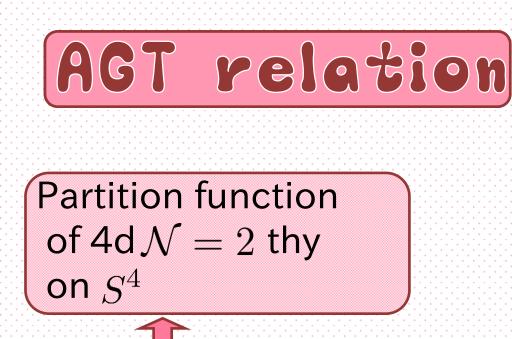
AGT relation in the light asymptotic Vikawa institute for Vikawa institute for Herorental Physics 11/20@PASCOS 2013 Naofumi Hama (YITP) Reference ·JHEP 1310, 152 (2013) arXiv:1307.8174[hep-th] with K. Hosomichi (YITP) ·JHEP 1209, 033 (2012) arXiv:1206.6359[hep-th] with K. Hosomichi



### equivalent Correlators in

2d CFT (LFT, Toda) whose coupling b=1 explained as :2 different limits in compactification of 6d (2,0) thy

[09 Alday-Gaiotto-Tachikawa]



### Localization method

### : deform the action by susy exact term

one way to compute path integral exactly
using nilpotent sym. of the system
up to bosonic sym.

### (gauge sym., Poincare, ·····)

#### here: susy

### only calculate susy closed observables

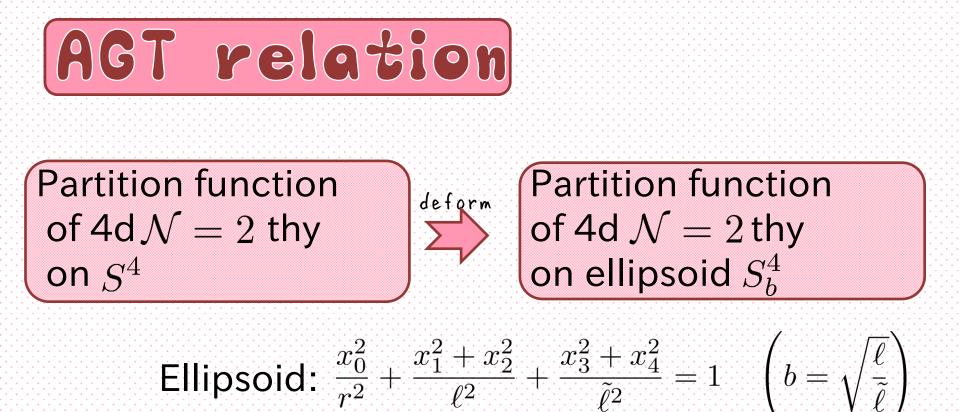
ex.) partition function  $Z = \int da_E e^{-S_{cl}(a_E)} Z_{1-loop}(a_E) |Z_{inst}|^2$ 

$$a_E\colon$$
 parameter of saddle Pt

$$e^{-S_{
m cl}}$$
: classical part

$$Z_{1-\mathrm{loop}}$$
 : 1-loop det.

 $Z_{
m inst}$  : Nekrasov partition function

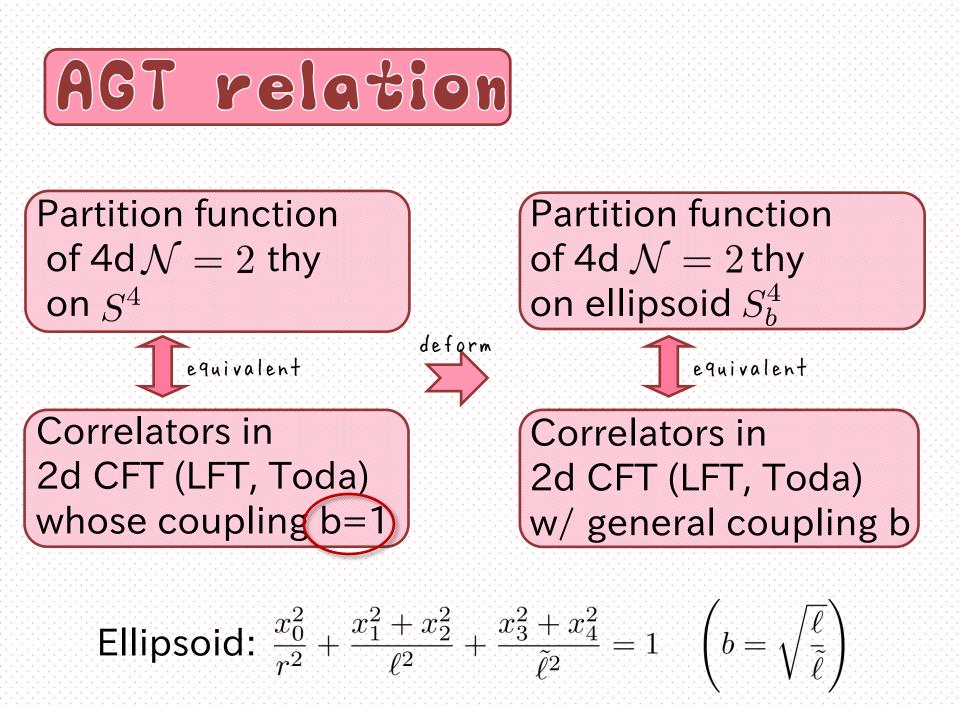


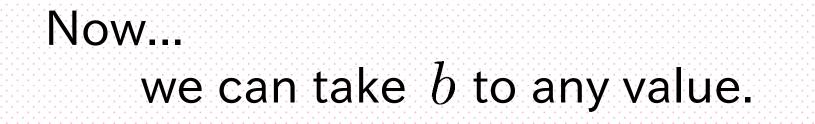
On the poles of these mfds, the system become  $\Omega$  -deformed  $\mathbb{R}^4$ 

With  $\Omega$ -deformation parameters  $\epsilon_1=\epsilon_2$ 

With  $\Omega$ -deformation parameters

$$\frac{\epsilon_1}{\epsilon_2} = b^2$$



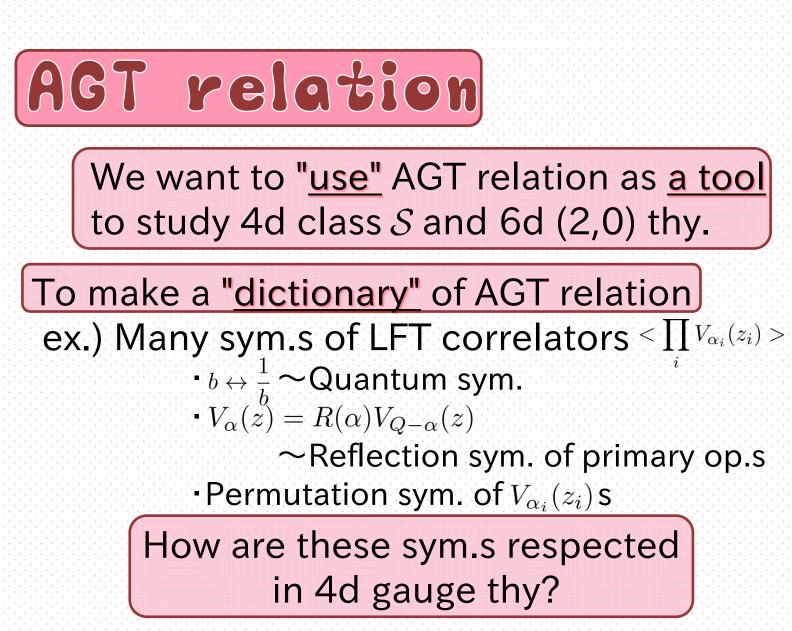


## Can't help taking limit!

We take  $b \rightarrow 0$  limit.

### LFT side: Semiclassical limit [Zamolodchikov '84]

### Gauge thy side: Nekrasov-Shatashvili limit



sym. of thy? only BPS observables? structure of mfd?

## AGT relation

Here we use **easiest situation** in AGT relation to take the **limit** where the dictionary can be filled by explicit calculations.

4d SU(2)  $N_f = 4$  SQCD w/ 4 (anti)fund.s on  $S_b^4$ 

LFT on sphere w/ 4 punctures

and take light asymptotic limit

## Light asymptotic limit

[84 Zamolodchikov]

# A particular example of classical limit $b \rightarrow 0$ in LFT path integral $\searrow$ finite dim. integral

break the quantum sym.  $b\leftrightarrow rac{1}{b}$ 

On gauge thy: In  $b \rightarrow 0$ , instanton counting become easy.

## Liouville Field Theory

Action:  $S_{\rm L} = \frac{1}{4\pi} \int \sqrt{g} d^2 \sigma \left( g^{ab} \partial_a \phi \partial_b \phi + QR \phi + 4\pi \mu e^{2b\phi} \right)$ 

where b: Liouville coupling,  $Q = b + \frac{1}{b}$ R: scalar curvature ~ here: sphere  $\rightarrow 0$ 

•2d CFT which realizes Virasoro algebra whose central charge is  $c = 1 + 6Q^2$ •Prim op.s are  $V_{\alpha} = e^{2\alpha\phi}$ , whose conformal dim. are  $\alpha(Q - \alpha)$ •Their correlators are  $\left\langle \prod V_{\alpha_i}(\sigma_i) \right\rangle = \int D\phi \exp\left(-S_{\rm L} + \sum 2\alpha_i\phi(\sigma_i)\right)$ 

## LFT in LA limit

### Semiclassical limit $b \rightarrow 0$ w/ rescaling $b\phi \equiv \varphi$ , $\mu$ ,

 $S_{\rm L} \to \frac{1}{\pi b_{
m N}^2} \int {\rm d}^2 z \Big( \partial \varphi \bar{\partial} \varphi + \tilde{\mu} \pi e^{2\varphi} \Big)$ 

### Only OVERALL FACTOR >> semiclassical limit

#### Furthermore,

LA limit: Inserted momenta  $\alpha_i = b\eta_i \rightarrow 0$  ( $\eta_i$ : fixed)  $\Rightarrow$  can be ignored  $\Rightarrow$  path integral in correlators localizes onto the solutions of EoM  $\partial \bar{\partial} \varphi = \pi \mu b^2 e^{2\varphi}$ 

## LFT in LA limit

### Solutions of EoM are labeled by $g \in H_3^+$



Path integral become an integration over  $H_3^+$ ;

$$\left\langle \prod_{i} V_{\alpha_{i}}(z_{i}) \right\rangle \Big|_{\mathrm{LA}} = \int_{H_{3}^{+}} \mathrm{d}g \prod_{i} \Phi_{z_{i}}^{\eta_{i}}(g) \equiv \left\langle \prod_{i} \Phi_{z_{i}}^{\eta_{i}} \right\rangle_{H_{3}^{+}}$$

where  $\Phi_z^{\eta}(g) \equiv \left[ (z \ 1) g \left( \begin{array}{c} \bar{z} \\ 1 \end{array} \right) \right]^{-2\eta}$  are wavefunctions:

 $\delta(g - g') = -\frac{1}{2\pi^3} \int d\eta \int d^2 z \ (2\eta - 1)^2 \Phi_z^{\eta}(g) \Phi_z^{1 - \eta}(g')$ 

### LFT in LA limit

### 4-pt. function is inserted complete set,

 $c = \frac{12}{2}$ 

 $\left\langle \Phi^{\eta_1}_{z_1} \Phi^{\eta_2}_{z_2} \Phi^{\eta_3}_{z_3} \Phi^{\eta_4}_{z_4} 
ight
angle_{H^+_2}$ 

on

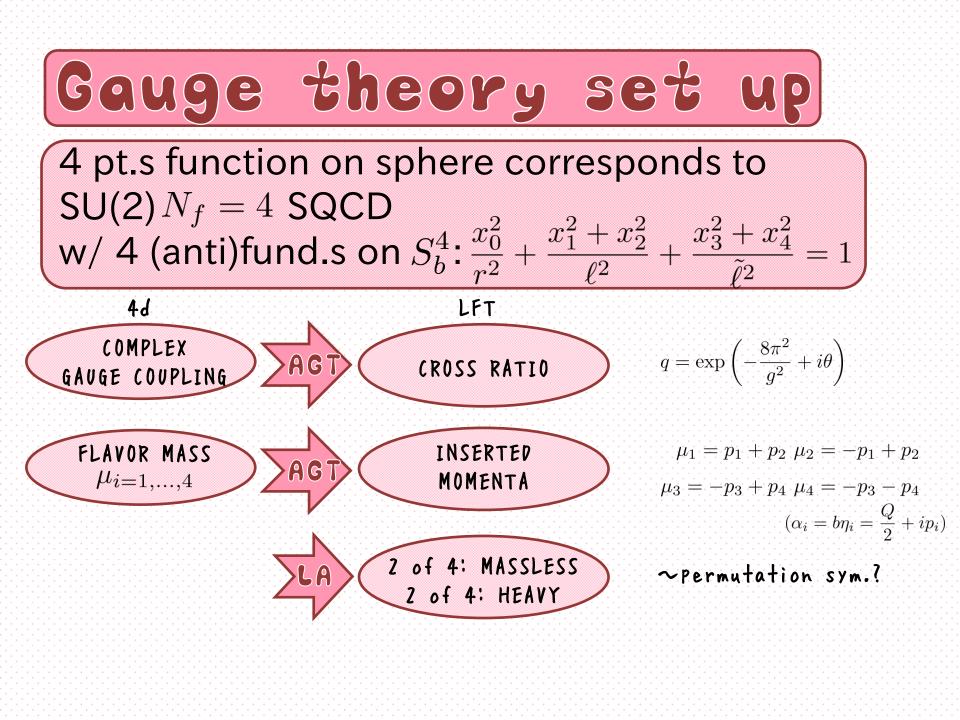
$$= -\int \frac{d\eta_0 d^2 z_0}{2\pi^3} (2\eta_0 - 1)^2 \left\langle \Phi_{z_1}^{\eta_1} \Phi_{z_2}^{\eta_2} \Phi_{z_0}^{\eta_0} \right\rangle_{H_3^+} \left\langle \Phi_{z_0}^{1-\eta_0} \Phi_{z_3}^{\eta_3} \Phi_{z_4}^{\eta_4} \right\rangle_{H_3^+}$$

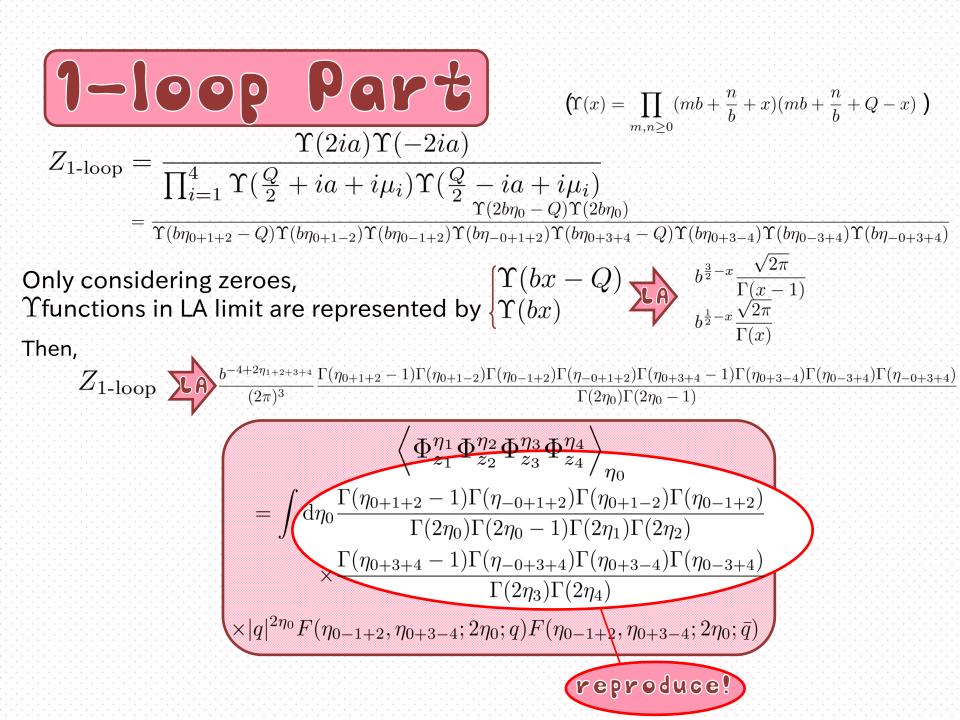
$$= \frac{1}{4} |z_{12}|^{-2\eta_{1+2}} |z_{13}|^{2\eta_{-1-2-3+4}} |z_{14}|^{4\eta_2+2\eta_{3-4}} |z_{23}|^{2\eta_{1+2}} |z_{24}|^{-4\eta_2} |z_{34}|^{-2\eta_{3+4}}$$

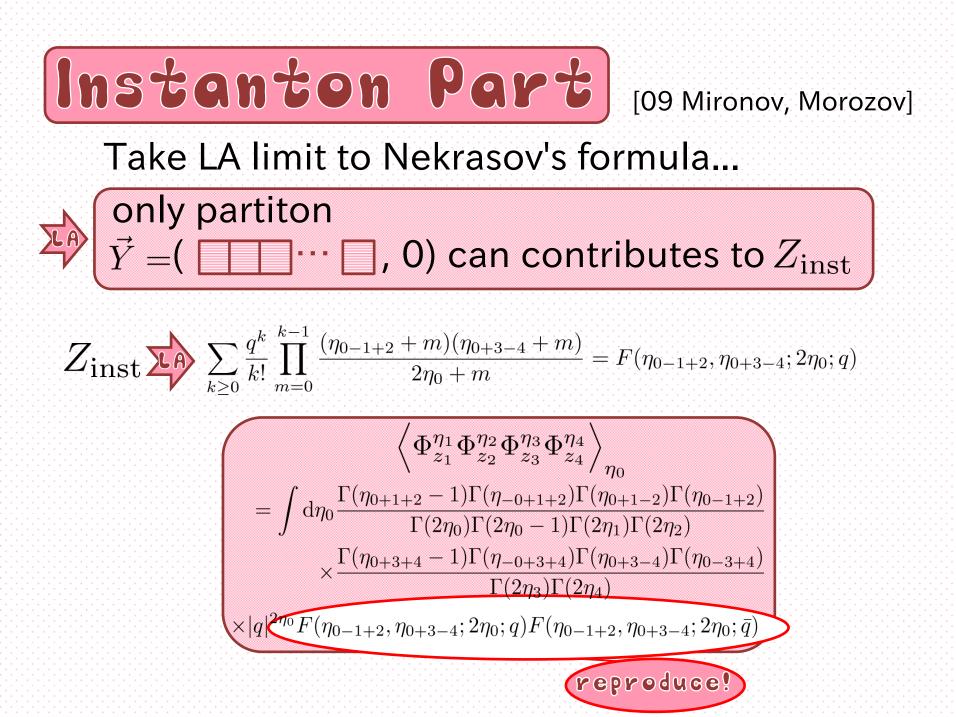
$$\times \int d\eta_0 \frac{\Gamma(\eta_{0+1+2} - 1)\Gamma(\eta_{-0+1+2})\Gamma(\eta_{0+1-2})\Gamma(\eta_{0-1+2})}{\Gamma(2\eta_0)\Gamma(2\eta_0 - 1)\Gamma(2\eta_1)\Gamma(2\eta_2)}$$

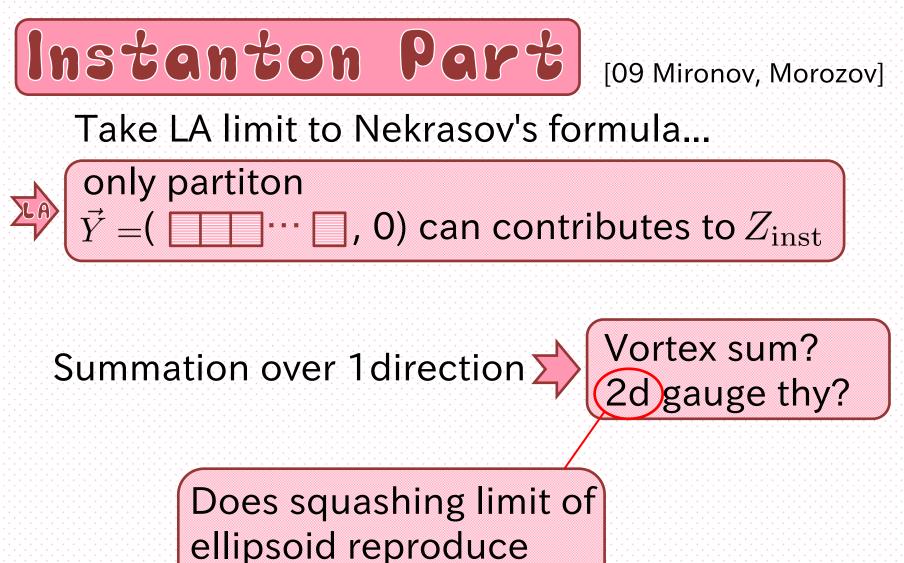
$$\times \frac{\Gamma(\eta_{0+3+4} - 1)\Gamma(\eta_{-0+3+4})\Gamma(\eta_{0+3-4})\Gamma(\eta_{0-3+4})}{\Gamma(2\eta_3)\Gamma(2\eta_4)}$$

$$\times |q|^{2\eta_0} F(\eta_{0-1+2}, \eta_{0+3-4}; 2\eta_0; q) F(\eta_{0-1+2}, \eta_{0+3-4}; 2\eta_0; \bar{q})$$
IN AGT RELATION, where  $q \equiv \frac{z_{43}z_{21}}{z_{42}z_{31}}$ : cross ratio by  $\eta_0$  related parts appear he partition function of 4d gauge thy









ellipsoid reproduce this 2d mfd?