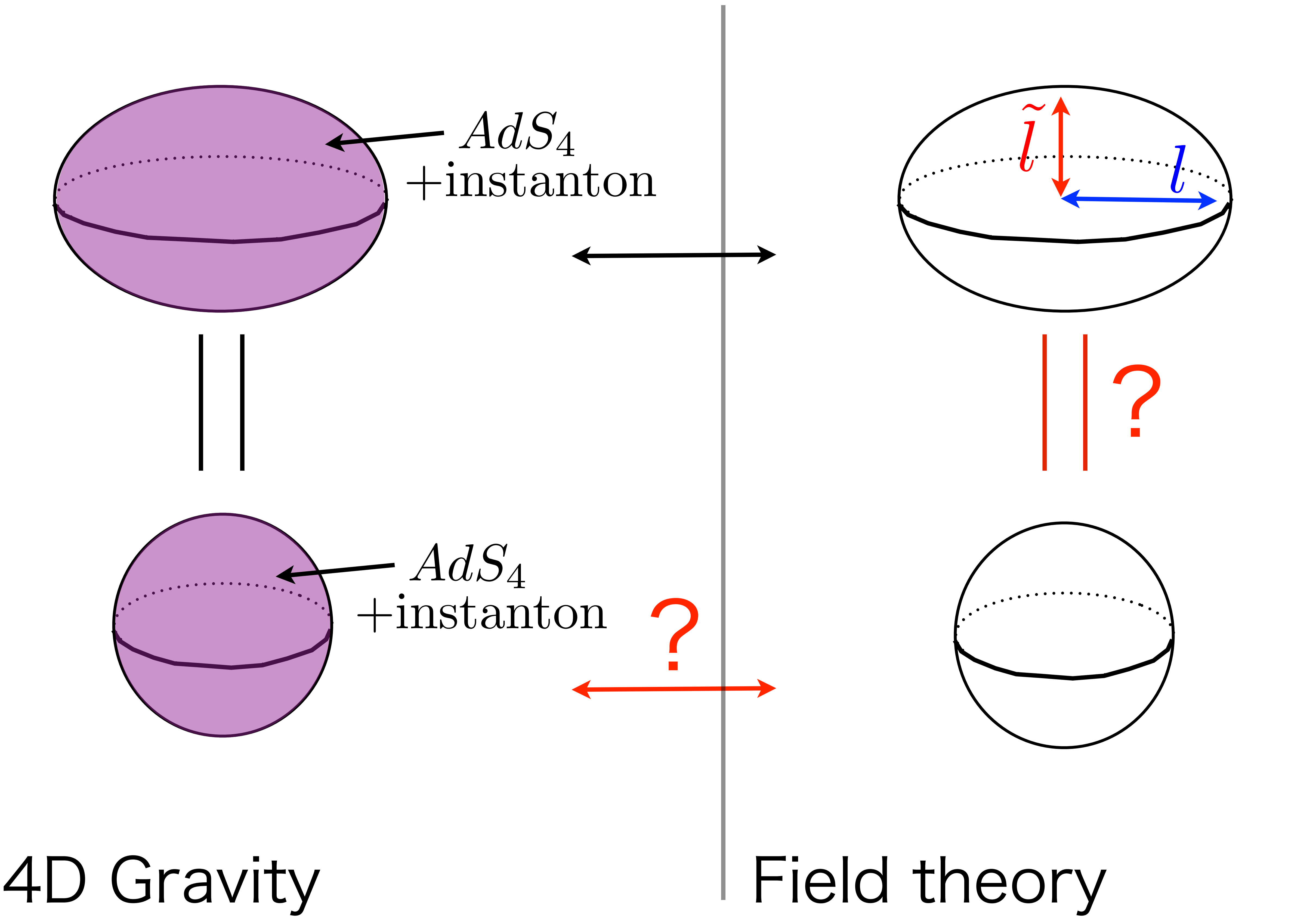
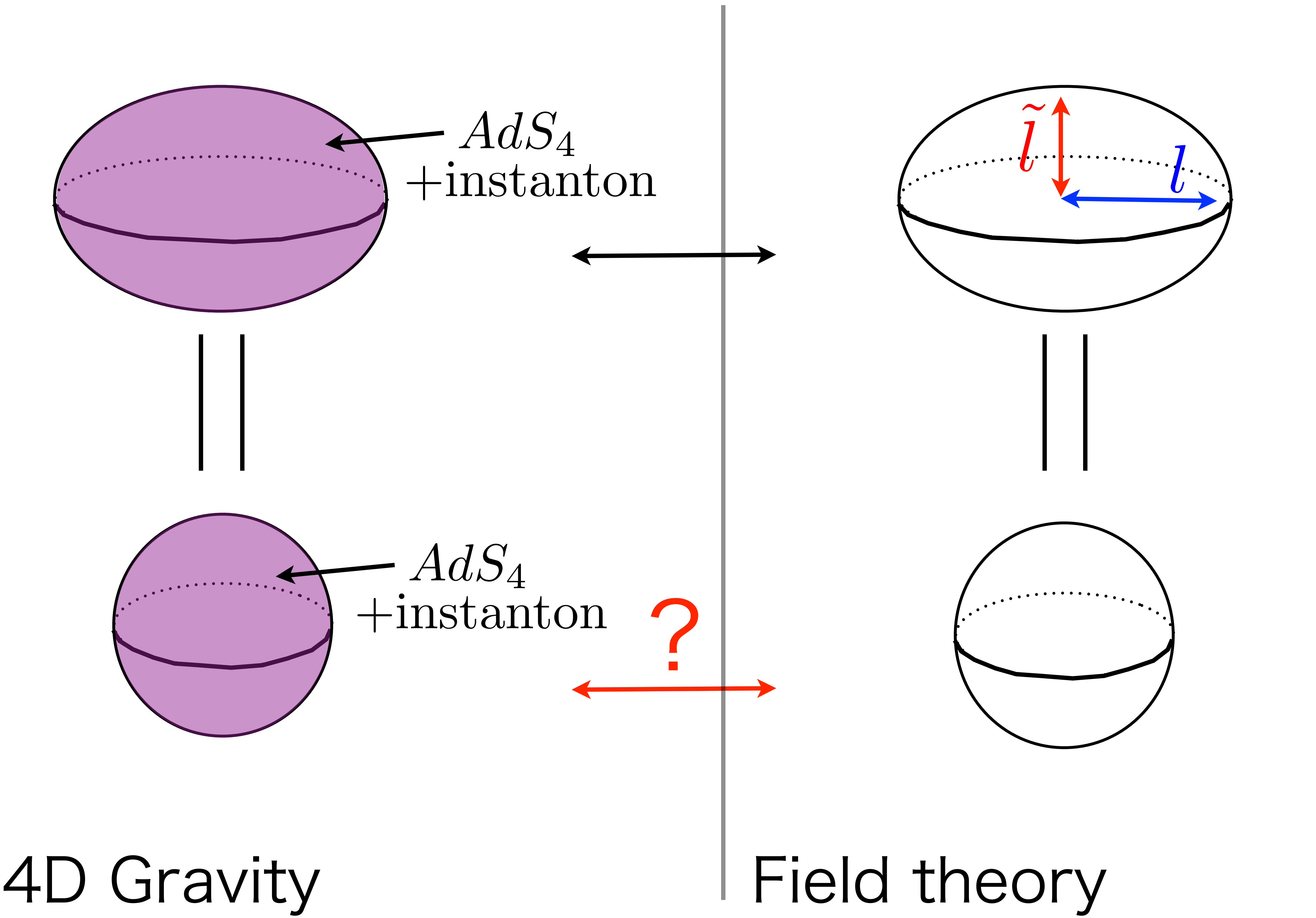
Double sine
(depends on s)



AdS_4 +instanton
$\tilde{l}$
l
?
AdS_4 +instanton
?
4D Gravity
Field theory



AdS_4
+instanton
$\tilde{l}$
l
AdS_4
+instanton
?
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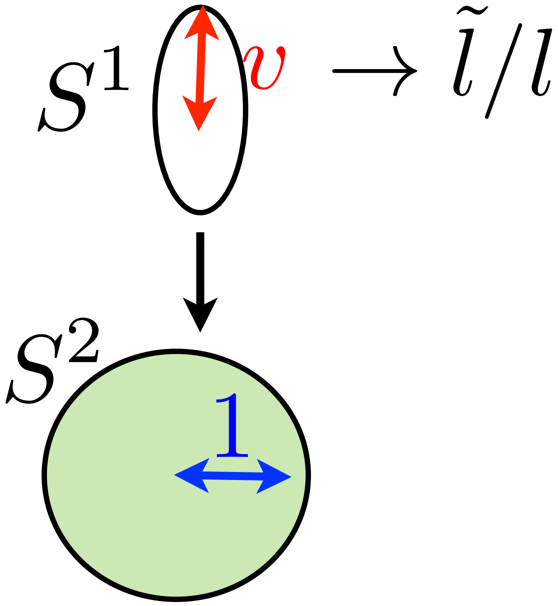
Related works:



Nian arXiv:1309.3266

←solving a puzzle for S^2

Alday, Martelli, Richmond, Sparks arXiv:1307.6848

←emergence of double sine function with a broader class of metrics

Closset, Dumitrescu, Festuccia, Komargodski arXiv:1309.5876

←(geometrical) parameter dependence for partition function in 4D and 3D

Contents

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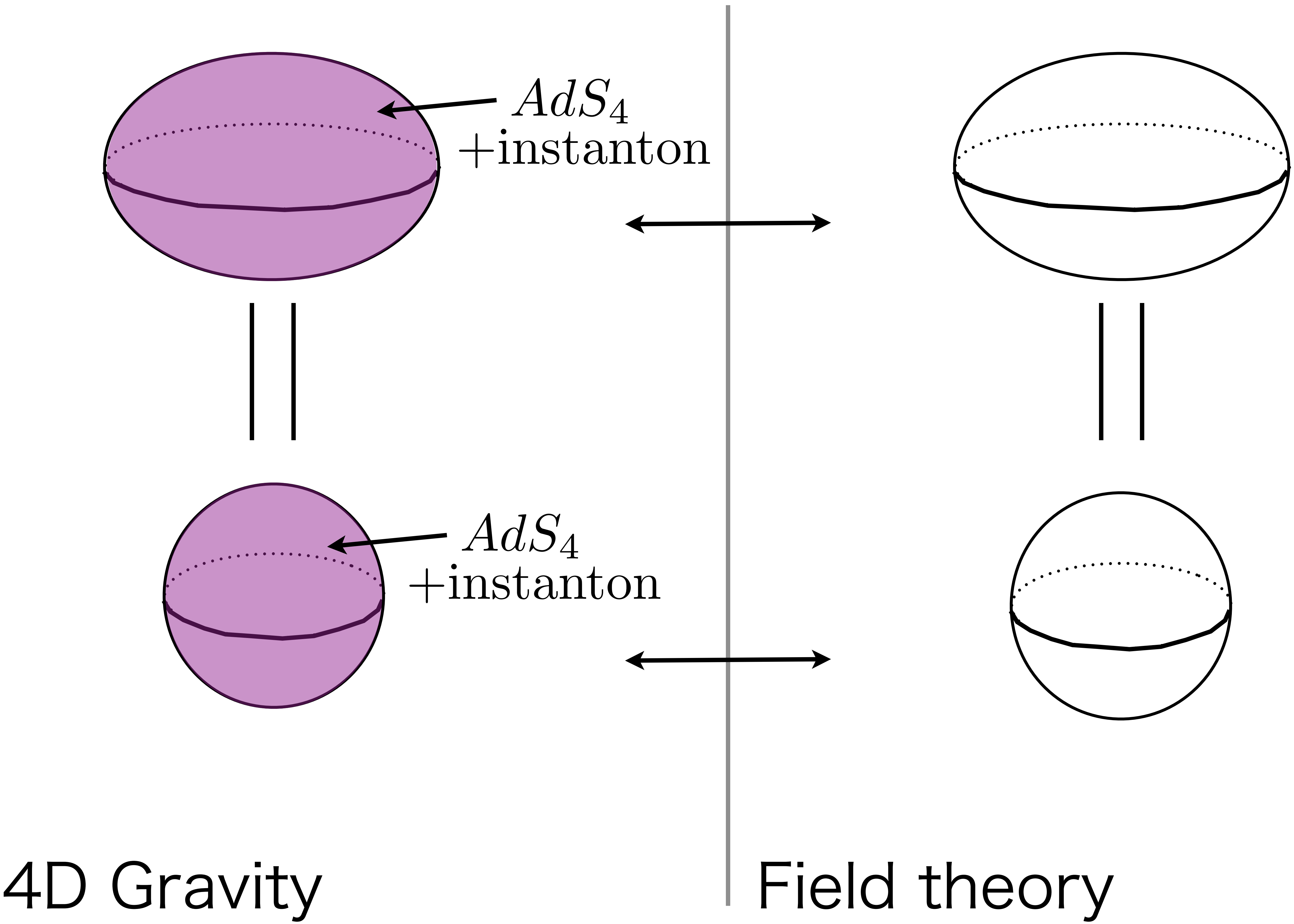
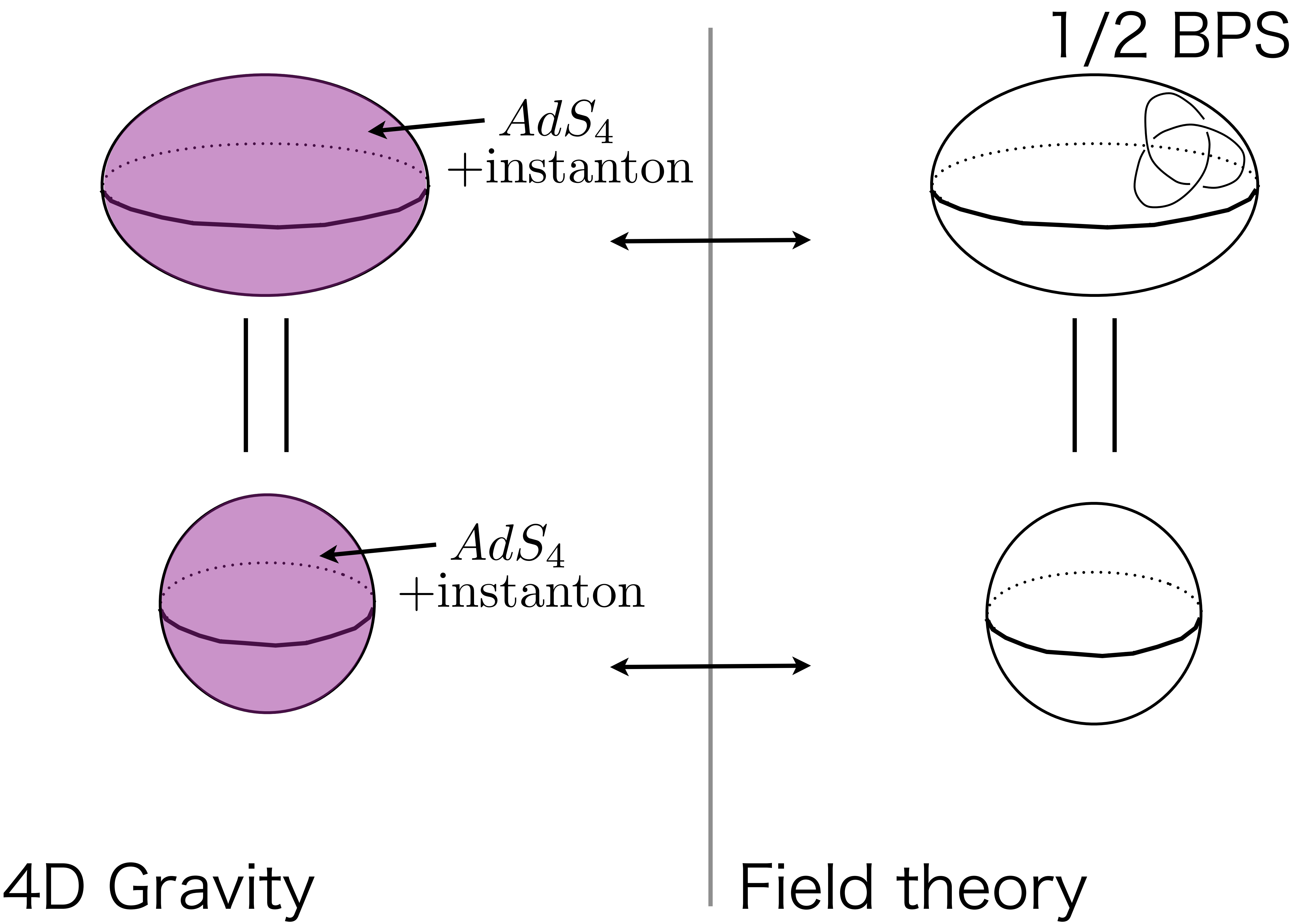
✔SUSY on round sphere

○Applications

1. Wilson loop



1. Wilson loop



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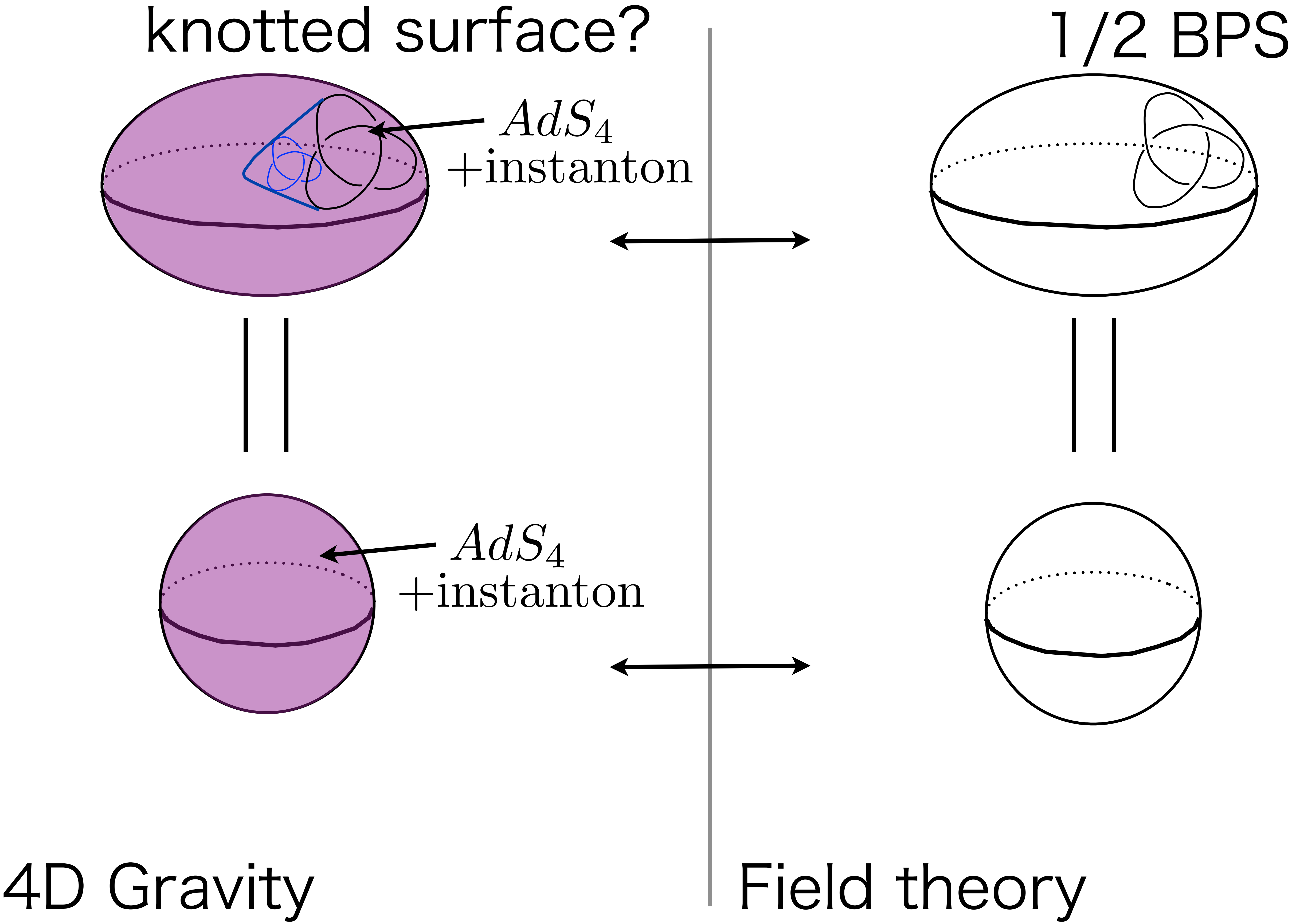

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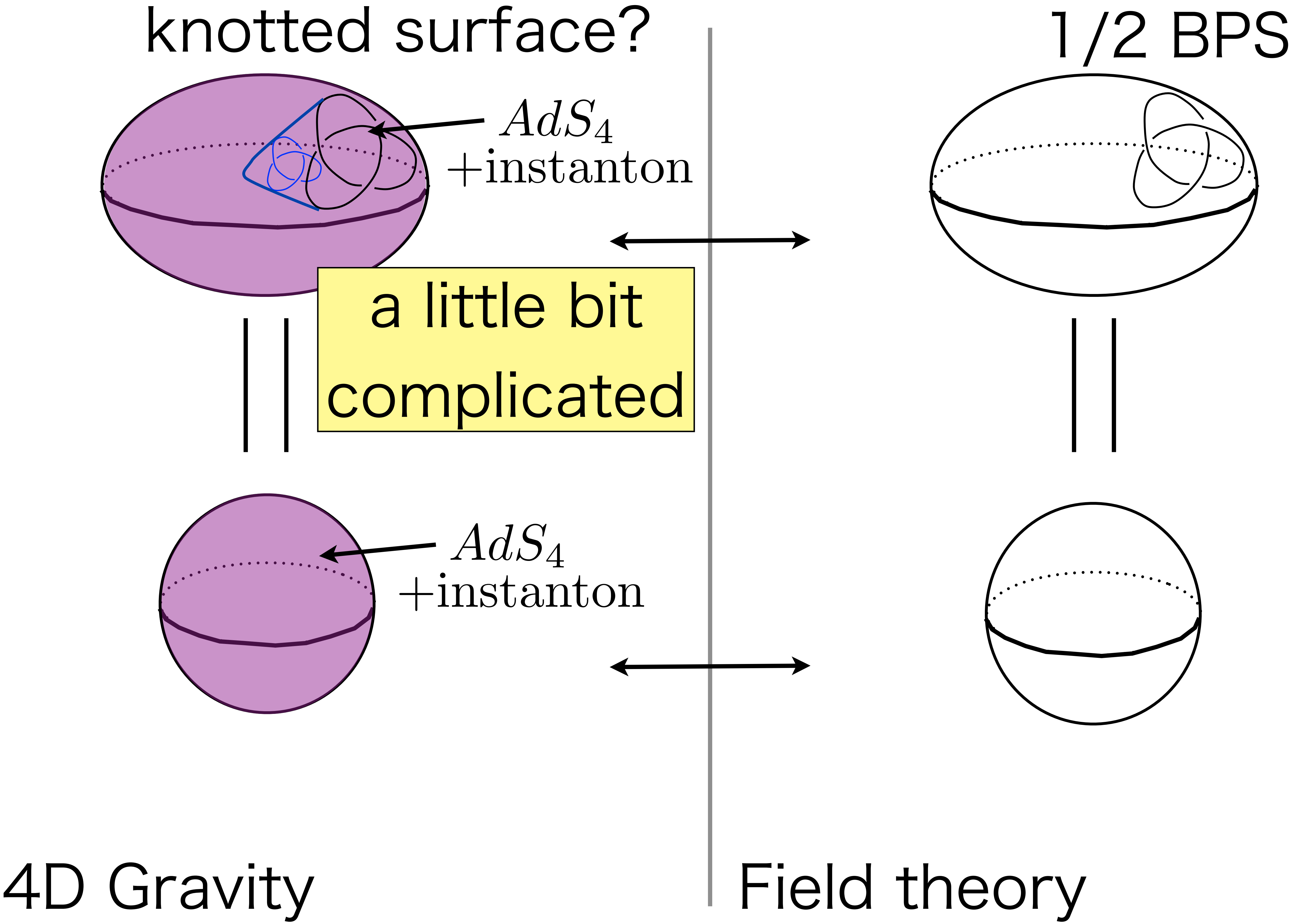

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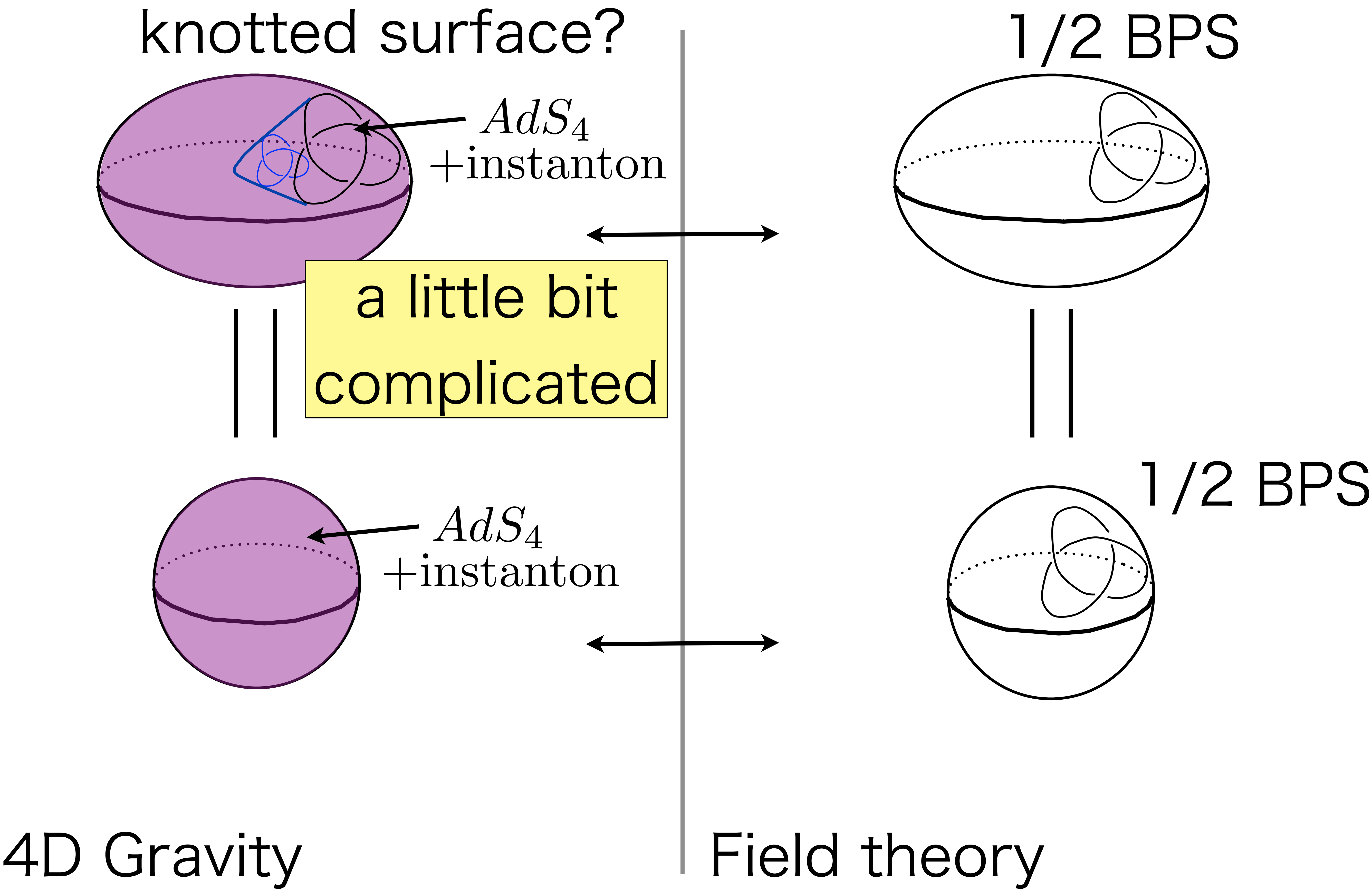

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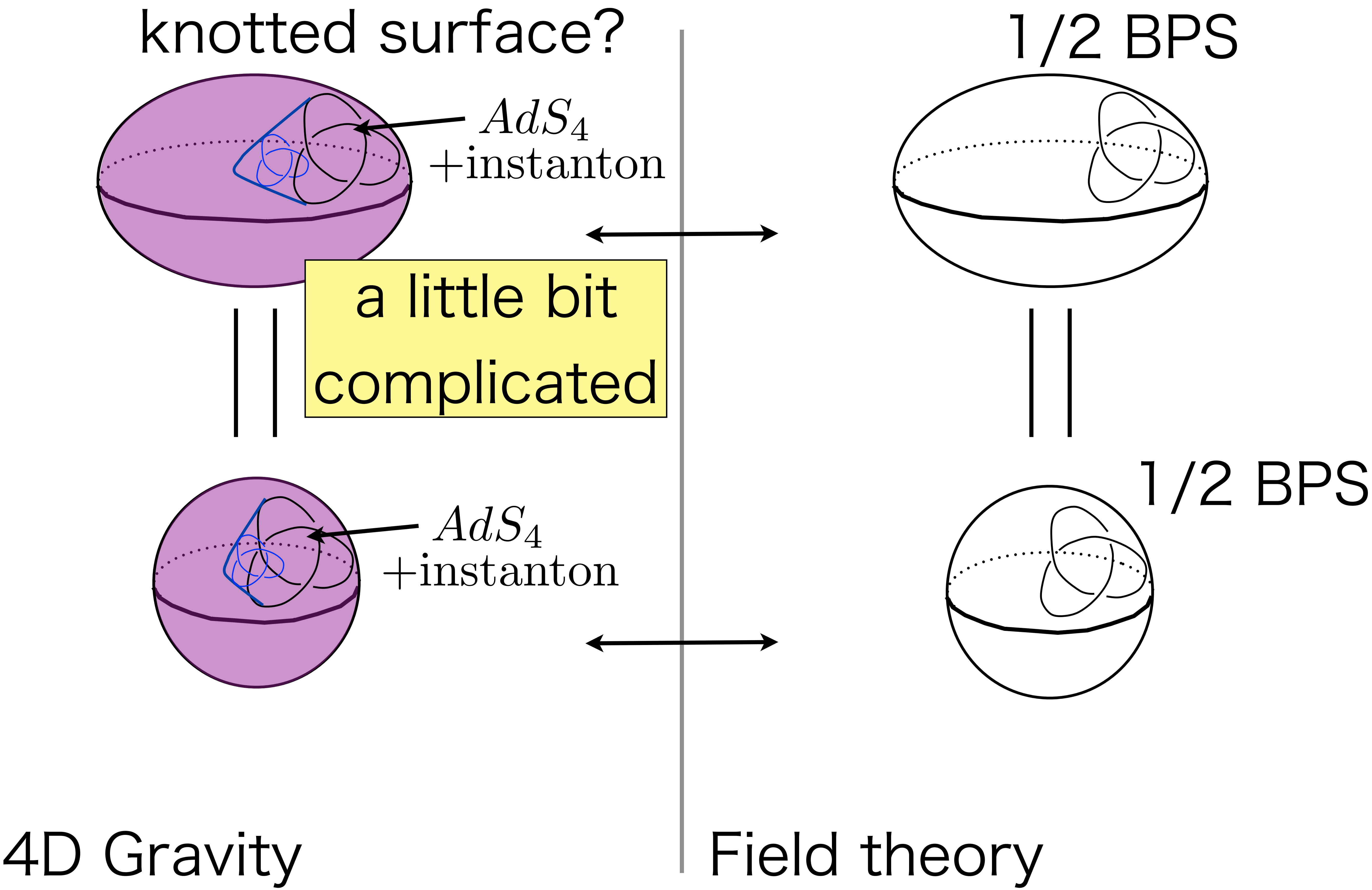
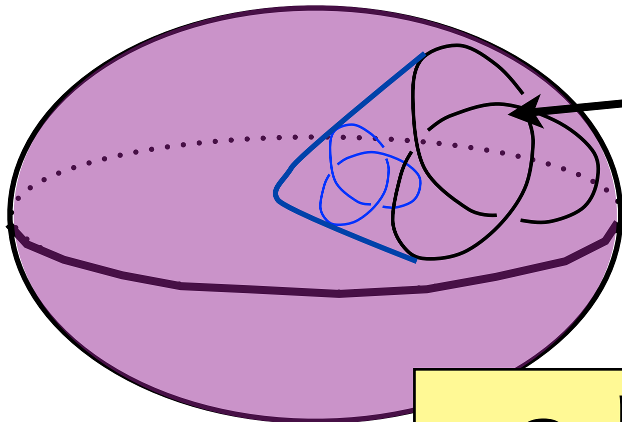


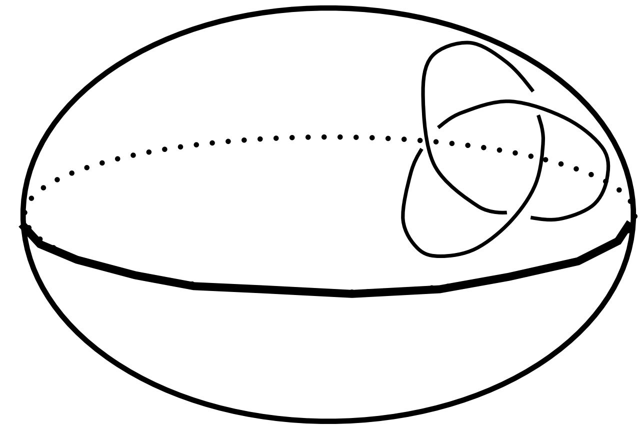
1. Wilson loop

knotted surface?

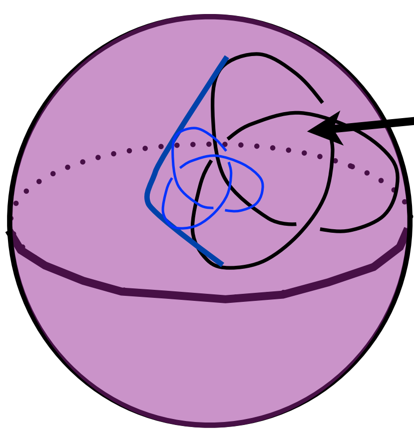
1/2 BPS



AdS_4
+instanton



a little bit complicated



AdS_4
+instanton

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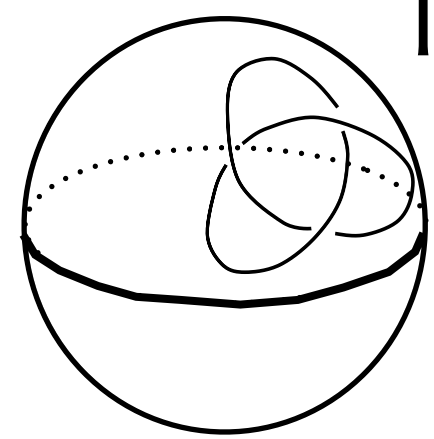


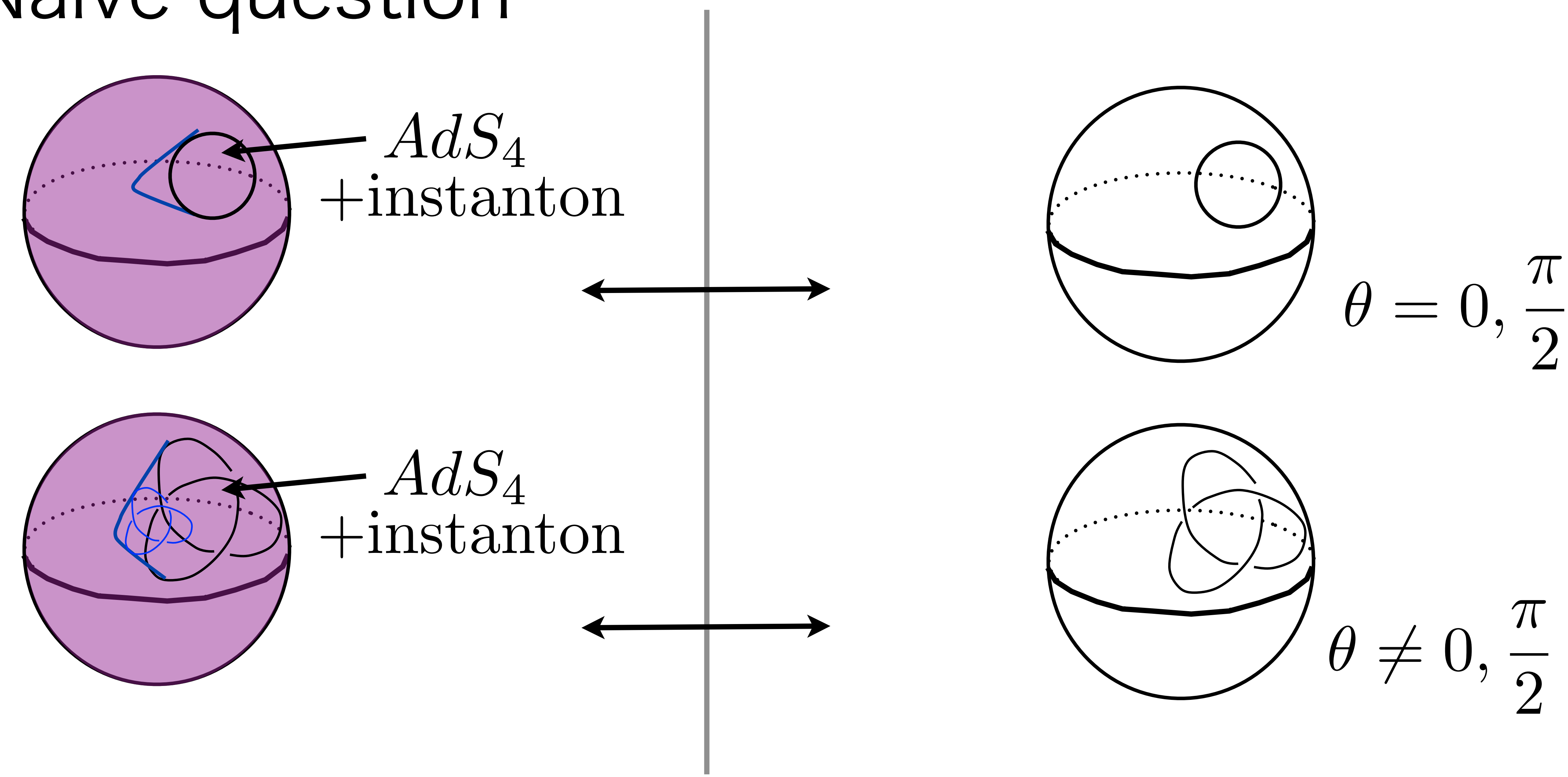
analysis is simplified?

4D Gravity

Field theory

1. Wilson loop

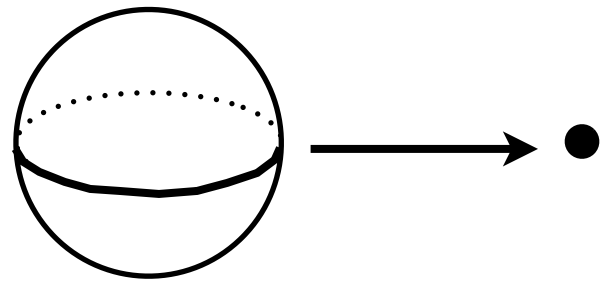
Naive question

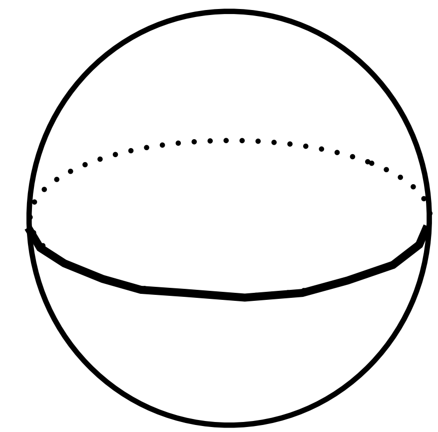


- Each surfaces are BPS? (expected so)
- Rotating in internal Sasaki-Einstein7-mfd?

<u>2013: Farquet, Sparks</u> arXiv:1304.0784

2. Large N reduction?



We can construct 1para SUSY on 

→It guarantees the uses of invariant 1-form.

(needed field redefinition)

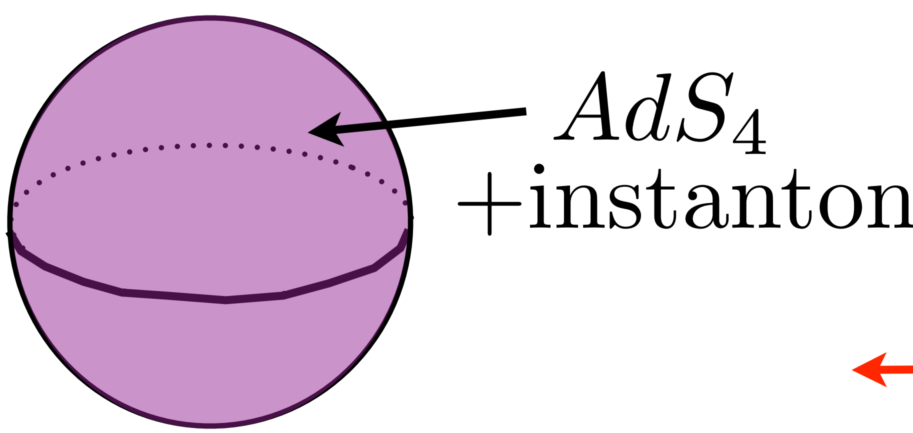
Kapustin et. al. case

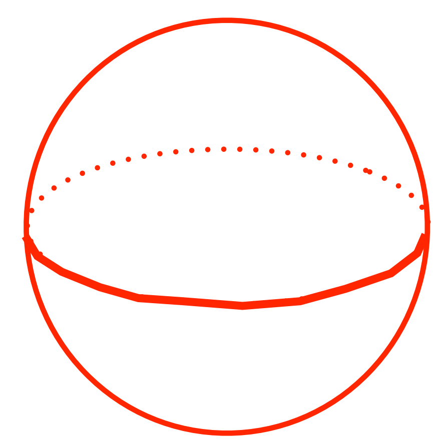
<u>2012: Honda, Yoshida</u> arXiv:1203.1019

<u>2012: Asano, Ishiki, Okada, Shimasaki</u> arXiv:1203.0559

3. Other dimension case via AdS/CFT

We construct it inspired by AdS/CFT.

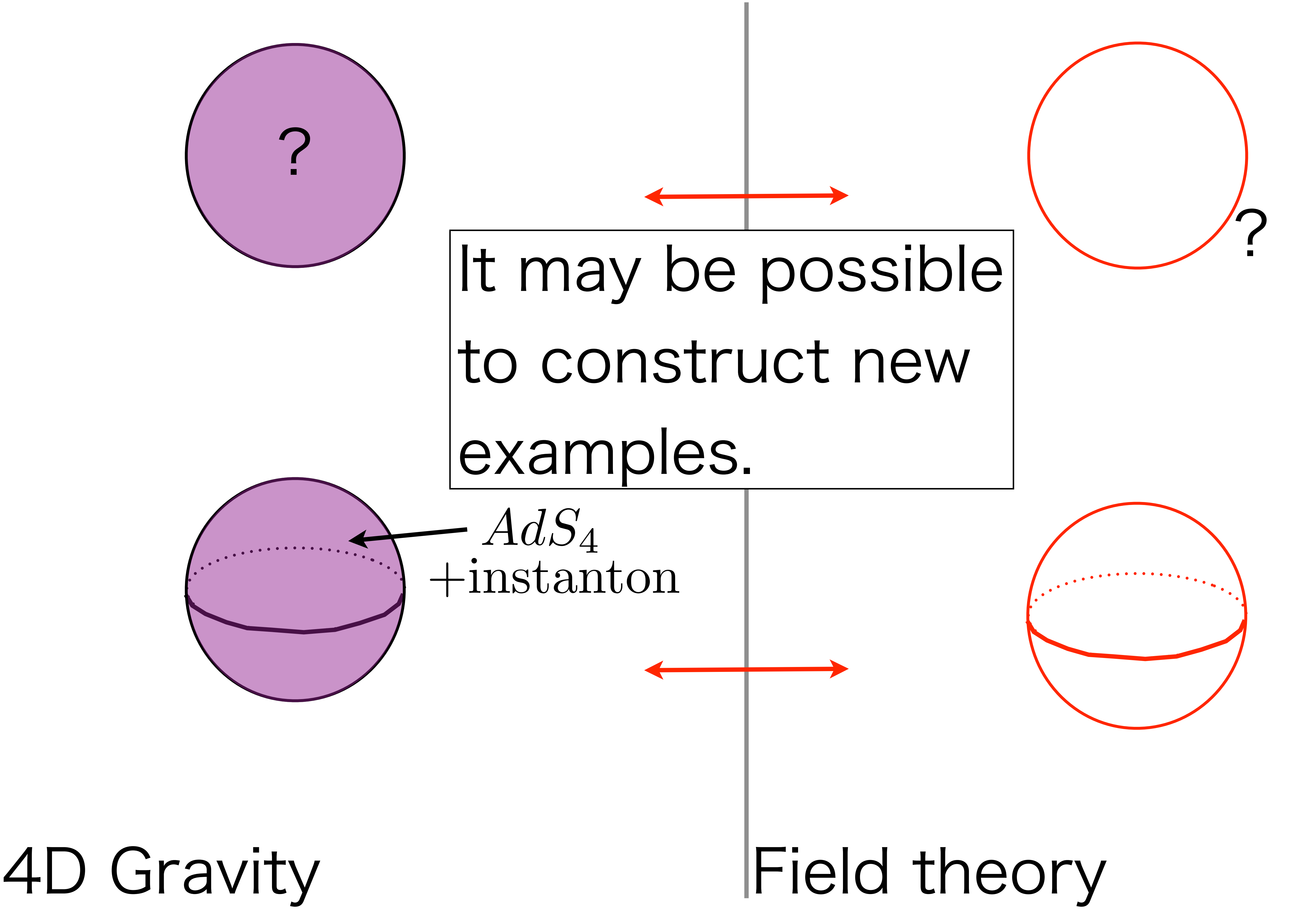




4D Gravity

Field theory

3. Other dimension case via AdS/CFT



It may be possible to construct new examples.

Thank you
for your attention.

SUSY on round sphere

$$\epsilon = \begin{pmatrix} -(\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$$\bar{\epsilon} = \begin{pmatrix} (\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$\mathcal{N}=2$ Vector multiplet

$\mathcal{N}=2$ Matter multiplet

$$Z(t) = \int [\mathcal{D}A \ldots \mathcal{D}\phi \ldots] e^{\frac{ik}{4\pi} S_{\rm CS} - t(\delta_{\bar\epsilon}\text{-exact})}$$

$\boxed{t \to \infty}$ steepest decent method around $(\delta_{\bar\epsilon}\text{-exact}) = 0$ turns to be exact.

$$Z(\infty) = \int d\sigma_0 \, e^{\frac{ik}{4\pi} S_{\rm CS}^{cl}} \frac{\det \Delta_f}{\det \Delta_b}$$

$$A = 0, \bar\lambda = \lambda = 0, \sigma = \frac{\sigma_0}{f}, D = -\frac{\sigma_0}{sf^3}$$

$$\Delta_b \sim i\alpha(\sigma_0) - *df$$

$$\Delta_f = if\gamma^\mu \mathcal{D}_\mu + \frac{if'}{2}\gamma_3 + i\alpha(\sigma_0) - \frac{1}{2sf}$$

$$\mathcal{L}_{\rm YM} = \delta_{\bar\epsilon}\text{-exact}$$

$$\phi = \bar\phi = \ldots = 0$$

$$\Delta_b = -f^2\mathcal{D}^2 - (2\Delta - 1)ff'\mathcal{D}_\theta - \ldots$$

$$\Delta_f = -if\gamma^\mu \mathcal{D}_\mu - if'(\Delta - \frac{1}{2})\gamma_3 - \ldots$$

$$\mathcal{L}_{\rm mat} = \delta_{\bar\epsilon}\text{-exact}$$

SUSY on round sphere

$$\epsilon = \begin{pmatrix} -(\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

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$\mathcal{N}=2$ Vector multiplet

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$$\Delta_f = if\gamma^\mu \mathcal{D}_\mu + \frac{if'}{2}\gamma_3 + i\alpha(\sigma_0) - \frac{1}{2sf}$$

$$\frac{\det \Delta_f}{\det \Delta_b}$$

$$\Delta_b \mathcal{B} = M\mathcal{B}$$

$$\Delta_f \Lambda = M\Lambda$$

SUSY on round sphere

$$\epsilon = \begin{pmatrix} -(\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

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$\mathcal{N} = 2$ Vector multiplet

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$$\frac{\det \Delta_f}{\det \Delta_b}$$

cf. arXiv:1012.3512

Pairing structure

$$\Delta_b \mathcal{B} = M\mathcal{B}$$

$$\Delta_f \Lambda = M\Lambda$$

SUSY on round sphere

$$\epsilon = \begin{pmatrix} -(\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$$\bar{\epsilon} = \begin{pmatrix} (\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$\mathcal{N}=2$ Vector multiplet

$$\Delta_b \sim i\alpha(\sigma_0) - *df$$

$$\Delta_f = if\gamma^\mu \mathcal{D}_\mu + \frac{if'}{2}\gamma_3 + i\alpha(\sigma_0) - \frac{1}{2sf}$$

$$\frac{\det \Delta_f}{\det \Delta_b}$$

cf. arXiv:1012.3512

Pairing structure

$$\Delta_b \mathcal{B} = M\mathcal{B}$$

$$\Big\downarrow \; \Lambda = \gamma^\mu \epsilon \mathcal{B}_\mu$$

$$\Delta_f \Lambda = M\Lambda$$

SUSY on round sphere

$$\epsilon = \begin{pmatrix} -(\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$$\bar{\epsilon} = \begin{pmatrix} (\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$\mathcal{N} = 2$ Vector multiplet

$$\Delta_b \sim i\alpha(\sigma_0) - *df$$

$$\Delta_f = if\gamma^\mu \mathcal{D}_\mu + \frac{if'}{2}\gamma_3 + i\alpha(\sigma_0) - \frac{1}{2sf}$$

$$\frac{\det \Delta_f}{\det \Delta_b}$$

cf. arXiv:1012.3512

Pairing structure

$$\Delta_b \mathcal{B} = M\mathcal{B}$$

$$\mathcal{B} = f^{-1}\Big(d(f\bar{\epsilon}\Lambda) + [iM + \alpha(\sigma_0)]\bar{\epsilon}\gamma_\mu \Lambda dx^\mu\Big)$$

$$\Delta_f \Lambda = M\Lambda$$

SUSY on round sphere

$$\epsilon = \begin{pmatrix} -(\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$$\bar{\epsilon} = \begin{pmatrix} (\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$\mathcal{N}=2$ Vector multiplet

$$\Delta_b \sim i\alpha(\sigma_0) - *df$$

$$\Delta_f = if\gamma^\mu \mathcal{D}_\mu + \frac{if'}{2}\gamma_3 + i\alpha(\sigma_0) - \frac{1}{2sf}$$

$$\frac{\det \Delta_f}{\det \Delta_b}$$

cf. arXiv:1012.3512

Pairing structure

$$\Delta_b \mathcal{B} = M\mathcal{B}$$

$$\Delta_f \Lambda = M\Lambda$$

SUSY on round sphere

$$\epsilon = \begin{pmatrix} -(\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$$\bar{\epsilon} = \begin{pmatrix} (\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$\mathcal{N} = 2$ Vector multiplet

$$\Delta_b \sim i\alpha(\sigma_0) - *df$$

$$\Delta_f = if\gamma^\mu \mathcal{D}_\mu + \frac{if'}{2}\gamma_3 + i\alpha(\sigma_0) - \frac{1}{2sf}$$

$$\frac{\det \Delta_f}{\det \Delta_b}$$

cf. arXiv:1012.3512

Pairing structure

$$\Delta_b \mathcal{B}_{rel} = M_b \mathcal{B}_{rel} \quad \Delta_b \mathcal{B} = M\mathcal{B}$$

$$0 \quad \Delta_f \Lambda = M\Lambda$$

SUSY on round sphere

$$\epsilon = \begin{pmatrix} -(\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$$\bar{\epsilon} = \begin{pmatrix} (\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$\mathcal{N}=2$ Vector multiplet

$$\Delta_b \sim i\alpha(\sigma_0) - *df$$

$$\Delta_f = if\gamma^\mu \mathcal{D}_\mu + \frac{if'}{2}\gamma_3 + i\alpha(\sigma_0) - \frac{1}{2sf}$$

$$\frac{\det \Delta_f}{\det \Delta_b}$$

cf. arXiv:1012.3512

Pairing structure

$$\Delta_b \mathcal{B}_{rel} = M_b \mathcal{B}_{rel} \qquad \Delta_b \mathcal{B} = M\mathcal{B} \qquad 0$$

$$0 \qquad \Delta_f \Lambda = M\Lambda \qquad \Delta_f \Lambda_{rel} = M_f \Lambda_{rel}$$

SUSY on round sphere

$$\epsilon = \begin{pmatrix} -(\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$$\bar{\epsilon} = \begin{pmatrix} (\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$\mathcal{N}=2$ Vector multiplet

$$\Delta_b \sim i\alpha(\sigma_0) - *df$$

$$\Delta_f = if\gamma^\mu \mathcal{D}_\mu + \frac{if'}{2}\gamma_3 + i\alpha(\sigma_0) - \frac{1}{2sf}$$

$$\frac{\det \Delta_f}{\det \Delta_b}$$

cf. arXiv:1012.3512

Pairing structure

$$\Delta_b \mathcal{B}_{rel} = M_b \mathcal{B}_{rel} \;\longrightarrow\; 0$$

$$\Delta_b \mathcal{B} = M\mathcal{B}$$

$$\Delta_f \Lambda = M\Lambda$$

$$\Delta_f \Lambda_{rel} = M_f \Lambda_{rel} \;\longrightarrow\; 0$$

$$\frac{M}{M} = 1$$

SUSY on round sphere

$$\epsilon = \begin{pmatrix} -(\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$$\bar{\epsilon} = \begin{pmatrix} (\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$\mathcal{N}=2$ Vector multiplet

$$\frac{\det\Delta_f}{\det\Delta_b} = \prod_{m,n} \frac{M_f}{M_b}$$

$$\Delta_b \mathcal{B}_{rel} = M_b \mathcal{B}_{rel} \longrightarrow 0$$

$$\Delta_f \Lambda_{rel} = M_f \Lambda_{rel} \longrightarrow 0$$

$$\boxed{\begin{array}{l} M_b = \frac{1}{s}m + n + i\alpha\cdot\sigma_0 \\ m,n \leq -1 \end{array}}$$

$$\boxed{\begin{array}{l} M_f = \frac{1}{s}m + n + i\alpha\cdot\sigma_0 \\ m,n \geq 0 \text{ except for } m=n=0 \end{array}}$$

SUSY on round sphere

$$\epsilon = \begin{pmatrix} -(\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$$\bar{\epsilon} = \begin{pmatrix} (\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$\mathcal{N}=2$ Vector multiplet

$$\frac{\det \Delta_f}{\det \Delta_b} = \prod_{m,n} \frac{M_f}{M_b}$$

$$= \prod_{\alpha\in\Delta_+} 4\sinh(\pi\alpha(\sigma_0))\sinh s(\pi\alpha(\sigma_0))$$

$$\Delta_b \mathcal{B}_{rel} = M_b \mathcal{B}_{rel} \longrightarrow 0$$

$$M_b = \frac{1}{s}m + n + i\alpha\cdot\sigma_0$$

$m, n \leq -1$

$$\Delta_f \Lambda_{rel} = M_f \Lambda_{rel} \longrightarrow 0$$

$$M_f = \frac{1}{s}m + n + i\alpha\cdot\sigma_0$$

$m, n \geq 0$ except for $m = n = 0$

SUSY on round sphere

$$\epsilon = \begin{pmatrix} -(\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$$\bar{\epsilon} = \begin{pmatrix} (\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$\mathcal{N}=2$ Vector multiplet

$$\frac{\det \Delta_f}{\det \Delta_b} = \prod_{m,n} \frac{M_f}{M_b}$$

$$= \prod_{\alpha\in\Delta_+} 4\sinh(\pi\alpha(\sigma_0))\sinh s(\pi\alpha(\sigma_0))$$

We recover the result on squashed sphere!

$$\Delta_b \mathcal{B}_{rel} = M_b \mathcal{B}_{rel}$$

$$\downarrow$$

$$0$$

$$\boxed{M_b = \frac{1}{s}m + n + i\alpha\cdot\sigma_0 \quad m,n \le -1}$$

$$0$$

$$\uparrow$$

$$\Delta_f \Lambda_{rel} = M_f \Lambda_{rel}$$

$$\boxed{M_f = \frac{1}{s}m + n + i\alpha\cdot\sigma_0 \quad m,n \ge 0 \text{ except for } m=n=0}$$

SUSY on round sphere

$$\epsilon = \begin{pmatrix} -(\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$$\bar{\epsilon} = \begin{pmatrix} (\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$\mathcal{N}=2$ Vector multiplet

$\mathcal{N}=2$ Matter multiplet

$$Z(t) = \int [\mathcal{D}A\ldots\mathcal{D}\phi\ldots] e^{\frac{ik}{4\pi}S_{\mathrm{CS}} - t(\delta_{\bar{\epsilon}}\text{-exact})}$$

$\boxed{t \to \infty}$ steepest decent method around $(\delta_{\bar{\epsilon}}\text{-exact}) = 0$ turns to be exact.

$$Z(\infty) = \int d\sigma_0\, e^{\frac{ik}{4\pi}S^{cl}_{\mathrm{CS}}} \frac{\det \Delta_f}{\det \Delta_b}$$

$$A=0, \bar{\lambda}=\lambda=0, \sigma=\frac{\sigma_0}{f}, D=-\frac{\sigma_0}{sf^3}$$

$$\frac{\det \Delta_f}{\det \Delta_b} = \prod_{\alpha\in\Delta_+} 4\sinh(\pi\alpha(\sigma_0))\sinh s(\pi\alpha(\sigma_0))$$

$$\mathcal{L}_{\mathrm{YM}} = \delta_{\bar{\epsilon}}\text{-exact}$$

$$\phi = \bar{\phi} = \ldots = 0$$

$$\Delta_b = -f^2\mathcal{D}^2 - (2\Delta - 1)ff'\mathcal{D}_\theta - \ldots$$

$$\Delta_f = -if\gamma^\mu\mathcal{D}_\mu - if'(\Delta - \frac{1}{2})\gamma_3 - \ldots$$

$$\mathcal{L}_{\mathrm{mat}} = \delta_{\bar{\epsilon}}\text{-exact}$$

SUSY on round sphere

$$\epsilon = \begin{pmatrix} -(\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$$\bar{\epsilon} = \begin{pmatrix} (\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$\mathcal{N}=2$ Vector multiplet

$\mathcal{N}=2$ Matter multiplet

$$Z(t) = \int [\mathcal{D}A \ldots \mathcal{D}\phi \ldots] e^{\frac{ik}{4\pi} S_{\rm CS} - t(\delta_{\bar{\epsilon}}\text{-exact})}$$

$\boxed{t \to \infty}$ steepest decent method around $(\delta_{\bar{\epsilon}}\text{-exact}) = 0$ turns to be exact.

$$Z(\infty) = \int d\sigma_0 \, e^{\frac{ik}{4\pi} S^{cl}_{\rm CS}} \, \frac{\det \Delta_f}{\det \Delta_b}$$

$$A=0, \overline{\lambda} = \lambda = 0, \sigma = \frac{\sigma_0}{f}, D = -\frac{\sigma_0}{sf^3}$$

$$\frac{\det \Delta_f}{\det \Delta_b} = \prod_{\alpha\in\Delta_+} 4\sinh(\pi\alpha(\sigma_0)) \sinh s(\pi\alpha(\sigma_0))$$

$$\mathcal{L}_{\rm YM} = \delta_{\bar{\epsilon}}\text{-exact}$$

$$\phi = \overline{\phi} = \ldots = 0$$

$$\Delta_b = -f^2\mathcal{D}^2 - (2\Delta - 1) f f' \mathcal{D}_\theta - \ldots$$

$$\Delta_f = -i f \gamma^\mu \mathcal{D}_\mu - i f'(\Delta - \frac{1}{2})\gamma_3 - \ldots$$

$$\mathcal{L}_{\rm mat} = \delta_{\bar{\epsilon}}\text{-exact}$$

SUSY on round sphere

$$\epsilon = \begin{pmatrix} -(\cos\theta + i s \sin\theta)^{1/2} \\ (\cos\theta - i s \sin\theta)^{1/2} \end{pmatrix}$$

$$\bar{\epsilon} = \begin{pmatrix} (\cos\theta + i s \sin\theta)^{1/2} \\ (\cos\theta - i s \sin\theta)^{1/2} \end{pmatrix}$$

$\mathcal{N}=2$ Matter multiplet

$$\Delta_b = -f^2\mathcal{D}^2 - (2\Delta - 1) f f' \mathcal{D}_\theta - \ldots$$

$$\Delta_f = -i f \gamma^\mu \mathcal{D}_\mu - i f'(\Delta - \frac{1}{2})\gamma_3 - \ldots$$

$$\frac{\det \Delta_f}{\det \Delta_b}$$

$$\Delta_b \Phi = M(M - 2i\sigma_0)\Phi$$

$$\Psi_1, \Psi_2 : \Delta_f = M, M - 2i\sigma_0$$

SUSY on round sphere

$$\epsilon = \begin{pmatrix} -(\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$$\bar{\epsilon} = \begin{pmatrix} (\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$\mathcal{N}=2$ Matter multiplet

$$\Delta_b = -f^2\mathcal{D}^2 - (2\Delta - 1)ff'\mathcal{D}_\theta - \ldots$$

$$\Delta_f = -if\gamma^\mu\mathcal{D}_\mu - if'(\Delta - \frac{1}{2})\gamma_3 - \ldots$$

$$\frac{\det\Delta_f}{\det\Delta_b}$$

Pairing structure

$$\Delta_b\Phi = M(M - 2i\sigma_0)\Phi$$

$$\Psi_1, \Psi_2 : \Delta_f = M, M - 2i\sigma_0$$

SUSY on round sphere

$$\epsilon = \begin{pmatrix} -(\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$$\bar{\epsilon} = \begin{pmatrix} (\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$\mathcal{N}=2$ Matter multiplet

$$\Delta_b = -f^2\mathcal{D}^2 - (2\Delta - 1)ff'\mathcal{D}_\theta - \ldots$$

$$\Delta_f = -if\gamma^\mu\mathcal{D}_\mu - if'(\Delta - \frac{1}{2})\gamma_3 - \ldots$$

$$\frac{\det \Delta_f}{\det \Delta_b}$$

Pairing structure

$$\Delta_b\Phi = M(M - 2i\sigma_0)\Phi$$

$$\Psi_1 = f^{-1}\epsilon\Phi$$

$$\Psi_2 = i\gamma^\mu\epsilon\mathcal{D}_\mu\Phi + i\frac{\sigma_0}{f}\epsilon\Phi - \frac{\Delta}{sf^2}\epsilon\Phi - i\Delta\frac{f'}{f}\bar{\epsilon}\Phi$$

$$\Psi_1, \Psi_2 : \Delta_f = M, M - 2i\sigma_0$$

SUSY on round sphere

$$\epsilon = \begin{pmatrix} -(\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$$\bar{\epsilon} = \begin{pmatrix} (\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$\mathcal{N}=2$ Matter multiplet

$$\Delta_b = -f^2\mathcal{D}^2 - (2\Delta - 1)ff'\mathcal{D}_\theta - \dots$$

$$\Delta_f = -if\gamma^\mu\mathcal{D}_\mu - if'(\Delta - \frac{1}{2})\gamma_3 - \dots$$

$$\frac{\det \Delta_f}{\det \Delta_b}$$

Pairing structure

$$\Delta_b\Phi = M(M - 2i\sigma_0)\Phi$$

$$\Phi = \bar{\epsilon}\Psi$$

$$\Psi_1, \Psi_2 : \Delta_f = M, M - 2i\sigma_0$$

SUSY on round sphere

$$\epsilon = \begin{pmatrix} -(\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$$\bar{\epsilon} = \begin{pmatrix} (\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$\mathcal{N} = 2$ Matter multiplet

$$\Delta_b = -f^2\mathcal{D}^2 - (2\Delta - 1)ff'\mathcal{D}_\theta - \ldots$$

$$\Delta_f = -if\gamma^\mu\mathcal{D}_\mu - if'(\Delta - \frac{1}{2})\gamma_3 - \ldots$$

$$\frac{\det\Delta_f}{\det\Delta_b}$$

Pairing structure

$$\Delta_b\Phi = M(M - 2i\sigma_0)\Phi$$

$$\Psi_1, \Psi_2 : \Delta_f = M, M - 2i\sigma_0$$

SUSY on round sphere

$$\epsilon = \begin{pmatrix} -(\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$$\bar{\epsilon} = \begin{pmatrix} (\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$\mathcal{N}=2$ Matter multiplet

$$\Delta_b = -f^2\mathcal{D}^2 - (2\Delta - 1)ff'\mathcal{D}_\theta - \ldots$$

$$\Delta_f = -if\gamma^\mu\mathcal{D}_\mu - if'(\Delta - \frac{1}{2})\gamma_3 - \ldots$$

$$\frac{\det \Delta_f}{\det \Delta_b}$$

Pairing structure

$$\Delta_b\Phi_{rel} = M_b(M_b - 2i\sigma_0)\Phi_{rel} \qquad \Delta_b\Phi = M(M - 2i\sigma_0)\Phi$$

$$\downarrow \qquad\qquad \uparrow \quad \downarrow$$

$$M\Psi_1 = \Psi_2 : \Delta_f = M_b, M_b - 2i\sigma_0 \qquad \Psi_1, \Psi_2 : \Delta_f = M, M - 2i\sigma_0$$

SUSY on round sphere

$$\epsilon = \begin{pmatrix} -(\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$$\bar{\epsilon} = \begin{pmatrix} (\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$\mathcal{N} = 2$ Matter multiplet

$$\Delta_b = -f^2\mathcal{D}^2 - (2\Delta - 1)ff'\mathcal{D}_\theta - \dots$$

$$\Delta_f = -if\gamma^\mu\mathcal{D}_\mu - if'(\Delta - \frac{1}{2})\gamma_3 - \dots$$

$$\frac{\det \Delta_f}{\det \Delta_b}$$

Pairing structure

$$\Delta_b\Phi_{rel} = M_b(M_b - 2i\sigma_0)\Phi_{rel}$$

$$\Delta_b\Phi = M(M - 2i\sigma_0)\Phi$$

$$M\Psi_1 = \Psi_2 : \Delta_f = \cancel{M_b}, M_b - 2i\sigma_0$$

$$\Psi_1, \Psi_2 : \Delta_f = M, M - 2i\sigma_0$$

SUSY on round sphere

$$\epsilon = \begin{pmatrix} -(\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$$\bar{\epsilon} = \begin{pmatrix} (\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$\mathcal{N} = 2$ Matter multiplet

$$\Delta_b = -f^2\mathcal{D}^2 - (2\Delta - 1)ff'\mathcal{D}_\theta - \ldots$$

$$\Delta_f = -if\gamma^\mu\mathcal{D}_\mu - if'(\Delta - \frac{1}{2})\gamma_3 - \ldots$$

$$\frac{\det\Delta_f}{\det\Delta_b}$$

Pairing structure

$$\Delta_b\Phi_{rel} = M_b(\cancel{M_b - 2i\sigma_0})\Phi_{rel}$$

$$\Delta_b\Phi = M(M - 2i\sigma_0)\Phi$$

$$M\Psi_1 = \Psi_2 : \Delta_f = \cancel{M_b}, \cancel{M_b - 2i\sigma_0}$$

$$\Psi_1, \Psi_2 : \Delta_f = M, M - 2i\sigma_0$$

SUSY on round sphere

$$\epsilon = \begin{pmatrix} -(\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$$\bar{\epsilon} = \begin{pmatrix} (\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$\mathcal{N} = 2$ Matter multiplet

$$\Delta_b = -f^2\mathcal{D}^2 - (2\Delta - 1)ff'\mathcal{D}_\theta - \ldots$$

$$\Delta_f = -if\gamma^\mu\mathcal{D}_\mu - if'(\Delta - \frac{1}{2})\gamma_3 - \ldots$$

$$\frac{\det \Delta_f}{\det \Delta_b}$$

Pairing structure

$$\Delta_b\Phi_{rel} = M_b\cancel{(M_b - 2i\sigma_0)}\Phi_{rel}$$

$$\Delta_b\Phi = M(M - 2i\sigma_0)\Phi$$

$$M\Psi_1 = \Psi_2 : \Delta_f = \cancel{M_b}, \cancel{M_b - 2i\sigma_0}$$

$$\Psi_1, \Psi_2 : \Delta_f = M, M - 2i\sigma_0$$

SUSY on round sphere

$$\epsilon = \begin{pmatrix} -(\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$$\bar{\epsilon} = \begin{pmatrix} (\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$\mathcal{N}=2$ Matter multiplet

$$\Delta_b = -f^2\mathcal{D}^2 - (2\Delta - 1)ff'\mathcal{D}_\theta - \dots$$

$$\Delta_f = -if\gamma^\mu\mathcal{D}_\mu - if'(\Delta - \frac{1}{2})\gamma_3 - \dots$$

$$\frac{\det \Delta_f}{\det \Delta_b}$$

Pairing structure

$$\Delta_b\Phi_{rel} = M_b\cancel{(M_b - 2i\sigma_0)}\Phi_{rel}$$

$$\Delta_b\Phi = M(M - 2i\sigma_0)\Phi$$

$$0 = \bar{\epsilon}\Psi_{rel}$$

$$M\Psi_1 = \Psi_2 : \Delta_f = \cancel{M_b}, \cancel{M_b - 2i\sigma_0}$$

$$\Psi_1, \Psi_2 : \Delta_f = M, M - 2i\sigma_0$$

$$\Delta_f\Psi_{rel} = M_f\Psi_{rel}$$

SUSY on round sphere

$$\epsilon = \begin{pmatrix} -(\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$$\bar\epsilon = \begin{pmatrix} (\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$\mathcal{N}=2$ Matter multiplet

$$\Delta_b = -f^2\mathcal{D}^2 - (2\Delta - 1)ff'\mathcal{D}_\theta - \ldots$$

$$\Delta_f = -if\gamma^\mu\mathcal{D}_\mu - if'(\Delta - \frac{1}{2})\gamma_3 - \ldots$$

$$\frac{\det\Delta_f}{\det\Delta_b}$$

cf. arXiv:1012.3512

Pairing structure

$$\Delta_b\Phi_{rel} = M_b\cancel{(M_b - 2i\sigma_0)}\Phi_{rel}$$

$$M\Psi_1 = \Psi_2 : \Delta_f = \cancel{M_b}, \cancel{M_b - 2i\sigma_0}$$

$$\Delta_b\Phi = M(M - 2i\sigma_0)\Phi$$

$$\Psi_1, \Psi_2 : \Delta_f = M, M - 2i\sigma_0$$

$$\frac{M(M-2i\sigma_0)}{M(M-2i\sigma_0)} = 1$$

$$0 = \bar\epsilon\Psi_{rel}$$

$$\Delta_f\Psi_{rel} = M_f\Psi_{rel}$$

SUSY on round sphere

$$\epsilon = \begin{pmatrix} -(\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$$\bar{\epsilon} = \begin{pmatrix} (\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$\mathcal{N}=2$ Matter multiplet

$$\frac{\det \Delta_f}{\det \Delta_b} = \prod_{m,n} \frac{M_f}{M_b}$$

$$\Delta_b \Phi_{rel} = M_b \cancel{(M_b - 2i\sigma_0)}\Phi_{rel}$$

$$0 = \bar{\epsilon}\Psi_{rel}$$

$$M\Psi_1 = \Psi_2 : \Delta_f = \cancel{M_b}, \cancel{M_b - 2i\sigma_0}$$

$$\Delta_f \Psi_{rel} = M_f \Psi_{rel}$$

$$\boxed{M_b = i\rho\cdot\sigma_0 + \frac{1}{s}m + n - \frac{\Delta-2}{2}\left(\frac{1}{s}+1\right) \qquad m,n \geq 0}$$

$$\boxed{M_f = i\rho\cdot\sigma_0 - \frac{1}{s}m - n - \frac{\Delta}{2}\left(\frac{1}{s}+1\right) \qquad m,n \leq 0}$$

SUSY on round sphere

$$\epsilon = \begin{pmatrix} -(\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$$\bar{\epsilon} = \begin{pmatrix} (\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$\mathcal{N}=2$ Matter multiplet

$$\frac{\det \Delta_f}{\det \Delta_b} = \prod_{m,n} \frac{M_f}{M_b} = \prod_{\rho\in\text{weight}} s_b\Big(\frac{i(\sqrt{\frac{1}{s}}+\sqrt{s})}{2}(1-\Delta) - \sqrt{s}\rho\cdot\sigma_0\Big)$$

$$\Delta_b\Phi_{rel} = M_b\cancel{(M_b - 2i\sigma_0)}\Phi_{rel}$$

$$M\Psi_1 = \Psi_2 : \Delta_f = \cancel{M_b}, \cancel{M_b - 2i\sigma_0}$$

$$\boxed{M_b = i\rho\cdot\sigma_0 + \frac{1}{s}m + n - \frac{\Delta-2}{2}\Big(\frac{1}{s}+1\Big) \quad m,n\geq 0}$$

$$0 = \bar{\epsilon}\Psi_{rel}$$

$$\Delta_f\Psi_{rel} = M_f\Psi_{rel}$$

$$\boxed{M_f = i\rho\cdot\sigma_0 - \frac{1}{s}m - n - \frac{\Delta}{2}\Big(\frac{1}{s}+1\Big) \quad m,n\leq 0}$$

SUSY on round sphere

$$\epsilon = \begin{pmatrix} -(\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$$\bar{\epsilon} = \begin{pmatrix} (\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$\mathcal{N}=2$ Matter multiplet

$$\frac{\det \Delta_f}{\det \Delta_b} = \prod_{m,n} \frac{M_f}{M_b} = \prod_{\rho \in \text{weight}} s_b\Big(\frac{i(\sqrt{\frac{1}{s}} + \sqrt{s})}{2}(1-\Delta) - \sqrt{s}\rho\cdot\sigma_0\Big)$$

We recover the result on squashed sphere!

$$\Delta_b \Phi_{rel} = M_b \cancel{(M_b - 2i\sigma_0)}\Phi_{rel}$$

$$0 = \bar{\epsilon}\Psi_{rel}$$

$$M\Psi_1 = \Psi_2 : \Delta_f = \cancel{M_b}, \cancel{M_b - 2i\sigma_0}$$

$$\Delta_f \Psi_{rel} = M_f \Psi_{rel}$$

$$\boxed{M_b = i\rho\cdot\sigma_0 + \frac{1}{s}m + n - \frac{\Delta - 2}{2}\Big(\frac{1}{s}+1\Big) \quad m,n \geq 0}$$

$$\boxed{M_f = i\rho\cdot\sigma_0 - \frac{1}{s}m - n - \frac{\Delta}{2}\Big(\frac{1}{s}+1\Big) \quad m,n \leq 0}$$

SUSY on round sphere

$$\epsilon = \begin{pmatrix} -(\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$$\bar{\epsilon} = \begin{pmatrix} (\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$\mathcal{N}=2$ Vector multiplet

$\mathcal{N}=2$ Matter multiplet

$$Z(t) = \int [\mathcal{D}A\ldots\mathcal{D}\phi\ldots] e^{\frac{ik}{4\pi}S_{\mathrm{CS}} - t(\delta_{\bar{\epsilon}}\text{-exact})}$$

$\boxed{t \to \infty}$ steepest decent method around $(\delta_{\bar{\epsilon}}\text{-exact}) = 0$ turns to be exact.

$$Z(\infty) = \int d\sigma_0\, e^{\frac{ik}{4\pi}S^{cl}_{\mathrm{CS}}} \frac{\det\Delta_f}{\det\Delta_b}$$

$$A = 0, \bar{\lambda} = \lambda = 0, \sigma = \frac{\sigma_0}{f}, D = -\frac{\sigma_0}{sf^3}$$

$$\frac{\det\Delta_f}{\det\Delta_b} = \prod_{\alpha\in\Delta_+} 4\sinh(\pi\alpha(\sigma_0))\sinh s(\pi\alpha(\sigma_0))$$

$$\mathcal{L}_{\mathrm{YM}} = \delta_{\bar{\epsilon}}\text{-exact}$$

$$\phi = \bar{\phi} = \ldots = 0$$

$$\frac{\det\Delta_f}{\det\Delta_b} = \prod_{\rho\in\mathrm{weight}} s_b\left(\frac{i(\sqrt{\frac{1}{s}} + \sqrt{s})}{2}(1-\Delta) - \sqrt{s}\rho\cdot\sigma_0\right)$$

$$\mathcal{L}_{\mathrm{mat}} = \delta_{\bar{\epsilon}}\text{-exact}$$

How about S^{cl}_{CS} ?

Classical configuration : $A=0, \bar{\lambda}=\lambda=0, \sigma=\frac{\sigma_0}{f}, D=-\frac{\sigma_0}{sf^3}$

$$Z(\infty)=\int d\sigma_0\, e^{\frac{ik}{4\pi}S_{\mathrm{CS}}^{cl}}\ \frac{\det\Delta_f}{\det\Delta_b}$$

$$\mathcal{L}_{\mathrm{CS}}=\mathrm{Tr}\Big[\frac{1}{\sqrt{g}}\varepsilon^{\mu\nu\lambda}(A_\mu\partial_\nu A_\lambda-\frac{2i}{3}A_\mu A_\nu A_\lambda)-\bar{\lambda}\lambda+2D\sigma\Big]=\mathrm{Tr}\Big[-2\frac{\sigma_0^2}{sf^4}\Big]$$

$$\begin{aligned}S_{\mathrm{CS}}^{cl}&=\int\sqrt{g}d^3x\ \mathcal{L}_{\mathrm{CS}}\\&=(2\pi)^2\int\sin\theta\cos\theta d\theta\ \mathrm{Tr}\Big[-2\frac{\sigma_0^2}{sf^4}\Big]\\&=(2\pi)^2\int\sin\theta\cos\theta d\theta\ \mathrm{Tr}\Big[-2\frac{\sigma_0^2}{s\big(\sin^2\theta+\frac{1}{s^2}\cos^2\theta\big)^2}\Big]\\&=-4\pi^2 s\ \mathrm{Tr}\sigma_0^2\end{aligned}$$

: Equivalent to squashed one

Classical configuration : $A=0, \bar{\lambda}=\lambda=0, \sigma=\frac{\sigma_0}{f}, D=-\frac{\sigma_0}{sf^3}$

$$Z(\infty)=\int d\sigma_0\, e^{\frac{ik}{4\pi}S^{cl}_{\mathrm{CS}}}\frac{\det\Delta_f}{\det\Delta_b}$$

$$\mathcal{L}_{\mathrm{CS}}=\mathrm{Tr}\Big[\frac{1}{\sqrt{g}}\varepsilon^{\mu\nu\lambda}(\underbrace{A_\mu\partial_\nu A_\lambda}_{0}-\frac{2i}{3}\underbrace{A_\mu A_\nu A_\lambda}_{0})-\underbrace{\bar{\lambda}\lambda}_{0}+2D\sigma\Big]=\mathrm{Tr}\Big[-2\frac{\sigma_0^2}{sf^4}\Big]$$

$$S^{cl}_{\mathrm{CS}}=\int\sqrt{g}d^3x\ \mathcal{L}_{\mathrm{CS}}$$

$$=(2\pi)^2\int\sin\theta\cos\theta d\theta\ \mathrm{Tr}\Big[-2\frac{\sigma_0^2}{sf^4}\Big]$$

$$=(2\pi)^2\int\sin\theta\cos\theta d\theta\ \mathrm{Tr}\Big[-2\frac{\sigma_0^2}{s\big(\sin^2\theta+\frac{1}{s^2}\cos^2\theta\big)^2}\Big]$$

$$=-4\pi^2 s\ \mathrm{Tr}\sigma_0^2$$

: Equivalent to squashed one

Classical configuration : $A=0, \bar{\lambda}=\lambda=0, \sigma=\frac{\sigma_0}{f}, D=-\frac{\sigma_0}{sf^3}$

$$Z(\infty)=\int d\sigma_0\, e^{\frac{ik}{4\pi}S^{cl}_{\rm CS}}\frac{\det\Delta_f}{\det\Delta_b}$$

$$\mathcal{L}_{\rm CS}={\rm Tr}\Big[\frac{1}{\sqrt{g}}\varepsilon^{\mu\nu\lambda}(\underbrace{A_\mu\partial_\nu A_\lambda}_{0}-\frac{2i}{3}\underbrace{A_\mu A_\nu A_\lambda}_{0})-\underbrace{\bar{\lambda}\lambda}_{0}+2\underbrace{D\sigma}_{\frac{\sigma_0}{f}\cdot\left(-\frac{\sigma_0}{sf^3}\right)}\Big]={\rm Tr}\Big[-2\frac{\sigma_0^2}{sf^4}\Big]$$

$$\begin{aligned}
S^{cl}_{\rm CS}&=\int\sqrt{g}d^3x\;\mathcal{L}_{\rm CS}\\
&=(2\pi)^2\int\sin\theta\cos\theta d\theta\;{\rm Tr}\Big[-2\frac{\sigma_0^2}{sf^4}\Big]\\
&=(2\pi)^2\int\sin\theta\cos\theta d\theta\;{\rm Tr}\Big[-2\frac{\sigma_0^2}{s\big(\sin^2\theta+\frac{1}{s^2}\cos^2\theta\big)^2}\Big]\\
&=-4\pi^2 s\;{\rm Tr}\sigma_0^2
\end{aligned}$$

: Equivalent to squashed one

Classical configuration : $A=0, \bar{\lambda}=\lambda=0, \sigma=\frac{\sigma_0}{f}, D=-\frac{\sigma_0}{sf^3}$

$$Z(\infty)=\int d\sigma_0\, e^{\frac{ik}{4\pi}S_{\mathrm{CS}}^{cl}}\frac{\det\Delta_f}{\det\Delta_b}$$

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$$Z(\infty)=\int d\sigma_0\, e^{\frac{ik}{4\pi}S_{\rm CS}^{cl}}\frac{\det\Delta_f}{\det\Delta_b}$$

$$\mathcal{L}_{\rm CS}={\rm Tr}\Big[\frac{1}{\sqrt{g}}\varepsilon^{\mu\nu\lambda}(\underbrace{A_\mu}_{0}\partial_\nu A_\lambda-\frac{2i}{3}\underbrace{A_\mu}_{0} A_\nu A_\lambda)-\bar{\lambda}\underbrace{\lambda}_{0}+2\underbrace{D\sigma}_{\frac{\sigma_0}{f}\cdot\left(-\frac{\sigma_0}{sf^3}\right)}\Big]={\rm Tr}\Big[-2\frac{\sigma_0^2}{sf^4}\Big]$$

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&=-4\pi^2 s\,{\rm Tr}\sigma_0^2
\end{aligned}$$

: Equivalent to squashed one

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$$Z(\infty)=\int d\sigma_0\, e^{\frac{ik}{4\pi}S_{\rm CS}^{cl}}\frac{\det\Delta_f}{\det\Delta_b}$$

$$\mathcal{L}_{\rm CS}={\rm Tr}\Big[\frac{1}{\sqrt{g}}\varepsilon^{\mu\nu\lambda}(A_\mu\partial_\nu A_\lambda-\frac{2i}{3}A_\mu A_\nu A_\lambda)-\bar{\lambda}\lambda+2D\sigma\Big]={\rm Tr}\Big[-2\frac{\sigma_0^2}{sf^4}\Big]$$

0

0

0

$\frac{\sigma_0}{f}\cdot\Big(-\frac{\sigma_0}{sf^3}\Big)$

$$S_{\rm CS}^{cl}=\int\sqrt{g}d^3x\,\mathcal{L}_{\rm CS}$$

$$=(2\pi)^2\int\sin\theta\cos\theta d\theta\ {\rm Tr}\Big[-2\frac{\sigma_0^2}{sf^4}\Big]$$

$$=(2\pi)^2\int\sin\theta\cos\theta d\theta\ {\rm Tr}\Big[-2\frac{\sigma_0^2}{s\big(\sin^2\theta+\frac{1}{s^2}\cos^2\theta\big)^2}\Big]$$

$$=-4\pi^2 s\,{\rm Tr}\sigma_0^2$$

: Equivalent to squashed one

SUSY on round sphere

$$\epsilon = \begin{pmatrix} -(\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$$\bar{\epsilon} = \begin{pmatrix} (\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

$\mathcal{N}=2$ Vector multiplet

$\mathcal{N}=2$ Matter multiplet

$$Z(t) = \int [\mathcal{D}A\ldots\mathcal{D}\phi\ldots] e^{\frac{ik}{4\pi}S_{\mathrm{CS}} - t(\delta_{\bar{\epsilon}}\text{-exact})}$$

$\boxed{t \to \infty}$ steepest decent method around $(\delta_{\bar{\epsilon}}\text{-exact}) = 0$ turns to be exact.

$$Z(\infty) = \int d\sigma_0\, e^{\frac{ik}{4\pi}S^{cl}_{\mathrm{CS}}} \frac{\det \Delta_f}{\det \Delta_b}$$

$$A = 0, \bar{\lambda} = \lambda = 0, \sigma = \frac{\sigma_0}{f}, D = -\frac{\sigma_0}{sf^3}$$

$$\frac{\det \Delta_f}{\det \Delta_b} = \prod_{\alpha\in\Delta_+} 4\sinh(\pi\alpha(\sigma_0))\sinh s(\pi\alpha(\sigma_0))$$

$$\phi = \bar{\phi} = \ldots = 0$$

$$\frac{\det \Delta_f}{\det \Delta_b} = \prod_{\rho\in\mathrm{weight}} s_b\left(\frac{i(\sqrt{\frac{1}{s}} + \sqrt{s})}{2}(1-\Delta) - \sqrt{s}\rho\cdot\sigma_0\right)$$

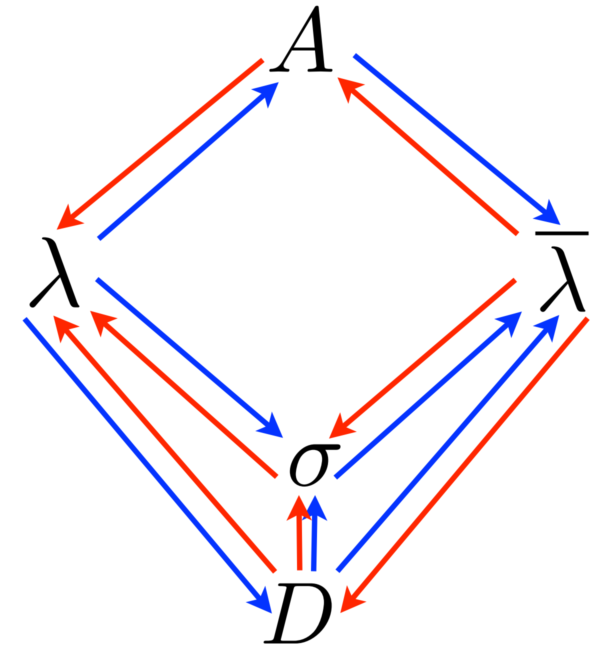
$\mathcal{L}_{\mathrm{YM}} = \delta_{\bar{\epsilon}}\text{-exact}$

$\mathcal{L}_{\mathrm{mat}} = \delta_{\bar{\epsilon}}\text{-exact}$

How about Wilson loop?

$$\delta_{\bar{\epsilon}}(\text{Wilson loop}) = 0$$

$$\text{Wilson loop} = \mathrm{Tr}_R \mathcal{P} \exp\left(\oint_C dt (i A_\mu \dot{x}^\mu + \sigma |\dot{x}|) \right)$$



$$\delta_{\bar{\epsilon}}(\text{Wilson loop}) = 0$$

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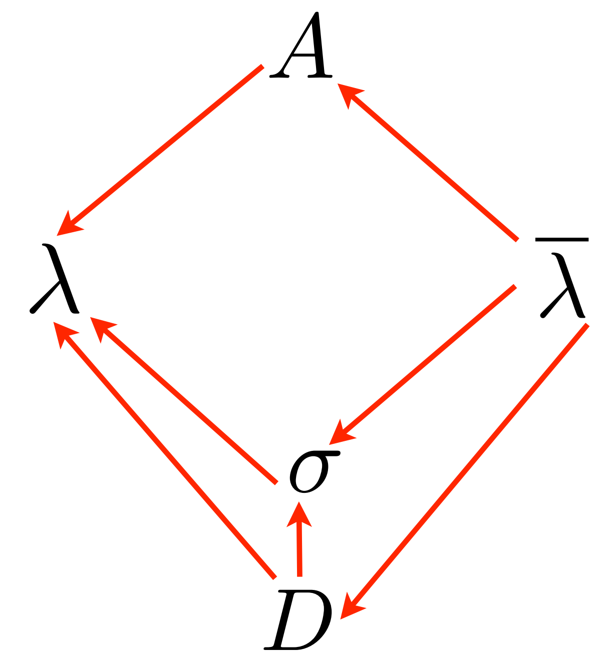
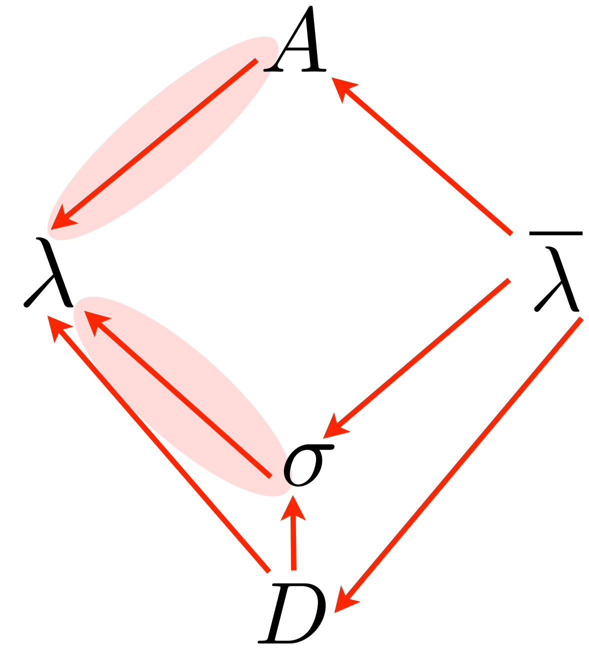


$$\delta_{\bar{\epsilon}}(\text{Wilson loop}) = 0$$

$$\text{Wilson loop} = \mathrm{Tr}_R \mathcal{P} \exp\left(\oint_C dt (iA_\mu \dot{x}^\mu + \sigma |\dot{x}|) \right)$$

$$\lambda$$

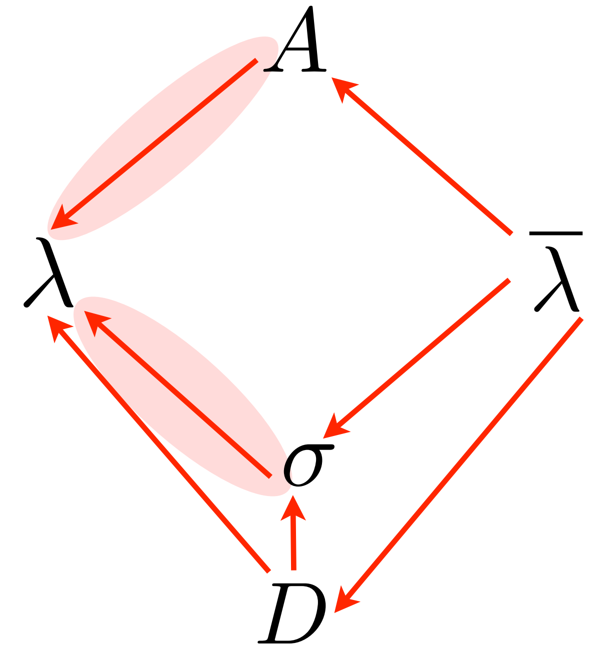


$$\delta_{\overline{\epsilon}}(\text{Wilson loop}) = 0$$

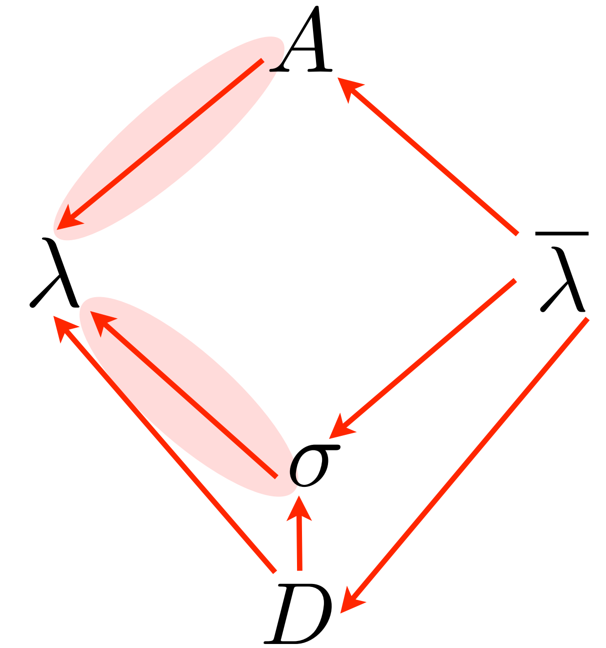
$$\text{Wilson loop} = \mathrm{Tr}_R \mathcal{P} \exp\left(\oint_C dt (i A_\mu \dot{x}^\mu + \sigma |\dot{x}|) \right)$$

$$\lambda$$

$$\delta_{\overline{\epsilon}}(\text{Wilson loop}) \propto \overline{\epsilon}(\gamma_\mu \dot{x}^\mu + |\dot{x}|)\,\lambda$$



$$\delta_{\bar{\epsilon}}(\text{Wilson loop}) = 0$$



$$\text{Wilson loop} = \mathrm{Tr}_R \mathcal{P} \exp\left(\oint_C dt (i A_\mu \dot{x}^\mu + \sigma |\dot{x}|) \right)$$

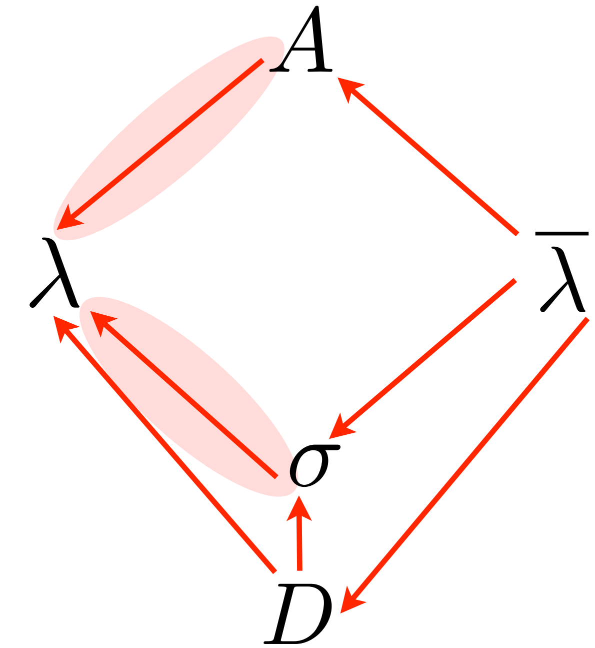
$$\lambda$$

$$\delta_{\bar{\epsilon}}(\text{Wilson loop}) \propto \bar{\epsilon}(\gamma_\mu \dot{x}^\mu + |\dot{x}|)\, \lambda \quad = 0$$

Equations for the Wilson loop's contour!

$$\delta_{\overline{\epsilon}}(\text{Wilson loop}) = 0$$

$$\text{Wilson loop} = \mathrm{Tr}_R \mathcal{P} \exp\left(\oint_C dt(iA_\mu \dot{x}^\mu + \sigma|\dot{x}|) \right)$$

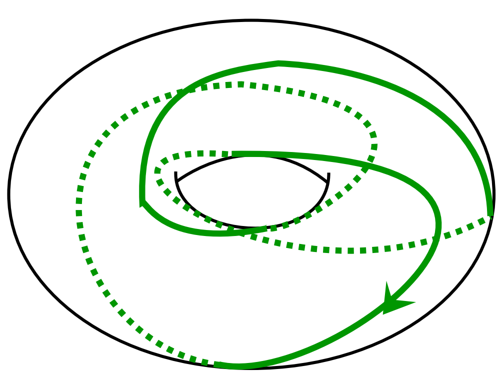


$$\lambda$$

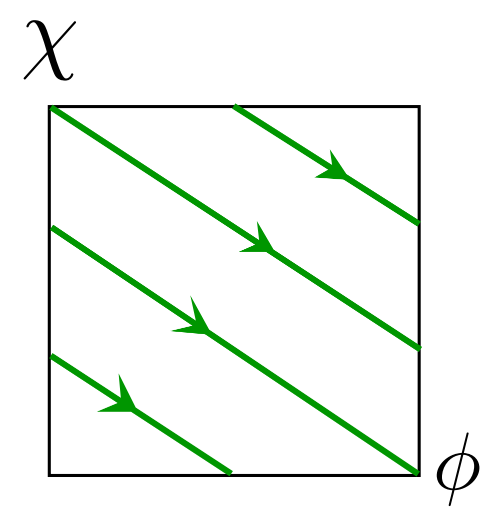
$$\propto \underline{\overline{\epsilon}(\gamma_\mu \dot{x}^\mu + |\dot{x}|) = 0}$$

Equations for the Wilson loop's contour!

Torus knot



$=$



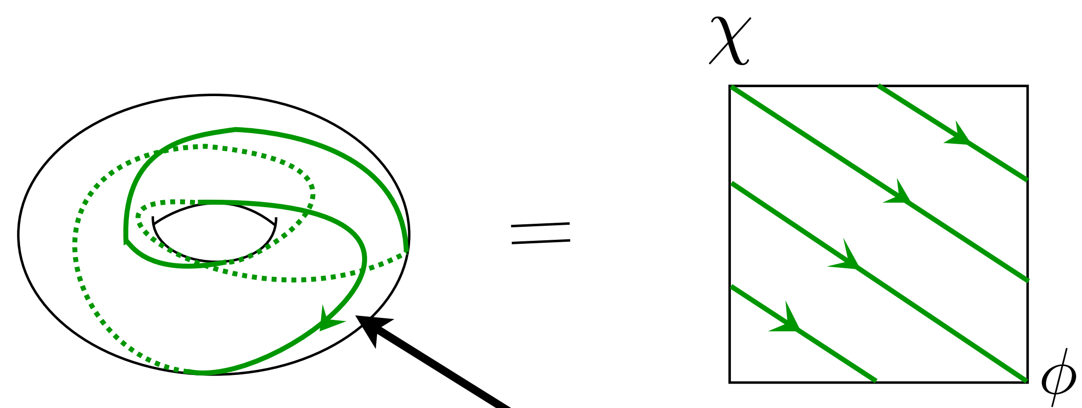
SUSY on round sphere

$$\epsilon = \begin{pmatrix} -(\cos\theta + i{\color{red}s}\sin\theta)^{1/2} \\ (\cos\theta - i{\color{red}s}\sin\theta)^{1/2} \end{pmatrix}$$

$$\bar{\epsilon} = \begin{pmatrix} (\cos\theta + i{\color{red}s}\sin\theta)^{1/2} \\ (\cos\theta - i{\color{red}s}\sin\theta)^{1/2} \end{pmatrix}$$

$\mathcal{N}=2$ Vector multiplet

$\mathcal{N}=2$ Matter multiplet



1/2 BPS

$$Z({\color{red}\infty}) = \int d\sigma_0 \, e^{\frac{ik}{4\pi} S^{cl}_{\rm CS}} \frac{\det \Delta_f}{\det \Delta_b}$$

$$A=0, \overline{\lambda}=\lambda=0, \sigma=\frac{\sigma_0}{f}, D=-\frac{\sigma_0}{sf^3}$$

$$\frac{\det \Delta_f}{\det \Delta_b} = \prod_{\alpha\in\Delta_+} 4\sinh(\pi\alpha(\sigma_0))\sinh s(\pi\alpha(\sigma_0))$$

$$\phi = \overline{\phi} = \ldots = 0$$

$$\frac{\det \Delta_f}{\det \Delta_b} = \prod_{\rho\in\text{weight}} s_b\left(\frac{i(\sqrt{\frac{1}{s}}+\sqrt{s})}{2}(1-\Delta) - \sqrt{s}\rho\cdot\sigma_0\right)$$

$$\mathcal{L}_{\rm YM} = \delta_{\bar{\epsilon}}\text{-exact}$$

$$\mathcal{L}_{\rm mat} = \delta_{\bar{\epsilon}}\text{-exact}$$

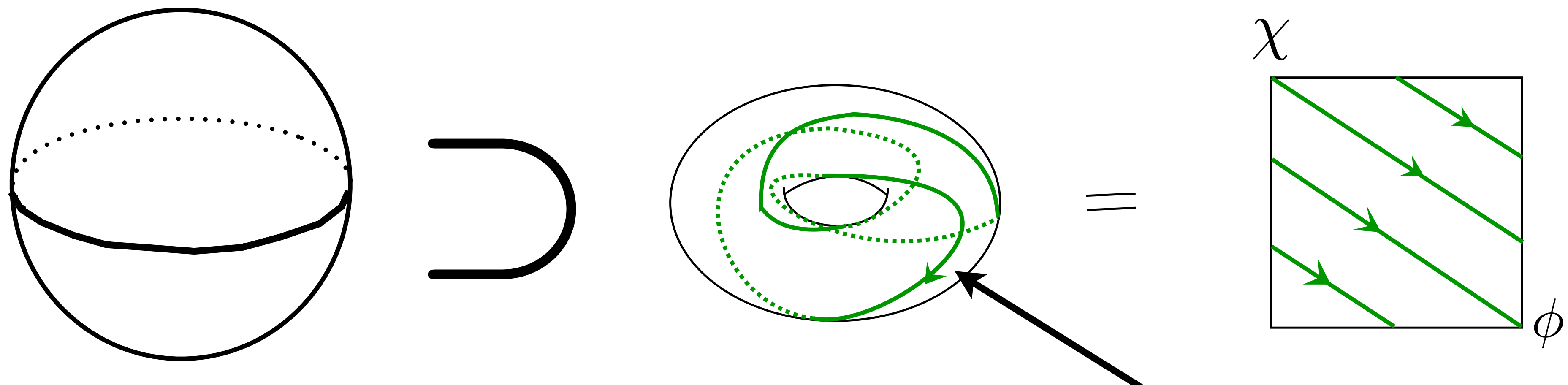
SUSY on round sphere

$$\epsilon = \begin{pmatrix} -(\cos\theta + i s \sin\theta)^{1/2} \\ (\cos\theta - i s \sin\theta)^{1/2} \end{pmatrix}$$

$$\bar{\epsilon} = \begin{pmatrix} (\cos\theta + i s \sin\theta)^{1/2} \\ (\cos\theta - i s \sin\theta)^{1/2} \end{pmatrix}$$

$\mathcal{N}=2$ Vector multiplet

$\mathcal{N}=2$ Matter multiplet



1/2 BPS

$$Z(\infty) = \int d\sigma_0 \, e^{\frac{ik}{4\pi} S^{cl}_{\mathrm{CS}}} \frac{\det \Delta_f}{\det \Delta_b}$$

$$A = 0, \overline{\lambda} = \lambda = 0, \sigma = \frac{\sigma_0}{f}, D = -\frac{\sigma_0}{s f^3}$$

$$\frac{\det \Delta_f}{\det \Delta_b} = \prod_{\alpha \in \Delta_+} 4 \sinh(\pi\alpha(\sigma_0)) \sinh s(\pi\alpha(\sigma_0))$$

$\mathcal{L}_{\mathrm{YM}} = \delta_{\bar{\epsilon}}$-exact

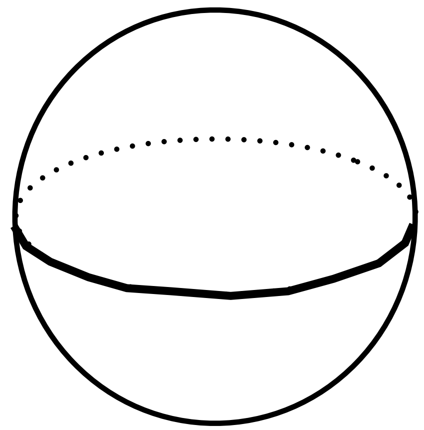
$$\phi = \overline{\phi} = \ldots = 0$$

$$\frac{\det \Delta_f}{\det \Delta_b} = \prod_{\rho \in \mathrm{weight}} s_b\left(\frac{i(\sqrt{\frac{1}{s}} + \sqrt{s})}{2}(1-\Delta) - \sqrt{s}\rho\cdot\sigma_0\right)$$

$\mathcal{L}_{\mathrm{mat}} = \delta_{\bar{\epsilon}}$-exact

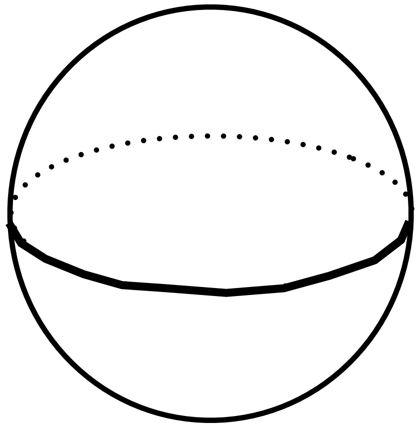
In introduction

Is it possible to get deformed results

even on  ?

In introduction

Is it possible to get deformed results

even on  ?

Now

The answer is YES.

(Based on direct check)

vector

SUSY on round sphere

$$f^2(\theta) = \sin^2\theta + \frac{1}{s^2}\cos^2\theta$$

$$\mathcal{D}_\mu \epsilon = \frac{i}{2sf^2}\gamma_\mu \epsilon - \frac{1}{2}\frac{f'}{f}\gamma_\mu \bar{\epsilon}$$

$$\mathcal{D}_\mu \bar{\epsilon} = \frac{i}{2sf^2}\gamma_\mu \bar{\epsilon} - \frac{1}{2}\frac{f'}{f}\gamma_\mu \epsilon$$

$\mathcal{N}=2$ Vector multiplet

- SUSY transformation

$\rightarrow$ Just Replacing usual Killing spinors
with our unusual Killing spinors.

$$\delta_\epsilon A_\mu = -\frac{i}{2}\bar{\lambda}\gamma_\mu\epsilon, \quad \delta_{\bar{\epsilon}} A_\mu = -\frac{i}{2}\bar{\epsilon}\gamma_\mu\lambda,$$

$$\delta_\epsilon \sigma = +\frac{1}{2}\bar{\lambda}\epsilon, \quad \delta_{\bar{\epsilon}}\sigma = +\frac{1}{2}\bar{\epsilon}\lambda,$$

$$\delta_\epsilon \lambda = \frac{1}{2}\gamma^{\mu\nu}\epsilon F_{\mu\nu} - D\epsilon + i\gamma^\mu \epsilon \mathcal{D}_\mu \sigma + \frac{2i}{3}\sigma\gamma^\mu \mathcal{D}_\mu \epsilon, \quad \delta_{\bar{\epsilon}}\lambda = 0,$$

$$\delta_\epsilon \bar{\lambda} = 0, \quad \delta_{\bar{\epsilon}}\bar{\lambda} = \frac{1}{2}\gamma^{\mu\nu}\bar{\epsilon}F_{\mu\nu} + D\bar{\epsilon} - i\gamma^\mu\bar{\epsilon}\mathcal{D}_\mu\sigma - \frac{2i}{3}\sigma\gamma^\mu\mathcal{D}_\mu\bar{\epsilon},$$

$$\delta_\epsilon D = +\frac{i}{2}\mathcal{D}_\mu\bar{\lambda}\gamma^\mu\epsilon - \frac{i}{2}[\bar{\lambda}\epsilon, \sigma] + \frac{i}{6}\bar{\lambda}\gamma^\mu\mathcal{D}_\mu\epsilon, \delta_{\bar{\epsilon}} D = -\frac{i}{2}\bar{\epsilon}\gamma^\mu\mathcal{D}_\mu\lambda + \frac{i}{2}[\bar{\epsilon}\lambda, \sigma] - \frac{i}{6}\mathcal{D}_\mu\bar{\epsilon}\gamma^\mu\lambda.$$

SUSY on round sphere

$$f^2(\theta) = \sin^2\theta + \frac{1}{s^2}\cos^2\theta$$

$$\mathcal{D}_\mu \epsilon = \frac{i}{2sf^2}\gamma_\mu \epsilon - \frac{1}{2}\frac{f'}{f}\gamma_\mu \bar{\epsilon}$$

$$\mathcal{D}_\mu \bar{\epsilon} = \frac{i}{2sf^2}\gamma_\mu \bar{\epsilon} - \frac{1}{2}\frac{f'}{f}\gamma_\mu \epsilon$$

$\mathcal{N}=2$ Vector multiplet

- SUSY transformation

$\rightarrow$ Just Replacing usual Killing spinors with our unusual Killing spinors.

$$\{\delta_\epsilon, \delta_{\bar{\epsilon}}\} A_\mu = i v^\nu \partial_\nu A_\mu + i \partial_\mu v^\nu A_\nu + \mathcal{D}_\mu \Lambda,$$

$$\{\delta_\epsilon, \delta_{\bar{\epsilon}}\} \sigma = i v^\mu \partial_\mu \sigma + i[\Lambda, \sigma]$$

$$\{\delta_\epsilon, \delta_{\bar{\epsilon}}\} \lambda = i v^\mu \partial_\mu \lambda + \frac{i}{4}\Theta_{\mu\nu}\gamma^{\mu\nu}\lambda + i[\Lambda, \lambda] + \alpha\lambda,$$

$$\{\delta_\epsilon, \delta_{\bar{\epsilon}}\} \bar{\lambda} = i v^\mu \partial_\mu \bar{\lambda} + \frac{i}{4}\Theta_{\mu\nu}\gamma^{\mu\nu}\bar{\lambda} + i[\Lambda, \bar{\lambda}] - \alpha\bar{\lambda},$$

$$\{\delta_\epsilon, \delta_{\bar{\epsilon}}\} D = i v^\mu \partial_\mu D + i[\Lambda, D],$$

($v^\mu, \Lambda,$ $\Theta_{\mu\nu}, \alpha$ same definition in Hama, Hosomichi, Lee) matter

SUSY on round sphere

$$f^2(\theta) = \sin^2\theta + \frac{1}{s^2}\cos^2\theta$$

$$\mathcal{D}_\mu \epsilon = \frac{i}{2sf^2}\gamma_\mu \epsilon - \frac{1}{2}\frac{f'}{f}\gamma_\mu \bar{\epsilon}$$
$$\mathcal{D}_\mu \bar{\epsilon} = \frac{i}{2sf^2}\gamma_\mu \bar{\epsilon} - \frac{1}{2}\frac{f'}{f}\gamma_\mu \epsilon$$

$\mathcal{N}=2$ Matter multiplet

- SUSY transformation

$\rightarrow$ Just Replacing usual Killing spinors with our unusual Killing spinors.

$$\delta_\epsilon \phi = 0, \qquad \delta_{\bar{\epsilon}} \phi = \bar{\epsilon}\psi,$$
$$\delta_\epsilon \overline{\phi} = \epsilon\overline{\psi}, \qquad \delta_{\bar{\epsilon}} \overline{\phi} = 0,$$
$$\delta_\epsilon \psi = i\gamma^\mu \epsilon \mathcal{D}_\mu \phi + i\epsilon\sigma\phi + \frac{2\Delta i}{3}\gamma^\mu \mathcal{D}_\mu \epsilon \phi, \qquad \delta_{\bar{\epsilon}} \psi = \bar{\epsilon} F,$$
$$\delta_\epsilon \overline{\psi} = \overline{F}\epsilon, \qquad \delta_{\bar{\epsilon}} \overline{\psi} = i\gamma^\mu \bar{\epsilon} \mathcal{D}_\mu \overline{\phi} + i\overline{\phi}\sigma\bar{\epsilon} + \frac{2\Delta i}{3}\overline{\phi}\gamma^\mu \mathcal{D}_\mu \bar{\epsilon},$$
$$\delta_\epsilon F = \epsilon(i\gamma^\mu \mathcal{D}_\mu \psi - i\sigma\psi - i\lambda\phi) + \frac{i}{3}(2\Delta - 1)\mathcal{D}_\mu \epsilon \gamma^\mu \psi, \qquad \delta_{\bar{\epsilon}} F = 0,$$
$$\delta_\epsilon \overline{F} = 0, \qquad \delta_{\bar{\epsilon}} \overline{F} = \bar{\epsilon}(i\gamma^\mu \mathcal{D}_\mu \overline{\psi} - i\overline{\psi}\sigma + i\overline{\phi}\overline{\lambda}) + \frac{i}{3}(2\Delta - 1)\mathcal{D}_\mu \bar{\epsilon}\gamma^\mu \overline{\psi}.$$

SUSY on round sphere

$$f^2(\theta) = \sin^2\theta + \frac{1}{s^2}\cos^2\theta$$

$$\mathcal{D}_\mu\epsilon = \frac{i}{2sf^2}\gamma_\mu\epsilon - \frac{1}{2}\frac{f'}{f}\gamma_\mu\bar{\epsilon}$$

$$\mathcal{D}_\mu\bar{\epsilon} = \frac{i}{2sf^2}\gamma_\mu\bar{\epsilon} - \frac{1}{2}\frac{f'}{f}\gamma_\mu\epsilon$$

$\mathcal{N}=2$ Matter multiplet

- SUSY transformation

$\rightarrow$ Just Replacing usual Killing spinors with our unusual Killing spinors.

$$\{\delta_\epsilon, \delta_{\bar{\epsilon}}\}\phi = iv^\mu\partial_\mu\phi + i\Lambda\phi - \Delta\alpha\phi,$$

$$\{\delta_\epsilon, \delta_{\bar{\epsilon}}\}\overline{\phi} = iv^\mu\partial_\mu\overline{\phi} - i\overline{\phi}\Lambda + \Delta\alpha\overline{\phi},$$

$$\{\delta_\epsilon, \delta_{\bar{\epsilon}}\}\psi = iv^\mu\partial_\mu\psi + \frac{i}{4}\Theta_{\mu\nu}\gamma^{\mu\nu}\psi + i\Lambda\psi + (1-\Delta)\alpha\psi,$$

$$\{\delta_\epsilon, \delta_{\bar{\epsilon}}\}\overline{\psi} = iv^\mu\partial_\mu\overline{\psi} + \frac{i}{4}\Theta_{\mu\nu}\gamma^{\mu\nu}\overline{\psi} - i\overline{\psi}\Lambda + (\Delta-1)\alpha\overline{\psi},$$

$$\{\delta_\epsilon, \delta_{\bar{\epsilon}}\}F = iv^\mu\partial_\mu F + i\Lambda F + (2-\Delta)\alpha F,$$

$$\{\delta_\epsilon, \delta_{\bar{\epsilon}}\}\overline{F} = iv^\mu\partial_\mu\overline{F} - i\overline{F}\Lambda + (\Delta-2)\alpha\overline{F},$$

Poincare-Hopf

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Simple but interesting example $(\mu = 1, 2)\, x^\mu \in \mathcal{M}$ (2D manifold)

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$$\text{Ferminic d.o.f} :(\psi^\mu, \overline{\psi}_\mu)$$

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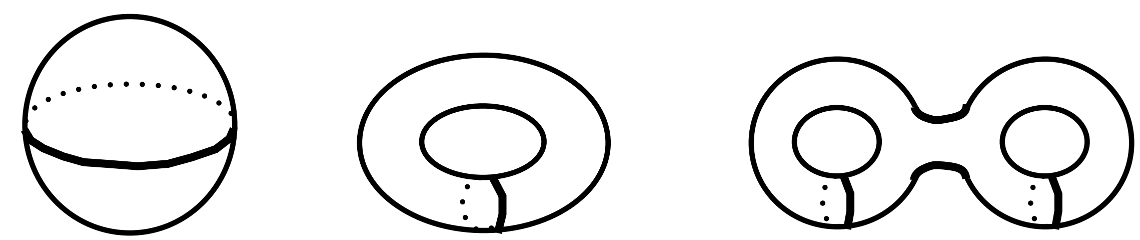

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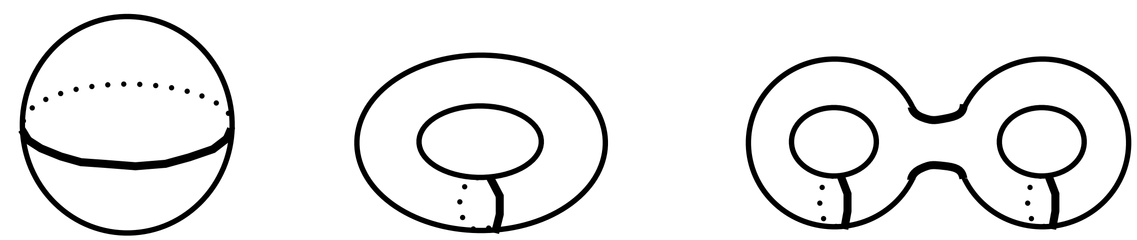

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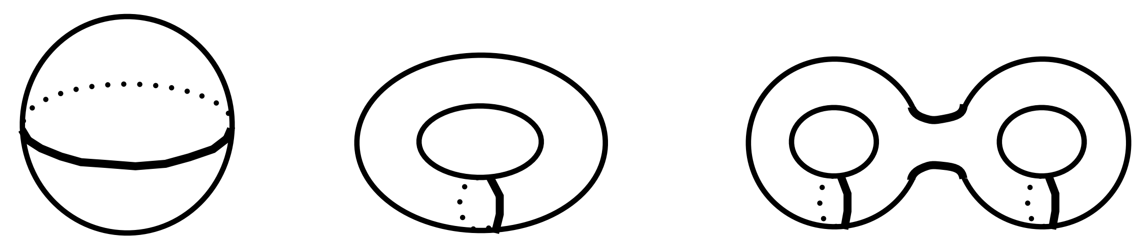

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Steepest decent method is exact

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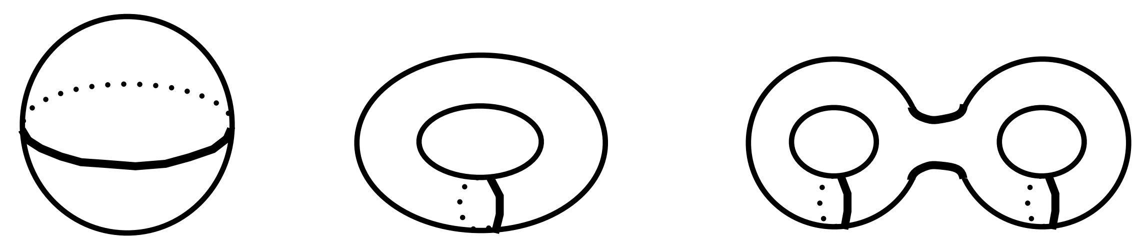

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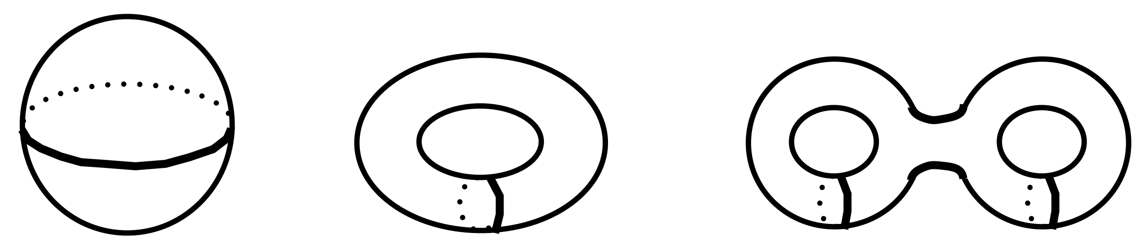

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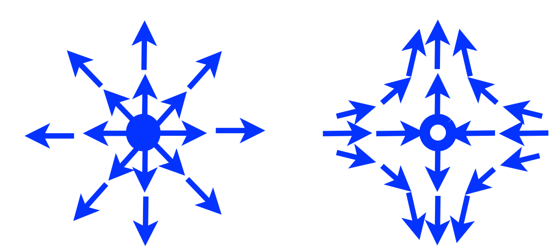

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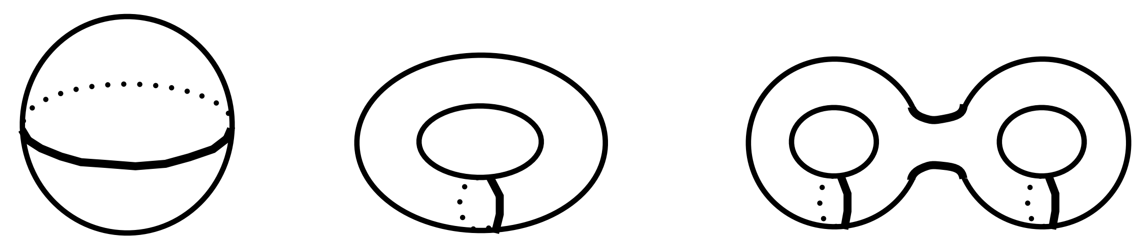

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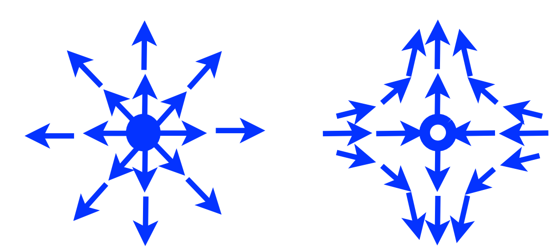

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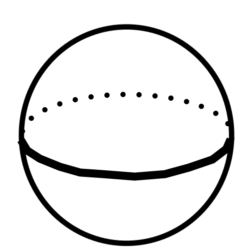

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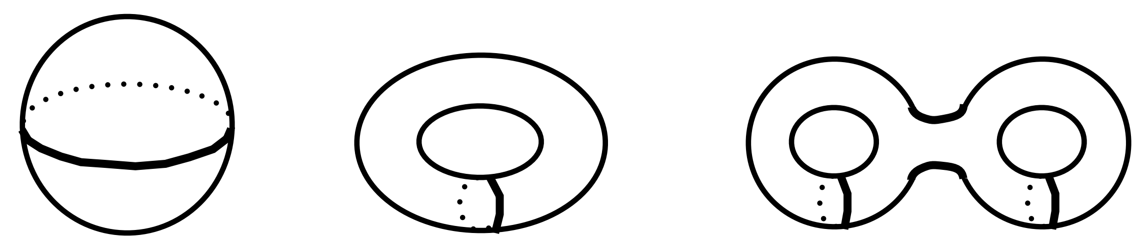

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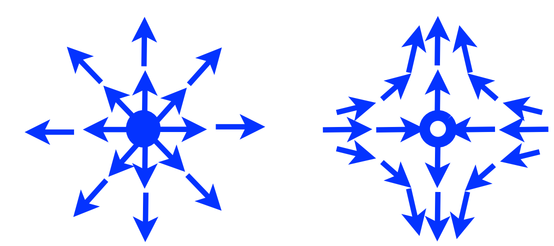

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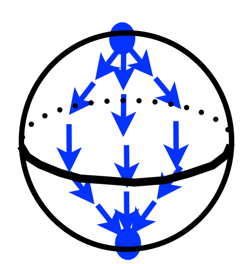

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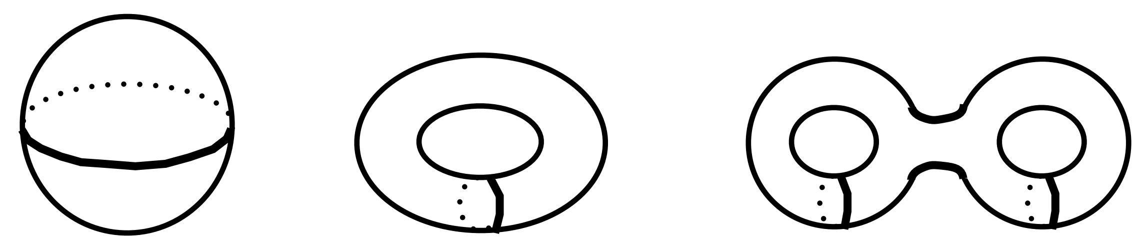

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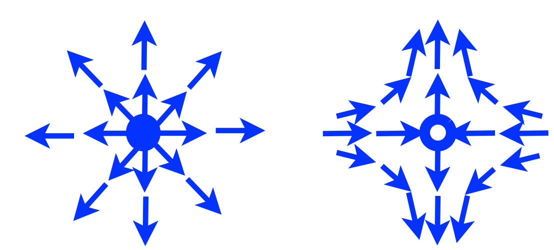

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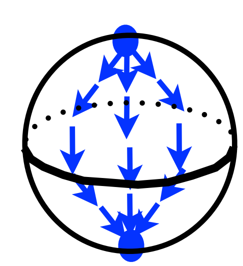

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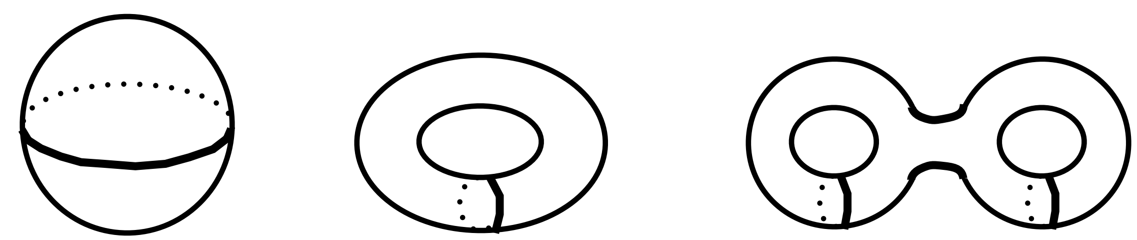

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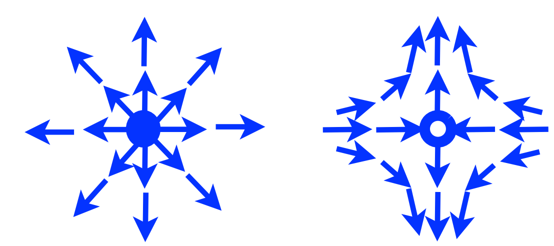

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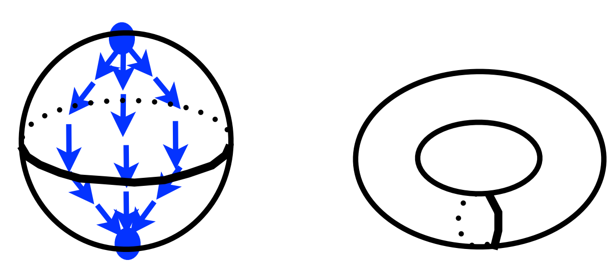

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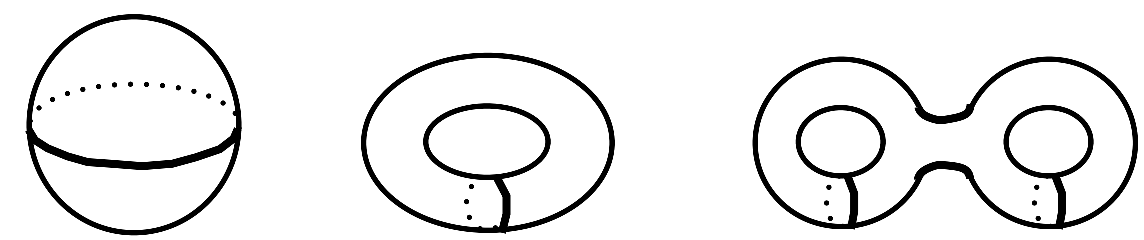

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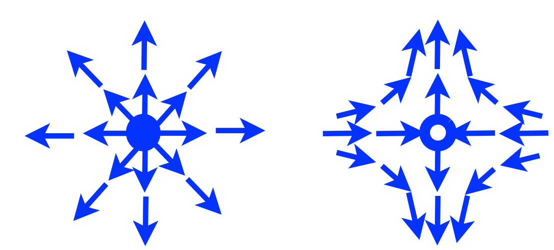

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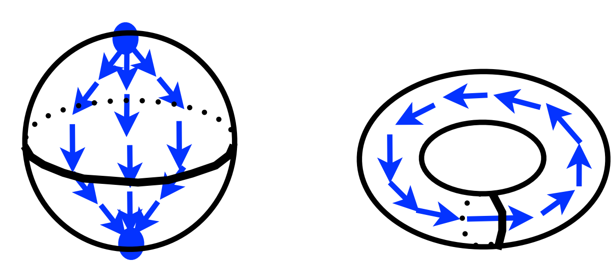

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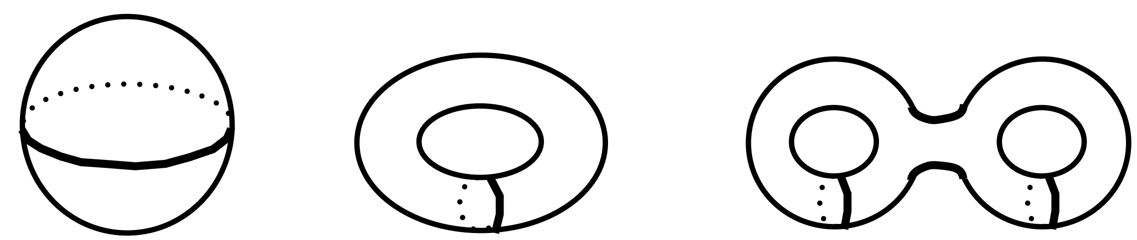

2 0 -2

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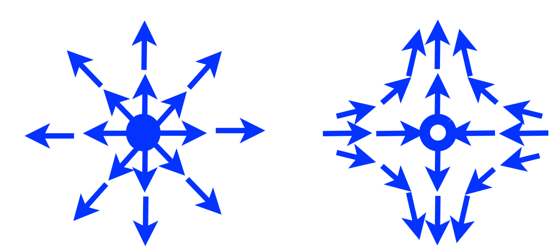

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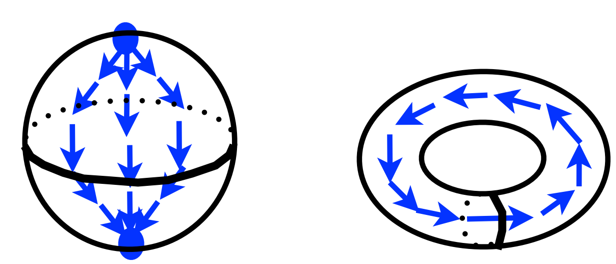

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What is the spirit of localization?

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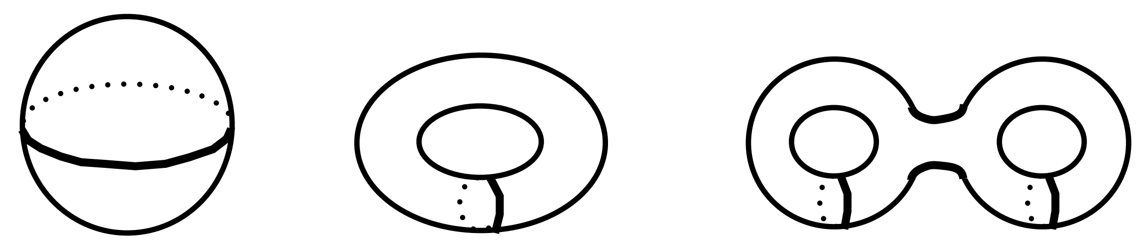

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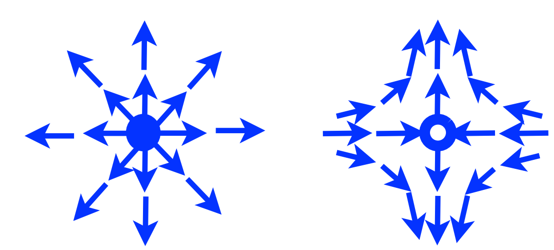

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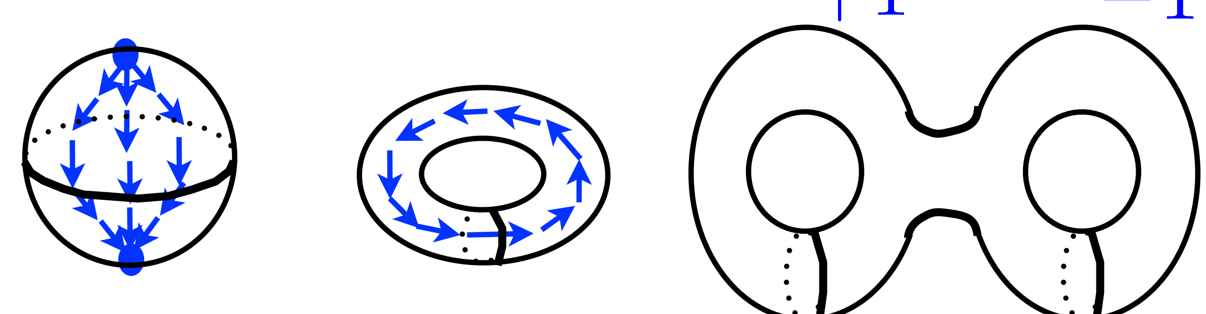

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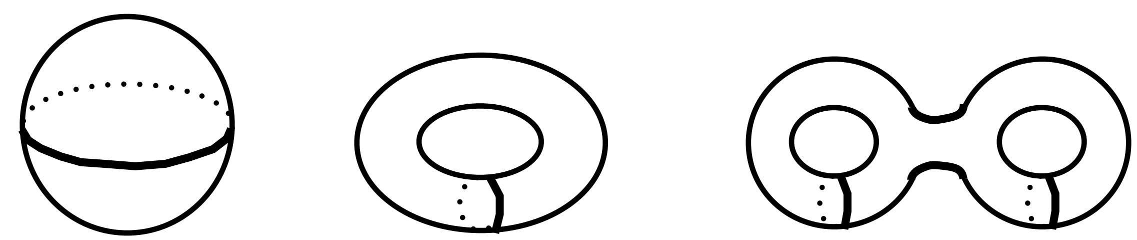

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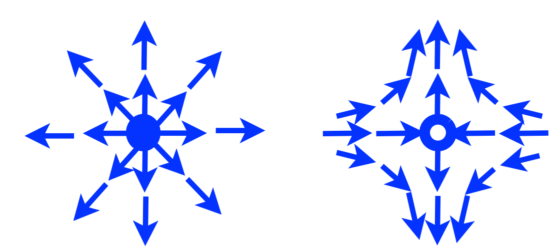

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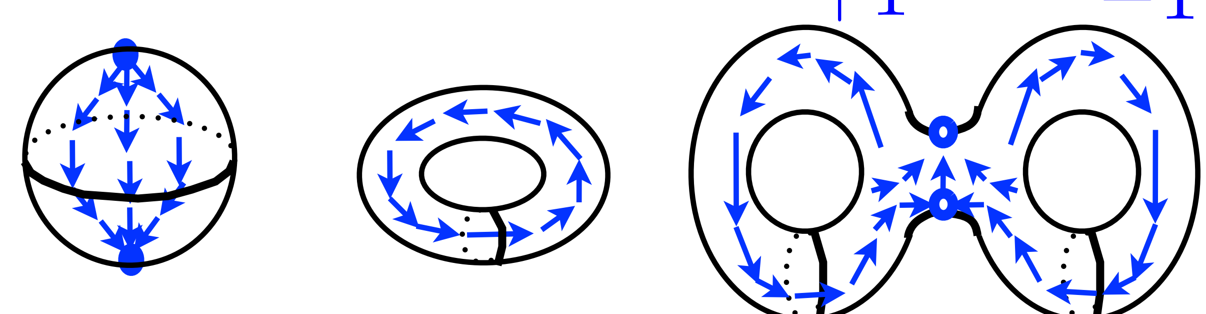

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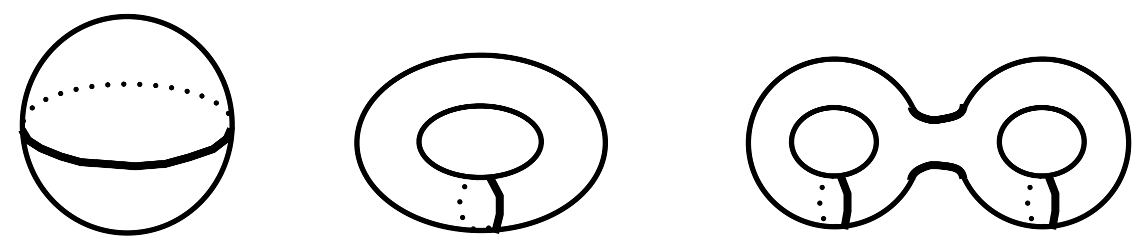

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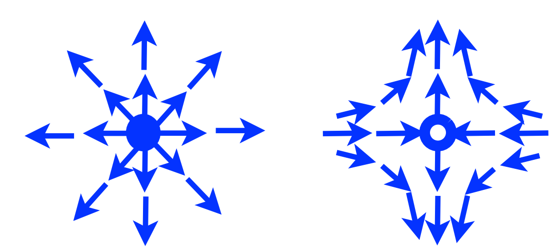

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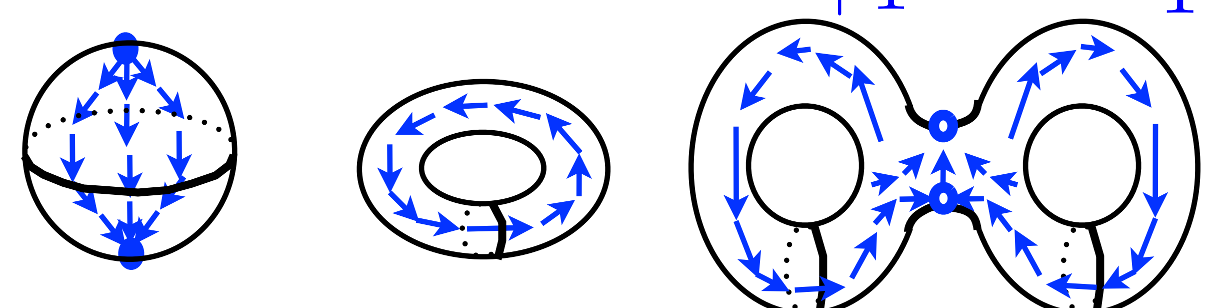

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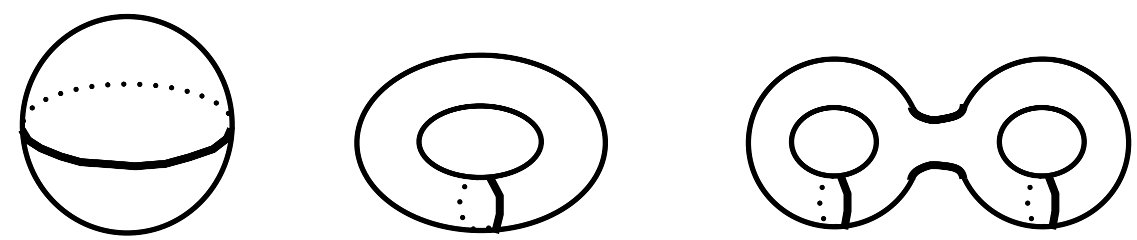

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does not depend on t !?

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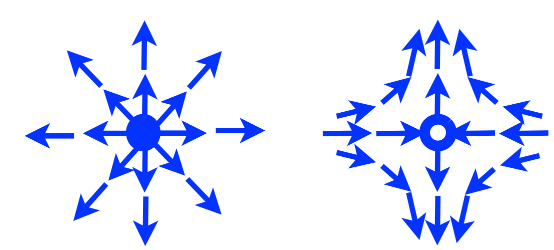

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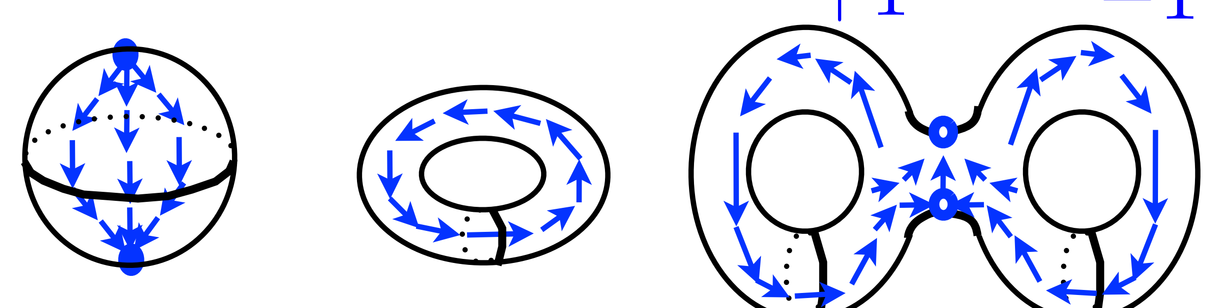

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1-loop details

SUSY on round sphere

$$\epsilon = \begin{pmatrix} -(\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

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$\mathcal{N}=2$ Vector multiplet

$$\Delta_b \sim i\alpha(\sigma_0) - *df$$

$$\Delta_f = if\gamma^\mu \mathcal{D}_\mu + \frac{if'}{2}\gamma_3 + i\alpha(\sigma_0) - \frac{1}{2sf} \qquad \frac{\det \Delta_f}{\det \Delta_b} = \frac{\prod_f M_f}{\prod_b M_b}$$

$$\Delta_b \mathcal{B}_{rel} = M_b \mathcal{B}_{rel}$$

$$f^{-1}\Big(d(f\bar{\epsilon}\Lambda_{rel}) + [iM_f + \alpha(\sigma_0)]\bar{\epsilon}\gamma_\mu\Lambda_{rel}dx^\mu\Big) = 0$$

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$$\boxed{\begin{aligned} M_f &= \frac{1}{s}m + n + i\alpha(\sigma_0), \quad (2sf' + sf\partial_\theta)\varphi_0 = i(m+ns)\varphi_2, \\ &\left(2sf' + sf\partial_\theta + sm\frac{\sin\theta}{\cos\theta} - n\frac{\cos\theta}{\sin\theta}\right)\varphi_0 = 0. \end{aligned}}$$

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$$\boxed{\begin{aligned} M_b &= \frac{1}{s}m + n + i\alpha(\sigma_0), \\ 0 &= \left(\partial_\theta - \frac{\sin\theta}{\cos\theta}\left(\frac{1}{f}m + 1\right) + \frac{\cos\theta}{\sin\theta}\left(\frac{1}{sf}n + 1\right)\right)y. \end{aligned}}$$

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these terms does not affect

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$$\boxed{\varphi_0 \sim \begin{cases} (\sin\theta)^n & \theta \sim 0 \\ (\cos\theta)^m & \theta \sim \frac{\pi}{2} \end{cases} \quad \to \quad m \geq 0, n \geq 0 (\text{without } m = n = 0)}$$

$$\boxed{M_f = \frac{1}{s}m + n + i\alpha(\sigma_0), \quad (2sf' + sf\partial_\theta)\varphi_0 = i(m+ns)\varphi_2, \quad \left(2sf' + sf\partial_\theta + sm\frac{\sin\theta}{\cos\theta} - n\frac{\cos\theta}{\sin\theta}\right)\varphi_0 = 0.}$$

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$$\Delta_f \Lambda_{rel} = M_f \Lambda_{rel}$$

$$\boxed{M_b = \frac{1}{s}m + n + i\alpha(\sigma_0), \quad 0 = \left(\partial_\theta - \frac{\sin\theta}{\cos\theta}(\frac{1}{f}m + 1) + \frac{\cos\theta}{\sin\theta}(\frac{1}{sf}n + 1)\right)y.}$$

$$\boxed{y \sim (\cos\theta)^{(-m-1)}(\sin\theta)^{(-n-1)} \quad \to \quad m, n \leq -1}$$

SUSY on round sphere

$$\epsilon = \begin{pmatrix} -(\cos\theta + is\sin\theta)^{1/2} \\ (\cos\theta - is\sin\theta)^{1/2} \end{pmatrix}$$

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$\mathcal{N}=2$ Vector multiplet

$$\Delta_b \sim i\alpha(\sigma_0) - *df$$

$$\Delta_f = if\gamma^\mu \mathcal{D}_\mu + \frac{if'}{2}\gamma_3 + i\alpha(\sigma_0) - \frac{1}{2sf}$$

$$\frac{\det \Delta_f}{\det \Delta_b} = \frac{\prod_f M_f}{\prod_b M_b} = \prod_{\alpha\in\Delta_+} 4\sinh(\pi\alpha(\sigma_0))\sinh s(\pi\alpha(\sigma_0))$$

$$\varphi_0 \sim \begin{cases} (\sin\theta)^n & \theta \sim 0 \\ (\cos\theta)^m & \theta \sim \frac{\pi}{2} \end{cases}$$

$$\to \quad m \geq 0, n \geq 0 (\text{without } m=n=0)$$

$$M_f = \frac{1}{s}m + n + i\alpha(\sigma_0), \quad (2sf' + sf\partial_\theta)\varphi_0 = i(m+ns)\varphi_2,$$

$$\left(2sf' + sf\partial_\theta + sm\frac{\sin\theta}{\cos\theta} - n\frac{\cos\theta}{\sin\theta}\right)\varphi_0 = 0.$$

$$\Delta_b \mathcal{B}_{rel} = M_b \mathcal{B}_{rel}$$

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$\mathcal{N} = 2$ Matter multiplet

$$\Delta_b = -f^2\mathcal{D}^2 - (2\Delta - 1)ff'\mathcal{D}_\theta - \ldots$$

$$\Delta_f = -if\gamma^\mu\mathcal{D}_\mu - if'(\Delta - \frac{1}{2})\gamma_3 - \ldots$$

$$\frac{\det\Delta_f}{\det\Delta_b} = \frac{\prod_f M_f}{\prod_b M_b}$$

$$\Delta_b\Phi_{rel} = M_b\cancel{(M_b - 2i\sigma_0)}\Phi_{rel}$$

$$M\Psi_1 = \Psi_2 : \Delta_f = \cancel{M_b}, \cancel{M_b - 2i\sigma_0}$$

$$0 = \bar{\epsilon}\Psi_{rel}$$

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SUSY on round sphere

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$$\frac{\det\Delta_f}{\det\Delta_b} = \frac{\prod_f M_f}{\prod_b M_b}$$

$$M_f = i\sigma_0 - \frac{m}{s} - n - \frac{\Delta}{2}\Big(\frac{1}{s}+1\Big)$$

$$(f\mathcal{D}_\theta + f'\Delta)\Phi = -i\frac{\sin\theta}{\cos\theta}\mathcal{D}_\phi\Phi - \frac{i}{s}\frac{\cos\theta}{\sin\theta}\mathcal{D}_\chi\Phi,$$

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$$\Delta_b\Phi_{rel} = M_b\cancel{(M_b - 2i\sigma_0)}\Phi_{rel}$$

$$M\Psi_1 = \Psi_2 : \Delta_f = \cancel{M_b}, \cancel{M_b - 2i\sigma_0}$$

$$M_b = i\sigma_0 + \frac{m}{s} + n - \frac{\Delta - 2}{2}\Big(\frac{1}{s}+1\Big)$$

$$\Big(f\mathcal{D}_\theta + f'(\Delta+1)\Big)F = i\frac{\sin\theta}{\cos\theta}\mathcal{D}_\phi F + \frac{i}{s}\frac{\cos\theta}{\sin\theta}\mathcal{D}_\chi F,$$

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$$\frac{\det \Delta_f}{\det \Delta_b} = \frac{\prod_f M_f}{\prod_b M_b}$$

$$\boxed{\begin{aligned} M_f &= i\sigma_0 - \frac{m}{s} - n - \frac{\Delta}{2}\left(\frac{1}{s} + 1\right) \\ (f\mathcal{D}_\theta + f'\Delta)\Phi &= -i\frac{\sin\theta}{\cos\theta}\mathcal{D}_\phi\Phi - \frac{i}{s}\frac{\cos\theta}{\sin\theta}\mathcal{D}_\chi\Phi, \end{aligned}}$$

$$\Delta_b \Phi_{rel} = M_b \cancel{(M_b - 2i\sigma_0)}\Phi_{rel}$$

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these terms does not affect

$$M\Psi_1 = \Psi_2 : \Delta_f = \cancel{M_b}, \cancel{M_b - 2i\sigma_0}$$

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SUSY on round sphere

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$$\Phi \sim \begin{cases} (\sin\theta)^{-n} & \theta\sim 0 \\ (\cos\theta)^{-m} & \theta \sim \frac{\pi}{2} \end{cases}$$

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SUSY on round sphere

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